A study into loyalty-inducing programmes which do not induce loyalty

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1 Introduction

Keeping up with its reputation as a pioneer in the airline industry, in May 1981 American Airlines launched AAdvantage, the first frequent-flyer programme (FFP). Within a week, United Airlines countered by introducing a similar programme and in the next few months all of the main American carriers followed.[1] The response outside the United States was slower though today all of the more important players in the industry have their own FFP.

The concept behind these programmes is that of rewarding passenger loyalty to a carrier. Rewards come in the shape of free flights, gifts or upgrades and loyalty is measured in air miles, which are calculated as a combination of money spent and distance travelled, so that business and first-class tickets generate more air miles than economy tickets on the same route. Membership in a FFP is, almost always, free.

The popularity of $FFPs$ is considerable. Today, AAdvantage can boast over 30 million members whilst its main American competitors, United Airlines and Delta, have in their loyalty programmes around 23 million members each.[2] In Europe, British Airways has enrolled in its Air Miles scheme over

[1] Chapter 1 in Nako (1992) provides a detailed description of the early developments of $FFPs$ in the United States.

[2] Frequent Flier Newsletter available on the Internet address, http://www.frequentflier.com.

four million people and in 1996 half a million of these had made use of their rewards and flown for free.[3]

Throughout this chapter I will refer to airlines, passengers and to frequent-flyers. It should be understood, however, that the analysis aims to be pertinent to the class of loyalty-inducing programmes as a whole and hence to go beyond the air transport industry. As with airlines, the proliferation of these schemes across supermarkets, fuel retailers, hotel chains, car rental companies and other retailers has been phenomenal. In 1995, according to Andersen Consulting a quarter of American consumers had access to frequent-shopping programmes at their local supermarket. In Britain, by the end of 1997, the three largest supermarkets, Tesco, Sainsbury and Safeway, had 25 million card holders between them, which accounted for more than two thirds of their customers.[4]

How can the proliferation of $FFPs$ be accounted for? Why have they become an industry standard and why are they so popular with travellers? At first sight the answers to these questions appear straight-forward.

The airlines' rationale to implement a loyalty-scheme, as announced by the carriers themselves, is to increase the repeat purchase rate of its customers. The mechanism by which $FFPs$ achieve this is the following. Due to the equity customers build in the programmes via the collection of air miles, a customer faces the opportunity cost of foregone miles when he decides to patronise a second airline. In order to avoid this cost, travellers stick to one airline. On the other hand, the prospect of a free trip to Paris or an upgrade to Executive Class provides a clear motivation for travellers to join an airline's programme.

As it stands, the above explanation is backed by strong intuition. Upon scrutiny, however, the intuition wobbles. Firstly, the causal relation between loyalty inducing schemes and locked-in consumers does not perform well when confronted with empirical evidence. Secondly, it is not clear that the possibility of receiving an award in the future is the best way through which to raise the value of an airline in the eyes of travellers. I now turn to these two points.

While advertising, improved distribution or sales promotion aim, through different routes, to raise a company's market penetration, loyalty programmes

[3] Skypala, P. (1997). *How taking off is taking off: Air Miles.* Financial Times, 18th October, p. 3, London Edition.

[4] Brown-Humes, C. (1997), *Points have a blunt edge*, Financial Times (p.24), 25th October, London Edition.

have the precise goal of increasing the purchase frequency of customers. As such an evaluation of a FFP should hinge on its success on this front.

To the best of my knowledge, the only such appraisal of a loyalty-programme carried out within academic circles is that conducted by Sharpe and Sharpe (1997). These authors investigate the success of the Australian Fly Buy programme in raising the loyalty of customers of the participating brands to levels that are in excess of what would be expected.[5] Their findings leave little room for enthusiasm: the results show that although there is a weak level of excess loyalty the expected deviation is not consistently observed for all the participating brands.[6] The findings of Sharpe and Sharpe (1997) second a 1997 report from the Mintel research group which finds that consumers do not become more loyal to a retailer despite being a member of its loyalty-scheme.[7]

An informal confirmation of the lukewarm performance of FFP can be read from what is not said by the industry practitioners in comments and interviews to the press. Indeed, while they are keen to herald the launch of loyalty schemes as a means to obtain a loyal clientele, they are suspiciously silent on the actual outcome of such programmes. It is also notable that in Reichheld's (1996) extensive examination of companies that have implemented (successful) policies to induce loyalty no mention is made of $FFPs$.

The few empirical studies and the silence of airline executives are far from being watertight evidence of the shortcomings of $FFPs$ in fulfilling the proclaimed objective of raising repeat purchase rates. However, they do cast doubts. These doubts are reinforced by noting that the strength of the intuitive link between $FFPs$ and high repeat purchase rate is eroded in light of the following observations.

1. All of the main airlines have their own FFP.

2. An individual is able to join more than one programme. Furthermore, membership is generally free and the time cost involved in filling in forms is negligible. Not surprisingly, individuals join the programmes of more than one airline. A survey carried out by Toh and Hu (1988)

[5] The Fly Buy programme is a multi-collection scheme whereby points can be collected for the same reward scheme from any of the participating suppliers.

[6] The *excess* is in relation to the degree of loyalty predicted by the Dirichlet and by the Negative Binomial Distribution models of repeat buying as set out in Ehrenberg (1988) and applied widely in the marketing literature.

[7] Loyalty cards fail to impress, *Financial Times,* 7th January, p.7, London edition.

reported that the average number of multiple membership among frequent flyers was 2.3. A 1999 survey amongst business travelers found that on average travelers belonged to three separate loyalty-programs and suggested that this figure was rising.[8]

3. Holding multiple FFP membership cards implies that a traveller may vary his choice of airline without necessarily losing out on air miles. The opportunity costs which FFP seek to induce onto consumers are, in this way, mitigated, although the non-linearity of most schemes and the existence of a mileage expiration date in most programmes ensure that they are not totally lost.

The above exposition casts doubts on the effectiveness of loyalty-programmes in fulfilling the objective announced by their implementers - raising the repeat purchase rate of customers. There remains the suggestion that $FFPs$ are tools through which to raise the general value of the airline in the eyes of travellers and so contribute to an airline's market penetration. After all, travellers are attracted by discounts and gifts and their demand can be competed for via the generosity of $FFPs$. However, it is questionable whether loyalty programmes are the most effective tools with which to lever customer value. Are there no better policies with which to motivate consumers to select a given airline?

Surveys carried out amongst travellers consistently report that price levels, punctuality and on-board service are the three criteria to which passengers pay more attention to in their choice of airline.[9]

In addition, loyalty-schemes are costly. A recent estimate placed the costs of running a FFP between 3 and 6% of an airline's revenue.[10] Furthermore, airlines should cost the lost revenue that comes about from the award of free flights. On the one hand, some passengers use the collected air miles to go on a flight they would otherwise have been willing to pay for. On the other hand, passengers flying on their awards might displace regularly paying travellers. Admittedly, this problem is limited by the general excess capacity

[8] OAG Business Travel Lifestyle Survey, quoted in *Insideflyer* January 1999.

[9] Survey carried out by the International Air Transport Association (IATA) referred to in Roger Bray, *When Work Gets the Perks,* in The Financial Times, 9th February 1998, p.14, London Edition. The survey of Toh and Hu (1988) arrives at the same conclusions.

[10] "Extra Lift for Airlines", *Asian Business,* August 1993, pp.44-46, quoted in Dowling and Uncles (1997).

in the airline industry as well as by the restrictions imposed by airlines on the flights against which an award may be claimed.[11]

The discussion presented so far has aimed at dismembering the frequently heralded view of $FFPs$ as win-win arrangements between airlines and travellers. As was sketched above, it is not clear that these schemes succeed in providing airlines with a portfolio of loyal customers nor are they the most direct means of raising the value proposition to customers and, through this, an airline's market share.

Banerjee and Summers (1987) and Caminal and Matutes (1990) offer an alternative and insightful explanation for the ubiquity of loyalty schemes. Their central idea is that $FFPs$ are a tool for airlines to transfer some of the consumer surplus to themselves - they are win-lose arrangements where airlines are on the winning side. In both papers, the ability of airlines to reap the surplus arises from the switching costs that such programmes induce on consumers. In turn, the switching costs emerge due to the following reason. The models analysed in both of these papers consider a two-period time horizon so that travellers must patronise, by construction, the same airline in the second period as they did in the first period in order to collect the discount offered to repeat buyers. A traveller who switches airline, on the other hand, foregoes the entitlement to the discount. It follows that in the second period travellers are induced to stick to their first period choice. In the second period, therefore, airlines compete less aggressively as it becomes harder to attract passengers who chose the rival in the past. In addition, there is an incentive to be less aggressive in the first period as well. It is in an airline's interest to ensure that the competitor has a sufficient share of 'old customers' to induce it not to behave aggressively in the second period. To ensure this, an airline will resist lowering prices in the first period and take over the entire market.[12] Hence, the benefit of $FFPs$ to airlines accrues from the higher prices which the segmentation of the market allows carriers to set, rather than from the rewards of holding a portfolio of loyal customers *per se*.

While some travellers will choose their airline according to the balance on their air miles account, the arguments presented earlier suggest that the behaviour of others runs against the behaviour hypothesized by the two pa-

[11] A general rule offered by the Frequent Flier Newsletter is that 5% of an airline's seats are allocated for use by FFP members making use of a reward.

[12] This intuition for the less aggressive first period behaviour is valid in the setting of Banerjee and Summers (1987) although it does not apply in Caminal and Matutes (1990).

pers. As mentioned earlier, travellers are generally enrolled in more than one loyalty scheme and tend to distribute their purchases over several airlines. This conduct suggests that even if $FFPs$ give rise to switching costs, travellers do not seem to be greatly limited by them. Given this, the mechanism identified by Banerjee and Summers (1987) and Caminal and Matutes (1990) which allows airlines to charge higher prices is no longer present. An appropriate question which follows is whether airlines still find $FFPs$ appealing if these schemes are not successful in imposing switching costs on travellers. Are airlines still able to extract consumer welfare through such programmes?

In this chapter I examine the role of loyalty-programmes in a setting where the schemes do not induce a switching-cost on consumers. This will be carried out by analysing a model which extends the two-period horizon of the papers mentioned above to a three-period setting. The relevance to the analysis, however, is not in the number of periods *per se*, but rather in the notion that to benefit from the discount offered by the FFP a traveller does not have to choose the same airline in all periods.[13]

Running parallel with the above inquiry, the analysis in this chapter also attempts to shed some light on the welfare of travellers who participate in the market rarely *vis-à-vis* those who fly frequently. How do these two groups fare when $FFPs$ are launched? The interest in this question is grounded on the idea that $FFPs$ are targeted at rewarding frequent customers. It follows that one would expect this group to benefit from these programmes. Here, I will show that these two groups of consumers do benefit differently from the implementation of a loyalty-scheme. The results obtained point out that the group of occasional travellers, those that fly rarely, invariably lose. On the other hand, whether frequent-flyers benefit or not from the introduction of a FFP will be shown to depend on the weight that this group has in the population of consumers as a whole.

The following questions, reflecting the above discussion, summarize the points which I will seek to address in the chapter.

1. Can a FFP which does not lock in customers be the outcome of competitive practice?

2. How do prices compare between the scenario in which firms launch

[13] Banerjee and Summers (1987) on p. 16 extend their model to T periods. They maintain, however, the restriction that consumers must have patronised the airline in all $T-1$ periods before the coupons can be used.

a FFP and one where firms are unable to discriminate between customers (have no record of their past behaviour) and are therefore unable to offer special treatment to loyal customers?

3. Do travellers benefit or lose from the implementation of a FFP ? Do occasional and frequent traveller benefit or lose differently?

4. How does the composition of the consumer population - the ratio of frequent to occasional travellers- influence the prices set and the coupon values set by airlines?

The role of discount coupons as a means to discriminate between consumers involved in repeat buying has been studied in contexts outside loyalty programmes.[14] Fudenburg and Tirole (1997) and Chen (1997) turn loyalty programmes on their head and study the behaviour of firms poaching customers of competitors by offering a discount to these if they switch. As expected, the ability to poach affects the degree to which customers switch between firms. Whether poaching leads to too much or too little switching in comparison to the socially efficient level is shown to depend on the nature of consumers' relative preferences for the two brands - whether these are constant or independent over time - and on whether the firm is able to commit in the first period to its behaviour in second period.

The rest of this chapter is structured as follows. In Section 2.2, I describe the model with which I intend to tackle the questions laid out above. Section 2.3 solves this model under the special case where airlines are unable to discriminate passengers by their past choices. The results derived there will serve as an appropriate benchmark for the findings obtained in Section 2.4 where, in contrast, airlines are allowed to discriminate in favour of repeat travellers. Section 2.5 ties the results obtained with those of the relevant literature and Section 2.6 concludes.

[14] There is also an interesting literature examining those coupons generally distributed through newspaper or mail and unrelated to repeat buying. Such coupons have been regarded as a means to price discriminate, both in settings where the coupons are targetted at a particular group (Bester and Petrakis, 1994) and where they are untargetted (Narasimhan, 1984 and Caminal , 1996).

2 Model

The model borrows its basic structure from von Weizsäcker (1984) and Klemperer (1987) who, to my knowledge, first grafted inter-temporally changing tastes onto a Hotelling-like model of product differentiation.

There are two competing airlines to be denoted by A and B who offer services at a constant marginal cost c. The services offered by airline A differ from those of airline B along dimensions such as the menu of travel schedules offered and the available connecting flights.[15] Following Hotelling (1929), these differences are captured by picturing the two airlines as if located at the end-points of the interval $I \equiv [0, 1]$. Let airline A be located at point 0 and airline B at point 1. Consumers are distributed along I. The location of a consumer at time t is given by $i_t \in I$. The location i_t reflects a consumer's ideal point, and the distance from it to the end-points measures the disutility from purchasing a less preferred ticket. A consumer who purchases a ticket from airline A in Period t enjoys a utility $R - p_t^A - i_t$, where R is the reservation price, p_t^A is the effective price charged by A and i_t is the distance that separates the consumer from A. Similarly, if he buys the service from B, the benefit will be given by $R - p_t^B - 1 + i_t$. In line with the terminology of models employing the Hotelling (1929) setting, though at the risk of causing some confusion, I shall refer to the disutility associated with buying a less preferred ticket as the transportation cost.

The model considers a 3-period setting. If present in the market in period t, a consumer will demand one unit of the service offered by A or one unit of the service offered by B.

There are two types of consumers. The *frequent travellers* take part in the market in each of the three periods. It is assumed that these consumers are uniformly distributed along I in each period and that their location in one period is independent of that in the previous period. As in Caminal and Matutes (1990) the change in the location of these travellers can be interpreted as "a change in travel plans: connecting flights and time schedules are more or less appropriate in one airline or the other depending on the origin and destination of the plane" (Caminal and Matutes 1990, p. 356). Hence, *ceteris paribus*, which of the two airlines is more attractive may vary from one period to the other.

[15] What I wish to exclude are differences in services which give rise to vertical product differentiation.

The *occasional travellers* are the second type of consumers, and these take part in the market for one period only. It is assumed that this group of consumers is also uniformly distributed along I. At the end of each period, however, they exit the market and are replaced by a new mass of occasional travellers whose locations are independent of the consumers they replace. To ease computation, I assume that the market serves a constant unit mass of consumers over time which requires that the density of consumers joining the market equals the density of consumers leaving the market at the end of a period. The share of regular and occasional travellers in the market is, therefore fixed. Let μ be the proportion of frequent travellers so that $1-\mu$ is the proportion of occasional ones. The value of the parameter μ is common knowledge. Lastly, I assume that a consumer knows which type of traveller he is: whether he will leave after one period or whether he will be in the market for all 3 periods.

The two categories of travellers described above are not an exhaustive description of the types of travellers one might wish to consider. The absence of travellers who are present in the market over the three periods and have a fixed location throughout seems particularly critical. Such a set of consumers corresponds to those who need to take the same trip - at the same hour, to and from the same airports- and hence are likely to hold a constant relative preference between the two airlines. I have not made room for them in order to keep the analysis tractable.[16]

In addition, there are dimensions other than that of frequency of consumption and location on the interval I along which passengers can be distinguished. An obvious one is that between business class and economy class passengers. In the framework of the model, making this distinction would call for the modelling of consumers with different reservation prices and heterogeneous unit transportation costs.[17] A second, closely related, distinction is that between travellers whose tickets are paid for by their employer and those who have to cover the cost themselves. Characterizing travellers along either or both of these lines appears natural in the context of a study on

[16] On the other hand, the model purposefully rules out travellers who are present in the market for two out of the three periods. Their presence would have attributed to the $FFPs$ the ability to create switching costs which would run against the premise of the model.

[17] Different reservation prces could be easily incoorporated into the above model. Provided these prices were such that they guranteed that consumers alwyas purchased a unit of the good, the analysis carries through unaltered.

$FFPs$.[18] Doing so, however, would burden the analysis and put at risk the ability to yield any clear answers to the questions laid out at the end of Section 1.

Having added the above parenthesis, I now return to the description of the model. The two airlines recognise past customers and are, accordingly, able to discriminate between travellers on the basis of the revenue that they have generated to the airline in the past. In other words, airlines are able to launch $FFPs$. Here, the structure of the $FFPs$ which airlines are allowed to implement is restricted to the following class: customers receive a discount - a coupon - on patronising an airline for the second time. In restricting the class of admissible $FFPs$ to this I have aimed at finding a compromise between parsimony and the need to portray the most salient features of a FFP. Hence, and in line with the discussion in Section 1, the class of $FFPs$ that I consider does not impose switching costs as it allows consumers to collect a discount from an airline even if he has addressed the rival in the past. Furthermore, the $FFPs$ considered are such that they ensure that frequent travellers collect a discount over the three periods.

The timing of the decision taken by the players in the model is as follows. Prior to Period 1, airline A selects its price p_1^A and the (absolute) value of the coupon α it offers to repeat buyers. Simultaneously, airline B selects its price p_1^B and its coupon β. Whilst the value of the coupons remain unaltered throughout the 3 periods, the price levels are reviewed at the start of both Period 2 and Period 3 before the redistribution of consumers along I takes place. Let p_2^A be the price selected by A in the second period and p_3^A that chosen by this same airline in Period 3. The analogous prices chosen by airline B will be denoted by p_2^B and p_3^B respectively. Consumers, on the other hand, must decide at the beginning of each period which airline to patronise. Their choices are made once they have been distributed along I and, hence, once their location i_t is known. It follows that consumers base their choice on their relative distance to the end-points as well as on the relative prices and coupons offered by the two airlines.

In selecting its price and coupon airlines aim to maximise expected profits over the three periods. Travellers, on the other hand, intend to maximise the sum of the expected utility gained during their stay in the market. Future income and utility are not discounted.

[18] See Cairns and Galbraith (1990) for a study of $FFPs$ which pivots on the existence of travellers whose fare is partially covered by a third party.

3 Benchmark: no discrimination

An appropriate benchmark for the analysis that follows is to consider the above model in a setting where airlines are unable monitor consumers' past decisions. In such a scenario, airlines cannot identify repeat customers and, hence, are unable to offer them coupons. The absence of a reward for repeat-buyers breaks the inter-temporal link in the (frequent)travellers' decision rules thereby making consumers' decision in one period independent of that in the others. The independence of consumers' behaviour over time implies that the policy which maximises airlines' profits over the 3 periods coincides with that which maximises profits over a single period.

Consider then the behaviour of consumers and airlines in Period 1, say. A consumer present in the market in this period collects a utility level of $R - p_1^A - i_1$ if he patronises airline A and a level $R - p_1^B - 1 + i_1$ if he chooses airline B. Comparing the two expressions, it follows that the optimal behaviour is to address A if $i_1 \in (0, \Psi)$, where $\Psi \equiv \frac{1+p_1^B-p_1^A}{2}$ and address B otherwise. Given this behaviour, it is simple to show that price competition between the two airlines gives rise to the unique equilibrium prices $p_1^{A*} = p_1^{B*} = 1 + c$. The excess of price over marginal cost arises from the local monopoly power that airlines possess due to the spatial setting of the model.[19] In turn, the symmetric solution implies that $\Psi = \frac{1}{2}$ in equilibrium. Travellers located in the first half of I address A and the remaining address B. Given this, airlines expect to collect a revenue of $\frac{1+c}{2}$ in each period and thereby expect to make profits over the three periods equal to $\frac{3}{2}$.

4 Solving the general model

I now turn to the case where airlines are able to discriminate between consumers on the basis of their past purchases.

The model is solved by working backwards from Period 3. For each of the periods, the analysis establishes the consumers' set of optimal decision rules. For a frequent traveller, an example of a typical rule within this set takes the form: "In Period 2, given that prices and the coupon levels offered by airlines are p_2^A, p_2^B, α and β , and given that in Period 1, I addressed airline A and that

[19] More generally, equilibiurm prices are given by $t + c$, where t is the unit cost of transport. The higher the cost of transport faced by travellers the greater is the ability of the two airlines to extract consumer surplus.

in the following period I expect to be located at i_3 and expect to face prices p_3^A and p_3^B then I will return to A if $i_2 \leq i^*$ and go to B otherwise." The set of rules are optimal in the sense that they ensure consumers maximise their expected future utility at each point in time. It is assumed that travellers have completely rational expectations. Given that consumers abide by these optimal rules, I then calculate the prices and coupon levels that airlines set in order to maximise the sum of their expected profits over the three periods.

To make the presentation as fluid as possible I leave to Appendix 2.A many of the algebraic stepping-stones involved.

4.1 Period 3

Consider the behaviour of an occasional traveller who takes part in the market in Period 3. Given prices p_3^A and p_3^B, an occasional traveller located at i_3 will address airline A if $i_3 \leq \frac{1+p_3^B-p_3^A}{2}$. Otherwise he will purchase from airline B.

The behaviour of a frequent traveller, on the other hand, depends on the history of his past purchases. By Period 3, a frequent traveller will have either bought a ticket once from each airline or he will have bought twice from the same. If the latter, he will have received the discount offered to repeat buyers already and, by construction, will not be able to benefit from a further coupon. Accordingly, the decision of such a consumer will depend only on his location i_3 and on the relative prices of tickets. It follows that he will address airline A if $i_3 \leq \frac{1+p_3^B-p_3^A}{2}$[1] and airline B otherwise. On the other hand, if the consumer has addressed different airlines in the past, then his choice in Period 3 determines the airline from which the discount is to be received. Consequently, his decision takes into account the relative generosity of the two coupons. Patronising airline A yields him a utility level of $R - p_3^A + \alpha - i_3$ while the utility from choosing B is $R - p_3^B + \beta - 1 + i_3$. Comparing the two utility levels, it is straight-forward to see that he will address A if $i_3 \leq \frac{1+p_3^B-\beta-p_3^A+\alpha}{2}$ and address B otherwise. In sum, the optimal decision rule for a frequent traveller in Period 3 can be written as, Period 3

$$\text{If}\begin{cases}\text{have received discount in Period 2 and}\begin{cases} i_3 \leq \frac{1+p_3^B-p_3^A}{2} \equiv \Omega_1 & \text{then go to } A. \\ i_3 > \Omega_1 & \text{then go to } B.\end{cases} \\ \\ \text{have not yet received discount and}\begin{cases} i_3 \leq \frac{1+p_3^B-\beta-p_3^A+\alpha}{2} \equiv \Omega_2 & \text{then go to } A. \\ i_3 > \Omega_2 & \text{then go to } B.\end{cases}\end{cases} \quad (1)$$

Using the above decision rules it is possible to construct the expressions for the airlines' profits in Period 3. For airline A this will be given by,

$$\Pi_3^A = \left(p_3^A - c\right)\left((1-\mu) + \mu\,(1-s)\right)\,\Omega_1 \, + \left(p_3^A - \alpha - c\right)\mu\, s\,\Omega_2 \qquad (2)$$

where s is the share of frequent travellers who have $s$witched between the two airlines in the first two periods. The first component of (2) reflects the contribution to A 's profits from those travellers who will not collect the discount α in the third period: that is to say, the occasional travellers and the share of frequent travellers who have already received the discount in the past. The second component, on the other hand, picks up the portion of A 's third period profits generated from those who are yet to receive the discount.

Airline A chooses p_3^A to maximise Π_3^A. Solving this optimization problem it is possible to derive A 's reaction function as,

$$p_3^A = \frac{1}{2}\left(1 + c + p_3^B + \mu s\,(2\alpha - \beta)\right). \qquad (3)$$

An analogous expression can be derived for airline B. Solving the two reaction functions simultaneously it is then possible to derive the equilibrium prices. These are described in Proposition 2.1.

Proposition 2.1 *The unique equilibrium in Period 3 is for airline A to set a price* $p_3^{A*} = 1 + c + \alpha\mu s$ *and airline B a price* $p_3^{B*} = 1 + c + \beta\mu s$.
Proof. The result is obtained by solving the two reaction functions simultaneously. The necessary second-order conditions are also met. ■

Proposition 2.1 establishes that the third-period equilibrium prices lie above the benchmark level. The intuition behind this result can be explained with reference to the airlines' reaction functions.

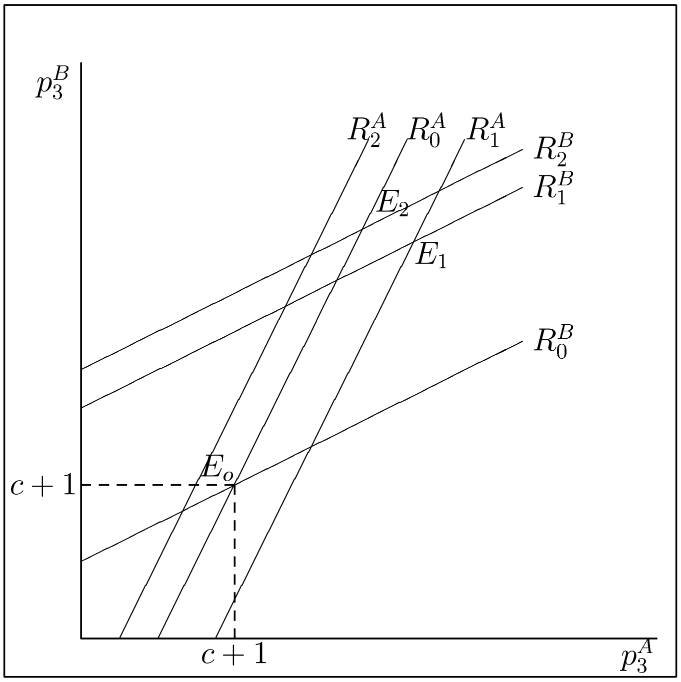

Figure 1: Shift of airlines' reaction functions due to $FFPs$

First, note from A 's best-reply (3) that airlines' prices are strategic complements - if airline B raises its price, airline A 's optimal policy requires that it raises p_3^A as well. Second, offering a discount affects an airline's behaviour in Period 3 through two distinct routes. On the one hand, the coupon acts as a second tool through which to compete for the demand of frequent travellers. Consequently, the higher the value of the discount offered, the less aggressive will an airline be in its price competition. On the other hand, the commitment to pay out a coupon raises the costs that airlines face compared to the benchmark case. Graphically, both of these effects - the less aggressive behaviour and the need to cover committed costs - lead to an outward shift of the airlines' reaction functions. For airline A, say, each of the two effects is responsible for a shift of size $\frac{\alpha\mu s}{2}$ (for airline B, the expression would be $\frac{\beta\mu s}{2}$). Third, given the value of airline A's discount, a higher value β leads to a more aggressive competition over base price by A. Graphically, this is represented by an inward shift of the best-reply curve by $\frac{\beta\mu s}{2}$ for airline A and by $\frac{\alpha\mu s}{2}$ for airline B.

The net effect of introducing FFP on the reaction functions depends on the relative values of α and β as seen in (3) . If an airline's discount is greater than half of that of the rival's then its reaction curve will shift outwards. Otherwise, the shift will be inwards. Figure 2.1 illustrates the former case by the shift of the best-reply curves from the benchmark case $\left(R_0^A, R_0^B\right)$ to $\left(R_1^A, R_1^B\right)$. The pair of curves $\left(R_2^A, R_2^B\right)$, on the other hand, depicts the case where $\alpha < \frac{\beta}{2}$.

Note that an airline's discount shifts that airline's best-reply curve outwards by twice the amount that it shifts the rival's inwards. Hence, and independent of whether the pair $\left(R_1^A, R_1^B\right)$ or $\left(R_2^A, R_2^B\right)$ is the appropriate one, the resulting equilibrium prices will be greater than those of the benchmark case.

The higher equilibrium prices do not imply that the profits earned by the two airlines are necessarily higher than those derived in the benchmark case since discounts must now be handed out. To see this, note that at the equilibrium prices, the third period profit of the two airlines are given by,

$$\begin{aligned} \Pi_3^{A*} &= \frac{1}{2}\left(1 + \alpha\mu s\left(1-\mu s\right)\left(\beta - \alpha\right)\right) \\ \Pi_3^{B*} &= \frac{1}{2}\left(1 + \beta\mu s\left(1-\mu s\right)\left(\alpha - \beta\right)\right) \end{aligned} \tag{4}$$

It follows, that an airline's expected profit increases with the value of the discount offered by its rival. Furthermore, they will be higher, equal to or lower than the benchmark level - $\frac{1}{2}$ - depending on whether the coupon it offers is lower, equal to or higher than that of its rival. The intuition behind this is the following. Say $\beta > \alpha$. For travellers who will benefit from a discount in Period 3, the effective price will be lower at airline B since $p_3^{B*} - \beta = 1 + c - \beta\left(1-\mu s\right) < 1 + c - \alpha\left(1-\mu s\right) = p_3^{A*} - \alpha$. Given this, a greater proportion of these travellers will opt to address airline B. This airline must increase its base price, p_3^B, to cover the costs of handing out discounts and, at the same time, to curtail the demand by this set of travellers. However, raising base price also induces a greater share of those consumers who will not benefit from the discount in Period 3 to choose airline A. While the former set of travellers pay an effective price lower than $1 + c$, the effective price paid by the latter is above the competitive level. To sum up, offering a more generous coupon attracts those who pay a lower effective price and repels those who pay the higher price.

Lastly, it should be pointed out that at the start of Period 3, the airline burdened with the more geneorus discount would have the incentive to review the value of the discount it had chosen in Period 1 and set it equal to 0. In the context of this model, such an action is assumed to be not possible. In turn, the assumption can be supported on the grounds of reputation effects and on the fact that it would damage the airline's ability to set up a new FFP in the future.[20]

4.2 Period 2

Using the equilibrium prices of Period 3, I now work backwards to determine the optimal choice rules of consumers in Period 2 which, in turn, will allow me to solve the optimization problem of the two airlines at the start of Period 2.

Like his Period 3 counterpart, an occasional traveller in Period 2 will be unaffected by the generosity of the discounts offered. Therefore, he will patronise airline A if $i_2 \leq \frac{1+p_2^B-p_2^A}{2}$ and airline B otherwise.

Now consider the behaviour of a frequent traveller in Period 2. It is necessary to distinguish these travellers by their choice in Period 1.

Consider first a frequent traveller who purchased from A in Period 1. If he returns to airline A in the second period he will receive the discount offered by A to repeat buyers. The expected utility of a consumer returning to A in Period 2 is therefore given by $R-p_2^A+\alpha-i_2+E\left(U^3|xx\right)$, where $E\left(U^3|xx\right)$ is the expected utility enjoyed in Period 3 by a consumer who has in the previous two periods visited the same airline. Similarly, the expected utility of a consumer addressing airline B in Period 2, conditional on having bought a ticket from A in Period 1, is equal to $R-p_2^B-(1-i_2)+E\left(U^3|xy\right)$. The term $E\left(U^3|xy\right)$ reflects the utility a consumer can expect in Period 3 given that he has addressed different airlines in the first two periods. Therefore, a consumer who has addressed A in Period 1 will return to it in Period 2 if $i_2 \leq \frac{1+p_2^B-p_2^A+\alpha+E\left(U^3|xx\right)-E\left(U^3|xy\right)}{2}$ and will address B otherwise.

[20]Interestingly, airlines are typically within their rights to review the discounts offered in their schemes. In the conditions laid out by $FFPs$ that I have come across, airlines reserve the right to change the awards, the rules for earning mileage credit and, with a few month's notice, to end the programme (see for example AAdvantage 2000 and Qualiflyer 2000).

Following an analogous reasoning, the decision rule for a consumer who addressed B in Period 1 will be given by: patronise A if $i_2 \leq \frac{1+p_2^B-\beta-p_2^A+E\left(U^3|xy\right)-E\left(U^3|xx\right)}{2}$ and B otherwise.

The expressions $E\left(U^3|xx\right)$ and $E\left(U^3|xy\right)$ are given by,[21]

$$\begin{aligned} E\left(U^3|xx\right) &= \int_0^{\Omega_1} \left(R - p_3^A - i\right) di + \int_{\Omega_1}^1 \left(R - p_3^B - 1 + i\right) di \\ &= R - p_3^B - \frac{1}{2} + \Omega_1^2 \end{aligned}$$

$$\begin{aligned} E\left(U^3|xy\right) &= \int_0^{\Omega_2} \left(R - p_3^A + \alpha - i\right) di + \int_{\Omega_2}^1 \left(R - p_3^B + \beta - 1 + i\right) di \\ &= R - p_3^B + \beta - \frac{1}{2} + \Omega_2^2 \end{aligned}$$

where, recall, $\Omega_1 = \frac{1+p_3^B-p_3^A}{2}$ and $\Omega_2 = \frac{1+p_3^B-\beta-p_3^A+\alpha}{2}$. Note that a consumer who addresses the same airline in the first two periods, benefits from the discount offered by the FFP in the second period and hence cannot expect to benefit further in Period 3. Consequently, the expression for $E\left(U^3|xx\right)$ does not include either of the terms α and β. On the other hand, a consumer who has chosen different airlines in the first two periods can still expect to receive the coupon offered by the $FFPs$. It follows that $E\left(U^3|xy\right)$, is a function of the coupon values.

Making use of the expressions derived for $E\left(U^3|xx\right)$ and $E\left(U^3|xy\right)$, the second period optimal decision rules can be written as,

Period 2

$$\text{If} \begin{cases} \text{went to } A \text{ in } t=1 \text{ and} \begin{cases} i_2 \leq \frac{1+p_2^B-\beta-p_2^A+\alpha+\Omega_1^2-\Omega_2^2}{2} \equiv \Omega_3 & \text{then go to } A. \\ i_2 > \Omega_3 & \text{then go to } B. \end{cases} \\ \\ \text{went to } B \text{ in } t=1 \text{ and} \begin{cases} i_2 \leq \frac{1+p_2^B-p_2^A-\Omega_2^2+\Omega_1^2}{2} \equiv \Omega_4 & \text{then go to } A. \\ i_2 > \Omega_4 & \text{then go to } B. \end{cases} \end{cases} \tag{5}$$

[21] It is assumed that the values of Ω_1 and of Ω_2 lie in the unit interval. In equilibrium this condition is assured.

Both Ω_3 and Ω_4 are functions of the prices in the third period and hence a function of s, the share of frequent travellers who address different airlines in the first two periods. By construction,

$$s = \sigma\left(1 - \Omega_3\right) + \left(1 - \sigma\right)\Omega_4 \tag{6}$$

where σ is the share of frequent travellers who addressed airline A in Period 1. Using (5), equation (6) can be written as,

$$s = \frac{3 + 4\left(1 - 2\sigma\right)\left(p_2^B - p_2^A\right) + \left(1 + \alpha - \beta\right)^2 + 4\sigma\left(\beta - \alpha\right)}{2\left(4 + \mu\left(\alpha - \beta\right)^2\right)} \tag{7}$$

The problem facing the two airlines at the start of Period 2 -to maximize profits over Periods 2 and 3 - can now be written out. For airline A this problem is given by,

$$\begin{aligned} \underset{\left\{p_2^A\right\}}{Max}\Pi_2^A &= \left(p_2^A - c\right)\left(1 - \mu\right)\left(\frac{1 + p_2^B - p_2^A}{2}\right) + \left(p_2^A - c\right)\mu\left(1 - \sigma\right)\Omega_4 \\ &+ \left(p_2^A - \alpha - c\right)\mu\sigma\Omega_3 + \Pi_3^A \end{aligned}$$

The first three terms on the right hand side of (8) pick up the contribution to A 's profits respectively from: the occasional travellers, the frequent travellers who addressed B in Period 1 and the frequent travellers who addressed A in Period 1. An analogous expression can be written down for airline B and the equilibrium second period prices are given by solving the two maximization problems simultaneously.

Proposition 2.2. *There is a unique pair of p_2^{A*} and p_2^{B*} which forms the equilibrium to the second period pricing game.*
Proof. See Appendix 2.A. ■

The expressions defining the equilibrium prices assured by Proposition 2.2 are, however, unwieldy and offer little insight. Instead, I will draw attention to two special cases.

Corollary 2.1. **a)** *If the demand generated by the frequent travellers in Period* 1 *is divided equally between the two airlines,* $\sigma = \frac{1}{2}$, *then the equilibrium second period prices are given by* $p_2^{A*} = 1 + c + \frac{\mu\alpha}{2}$ and $p_2^{B*} = 1 + c + \frac{\mu\beta}{2}$.
b) *If the value of the discounts set by the two airlines is equal, say* λ, *then the equilibrium second period prices are given by* $p_2^{A*} = 1 + c + \mu\lambda\frac{1+\sigma}{3}$ and $p_2^{B*} = 1 + c + \mu\lambda\frac{2-\sigma}{3}$.
Proof. See Appendix 2.A. ■

As is clear in the two special cases considered in Corollary 2.1, equilibrium prices are set above the competitive level. The intuition for this result is identical to the one presented in Section 2.4.1 for the third period equilibrium prices. Here, as was the case in Period 3, the best-reply curves of the two airlines are also shifted out due to the $FFPs$. The forces behind these shifts are the same as those identified earlier. Recall that an airline's own discount lessens its competitive aggressiveness in prices and raises its costs. Both effects lead to an outward shift of its best-reply curve. On the other hand, the discount offered by the rival leads to an inward movement of an airline's best-reply curve, representing the incentive to price more aggressively. A graphical representation of the effect of $FFPs$ on the second period best-reply curves of the two airlines is similar, therefore, to those drawn in Figure 2.1.

The second special case offers an added insight to the model as it illustrates that the price set by an airline in Period 2 increases with σ, its first period market share of frequent travellers. Althought, this feature is shared with the model of Caminal and Matutes (1990), the intuition behind it is markedly different. In the latter paper, a higher market share in the first period increases the incentive of an airline to exploit their repeat-buyers and lowers that of attracting first time buyers so that price competition is less aggressive. In the model considered here, on the other hand, the relation between second period prices and first period market share comes about because the mass of repeat buyers, and therefore the mass of travellers entitled to a discount in Period 2 is increasing in an airline's first period share of frequent travellers. The higher this share is, the greater the total value of discounts to be handed out in Period 2. Faced with this burden, the optimal response of an airline is to raise its price.

4.3 Period 1

I now examine the choices of travellers and airlines in Period 1.

An occasional traveller in Period 1 will base his choice on the relative prices of the two airlines. In particular, he will address airline A in Period 1 if his location $i_1 \leq \frac{1+p_1^B-p_1^A}{2}$ and will address airline B otherwise.

A frequent traveller, on the other hand, will take into account the effect of his decision on expected future utility. Let $E\left(U^2|A\right)$ be the expected surplus a traveller expects to collect in Periods 2 and 3 given that he addresses airline A in Period 1 and let $E\left(U^2|B\right)$ be the analogous term for a passenger who addresses airline B in Period 1. It follows that a frequent traveller will patronise airline A in Period 1 if and only if,

$$R - p_1^A - i_1 + E\left(U^2|A\right) \geq R - p_1^B - (1 - i_1) + E\left(U^2|B\right) \tag{8}$$

The terms $E\left(U^2|A\right)$ and $E\left(U^2|B\right)$ can be explicitly worked out as,[22]

$$\begin{aligned} E\left(U^2|A\right) &= \int_0^{\Omega_3}\left(R-p_2^A+\alpha-i+E\left(U^3|xx\right)\right)di+\int_{\Omega_3}^1\left(R-p_3^B-(1-i)+E\left(U^3|xy\right)\right)di \\ &= R+\Omega_3^2-p_2^B-\tfrac{1}{2}+E\left(U^3|xy\right) \end{aligned}$$

and

$$\begin{aligned} E\left(U^2|B\right) &= \int_0^{\Omega_4}\left(R-p_2^A-i+E\left(U^3|xy\right)\right)di+\int_{\Omega_4}^1\left(R-p_2^B+\beta-(1-i)+E\left(U^3|xx\right)\right)di \\ &= R+\Omega_4^2-p_2^B+\beta-\tfrac{1}{2}+E\left(U^3|xx\right) \end{aligned}$$

Substituting these expressions into (8) yields after some simplification the following first period optimal decision rule for frequent travellers,

<u>Period 1</u> (9)

$$\text{If} \begin{cases} i_1 \leq \frac{1+p_1^B-p_1^A+\Omega_3^2-\Omega_4^2+\Omega_2^2-\Omega_1^2}{2} \equiv \Omega_5 & \text{then go to } A. \\ i_1 > \Omega_5 & \text{then go to } B. \end{cases}$$

Recall that the terms $\Omega_1, \Omega_2, \Omega_3$ and Ω_4 are a function of $E(\sigma)$. However, by construction, Ω_5 is equal to σ. It follows that an expression for σ can be computed through the implicit function J,

[22] Again it is implictly assumed that both Ω_3 and Ω_4 lie in the unit interval.

$$J \equiv \frac{1+p_1^B - p_1^A + \Omega_3^2 - \Omega_4^2 + \Omega_2^2 - \Omega_1^2}{2} - \sigma \quad (10)$$

and then substituted into the definition of Ω_5.

The rule defined in (9) completes the description of the optimal behaviour of frequent travellers at each point in time. Using the full set of optimal rules, it is straight-forward to derive the expected demand facing the two airlines over the 3 periods and hence to formulate the optimization problem of the two airlines. For airline A, this problem is given by,

$$\underset{\{p_1^A,\alpha\}}{Max}\, \Pi_1^A = \left(p_1^A - c\right)\left((1-\mu)\left(\frac{1+p_1^B-p_1^A}{2}\right)+\mu\Omega_5\right)+\Pi_2^A \quad (11)$$

while an analogous problem can be constructed for airline B.

Given that I restrict the search for an equilibrium to the class of symmetric equilibria, it will be sufficient to limit the work to the optimization problem of one of the airlines. Appendix 2.A presents a full description of the work involved. Here, only the result is presented.

Proposition 2.3. *In the unique symmetric equilibrium airlines set first period prices* $p_1^{A*} = p_1^{B*} = 1 + c + \frac{4\mu}{13-10\mu}$ *and issue coupons with a value of* $\alpha^* = \beta^* = \frac{6}{13-10\mu}$. *It then follows from Propositions 2.1 and 2.2 that in the second and third period* $p_2^{A*} = p_2^{B*} = p_3^{A*} = p_3^{A*} = 1 + c + \frac{3\mu}{13-10\mu}$.

Proof. The first step is to derive the candidate symmetric equilibrium by solving the first order conditions of the maximization problem once the symmetry conditions are imposed. To then show that the pair of prices and discounts thus obtained constitute an equilibrium it is necessary to show that neither airline has an incentive to deviate from them. To carry out this second step, I resort to numerical simulations and show that the equilibrium is indeed robust to small deviations as well as to the deviation of an airline opting to offer no discount. Appendix 2.A presents the details of the work involved. ■

It follows from Proposition 2.3 that at equilibrium, airlines will earn the benchmark profit of $\frac{1}{2}$ in Period 2 and in Period 3. Although prices are above $1+c$ in both of these periods, profits are kept down to the competitive

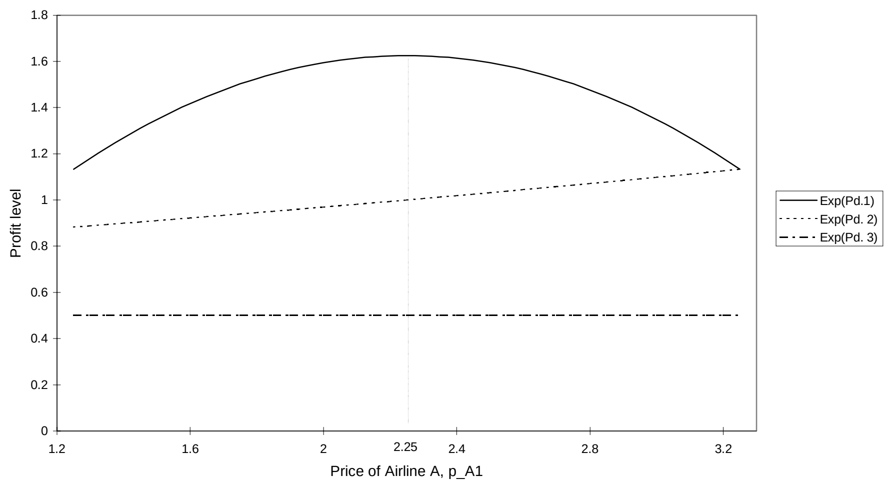

Figure 2: Expected profit of airline A at the start of each of the 3 periods. $\left(c = 1, \mu = 0.5, \alpha = \beta = 0.75, p_1^B = 2.25\right)$.

level due to the discounts that are handed out. In Period 1, however, this reasoning does not apply. There, prices are above the competitive level and, by construction, no discounts are given. It follows that airlines yield non-competitive levels of profits in this period. To see how this can be sustained consider the effects of an airline deviating from the equilibrium.

Other things equal, a reduction from the equilibrium level of, say, airline A's price would increase the share of travellers - both occasional and frequent - that this airline would attract in Period 1. However, this would imply that in Period 2, airline B would be facing a smaller mass of frequent travellers that would qualify for its discount. Accordingly, B would be willing to compete more aggressively in price, which would have negative effects on A's Period 2 profits. First, airline A would be unable to compete as aggressively as airline B, as it faces a greater mass of travellers qualifying for discounts and would therefore attract a lower share of travellers. Second, of the mass of frequent travellers that do address A in Period 2 a greater proportion of these will qualify for A 's discount thereby reducing A 's profits from its non-deviation level.

Figure 2.2 illustrates the previous discussion. The figure draws, for $\mu =$

0.5, the profit that airline A can expect at the start of each period as a function of the price it sets at the start of Period 1, given that its rival sets the equilibrium strategy. As shown in the graph for $\mu = 0.5$ and $c = 1$, the equilibrium discount is 0.75 and the first period price is 2.25. A deviation to a price below this level, but above 2, the benchmark price, allows airline A to collect a higher profit in the first period - the vertical distance between the two upper curves - though a smaller one in Period 2 - given by the distance between the two lower curves. The profit over the 3 periods, given by the upper curve, falls.

In sum, prices are sustained above the competitive benchmark level in Period 1 as it is in the airlines' interest to ensure that the rival attracts a sufficiently large market share in that period, so that the incentive to compete aggressively in the subsequent period is reduced.

Note that the ability to charge prices above $1 + c$ in the first period hinges on the coupon having a non-zero value. If this was not the case, *i.e.* $\alpha = \beta = 0$, a firm which deviates by undercutting its rival in the first period would go unpunished in Period 2 since it would not suffer from serving a higher mass of repeat buyers. It is in the interest of both airlines, therefore, to set non-zero coupons.

5 Discussion

In this section I will discuss the results obtained above. Particular attention is paid to the effect of population mix ie the proportion of travellers in a period which are frequent travellers, on the equlibrium prices, discounts and airline profits. Lastly, the impact of the FFP on the welfare of each type of traveller and on social welfare as a whole is considered.

5.1 Effect of population mix on equilibrium prices and discounts

Equilibrium prices are the same in both Period 2 and Period 3 and they are above the benchmark prices. This result can be understood by noting that in equilibrium, as far as an individual airline is concerned, the proportion of different types of travellers is the same in both periods. In particular, there are $1 - \mu$ occasional travellers, $\frac{\mu}{2}$ frequent travellers that qualify for that airline's discount and $\frac{\mu}{2}$ that do not qualify.

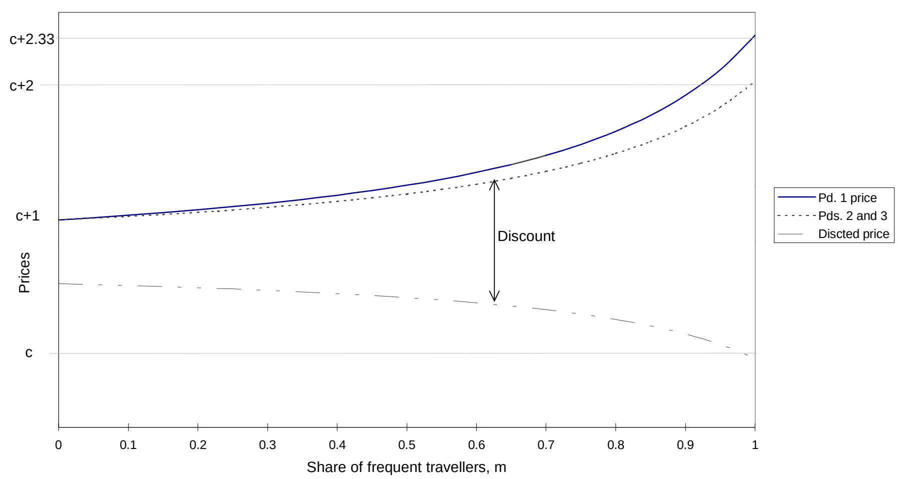

Figure 3: Equilibrium prices and discounts as a function of the share of frequent travellers.

It is immediate from Proposition 2.3 that the equilibrium price levels and the value of the discount are positively related to the share of frequent travellers in the market. This is graphed in Figure 2.3. Note that the curve describing the discounted price is not defined at $\mu = 0$ as the notion of a discount does not exist in such a setting. For this value of μ equilibrium prices are at their lowest and are equal to the benchmark level, $c+1$. On the other hand, when the entire mass of consumers are frequent travellers, $\mu = 1$, the equilibrium prices are at their highest. In Period 1, the price equals to $c+2\frac{1}{3}$ while in the last two period it is equal to $c+2$. The equilibrium discount level also reaches its maximum value, 2, when $\mu = 1$. Note that the effective price paid by a repeat buyer, base price minus discount is given by $c + \frac{7-7\mu}{13-10\mu}$ and is decreasing in μ.

As a percentage of the price mark-up, the discount is equal to $\frac{6}{13-7\mu}$ which is increasing in μ. For very low values of μ this percentage is below 50% but it rises with μ until it reaches 100% when $\mu = 1$. Hence, when the market is made up exclusively of frequent travellers, the effective price paid by a consumer benefiting from the discount is equal to an airline's marginal cost.

The intuition for the positive relation between μ with both equilibrium prices and coupon levels is the following. Low values of μ imply that, over the 3 periods, a greater proportion of purchases is carried out by occasional rather than by frequent travellers. Accordingly, airlines place a greater weight in competing for the former set of travellers. Given that the behaviour of an occasional traveller is not influenced by the discount offered, airlines can only compete for their business by offering low base prices. At the same time, airlines do not wish to offer higher discounts since these would raise base prices, as noted in Proposition 2.1,and hence drive away a share of the more numerous, by assumption, occasional travellers. On the other hand, when the share of frequent travellers is high, airlines are able to compete for their patronage through the generosity of their coupons and are less concerned with the adverse effect that higher prices have in the demand generated by the (small) group of occasional travellers.

5.2 Airline profits and travellers' welfare

The previous section noted that both equilibrium prices and discounts are positively related to the share of frequent travellers in the market. Other things equal, higher prices across the two airlines raise their profits and higher discounts lower them. The reverse is true with respect to travellers' welfare. What then is the net effect of μ on airline profits and travellers' welfare?

5.2.1 Travellers' welfare

When airlines are allowed to introduce $FFPs$, their equilibrium profits are above the benchmark level. It follows that the set of travellers, taken as a whole, are worse off when these loyalty schmes are introduced.[23] However, this does not imply that each individual fares badly from the introduction of the programmes. To see this consider the welfare of the two sets of travellers separately.

Occasional travellers are in the market for one period only. Given that they cannot receive a coupon and that the price paid by them is always above the benchmark level it follows that all occasional travellers are worse off if

[23] Provided the prices and discounts set by the two airlines are equal - so that the equilibrium is symmetric -the total surplus to be divided between travellers and airlines is equal to $3\left(R-c-\frac{1}{2}\right)$ and is independent of the prices and discounts offered. The constant level of total surplus follows from the assumption that travellers hold an inelastic demand for one unit of the good in each period.

the $FFPs$ are launched. Furthermore, given that prices increase with μ, the higher the share of frequent travellers the worse off will the occasional travellers be. This set of travellers will always prefer the benchmark scenario in which airlines are unable to launch $FFPs$.

To see the effect of the $FFPs$ on the welfare of a frequent traveller it is necessary to calculate his expected life-time utility. This expression is given by $3\left(R-c-\frac{1}{2}\right)-\frac{33-20\mu}{13-10\mu}$ which is a decreasing function of μ. It follows that a frequent traveller does better when he is one of the few frequent travellers. Compared to the expected lifetime surplus in the benchmark model, given by $3\left(R-c-\frac{1}{2}\right)-3$, it is quickly established that frequent travellers are better off in a market which offers $FFPs$ if and only if $\mu<\frac{6}{10}$. When the share of frequent travellers is above this level, the higher coupon value received from being a repeat buyer does not make up for the higher prices faced.

Corollary 2.2 summarizes the previous discussion.

Corollary 2.2. *Provided the set of frequent travellers is not empty, airlines are able to collect higher profits if they are allowed to launch $FFPs$. Introducing these programmes makes an occasional consumer necessarily worse off while frequent travellers benefit if and only if they account for less than $\frac{6}{10}$ of the passengers.*

5.2.2 Airline profits

In equilibrium, the airlines' profits over the 3 periods can be derived as,

$$\Pi_1^j=\frac{39-26\mu}{2\left(13-10\mu\right)},\ j=A,B. \tag{12}$$

and it follows directly from Proposition 2.3. Note that the profit earned over the three periods is increasing in μ. When $\mu=0$ profits are equal to the benchmark level of $\frac{3}{2}$ as one would expect. On the other hand, profits are at their highest, $\Pi_1^j=2\frac{1}{6}$, when the entire market is composed of frequent travellers.

The positive relation between μ and profits suggests that airlines would find it appealing to divide the market into two segments as described below in Corollary 2.3.

Corollary 2.3. *Consider the following policy.*

i) Split the market into two such that occasional travellers patronise one segment whilst frequent travellers patronise the other segment;

ii) Set equilibrium prices and discounts in each segment according to Proposition 2.3:

Segment	p_1^{J*}	p_2^{J*}	p_3^{J*}	$\alpha^* = \beta^*$
Occasional	$1+c$	$1+c$	$1+c$	$-$
Frequent	$2\frac{1}{3}+c$	$2+c$	$2+c$	2

In a symmetric equilibrium where both carriers adopt this policy, airline profits are given by $\frac{6+5\mu}{4}$. Airlines will find it attractive to split the market in the way described by the above policy.

Proof. By construction $\mu = 0$ n the segment of the market patronised by occasional travellers and $\mu = 1$ in the segment patronised entirely by frequent travellers. Given this, the equilibrium prices and discount set out in the Corollary follow immediately from Proposition 2.3. Using (12), the profits to an airline when it segments the market as described in the Corollary is given by, $\mu\frac{39-26(1)}{2(13-10(1))} + (1-\mu)\frac{3}{2} = \frac{9+4\mu}{6}$. To see that this profit level is higher than that which would be achieved if the market was not divided note that $\frac{9+4\mu}{6} = \frac{(9+4\mu)(13-10\mu)}{6(13-10\mu)} = \frac{117-38\mu-40\mu^2}{6(13-10\mu)} \geq \frac{117-78\mu}{6(13-10\mu)} = \frac{39-26\mu}{2(13-10\mu)}$ which is the profit level of the airlines when the market is not split. ■

Corollary 2.3 assumes that airlines are able to segment the market between frequent and occasional travellers. The menu of price and discounts laid out in the Corollary will not, by themselves, achieve such a segmentation - frequent travellers would prefer to pay the price charged occasional travellers and forefeit the chance of benefitting from a discount.[24] However, in the air transport industry, as indeed with passenger transportation in general, an imperfect segmentation is obtained through the offer of First/Business Class and Economy Class seats. The former tend to be occupied exclusively by frequent travellers who place greater value on comfor and flexibility whilst the latter are typically taken up by occasional passengers. Clearly, factors other than those related to FFP lead airlines to offer Business Class and

[24] In other words, the menu of prices and discounts are not a separating equilibrium.

Economy Class seats and to charge a higher price for the former. Nevertheless, Corollary 2.3 provides an addition reason why, in conjunction with launching a FFP, an airline find this segmentation profitable.

5.2.3 Social welfare

Given that consumers hold an inelastic demand for one unit of the good in each period, maximizing social welfare is tantamount to minimising total consumer transportation costs. In turn, the latter are minimised if, in each period, consumers address the airline closest to them: those located in the first half of I patronise A whilst those in the second half of the unit interval address airline B.

In the absence of $FFPs$ total transportation costs are minimized since consumers' optimal policy is to address the closest airline in each period. Introducing the $FFPs$ described by Proposition 2.3 does not alter this result. It can be checked that when the prices and coupons offered by the two airlines are equal, as they are in the symmetric equilibrium described above, then consumers will also patronise the closest airline in each period. Social welfare is, therefore, maximized.[25]

This result contrasts with that of Caminal and Matutes (1990). These authors report that the launch of $FFPs$ lead consumers to incur higher transportation costs than they would otherwise. Their result is driven by the fact that some consumers will be willing to travel further in order to address the same airline that they had done in the past and so be eligible for the coupon offered to repeat buyers. In the model presented here, on the other hand, consumers can always address the closest airline, minimising travelling costs and be sure that at some point in time - if not in the second period then in the third- they will be entitled to a discount. This result hinges on the assumption that airlines are symmetric and that travellers do not discount future gains.

To close this section, I should note - as do Caminal and Matutes (1990, p.361) - that in a more general model where consumers are endowed with an elastic demand function, the increase in the prices that results from the introduction of $FFPs$ will have negative welfare effects. In this light, the benchmark case which does not allows for discrimination between first-time and repeat buyers would be superior in terms of total social welfare.

[25] If the prices and discount offered by airlines are equal then it can be checked that $\Omega_j = \frac{1}{2}$ for $j = 1, 2, 3, 4, 5$.

To lock-in or not?[26]

The previous sections have argued that a FFP need not create switching costs to allows an airline to collect supra-competitive profits. Given this, one might expect that an airline can do even better if it implements a programme which *does* induce switching costs on travellers as this would add one other force driving airlines towards an outcome away from the competitive benchmark setting. In this section I ask whether such reasoning is valid.

To approach this question it is necessary to first recast the model of Section 2.2 so that it features $FFPs$ which induce switching-costs. The modified model is then solved and the equilibrium prices and profits of airlines compared with those that were derived in the original model.

There are two simple alternative ways of altering the model so that the $FFPs$ considered induce switching costs on travellers. Firstly, the generosity of the schemes may be tightened so that a discount is handed out to a traveller when he patronises the same airline for the third time. A second possibility is to reduce the time horizon of the model to two periods. Either modification would give rise to a setting where frequent travellers must patronise the same airline at every purchasing opportunity in order to benefit from the discount offered.

However, it should be noted that neither of these alternatives is ideal. In both cases, the *locking-in feature* is introduced at the cost of altering other aspects of the model. If the first route is followed, then the number of purchases required for a traveller to earn the discount increases from two to three. On the other hand, if the second suggestion is taken, the time horizon over which airlines compete is shortened to two periods. In either case, the structure of the model is altered. Therefore, these structural differences must be kept in mind when the results of the modified model are compared with those of the original one, as it would be wrong to attribute the differences in the outcome of the models entirely to the presence or absence of a *locking-in feature*.

The above problem cannot, however, be overcome. Introducing $FFPs$ which induce switching costs will necessarily alter other features of the model.[27]

[26] To be rigorous, the term *lock-in* should be replaced by *induce switching costs* since travellers' choices are never forcefully tied to their past actions. With this in mind, I will use in this section the term *lock-in* as it makes the exposition easier.

[27] A third alternative is to alter the original model in the following way. Let the population of frequent travellers be composed of three groups, -label them F, G, H - which take

Of the two possibilities described above, the second one is chosen - shortening the time horizon of the model to two periods - as it corresponds to the setting described in Banerjee and Summers (1987) and Caminal and Matutes (1990).

5.3 Modified model

The modified model differs from the one described in Section 2.2 due to the shortening of the time horizon from 3 to 2 periods. Hence, airlines set prices at the start of both periods while the coupon value is decided at the start of Period 1. The occasional travellers are in the market for only one of the periods while the frequent travellers participate in both. Lastly, note that - as before - a frequent traveller who patronises an airline for the second time receives that airline's discount.[28]

The steps involved in solving the modified model are analogous to those taken in solving the 3-period model which were presented in Section 2.3. To avoid repeating the presentation of similar reasoning most of the work involved in solving the model is left to the Appendix 2.B.

As before, the model is solved by working backwards. By solving the optimization problems of the two airlines in Period 2, their reaction functions can be derived. For airline A, this is given by,

$$p_2^{A'} = \frac{1}{2}\left(1 + c + p_2^{B'} + 2\mu\alpha'\sigma' + \mu\beta'\left(1 - \sigma'\right)\right)$$

where the notation is equivalent to the one used in Sections 2.2 and 2.3 and the suffix $'$ is used to denote the modified model. An analogous expression can be derived as B 's reaction function. Solving the two equations simultaneously yields the second period equilibrium prices,

$$\begin{aligned} p_2^{A'} &= 1 + c + \mu\alpha'\sigma' \\ p_2^{B'} &= 1 + c + \mu\beta'\left(1 - \sigma'\right) \end{aligned} \quad (13)$$

turns in participating in the market: in Period 1, groups F and G take part, in Period 2 groups G and H and in the last period groups F and H. This alternative has the merit of not altering the time horizon nor the generosity of the $FFPs$. This alternative was explored but it proved hard to derive any results from it.

[28] When all travellers are frequent travellers, $\mu = 1$, this model is identical to one of the models examined by Caminal and Matutes (1990).

This result points out that an airline's second period price is above the benchmark level and increases both with the size of the discounts set in the first period and with the mass of travellers eligible to receive the discount in Period 2 - given by the product of μ and the airline's share of the demand generated by frequent travellers in Period 1.

Using the above equilibrium prices, it is instructive to construct the expressions which describes the airlines' profits in the second period. For airline A, this will be given by,

$$\Pi_2^{A'} = \frac{1}{2} \left(1 + \alpha' \mu \sigma' \left(\mu \sigma' - 1\right) \left(1 + \beta \mu\right)\right) \tag{14}$$

and a similar expression holds for airline B. Given that $\mu\sigma' \leq 1$, it follows that $\Pi_2^{A'} \leq \frac{1}{2}$, which is competitive benchmark level..

Now consider the airlines' problems in Period 1. Here, the airlines' objective is to maximise the sum of its first period profits and those expected in the second period - given by (14)for airline A. The equilibrium prices and disounts will be given by solving the reaction function of each airline simultaneously. It is shown in Appendix 2.A, that equilibrium prices and discounts ds exist though it is not possible to derive an explicit expression for them. Instead, for a given value of the parameter μ their values can be worked out numerically. Figure 2.4 summarizes the results by plotting the base prices as well as the discounted price paid by repeat buyers in Period 2. Note that the curve drawing the discounted price is not defined at $\mu = 0$.

It is clear from Figure 2.4 that equilibrium prices increase with the share of frequent travellers and that they are above the competitive level, $c + 1$, for all $\mu > 0$. The discount offered to repeat buyers also increases with μ although only just slightly. For $\mu = \varepsilon$ - a share just above 0 - the discount is equal to 0.65 while for $\mu = 1$, its value is 0.67. Other readings of the results summarized in Figure 2.4, and an interpretation of them, are left to the next section where a comparison with the results derived in Section 2.3 for the three-period model is carried out.

5.4 Comparing the two models

In order to contrast the outcome of the two models more easily and given that the symmetric equilibrium of the modified model can only be characterized numerically, it is appropriate to compare the prices, discounts and profits resulting from the two models for a given value of μ.

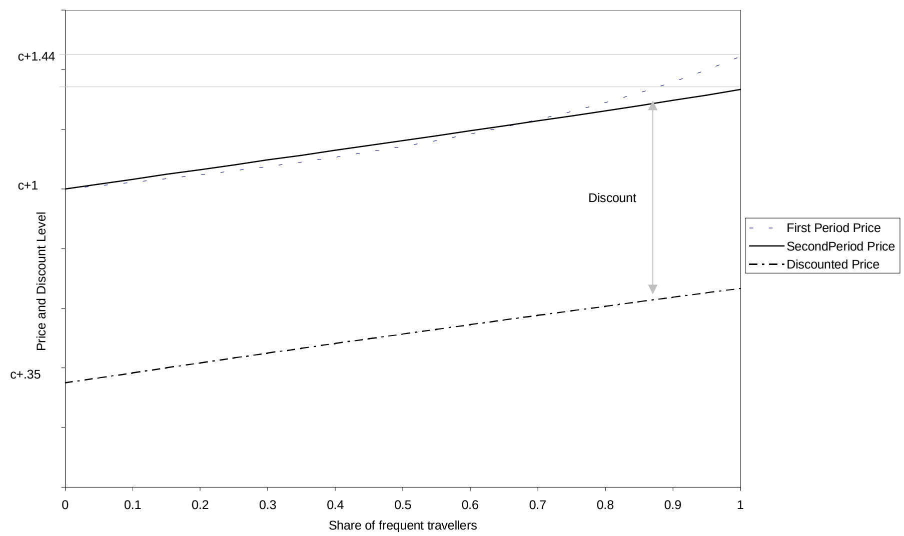

Figure 4: Two-period model equilibrium prices as a function of the share of frequent travellers.

Table 2.1 below characterizes the symmetric equilibrium of the games described in the two models for $\mu = 1$. The figures presented in the table would be different if other values of μ had been chosen, although the qualitative comparison which follows would still be valid. The first three columns report the equilibrium mark-up of the airlines in each of the periods. The absolute value of the discount as well as the percentage of the mark-up which it represents are given in the subsequent two columns. The remaining four columns show the equilibrium profits earned by the airlines over the time horizon of the model and the average profit per period. To make the following discussion simpler, the original three-period model is referred to as Model I while the modified, two-period model will be labelled as Model II.

Table 2.1: Comparison of the three models for $\mu = 1$.

Model	Price mark-up			Discount		Profit per Period			
	Pd. 1	Pd. 2	Pd. 3	Abs.	%	Pd. 1	Pd. 2	Pd. 3	Avg.
Benchmark	1	1	1	–	–	0.50	0.50	0.50	0.50
Model I	2.33	2	2	2	100	1.17	0.50	0.50	0.72
Model II	1.44	1.33	–	0.67	50	0.72	0.39	–	0.56

As had been noted earlier, the second period price in Model II is above the benchmark level. The intuition for this was described in Section 1 and it stems from the airlines' ability to exploit the switching costs which their $FFPs$ induce on travellers.

The second period price in Model II is considerably lower than the price level in Period 2 and 3 in Model I. Thisdifference reflects the more generous discounts offered in Model I. Give equal discounts, both settings would give rise to equal second period prices.

To see how airlines are able to sustain higher equilibrium prices in the first period in Model I than in Model II, recall equation (14) describing the second period profits of airline A in Model II,

$$\Pi_2^{A'} = \frac{1}{2}\left(1 + \alpha'\mu\sigma'\left(\mu\sigma' - 1\right)\left(1 + \beta\mu\right)\right) \tag{15}$$

In the symmetric equilibrium $\sigma' = \frac{1}{2}$. Any deviation in the first period prices away from the symmetric equilibrium will shift σ' away from $\frac{1}{2}$ and, given (14) , will increase airline A 's second period profits. If the symmetric equilibrium is to be sustained it is necessary that a downward deviation price from the equilibrium level harms an airline's first-period profits. In other words, the symmetric equilibrium prices must be such that the increase in the first-period market share of such a deviant does not make up for the lower price charged. In turn, this implies that prices cannot be sustained at high values.

On the other hand, as was discussed in Section 2.3.4, in Model I,the temptation of an airline to cut its first-period price from the equilibrium value is checked by the negative effects that this has on its second-period profits. This allows for higher prices to be sustained in Period 1 than those of Model II.

The figures provided in Table 2.1 show that average profits per period are higher in Model I than in Model II for $\mu = 1$. This is in fact a general result as be read from Figure 2.5 which contrasts the average profit per period earned by an airline in the benchmark case, under Model I and under Model II. The relative ranking of the models in terms of average profit per period is largely accounted by the fact that airlines are able to sustain higher prices in Model I than in Model II. In addition in Model II, a large proportion, $\frac{5}{6}$, of the frequent travellers addressing an airline in Period 2 are repeat buyers

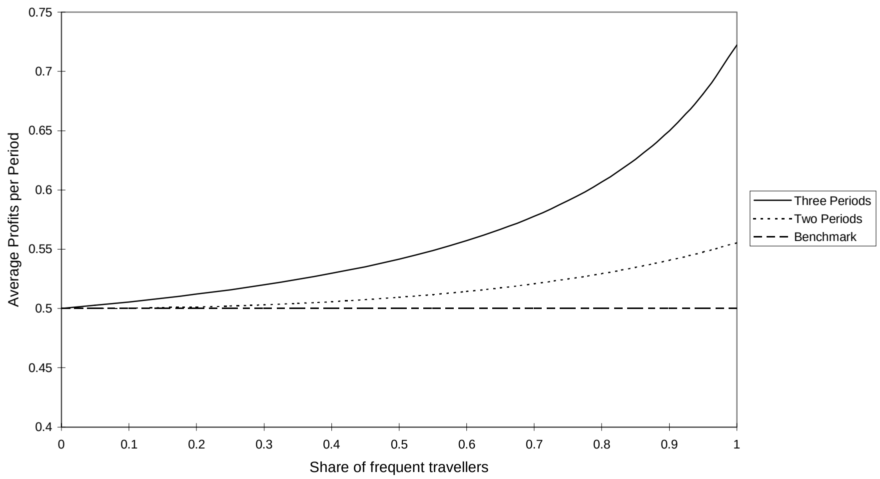

Figure 5: Average profit per period in three settings: the two-, the three-period and the benchmark model.

who qualify for a discount and accordingly, pay a low effective price.[29] On the other hand, in Model I, in each of the Periods 2 and 3, only half of the frequent travellers qualify for the discount and the revenue foregone due to these discounts is made up by those paying the high full price. The net result is that, both in Period 2 and in Period 3, airlines' profits do not fall below the benchmark level as occurs in the Period 2 of Model II.

Before closing this section, it is appropriate to consider a setting where airlines compete over three periods, as in Model I, and frequent travellers qualify for a discount only if they patronise the same airline throughout. This was an alternative modification discussed at the start of this section and it reflects the setting discussed briefly in Banerjee and Summers (1997). While the formal treatement of this model proved unworkable, it is possible, through intuition, to characterise the outcome in such a setting. It is in the interest of an airline to ensure that the rival attracts a mass of frequent travellers that will qualify for a discount in the third period. If this were not the case, the airline with no loyal travelllers would have the incentive to

[29]The share of repeat buyers in Period 2 is given by the sum of the terms $\Lambda_1 + (1 - \Lambda_2)$ which are defined in Appendix 2.B For $\mu = 1$, this sum is equal to $\frac{5}{6}$.

undercut and prices would be driven to the benchmark level. Compared to the two period setting of Model II, price competition would be less aggressive and lower discounts would be necessary to sustain the cooperation between the airlines. This intuition points to the result that the airlines fare better when the number of periods during which customers must be loyal increases. It does not, however, establish a comparison with the profit level earned in the setting described in the original model described in Section 2.4 where the FFP induced no switching costs.

So how can the heading of Section 2.5 be answered? Are airlines better off with a FFP which imposes switching-costs on its travellers or not? The above discussion does not allow this question to be answered. However, one - trivial - point is apparent: the structure of the $FFPs$ plays a significant impact on the profitability of the scheme.

6 Conclusion

This chapter aimed at exploring some issues surrounding customer loyalty schemes. Its main concern was to examine whether loyalty-schemes need to induce switching costs on travellers in order to have a *raison d'être.* The answer is in the negative. It was shown that even when $FFPs$ do not induce such costs, they are a tool which facilitate tacit-collusion amongst airlines.

The composition of the population was seen to influence equilibrium prices and the level of the coupons awarded to repeat buyers. Both increased with the share of frequent travellers in the population.

The analysis also pointed out that, typically, travellers have little to be enthusiast about $FFPs$. Those who participate in the market rarely, and hence cannot hope to benefit from any coupon, lose out due to the high prices. On the other hand, for frequent travellers to benefit from the launch of a FFP it is necessary that the share of travellers which they account for is not very high so that the coupon received offsets the higher prices practised.

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Appendices

2.A Solving the Three Period Model

The proof is divided into two parts. Part 1 sets out the candidate symmetric equilibrium. Part 2 then shows that this candidate equilibrium is robust to deviations and so does indeed form an equilibrium.

In the process of going through the solution to the model Propositions 2.2 and 2.3 and Corollary 2.1 will be proved.

2.A.1 Part 1 - Finding the candidate symmetric equilibrium

The optimal decision rules of travellers and airlines are traced backwards in time, as presented in the main body of the text.

Period 3 Recall that the equilibrium prices in Period 3 were shown in Section 2.4.1 to be,

$$\begin{aligned} p_3^{A*} &= 1 + c + \alpha\mu s \\ p_3^{B*} &= 1 + c + \beta\mu s \end{aligned}$$

This was proven in the main text and is not repeated here.

Given the third period equilibrium prices, it will be useful to re-write the terms Ω_1 and Ω_2 as,

$$\begin{aligned} \Omega_1 &= \frac{1 + p_3^{B*} - p_3^{A*}}{2} = \frac{1 + \mu s\,(\beta - \alpha)}{2} \\ \Omega_2 &= \frac{1 + p_3^{B*} - \beta - p_3^{A*} + \alpha}{2} = \frac{1 + (\mu s - 1)\,(\beta - \alpha)}{2} \end{aligned} \tag{16}$$

Period 2 Section 2.4.2 defined the terms Ω_3 and Ω_4 as,

$$\begin{aligned} \Omega_3 &= \frac{1 + p_2^B - p_2^A + \alpha + \Omega_1^2 - \Omega_2^2 - \beta}{2} \\ \Omega_4 &= \frac{1 + p_2^B - p_2^A + \Omega_1^2 - \Omega_2^2}{2} \end{aligned} \tag{17}$$

These terms will be useful for later.

Following the presentation in Section 2.4.2, the maximization problems of the two airlines at the start of Period 2 are given by,

$$
\begin{aligned}
\underset{\{p_2^A\}}{Max}\Pi_2^A &= \left(p_2^A - c\right)(1-\mu)\left(\frac{1+p_2^B-p_2^A}{2}\right) + \left(p_2^A - c\right)\mu(1-\sigma)\Omega_4 + \\
&\quad \left(p_2^A - \alpha - c\right)\mu\sigma\Omega_3 + \Pi_3^A
\end{aligned}
$$

$$
\begin{aligned}
\underset{\{p_2^B\}}{Max}\Pi_2^B &= \left(p_2^B - c\right)(1-\mu)\left(\frac{1-p_2^B+p_2^A}{2}\right) + \left(p_2^B - c\right)\mu\sigma(1-\Omega_4) + \\
&\quad \left(p_2^B - \beta - c\right)\mu(1-\sigma)(1-\Omega_3) + \Pi_3^B
\end{aligned}
$$

The second period equilibrium prices are found by solving the two problems simultaneously. By solving the two first order conditions, it can be shown that the reaction functions of the two airlines are given by the following algebraically cumbersome expressions:

$$
\begin{aligned}
p_2^A &= \frac{1}{D}\left(A + Bc + Cp_2^B\right) \\
p_2^B &= \frac{1}{G}\left(E + Bc + Fp_2^A\right)
\end{aligned}
\tag{18}
$$

where,

$$
\begin{aligned}
A = 2\sigma \quad & (\sigma-1)\,\mu^3\left((\alpha-\beta)^5+\alpha\,(\alpha-\beta)^4\right)+8\sigma^2\mu^2\left((\alpha-\beta)^3+3\alpha\,(\alpha-\beta)^2\right)+\\
& 2\sigma\begin{pmatrix} (\mu^2-\mu^3)\,(\alpha-\beta)^4-4\mu^2\,(\beta-\alpha)\,(\beta-3\alpha)+\\ 2\mu^2\left(2\,(\beta-\alpha)^3-7\alpha\,(\alpha-\beta)^2\right)+4\mu\,((\beta-\alpha)\,(\beta-3\alpha)-2\alpha)\end{pmatrix}+\\
& (\alpha-\beta)^4\left(\mu^3-2\mu^2\right)+2\mu^2\left((\beta-\alpha)\left(\beta^2-\alpha^2\right)+2\,(\beta-\alpha)\,(\beta-3\alpha)\right)+\\
& 4\mu\,(2\,(\beta-\alpha)-(\alpha-\beta)\,(5\alpha-3\beta))-16\\
B = 4\sigma \quad & (\sigma-1)\,\mu^2\left(\mu\,(\alpha-\beta)^4+4\,(\alpha-\beta)^2\right)+(\alpha-\beta)^4\left(\mu^3-\mu^2\right)+\\
& 4\mu\,(\mu-2)\,(\alpha-\beta)^2-16\\
C = 4\sigma \quad & (\sigma-1)\,\mu^2\left(\mu\,(\alpha-\beta)^4+4\,(\beta-3\alpha)\,(\beta-\alpha)\right)+(\alpha-\beta)^4\left(\mu^3-\mu^2\right)+\\
& 4\mu\,(\mu-2)\,(\alpha-\beta)^2-16\\
D = 8\sigma \quad & (\sigma-1)\,\mu^2\left(\mu\,(\alpha-\beta)^4+4\,(2\alpha-\beta)\,(\alpha-\beta)\right)+2\,(\alpha-\beta)^4\left(\mu^3-\mu^2\right)+\\
& 8\mu^2\,(\beta-2\alpha)\,(\alpha-\beta)-16\mu\,(\alpha-\beta)^2-32\\
E = 2\sigma \quad & (\sigma-1)\,\mu^3\left((\beta-\alpha)^5+\beta\,(\alpha-\beta)^4\right)+8\sigma^2\mu^2\left((\beta-\alpha)^3+3\beta\,(\alpha-\beta)^2\right)+\\
& 2\sigma\left(\mu^3-\mu^2\right)(\alpha-\beta)^4+2\sigma\mu^2\left(4\,(\alpha-3\beta)\,(\alpha-\beta)+4\,(\alpha-\beta)^3\right)-\\
& 2\sigma\mu^2\left(10\,(\alpha-\beta)^2-4\mu\,((\alpha-3\beta)\,(\alpha-\beta)-2\beta)\right)-\\
& \mu^3\,(\alpha-\beta)^4+2\mu^2\left((\alpha-\beta)^3-2\,(3\beta-\alpha)\,(\beta-\alpha)\right)+\\
& 4\mu\left(2\alpha-\alpha^2+\beta^2-6\beta\right)-16\\
F = 4\sigma \quad & (\sigma-1)\,\mu^2\left(\mu\,(\alpha-\beta)^4+4\,(\alpha-3\beta)\,(\alpha-\beta)\right)+(\alpha-\beta)^4\left(\mu^3-\mu^2\right)+\\
& 4\mu^2\,(\alpha-3\beta)\,(\alpha-\beta)-8\mu\,(\alpha-\beta)^2-16\\
G = 8\sigma \quad & (\sigma-1)\,\mu^2\left(\mu\,(\alpha-\beta)^4+4\,(2\beta-\alpha)\,(\beta-\alpha)\right)+2\,(\alpha-\beta)^4\left(\mu^3-\mu^2\right)+\\
& 8\mu^2\,(\alpha-2\beta)\,(\alpha-\beta)-16\mu\,(\alpha-\beta)^2-32
\end{aligned}
$$

The equilibrium prices are found by solving the system of equations given by the two reaction functions, (18). It follows that the second period equiliobrium prices are given by,

$$p_2^{A*}=\frac{AG+CE+(C+G)\,Bc}{DG-FC},\qquad p_2^{B*}=\frac{DE+FA+(F+D)\,Bc}{DG-FC}\tag{19}$$

It was checked that at this equilibrium point the necessary second-order conditions are met.

The expressions for p_2^{A*} and p_2^{B*} in terms of the model's parameters are too unwieldy to provide any insight. However, two special cases can be considered.

First, when $\alpha = \beta = \lambda$ it can be shown that $A = -16\sigma\mu\lambda - 16$, $B = C = F = -16$, $D = G = -32$ and $E = 16\sigma\mu\lambda - 16\mu\lambda - 16$. Substituting these terms into (19), the equilibrium second prices simplify to $p_2^{A*} = 1 + c + \mu\lambda\frac{(1+\sigma)}{3}$ and $p_2^{B*} = 1 + c + \mu\lambda\frac{(2-\sigma)}{3}$.

Second, when $\sigma = 0.5$ it can be shown that $A =$, $B =$, $C =$, $D =$, $E =$, $F =$ and $G =$. Substituting these terms into (19), the equilibrium second prices simplify to $p_2^{A*} = 1 + c + \frac{\mu\alpha}{2}$ and $p_2^{B*} = 1 + c + \frac{\mu\beta}{2}$.

This proves Corollary 2.1.

Period 1 The maximization problem facing airline A at the start of Period 1 is given by,

$$\begin{aligned}
\underset{\{p_1^A,\alpha\}}{Max}\, \Pi_1^A &= \left(p_1^A - c\right)\left(\mu\sigma + (1-\mu)\left(\frac{1+p_1^B-p_1^A}{2}\right)\right) + \Pi_2^A \\
&= \left(p_1^A - c\right)\left(\mu\sigma + (1-\mu)\left(\frac{1+p_1^B-p_1^A}{2}\right)\right) + \\
&\quad \left(p_2^A - c\right)\left(\mu(1-\sigma)\Omega_4 + (1-\mu)\left(\frac{1+p_2^B-p_2^A}{2}\right)\right) + \\
&\quad \left(p_2^A - \alpha - c\right)\mu\sigma\Omega_3 + \\
&\quad \left(p_3^A - c\right)\left(\mu(1-s)\Omega_1 + (1-\mu)\left(\frac{1+p_3^B-p_3^A}{2}\right)\right) + \\
&\quad \left(p_3^A - \alpha - c\right)\mu s\Omega_2
\end{aligned}$$

An analogous expression can be written for airline's B maximization problem. However, given that the I will be searching for a symmetric equilibrium it will be sufficient to work with (20).

The two first order conditions of A 's optimisation problem are given by,

$$
\begin{aligned}
\frac{d\Pi_1^A}{dp_1^A} &= \mu\sigma + (1-\mu)\left(\frac{1+p_1^B - p_1^A}{2}\right) + \left(p_1^A - c\right)\left(\mu\frac{d\sigma}{dp_1^A} - \frac{(1-\mu)}{2}\right) + \\
&\left(\mu(1-\sigma)\Omega_4 + (1-\mu)\left(\frac{1+p_2^B - p_2^A}{2}\right)\right)\frac{dp_2^A}{dp_1^A} + \mu\sigma\Omega_3\frac{dp_2^A}{dp_1^A} \\
&\left(p_2^A - c\right)\left(-\mu\Omega_4\frac{d\sigma}{dp_1^A} + \mu(1-\sigma)\frac{d\Omega_4}{dp_1^A}\right) + \mu\left(p_2^A - \alpha - c\right)\left(\sigma\frac{d\Omega_3}{dp_1^A} + \Omega_3\frac{d\sigma}{dp_1^A}\right) + \\
&\left(p_2^A - c\right)\left(\frac{1-\mu}{2}\right)\left(\frac{dp_2^B}{dp_1^A} - \frac{dp_2^A}{dp_1^A}\right) + \left(p_3^A - c\right)\left(-\mu\Omega_1\frac{ds}{dp_1^A} + \mu s\frac{d\Omega_1}{dp_1^A}\right) + \\
&\left(\mu(1-s)\Omega_1 + (1-\mu)\left(\frac{1+p_3^B - p_3^A}{2}\right)\right)\frac{dp_3^A}{dp_1^A} + \mu s\Omega_2\frac{dp_3^A}{dp_1^A} + \\
&\left(p_3^A - c\right)\left(\frac{1-\mu}{2}\right)\left(\frac{dp_3^B}{dp_1^A} - \frac{dp_3^A}{dp_1^A}\right) + \left(p_3^A - \alpha - c\right)\left(\mu\Omega_2\frac{ds}{dp_1^A} + \mu s\frac{d\Omega_2}{dp_1^A}\right)
\end{aligned}
$$

$$
\begin{aligned}
\frac{d\Pi_1^A}{d\alpha} &= \left(p_1^A - c\right)\mu\frac{d\sigma}{d\alpha} + \left((1-\sigma)\Omega_4\mu + (1-\mu)\left(\frac{1+p_2^B - p_2^A}{2}\right)\right)\frac{dp_2^A}{d\alpha} + \\
&\left(p_2^A - c\right)\left(-\Omega_4\mu\frac{d\sigma}{d\alpha} + (1-\sigma)\mu\frac{d\Omega_4}{d\alpha} + \frac{(1-\mu)}{2}\left(\frac{dp_2^B}{d\alpha} - \frac{dp_2^A}{d\alpha}\right)\right) + \\
&\sigma\Omega_3\mu\left(\frac{dp_2^A}{d\alpha} - 1\right) + \left(p_2^A - \alpha - c\right)\mu\left(\Omega_3\frac{d\sigma}{d\alpha} + \sigma\frac{d\Omega_3}{d\alpha}\right) + \\
&\left((1-s)\Omega_1\mu + (1-\mu)\left(\frac{1+p_3^B - p_3^A}{2}\right)\right)\frac{dp_3^A}{d\alpha} + \\
&\left(p_3^A - c\right)\mu\left(-\Omega_1\frac{ds}{d\alpha} + (1-s)\frac{d\Omega_1}{d\alpha}\right) + \left(\frac{dp_3^A}{d\alpha} - 1\right)s\Omega_2\mu + \\
&\left(p_3^A - c\right)\left(\frac{1-\mu}{2}\right)\left(\frac{dp_3^B}{d\alpha} - \frac{dp_3^A}{d\alpha}\right) + \left(p_3^A - \alpha - c\right)\mu\left(\Omega_2\frac{ds}{d\alpha} + s\frac{d\Omega_2}{d\alpha}\right)
\end{aligned}
$$

I now set out to evaluate the derivatives of various terms with respect to the two choice variable which build up the two first order conditions

To derive $\frac{d\sigma}{dp_1^A}$ and $\frac{d\sigma}{d\alpha}$ recall the implicit equation (10) described in Section 2.4.3 ,

$$
J \equiv \frac{1 + p_1^B - p_1^A + \Omega_3^2 - \Omega_4^2 + \Omega_2^2 - \Omega_1^2}{2} - \sigma \tag{20}
$$

where the terms $\Omega_1, \Omega_2, \Omega_3$ and Ω_4 are functions of σ, p_1^A and α. Based on (20) , it is possible to use the implicit function theorem to calculate $\frac{d\sigma}{dp_1^A}$ and $\frac{d\sigma}{d\alpha}$. The partial derivatives of J - after imposing the symmetry conditions, *i.e.* $\sigma = 0.5$, $\alpha = \beta$, $p_1^A = p_1^B$ - are given by $\frac{\partial J}{\partial p_1^A} = -\frac{1}{2}, \frac{\partial J}{\partial \alpha} = \frac{1}{4}$ and $\frac{\partial J}{\partial \sigma} = -1$. Therefore, given the symmetry,

$$\begin{aligned}\frac{d\sigma}{dp_1^A} &= -\frac{\partial J}{\partial p_1^A} \div \frac{\partial J}{\partial \sigma} = -\frac{1}{2} \\ \frac{d\sigma}{d\alpha} &= -\frac{\partial J}{\partial \alpha} \div \frac{\partial J}{\partial \sigma} = \frac{1}{4}\end{aligned}$$

To derive the expressions for $\frac{dp_2^A}{dp_1^A}, \frac{dp_2^B}{dp_1^A}$ note that $\frac{dp_2^A}{dp_1^A} = \frac{\partial p_2^A}{\partial p_1^A} + \frac{d\sigma}{dp_1^A}\frac{\partial p_2^A}{\partial \sigma}$ and $\frac{dp_2^B}{dp_1^A} = \frac{\partial p_2^B}{\partial p_1^A} + \frac{d\sigma}{dp_1^A}\frac{\partial p_2^B}{\partial \sigma}$. From (19) it follows that $\frac{\partial p_2^A}{\partial p_1^A} = \frac{\partial p_2^B}{\partial p_1^A} = 0$.

Obtaining the expressions for $\frac{\partial p_2^A}{\partial \sigma}$ and $\frac{\partial p_2^B}{\partial \sigma}$, requires a bit more work as it requires to look at the terms A, B, C, D, E, F and G which define p_2^A and p_2^B. Under symmetry $\frac{\partial A}{\partial p_1^A} = -16\mu\alpha$, $\frac{\partial E}{\partial p_1^A} = 16\mu\alpha$ and $\frac{\partial B}{\partial p_1^A} = \frac{\partial C}{\partial p_1^A} = \frac{\partial D}{\partial p_1^A} = \frac{\partial F}{\partial p_1^A} = \frac{\partial G}{\partial p_1^A} = 0$. In addition it is straight-forward to calculate that under symmetry, $A = E = -8\mu\alpha - 16$, $B = C = F = -16$ and $D = G = -32$. It then follows from (19) that $\frac{\partial p_2^A}{\partial p_1^A} = \frac{\left(G\frac{\partial A}{\partial p_1^A} + C\frac{\partial E}{\partial p_1^A}\right)}{(DG - FC)} = \frac{\mu\alpha}{3}$ and that $\frac{\partial p_2^B}{\partial p_1^A} = \frac{\left(D\frac{\partial E}{\partial p_1^A} + F\frac{\partial A}{\partial p_1^A}\right)}{(DG - FC)} = -\frac{\mu\alpha}{3}$.

Putting these previous results together, it follows that,

$$\begin{aligned}\frac{dp_2^A}{dp_1^A} &= -\frac{\mu\alpha}{6} \\ \frac{dp_2^B}{dp_1^A} &= \frac{\mu\alpha}{6}\end{aligned}$$

An analogous procedre can be followed to obtain the following expressions for $\frac{dp_2^A}{d\alpha}$ and $\frac{dp_2^B}{d\alpha}$,

$$\begin{aligned}\frac{dp_2^A}{d\alpha} &= \frac{\partial p_2^A}{\partial \alpha} + \frac{d\sigma}{d\alpha}\frac{\partial p_2^A}{\partial \alpha} = \frac{\mu}{2} + \frac{1}{4}\frac{\mu\alpha}{3} \\ &= \frac{\mu}{2} + \frac{\mu\alpha}{12} \\ \frac{dp_2^B}{d\alpha} &= \frac{\partial p_2^B}{\partial \alpha} + \frac{d\sigma}{d\alpha}\frac{\partial p_2^B}{\partial \alpha} = 0 + \frac{1}{4}\frac{-\mu\alpha}{3} \\ &= \frac{-\mu\alpha}{12}\end{aligned}$$

After imposing the symmetry conditions it can be shown that $\frac{d\Omega 1}{dp_1^A} = \frac{d\Omega_2}{dp_1^A} = 0$, $\frac{d\Omega_3}{dp_1^A} = \frac{d\Omega_4}{dp_1^A} = \frac{\mu\alpha}{6}$ and $\frac{dp_3^A}{dp_1^A} = \frac{dp_3^B}{dp_1^A} = \frac{ds}{dp_1^A} = 0$. Similarly, it can be shown that $\frac{d\Omega_1}{d\alpha} = -\frac{\mu}{4}$, $\frac{d\Omega_2}{d\alpha} = \frac{2-\mu}{4}$, $\frac{d\Omega_3}{d\alpha} = \frac{d\Omega_4}{d\alpha} = \frac{1}{4} - \frac{\mu}{4} - \frac{\mu\alpha}{12}$, $\frac{dp_3^A}{d\alpha} = \frac{\mu}{2}$,$\frac{dp_3^B}{d\alpha} = 0$ and $\frac{ds}{d\alpha} = 0$. Lastly, it is easy to see that under symmetry $\Omega_i = \frac{1}{2}$ $i = 1, 2, 3, 4$ and $s = \frac{1}{2}$

Substituting the various expressions obtained into the two first-order conditions yields after some simplifcation the following system of equations,

$$\begin{aligned}\frac{d\Pi_1^A}{dp_1^A} &= \frac{1}{2} - \frac{p_1^A}{2} + \frac{c}{2} + \frac{\mu\alpha}{3} \\ \frac{d\Pi_1^A}{d\alpha} &= \frac{\left(p_1^A - c\right)\mu}{4} - \frac{13\mu\alpha}{24} + \frac{\mu^2\alpha}{4}\end{aligned} \tag{21}$$

The equilibrium price and coupon level is obtained by equating the two conditions to 0 and solving the system. Carrying this out yields, for $\mu \neq 0$,

$$\begin{aligned}p_1^{A*} &= 1 + c + \frac{4\mu}{13 - 10\mu} \\ \alpha^* &= \frac{6}{13 - 10\mu}\end{aligned} \tag{22}$$

Given the symmetry, it follows $p_1^{B*} = 1 + c + \frac{4\mu}{13-10\mu}$ and $\beta^* = \frac{6}{13-10\mu}$. Substituting these values into the previously derived expressions for the equilibrium prices in Period 2 and in Period 3, gives the equilibrium prices described in Proposition 2.3.

The second-order have been seen to be satisfied locally. It is necessary to check that the candidate symmetric equilibrium described in (22) is robust to deviations. This is set out below, in Part 2 of the proof.

2.A.2 Part 2 - Robustness of candidate equilibrium

With no loss of generality, it is assumed that the deviating airline in Period 1 is A whilst airline B selects the candidate equilibrium's price and discount, $\beta^* = \frac{6}{13-10\mu}$ and $p_1^{B*} = 1 + c + \frac{4\mu}{13-10\mu}$.

In Part 1 of this proof, it was implicitly assumed that the terms Ω_j, $j = 1, ..., 5$ lied within the unit interval. At the proposed equilibrium point, such conditions are indeed met. However, when testing for the robustness of the candidate equilibrium, it is necessary to consider the possibility that following A 's initial deviation, the optimal behaviour of airlines in subsequent periods will be such that, Ω_j lies outside the unit interval. This alters the functional form of the airlines' profit function and therefore requires that the reaction function be re-examined.

Following on from this consideration, the rest of this section is structured as follows. First, it is shown numerically that there is no profitable first-period deviation by airline A provided airlines' choices of prices and discount are such that $0 \leq \Omega_j \leq 1$, $j = 1, ..., 5$ .This restriction is then set aside at the cost, however, of confining the space of A 's potential deviations to those where $\alpha = 0$. As before, it will be shown that under the new equilibrium prices following A 's deviation, airline A earns a lower profit level than if it followed the strategies described by the (candidate) symmetric equilibrium.

2.A.2.1 Restrict Ω_j to the unit interval, $j = 1, 2..., 5$. When the choice of A 's first period deviation and the choice of ensuing equilibrium prices in Periods 2 and 3 are limited such that $0 \leq \Omega_j \leq 1$, then the airlines' profit functions described above are correct. It follows, that it is then possible to use the reaction functions derived above to establish the equilibrium prices in the last two periods for a given deviation by A. Due to the cumbersome expressions involved I carried out this task numerically using the following algorithm:

1. Let $\mu = 0$.

2. In 0.1 fine grid, consider all pairs of α and p_1^A, and for each pair use the expressions derived in Part 1 of the proof to work out the equilibrium prices in Periods 2 and 3.

3. If the equilibrium prices derived are such that any of the terms Ω_j, $j = 1, 2, ..., 5$ lie outside the unit interval, then discard the relevant

pair $\{\alpha, p_1^A\}$ as a possible deviation.

4. For each admissible deviation, use the respective equilibrium prices to construct the expected profit of airline A over the 3 period and check whether this is higher or lower than the expected profits earned at the candidate symmetric equilibrium.

5. Let $\mu = \mu + 0.05$.

6. Repeat steps 2 - 5, until $\mu = 1$.

The results from this programme show that there is no admissible deviation, where admissible has the peculiar definition described above, from the candidate symmetric equilibrium which is profitable to airline A.

2.A.2.2 Allowing $\Omega_j, j = 1, 2, ..., 5$ to lie outside the unit interval

As mentioned previously the space of A 's deviations will be restricted to those where it chooses to offer no discount, *ie* $\alpha = 0$..

To derive the new equilibrium strategies following A 's deviation in the first period, it is necessary to work backwards from Period 3.

Period 3 Given that $\alpha = 0$, the objective functions of the two airlines in Period 3 can be written as,

$$\begin{aligned} \Pi_3^A &= \left(p_3^A - c\right)(1 - \mu s)\,\widehat{\Omega}_1 + \mu s \widehat{\Omega}_2 \\ \Pi_3^B &= \left(p_3^B - c\right)(1 - \mu s)\left(1 - \widehat{\Omega}_1\right) + \left(p_3^B - \beta^* - c\right)\mu s \widehat{\Omega}_2 \end{aligned} \tag{23}$$

where,

$$\widehat{\Omega}_i = \begin{cases} 0 & \text{if } \Upsilon_i < 0 \\ \Upsilon_i & \text{if } \Upsilon_i \in [0,1] \\ 1 & \text{if } \Upsilon_i > 1 \end{cases}, i = 1, 2 \text{ where } \Upsilon_1 = \frac{1 + p_3^B - p_3^A}{2}, \Upsilon_2 = \frac{1 + p_3^B - p_3^A - \beta^*}{2}. \tag{24}$$

The objective functions of the two airlines are piece-wise functions in third period prices. This complicates matters as it becomes necessary to consider $3 * 3 = 9$ cases and solve the maximization problem of each airline. Note, however, that $\widehat{\Omega}_2 \leq \widehat{\Omega}_1$ so that the cases to be considered are reduced to 6 as tabled below,

	Range $\widehat{\Omega}_1$	Range $\widehat{\Omega}_2$
Case A_3	0	0
Case B_3	$\in I$	0
Case C_3	1	0
Case D_3	$\in I$	$\in I$
Case E_3	1	$\in I$
Case F_3	1	1

(25)

The Period 3 equilibrium prices are found by carrying out the following steps.

First, calculate for each of the six cases the *constrained* reaction functions of the two airlines. The constrained reaction function gives an airline's best response to its rival's price conditional on the values of $\widehat{\Omega}_1$ and $\widehat{\Omega}_2$ remaining within the range defining the relevant case.

Second, use the six constrained reaction functions to construct the overall best reply function for each airline. This is done by comparing the profit level obtained by following the constrained reaction function across each of the six cases and selecting the price response which yields the highest profit.

Third, the equilibrium to the pricing game is given by the intersection of the two overall best reply functions

The work involved in constructing the six constrained reaction functions will not be presented here. However, to act as an illustration case D_3 is considered in detail.

Case $\mathbf{D}_3$

It follows from (24) and from (25) that the conditions which define Case D_3 are given by $0 \leq \frac{1+p_3^B-p_3^A}{2} \leq 1$ and $0 \leq \frac{1+p_3^B-p_3^A-\beta^*}{2} \leq 1$. These conditions can be re-written as,

$$p_3^A + \beta^* - 1 \leq p_3^B \leq 1 + p_3^A \tag{26}$$

Solving the first-order conditions of the airlines' optimization problem (23) gives,

$$\left.\frac{d\Pi_3^A}{dp_3^A}\right|_{case.D_3} = \frac{1}{2}\left(1+c+p_3^B-2p_3^A-\beta^*\mu s\right)=0$$
$$\Rightarrow \quad p_3^A=\frac{1}{2}\left(1+c+p_3^B-\beta^*\mu s\right)$$
$$\left.\frac{d\Pi_3^B}{dp_3^B}\right|_{case.D_3} = \frac{1}{2}\left(1+c+p_3^A-2p_3^B+2\beta^*\mu s\right)=0$$
$$\Rightarrow \quad p_3^B=\frac{1}{2}\left(1+c+p_3^A+2\beta^*\mu s\right) \tag{27}$$

The system of equations (27) together with the conditions (26) and the fact that the third period profit functions of the airlines are concave in own prices allow me to describe the optimal behaviour of the two airlines conditional on $\widehat{\Omega}_1$ and $\widehat{\Omega}_2$ being in the unit interval, *ie* remain within case D_3. For a given p_3^B, airline A will set its price p_3^A according to its reaction function given in (27). However, if this choice of p_3^A is below the smallest value of p_3^A necessary to satisfy (26) then A will set the lowest price consistent with (26). On the other hand, if the price is above the maximum price consistent with(26), then it will set the highest admissible price. Airline B follows a similar behaviour. Figure 2.5 below illustrates this discussion. The kinked solid curves graph the reaction functions of the two airlines and they are constructed from the 4 dotted lines which describe the two conditions (26) and the two equations in (27).

An analogous procedure can be followed for the other five cases. The following tables summarize the results for all 6 cases by reporting the best-reply of each airline to a rival's price. The tables should be read as follows. If the price of the rival is below the critical value *crit.* 1, then the best-reply of an airline is to the set its price as given in $BR_{low.}$ On the other hand, if the rival sets a price above the critical value *crit.* 2, then the airline should reply according to $BR_{high.}$ Lastly, if the price of the competitor lies between the two critical values, the best response id given by BR_{mid}. Note that while in cases A_3 and F_3 only one of the airlines is active, in case C_3, the best-reply of the airlines is invariant to the rival's strategy.

Case $\mathbf{A}_3$	Airline A	Airline B
BR	-	p_3^A-1

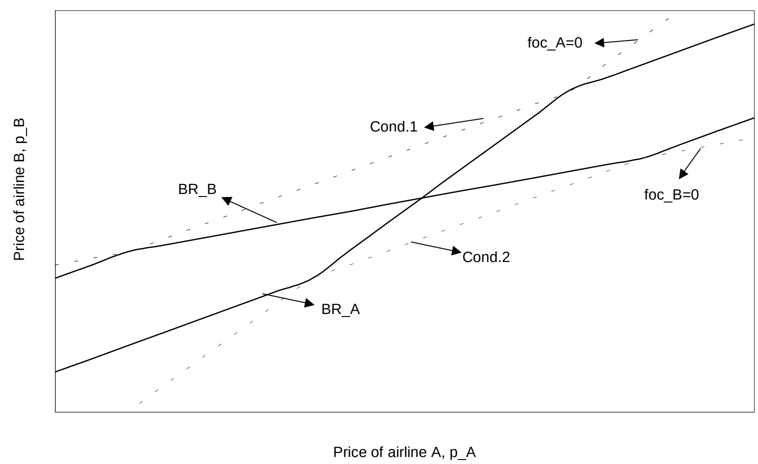

Figure 6: Airlines' reaction functions under case D.

Case B$_3$	Airline A	AirlineB
$crit.1$	$c-1$	$c+3-2\beta^*+\frac{2\mu s}{1-\mu s}$
$crit.2$	$c+2\beta^*-1$	$c+3+\frac{2\mu s}{1-\mu s}$
BR_{low}	p_3^B+1	$p_3^A+\beta^*-1$
BR_{mid}	$\frac{1}{2}\left(p_3^B+c+1\right)$	$\frac{1}{2}\left(p_3^A+c+1+\frac{2\mu s}{1-\mu s}\right)$
BR_{high}	$p_3^B+1-\beta^*$	p_3^A-1

Case C$_3$	Airline A	Airline B
BR	p_3^B-1	$p_3^A+1+\beta^*$

Case D$_3$	Airline A	Airline B
$crit.$ 1	$c-1+\beta^*(2-\mu s)$	$c-1+2\beta^*\mu s$
$crit.2$	$c+3-\beta^*\mu s$	$c+3-2\beta^*(1-\mu s)$
BR_{low}	$p_3^B+1-\beta^*$	p_3^A+1
BR_{mid}	$\frac{1}{2}\left(p_3^B+c+1-\beta^*\mu s\right)$	$\frac{1}{2}\left(p_3^A+c+1+2\beta^*\mu s\right)$
BR_{high}	p_3^B-1	$p_3^A-1+\beta^*$

Case E$_3$	Airline A	Airline B
crit. 1	$c+1+\frac{2}{\mu s}-\beta^*$	$c-1$
crit . 2	$c+1+\frac{2}{\mu s}+\beta^*$	$c-1+2\beta^*$
BR_{low}	p_3^B-1	$p_3^A+1+\beta^*$
BR_{mid}	$\frac{1}{2}\left(p_3^B+c-1-\beta^*+\frac{2}{\mu s}\right)$	$\frac{1}{2}\left(p_3^A+c+1+2\beta^*\right)$
BR_{high}	$p_3^B-1-\beta^*$	p_3^A+1

Case F$_3$	Airline A	Airline B
BR	$p_3^B-1-\beta^*$	$-$

Equipped with the 6 constrained reaction functions it is possible to construct the overall reaction curve for each airline and therfore solve for the equilibrium prices. This is a particularly cumbersome task in terms of the algebra involved and, for the sake of exposition, the working is not shown here. Nevertheless, it can be shown that for $\beta^* \leq 1 \Leftrightarrow \mu \leq \frac{7}{10}$ an equilibrium exists and it is given by $p_3^{A*}=1+c$ and $p_3^B=1+c+\beta^*\mu s$. At these prices, the relevant case is Case D_3. For $\mu > \frac{7}{10}$, no equilibrium exists.

Period 2 Henceforth, I assume that the solutions to the Period 3 subgame are given by $p_3^{A*}=1+c$ and $p_3^{B*}=1+c+\beta^*\mu s$. In other words, I will restrict my attention to the case where $\mu < \frac{7}{10}$ as this allows me to proceed analytically with the proof.

Recall, that A 's objective function at the start of Period 2 is,

$$\Pi_2^A=\left(p_2^A-c\right)\left(\left(1-\mu\right)\Psi_1+\mu\sigma\widehat{\Omega}_3+\mu\left(1-\sigma\right)\widehat{\Omega}_4\right)+\Pi_3^A$$

where,,

$$\widehat{\Omega}_j = \begin{cases} 0 & \text{if} \quad \Upsilon_j < 0 \\ \Upsilon_j & \text{if} \quad \Upsilon_j \in [0,1] \\ 1 & \text{if} \quad \Upsilon_j > 1 \end{cases}, \quad j = 3,4 \tag{28}$$

$$\text{where } \Upsilon_3 = \frac{1 + p_2^B - p_2^A + E\left(U^3|xx\right) - E\left(U^3|xy\right)}{2},$$

$$\Upsilon_4 = \frac{1 + p_2^B - p_2^A - \beta^* - E\left(U^3|xx\right) + E\left(U^3|xy\right)}{2}.$$

$$\text{and } \Psi_1 = \begin{cases} 0 & \text{if} \quad \frac{1+p_2^B-p_2^A}{2} < 0 \\ \frac{1+p_2^B-p_2^A}{2} & \text{if} \quad \frac{1+p_2^B-p_2^A}{2} \in [0,1] \\ 1 & \text{if} \quad \frac{1+p_2^B-p_2^A}{2} > 1 \end{cases}$$

Using the Period 3 equilibrium prices established above, it follows that $\widehat{\Omega}_1 = \frac{1+\beta^*\mu s}{2}$, $\widehat{\Omega}_2 = \frac{1-\beta^*(1-\mu s)}{2}$ and consequently that $E\left(U^3|xx\right) - E\left(U^3|xy\right) = \frac{\beta^*}{2}\left(\beta^*\mu s - \frac{\beta^*}{2} - 1\right)$. In turn, it is straightforward to establish that $\Upsilon_3 \leq \Upsilon_4$ if and only if $\mu s \leq \frac{1}{2}$. Below I will assume that the latter condition is satisfied. In equilibrium this will indeed be the case. Lastly, note that Ψ_1 is always greater than both $\widehat{\Omega}_3$ and $\widehat{\Omega}_{4.}$

The procedure followed to solve the pricing game in this period is similar to that adopted for Period 3.

Firstly, it is necessary to distinguish the cases where the airlines' objective functions change due to the discontinuities that arise from (28).There are five possible cases that must be analysed as tabled below.

	$\widehat{\Omega}_3$	$\widehat{\Omega}_4$	Ψ_1	Constraint 1	Constraint 2
Case A_2	0	0	$\in I$	$\Upsilon_3 < 0$	$\frac{1+p_2^B-p_2^A}{2} > 0$
Case B_2	0	$\in I$	$\in I$	$\Upsilon_3 < 0$	$\Upsilon_4 > 0$
Case C_2	$\in I$	$\in I$	$\in I$	$\frac{1+p_2^B-p_2^A}{2} < 1$	$\Upsilon_3 > 0$
Case D_2	$\in I$	$\in I$	1	$\Upsilon_4 < 1$	$\frac{1+p_2^B-p_2^A}{2} > 1$
Case E_2	$\in I$	1	1	$\Upsilon_3 < 1$	$\Upsilon_4 > 1$

The constraints 1 and 2 described in the table are those constraints which define the relevant case.

Secondly, for each of the 5 cases, it is necessary to work out the *constrained* reaction functions. To do so, the following procedure must be carried out for each of the 5 cases. Using the equality $s = \sigma\left(1-\widehat{\Omega}_3\right) + (1-\sigma)\,\widehat{\Omega}_4$, and the relevant expressions for $\widehat{\Omega}_3$ and $\widehat{\Omega}_4$, derive an expression for s in terms of p_3^A, p_3^B, μ and σ . This expression is then substituted in for s in the definitions of $\widehat{\Omega}_3$, $\widehat{\Omega}_4$ and Ψ_1. In a similar way to what was done above, the constrained reaction functions of each airline are obtained by maximizing the relevant Period 2 profit function with respect to their second-period price subject to the two constraints defining the case at hand. The expressions describing these functions are too cumbersome and will not be present here. Graphically, however, the constrained reaction functions of the two airlines are similar to those presented in Figure 2.5.

Lastly, to obtain the global reaction function, it is necessary to *i*) calculate the best reply to a rival's price under the five different cases and *ii*) select the reply which yields the highest profit. The solution to the game is then given by the intersection of the two overall reaction functions.

As it was not feasible to carry out these steps algebraically, I did so by resorting to numerical simulations. The result obtained is the following. For $\mu \leq \frac{7}{10}$, the region being considered, an equilibrium to the second period sub-game is given by the set of prices described in Proposition 2.2 and in (19) once the substitutions $\alpha = 0$ and $\beta = \beta^*$ are made.

Period 1 By assumption, $p_1^B = p_1^{B*}$, $\beta = \beta^*$ and $\alpha = 0$. It is sufficient to the consider the optimisation problem facing airline A. The objective function of this airline is,

$$\underset{\{p_1^A\}}{Max} \quad \Pi_1^A = \left(p_1^A - c\right)\left(\mu\sigma + (1-\mu)\,\Psi_2\right) + \Pi_2^A,$$

$$\text{where } \Psi_2 = \begin{cases} 0 & \text{if } \frac{1+p_1^{B*}-p_1^A}{2} < 0 \\ 1 & \text{if } \frac{1+p_1^{B*}-p_1^A}{2} > 1 \\ \frac{1+p_1^{B*}-p_1^A}{2} & \text{otherwise} \end{cases}$$

$$\text{and} \quad \sigma = \begin{cases} 0 & \text{if } \frac{1+p_1^{B*}-p_1^A+E\left(U^2|A\right)-E\left(U^2|B\right)}{2} < 0 \\ 1 & \text{if } \frac{1+p_1^{B*}-p_1^A+E\left(U^2|A\right)-E\left(U^2|B\right)}{2} > 1 \\ \frac{1+p_1^{B*}-p_1^A+E\left(U^2|A\right)-E\left(U^2|B\right)}{2} & \text{otherwise} \end{cases}$$

As before, the piece-wise nature of the objective function, forces me to resort to numerical calculations. The price p_1^A was allowed to take values

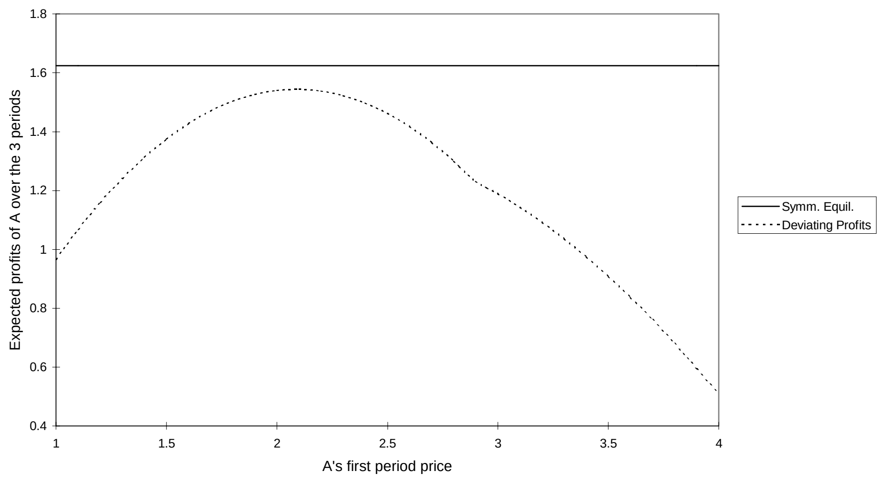

Figure 7: Expected profits of airline A as a function of its first period price.$(c = 1, \mu = 0.5)$

between $c + 1$ and $c + 4$ at intervals of 0.05. For each level of p_1^A, I derived the respective value of σ and consequently the profits of A over the three periods. The results derived show that A 's profits are everywhere below what it would receive if it did not deviate from the candidate symmetric equilibrium. This is illustrated by Figures 2.6 and 2.7. For $\mu = 0.5$, Figure 2.6 graphs A 's profit level as a function of its first period price, p_1^A, when this airline offers no discount and follows in subsequent periods the equilibrium pricing strategies that were derived above. There is an optimal price to be charged by A, though it is clear that its profits are below those that it would receive had it not deviated - this profit level is given by the horizontal line. Figure 2.7 summarizes the numerical simulations carried out by showing how the equilibrium profits of airline A varies with μ. For the sake of comparison, the figure also draws out the profits that this airline would earn had it not deviated. It is clear that A is better off if it does not deviate from the candidate symmetric equilibrium. This completes the proof.

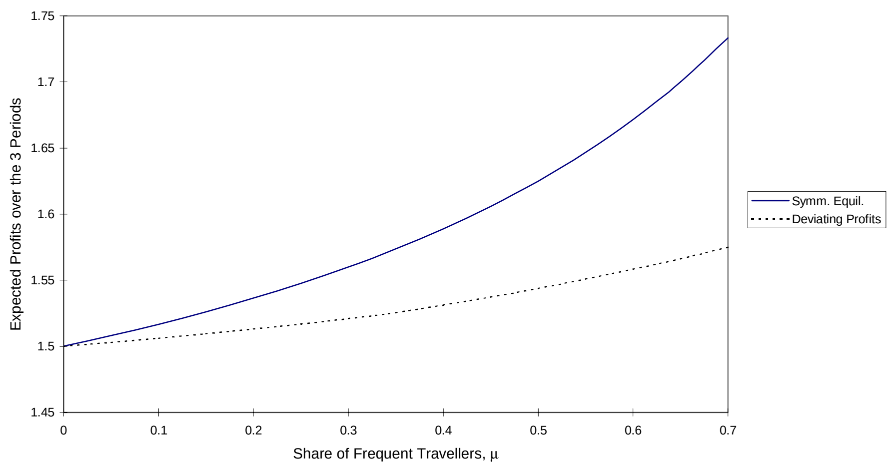

Figure 8: Profits of airline A in the symmetric equilibrium and under its optimal deviating behaviour given $\alpha = 0$, as a function of μ.

2.B Solving the Two Period Model

The steps involved in solving the two-period model follow closely the presentation in Caminal and Matutes $(1990, p.370)$. The same notation is used as in the three-period model, though the suffix $'$ (prime) is added to distinguish the two. The model is solved by working backwards starting in Period 2.

2.B.1 Period 2

An occasional traveller will patronise airline A if and only if $i_2 \leq \frac{1+p_2^{B'}-p_2^{A'}}{2} \equiv \Lambda_1$. Otherwise he will address B. A frequent traveller conditional on having address A in Period 1 will return to it in Period 2 if and only if $i_2 \leq \frac{1+p_2^{B'}-p_2^{A'}+\alpha'}{2} \equiv \Lambda_2$. On the other hand, if he addressed B in the first period, he will purchase from A in Period 2 if and only if $i_2 \leq \frac{1+p_2^{B'}-\beta-p_2^{A'}}{2} \equiv \Lambda_3$. Let σ' be the share of frequent travellers who selected A in Period 1. The profit of airline A in the second period is given by,

$$\Pi_2^{A'} = \left(p_2^{A'} - c\right)\left((1-\mu)\,\Lambda_1 + \mu\,(1-\sigma')\,\Lambda_3\right) + \left(p_2^{A'} - \gamma - c\right)\mu\,(\sigma' * \Lambda_2)$$

After substituting out the terms Λ_1, Λ_2 and Λ_3 in the above and upon simplification, one can write the profit function of A in the second period as,

$$\Pi_2^{A'} = \frac{1}{2}\left(p_2^{A'} - c\right)\left(1 + p_2^{B'} - p_2^{A'} + \mu\left(\beta'\left(\sigma' - 1\right) + \alpha'\right)\right) - \frac{\mu\sigma'\alpha'}{2}\left(1 + \alpha' + p_2^B - p_2^A\right)$$

An analogous expression can be derived for airline B. Differentiating the two profit functions with respect to the choice variables, p_2^A and p_2^B yields the following first order conditions,

$$\begin{aligned}
\frac{d\Pi_2^{A'}}{dp_2^{A'}} &= 1 - 2p_2^{A'} + p_2^{B'} + c - \mu\beta' + \mu\sigma'\beta + 2\mu\sigma'\alpha' \\
\frac{d\Pi_2^{B'}}{dp_2^{B'}} &= 1 + p_2^{A'} - 2p_2^{B'} + c + 2\mu\beta' - 2\mu\sigma'\beta' - \mu\sigma'\alpha'
\end{aligned}$$

Setting the two first order conditions to 0 and solving the ensuing system gives the equilibrium second period prices,

$$\begin{aligned}
p_2^{A'*} &= 1 + c + \mu\sigma'\alpha \\
p_2^{B'*} &= 1 + c + \mu\left(1 - \sigma'\right)\beta'
\end{aligned}$$

It is straight-forward to see that the second-order conditions are met since $\frac{d^2\Pi_2^{A'}}{dp_2^{A'}} = \frac{d^2\Pi_2^{B'}}{dp_2^{B'}} = -2 < 0$.

2.B.2 Period 1

An occasional traveller patronises A in Period 1 if and only if $i_1 \leq \frac{1+p_1^{B'}-p_1^{A'}}{2}$.
On the other hand a frequent traveller will address A if $i_1 \leq \frac{1+p_1^{B'}-p_1^{A'}+E\left(U^2|A\right)-E\left(U^2|B\right)}{2} \equiv \Lambda_4$ and will address B otherwise. The term $E\left(U^2|J\right)$, $J = \{A, B\}$, is the expected utility gained by a frequent traveller in Period 2 given that he addressed J in Period 1. The value of these expected utilities can be calculated as,

$$
\begin{aligned}
E\left(U^2|A\right) &= \int_0^{\Lambda_2}\left(R-p_2^{A'}+\alpha'-i\right)di+\int_{\Lambda_2}^1\left(R-p_2^{B'}-1+i\right)di \\
&= R-p_2^{B'}-\frac{1}{2}+\Lambda_2^2 \\
E\left(U^2|B\right) &= \int_0^{\Lambda_3}\left(R-p_2^{A'}-i\right)di+\int_{\Lambda_3}^1\left(R-p_2^{B'}+\beta'-1+i\right)di \\
&= R-p_2^{B}+\beta'-\frac{1}{2}+\Lambda_3^2
\end{aligned}
$$

Using these results it is possible to express Λ_4 as

$$
\Lambda_4=\frac{1+p_1^{B'}-p_1^{A'}-\beta'+\Lambda_2^2-\Lambda_3^2}{2} \tag{29}
$$

The profit earned by airline A over the two periods is given by,

$$
\Pi_1^{A'}=\left(p_1^{A'}-c\right)\left(\mu\sigma'+(1-\mu)\left(\frac{1+p_1^{B'}-p_1^{A'}}{2}\right)\right)+\Pi_A^2
$$

where, by construction, $\sigma'=\Lambda_4$.

Maximizing $\Pi_1^{A'}$ with respect to $p_1^{A'}$ and α' and then imposing the symmetry conditions $p_1^{B'}=p_1^{A'}$, $\beta'=\alpha'$ and $\sigma'=\frac{1}{2}$ yields,

$$
\begin{aligned}
\frac{d\Pi_1^{A'}}{dp_1^{A'}} &= \frac{1}{2}\left(1-\left(p_1^{A'}-c\right)(1-\mu)\right)+\left(p_1^{A'}-c\right)\mu\frac{d\sigma'}{dp_1^{A'}}-\frac{\mu\alpha'^2}{2}(1-\mu)\frac{d\sigma'}{dp_1^{A'}} \\
\frac{d\Pi_1^{A'}}{d\alpha'} &= \left(p_1^{A'}-c\right)\mu\frac{d\sigma'}{d\alpha'}+\frac{\mu^2\alpha'}{8}-\frac{\mu\alpha'}{2}-\frac{\mu\alpha'^2}{2}(1-\mu)\frac{d\sigma'}{d\alpha'}
\end{aligned} \tag{30}
$$

Using (29) and recalling that $\Lambda_4=\sigma'$, it is straight-forward to compute $\frac{d\sigma'}{dp_1^{A'}}$ and $\frac{d\sigma'}{d\alpha}$ at the symmetric point as,

$$
\begin{aligned}
\frac{d\sigma'}{dp_1^{A'}} &= -\frac{1}{2\left(1+\mu\alpha'^2\right)} \\
\frac{d\sigma'}{d\alpha'} &= \frac{1+\alpha'(1-\mu)}{4\left(1+\mu\alpha'^2\right)}
\end{aligned}
$$

Substituting these expressions into (30) and solving the first order conditions gives the following system of equations,

$$
\begin{aligned}
0 &= p_1^{A'} - c - \frac{2 + \mu\alpha'^2 (3 - \mu)}{2 + 2\mu\alpha'^2 (1 - \mu)} \\
0 &= \mu \frac{2\left(p_1^{A'} - c\right) + \alpha (1 - \mu) \left(2p_1^{A'} - 2c - \alpha'\right) + \mu\alpha - 4\alpha - \alpha'^3 (2\mu + 1)}{8 (1 + \mu\alpha'^2)}
\end{aligned}
\tag{31}
$$

The candidate symmetric equilibrium prices and coupon levels are given by the solution to this system of equations. This systemn can not be solved analytically and it is necessary to resort to numerical computations to derive the equilibrium price and discount levels.

The equilibrium values are those plotted in Figure 2.4 in Section 2.5.1.

The equilibrium prices and discount are a function of μ. It was checked numerically that for each μ in a 0.02 grid in the $[0, 1]$ interval, the second-order conditions were met at the symmetric equilibrium point.

As with the three-period model, it is necessary to check that the solutions to (31) actually form an equilibrium. In other words, it is necessary to check that neither airline has the incentive to deviate. The steps involved to carry this out were similar to those presented in Part 2 of Appendix 2.A. For the sake of exposition, that work is not presented here. It is noted, however, that the analysis concluded that the symmetric equilibrium is robust to deviations to corner solutions, ie. to deviations by a firm not to offer no discount. This completes the proof.