

The complementarity of RJDs and subsidies when absorptive capacity matters

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Abstract

This paper introduces a model of coordination of research paths into the theory of Research Joint Ventures (RJDs). A comparison of RJDs and licensing by non-cooperative firms shows that the latter may be unable to co-ordinate on complementary research paths whereas RJDs are always able to do this. This happens when the acquisition of absorptive capacity is costly. It is shown that the policies of promoting RJDs and subsidising R&D are complementary, whereas either policy by itself may reduce welfare. Finally it is shown that costly absorptive capacity gives rise to a welfare loss that is very difficult to address.

Key Words: Research Joint Venture; Licensing; Information-Sharing;

JEL Classification: F13, L13

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1 Introduction

In most research intensive industries the formation of joint ventures to facilitate the collaborative R&D efforts of firms that compete in the output market has become commonplace¹. Meanwhile the burgeoning literature on such Research Joint Ventures (RJVs) seems to be reaching a consensus that the social benefits, which will arise from RJV formation are slight² if not negative³. This contrast is troubling from a public policy point of view because it questions the rationale for tolerating RJV formation.

This paper seeks to establish what public policy towards firms that collaborate on R&D should look like in industries in which firms' ability to learn from one another arises from costly investments. I compare R&D collaboration through RJV formation with collaboration through licensing. My main finding is that RJV formation is a complement to the provision of R&D subsidies. This result differs from the existing literature on research collaboration which generally considers these two policies to be substitutes⁴.

Here I extend the model of an RJV by introducing a coordination problem into the game that the firms play and considering how this is resolved in differing modes of R&D collaboration. I assume that firms undertaking R&D have a choice about the direction in which they undertake research. The idea that a firm's research can be described as mapping out a path in a space of possible research directions is widespread in the empirical literature on innovation⁵ and is also beginning to be used in theoretical work⁶.

Once the differentiation of research paths is considered it is natural to complement this with the assumption that such differentiation imposes a cost on the firms' ability to transfer innovations between different research paths. The idea that the capacity to absorb others' research results could be costly originates with Cohen and Levinthal (1989) who show that such costs may lead to a reversal of Arrow's⁷ result that involuntary spillovers of knowledge will reduce investments in innovation. They suggest that this will have important implications for public policy toward innovative industries in which the acquisition of absorptive capacity matters.

I use the simplest possible formulation of the costs that arise from achieving the ability to absorb from rival firms here. Using this model I show that the need for investment in absorptive capacity increases the complexity of the regulatory problem. It is for this reason that a policy of tolerating RJV formation is shown to be a complement to the introduction of an R&D subsidy below .

The effects of introducing absorptive capacity into the welfare evaluation of RJV's have previously been studied by Kamien and Zang (2000). Whilst they also consider a coordination problem their modelling of this restricts the decision problem to the choice between being able to absorb outside R&D whilst facing high spillovers of own R&D, or being unable to absorb outside R&D but reducing spillovers of own R&D. In contrast I do not

¹This is documented by Hagedorn (2000). The figure on the next page also provides a flavour of this.

²This is the conclusion arrived at by Leahy and Neary (1997).

³Katsoulacos and Ulph (1999) compare RJVs and licensing as I do here. They find that the welfare losses from RJV formation can exceed those under licensing.

⁴Compare for instance Hinloopen (1997, 2000) or Spence (1984).

⁵Compare Jaffe (1986); Mowery et al. (1996)

⁶An instance of this for a dynamic framework can be found in Vega-Redondo (1999)

⁷This result is first derived in Arrow (1962). For a further discussion of its implications see Spence (1984).

assume that the level of spillovers of own R&D is affected by the decision to invest in absorptive capacity. This allows me to study the effects that arise from exogenous variation in the costs of absorbing outside R&D.

In this paper I compare RJV formation and licensing of innovations. Although licensing is not usually adopted as a counterfactual to RJV formation I believe that this approach may be justified both theoretically and empirically.

It is clear that one important aim of theoretical modelling of RJV's is to derive the welfare effects of RJV formation. In order to conclude that RJV formation is undesirable, we should also be able to explain why alternative methods of sharing technologies between firms are more desirable. This paper provides such a comparative analysis and shows that RJV formation can improve on licensing in most cases.

The comparison of RJV formation and licensing is empirically interesting because recent studies have shown that these two methods of sharing research results are frequently employed by firms operating in the same industries and are therefore real alternatives for these firms.

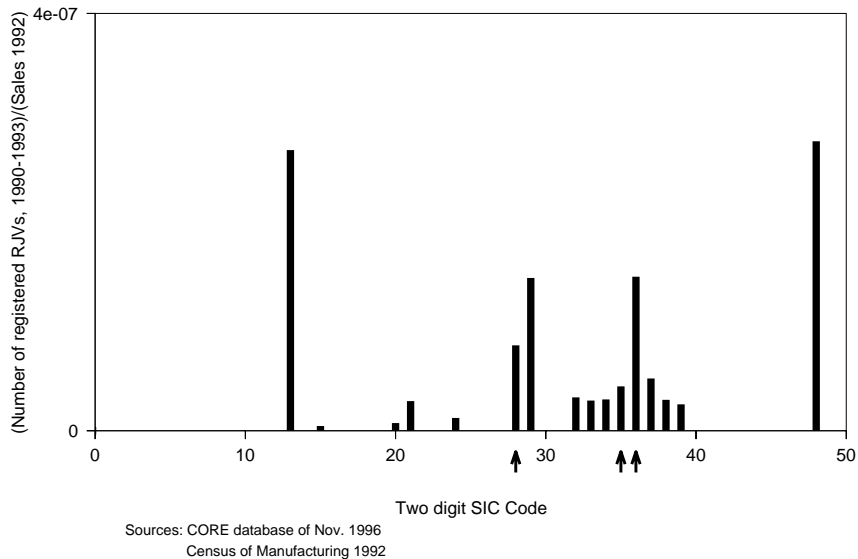


Figure 1: **The distribution of RJVs across two digit SIC industries for the period 1990 – 1993**

Figure 1 above provides a histogram of the number of RJVs registered in the United States between 1990 – 1993 by two digit SIC industry^{8,9}.

These RJVs were registered under the provisions of the NCRA act. The *arrows* in the diagram point to those industries in which licensing was found to be particularly impor-

⁸The data used for this histogram is based on the CORE database described in Link (1996) weighted by the total sales of the two digit SIC industry as measured in the 1992 Census of Manufacturing. The histogram is based on the 1999 CORE database which is available from A. Link and the 1992 Census of Manufacturing which can be found at: <http://www.census.gov/epcd/ec97sic/E97sus.htm>

⁹It should be noted that this distribution of RJVs across the two digit SIC industries is not stable over time. For instance by 1999 more RJVs had been registered in SIC 36 than in SIC 48. Nonetheless the industries that saw the majority of RJV filings are the same for the periods 1985 – 1999 and 1990 – 1993.

tant by Anand and Khanna (2000)¹⁰. They provide evidence on the incidence of licensing contracts in the United States. They show that roughly 80% of all licensing contracts in their sample fall within three two digit SIC industries. These are: Chemicals (SIC 28), Computers (SIC 35) and Electronics (SIC 36)¹¹.

This simple comparison of findings suggests that licensing is often an alternative to RJV formation. Notice finally that Cohen and Levinthal (1989) find evidence that absorptive capacity is an important factor in the Chemicals (SIC 28) and the Electronics industries (SIC 36). The modelling of R&D collaboration in the context of investments in absorptive capacity would therefore seem to be very relevant to public policy towards these industries.

In the following section I discuss the welfare effects which I consider in this paper. In section 3 below I introduce the model, characterise the research strategies open to firms in this model and derive the cost reductions each firm can achieve under these strategies. Then in section 4 I solve the model and characterise the welfare losses that arise in its context. I also provide a comparison of the allocative efficiency of RJVs and non-cooperative firms. In section 5 I discuss policies with which the welfare losses described in the previous section can be addressed. Finally section 6 contains my conclusions.

2 The resource allocation problem in the context of absorptive capacity

As noted above this paper provides an explicit welfare comparison between RJVs and non-cooperative firms in a model that encompasses absorptive capacity.

I derive the socially optimal degree of complementarity between firms' R&D, the socially optimal decisions on absorptive capacity and the socially optimal decision on the degree of information sharing. The performance of firms that form an RJV and firms in a non-cooperative equilibrium is measured against this standard. As pointed out in the introductory chapter I assume that firms in a non-cooperative equilibrium can share information with one-another through licensing.

Note that this standard of social optimality is not concerned with the decisions of the firms in the product market. I focus exclusively on the ability of RJVs and non-cooperative firms to co-ordinate and allocate resources, to R&D.

The firms in an RJV will maximise joint profits. This implies that they do not take into account the effects of their actions on consumers' surplus. In particular the firms will deviate from the socially optimal choices on R&D coordination. Furthermore they will *underinvest* in R&D relative to the social optimum. I refer to this as the *undervaluation effect*.

The non-cooperative firms will maximise only their own profits. Therefore they fail to take into account both the effects of their actions for consumers' surplus and for other firms. In addition to the undervaluation effect that is common to non-cooperative firms

¹⁰They study licensing contracts on the basis of a different database based on data provided by the Securities Data Corporation, which also includes information on joint venture formation. I have been unable to obtain this information. Their dataset is restricted to manufacturing industries, i.e. SIC 20 – 39. It was collected for the period 1990 – 1993.

¹¹In a separate study on licensing in France, Bessy and Brousseau (1998) find that licensing is concentrated in a small number of large firms and in the following industries: Chemicals, Pharmaceuticals, Electronics, Electrical Equipment and Professional Machinery.

and the RJV we therefore have a loss from the *stand-alone effect*. This extra welfare loss may be compensated by the *strategic incentive* that arises from the competition between non-cooperative firms to be the first to further reduce their costs.

Just as the firms in the RJV the non-cooperative firms will be shown to make suboptimal choices about the co-ordination of their research strategies.

To keep the model I employ simple I assume that firms face a threshold investment level beyond which they acquire the ability to absorb R&D¹². As I show below firms will find it optimal not to acquire absorptive capacity for parameter combinations that make it socially optimal for them to do so. This gives rise to a welfare loss which I will refer to as the *absorption problem*.

A further complication in our welfare analysis arises from the fact that the non-cooperative firms are shown to be unable to coordinate their R&D in the socially optimal manner where the acquisition of absorptive capacity is very costly and involuntary spillovers are high.

In order to disentangle the interaction of these different welfare effects I use the model set out in the following section.

3 The Model

In this paper I model explicitly the choices of firms that coordinate the paths of their R&D efforts. The literature on RJV's has usually modelled R&D efforts, by assuming that they can be added together to yield the total cost reduction of the firms. This presupposes that the R&D undertaken by the firms is already complementary¹³.

Here I seek to model the R&D process in such a way that the choice of the R&D paths and their degree of complementarity is endogenous to the model. In doing so I also take seriously the notion that differentiation in R&D imposes a cost. This is the cost of maintaining sufficient expertise along the research path chosen by another firm to be able to absorb its innovations.

At the product market stage this model is of a Cournot duopoly producing homogeneous goods. This rather special setting is not essential for the results here, but I maintain it in order not to overload the analysis with detail. The model I consider has four stages:

Stage 1: Research Co-Ordination The firms choose one of two research strategies: *non-specialisation* or *partial specialisation*, where a firm is called 'specialised' when it cannot absorb outside innovation. Given their choice of research strategy the firms choose a research vector in a two dimensional technology space. This space has the dimensions x and y . The relative position of the firms' vectors determines the degree of additivity between the R&D of the two firms. I will make this notion more precise below.

The firms undertake research in order to lower their marginal cost of production. At the outset the firms are symmetrical and both face a marginal cost of production \bar{c} . I

¹²A more complete model might assume that firms trade-off the degree of absorption they are capable of against a cost of investment in absorptive capacity. In such a context the welfare loss I describe here will always arise. This would come closer to a model such as that of Cohen and Levinthal (1989)

¹³This additivity assumption is pointed out by Kamien and Zang (2000) in their discussion of the Ruff equation, as they call it.

assume that the two firms, can both achieve a total cost reduction $\mathbf{G} = \bar{c} - \underline{c}$. Each firm faces the choice as to how to distribute its R&D efforts among the two dimensions. I express the resulting research vectors as:

$$\bar{\mathbf{v}}_{1,2} \equiv G \begin{bmatrix} \lambda_{1,2} \\ (1-\lambda_{1,2}) \end{bmatrix}, \quad \text{where I introduce the convention that}^{14} : \lambda_1 \geq \lambda_2 \quad (1)$$

Here $0 \leq \lambda_i \leq 1$ stands for the proportion of its R&D investment that firm i directs toward research in dimension \mathbf{x} . For reasons that will become apparent below define:

$$\Lambda \equiv \lambda_1 - \lambda_2, \quad 0 \leq \Lambda \leq 1$$

Then Λ captures the difference between the investment in R&D by firms 1 and 2 in each dimension of the technology space. As I show below the greater is this difference the greater will be the differentiation between the firms' research vectors.

I make two important assumptions about the role of absorptive capacity in the choice of the research vectors:

- i I assume that a firm must invest at least a fraction μ (where $0 < \mu \leq 1$) of its R&D in a dimension of the technology space to be able to absorb an innovation which the other firm achieves in that dimension.
- ii I assume that absorptive capacity is only acquired where firms successfully innovate.

These assumptions¹⁵ imply that when R&D vectors are differentiated firms can only share information if at least one invests beyond the absorption threshold in the relevant dimension of technology space. When R&D vectors are not differentiated firms will either be unable to share or will not gain anything from doing so.

Given the notation introduced above I can characterise the research strategies between which the firms choose as follows:

- Nonspecialisation: Both firms choose their research vectors such that they are able to absorb the innovation of the other firm. This implies that:

$$\lambda_1, [1 - \lambda_2] > \lambda_2, [1 - \lambda_1] \geq \mu$$

- Partial specialisation: One firm (Firm 2) remains a generalist, whereas the other becomes a specialist. This implies that either :

$$[1 - \lambda_1] > \lambda_2 \geq [1 - \lambda_2] \geq \mu > \lambda_1 \quad \text{and} \quad \mu < \frac{1}{2}, \quad \text{or:}$$

$$[1 - \lambda_1] > [1 - \lambda_2] \geq \mu \geq \frac{1}{2} > \lambda_2 > \lambda_1$$

This case arises when absorption costs are fairly high. Joint profits can then be increased if only one firm chooses to invest in absorptive capacity.

¹⁴This convention allows us to avoid analysing identical cases that differ only in the role of firm 1 and 2.

¹⁵These two assumptions are the most restrictive possible assumptions in that they rule out the possibility of information sharing where only one firm has innovated and the other has at least attempted to acquire absorptive capacity. Although such a model is richer and might be seen as more realistic the added complexity is not rewarded with further insights.

An illustration of these ideas is provided in section 3.1 below.

Stage 2: R&D Investment The firms choose the probability with which their research vectors $\bar{v}_{1,2}$ become progress vectors $v_{1,2}$. In doing so they face the constraint that the cost of achieving a given probability of innovation ρ is given by the R&D cost function $\gamma(\rho)$. This cost function has the properties that no firm will ever innovate with certainty but both firms always have an incentive to invest a positive amount in R&D, i.e. $0 < \rho < 1$. This implies that the R&D cost function satisfies the following conditions:

$$\gamma(0) = 0 \quad \gamma'(0) = 0 \quad \forall \rho, 0 < \rho \leq 1 \quad \gamma'(\rho) > 0, \quad \gamma''(\rho) > 0 \quad (2)$$

On page 15 below I introduce a further condition under which it is neither socially optimal nor profit maximising to innovate with certainty.

Once the firms have invested in a probability ρ of innovation, the innovations are realized simultaneously. The firms' probabilities of innovation are assumed to be independent. There are three possible outcomes of the R&D investment stage: both, one or no firm innovates. I set out the resulting cost reduction in each of these cases on page 10 below.

Stage 3: Technology Sharing Given the outcome of the innovation process the firms may be able to share newly discovered technologies with one another. This is the case *iff* both firms have innovated and at least one of the firms has invested in sufficient absorptive capacity. Logically there are five different possible outcomes, which I characterise below¹⁶:

- | | |
|------------------------------------|------------------------------------|
| <i>a</i> : Both firms innovate. | Both firms share technologies. |
| <i>b</i> : Both firms innovate. | Only firm 1 shares its technology. |
| <i>c</i> : Both firms innovate. | No firm shares its technology. |
| <i>d</i> : Firm 1 innovates. | Firm 2 cannot absorb. |
| <i>e</i> : Neither firm innovates. | No technology can be shared. |

Stage 4: Product Market Competition In the final stage of the game the firms choose their output and compete in the product market. I assume that the firms are engaged in Cournot competition. In order to simplify the analysis as much as possible that demand is linear:

$$P = a - Q \quad \text{where } a > 0 \quad (3)$$

Here $Q = q_1 + q_2$ is aggregate output in the industry and P is price. I set out the expressions for output, profits and social surplus that follow from these assumptions in appendix 7.1 below. I assume that both firms are always active in equilibrium, which implies that: $G < a - \bar{c}$.

This game is solved by backward induction.

¹⁶There are two other cases which differ trivially from these only in the roles of firms 1 and 2 where these firms are no longer symmetrical.

3.1 An illustration of the role of absorptive capacity

In the description of the first stage of the model two possible equilibrium configurations for the firms' research vectors were set out. Either the firms both invest in absorptive capacity or only one of them does so. As I will show below this choice depends on the costs of absorbing outside R&D.

In order to keep the model simple I assume that an investment in absorptive capacity lowers the total cost reduction which a firm can achieve through innovation by a fixed proportion μG , where $0 < \mu < 1$.

I represent the tradeoff between investing in absorptive capacity in order to learn about the R&D which the other firm has undertaken as the choice of a vector in a two dimensional space. The two firms choose their vectors simultaneously. In the diagrams below I have marked on each axis the threshold investment level in each dimension which a firm must supercede in order to be able to learn from the other.

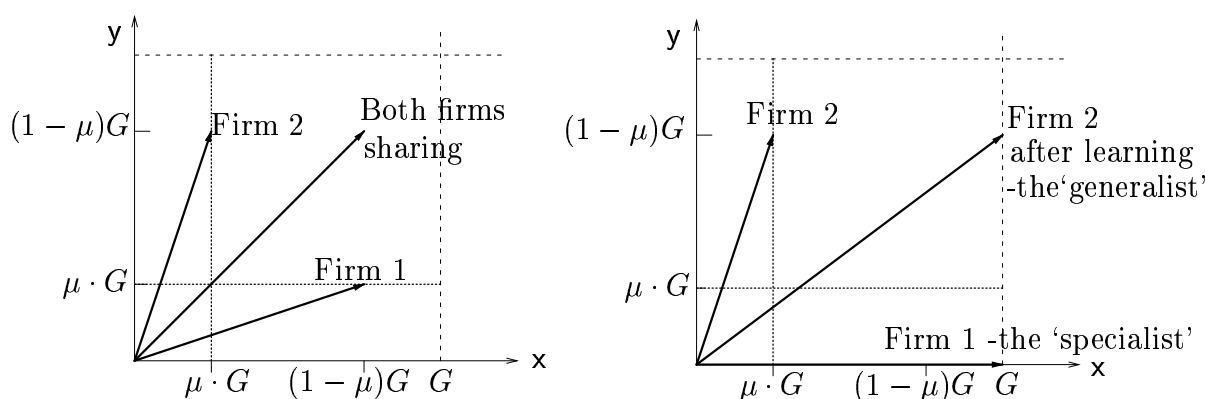


Figure 2: **Symmetric and asymmetric progress vectors**

If the firms have invested beyond the absorption threshold in the other firm's field of expertise and both innovate, then they can learn from one another. In the diagram this is represented as the addition of both firms' progress vectors. Note that the total cost reduction represented by any vector is measured as the sum of the lengths of the two orthogonal vectors along the axes x and y which it can be decomposed into.

The left hand diagram above represents the case in which both firms have chosen to invest the minimum necessary to be able to absorb one another's innovations. This will be shown to be an equilibrium, as long as the costs of absorption are not too high. If the costs of absorption become large, the firms can do better if one of them specialises in R&D provision and the other remains able to absorb. In such cases I refer to the two types of firms as 'specialists' and as 'generalists' respectively. An example of this is depicted in the right hand diagram above.

Clearly the two firms will become asymmetrical in the case of partial specialisation. For this reason it is not always possible for the non-cooperative firms to sustain such equilibria. I will show below that this has implications for the relative efficiency of RJV and licensing equilibria when costs of absorption are high.

The main difference between this framework and that of Kamien and Zang (2000) arises

very clearly here in that I investigate the effects of a variation in the costs of absorption on firms' innovation strategies whereas they do not.

3.2 Deriving the cost reduction for each of the innovation outcomes

Here I set out how the progress vectors of each firm translate into cost reductions. I do this for any conceivable combination of progress vectors under the model set out above.

Given that each firm can achieve a total cost reduction of G , a firm i will choose a two dimensional research vector \bar{v}_i and the outcome of the innovation process will then be a progress vector v_i :

$$\bar{v}_i = G \begin{bmatrix} \lambda_i \\ (1 - \lambda_i) \end{bmatrix} \rightarrow v_i = \begin{bmatrix} x_i \\ y_i \end{bmatrix} \in \left\{ \begin{bmatrix} \lambda_i \cdot G \\ (1 - \lambda_i) \cdot G \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\} \quad \text{where } 0 \leq \lambda_i \leq 1 \quad i = 1, 2$$

If at least one firm has chosen a research vector allowing it to absorb the innovation of the other and both firms have innovated successfully the firms can exchange information. Then the total progress both firms will make in any given dimension will be the greatest progress that any one firm has achieved in that dimension. I denote the progress in any dimension after information exchange with capitals, i.e. X, Y . Formally I express all of the above as follows:

For the choice of progress along dimension $z \in \{x, y\}$ by a firm i the total progress $Z \in \{X, Y\}$ in that dimension will be:

A.1 $Z_i = z_i, \quad i = 1, 2 \quad \text{if } 0 \leq z_i \leq \mu \cdot G$

A.2 $Z_i = \text{MAX}[z_i, z_j], \quad i \neq j, \quad i, j \in \{1, 2\} \quad \text{if } z_i \geq \mu \cdot G$

A.3 The total cost reduction achieved by i is $\bar{c} - c_i = X_i + Y_i$

Here the use of the MAX operator captures the idea that within one dimension firms' discoveries are perfect substitutes.

In the table on the following page I set out how firms' progress vectors are translated into cost reductions for the cases *a-d* set out on page 7 above. Case *e* means that the firms do not reduce their costs at all.

Given the cost reduction for each configuration of progress vectors and innovation outcomes I can proceed to set out the corresponding expressions for output, profits and Social surplus. These can be found in appendix 7.1.

4 Solutions and Welfare Losses

In this section I solve and analyse the model set out in the previous section.

Translating progress vectors into cost reductions		
	Cost reduction if:	
	-Both firms innovate	- Only firm 1 innovates
Nonspecialization	$a : \mathbf{v}_{1,2} = G \begin{bmatrix} \lambda_1 \\ 1-\lambda_2 \end{bmatrix} \rightarrow \bar{c} - c_{1,2} = G(1 + \Lambda)$	$d : \mathbf{v}_1 = G \begin{bmatrix} \lambda_1 \\ 1-\lambda_1 \end{bmatrix} \rightarrow \bar{c} - c_1 = G$
	$b : \mathbf{v}_1 = G \begin{bmatrix} \lambda_1 \\ 1-\lambda_1 \end{bmatrix} \rightarrow \bar{c} - c_1 = G$	$d : \mathbf{v}_2 = G \begin{bmatrix} 0 \\ 0 \end{bmatrix} \rightarrow \bar{c} - c_2 = 0$
	$b : \mathbf{v}_2 = G \begin{bmatrix} \lambda_1 \\ 1-\lambda_2 \end{bmatrix} \rightarrow \bar{c} - c_2 = G(1 + \Lambda)$	
	$c : \mathbf{v}_i = G \begin{bmatrix} \lambda_i \\ 1-\lambda_i \end{bmatrix} \rightarrow \bar{c} - c_i = G \quad i \in \{1, 2\}$	
Partial-spec.	$a, b : \mathbf{v}_1 = G \begin{bmatrix} \tilde{\lambda}_1 \\ 1-\tilde{\lambda}_2 \end{bmatrix} \rightarrow \bar{c} - c_1 = G(1 + \tilde{\Lambda})$	$d : \mathbf{v}_1 = G \begin{bmatrix} \tilde{\lambda}_1 \\ 1-\tilde{\lambda}_1 \end{bmatrix} \rightarrow \bar{c} - c_1 = G$
	$a, b : \mathbf{v}_2 = G \begin{bmatrix} \tilde{\lambda}_2 \\ (1-\tilde{\lambda}_2) \end{bmatrix} \rightarrow \bar{c} - c_2 = G$	$d : \mathbf{v}_2 = G \begin{bmatrix} 0 \\ 0 \end{bmatrix} \rightarrow \bar{c} - c_2 = 0$
	$c : \mathbf{v}_i = G \begin{bmatrix} \lambda_i \\ 1-\lambda_i \end{bmatrix} \rightarrow \bar{c} - c_i = G \quad i \in \{1, 2\}$	

Table 1

4.1 Stage 4: How research vectors translate into cost reductions

Solving for equilibrium output in a Cournot duopoly I get the well know expression:

$$q_i = \frac{a - 2c_i + c_j}{3} = A \cdot \left[1 + \frac{\bar{c} - c_i}{a - \bar{c}} + \frac{c_j - c_i}{a - \bar{c}} \right] \cdot \frac{1}{3} \quad \text{where } A \equiv a - \bar{c} > 0 \quad (4)$$

I insert the cost reduction terms set out in table 3.1 above into equation (4) to derive the equilibrium outputs for each configuration of progress vectors and innovation outcomes. Define $\mathbf{g} \equiv \frac{G}{a - \bar{c}}$. The resulting expressions for outputs, profits and the Social surplus are relegated to Appendix 7.1.

4.2 Stage 3: Technology Sharing

Given that the firms have chosen to be able to absorb information from one another they face a choice about information sharing only where both innovate simultaneously. Any decision not to share information is made in order to maximise profits, and we assume that the firm which does not receive knowledge from its partner will not make use of any involuntary spillovers.

The proof of the following result is relegated to appendix 7.2 due to its length. There I show that:

Result 1

- *If both firms have innovated:*
 - *The firms will never find it optimal not to share technologies at all.*
 - *The firms will find it optimal for one firm to share technology with the other but not vice versa when the following condition holds:*

$$g > \frac{2}{3\Lambda - 2} \text{ where } 3\Lambda - 2 > 0 \quad \text{never, where } 3\Lambda - 2 < 0$$

The upper bound for Λ is 1. This implies that g must always be greater or equal to at least 2 if the left hand inequality is to hold. I have to restrict g to be smaller than 1 to exclude exit¹⁷. This implies that the firms will always share technologies entirely when both are able to absorb them.

- If only one firm innovates then by assumption the firms cannot share technologies at all.

Note that it will always be socially optimal to share as much technology as possible. The more technology is shared the lower will be average industry costs and this will maximise the social surplus.

4.3 Stage 2: Equilibrium R&D investment

In this section I show that the firms in the RJV under-invest more relative to the social optimum than the firms in a non-cooperative equilibrium. Furthermore I show that the non-cooperative firms are not always able to choose the research strategy of partial specialisation.

In the previous section I discussed two possible strategies of specialisation for the firms in either type of equilibrium. Here I analyse each of these strategies in turn, providing comparisons of the cooperative and non-cooperative outcomes with the socially optimal outcomes.

4.3.1 Non-specialisation

This strategy implies that both firms wish to maintain an ability to absorb one-another's' research. I set out the objective functions for the two types of equilibria below and derive the corresponding first order conditions¹⁸.

I assume that the non-cooperative firms engage in cross-licensing of technologies when both of them innovate. This means that there is no license fee and the firms simply exchange technologies.

The Social Optimum:

$$W = \rho_O^2 S_a(\Lambda) + 2\rho_O (1 - \rho_O) S_d + (1 - \rho_O)^2 S_e - 2\gamma(\rho_O) \quad (5)$$

The RJV-equilibrium:

$$\Pi_R = \rho_R^2 \Sigma_a(\Lambda) + 2\rho_R (1 - \rho_R) \Sigma_d + (1 - \rho_R)^2 \Sigma_e - 2\gamma(\rho_R) \quad (6)$$

The non-cooperative equilibrium:

$$\begin{aligned} \pi_N^1 = & \rho_N^1 \rho_N^2 \pi_a(\Lambda) + \rho_N^1 (1 - \rho_N^2) \pi_d + (1 - \rho_N^1) \rho_N^2 \pi_{\bar{d}} \\ & + (1 - \rho_N^1) (1 - \rho_N^2) \pi_e - \gamma(\rho_N^1) \end{aligned} \quad (7)$$

¹⁷Compare the expressions for output in case d on page 32 below.

¹⁸All variables should carry a superscript ^{*g*} here to indicate that they are specific to the equilibrium in which both parties choose to be generalists. I suppress the superscripts for the time being in order to unclutter the notation.

From these objective functions I derive the first order conditions:

$$\rho_O (S_a(\Lambda) - S_d) + (1 - \rho_O) (S_d - S_e) = \gamma'(\rho_O) \quad (8)$$

$$\rho_R (\Sigma_a(\Lambda) - \Sigma_d) + (1 - \rho_R) (\Sigma_d - \Sigma_e) = \gamma'(\rho_R) \quad (9)$$

$$\rho_N (\pi_a(\Lambda) - \pi_d) + (1 - \rho_N) (\pi_d - \pi_e) = \gamma'(\rho_N) \quad (10)$$

For the moment I assume that a unique interior solution to these expressions exists. I shall set out the expressions that guarantee this below.

We can identify the ‘competitive threat’ and the ‘profit incentive’¹⁹ for each of these equilibria, where these are defined as follows:

The competitive threat This is the payoff increase from innovation, conditional on the second firm having innovated.

The profit incentive This is the payoff increase from innovation, conditional on the second firm failing to innovate.

These are set out in the table below:

Comparing Innovation Incentives		
	The Competitive Threats	The Profit Incentives
Social Optimum	$S_a(\Lambda) - S_d =$ $[CS_a(\Lambda) - CS_d] + [\Sigma_a(\Lambda) - \Sigma_d]$	$S_d - S_e =$ $[CS_d - CS_e] + [\Sigma_d - \Sigma_e]$
RJV	$\Sigma_a(\Lambda) - \Sigma_d$	$\Sigma_d - \Sigma_e$
Non-Coop. Firms	$\pi_a(\Lambda) - \pi_{\bar{d}} =$ $\frac{1}{2} (\Sigma_a(\Lambda) - \Sigma_d) + \frac{1}{2} (\pi_{\underline{d}} - \pi_{\bar{d}})$	$\pi_{\underline{d}} - \pi_e =$ $\frac{1}{2} [\Sigma_d - \Sigma_f] + \frac{1}{2} (\pi_{\underline{d}} - \pi_{\bar{d}})$

Table 2

The table shows that the firms in the RJV invest less in R&D than the non-cooperative firms. We can see how the market failures discussed in section 2 interact to give this result. The comparison of the RJV and the social optimum shows that both the profit incentive and the competitive threat faced by the RJV are smaller than those in the social optimum. The difference between the incentives captures the *undervaluation effect*.

The profit incentive and competitive threat faced by the non-cooperative firms can be decomposed into two terms. The first is half of the profit incentive or competitive threat faced by the RJV. This captures the *stand alone effect*. The second effect arises from the firms’ preference for being the sole innovator and the higher profits that this implies. The difference in the profits of the sole innovator and the non-innovating firm is $\frac{1}{2} [\pi_{\underline{d}} - \pi_{\bar{d}}]$. This term captures the *strategic investment effect*. It arises where firms care whether it is they or another firm which is the sole innovator, i.e. when the firms face the prospect of asymmetrical costs.

¹⁹These terms are discussed at length by Beath et al. (1994)

Using the expressions set out on page 32 in appendix 7.1 it can easily be shown that the *strategic investment incentive* always dominates the *stand alone effect*. This can be done for both of the comparisons arising in table 3.2 above.

I conclude that where the firms choose a strategy of non-specialisation the firms in the RJV will always face a lower incentive to innovate than non-cooperative firms if both choose the strategy of non-specialisation.

4.3.2 Partial specialisation

I show below that joint profits may be higher where one of the two firms in the RJV specialises and the other remains a generalist when μ is large. This *partial specialisation* allows the firms to reduce the costs of acquiring absorptive capacity. As before I set out the objective functions, first order conditions and the innovation incentives.

Notice that because only one of the two firms will now be acquiring a new technology from the other, the non-cooperative equilibrium involves the payment of a license fee F . I will characterise this fee in the discussion below.

The objective functions in this case are²⁰:

The Social Optimum:

$$W = \rho_O^2 S_{ab}(\tilde{\Lambda}) + \rho_O (1 - \rho_O) S_d + (1 - \rho_O)^2 S_e - 2\gamma(\rho_O) \quad (11)$$

The RJV-equilibrium:

$$\Pi_R = \rho_R^2 \Sigma_{ab}(\tilde{\Lambda}) + 2\rho_R (1 - \rho_R) \Sigma_d + (1 - \rho_R)^2 \Sigma_e - 2\gamma(\rho_R) \quad (12)$$

The non-cooperative equilibrium:

$$\begin{aligned} \pi_N^s = & \rho_N^s \rho_N^g \left(\pi_{ab}^s(\tilde{\Lambda}) + F \right) + \rho_N^s (1 - \rho_N^g) \pi_{\underline{d}} + (1 - \rho_N^s) \rho_N^g \pi_{\bar{d}} \\ & + (1 - \rho_N^s) (1 - \rho_N^g) \pi_e - \gamma(\rho_N^s) \end{aligned} \quad (13)$$

$$\begin{aligned} \pi_N^g = & \rho_N^g \rho_N^s \left(\pi_{ab}^g(\tilde{\Lambda}) - F \right) + \rho_N^g (1 - \rho_N^s) \pi_{\underline{d}} + (1 - \rho_N^g) \rho_N^s \pi_{\bar{d}} \\ & + (1 - \rho_N^g) (1 - \rho_N^s) \pi_e - \gamma(\rho_N^g) \end{aligned} \quad (14)$$

From these objective functions we can derive the first order conditions:

The Social Optimum:

$$\rho_O \left(S_{ab}(\tilde{\Lambda}) - S_d \right) + (1 - \rho_O) (S_d - S_e) = \gamma'(\rho_O)$$

The RJV-equilibrium:

$$\rho_R \left(\Sigma_{ab}(\tilde{\Lambda}) - \Sigma_d \right) + (1 - \rho_R) (\Sigma_d - \Sigma_e) = \gamma'(\rho_R) \quad (15)$$

The non-cooperative equilibrium:

$$\rho_N^g \left(\pi_{ab}^s(\tilde{\Lambda}) + F - \pi_{\bar{d}} \right) + (1 - \rho_N^g) (\pi_{\underline{d}} - \pi_e) = \gamma'(\rho_N^s) \quad (16)$$

²⁰All variables should carry a superscript p here to indicate that they are specific to the equilibrium in which one firm chooses the role of a specialist whilst the other chooses the role of a generalist. I suppress the superscripts for the time being in order to unclutter the notation.

$$\rho_N^s \left(\pi_{ab}^g(\tilde{\Lambda}) - F - \pi_{\bar{d}} \right) + (1 - \rho_N^s) (\pi_{\underline{d}} - \pi_e) = \gamma'(\rho_N^g) \quad (17)$$

Once more I assume that a unique interior solution to the above expressions exists. I will return to this assumption below. Notice that as $\mu \rightarrow 1$ the strategy of partial specialisation becomes a strategy of duplicating research vectors.

The license fee F has to leave the licensor at least as well off as they would be without issuing their license and can be no greater than the added profit for the licensee. I will assume that the firms have equal bargaining power²¹ The license fee to be payed when both firms innovate will be:

$$F \equiv \frac{1}{2} \left(\pi_{ab}^g - \pi_{\underline{d}} \right) + \frac{1}{2} \left(\pi_{\underline{d}} - \pi_{ab}^s \right) = \frac{\pi_{ab}^g - \pi_{ab}^s}{2} \quad (18)$$

Is partial specialisation feasible? In this model the two firms are *ex ante* identical. This implies that if in a non-cooperative equilibrium one of the two firms agrees to specialize, whilst the other remains a generalist the firms' profits will have to be *ex post* identical too.

The license fee F set out above is sufficient for the profit incentives and competitive threats of the generalist and the specialist firm to be equal. Therefore $\rho_N^s = \rho_N^g$ and the firms' expected profits will also be equal:

$$\pi_N^p = \pi_N^g(F) = \pi_N^s(F) \quad (19)$$

This also has the implication that the expected profits from the research strategies of non-specialisation and partial specialisation will be equal when the following condition is satisfied:

$$\pi_N^p = \pi_N^n \Leftrightarrow \pi_{ab} - F = \pi_a \Leftrightarrow \pi_{ab} + \pi_{\bar{a}b} = 2\pi_a$$

This condition is the same as that for indifference between the two research strategies by the firms in an RJV. The corresponding boundary in μ, g space is derived in Appendix 7.6.

The innovation incentives that follow from this discussion are set out in the following table:

Comparing Innovation Incentives		
	The Competitive Threats	The Profit Incentives
Social Optimum	$S_{ab}(\tilde{\Lambda}) - S_d = [CS_{ab}(\tilde{\Lambda}) - CS_d] + [\Sigma_{ab}(\tilde{\Lambda}) - \Sigma_d]$	$S_d - S_e = [CS_d - CS_e] + [\Sigma_d - \Sigma_e]$
RJV	$\Sigma_{ab}(\tilde{\Lambda}) - \Sigma_d$	$\Sigma_d - \Sigma_e$
Non-Coop. Firms	$\frac{1}{2} (\pi_{ab} + \pi_{\bar{a}b}) - \pi_{\bar{d}} = \frac{1}{2} (\Sigma_{ab}(\tilde{\Lambda}) - \Sigma_d) + \frac{1}{2} (\pi_{\underline{d}} - \pi_{\bar{d}})$	$\pi_{\underline{d}} - \pi_e = \frac{1}{2} (\Sigma_d - \Sigma_e) + \frac{1}{2} (\pi_{\underline{d}} - \pi_{\bar{d}})$

Table 3

²¹The assumption that both firms have equal bargaining power, seems defensible in light of their *ex ante* symmetry.

This table presents a similar set of results to those in table 2 above. The *undervaluation effect* once more reduces the profit incentive and competitive threats of the RJV below the socially optimal level.

Furthermore the the *stand alone effect* and the *strategic investment effect* again interact to provide a stronger incentive for innovation to the non-cooperative firms than to the RJV. This can be easily shown using the expressions set out on page 32 below.

Therefore I have now demonstrated that independently of the research strategy the firms in the RJV face lower innovation incentives than the firms in a non-cooperative licensing equilibrium, when we compare the two modes of R&D cooperation under the same research strategy. I have yet to determine whether the non-cooperative firms and the RJV will always pick the same research strategies. This is done in the following section.

4.3.3 Conditions for the existence of a unique interior equilibrium

Here I introduce two further assumptions that guarantee the existence of a unique interior solution for all of the above expressions which I have so far assumed. Notice that the first order conditions (8)-(10) and (15)-(17) all share an identical structure. In each case the expression on the LHS is the marginal benefit of an increase in the probability of discovery by a firm. This expression is always linear in ρ and strictly positive when $\rho = 0$. Given my assumption at (2) a solution with $\rho = 0$ is impossible. From now on I also assume that:

- for all $0 < \Lambda < 1$, a solution with $\rho = 1$ is impossible:

$$\gamma'(1) > \text{MAX} \left[(S_a(\Lambda) - S_d), (S_{ab}(\tilde{\Lambda}) - S_d), (\Sigma_a(\Lambda) - \Sigma_d), (\Sigma_{ab}(\tilde{\Lambda}) - \Sigma_d), (\pi_a(\Lambda) - \pi_d), (\pi_{\underline{ab}} - \pi_{\bar{d}}) \right]$$

There must therefore be at least one interior value of ρ for which the above first order conditions are fulfilled.

- I also impose the assumption that the interior equilibrium is unique. This implies that the cost function has the property that the marginal cost curve always cuts the marginal benefit curve from below. For the case of the non-cooperative equilibrium this implies that:

$$\Delta \equiv \gamma''(\rho) + h > 0 \text{ where } h = \text{MAX} \left\{ (2\pi_d - \pi_a(\Lambda) - \pi_e), (\pi_{\bar{d}} + \pi_{\underline{d}} - \pi_{ab}(\tilde{\Lambda}) - \pi_e) \right\}$$

4.4 Stage 1: Choosing the optimal research vectors

In this section I show that the formation of an RJV sometimes allows firms to co-ordinate their R&D more effectively than they could in a non-cooperative equilibrium with licensing.

The choice of the optimal research vectors by the two firms depends on the equilibrium they are in and on the level of the absorption threshold.

Beginning with the RJV I show that irrespective of the absorption costs the firms in the RJV will always choose maximally differentiated research vectors. Then I demonstrate below that firms in an RJV will choose an asymmetric configuration of research vectors when the costs of absorption are high.

Turning to the non-cooperative equilibrium I demonstrate that firms may choose minimally or maximally differentiated R&D vectors²². I then demonstrate that firms in this

²²This result is first derived in a companion paper Ulph and von Graevenitz (2000)

type of equilibrium may be unable to choose an asymmetric equilibrium because it is not incentive compatible. Finally I discuss the welfare implications of these findings through comparisons of the RJV and licensing equilibrium with the socially optimal choice of R&D vectors.

4.4.1 The Socially Optimal Choice of R&D Vectors

In the social optimum the firms would choose their R&D vectors while taking into account the social surplus thereby generated and not simply their own- or joint-profits. In this section I demonstrate that it is always socially optimal to maximally differentiate R&D vectors. I also investigate whether it is ever socially optimal for firms to choose asymmetrical R&D vectors.

To do this I differentiate the welfare functions for symmetrical and asymmetrical equilibria with respect to the measure of distance between the firms' R&D vectors Λ that was introduced above. Note that this measure will have slightly different domains depending on the research strategy which the firms adopt:

$$\text{Symmetric absorption capability:} \quad 0 \leq \Lambda \leq (1 - 2\mu) \quad (20)$$

$$\text{Partial specialisation:} \quad 0 \leq \tilde{\Lambda} \leq (1 - \mu) \quad (21)$$

The expected social surplus arising from the two research strategies is²³:

$$W^n = (\rho^n)^2 S_a(\Lambda) + 2\rho^n (1 - \rho^n) \cdot S_d + (1 - \rho^n)^2 \cdot S_e - 2\gamma(\rho^n) \quad (5)$$

$$W^p = (\rho^p)^2 S_{ab}(\tilde{\Lambda}) + 2\rho^p (1 - \rho^p) S_d + (1 - \rho^p)^2 \cdot S_e - 2\gamma(\rho^p) \quad (11)$$

where the probabilities of innovation are functions of the degree of differentiation: $\rho^n(\Lambda)$, $\rho^p(\tilde{\Lambda})$.

I derive the following Result in Appendix 7.3:

Result 2

It is always socially optimal to differentiate R&D vectors maximally whilst maintaining the ability to absorb outside R&D to the extent that this is optimal. This implies that:

$$\Lambda = 1 - 2\mu \quad \tilde{\Lambda} = 1 - \mu$$

At the first stage of the game described above the firms choose both their research vector and their research strategy. The research vector the firms can choose is dependent on the choice of the research strategy as Result 2 shows. The firms' choice of research strategy is a function of the size of the innovation which they are attempting and the cost of absorptive capacity which they face.

The following result characterises the socially optimal choice of research strategy:

Result 3

It will be socially optimal to choose the research strategy of partial specialisation whenever

$$g > \frac{8(1-3\mu)}{(34\mu-5-21\mu^2)}.$$

This result is derived in Appendix 7.4.

To illustrate the result I plot the boundary in μ, g space between the parameters for which non-specialisation is optimal and those for which partial specialisation is optimal.

²³I suppress the o subscripts here.

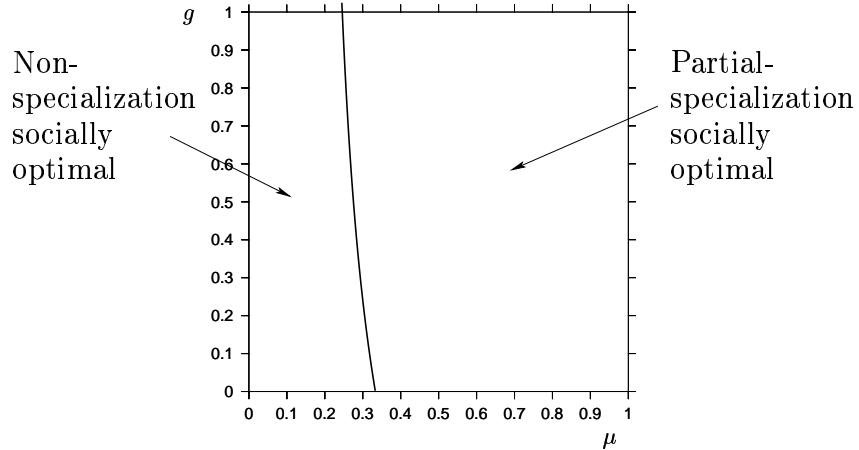


Figure 3: **When it is socially optimal to choose asymmetric research vectors**

The plot shows that, it will be optimal to choose asymmetrical R&D vectors for lower absorption thresholds, the greater the size of innovation g . In interpreting the diagram we should bear in mind that given g , an increase in μ represents a greater fraction of innovative activity going towards the ability to exchange information.

We showed above that it will always be socially optimal for both firms to share information with one another. In this sense the inability of one firm to absorb the innovations of the other, that would arise from specialisation, represents a loss to society. However as innovations become larger, a given μ represents a growing absolute expenditure on absorptive capacity by both firms. Thus there is a gain if one firm specialises in cost reduction only, and this gain increases with g for a given μ . The boundary plotted in figure 3 represents the locus of those points where this gain exactly outweighs the loss to society that arises from asymmetry.

4.4.2 The choice of research vectors by the RJV

The firms in the RJV will always find it profit maximising to differentiate their research paths as much as possible. We can show this by analysing their expected profits for equilibria with symmetric and asymmetric absorption capacities.

The firms expected profits under the two research strategies are:

$$\Pi^n = (\rho^n)^2 \Sigma_a + 2\rho^n (1 - \rho^n) \cdot \Sigma_d + (1 - \rho^n)^2 \cdot \Sigma_e - 2\gamma(\rho^n) \quad (22)$$

$$\Pi^p = (\rho^p)^2 \Sigma_{ab} + 2\rho^p (1 - \rho^p) \cdot \Sigma_d + (1 - \rho^p)^2 \cdot \Sigma_e - 2\gamma(\rho^p) \quad (23)$$

where the probabilities of innovation are once again functions of the degree of differentiation of the firms' research vectors $\rho^n(\Lambda)$, $\rho^p(\tilde{\Lambda})$. I drop the R subscripts on the probabilities here to ease notation. Given these expressions I can show that:

Result 4

The firms in the RJV will always choose maximal differentiation of R&D vectors.

Result 4 is proved in Appendix 7.5 where I show that:

$$\frac{d\Pi^n}{d\Lambda} = (\rho^n)^2 \frac{d\Sigma_a}{d\Lambda} > 0 \qquad \frac{d\Pi^p}{d\tilde{\Lambda}} = (\rho^p)^2 \frac{d\Sigma_{ab}}{d\tilde{\Lambda}} > 0$$

The firms in the RJV must choose which research strategy to follow. I showed above (Result 1) that the firms will always prefer to share information with one another. This implies that there is a loss in joint profits if one firm is unable to absorb the innovation of the other. However by choosing a strategy of partial specialisation the RJV will reduce expenditure on absorptive capacity that may be employed in further reducing the costs of both firms. I can show that a boundary exists along which these two effects exactly offset each other.

In order to determine when asymmetric research vectors are optimal for the firms I compare the expected profits under symmetric and asymmetric research vectors:

$$\Pi^n = (\rho^n)^2 \Sigma_a + 2\rho^n (1 - \rho^n) \cdot \Sigma_d + (1 - \rho^n)^2 \cdot \Sigma_e - 2\gamma(\rho^n) \qquad (6)$$

$$\Pi^s = (\rho^p)^2 \Sigma_{ab} + 2\rho^p (1 - \rho^p) \cdot \Sigma_d + (1 - \rho^p)^2 \cdot \Sigma_e - 2\gamma(\rho^p) \qquad (12)$$

where the probabilities are determined by the following f.o.c.'s:

$$\rho^n \cdot (\Sigma_a - \Sigma_d) + (1 - \rho^n) \cdot (\Sigma_d - \Sigma_e) = \gamma'(\rho^n)$$

$$\rho^p \cdot (\Sigma_{ab} - \Sigma_d) + (1 - \rho^p) \cdot (\Sigma_d - \Sigma_e) = \gamma'(\rho^p)$$

The first order conditions show us that the probability of innovation in an asymmetric equilibrium ρ^p will be greater than that in a symmetric equilibrium as soon as $\Sigma_{ab} > \Sigma_a$. The expected profits for the RJV from an asymmetric equilibrium will also be greater as soon as this condition is fulfilled.

Result 5

Whenever $g > \frac{6\mu-2}{3\mu^2-4\mu-1}$ the firms in the RJV will choose asymmetric research vectors.

This Result is derived in Appendix 7.6. Figure 4 below shows this boundary between the parameter combinations for which RJVs choose symmetrical and asymmetrical R&D vectors.

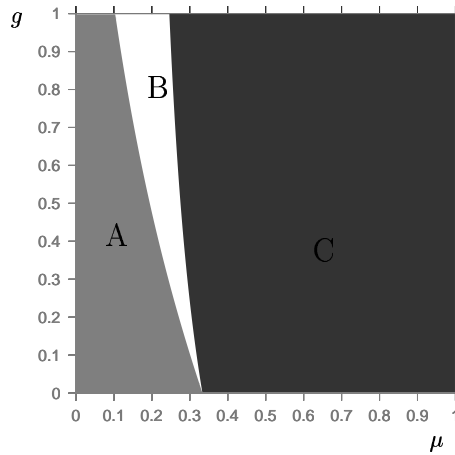


Figure 4: **Boundaries for partial specialisation to be optimal and profitable.**

The figure not only plots the boundary between those parameter combinations for which symmetrical and asymmetrical research vectors are profit maximising for the RJV, but also the boundary beyond which it is socially optimal for firms to choose asymmetrical research vectors. The figure divides the parameter space μ, g into three segments:

- A** Here the firms choose symmetric absorptive capacities and this is socially optimal.
- B** Here the firms choose asymmetric absorptive capacities whilst this is socially sub-optimal.
- C** Here the firms choose asymmetric absorptive capacities which is socially optimal.

Firms clearly specialise sooner than is socially optimal. RJVs that are described by the parameter combinations in segment **B** will produce the welfare loss from undervaluation, and an additional welfare loss due to inappropriate specialisation. This welfare loss arises because the social and private incentives for both firms to invest in absorptive capacity differ. It captures the *absorption problem*.

In the table below I set out the innovation incentives in the social optimum and those faced by the RJV for parameter combinations μ, g in area **B**. The table allows me to illustrate how the *absorption problem* arises. The bold term $(S_a - S_{ab})$ captures the social incentive to invest in absorptive capacity for both firms, which the RJV does not face.

When the RJV faces parameter combinations in area **A** it can be shown that the RJVs incentive to invest in absorptive capacity $(\Sigma_a - \Sigma_{ab})$ falls short of the social incentive. However the discrete nature of the investment choice in this model has the consequence that the overall welfare loss which can be ascribed to the RJV is unaffected by the difference between the private and social incentive to invest in absorptive capacity, because the firms in the RJV make the correct investment decision. However once the RJVs incentive to invest in absorptive capacity for both firms is nil, the *absorption problem* arises because $S_a - S_{ab} > 0$ at this point.

The Absorption Problem		
	The Competitive Threats	The Profit Incentives
Social Optimum	$S_a(\Lambda) - S_d = (\mathbf{S_a} - \mathbf{S_{ab}}) +$ $[CS_{ab}(\Lambda) - CS_d] + [\Sigma_{ab}(\Lambda) - \Sigma_d]$	$S_d - S_e =$ $[CS_d - CS_e] + [\Sigma_d - \Sigma_e]$
RJV	$\Sigma_{ab}(\Lambda) - \Sigma_d = \Sigma_a - \Sigma_{ab} +$ $\Sigma_{ab} - \Sigma_d$	$\Sigma_d - \Sigma_e$

Table 4

It now remains to determine whether the non-cooperative licensing equilibrium gives rise to a similar welfare loss.

4.4.3 The choice of research vectors in a non-cooperative equilibrium

The expected profits of each individual firm in a non-cooperative equilibrium are exactly the same as those of each individual firm in the RJV. Asymmetric R&D vectors therefore have the potential to increase the joint profits of the firms in a non-cooperative equilibrium for the same parameter range as for an RJV.

In contrast to the RJV which maximised joint profits the firms in a non-cooperative equilibrium maximise only their own profits. When both firms choose symmetrical research vectors the value of the exchanged technology will be the same to both firms and thus no payments will be necessary.

However in a case in which one of the firms chooses to forgo the ability to absorb the innovations of its rival (asymmetrical research vectors), that firm will charge the other a license fee for any technology it passes on. Assuming equal bargaining power of the two firms neither firm will be willing to become a ‘specialist’, unless it can expect to become at least as well off as its rival in the process.

Below I set out the expected profits of both firms in a non-cooperative equilibrium in which they choose symmetrical research vectors as well as for an equilibrium in which the firms choose asymmetrical research vectors²⁴.

$$\pi^n = (\rho^n)^2 \pi_a(\Lambda) + \rho^n (1 - \rho^n) \cdot [\pi_{\underline{d}} + \pi_{\bar{d}}] + (1 - \rho^n)^2 \cdot \pi_e - \gamma(\rho^n) \quad (7)$$

Asymmetrical R&D Strategies:

$$\pi_1^p = (\rho^p)^2 \left(\pi_{\underline{ab}}(\tilde{\Lambda}) + F \right) + \rho^p (1 - \rho^p) [\pi_{\underline{d}} + \pi_{\bar{d}}] + (1 - \rho^p)^2 \pi_e - \gamma(\rho^p) \quad (13)$$

$$\pi_2^p = (\rho^p)^2 \left(\pi_{\underline{ab}}(\tilde{\Lambda}) - F \right) + \rho^p (1 - \rho^p) [\pi_{\underline{d}} + \pi_{\bar{d}}] + (1 - \rho^p)^2 \pi_e - \gamma(\rho^p) \quad (14)$$

Here firm 1 is the specialist and firm 2 is the generalist. The probabilities of innovation are functions of the degree of differentiation between the firms’ research vectors: $\rho^n(\Lambda)$, $\rho^p(\tilde{\Lambda})$. The license fee F is defined in equation (18) above.

The incentives to differentiate R&D vectors facing the firms in a noncooperative equilibrium are set out next. Then I show that not all equilibria involving asymmetrical R&D vectors are incentive compatible in a non-cooperative equilibrium.

The following differentiation result is derived in Appendix 7.7:

Result 6

In a non-cooperative equilibrium the firms will choose either minimal or maximal differentiation of R&D vectors. They will choose minimal differentiation if diminishing returns to R&D are small.

Formally I show that:

$$\frac{d\Pi^n}{d\Lambda} = \frac{d\pi_a}{d\Lambda} \cdot \frac{\rho^n}{\Delta} \left\{ \rho^n \cdot \gamma''(\rho^n) + [\pi_{\bar{d}} - \pi_e] \right\} \quad (24)$$

$$\frac{d\Pi^p}{d\tilde{\Lambda}} = \frac{d\pi_{ab}}{d\tilde{\Lambda}} \cdot \frac{\rho^p}{\tilde{\Delta}} \left\{ \rho^p \cdot \gamma''(\rho^p) + [\pi_{\bar{d}} - \pi_e] \right\} \quad (25)$$

²⁴In order to keep the notation simple I suppress all the $_N$ subscripts here. I have also deviated from the superscripts g and s for the same reason below.

$$\text{where } \Delta = \gamma''(\rho_N^p) + [\pi_{\underline{d}} + \pi_{\bar{d}} - \pi_e - \pi_a]$$

$$\text{and where } \tilde{\Delta} = \gamma''(\rho_N^p) + [\pi_{\underline{d}} + \pi_{\bar{d}} - \pi_e - \pi_a]$$

The above result shows us that under fairly strong conditions on the R&D cost function we can show that firms in non-cooperative equilibria will restrict their R&D activity in order to reduce competition between them²⁵. In the following I assume that the R&D cost function is sufficiently steep, as to rule out cases in which the non-cooperative firms choose minimal differentiation. This will bias my welfare comparisons very slightly in favour of licensing equilibria²⁶.

In those cases in which firms in a non-cooperative equilibrium do choose to maximally differentiate their research vectors I can demonstrate the following result:

Result 7

The firms in a non-cooperative equilibrium may be unable to choose asymmetrical R&D vectors because the payment of a license fee is not incentive compatible.

I showed above that firms in a noncooperative equilibrium will only be able to achieve equilibria with asymmetrical research vectors if there exists a license fee F that the ‘generalist’ firm is willing to pay and that the ‘specialist’ firm is willing to accept.

The license fee that the ‘specialist’ firm is willing to accept is F . The ‘generalist’ firm will have no incentive *ex post* to pay this license fee if this lowers its profits below the level of profits $\pi_{\underline{ab}}^1(\sigma)$ which it can achieve given a level of involuntary spillovers σ . In the event of simultaneous discovery a non-cooperative equilibrium with asymmetrical R&D vectors is not incentive compatible when:

$$\pi_{\underline{ab}}^1(\sigma) > \pi_{\underline{ab}}^1(1) - F \quad (26)$$

In Appendix 7.8 I show that this condition implies that the non-cooperative firms will sometimes be unable to choose the research strategy of partial specialisation because the generalist would have no incentive to pay the specialist a license fee.

Put in another way this implies that neither firm will be willing to become a specialist, as it will fear that the other firm as ‘generalist’ will take advantage of its R&D without it receiving any compensation for the resulting cost asymmetry. The resulting inability of the firms in the non-cooperative case to choose the strategy of partial specialisation arises from the interaction of the *appropriability problem* and the barrier to technological absorption which the absence of absorptive capacity erects for the specialist firm. Once a firm has gone down the route of specialisation its ability to extract a license fee depends on its ability to exclude the other firm from its technology. Where the *appropriability problem* is large it undermines this ability and the attraction of becoming a specialist.

The boundary between the sections of the parameter space in which the non-cooperative firms are able to choose asymmetrical research vectors and in which they are not is given by the following expression, which is derived in Appendix 7.8:

$$g \left[2(4\sigma - 1) + (1 - \mu) [8\sigma^2 - 5] \right] \geq 2 [1 - 4\sigma] \quad (27)$$

²⁵Compare Ulph and von Graevenitz (2000)

²⁶Remember that we showed in the previous chapter that the decreasing returns to scale in the R&D cost function have to be very low for the firms to choose minimal differentiation.

This inequality defines a surface in the three dimensional parameter space with the dimensions μ , g , σ .

In the representation of the parameter space below I demonstrate for which parameter combinations the non-cooperative firms must choose an equilibrium in which neither specialises, although it would be socially optimal and sometimes also more profitable to do otherwise. The figure shows the parameter space from two different angles. The face of the

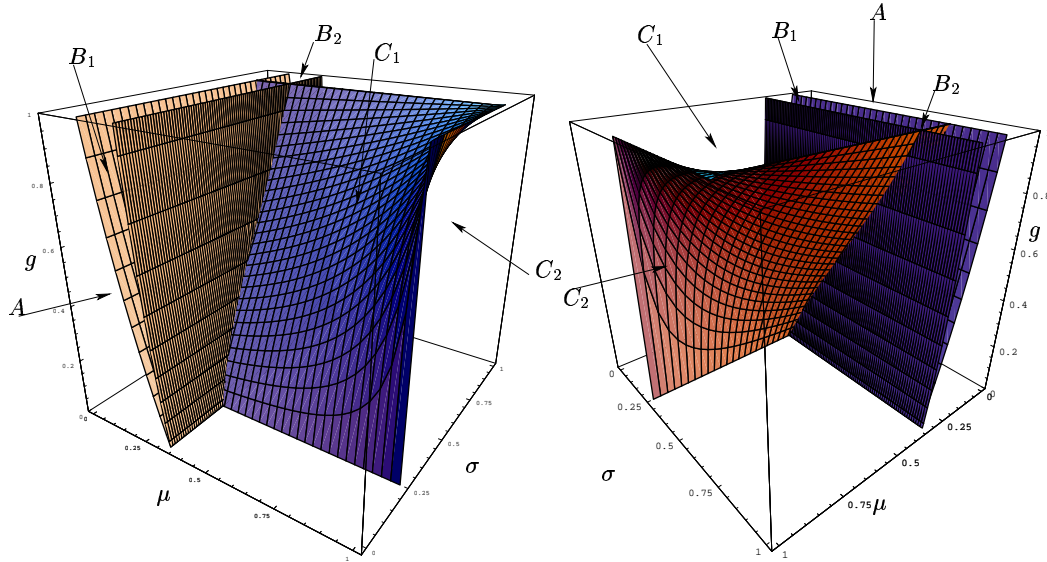


Figure 5: **The parameter space: Showing the region within which non-cooperative firms cannot choose partial specialisation.**

parameter space given by the axes μ , g is the two dimensional space of figures 3 and 4. The surface defined by (27) separates the regions marked as $B1, C1$ from those marked $B2, C2$ in the figure above. The regions A, B, C in the figure correspond to A, B, C in figure 4. Notice that regions $B1, C1$ are characterised by lower values of σ , than regions $B2, C2$.

Region:

- A** is the set of all parameter combinations for which the firms in an RJV and the non-cooperative firms choose the research strategy of non-specialisation and this is socially optimal.
- B1** is the set of all parameter combinations for which the firms in an RJV and the non-cooperative firms choose the research strategy of partial specialisation and this is socially suboptimal.
- B2** is the set of all parameter combinations for which the RJV chooses partial specialisation and the non-cooperative firms are unable to do so and are forced into the socially optimal strategy of non-specialisation.
- C1** is the set of all parameter combinations for which the firms in an RJV and the non-cooperative firms choose the research strategy of partial specialisation and this is socially optimal.

C2 is the set of all parameter combinations for which the RJV chooses partial specialisation and the non-cooperative firms are unable to do so and are forced into the socially suboptimal research strategy of non-specialisation.

I now investigate what consequences this inability to partially differentiate has for the relative efficiency of allocation of resources to R&D investment, in the cooperative and non-cooperative equilibria.

I have demonstrated above that the non-cooperative firms always under-invest by a smaller amount than the firms in an RJV given the same research strategy. It remains to determine whether or not the inability of the non-cooperative firms to choose asymmetrical research vectors can change this conclusion for some of the parameter space. I pursue this question in the following section.

5 Achieving the Social Optimum

In this section I establish the relative level of the welfare losses that arise in the non-cooperative and the RJV-equilibria. I also investigate whether an R&D subsidy is useful as a means to address these welfare losses.

In the previous sections I have established that wherever the RJV and the non-cooperative firms choose the same research strategies, the firms in the RJV face weaker incentives to invest than the non-cooperative firms. However I have also shown that the firms in the RJV and the non-cooperative firms will make different choices regarding their research strategies for segments **B2** and **C2** of the parameter space as it is depicted in figure 5.

The question that now arises is whether the welfare loss in a non-cooperative equilibrium becomes greater than that in the cooperative equilibrium where the non-cooperative firms are forced to choose the socially suboptimal research strategy of non-specialisation. In table 5 below I set out the innovation incentives for the case in which it is socially optimal for the firms to adopt non-specialisation and the non-cooperative firms are unable to do this.

Notice that the inability of the non-cooperative firms to partially specialise affects only the competitive threats they are facing. The profit incentives facing the non-cooperative firms will always be greater than those facing the firms in an RJV as was set out in the discussion of tables 2 and 3 above. As long as the non-cooperative firms face at least as great a competitive threat as the firms in the RJV, they will innovate with a greater probability and the welfare loss associated with the RJV will outweigh that associated with the non-cooperative firms.

If the competitive threat facing the non-cooperative firms is lower than that facing the firms in the RJV it is possible but not necessary that the probability of innovation of the RJV ρ_R^p is greater than that of the non-cooperative firms ρ_N^n . This depends on the R&D cost function though. Therefore we can only delineate for which parameters it is possible that the welfare loss in an RJV equilibrium might be lower than in a non-cooperative equilibrium.

The table below sets out how the competitive threats of the noncooperative firms are affected by their inability to partially specialise.

Notice that the bold terms are both negative and capture the reduction in the competitive threats of the non-cooperative firms which arise because these cannot choose to partially specialize.

Comparing Competitive Threats		
	The Competitive Threats	The Profit Incentives
Social Optimum	$S_{ab}(\tilde{\Lambda}) - S_d =$ $[CS_{ab}(\Lambda) - CS_d] + [\Sigma_{ab}(\Lambda) - \Sigma_d]$	$S_d - S_e =$ $[CS_d - CS_e] + [\Sigma_d - \Sigma_e]$
RJV	$\Sigma_{ab}(\Lambda) - \Sigma_d$	$\Sigma_d - \Sigma_e$
$\mu < \frac{1}{2}$ Non-Coop.	$\pi_a(\Lambda) - \pi_{\bar{d}} = \frac{1}{2} \left(\Sigma_{ab}(\tilde{\Lambda}) - \Sigma_d \right) +$ $\frac{1}{2} \left(\Sigma_{\mathbf{a}}(\Lambda) - \Sigma_{\mathbf{ab}}(\tilde{\Lambda}) \right) + \frac{1}{2} (\pi_{\underline{d}} - \pi_{\bar{d}})$	$\pi_{\underline{d}} - \pi_e =$ $\frac{1}{2} (\Sigma_d - \Sigma_e) + \frac{1}{2} (\pi_{\underline{d}} - \pi_{\bar{d}})$
Firms $\mu \geq \frac{1}{2}$	$\pi_a(0) - \pi_{\bar{d}} = \frac{1}{2} \left(\Sigma_{ab}(\tilde{\Lambda}) - \Sigma_d \right) +$ $\frac{1}{2} \left(\Sigma_{\mathbf{a}}(\mathbf{0}) - \Sigma_{\mathbf{ab}}(\tilde{\Lambda}) \right) + \frac{1}{2} (\pi_{\underline{d}} - \pi_{\bar{d}})$	$\pi_{\underline{d}} - \pi_e =$ $\frac{1}{2} (\Sigma_d - \Sigma_e) + \frac{1}{2} (\pi_{\underline{d}} - \pi_{\bar{d}})$

Table 5

As long as the difference between these terms and the underlined terms is not negative the competitive threats facing the non-cooperative firms are still at least as great as that facing the firms in an RJV. Given the expressions set out in Appendix 7.1 below it is easily shown that even where the non-cooperative firms can no longer cross-license the competitive threat they face is greater than that faced by the firms in an RJV:

$$\Sigma_{ab}(\Lambda) - \Sigma_d - \pi_a(0) + \pi_{\bar{d}} = 2(1+g) \left[\tilde{\Lambda} - 1 \right] + 5g \left[\tilde{\Lambda} \right]^2 - g < 0 \quad (28)$$

This shows that even when the welfare losses which arise in the non-cooperative equilibrium as a result of non-specialisation are maximal the *strategic investment effect* is strong enough to more than compensate these. Therefore I can now put forward the following result:

Result 8

The firms in an RJV will always under-invest in R&D to a greater extent than the non-cooperative firms because of the strength of the strategic investment effect.

5.1 Two alternative subsidy regimes

The discussion in the previous sections has shown that there are two separate sources of welfare losses where absorptive capacity matters:

- I The first source of welfare loss is that both the firms in the RJV and the non-cooperative firms face incentives to innovate that are too low. In particular both the competitive threats and the profit incentives are too low, so that the firms under-invest in R&D.
- II The second source of welfare loss is that the firms in the RJV adopt the strategy of partial differentiation for a lower absorption threshold than is optimal, this is the *absorption problem* as discussed in section 4.4.2 on page 17 . In a non-cooperative equilibrium the firms adopt the strategy of partial specialisation for lower absorption thresholds than is socially optimal if spillovers are not too high.

Although I make no attempt to model RJV formation here it should be noted that we can expect firms to find RJV formation to be particularly attractive in those cases in which the non-cooperative firms cannot partially differentiate. Firms will be able to raise their profits by forming RJVs in these cases, so that even if RJV formation were more costly than licensing it might nonetheless become more attractive.

Oxley (1997) sets out a transactions cost approach to the comparison of licensing and RJV formation and suggests that RJV formation will be costlier because it involves the creation of a more hierarchical structure to manage the relationship between two firms. However she also points out that it is this more hierarchical structure which allows the firms to co-operate in the context of greater asymmetries between the firms. She goes on to test this hypothesis and finds some support for it.

The theory developed here suggests that RJV formation will be particularly attractive to firms when these wish to adopt a research strategy which leads to asymmetries between them. It provides a further reason to expect the formation of RJVs to take place in the context of firm asymmetries or to lead to firm asymmetries.

The implication of this discussion is that the *absorption problem* is likely to arise quite regularly because it is profit maximising for firms to adopt a mode of R&D cooperation, the RJV, that gives rise to it.

As put forward in Spence (1984) the solution to the problem of under-investment in R&D is to provide an R&D subsidy to the firms. Such a subsidy can in principle make the firms in an RJV or a non-cooperative equilibrium choose the correct level of R&D investment. In reality it will be impossible to determine the correct level of the subsidy but as Spence (1984) shows a subsidy that is chosen at a fixed rate can certainly reduce welfare losses substantially.

Notice however that such a subsidy will not affect the incentives of firms in an RJV to adopt a policy of partial specialisation. This is because an R&D subsidy will change the incentive to invest in R&D and thereby the probability of innovation but it will not affect the incentive to choose a particular research strategy which depends on the tradeoff of profits in the case in which both firms innovate.

An alternative kind of subsidy regime would be to pay the firms an additional prize upon successful innovation by both firms²⁷. The introduction of an innovation prize T in the case where both firms innovate simultaneously can help to address both the problem of under-investment in R&D (I) and the absorption problem (II). I compare this approach to a simple R&D subsidy below.

In what follows I focus on subsidy programs for RJVs. The discussion above and here has already shown that the formation of an RJV will enable firms to adopt the research strategy of *partial specialisation* where non-cooperative firms would otherwise be unable to do so. This is especially important because the proposed subsidy regimes do not affect the root of the inability to adopt partial specialisation on the part of the non-cooperative firms. Any policy seeking to do this would have to reduce the involuntary spillover of technology σ towards zero²⁸. The costs and benefits of such a policy are much harder to quantify that

²⁷A similar regime is suggested by Pérez-Castrillo and Sandonís (1996) in the context of an entirely different model of RJVs.

²⁸This is the policy approach favoured by Katz and Ordover (1990), although they derive it in the context of a set of very different models. In the context of a model in which the non-cooperative firms engage in cross-licensing this policy is much less damaging than where firms do not do this.

those of the subsidy regimes I discuss below. Therefore I ignore such a policy here.

To summarise the findings in this section I can say that:

- The RJV and the non-cooperative firms always underinvest in R&D.
- The RJV will always underinvest more than the non-cooperative firms. Compare result 8.
- The firms in the RJV choose the strategy of partial specialisation too early (segment **B** in figure 5) and the non-cooperative firms may choose it too early too (segment **B1**).
- The firms in a non-cooperative equilibrium will be unable to choose the strategy of partial specialisation for a large part of parameter space in which this would be socially optimal (section **C2**).
- A subsidy set to address the undervaluation effect will not address either of the last two problems.
- Such a subsidy joined with the policy of permitting RJV formation will address the undervaluation effect and the inability of the non-cooperative firms to partially specialise. It will not address the *absorption problem*.

Result 9

The policies of permitting RJV formation and subsidising R&D by RJVs are complements. Instrumenting one without the other can be counterproductive in augmenting already existing welfare losses. However these two policies cannot always achieve the social optimum.

I discuss the policy that will achieve the social optimum below.

5.1.1 An R&D subsidy

I assume that the government will set a subsidy to the R&D expenditure of the RJV before the firms in it determine the degree of complementarity of their research vectors. This subsidy will be denoted τ here. I will follow our approach in the previous chapter as well as Hinloopen (1997), Hinloopen (2000) and Stenbacka and Tombak (1998) in ignoring the distortions caused by the need to fund an R&D subsidy.

It is easily shown that an optimal R&D subsidy $\hat{\tau}$ exists which induces the RJV to innovate with the socially optimal probability ρ_O . I set out the optimal level of subsidy for the cases in which both firms remain generalists and for the partial specialisation case below. The first order conditions for the RJV in the case of subsidisation follow directly from those set out above in equations (9) and (15):

$$\begin{aligned} \rho_R (\Sigma_a(\Lambda) - \Sigma_d) + (1 - \rho_R) (\Sigma_d - \Sigma_e) &= (1 - \tau) \gamma'(\rho_R) && \text{Non-specialisation} \\ \rho_R (\Sigma_{ab}(\tilde{\Lambda}) - \Sigma_d) + (1 - \rho_R) (\Sigma_d - \Sigma_e) &= (1 - \tau) \gamma'(\rho_R) && \text{Part. Specialisation} \end{aligned}$$

From these we can derive the optimal subsidies as:

$$\hat{\tau} = \frac{\rho_O (CS_a(\Lambda) - CS_d) + (1 - \rho_O) (CS_d - CS_e)}{\rho_O (S_a(\Lambda) - S_d) + (1 - \rho_O) (S_d - S_e)} \quad \text{Non-specialisation}$$

$$\hat{\tau} = \frac{\rho_O (CS_{ab}(\tilde{\Lambda}) - CS_d) + (1 - \rho_O) (CS_d - CS_e)}{\rho_O (S_{ab}(\Lambda) - S_d) + (1 - \rho_O) (S_d - S_e)}$$

Part. Specialisation

The optimal subsidy is defined as that subsidy which induces the RJV to innovate with the probability that is socially optimal.

Notice that there are now two optimal subsidy rates, one for each of the cases in which one of the two research strategies is socially optimal. This complicates the problem of a subsidising agent because they not only need to form a judgement about the level of overall underinvestment in R&D, but also need to determine when a given subsidy rate is appropriate.

However as discussed above the R&D subsidy has a serious drawback in the context of this model as it does not affect the incentives according to which the firms in the RJV choose their research strategies. In figure 4 on page 18 I showed that the firms in an RJV may choose a research strategy of partial specialisation when this is not socially optimal.

Whenever the RJV chooses the social suboptimal research strategy neither of the two subsidies set out above will reduce the welfare loss to zero in the RJV equilibrium. Without undertaking simulations it is difficult to tell whether the residual welfare losses arising from the *absorption problem* are important.

It is clear without the need for simulations that the optimal subsidy policy in the presence of important effects of absorptive capacity and the choice of the degree of differentiation of research paths is more complex than the optimal policy where absorptive capacity does not matter, as in the previous chapter for instance. Although I have modelled absorptive capacity in a discontinuous manner in this paper it is not difficult to see that this result will carry over to a continuous model of absorptive capacity. The reason being that the social planner must seek to achieve two ends:

- the socially optimal level of investment in R&D
- the socially optimal degree of differentiation of research paths

with just one instrument.

An optimal subsidy to the non-cooperative firms will achieve the social optimum for all cases except those in which the non-cooperative firms choose a socially suboptimal research strategy. As figure 5 on page 22 shows the non-cooperative firms choose a suboptimal research strategy for segments B_1 and C_2 of the parameter space.

The non-cooperative firms should be preferred to the RJV where that chooses the socially suboptimal research strategy (segment B_2 of figure 5). However as soon as the research strategy of partial specialisation is socially optimal the non-cooperative equilibrium involves the choice of a socially suboptimal research strategy for a wide range of parameters.

An R&D subsidy of the kind described in this section will not alleviate the incentive problem which the non-cooperative firms face when choosing their research strategy in the context of spillover levels that are high (Compare result 7 on page 21). There could therefore always be a welfare loss from the suboptimal choice of research strategy in a non-cooperative equilibrium even where an optimal subsidy is provided.

I do not set out what this optimal subsidy would look like for this reason. In the absence of simulations which show us how large the welfare losses from the suboptimal choice of research strategy will be, it is possible that these are insignificant and that therefore an

optimal R&D subsidy to non-cooperative firms could come close to a social optimum for all of the parameter space.

5.1.2 A two part subsidy

Here I discuss the policy of providing an innovation prize T along with the R&D subsidy. This combined policy addresses both the *absorption problem*, which the R&D subsidy alone does not do and the underinvestment in R&D which the R&D subsidy is directed towards.

As the discussion here will show it is a more complex policy than just an R&D subsidy, which will reduce its usefulness. In particular the subsidising agent will need to be able to distinguish between the adoption of *non-specialisation* and *partial specialisation* by the firms. The agent will also need to be able to distinguish cases in which both firms innovate from those in which only one firm innovates successfully.

I showed on page 18 above that the firms in an RJV will be indifferent between the two research strategies when $\Sigma_a = \Sigma_{ab}$. It is socially optimal to switch from non-specialisation to partial specialisation when $S_a \leq S_{ab}$. Therefore a research prize which is awarded in order to address the *absorption problem* should optimally be set as follows:

$$T \geq \Sigma_{ab} - \Sigma_a \text{ where } S_a > S_{ab} \quad (29)$$

and should only be awarded when both firms innovate and choose the research strategy of non-specialisation. The prize need only fulfil this condition for parameter combinations μ, g in area **B** of figure 4. For all other cases it could be set to zero. In particular it must be set to zero as soon as $S_a \leq S_{ab}$.

The optimal R&D subsidy in this case would be set as follows:

$$\hat{\tau} = \frac{\rho_O (CS_a(\Lambda) - T - CS_d) + (1 - \rho_O) (CS_d - CS_e)}{\rho_O (S_a(\Lambda) - S_d) + (1 - \rho_O) (S_d - S_e)} \quad (30)$$

This subsidy rate in conjunction with the innovation prize T I describe above will always attain the social optimum.

This is not a very practical subsidy scheme as the necessary information to implement it is not likely to be obtainable. The main difficulty in implementing this type of subsidy scheme is in establishing that the firms have indeed pursued a research strategy of non-specialisation. This is not likely to be a verifiable or even detectable action. Any subsidy scheme providing the firms with a fixed payment that is only conditional on both firms innovating will not be sufficient to change the firms' incentives to choose the socially optimal research strategy.

The only incentive compatible way in which to provide the firms with the socially optimal incentives to choose a research strategy is to allow the firms to appropriate all of the consumers' surplus, for instance via first degree price discrimination. The problems of such a policy are well known²⁹. It would be interesting to establish to what extent policies such as second degree price discrimination can help to reduce the *absorption problem*. In order to establish this I would need to simulate the model which I do not do here.

The conclusion of this discussion is that the *absorption problem* introduces a source of welfare loss into the discussion of R&D policy that is much more difficult to alleviate than

²⁹Tirole (1988) discusses perfect price discrimination.

the sources of welfare loss typically considered in the R&D literature. The reason for this is that any policy that even partially addresses the problem will require information that is unlikely to be available or policies that are more complex than those usually discussed in the context of R&D policy.

In this section I have shown that an R&D subsidy is enough to address the under-investment in R&D by the RJV but fails to address the *absorption problem*. I have also shown that in principle a combination of a research prize and an R&D subsidy can achieve the social optimum in all cases.

6 Conclusion

In this paper I have extended the model of research co-ordination introduced in the second chapter to investigate what effect non-negligible costs of absorptive capacity have on firms' co-ordination of their R&D efforts.

I find that firms inside and outside of RJVs will seek to co-ordinate their R&D efforts such that they exhibit maximum complementarity without compromising firms' ability to absorb outside innovation. Surprisingly I find that firms may choose asymmetrical absorption capacities when the costs of maintaining absorptive capacity are high. I also find that firms in RJVs are able to choose such asymmetric approaches to research co-ordination for a wider range of parameters than firms that do not cooperate at the R&D design stage. This suggests a possible motive for firms to choose the formation of an RJV over licensing.

Furthermore I show that a policy of permitting RJV formation by itself, will reduce welfare, where firms co-ordinate their R&D approaches. This is due to the fact that non-cooperative firms faced with the possibility of licensing generally have greater incentives to invest in R&D than firms in an RJV.

Finally I show that both firms in RJVs and non-cooperative firms will sometimes choose the wrong research strategy from a social point of view. However the non-cooperative firms will choose the wrong research strategy for a wider range of parameters. This leads me to discuss two separate subsidy schemes in conjunction with RJV formation that address the welfare losses I have identified to differing degrees. I show that the more complete subsidy scheme will be very difficult to implement.

I do not simulate this model. This could be a useful exercise in determining the relative importance of the welfare losses I have identified. In particular it would help to determine how important the welfare loss is, which the absorption problem gives rise to.

The model I employ here is a very simple one. It does not allow me to investigate what the effect of variations in product market competition may be for my results. This line of investigation seems promising as firms' choices of research strategies are likely to be affected by variations in product market competition.

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Appendix

7.1 The Stage 4 Solutions

In the cases in which the firms have asymmetrical costs a \bar{c} signifies a firm with the high costs and the index \underline{c} signifies the firm with the low costs.

Non-specialisation

Individual firm output:

$$q_a = \frac{1}{3} \cdot A (1 + g (1 + \Lambda))$$

$$q_b = \frac{1}{3} \cdot A (1 + g (1 + 2\Lambda))$$

$$q_{\bar{b}} = \frac{1}{3} \cdot A (1 + g (1 - \Lambda))$$

$$q_c = \frac{1}{3} \cdot A (1 + g)$$

$$q_d = \frac{1}{3} \cdot A (1 + 2g)$$

$$q_{\bar{d}} = \frac{1}{3} \cdot A (1 - g)$$

$$q_e = \frac{1}{3} \cdot A$$

Individual firm profits:

$$\pi_a = \frac{1}{9} \cdot A^2 (1 + g (1 + \Lambda))^2$$

$$\pi_b = \frac{1}{9} \cdot A^2 (1 + g (1 + 2\Lambda))^2$$

$$\pi_{\bar{b}} = \frac{1}{9} \cdot A^2 (1 + g (1 - \Lambda))^2$$

$$\pi_c = \frac{1}{9} \cdot A^2 (1 + g)^2$$

$$\pi_d = \frac{1}{9} \cdot A^2 (1 + 2g)^2$$

$$\pi_{\bar{d}} = \frac{1}{9} \cdot A^2 (1 - g)^2$$

$$\pi_e = \frac{1}{9} \cdot A^2$$

Aggregate output:

$$Q_a = \frac{2}{3} \cdot A (1 + g (1 + \Lambda))$$

$$Q_b = \frac{1}{3} \cdot A (2 + g (2 + \Lambda))$$

$$Q_c = \frac{2}{3} \cdot A (1 + g)$$

$$Q_d = \frac{1}{3} \cdot A (2 + g)$$

$$Q_e = \frac{2}{3} \cdot A$$

Aggregate profits:

$$\Sigma_a = \frac{2}{9} A^2 (1 + g (1 + \Lambda))^2$$

$$\Sigma_b = \pi_b + \pi_{\bar{b}}$$

$$\Sigma_c = \frac{2}{9} A^2 (1 + g)^2$$

$$\Sigma_d = \frac{2}{9} \cdot A^2 (1 + g + \frac{5}{2}g^2)$$

$$\Sigma_e = \frac{2}{9} \cdot A^2$$

Partial Specialisation

$$q_{ab} = \frac{1}{3} \cdot A (1 + g (1 + 2\tilde{\Lambda}))$$

$$q_{a\bar{b}} = \frac{1}{3} \cdot A (1 + g (1 - \tilde{\Lambda}))$$

$$q_c = \frac{1}{3} \cdot A (1 + g)$$

$$q_d = \frac{1}{3} \cdot A (1 + 2g)$$

$$q_{\bar{d}} = \frac{1}{3} \cdot A (1 - g)$$

$$q_e = \frac{1}{3} \cdot A$$

$$\pi_{ab} = \frac{1}{9} \cdot A^2 (1 + g (1 + 2\tilde{\Lambda}))^2$$

$$\pi_{a\bar{b}} = \frac{1}{9} \cdot A^2 (1 + g (1 - \tilde{\Lambda}))^2$$

$$\pi_c = \frac{1}{9} \cdot A^2 (1 + g)^2$$

$$\pi_d = \frac{1}{9} \cdot A^2 (1 + 2g)^2$$

$$\pi_{\bar{d}} = \frac{1}{9} \cdot A^2 (1 - g)^2$$

$$\pi_f = \frac{1}{9} \cdot A^2$$

$$Q_{ab} = \frac{1}{3} \cdot A (2 + g (2 + \tilde{\Lambda}))$$

$$Q_c = \frac{2}{3} \cdot A (1 + g)$$

$$Q_d = \frac{1}{3} \cdot A (2 + g)$$

$$Q_e = \frac{2}{3} \cdot A$$

$$\Sigma_{ab} = \frac{2}{9} A^2 \left(\left[1 + g + \frac{1}{2}g\tilde{\Lambda} \right]^2 + \left[\frac{3}{2}g\tilde{\Lambda} \right]^2 \right)$$

$$\Sigma_c = \frac{2}{9} A^2 (1 + g)^2$$

$$\Sigma_d = \frac{2}{9} \cdot A^2 (1 + g + \frac{5}{2}g^2)$$

$$\Sigma_e = \frac{2}{9} \cdot A^2$$

Non-specialisation

Social Surplus:

$$S_a = \frac{4}{9}A^2 (1 + g(1 + \Lambda))^2$$

$$S_b = \frac{1}{2}(Q_b)^2 + \Sigma_b$$

$$S_c = \frac{4}{9}A^2 (1 + g)^2$$

$$S_d = \frac{2A^2}{9} \left(2 \left[1 + \frac{1}{2}g \right]^2 + \frac{9}{4}g^2 \right)$$

$$S_e = \frac{4}{9}A^2$$

Partial Specialisation

$$S_{ab} = \frac{2A^2}{9} \left(2 \left[1 + g + \frac{1}{2}g\tilde{\Lambda} \right]^2 + \left[\frac{3}{2}g\tilde{\Lambda} \right]^2 \right)$$

$$S_c = \frac{4}{9}A^2 (1 + g)^2$$

Here the indices $a-e$ refer to the cases set out on page 7 above. For equilibria in which the firms choose partial specialisation there is no difference between cases a and b so that I refer to the case in which the firms share as much technology as possible as ab .

7.2 Proof of Result 1

By assumption the firms will only ever face the choice of sharing technologies when both have innovated. The firms then face the choice between the cases indexed a, b or c as set out on page 7.

In this proof I show that $\Sigma_a > \Sigma_b > \Sigma_c$, to show that the firms will always prefer sharing technologies completely. Notice that the ranking of joint profits is the relevant criterion for decisions about the firms' information sharing decisions in both the RJV and the licensing equilibria.

7.2.1 $\Sigma_a > \Sigma_b > \Sigma_c$

Using the expressions set out in section 7.1 it is easy to show that:

$$\begin{aligned} \Sigma_b > \Sigma_c &= \pi_b + \pi_{\bar{b}} > 2\pi_c \\ \Leftrightarrow \frac{1}{9} \cdot A^2 (1 + g(1 + 2\Lambda))^2 + \frac{1}{9} \cdot A^2 (1 + g(1 - \Lambda))^2 &> \frac{1}{9} \cdot A^2 (1 + g)^2 \\ \Leftrightarrow 5g\Lambda + 2(1 + g) &> 0 \end{aligned}$$

and that: $\Sigma_a > \Sigma_b = 2\pi_a > \pi_b + \pi_{\bar{b}}$

$$\begin{aligned} \Leftrightarrow \frac{1}{9} \cdot A^2 (1 + g(1 + \Lambda))^2 &> \frac{1}{9} \cdot A^2 (1 + g(1 + 2\Lambda))^2 + \frac{1}{9} \cdot A^2 (1 + g(1 - \Lambda))^2 \\ \Rightarrow \frac{2}{3\Lambda - 2} > g \wedge \Lambda > \frac{2}{3} \vee g > \frac{2}{3\Lambda - 2} \wedge \Lambda < \frac{2}{3} \end{aligned}$$

It is not hard to see that the condition for $\Sigma_a > \Sigma_b$ will hold for all values of Λ . In particular where $\Lambda = 1 \rightarrow 2 > g$ holds. Since we have assumed that $g < 1$ above, this will always be true. This shows that whenever we are in a case of *symmetric absorption* capabilities in which both firms innovate, they will choose to share information fully in both RJV and licensing equilibria.

Where the firms are in a case of *asymmetric absorption* capabilities in which both firms innovate we have to compare Σ_{ab} with Σ_c . As the expressions in section 7.1 show this is

equivalent to showing that $\Sigma_b > \Sigma_c$ which I have done above. For the purposes of the proof here the distinction between Λ and $\tilde{\Lambda}$ is not important.

Therefore I have established that whenever both firms innovate simultaneously they will share as much technology as possible.

7.3 Proof of Result 2

If it is socially optimal to adopt the research strategy of *non-specialisation* it can be shown that it is also optimal to differentiate the research vectors as far as possible. To do this I differentiate equation (5) w.r.t. Λ :

$$\frac{dS^n}{d\Lambda} = (\rho^n)^2 \frac{dS_a}{d\Lambda} + 2 \frac{d\rho^n}{d\Lambda} \left[\rho^n \cdot (S_a - S_d) + (1 - \rho^n) \cdot (S_d - S_e) - \gamma'(\rho^n) \right] = (\rho^n)^2 \frac{dS_a}{d\Lambda} > 0$$

From the expressions set out in Appendix 7.1 above it will be apparent that $\frac{dS_a}{d\Lambda} > 0$. Thus in a symmetrical RJV equilibrium it will always be true that: $\Lambda = (1 - 2\mu)$

If it is socially optimal to choose the research strategy of *partial specialisation* then I can also show that it is socially optimal to differentiate the research vectors as far as possible:

$$\frac{dS^p}{d\tilde{\Lambda}} = (\rho^p)^2 \frac{dS_{ab}}{d\tilde{\Lambda}} + 2 \frac{d\rho^p}{d\tilde{\Lambda}} \left[\rho^p \cdot (S_{ab} - S_d) + (1 - \rho^p) \cdot (S_d - S_e) - \gamma'(\rho^p) \right] = (\rho^p)^2 \frac{dS_{ab}}{d\tilde{\Lambda}} > 0$$

From the expressions set out in Appendix 7.1 above it will also be apparent that $\frac{dS_{ab}}{d\tilde{\Lambda}} > 0$. Thus in an asymmetrical RJV equilibrium it will always be true that: $\tilde{\Lambda} = (1 - \mu)$

7.4 Proof of Result 3

It will be socially optimal for the firms to pursue a research strategy of *partial specialisation* whenever $S_{ab} > S_a$. This becomes evident in the comparison of the innovation incentives in the social optimum which are set out in Tables 2 and 3. It is apparent that the profit incentive in each case is the same but that the competitive threats differ. When the competitive threat is greater under a particular research strategy, so are the probability of innovation and the profits attached to that strategy. This is clear from the comparison of equations (5) and (11) on page 16 above. The boundary along which the competitive threat is the same under both strategies is described by the following condition³⁰:

$$\begin{aligned} S_{ab} &= S_a \\ \Leftrightarrow \frac{2A^2}{9} \left(2 \left[1 + g + \frac{1}{2}g\tilde{\Lambda} \right]^2 + \left[\frac{3}{2}g\tilde{\Lambda} \right]^2 \right) &= \frac{4}{9}A^2 (1 + g(1 + \Lambda))^2 \\ \Leftrightarrow g \left[8 \left(\tilde{\Lambda} - 2\Lambda \right) + \left(11\tilde{\Lambda}^2 - 8\Lambda^2 \right) \right] &= 8 \left(2\Lambda - \tilde{\Lambda} \right) \end{aligned}$$

using Result 2 which is derived just above I replace $\Lambda, \tilde{\Lambda}$:

$$\begin{aligned} \Rightarrow g \left[8(3\mu - 1) + (11(1 - 2\mu + \mu^2) - 8(1 - 4\mu + 4\mu^2)) \right] &= 8(1 - 3\mu) \\ \Leftrightarrow g \left[34\mu - 5 - 21\mu^2 \right] &= 8(1 - 3\mu) \end{aligned}$$

I illustrate this in Figure 3 above.

³⁰Notice that $\mu^g = \mu^p$ along the boundary since μ is a common exogenous factor and therefore the probabilities of innovation are equal: $\rho^g = \rho^p$.

7.5 Proof of Result 4

If the firms in an RJV are pursuing a strategy of *nonspecialisation* they will find it optimal to maximally differentiate their research vectors. This can be demonstrated by differentiating their expected profits w.r.t. Λ :

$$\frac{d\Pi^n}{d\Lambda} = (\rho^n)^2 \frac{d\Sigma_a}{d\Lambda} + 2 \frac{d\rho^n}{d\Lambda} \left[\rho^n \cdot [\Sigma_a - \Sigma_d] + (1 - \rho^n) [\Sigma_d - \Sigma_e] - \gamma'(\rho^n) \right] = (\rho^n)^2 \frac{d\Sigma_a}{d\Lambda} > 0$$

From the expressions set out in Appendix 7.1 above it will be apparent that $\frac{d\Sigma_a}{d\Lambda} > 0$. Thus in a symmetrical RJV equilibrium it will always be true that: $\Lambda = (1 - 2\mu)$

In the case in which the firms in the RJV choose a strategy of *partial specialisation* I can show that they will also maximise profits by maximally differentiating their research vectors:

$$\frac{d\Pi^p}{d\tilde{\Lambda}} = (\rho^p)^2 \frac{d\Sigma_{ab}}{d\tilde{\Lambda}} + 2 \frac{d\rho^p}{d\tilde{\Lambda}} \left[\rho^p \cdot [\Sigma_{ab} - \Sigma_d] + (1 - \rho^p) \cdot [\Sigma_d - \Sigma_e] - \gamma'(\rho^p) \right] = (\rho^p)^2 \frac{d\Sigma_{ab}}{d\tilde{\Lambda}} > 0$$

From the expressions set out in Appendix 7.1 above it will also be apparent that $\frac{d\Sigma_{ab}}{d\tilde{\Lambda}} > 0$. Thus in an asymmetrical RJV equilibrium it will always be true that: $\tilde{\Lambda} = (1 - \mu)$

7.6 Proof of Result 5

Here I assume that maximal differentiation holds:

$$\begin{aligned} & \Sigma_{ab} > \Sigma_a \\ \Leftrightarrow & \frac{1}{9} \cdot A^2 \left(1 + g \left(1 + 2\tilde{\Lambda} \right) \right)^2 + \frac{1}{9} \cdot A^2 \left(1 + g \left(1 - \tilde{\Lambda} \right) \right)^2 > \frac{2}{9} \cdot A^2 \left(1 + g \left(1 + \Lambda \right) \right)^2 \\ \Leftrightarrow & 2\tilde{\Lambda} - 4\Lambda > g \left[2 \cdot (1 + \Lambda)^2 - \left(1 + 2\tilde{\Lambda} \right)^2 - \left(1 - \tilde{\Lambda} \right)^2 \right] \\ \Leftrightarrow & 6\mu - 2 > g \left[3\mu^2 - 4\mu - 1 \right] \\ \Leftrightarrow & g > \frac{6\mu - 2}{3\mu^2 - 4\mu - 1} \end{aligned} \tag{31}$$

7.7 Derivation of Result 6

When the non-cooperative firms both follow a strategy of nonspecialisation it is possible to show that it is not always profit maximising for the firms to differentiate R&D vectors maximally:

$$\begin{aligned} \frac{d\pi_N^n}{d\Lambda} &= (\rho_N^n)^2 \frac{d\pi_a}{d\Lambda} + \frac{d\rho_N^n}{d\Lambda} \left\{ 2\rho_N^n \pi_a - (1 - 2\rho_N^n) [\pi_{\underline{d}} + \pi_{\bar{d}}] - 2(1 - \rho_N^n) \pi_e - \gamma'(\rho_N^n) \right\} \\ &= (\rho_N^n)^2 \frac{d\pi_a}{d\Lambda} + \frac{d\rho_N^n}{d\Lambda} \left\{ \rho_N^n [\pi_a - \pi_{\underline{d}}] + (1 - \rho_N^n) [\pi_{\bar{d}} - \pi_e] \right\} \end{aligned}$$

Where the derivative $\frac{d\rho_N^n}{d\Lambda}$ is derived from equation (10) by the implicit function theorem:

$$\frac{d\rho_N^n}{d\Lambda} = \frac{\rho_N^n}{\Delta} \frac{d\pi_a}{d\Lambda} \quad (32)$$

which can be re-expressed as:

$$\frac{d\pi_N^n}{d\Lambda} = \frac{d\pi_a}{d\Lambda} \cdot \frac{\rho_N^n}{\Delta} \left\{ \rho_N^n \cdot \gamma''(\rho_N^n) + [\pi_{\bar{d}} - \pi_e] \right\} \quad (33)$$

The sign of this expression depends on the relative weight of the two terms in the brackets. As long as the first is small the expression will be negative and the firms in the non-cooperative equilibrium will choose not to differentiate at all. The first term in the brackets is small as long as diminishing returns to R&D are not too strong.

In analogous fashion it is easy to show that where the firms follow a research strategy of partial specialisation:

$$\frac{d\pi_N^p}{d\tilde{\Lambda}} = \frac{d\pi_{ab}}{d\tilde{\Lambda}} \cdot \frac{\rho_N^p}{\tilde{\Delta}} \left\{ \rho_N^p \cdot \gamma''(\rho_N^p) + [\pi_{\bar{d}} - \pi_e] \right\} \quad (34)$$

However this derivation can only hold where the firms are able to agree on the strategy of partial specialisation and this will be incentive compatible. This is because I am once more assuming that given the expected payment of the license fee the two firms have the same expected profits and the same probabilities of innovation.

7.8 Proof of Result 7

In order to understand when it is no longer incentive compatible for the ‘generalist’ to pay the specialist a license fee in a non-cooperative equilibrium with asymmetrical R&D vectors we must set out the profits of both in the event of non-sharing of information in such an equilibrium.

The R&D vectors that both firms will have in such a case are functions of the involuntary information spillover σ , where *sigma* represents a proportion of technological knowledge in the specialist’s technology dimension spilling over to the generalist. I define *sigma* to represent a spillover of *new* knowledge to the generalist, i.e. when σ is zero this could be because the generalist really learns nothing, or because the spillover that does take place is of knowledge which the generalist already possesses.

- The specialist is by definition restricted to the progress vector \mathbf{V}_2 :

$$\mathbf{V}_2 = \mathbf{v}_2 = (\lambda_2 \cdot g, (1 - \lambda_2) \cdot g)$$

and therefore achieves a total cost reduction of g .

- The generalist firm will have a progress vector \mathbf{V}_1 :

$$\mathbf{V}_1 = \left(g \cdot \lambda_1, g \cdot (1 - \lambda_1) + g\sigma\tilde{\Lambda} \right)$$

I am only interested in cases of maximal differentiation here and therefore I know that $\lambda_1 = 1 - \mu$ and $\lambda_2 = 0$ here, which implies that $\tilde{\Lambda} = 1 - \mu$.

This implies that the firms have the following profits in the case where both innovate, but share no knowledge:

$$\begin{aligned} \pi_{\underline{ab}}(\sigma) &= \frac{1}{9} \cdot A^2 (1 + g(1 + 2\sigma(1 - \mu)))^2 \\ \pi_{\bar{ab}}(\sigma) &= \frac{1}{9} \cdot A^2 (1 + g(1 - \sigma(1 - \mu)))^2 \end{aligned} \quad (35)$$

Given these expressions I can now calculate when it will be *ex post* more profitable for the generalist not to pay the license fee, but just to rely on the involuntary spillover of additional technology from the specialist as set out in equation (26) above:

$$\begin{aligned}
& \pi_{\underline{ab}}(\sigma) > \pi_{\underline{ab}}(1) - \hat{F} \\
& \Leftrightarrow \pi_{\underline{ab}}(\sigma) > \pi_{\underline{ab}}(1) - \frac{\pi_{\underline{ab}}(1) - \pi_{\bar{ab}}(1)}{2} \\
& \Leftrightarrow 2 \left[[1+g] + 2g\sigma(1-\mu) \right]^2 > \left[[1+g] + 2g(1-\mu) \right]^2 + \left[[1+g] - g(1-\mu) \right]^2 \\
& \Leftrightarrow g \left[2(4\sigma - 1) + (1-\mu) [8\sigma^2 - 5] \right] \geq 2[1 - 4\sigma]
\end{aligned}$$

This inequality is complex. The right hand side will be positive as long as $\sigma < \frac{1}{4}$. For this range the factor of g on the left hand side will always be negative. This implies that when $\sigma < \frac{1}{4}$ the condition requires that g be negative, which is ruled out by definition. Therefore the condition never holds where $\sigma < \frac{1}{4}$!

When $\sigma > \frac{1}{4}$ the left hand side of the expression will be negative. If the right hand side factor of g is positive then we can say that the condition always holds, because it requires g to be greater than a negative number. The right hand side expression will certainly be positive when $\sigma > \sqrt{\frac{5}{8}}$.

For $\frac{1}{4} \leq \sigma \leq \sqrt{\frac{5}{8}}$ the condition will be fulfilled if μ is large enough for the factor of g to be positive. If the factor is negative, then the condition states that g must be smaller than a positive number. The condition will be satisfied if:

$$g \left\{ \begin{array}{l}
> 0 > \frac{2[1-4\sigma]}{2(4\sigma-1)+(1-\mu)[8\sigma^2-5]}, \text{ for } \sigma \geq \sqrt{\frac{5}{8}} \text{ this always holds;} \\
> 0 > \frac{2[1-4\sigma]}{2(4\sigma-1)+(1-\mu)[8\sigma^2-5]}, \text{ for } 2(4\sigma-1) + (1-\mu)[8\sigma^2-5] > 0 \text{ this always holds;} \\
< \frac{2[1-4\sigma]}{2(4\sigma-1)+(1-\mu)[8\sigma^2-5]} \text{ for } 2(4\sigma-1) + (1-\mu)[8\sigma^2-5] < 0 \\
< 0 \text{ for } \sigma < \frac{1}{4}, \text{ which cannot hold by definition.}
\end{array} \right.$$

This set of inequalities defines a boundary in the parameter space given by μ , g , σ that separates the region of parameter combinations for which the non-cooperative firms can choose the strategy of partial specialisation and that for which they cannot. I illustrate this in figure 5 above.