Dual Regulation or Duelling Regulators?

The Welfare Impacts of Overlapping Regulatory Regimes:

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Abstract

Previous literature has examined the impact of a single regulatory constraint on the final product. However, an industry such as electricity requires two network inputs from naturally monopolistic industries i.e. gas transmission and electricity transmission. This paper models the impact of such dual regulation schemes finding: Firstly, the result of previous literature that tightening regulation increases prices to higher cost consumers may be reversed. Secondly, dual input regulation creates distortions if regulators do not explicitly co-operate. Where regulators place some weight on their respective sector's consumer surplus, these differing agendas lead to competition in regulatory strictness resulting in a significant sub optimal welfare outcome relative to a joint regulatory body.

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During the last decade the UK ex-nationalised utilities have increasingly been opened to competition. However, the natural monopolistic character of the network has limited the ability to introduce full competition to the industries. In general (with telecoms as the notable exception) the solution has been to separate the networks into their competitive and non-competitive elements, with the final consumer product generally requiring some elements of both. Some industries such as electricity, require two regulated inputs (gas and electricity transmission) from naturally monopolistic networks. It is the impact of electricity dual regulation in particular and more generally the case of overlapping regulatory schemes (i.e. international telecommunications with national regulatory bodies), that this paper analyses.

This paper builds on the small but significant literature on whether revenue regulation creates incentives for firms to manipulate prices across markets as well as the wider literature on distortions over time. The literature raises the important question of whether welfare is actually enhanced under price caps even within a static model. Bradley and Price (1988) first analysed the case of a regulated monopolist under an average revenue constraint, a constraint typical across many U.K. utilities. Under such a constraint, current demand, rather than previous consumption or revenue levels, weight prices. The firm is induced to manipulate these weights by restricting supply to the higher cost markets (through raising price) and expanding supply in the lower cost markets (through lowering price). They showed that for high cost markets, the constraint may increase prices above the unconstrained level. Crew and Kleindorfer (1996) using a similar model analyse a total revenue constraint. They show that a total revenue cap has a much larger potential than an average revenue constraint to distort output incentives, producing prices above the monopoly level in some markets. Armstrong and Vickers (1991) compare the welfare results of price discrimination with a uniform pricing scheme, both facing an average revenue constraint. They find the welfare result depends upon the tightness of the price constraint, with some degree of price discrimination increasing welfare as the constraint is relaxed.

Sappington and Sibley (1992) show that for an average revenue lagged tariff, the strategic incentive to manipulate prices through a non-linear tariff may result in a loss of welfare even though the linear tariff may enhance welfare. Armstrong, Cowan and Vickers (1995) strengthen this result by showing that the optimal non-linear tariff is distorted and other types of regulatory constraint may be preferable to a tight average revenue constraint. Law (1995) returns to the Bradley and Price framework t show that tightening a price cap can lower aggregate consumer surplus, confirming their result that tighter regulation induces the firm to reduce the number of high cost consumers by raising the price in this market and lowering the price in the low cost market. Cowan (1997a) developed this showing that not only consumer welfare but total welfare may fall as a result of an average revenue cap that is “too tight”, a result that this paper shows may be reversed for certain consumers under dual regulation. Cowan (1997b) compares the dynamic case for three different types of regulatory constraints: average revenue, Laspeyres base weighted tariff basket constraint and the average revenue lagged regulation first studied by Sappington and Sibley (1992). He confirms that the average revenue lagged constraint and average revenue may not only be inefficient but are likely to reduce overall levels of welfare, while a Laspeyres index based constraint can induce efficient prices even when the firm is not myopic.

The paper is structured as follows; the first section develops a simple model of a vertically structured market in which two firms, a downstream (electricity) and upstream (gas), provide inputs to the final product. Each firm is regulated with the regulator’s remit extending as far as its respective industry. Section two examines comparative statics for optimal prices, finding that tightening the downstream (electricity) regulatory constraint increases electricity prices to high cost consumers confirming previous literature. However, tightening the upstream industry constraint (gas transmission) allows a relaxation in the downstream regulatory constraint. For some consumers this effect dominates and reverses the previous process, causing lower prices contrary to the findings in previous literature. The final section analyses the welfare impact of tighter constraints and compares them to a market where the two regulators are integrated.[2] It shows that dual regulation drives a more interesting result than simply an example of the theory of the second best. Unlike the standard theory that assumes exogenous distortions, the two regulators may compete strategically and consciously introduce sub-optimal distortions in the other market to maximise their own consumer's welfare. As such, this problem is a special case of the theory of the second best where agents choose the level of distortion. Welfare in the two industries may only be maximised under joint regulation providing economic justification beyond simple cost savings into regulatory mergers.

Section 1: A Model of Dual Vertical Regulation in Electricity

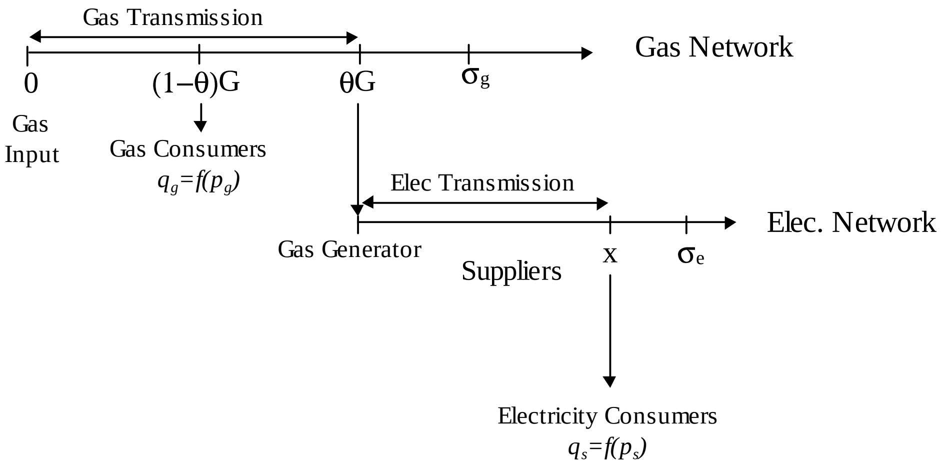
This paper analyses generation of electricity using gas rather than other fuels such as oil or coal for two main reasons: First and primarily, the latter two inputs do not involve transmission regulatory constraints, secondly, new gas powered generation plants are rising in incidence relative to other technologies.[3] Given gas generation, electricity delivered to consumers requires inputs from both the gas network (for generation) and the electricity network which we consider in turn.

The gas market is modelled as two groups of users with independent demands; those using gas for electricity generation and those using gas for all other purposes (gas consumers). The segmentation of the market into direct consumers and generators precisely defines the gas regulator's remit; welfare of all firms or consumers connected to the gas transmission network. The gas transmission agent supplies gas to both direct consumers and generators in the proportion 1-θ and θ respectively. The network is modelled by a continuum of supply points (generators and consumers) distributed uniformly on a straight line of distance $G \in [0, \infty)$.[4] The gas supply origin (the beachhead) is situated at $G = 0$. The gas transmission agent charges an identical price structure for both direct consumers and generators that increases with distance from the supply origin. The total distance served by the gas transmission agent is denoted σ_g. At the point $G = \sigma_g$ the transmission price is equal to the maximum price any consumer is willing to pay, resulting in a demand for gas equal to 0.

The second network transmits the generated electricity from the generator (G) to electricity suppliers located some geographic distance (x) from the generator. These suppliers are again distributed continuously and uniformly across the electricity network represented by a line of distance $x \in [G, \infty)$. As gas transmission, the electricity transmission agent's market distance is defined by the point σ_e. At this point the price of transmission is sufficiently high enough to reduce consumer demand for electricity to 0 thus $q_s(\sigma_e) = 0$. The framework of both these networks and their interactions is illustrated in figure 1.1. In this simple model, locations are taken to focus on the choices of prices and distance served. The vertical model is solved using backward induction. First the downstream consumer market demand is solved in terms of the endogenous upstream parameters and then each upstream element electricity, generation and gas respectively are solved taking all preceding prices, distances and quantities demanded as given.

Figure 1.1
Spatial Set-up of Model.

[2] This scenario is particularly relevant in the context of the UK when Ofgas and Offer were combined to create a single regulatory body Ofgem in 1999.
[3] The oil or coal problem may be thought of as a more specific case where the input is no longer regulated and competition exists within the transmission (i.e. freight transport).
[4] Whilst in reality supply points are discretely distributed across distance, a continuous distribution simplifies the problem, retaining the key element of an increasing transmission costs with distance.



Looking first at the consumer electricity markets. The supplier points are uniformly distributed and serve identical aggregate consumer demands $q_s(p_s)$ over distance x. This assumes that aggregate consumer demand is the same regardless of its location. For simplicity demand is modelled by a simple linear function for electricity consumers within each supplier market in the form of $q_s = a - p_s$. For each supplier point it is assumed there are at least two or more firms competing for aggregate consumer demand. Under the beliefs that; supplier fixed costs are small, electricity is homogeneous, firms compete in price and there are no capacity constraints within a given supply market, it is consistent with theory to use Bertrand competition.[5] Supplier's price per unit of electricity to consumers is $p_s = p_G + p_e\,(x\text{-}G)$. Where; p_G is the cost of generated electricity per unit of electricity, p_e is the cost of transporting one unit of electricity one unit of distance, x is the location of the supplier (assumed to be to the right of the generator) and G is the distance from the generator to the source of gas ($G\text{-}0$).

The generator market is characterised by a number of generation nodes each containing one generator producing electricity at cost for a range of suppliers. This is comparable with reality in which the electricity national grid is arranged into a number of generation nodes each bidding generation capacity into a pool. Currently England and Wales use a pool system to price generator's output, where generators compete by bidding supply prices for different times of the day. Like the supplier element the generation element of the model is simplified assuming small costs of starting up or shutting down generation capacity, no capacity constraints, identical cost structures and no collusion.[6] The additional but non-essential simplifying assumptions of constant one for one production function gas to electricity and a gas unit cost of 0 are made. Under such assumptions generators bid the electricity supply price down to a marginal cost equal to the cost of gas transmission to the generator, $p_G = Gp_g$.

As in reality the electricity transmission agent carrying the electricity from generator to suppliers is a monopoly. It charges a price of p_e per unit per distance of electricity transmitted and incurs a cost of T_e per unit per distance. Total profit for the electricity transmission agent is thus equal to total revenue $p_e\, q_s\,(x\text{-}G)$ minus costs $T_e\, q_s\,(x\text{-}G)$ in each generation node between G and σ_e.

$$\pi_t = \int_G^{\sigma_e} \{(p_e - T_e)q_s(x-G)\}dx$$

[5] This simplification can be relaxed still leaving the paper's main findings unchanged. Assuming Cournot or differentiated price competition merely adds another layer of marginalisation to the problem, reducing total welfare but leaving the direction of the main findings unchanged. The model concentrates on the impact of the two regulatory constraints rather than supply competition and for this reason Bertrand competition is assumed.

[6] Whilst these assumptions are perhaps unrealistic, the additional complication of different forms of competition does not change the main results of this paper (see previous footnote).

As in the UK firms monopoly electricity transmission is regulated via an average revenue constraint. Thus total revenue over total output (average revenue), must be less than or equal to some exogenously determined electricity average revenue constraint AR_e.

$$AR_e \geq \int_G^{\sigma_e} [p_e q_s (x - G)]\, dx \Big/ \int_G^{\sigma_e} [q_s]\, dx$$

Solving for the optimal p_e and σ_e in terms of the unsolved gas parameters and exogenous parameters is a relatively straightforward problem and relegated to appendix 1. Using these solutions the upstream gas variables can be solved for in terms of only the exogenous parameters. For gas consumer demand we assume a similar linear form to consumer electricity demand incorporating the fact there is no gas supply cost. Thus the quantity demanded is simply the intercept minus the delivered price of gas; $q_g = a - Gp_g$. Total demand for gas transmission is thus the weighted summation of gas consumer demand and total generator demand Q_s (as determined in appendix 1) over all gas nodes. Gas transmission agents total profit function over all gas supply points is thus;

$$\pi_g = \theta \int_0^{\sigma_g} Q_s G(p_g - T_g) dG + (1-\theta) \int_0^{\sigma_g} q_g G(p_g - T_g) dG$$

Like the electricity transmission agent the gas transmission firm is subject to an average revenue constraint made up of the two revenue sources;

$$AR_g \geq (\theta \int_0^{\sigma_g} (p_g\ Q_s G)\, dG + (1-\theta) \int_0^{\sigma_g} (p_g\ q_g G)\, dG) / (\theta \int_0^{\sigma_g} (Q_s)\, dG + (1-\theta) \int_0^{\sigma_g} (q_g)\, dG)$$

The solution methodology is identical to that of the electricity transmission agent; first solve for optimal price in terms of distance, under the knowledge that $\partial L/\partial p_g$ must be equal to 0 for all generators supplied. Use this to substitute into the gas regulatory constraint and solve for the lagrangian λ_2. Once again the workings are relegated to appendix 2 with the final solutions shown and analysed in the next section.

Section 2: Analysis of Regulatory Constraints on Prices

This section examines comparative statics for the optimal prices solved previously. As solved for in appendix 2, optimal distance served by the gas transmission agent is $\sigma_g = 3AR_g/2T_g$ and price of gas in terms of distance is;

$$p_g = \frac{T_{gas}(a - AR_g)}{AR_g} - \frac{9\theta AR_e^2 T_g}{AR_g(9\theta AR_e + 8T_e(1-\theta))} + \frac{1}{G}\left(\frac{3AR_g - a}{2} + \frac{9\theta AR_e^2}{(9\theta AR_e + 8T_e(1-\theta))}\right)$$

Note as long as the gas constraint is not 'too tight' optimal price is non-linear and declining in distance from the generator (G).[7] This contrasts with previously mentioned literature by Bradley and Price (1988), Law (1995), and Cowan (1997a), all of which derive a linear price of the finished good in the form of $p = \alpha + \beta G$. The contrast derives from the fact that p_g is the transmission price *per unit distance* whilst the fore mentioned authors examine a final delivered price to consumers. At small distances from the beach head, it is optimal for the transmission agent to charge a relatively high

[7] Too tight is defined as $AR_g < a/3$ at which point the differential $\partial p_g/\partial AR_e$ becomes positive. This point is where the regulatory constraint is so restrictive that it forces the transmission agent to adopt negative prices for some markets in order to reduce the average revenue to below the constrained level.

price per unit distance as total distance is very low and hence the total cost to consumers is also small. At distances further from *G* it is optimal to reduce the price per unit distance as the total cost for consumers will be much higher. (see figure 2.1a) This ability to price discriminate means the firm is able to sell, albeit at a much lower margin, to consumer markets at larger distances than under a uniform price. Note, the firm never sells below $T_e(a\text{-}AR_e\text{-}Gp_g)/AR_g$ but becomes tangentially close to this as distance tends to σ_g.[8]

Looking at the generator weightings shows that as the proportion of gas transmission dedicated to generators (θ) increases, the price of gas decreases at all distances. This is intuitive as gas transmission is only a part of the costs of consumers (the other part being electricity transmission). Hence a greater proportion of the gas transmission agents demand is derived from the lower reservation price downstream industry. One way of thinking of this, is as if the residual demand for gas transmission becomes smaller as the number of generators increase.

Proposition 1:

For the upstream industry tightening either the upstream or downstream regulatory constraint results in higher prices to 'high' cost generators/consumers, lower prices to 'low' cost generators/consumers and a fall in the market distance served.

Proposition one is proved simply through the use of comparative statics. Setting $\partial p_g/\partial AR_e = 0$ and $\partial p_g/\partial AR_g = 0$ and solving for distance G provides the point at which the new and old gas prices intercept, and hence the point at which the impact of a change in the constraint reverses. These points for gas and electricity respectively are;

$$G = \sigma_g AR_g \frac{9\theta AR_e + 8T_e a(1-\theta)}{9\theta AR_e(a - AR_e) + 8T_e a(1-\theta)} \text{ and } G = \frac{AR_g}{2T_g} = \frac{1}{3}\sigma_g$$

The two points are denoted $\tau_{g,g}$ and $\tau_{g,e}$ respectively for use in later analysis.[9] Whilst these two points are different, for the purpose of referral and proposition one, generators/consumers at distances greater (smaller) than $\tau_{g,g}$ and $\tau_{g,e}$ are termed as 'high' ('low') cost generators/consumers. For 'high' cost generators/consumers both $\partial p_g/\partial AR_g < 0$ and $\partial p_g/\partial AR_e < 0$, hence tightening the respective constraint increases prices. The opposite is true for 'low' cost generators/consumers. Finally it is straightforward to show that both differentials $\partial\sigma_g/\partial AR_g$ and $\partial\sigma_g/\partial AR_g$ are negative and tightening either constraint reduces the market supplied, proving proposition one.

The intuition behind this result is similar to previous literature; as the average revenue constraint on gas or electricity tightens, it restricts (either directly via the gas constraint or indirectly via the electricity transmission constraint) the total revenue the gas transmission agent can make. Given this restriction on average revenue, the gas transmission agent increases the amount sold to low cost/low revenue supply nodes close to the gas origin by lowering their price. Those supply points further away face an increasing price for two reasons, firstly to restrict the quantity sold to high revenue and low profit generation nodes, and secondly to maintain the average profitability of these further generation nodes. It is worth briefly examining the generator price as, being a delivered price per unit (rather than a unit per distance price) linearises it to a similar form as existing literature.

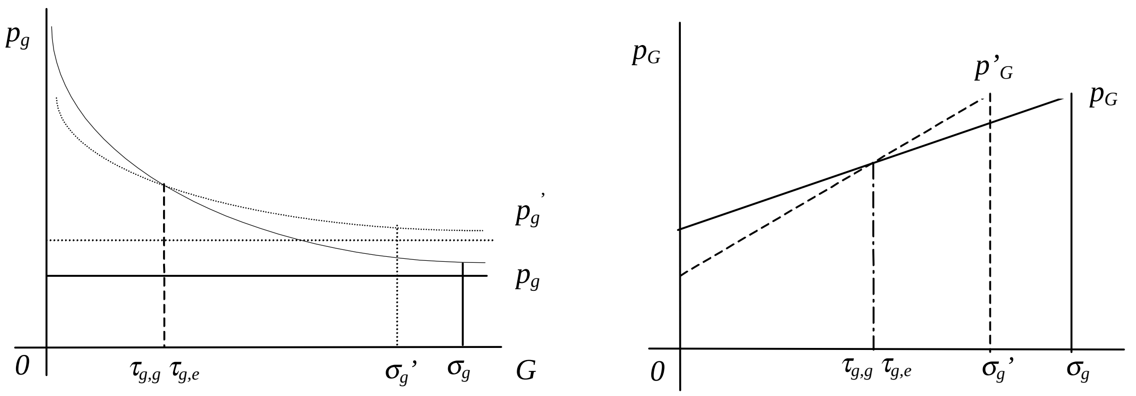
$$p_G = Gp_g = \frac{3AR_g - a}{2} + \frac{9\theta AR_e^2}{(9\theta AR_e + 8T_e(1-\theta))} + \left(\frac{T_g(a - AR_g)}{AR_g} - \frac{9\theta AR_e^2 T_g}{AR_g(9\theta AR_e + 8T_e(1-\theta))}\right)G$$

Note that tightening the constraints on this linearised price has of course the same impact as that of

[8] In the unconstrained case (which is left for the reader to verify), this 'price floor' is simply the cost T_e.
[9] It is useful to note for later analysis that $\tau_{g,g} < \sigma_g \; \forall \; \theta < 1$ using the fact that $(a - AR_e - AR_g) > 0$.

the non linear price and tightening the constraint increases price for high cost generation points and reduces price low cost. The impacts of tightening the constraint on the non-linear transmission and linear generation price are illustrated in figure 2.1a,b.

Figure 2.1a,b
Change in Non-Linear Gas and Linear Generation Price with Tightening of AR_g or AR_e Constraint



Looking at a tightening of the downstream (electricity) constraint on the electricity transmission industry; solving for the electricity transmission price yields intercept and slope terms both dependent and independent on the number of generators θ;

$$p_e = \frac{AR_g T_e(3a - 3AR_g - 2AR_e) - 2GT_g T_e(a - AR_g)}{4AR_e AR_g} - \frac{9\theta AR_e T_e(AR_g - 2GT_g)}{2AR_g(9\theta AR_e + 8T_e(1-\theta))}$$
$$+\left(\frac{2GT_g(a - AR_g) - 3AR_g(a - AR_e - 2AR_g)}{4AR_g} + \frac{9\theta AR_e^2(AR_g - 2GT_g)}{4AR_g(9\theta AR_e + 8T_e(1-\theta))}\right)\left(\frac{1}{x-G}\right)$$

The optimal market size (σ_e) for the electricity transmission agent to supply is $(2GT_e + 3AR_e)/2T_e$. Like Bradley and Price (1988), the optimal size of the market is not a function of the level of demand (a) nor (specific to this paper), the price of gas transmission p_g. Intuitively one would expect that as the gas price increases, so will generation price and hence supplier costs. For this reason it might be expected that the size of the market supplied would be a function of p_g as well as the exogenous cost of transmission. The explanation to this counter intuition lies in the price of gas as a function of the binding AR_e (see equation x.x). As the price of gas increases over time, so must the level of AR_e, else at some future point the constraint no longer binds, this causes σ_e to fall in line with intuition.

Like the transmission price for gas the electricity transmission price is non-linear in distance from generator to supplier (x-G). Again it is instructive to rearrange the electricity price into a linear form for analysis. This is most efficiently done by reinsertion back into the equation for p_s and rearranging to yield the delivered consumer price per electricity unit at each distance (x-G). Again the equation is arranged in terms of an intercept term (α_s) and slope coefficient (β_s).

$$p_s = \frac{6GT_g(a - AR_g) - AR_g(5a - 6AR_e - 9AR_g)}{4AR_g} + \frac{27\theta AR_e^2(AR_g - 2GT_g)}{4AR_g(9\theta AR_e + 8T_e(1-\theta))} +$$
$$\left(\frac{AR_g(3a - 3AR_g - 2AR_e) - 2GT_g(a - AR_g)}{2AR_e AR_g} - \frac{9\theta AR_e(AR_g - 2GT_g)}{2AR_g(9\theta AR_e + 8T_e(1-\theta))}\right)T_e(x-G)$$

Proposition 2

For the downstream industry tightening the downstream industry constraint has the same result as *proposition one, thus raising prices to 'high' cost customers and lowering prices to 'low' cost consumers and a fall in the market distance served.*

Proposition 2 is proved using comparative statics. Analysing changes in the electricity transmission constraint shows; $\partial\alpha_s/\partial AR_e > 0 \ \forall \ G < \sigma_g$, thus the intercept supply price is decreasing as the constraint (AR_e) becomes tighter. Differentiating the third term of p_s with respect to AR_e, $\partial\beta_s/\partial AR_e < 0 \ \forall \ G < \sigma_g$. In this case tightening the AR_e constraint increases the supply price slope coefficient. Like gas transmission, the counteracting effects of the change in slope coefficient and intercept point causes an anticlockwise rotation in the price line about the point where $\partial p_s/\partial AR_e = 0$ denoted $\tau_{e,e}$. As in proposition 1 those consumers to the left of this point are denoted 'low' cost consumers whilst those to the right are denoted 'high' cost consumers. This proves proposition 2.

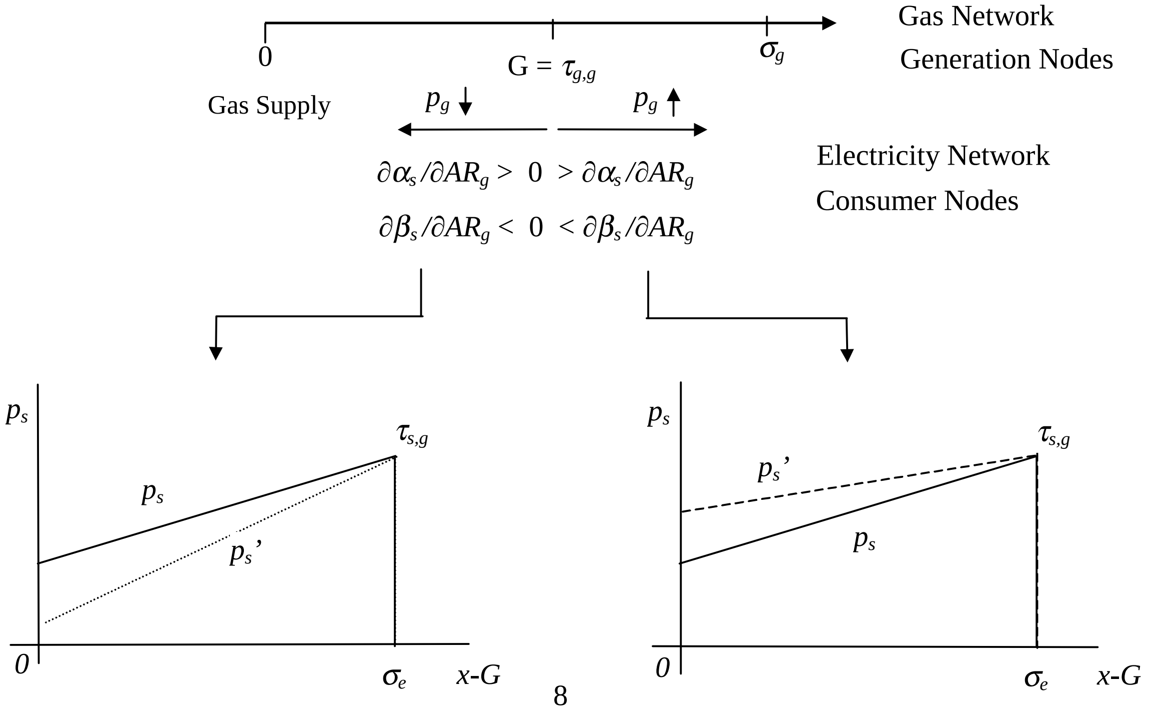
The intuition behind the impact of a tightening average revenue constraint for electricity transmission is the same as the previously discussed gas constraint. As the electricity constraint becomes tighter, the transmission agent is forced to reduce its average revenue, raising the price of those consumers furthest away which generate the lease amount of profit as a ratio of revenue, and lowering the price of the lower revenue suppliers closest.

Proposition 3

For the downstream industry tightening the upstream industry constraint results in:
(i) All consumers in 'Low' cost generation nodes (those to the left of $\tau_{g,g}$) face lower prices.
(ii) All consumers in 'High' cost generators nodes (those to the right of $\tau_{g,g}$) face higher prices.

The proof of proposition 3 is again simple and demonstrated via use of comparative statics on the consumer price. Differentiating the intersection terms (the first two terms) and the slope term of p_s with respect to AR_g shows (after some manipulation) $\partial\alpha_s/\partial AR_g > 0$ and $\partial\beta_s/\partial AR_g < 0$, $\forall \ G < \tau_{g,g}$. These counteracting impacts create a anti-clockwise movement around the point where $\partial p_s/\partial AR_g = 0$ denoted $\tau_{s,g}$. Solving for this point yields the total market distance for consumers served σ_e. This movement around σ_e reduces prices for all consumers and proves part (i). Returning to the comparative statics, $\partial\alpha_s/\partial AR_g < 0$ and $\partial\beta_s/\partial AR_g > 0$, $\forall \ G > \tau_{g,g}$. These counteracting effects create a clockwise movement around $\tau_{s,g}$ which again when solved for yields σ_e. This movement increases prices to all consumers within the generation node and proves part (ii). Figure 2.2 illustrates this.

Figure 2.2
The Effect of Generator Location on the Impact of Tightening AR_g on Consumer Prices



The reason for this relates to propositions 1 and 2. For generation nodes to the left of $\tau_{g,g}$, the gas transmission price is falling, thus generation price is falling for all distances. As supplier demand is an inverse function of generation price, consumer aggregate demand rises. The higher demand at all consumer nodes pushes average revenue past the electricity regulatory constrained level. From proposition two, the optimal response to come back within the constraint is to raise the price of distant 'costly' consumers, and reduce price to near 'low' cost suppliers. Given optimal market distance σ_e is fixed this means consumers prices can only fall, reducing average revenue to move back within the constraint (see left hand side of fig 2.2). For generation nodes to the right of $\tau_{g,g}$ gas transmission price is increasing, thus reduce aggregate consumer demand. This demand reduction shifts the electricity transmission agent well within the transmission regulatory constraint. This shift permits a move back towards the constraint by raising prices to those closest and reducing prices to those furthest away. Again as σ_e is fixed, all prices increase to takes the firm closer to the unconstrained optimum.

Concluding, this section has established that where a dual regulatory regime exists, the downstream (electricity) regulatory regime impacts consumer prices in a similar fashion to previous literature (ie Bradley and Price (1990), Law (1996) and Cowan (1997)). The impact of upstream product regulation on consumer prices depends on the location of the final consumers. In all cases prices for high 'cost' consumers are relatively constant, gaining neither the benefits nor the costs of an upstream regulatory change.

Section 3: The Optimal Welfare Levels of Dual Regulation:

Section two showed the finding of previous literature; tightening the price constraint reduces the 'near' consumer price and increases 'far' consumer prices, is not generally true. In this section the welfare effects of constraint changes are analysed to determine who benefits from a tightening constraint. Secondly it analyses the more important question of whether independent regulators maximise welfare, or whether independent regulatory regimes leads them to compete for consumer welfare resulting in lower overall welfare relative to a co-operative regime. To solve these questions this section is structured as follows; first consumer and producer gas welfare are determined and differentiated to yield the gas regulator's reaction function to changes in the electricity constraint. A similar reaction function for the gas consumer industry is constructed to determine how the optimum gas constraint changes with the electricity constraint. These two functions are treated as Cournot response functions and solved for the Nash equilibrium regulatory solution. Numerical simulations are used to examine the welfare levels under this Nash solution. This is then compared to welfare in the case where a single regulator maximises both gas and electricity constraints over total welfare within the network.[10]

Determination of Separate Regulators Nash Solution

As in reality regulators generally look to maximise welfare within their own industry. Thus the gas regulator maximises welfare only within the gas network, whilst the electricity regulator maximises within the electricity network. Both regulators are aware of the impact that changing their respective average revenue constraints have upon each other and for this reason their interaction may be thought of in terms of Nash reaction functions. The gas regulator's reaction function depends upon the measure of economic welfare the regulator chooses to optimise the regulatory constraint to. This paper uses a standard total welfare function of the form $TW_g = CS_g + \phi\pi_g$, where ϕ is the weighting given to transmission profits. Gas consumer surplus across all gas markets (CS_g) is;[11]

[10] It is important to note that combining these two constraints into a single constraint and maximising welfare would ignore the industry structure. As the two transmission companies are separate agents even under a single regulatory authority, they require separate constraints.

[11] Although generators are 'consumers' of gas, under the assumption of bertrand competition they make 0 profits and thus are not included within the gas consumer surplus. The electricity consumers which they sell onto are

$$CS_g = (1-\theta)\int_0^{\sigma_g}\frac{1}{2}(a-p_g)q_g\,dG = \frac{9AR_g(1-\theta)}{16T_g}\left[(a-AR_g)^2 + \frac{27\theta^2 AR_e^4}{[9\theta AR_e + 8T_e(1-\theta)]^2}\right] \tag{3.1}$$

When θ = 1 (no gas consumers), gas consumer welfare to 0. At the other extreme, setting θ to 0 equates market of only consumers and drops the second term with the brackets.

Proposition 4.

An 'Overly' tight gas regulatory constraint reduces gas consumer welfare.

Proof of proposition four is again through simple comparative statics. Differentiating gas consumer surplus firstly with respect to its own gas constraint shows that CS_g is increasing as AR_g becomes tighter. Solving for $\partial CS_g/\partial AR_g = 0$ yields two roots the first of which is a maximum the second a minimum. To the left of the maximum point, which is defined as 'overly' tight for the purpose of proposition 4, $\partial CS_g/\partial AR_g > 0$ and hence further decreases in the gas constraint reduces welfare.[12]

The intuition of proposition 4 can be seen through proposition 1. Tightening AR_g increases prices to the highest cost customers and reduces prices for low cost consumers. Secondly the total distance served falls, effectively removing some consumers from the market. Moving from low levels of constraint, the gains from the low cost consumers outweigh the losses, however at a certain point (past which the constraint is 'overly' tight) these losses outweigh the benefits. This result verifies that shown in Law 1995 and Cowan 1997a. Gas transmission agent's profits are derived from both the generator and consumer sectors in the ratios of θ and (1-θ) respectively.

$$\begin{aligned}\pi_g &= (1-\theta)\int_0^{\sigma_g}(Gp_g - T_g)Gq_g\,dG + \theta\int_0^{\sigma_g}(p_g - T_g)GQ_s\,dG \\ &= (1-\theta)\left(\frac{9(a+3AR_g-4T_g)(a-AR_g)AR_g^2}{64T_g^2} + \frac{81AR_e^2AR_g^2\theta(a+AR_g-4T_g)}{32T_g^2[9\theta AR_e + 8T_e(1-\theta)]}\right) + \\ &\quad \theta\left(\frac{81(a-AR_g)AR_eAR_g^2}{128T_g^2} + \frac{27AR_e^2AR_g[27\theta AR_eAR_g + 16aT_e(1-\theta)]}{128T_eT_g[9\theta AR_e + 8T_e(1-\theta)]}\right) - \frac{243AR_g\theta^2AR_e^4(1-\theta)(9AR_g-8T_g)}{64T_eT_g[9\theta AR_e + 8T_e(1-\theta)]}\end{aligned}$$

Whilst gas transmission profit looks messy, taking either of the extremes of θ = 1 or 0 removes two out of the three terms. Under the assumption that the gas constraint is binding, gas profits are always decreasing with the average revenue gas constraint, thus $\partial\pi_g/\partial AR_g > 0$. Note that as the optimal prices are functions of both the constraints, so the level of profit will also be related to the level of both constraints. Both consumer and producer welfare levels are graphically illustrated in Appendix 3 Combining these consumer and producer welfare elements into a weighted average allows the derivation of the best response function for total gas welfare. This is done through solving the differential $\partial TW_g/\partial AR_g$ is for AR_g. Unfortunately, the complexity of the best response is such that an analytical solution is not possible, however it is sufficient to not that such a solution exists and denote it as $AR_{gas}^*(AR_t)$, returning to it to solve for numerical solutions later.

Now a best response has been found for the gas regulator, it remain to solve for the best response function for the electricity regulator to find the Nash equilibrium regulatory solution. Looking first at electricity consumer surplus, within each generation node this is:

---

included in the electricity consumer surplus.

[12] This point is $2/3 + \sqrt{[9\theta AR_t + 8T_t(1-\theta) - 9\theta AR_t^2][9\theta AR_t + 8T_t(1-\theta) - 9\theta AR_t^2]}/3[9\theta AR_t + 8T_t(1-\theta)]$ simplifying to *a/3* for θ = 1.

$$CS_e(G) = \int_G^{\sigma_e} \frac{1}{2} q_s{}^2 dx$$

$$= \frac{9}{64} \frac{\theta AR_e{}^3 (a - AR_g - AR_e)^2 (2GT_g - 3AR_g)^2}{T_e AR_g{}^2 [9\theta AR_e + 8T_e(1-\theta)]^2} + 9 \frac{AR_e(1-\theta)^2 [(a - AR_g)(2GT_g - 3AR_g) + 2T_e AR_g AR_e]^2}{T_e AR_g{}^2 [9\theta AR_e + 8T_e(1-\theta)]^2}$$

Proposition 5

For a given downstream node tightening the upstream industry constraint results in:
(i) 'Low' cost generators (those to the left of $\tau_{g,g}$) see welfare strictly increasing.
(ii)'High' cost generators (those to the right of $\tau_{g,g}$) see welfare strictly falling.

Proof of proposition 5 can be done in either of two ways. The simplest is to recall proposition three in that for generation nodes to the left (right) of $\tau_{g,g}$ prices are declining (increasing) for *all* consumers. This simple proof can be verified by checking the differential $\partial CS_e/\partial AR_g$ and solving it equal to 0 for distance G. The result is G = $\tau_{g,g}$ further manipulation of the differential verifies that if G < $\tau_{g,g}$ welfare is increasing and vice versa. Total electricity consumer welfare is consumer welfare in a single generation market integrated across all generation markets served.

$$CS_e = \theta \int_0^{\sigma_g} \int_G^{\sigma_e} \frac{1}{2} (a - p_s) q_s dx dG$$

$$= \frac{243}{128} \frac{\theta AR_g AR_e (a - AR_g)(a - AR_g - 2AR_t)}{T_g T_e} + \frac{27}{128} \frac{256 AR_e^2 T_e^2 (1-\theta)^2 + 27\theta AR_e^3 [9\theta AR_e + 8T_e(1-\theta)]}{T_g T_e [9\theta AR_e + 8T_e(1-\theta)]^2}$$

Similar to proposition 4, differentiating CS_e with respect to AR_e provides a condition that depends upon the relative levels of the two constraints rather than the distance of the generation node from the gas supply. Up to the optimum point consumer welfare is increasing with the tightening of AR_e and after this point decreasing (see fig A3.3 in Appendix 3). Unfortunately analytical solutions for the electricity regulator's best response with a polynomial of this magnitude are again not possible and numerical simulations are run only on the weighted regulatory reaction function. The electricity transmission agent's profits are;

$$\pi_e = \theta \int_0^{\sigma_g} \int_G^{\sigma_e} (p_e q_s - T_e q_s)(x - G) dx dG$$

$$= \frac{27}{128} \left(\frac{\theta AR_g AR_e^2 (3a - 3AR_g - 4AR_e)}{T_g T_e} + \frac{9\theta AR_e^2}{T_g T_e [9\theta AR_e + 8T_e(1-\theta)]} \right)$$

Similar to gas, when the electricity constraint is binding electricity profits are declining with tighter gas constraints, $\partial\pi_g/\partial AR_g > 0$. (see fig A3.4 in Appendix 3). Total welfare in the electricity industry is a weighted combination of consumer surplus and profits. $TW_e = CS_e + \phi\pi_e$. Taking the differential $\partial TW_e/\partial AR_e$ and solving for AR_e yields the electricity regulator's best response function for any given level of gas constraint. Again the solution to the derivative of total welfare with respect to AR_e is a higher order polynomial in AR_e and analytical solutions are not possible. However to determine the relative efficiency of the two regulatory structures it is sufficient to note that a best response function $AR_e^*(AR_g)$ exists and can be solved via numerical solutions. Appendix 4 uses the simplification $\theta = 1$ to obtain analytical answers and give some intuition behind the results.

Nash Regulatory Solution

With both regulator's reaction functions identified, the mathematical packages Maple has been used to solve them simultaneously for numerical solutions and derive the optimal constraints for both regulators using the parameters discussed in appendix 3. These are shown in table 3.1

Table 3.1 Optimal Regulation for Separate Regulatory Regimes

Welfare Weight (ϕ)[13]	AR_g	AR_t	TW_g	TW_e	$TW_g + TW_g$
0	0.34	0.22	0.58	0.93	1.51
1/20	0.46	0.19	0.72	0.72	1.44
1/10	0.59	0.14	0.93	0.40	1.33

For $a = 1$, $T_t = 2/45$, $T_{gas} = 4/45$ $\theta = 4/10$

When ϕ increases, the electricity constraint becomes tighter. Here there are two effects working, the first is the same as the gas constraint; the constraint loosens as firm's profits are included into welfare. However the it is second indirect effect via the loosening gas constraint that is dominant. This is illustrated by plotting the iso-welfare curves for the relationship between the two constraints in figure 3.1.

Figure 3.1
Electricity Iso-Total Welfare Curves

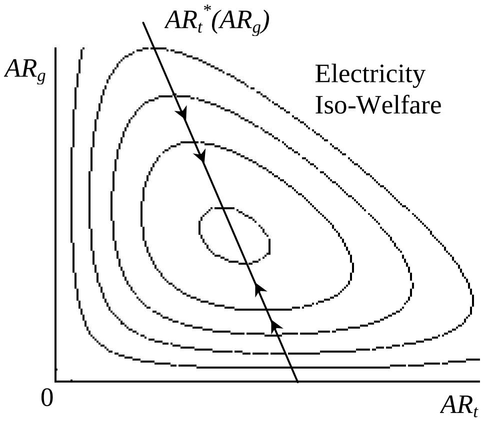
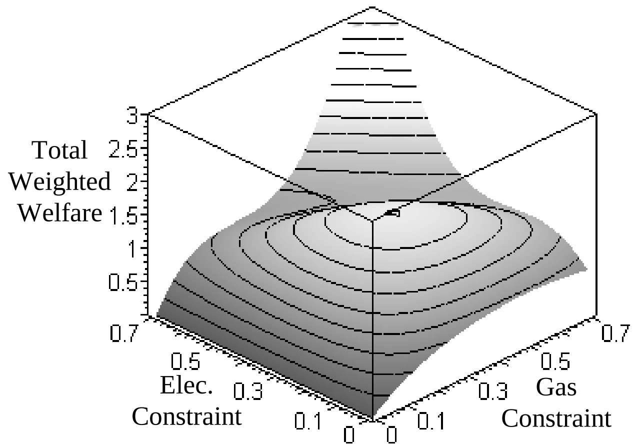


Figure 3.2
Total Weighted Welfare with Constraints



Joining the tangency points to the gas iso-welfare curves illustrates the best response line denoted by $AR_e^*(AR_g)$. Unlike normal reaction functions, the optimum movement along the line depends upon whether the movement is to the left or right of the electricity regulators optimum point. When looking at table 3.1 the gas constraint is slackening, and translates to a movement to the right of the optimum. The best response is then to tighten the electricity constraint to move to the highest possible iso-welfare curve, however both from the diagram and table we note that the new iso-welfare curve is lower than the previous. This interaction shows why relaxing the gas constraint causes a tightening of the electricity constraint and lowering of electricity welfare.

Joint Optimum

Now the Nash optimum solution is established, it can be compared with a co-operative equilibrium. The co-operative equilibrium is the equivalent of having the two regulators joining and maximising joint welfare across both industries rather than simply maximising their own consumer's welfare as previously. In this case total joint welfare is simply; $TW = CS_e + CS_g + \phi(\pi_e + \pi_g)$. Maximising total welfare is no longer a strategic game as there is only one player who simultaneously maximises both the constraints across the two transmission industries. The joint regulatory optimum is determined where $\partial TW/\partial AR_e = \partial TW/\partial AR_g = 0$. Again, as the solutions are higher order polynomials and analytical solutions are not possible, numerical solutions are used for illustration and comparison. Using the previous parameters, fig 3.2 illustrates how the two constraints interact to determine the total welfare within the system.

[13] For levels of ϕ >1/10, the gas regulatory constraint is no longer binding and thus such values are not considered.

The optimal constraints and welfare are sensitive to the two industry infrastructure parameters θ and ϕ. As the number of generators (θ) increase, the optimal joint constraints converge as the two industries become increasingly integrated. As ϕ tends towards 0, the relationship between constraints and welfare will tend away from the consumer welfare figures and towards the produce welfare figures illustrated in appendix 3. Like previously, total weighted welfare is subject to the binding condition for gas (equation x) ruling out the tail as a feasible solution. Numerical solutions using the same parameters in the previous sections are illustrated in Table 3.2. This shows that when the two constraints are determined simultaneously by a single regulatory body, the levels of constraints are much closer to each other than in the separate regimes.

Table 3.2 Optimal Regulation for Joint Regimes

Weight (ϕ)	AR_g	AR_e	TW_g	TW_e	$TW_g + TW_g$	*Relative Gain*
0	0.27	0.25	0.56	1.00	1.56	4%
1/20	0.30	0.25	0.67	1.01	1.68	16%
1/10	0.32	0.25	0.81	1.00	1.81	35%

For $a = 1$, $T_t = 2/45$, $T_{gas} = 4/45$ $\theta = 4/10$

Proposition 6

Integrating the two regulators with overlapping vertical markets, creates welfare efficiency gains relative to two independent regulators.

The last column of table 3.2 shows the welfare gain in changing the regulatory system from a separate regime to an integrated regime. Perhaps the 4% figure is the most realistic under the assumption that regulators generally concern themselves with consumer welfare and not profit. However, this 4% increase in welfare should be taken as a lower estimate for the following reason. It likely that the ratio of transmission costs of gas and electricity to reservation price chosen is an upper bound, lower ratios generate higher welfare gains in switching regulatory regimes regardless of the level of ϕ. This is because lower costs tighten the gas constraint under a joint regime but relax it to a sub-optimal level under the separate regime.

The gains in welfare by moving to a joint regulatory regime is not just a nice example of Lipsey and Lancaster's theory of the second best applying to regulation, although clearly this is the reason why a change in one industry effects the other. Where regulators regimes overlap as in the electricity industry, the overlap may drive regulators towards indirectly competing with each other to ensure maximisation of their own consumer's welfare. The regulators have conflicting aims. Without explicitly taking account of the impact they have on each other's markets, overall welfare is decreased. Only through explicitly looking at both own consumer welfare, and the impact that a change in the constraint has on the other sector, will regulators be able to obtain the higher levels of joint welfare maximisation. This reasons provide strong economic justification for the joining of the UK gas and electricity regulatory bodies OFGAS and OFFER under a single regulator OFGEM beyond simple bureaucracy cost reduction. Even when strong political or institutional reasons for separate regulators to exist, the result lends significant impetus for the two departments to work closely together, possibly bringing in other regulatory regimes when their regulatory remits overlap.

Conclusion

The model of dual input regulation for a final product presented within this paper provides some interesting results. In section two it established two main results. Firstly, changing the upstream (electricity) regulatory regime impacts upstream prices in an similar fashion to past literature with a single regulatory institution. Tightening the upstream average revenue constraint forces the transmission firm to restrict sales to high cost and hence high revenue consumer markets far from the generator by raising their price of transmission, and lowering demand. Secondly, the downstream product regulation has an impact on consumer prices different to previous literature such as Bradley and Price, Law and Cowan. Tightening the downstream constraint (gas transmission) increases the price of gas transmission for all ‘distant’ high cost generators. This causes a fall in generator demand which reduces the average revenue for the electricity transmission agent. The fall in average revenue allows the electricity transmission agent to bring its price structure closer to the unconstrained profit maximum thus increasing prices for *all* consumers. Thus *tighter upstream regulation raises all downstream consumer prices for ‘distant’ generators*. The opposite effect occurs for near generators.

Section three has two main results. At a basic level, the result that the regulatory regimes are closely interrelated is a nice illustration of Lipsey and Lancaster’s theory of the second best in regulation. It is not optimal for a regulator to merely maximise consumer welfare looking at the single transmission market it governs. It must look at both transmission markets and co-operate with the other regulator to ensure at best a joint maximum welfare, and at second best a minimisation of the other regimes distortion. Secondly, merely taking account of the other regulator’s impact on the own market optimal constraint does not ensure maximisation of welfare. This paper shows overlapping regulatory regimes provides a framework in which regulators reduce overall welfare for society in order to compete for own consumer’s welfare. Only when regulators explicitly look to maximise joint welfare will the optimum solution for the combined markets be reached. Numerical solutions showed that simply maximising own consumer welfare results in a 4% lower welfare relative to a joint regime. This figure increases as the level of overlap between regimes increases and as some degree of producer welfare is taken into account. Whilst 4% does not sound like a large welfare increase, it is a lower bound. More importantly when it is realised that the UK market for gas and electricity is measure in the tens of billions of pounds, even a figure of 4% becomes very substantial indeed.

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Appendix 1

Combining electricity profit and constraint into a lagrangian enables first order conditions for σ_e, p_e and λ_1 to be derived.

$$L_e = \int_G^{\sigma_e} \{(p_e\, q_s(x-G) - T_e q_s(x-G)\}dx + \lambda_1 (AR_e \int_G^{\sigma_e}[q_s]\,dx - \int_G^{\sigma_e}[p_e q_s(x-G)]\,dx) \qquad (1)$$

Noting $q_s = f(p_s, p_G(p_g))$, that is aggregate consumer demand is a function of supplier and generation (in turn a function of gas) prices, Supply demand can be rewritten in terms of generation and electricity price as $q_s = a - Gp_g + p_e(x\text{-}G)$. At the profit maximising level of price, a global maximum ($\partial L/\partial p_e = 0$) across all supplier markets exists only where every individual supplier market is at a local maximum. The electricity transmission agent's local maxima for each market is derived through differentiating with respect to p_e, at a given x, (thus dropping the integrals) and solving for p_e;

$$p_e = \frac{T_e}{2(1-\lambda_1)} + \frac{a-p_g}{x-G} - \frac{\lambda_1 AR_e}{2(1-\lambda_1)(x-G)} \qquad (2)$$

The range of the market the electricity transmission agent is derived via the first order condition for σ_e using the integrated lagrangian (equation 1). This yields:

$$\frac{\partial L_e}{\partial \sigma_e} = \{[(1-\lambda_1)p_e - T_e](\sigma_e - G) + \lambda_1 AR_e\}[a - p_e(\sigma_e - G) - Gp_g]$$

Solving the first order conditions for σ_e and p_e (σ_e), where at the optimal distance served $p_e(x) = p_e(\sigma_e)$, yields results for the optimal σ_e, and price at this point, $p_e(\sigma_e)$;

$$\sigma_e = \frac{(1-\lambda_1)(a-Gp_g) + \lambda_1 AR_e}{T_e} + G, \quad p_e(\sigma_e) = \frac{T_e(a-Gp_g)}{(1-\lambda_1)(a-Gp_g)+\lambda_1 AR_e} \qquad (3a,b)$$

Finally deriving the first order condition $\partial L/\partial \lambda_1$, and substituting for the optimal price as a function of distance (2), the optimal distance served (3a) in terms of λ_1, setting equal to 0 and solving for λ_1 yields two distinct roots. The first is identified as a minimum through re-insertion into the lagrangian function, whilst the second, identified as a maximum in the same manner, is;

$$\lambda_1 = 1 - \frac{AR_e}{2(a - AR_e - Gp_g)} \qquad (4)$$

Given $a > AR_e + Gp_g$ for all positive q_s, λ_1 is constrained to be less than 1 (similar to Waterson's (1992) result). At the optimum, a small change in AR_e results in a change of λ_1 to profit. Solving for AR_e when $\lambda_1 = 0$, derives the value for which the electricity average revenue constraint is binding at $AR_e = 2/3(a - Gp_g)$. Where AR_e is greater, the unconstrained monopoly price is lower than the constrained monopoly price and it is both optimal and feasible for the transmission firm to charge the unconstrained monopoly price. Substituting for the solved λ_1 and rearranging determines the optimal p_e with distance and total distance served σ_e;

$$p_e = \frac{T_e(a - AR_e - Gp_g)}{AR_e} + \frac{a - AR_e - Gp_g}{x-G}, \quad \sigma_e = \frac{2GT_e + 3AR_e}{2T_e} \qquad (5a,b)$$

As electricity transmission cost (T_e) rise, the optimal distance served (σ_e) falls and the final price $p_e(\sigma_e)$ rises, in line with intuition. Substituting p_e back into the equation for final price yields;

$$p_s = 3Gp_g + 3AR_e - a + \frac{T_e(x-G)(a - AR_e - Gp_g)}{AR_e} \qquad (6)$$

Finally total generation demand in each node, denoted Q_s, is simply the summation of demand in all generation nodes thus;

$$Q_s = \int_G^{\sigma_e} q_s\,dx = \frac{9AR_e(a - AR_e - Gp_g)}{8T_e} \qquad (7)$$

Appendix 2.

The gas Lagrangian is a weighted proportion of generators and consumers within the gas market;

$$L_g = \theta\int_0^{\sigma_g} Q_s G(p_g - T_g)dG + (1-\theta)\int_0^{\sigma_g} q_g G(p_g - T_g)dG +$$
$$\lambda_2\left[AR_g\left(\theta\int_0^{\sigma_g}(Q_s)\,dG + (1-\theta)\int_0^{\sigma_g}(q_g)\,dG\right) - \left(\theta\int_0^{\sigma_g}(p_g Q_s G)\,dG + (1-\theta)\int_0^{\sigma_g}(p_g q_g G)\,dG\right)\right]$$

Substituting for Q_s, (6) differentiating the electricity Lagrangian without the integrals with respect to p_g and solving for p_g in terms of G and λ_2 yields;

$$p_g = \frac{T_g}{2(1-\lambda_2)} + \theta\frac{(a-AR_e)(1-\lambda_2) - \lambda_2 AR_g}{2G(1-\lambda_2)} + (1-\theta)\frac{(a)(1-\lambda_2) - \lambda_2 AR_g}{2G(1-\lambda_2)} \qquad (8)$$

To solve for the optimal distance that is supplied by the gas transmission agent set $\partial L/\partial\sigma_g = 0$, yielding first order condition for σ_g in terms of λ_2 and p_g:

$$\frac{\partial\sigma_g}{\partial L} = AR_e\left(\frac{(1-\lambda_2)\sigma_g p_g + \lambda_2 AR_g - \sigma_g T_g}{8T_e}\right)\left((1-\theta)(a-\sigma_g p_g - AR_e) + \theta 8T_e(a-\sigma_g p_g)\right) \qquad (9)$$

Setting $G = \sigma_g$ and solving equations 8 and 9 simultaneously yields weighted expressions for total distance served σ_g in terms of λ_2 and the optimal price of gas at the furthest point served;

$$p_g(\sigma_g) = \theta\frac{T_g(a-AR_e)}{(1-\lambda_2)(a-AR_e) + \lambda_2 AR_g} + (1-\theta)\frac{T_g a}{(1-\lambda_2)a + \lambda_2 AR_g},\ \sigma_g = \frac{(1-\lambda_2)a + \lambda_2 AR_g}{T_e} + \frac{(1-\lambda_2)AR_e\theta}{T_e}$$

Finally to solve for λ_2 in terms of the exogenous parameters substitute the optimal σ_g and p_g in terms of distance *(8)* into the gas regulatory constraint, to derive the first order condition for λ_2. Solving for λ_2 yields a weighted result very similar (as expected) to the electricity regulation, where:

$$\lambda_2 = 1 - \left[\theta\left(\frac{AR_g}{2(a-AR_e-AR_g)}\right) + (1-\theta)\left(\frac{AR_g}{2(a-AR_g)}\right)\right] \qquad (1.19)$$

Solving the above yields the condition that must hold for the gas average revenue constraint to bind:

$$AR_{gas} < \frac{2}{3}a - \frac{9\theta AR_e^2}{9\theta AR_e + 8T_e(1-\theta)} \qquad (1.20)$$

Substituting λ_2 back into the equations for prices, and distances we can now derive solutions for all prices and quantities in terms of exogenous cost, demand and market parameters. After some manipulation into simpler forms, these are displayed in the main text of section two.

Appendix 3

To graphically illustrate the relationship between the welfare and the different constraints, values for the demand, cost and network gas/generator parameters are required. As relative efficiency rather than the absolute level of efficiency is of most interest the demand parameter *a* is simply normalised 1. Determining what the costs should be relative to *a* is more difficult and is facilitated through the use of previous studies. Green (1999) looks at numerical solutions for a simple model of the UK electricity contract market using a ratio of 2/45 for the ratio of costs to the demand parameter thus $T_e = 2/45$.[14] To determine the relative cost for transmission of gas, Waterson (1999) reports the cost of gas transmission as approximately twice that of electricity for a single unit of electricity supplied to the consumers thus $T_g = 4/45$. Lastly, the ratio of gas generators to total consumption of gas (θ) was approximately 40% in 1999.[15] This ratio can be thought of as a measure of the overlap between the gas and electricity industry, as the overlap increases the welfare gains from integrating the regulatory regimes also increase.

[14] Whilst Green's paper looks at generator's costs, because a proportion of transmission costs are borne by the generators and hence is incorporated into this ratio, it is used to approximate the electricity transmission cost parameter.

[15] As supplied by the Transco Transportation 10 Year Statement 2000 pg11

Figure A3.1
Gas Consumer Welfare with Constraints

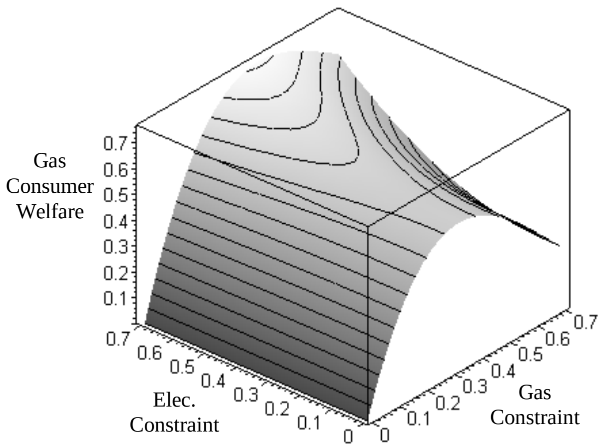
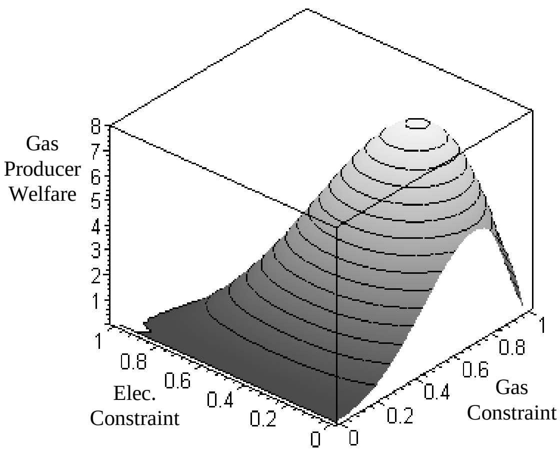


Figure A3.2
Gas Transmission Profit with Constraints



Looking at the graphical solutions to the numerical solutions in figures A3.1 and A3.2, the gas transmission agent makes far higher surplus (approximately 10 times) than gas consumers even at the consumer optimal level of constraint. The reason behind this is two fold, firstly there are only (1-θ) gas consumers being served, whilst the gas company serves both gas consumers and the entire generator market. Secondly, electricity transmission is half the gas transport cost, consequently σ_e is greater than σ_g and generator demand is larger than gas consumers. Both of these effects mean the firm is able to make substantial profits on sales. Because this unbalance in surplus exists, any simple combinations without weights on consumer surplus results in the gas producer's welfare strongly dominating consumer's. As will be shown in the later numerical simulations, ϕ must be in the order of 10/100 or less to prevent this domination occurring.

Using the same parameters the relationship between the two constraints and gas market welfare can also be illustrated.

Figure A3.3
Elec. Consumer Welfare with Constraints

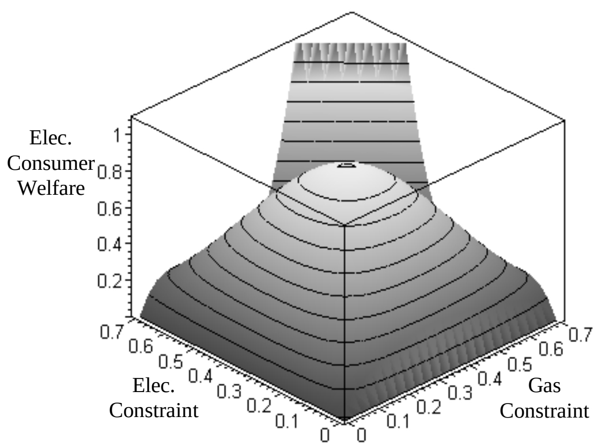
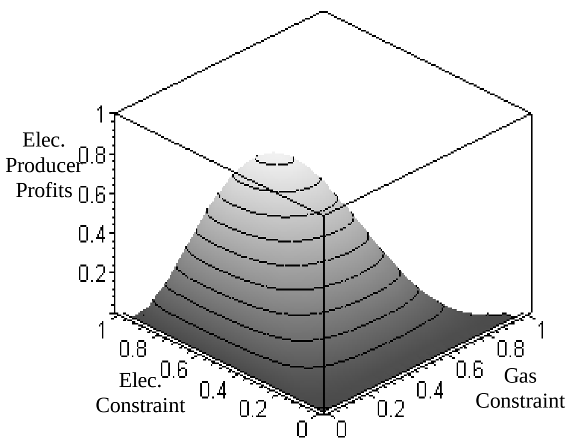


Figure A3.4
Elec. Transmission Welfare with Constraints



Both figures 3.3 and 3.4 are subject to the binding electricity constraint condition (equation 1.20) ruling out the tail as a feasible solution in Figure 3.3.[16]

[16] Naturally the transmission firm's optimal regulatory point (Figure 3.4) is non binding. Consequently the

Appendix 4

Weighted electricity industry welfare at θ = 1 (only generators) simplifies total electricity welfare to;

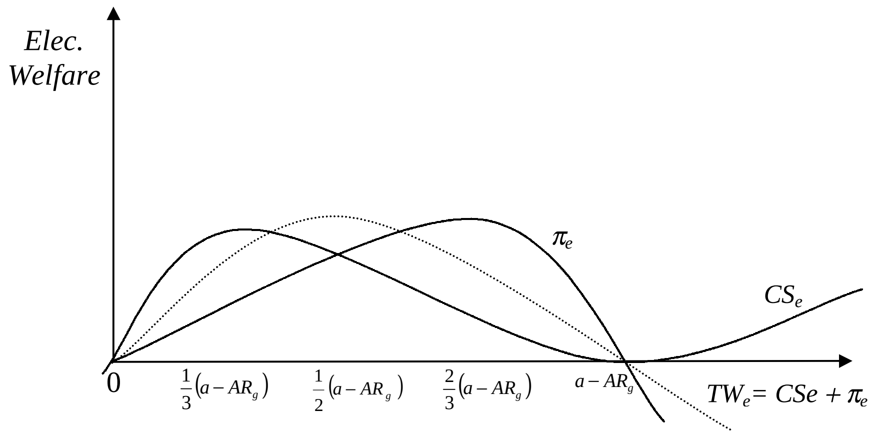
$$TW_e = \frac{81}{128}\frac{AR_g AR_t (a - AR_e - AR_g)^2}{T_g T_e} + \phi \frac{81}{128}\frac{AR_g AR_e^2 (a - AR_e - AR_g)}{T_g T_e}$$

Differentiating with respect to, and solving for AR_e and yields the electricity regulators best response function as;

$$AR_e = \begin{cases} \frac{1}{2}\frac{(a - AR_g)[4 - 2\phi \pm 2\sqrt{1-\phi(1-\phi)}]}{3(1-\phi)}, & 0 \le \phi < 1 \\ \frac{1}{2}(a - AR_g), & \phi = 1. \end{cases}$$

By differentiating only the second (ϕ) term of TW_e and setting ϕ = 1, the first order condition for a regulator only concerned with firms profit is derived. Setting this first order condition equal to 0 and solving for AR_e yields the best response for regulators unconcerned with consumers as; $AR_e = 2/3(a-AR_g)$. Although this is unrealistic as regulatory bodies do not optimise only according to firm profits, it does help in illustrating the two extremes of the firms and consumers optimal level of regulation. More specifically it illustrates the impact that the weighting variable ϕ has on the optimal level of electricity regulation. We illustrate the two extremes of optimising solely with respect to consumer welfare ϕ = 0, and solely with respect to producer profits in figure A.41

Figure A4.1
Optimal Electricity Welfares with Regulatory Constraint



This result is similar to Cowan's (1997) where an optimal point for consumer welfare maximisation exists and contrary to intuition tightening the average revenue constraint further may actually reduce consumer welfare as well as producer welfare. Secondly it shows that the impact of $\phi > 0$, is to shift the optimal best response to the right of $1/3(a-AR_g)$ until when $\phi = 1$, the optimal response is simply an average of the two extremes at $1/2(a-AR_g)$.

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simple maximising of joint welfare using identical weights (ϕ =1) results in non-binding regulatory constraints.