HIDDEN COINTEGRATION

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Abstract

Possibly hitherto unnoticed cointegrating relationships among integrated components of data series are identified. If the components are cointegrated, the data are said to have hidden cointegration. The implication of hidden cointegration on modeling data series themselves is discussed through what we call crouching error correction models. We show that hidden cointegration is a simple example of nonlinear cointegration. Economic examples are provided with U.S. short-term and long-term interest rates and output and unemployment, for which no evidence of standard cointegration is found.

KEY WORDS: Hidden cointegration; Crouching error correction models; Shocks; Interest rates; Hysteresis of unemployment

JEL classification: C32; E43; E24

Almost twenty years have passed since the introduction of the idea of cointegration into econometric literature by the first author of this paper; see Granger (1981, 1983), Engle and Granger (1987), and also the collection of papers in Engle and Granger (1991). It has become customary to investigate the existence of cointegrating relationships among integrated (often called nonstationary[1]) economic variables before conducting formal inference, like estimating parameters of interest or testing hypotheses. If the data are cointegrated, error correction models [ECMs] are estimated; otherwise, vector autoregressive [VAR] models are estimated in first differences. There are also numerous extensions of basic models; for instance, a discussion of nonlinear cointegration in Park and Phillips (2001), threshold cointegration in Balke and Fomby (1997), and nonlinear adjustment mechanism with asymmetric error correction in Enders and Granger (1998) and Enders and Siklos (2001). Granger and Swanson (1996) contain a summary of further

[1] An I(1) process is usually called nonstationary. However, while an I(1) series may be nonstationary, not all nonstationary series are I(1); see Granger (1997) for an example. Also, a stationary series will be I(0), but not all I(0) series are stationary. In this paper, keeping the distinction in mind, we use nonstationarity interchangeably with an I(1) or unit root process.

developments in cointegration analysis, many of which are widely used in empirical analysis.

Even though many economic series are routinely found to be cointegrated, it should be emphasized that cointegration is a very special phenominon indeed. Cointegration occurs because economic data share common stochastic trends, which are eliminated by cointegrating linear combinations. Common stochastic trends are usually expressed as a linear combination of the shocks of a system; see the Stock and Watson (1988) common trends representation. Putting it differently, economic data are cointegrated because they respond to shocks together. There would be no cointegration if they respond separately to shocks. However, what would happen if they respond together only to a certain kind of shocks? For instance, some series are known to be downwardly rigid; therefore, while they move together with others to positive shocks, they would respond differently to negative shocks. Further, we would observe an asymmetric response in unemployment to output changes according to the framework of the hysteresis hypothesis of unemployment. Also central banks may pay more attention to rising interest rates or exchange rates than falling ones, because they have different implications on inflation rates. Hence, the instruments of monetary policy like short-term interest rates may have relationships with only certain components of long-term interest rates or exchange rates. While such economic series are not cointegrated, there might be useful information hidden in their components to help understand their dynamic relationship. This possibility has been neglected until now. It is the purpose of this paper to fill the gap by testing if there are cointegrating relationships among the nonstationary components of data series. When the components are cointegrated, the data series are called to have hidden cointegration. Surprisingly rich information on their dynamics can be gathered from the approach. It becomes possible to investigate long-run relationship among non-cointegrated nonstationary data series. It will be shown that standard cointegration is a special case of hidden cointegration. Further, hidden cointegration is a simple example of nonlinear cointegration.

The remainder of this paper is organized as follows; in section 1, possible cointegration between nonstationary components of data series is discussed. In section 2, the implication of hidden cointegration is studied through error correction models associated with hidden cointegration. We call the error correction models implied by hidden cointegration crouching ECMs. In section 3, the size of conventional cointegration tests is investigated against hidden cointegration through Monte Carlo simulations. In section 4, empirical examples are provided for U.S. short-term and long-term interest rates and output and unemployment, for which no evidence of standard cointegration is found. Conclusions are provided in section 5.

1. Cointegration among nonstationary components of data series

Note initially that any I(1) series with an ARIMA(*p,1,q*) representation contain a random walk component possibly with drift; see Beveridge and Nelson (1981). We discuss only bivariate examples in this paper for simplicity. Consider the following two random walks without drifts;

$$X_t = X_{t-1} + \varepsilon_t = X_0 + \sum_1^t \varepsilon_i$$

and

$$Y_t = Y_{t-1} + \eta_t = Y_0 + \sum_1^t \eta_i \, ,$$

where t = 1, 2, …, and X_0 and Y_0 denote initial values. ε_i and η_i are white noises with zero means. At this stage, we do not discuss if *X* and *Y* are cointegrated or not. Define new variables;

$$\varepsilon_i^{\vee} = max(\varepsilon_i, d) \text{ and } \varepsilon_i^{\wedge} = min(\varepsilon_i, d) \, .$$

Note that $\varepsilon_i = \varepsilon_i^{\wedge} + \varepsilon_i^{\vee} - d$ and d will be called a threshold. Popular choice would be $\varepsilon_i^{+} = max(\varepsilon_i, 0)$ and $\varepsilon_i^{-} = min(\varepsilon_i, 0)$. The threshold should be chosen such that the situation where $\varepsilon_i = \varepsilon_i^{\wedge}$ or $\varepsilon_i = 0$ for all i should be excluded. Further, if $\varepsilon_i = \varepsilon_i^{\vee}$ for a small number of $i \in K$, while $\varepsilon_i = \varepsilon_i^{\wedge}$ for most of $i \in \mathbb{N}_+ - K$, it is similar to using a dummy variable for some $i \in K$. From now on, we assume that $\sum_1^t \varepsilon_i^{\wedge}$, $\sum_1^t \eta_i^{\wedge}$, $\sum_1^t \varepsilon_i^{\vee}$, and $\sum_1^t \eta_i^{\vee}$ are all I(1). Later, we will discuss how to choose *d* based on a certain criterion. There could be more than 2 components of interest in *X* and *Y*; however, for simplicity we consider only two. We now have

$$X_t = X_{t-1} + \varepsilon_t = X_0 + \sum_1^t \varepsilon_i^{\wedge} + \sum_1^t \varepsilon_i^{\vee} - dt$$

and

$$Y_t = Y_{t-1} + \eta_t = Y_0 + \sum_1^t \eta_i^{\wedge} + \sum_1^t \eta_i^{\vee} - dt.$$

It is much more convenient and intuitive if we set $d = 0$. For ease of exposition from now on, we use $d = 0$ and mention results for $d \neq 0$ when necessary. We also assume that X_0 is a constant and that $X_t = X_0 + X_t^+ + X_t^-$, where $X_t^+ = \sum_1^t \varepsilon_i^+$ and $X_t^- = \sum_1^t \varepsilon_i^-$. It follows that $\Delta X_t^+ = \varepsilon_t^+$ and $\Delta X_t^- = \varepsilon_t^-$. Note also that we do not need to estimate any of these terms; if $\Delta X_t > 0$, $\varepsilon_t^+ = \Delta X_t$ and $\varepsilon_t^- = 0$, for instance. We note the following observations on $\{\varepsilon_i^+ \quad \varepsilon_i^-\}$ under the assumption that $\varepsilon_i \sim N(0, 1)$. First, $\varepsilon_i^+ \sim d\left(\frac{1}{\sqrt{2\pi}}, \quad \frac{1}{2}\frac{\pi - 1}{\pi}\right)$ with $E[\varepsilon_i^+ \varepsilon_i^-] = 0$. $d(\cdot,\cdot)$ denotes mean and variance of a random variable. Second, define $\upsilon_i^+ = \varepsilon_i^+ - \frac{1}{\sqrt{2\pi}}$, then $X_t^+ = \frac{1}{\sqrt{2\pi}} t + \sum_1^t \upsilon_i^+$, where $\upsilon_i^+ \sim d\left(0, \ \frac{1}{2}\frac{\pi - 1}{\pi}\right)$. Hence, X_t^+ is a random walk with drift. Similarly for X_t^-. More general expressions are possible without assuming a specific distribution; see for instance Schorderet (2001). We will call $\left\{\sum_1^t \varepsilon_i^+ \quad \sum_1^t \eta_i^+\right\}$ = $\left\{X_t^+ \quad Y_t^+\right\}$ and $\left\{\sum_1^t \varepsilon_i^- \quad \sum_1^t \eta_i^-\right\}$ = $\left\{X_t^- \quad Y_t^-\right\}$ sums of positive and negative shocks, respectively.

We now consider possible cointegration between the nonstationary components of *X* and *Y*. First, a definition. We call *X* and *Y* to have <u>hidden cointegration</u> if their components are cointegrated each other. We will show that only under specific conditions hidden cointegration between nonstationary components of *X* and *Y* implies standard cointegrating relationship for data series themselves. We implicitly assume for convenience that neither $\left\{\sum_1^t \varepsilon_i^+ \quad \sum_1^t \eta_i^-\right\}$ nor $\left\{\sum_1^t \varepsilon_i^- \quad \sum_1^t \eta_i^+\right\}$ are cointegrated in the following discussion. In empirical implementation below, however, we will test for possible cointegration between them as well. First, we consider the case of no cointegration between the nonstationary components of data series.

Case 1: Neither $\left\{\sum_1^t \varepsilon_i^+ \quad \sum_1^t \eta_i^+\right\}$ nor $\left\{\sum_1^t \varepsilon_i^- \quad \sum_1^t \eta_i^-\right\}$ are cointegrated.

It follows that *X* and *Y* are not cointegrated. They are subject to positive and negative shocks, which have their own separate stochastic trends. More interesting cases follow;

Case 2: Either $\left\{\sum_1^t \varepsilon_i^+ \quad \sum_1^t \eta_i^+\right\}$ or $\left\{\sum_1^t \varepsilon_i^- \quad \sum_1^t \eta_i^-\right\}$, but not both, are cointegrated.

Hence, X and Y have either common positive or common negative shocks, but not both. For example, when sums of positive shocks are cointegrated, both X and Y are subject to common positive shocks. However, the sum of negative terms are not cointegrated due to, for instance, different degrees of downward rigidity in X and Y. Even though X and Y are still not cointegrated, they have more structure than available in the previous case 1. This information on hidden cointegrating relationship will not be utilized if we are interested only in the cointegrating relationship between X and Y.

Case 3: Both $\left\{\sum_1^t \varepsilon_i^+ \quad \sum_1^t \eta_i^+\right\}$ and $\left\{\sum_1^t \varepsilon_i^- \quad \sum_1^t \eta_i^-\right\}$ are cointegrated, but with different cointegrating vectors.

Still, X and Y are not cointegrated; even though they have common positive and negative shocks, the common shocks are not cointegrated. For X and Y to be cointegrated, an extra condition is necessary.

Case 4: Both $\left\{\sum_1^t \varepsilon_i^+ \quad \sum_1^t \eta_i^+\right\}$ and $\left\{\sum_1^t \varepsilon_i^- \quad \sum_1^t \eta_i^-\right\}$ are cointegrated with the same cointegrating vectors.

The positive and negative shocks are cointegrated with the same cointegrating vectors and there is only one common shock. It can be interpreted as a common stochastic trend of X

and Y and is responsible for their long-run dynamic behavior. In this case X and Y are cointegrated.

The above discussion demonstrates how special cointegration is. For X and Y to be cointegrated, their nonstationary components should be cointegrated each other with the same cointegrating vectors.[2] Table 1 summarizes the above discussion. In cases 1 to 3, X and Y are not cointegrated. It is current practice to model nonstationary variables with a VAR in first difference when they are not cointegrated. However, the degree of no-cointegration is different from each other. Therefore, if we pay attention only to cointegrating relationship between X and Y, much valuable information would be lost. Even if they are not cointegrated, there could be hidden structure that can be fruitfully utilized to help understand their dynamics and to produce possibly improved forecasts. It is the object of this research to find if such useful information is available.

It is easy to show that hidden cointegration is a simple example of nonlinear cointegration, which is actively studied these days; see for example, Granger (1995), Park

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[2] Further, if $\{X_t - kY_t\}$ is stationary where $k \neq 1$, the cointegrating relationship will contain a deterministic time trend, unless $d = 0$.

and Phillips (1999 and 2001) and Karlsen *et al.* (2000). Consider two I(1) variables $\{X_t\}$ and $\{Y_t\}$. They are linearly cointegrated if there exists α such that $\{Y_t - \alpha X_t\}$~ I(0). Further, they are nonlinearly cointegrated if there is β such that $\{f(Y_t) - \beta g(X_t)\}$~ I(0) for certain nonlinear functions *f* and *g*. In the hidden cointegration framework, we take $f(Y_t)$ to be either $\sum_{i=1}^{t} max(\Delta Y_i, d_Y)$ or $\sum_{i=1}^{t} min(\Delta Y_i, d_Y)$, with a threshold d_Y. Similarly for function *g*. Finally, our hidden cointegration should not be confused with that discussed in Nowak (1991) in the framework of errors-in-variables. He is interested in a situation in which cointegration of time series of interest can be partially or totally hidden in the observed data due to measurement errors. Note that his proposition 1 is analogous to our case 4 in table 1. He assumes that measurement errors are I(1), which implies that observed data are measured very imprecisely.

2. Crouching error correction models

An error correction model implied by hidden cointegration will be called a crouching error correction model. In this section we discuss it, starting with case 2 of the previous section. Assume that $\left\{\sum_1^t \varepsilon_i^+ \quad \sum_1^t \eta_i^+\right\}$ are cointegrated with a cointegrating vector of (1, -1) for convenience and that they have the following ECM;

$$\begin{aligned} \varepsilon_t^+ &= \gamma_0 + \gamma_1 \left(\sum_1^{t-1} \varepsilon_i^+ - \sum_1^{t-1} \eta_i^+ \right) + lags(\varepsilon_{t-1}^+, \eta_{t-1}^+) + \xi_t \\ \eta_t^+ &= \delta_0 + \delta_1 \left(\sum_1^{t-1} \varepsilon_i^+ - \sum_1^{t-1} \eta_i^+ \right) + lags(\varepsilon_{t-1}^+, \eta_{t-1}^+) + \zeta_t \end{aligned},$$

where $lags(\varepsilon_{t-1}^+, \eta_{t-1}^+)$ indicates additional terms with various lags of ε_{t-1}^+ and η_{t-1}^+. ξ_t and ζ_t are white noises. Note that for cointegration γ_1 and δ_1 cannot be both zero simultaneously. In terms of ΔX_t^+ and ΔY_t^+, we have

$$\Delta X_t^+ = \gamma_0 + \gamma_1 \left(X_{t-1}^+ - Y_{t-1}^+ \right) + lags(\Delta X_{t-1}^+, \Delta Y_{t-1}^+) + \xi_t .$$

Similarly for ΔY_t^+. *X* and *Y* do not have an error correction model in the usual sense. However, the hidden cointegration relationship between their nonstationary components reveals until now unnoticed structure.

For case 3, assume that $\left\{ \sum_1^t \varepsilon_i^- \quad \sum_1^t \eta_i^- \right\}$ are cointegrated as well, with a cointegrating vector of (1, -*k*), $k \neq 1$ and that they have the following error correction models;

$$\begin{aligned} \varepsilon_t^- &= \gamma_0^{'} + \gamma_2 \left(\sum_1^{t-1} \varepsilon_i^- - k \sum_1^{t-1} \eta_i^- \right) + lags(\varepsilon_{t-1}^-, \eta_{t-1}^-) + \xi_t \\ \eta_t^- &= \delta_0^{'} + \delta_2 \left(\sum_1^{t-1} \varepsilon_i^- - k \sum_1^{t-1} \eta_i^- \right) + lags(\varepsilon_{t-1}^-, \eta_{t-1}^-) + \zeta_t \end{aligned}.$$

X and *Y* are not cointegrated. However, they posses the following representation derived from the above crouching ECMs;

$$\Delta X_t = \gamma_0 + \gamma_1 \left(X_{t-1}^+ - Y_{t-1}^+ \right) + \gamma_2 \left(X_{t-1}^- - k Y_{t-1}^- \right) + lags(\Delta X_{t-1}^+, \Delta Y_{t-1}^+, \Delta X_{t-1}^-, \Delta Y_{t-1}^-) + \xi_t \quad (1)$$

and

$$\Delta Y_t = \delta_0 + \delta_1 \left(X_{t-1}^{+} - Y_{t-1}^{+} \right) + \delta_2 \left(X_{t-1}^{-} - kY_{t-1}^{-} \right) + lags(\Delta X_{t-1}^{+}, \Delta Y_{t-1}^{+}, \Delta X_{t-1}^{-}, \Delta Y_{t-1}^{-}) + \zeta_t \ .$$

These models are more general than the conventional error correction models corresponding to case 4 below because they put fewer restrictions on their coefficients. Finally for case 4, assume that (1, -1) is a common cointegrating vector and that X and Y have the following standard error correction models;

$$\Delta X_t = \gamma_0 + \gamma \left(X_{t-1} - Y_{t-1} \right) + lags(\Delta X_{t-1}, \Delta Y_{t-1}) + \xi_t \qquad (2)$$

and

$$\Delta Y_t = \delta_0 + \delta \left(X_{t-1} - Y_{t-1} \right) + lags(\Delta X_{t-1}, \Delta Y_{t-1}) + \zeta_t \ .$$

We may easily examine the restrictions that standard cointegration puts on equation (1) by rewriting equation (2) as follows;

$$\Delta X_t = \gamma_0 + \gamma \left(X_{t-1}^{+} - Y_{t-1}^{+} \right) + \gamma \left(X_{t-1}^{-} - Y_{t-1}^{-} \right) + lags(\Delta X_{t-1}^{+} + \Delta X_{t-1}^{-}, \Delta Y_{t-1}^{+} + \Delta Y_{t-1}^{-}) + \xi_t \ .$$

Therefore, $\gamma_1 = \gamma_2 = \gamma$ and the coefficients associated with ΔX_{t-k}^{+} and ΔX_{t-k}^{-}, $k = 1, 2, \ldots$, should be the same. Similarly for ΔY_{t-k}^{+} and ΔY_{t-k}^{-}.

Recall that couching ECMs are standard error correction models, except for the fact that they show long-run equilibrium relationship and short-run dynamics of nonstationary components of data series, rather than data themselves. We offer a brief comparison to other nonstandard error correction models considered in the literature. Note however that all of the ECMs discussed below are concerned with the data series themselves, not their components. Granger and Lee (1989) allow the effects of error correcting term to be different depending on its signs, within the framework of a nonsymmetric error correction model. Siklos and Granger (1997) introduce regime sensitive cointegration by allowing on-and-off cointegrating relationship induced by changes in monetary policy. They allow data series to be stationary depending on policy regimes. In contrast, we assume that the data series are always nonstationary and allow possibly different cointegrating relationships among the components of data series corresponding to different shocks or different regimes. Escribano and Pfann (1998) introduce nonlinear error correction model embodying asymmetric costs of adjustment. Finally, Enders and Granger (1998) and Enders and Siklos (2001) consider threshold adjustment to long-run equilibrium relationships depending on the signs and magnitude of changes in error correcting terms. In the hidden cointegration framework, the adjustment to long-run equilibrium is linear, even though the asymmetric threshold adjustment can be easily accommodated. It is also possible to provide theoretical justifications to the widely used non-linear error correction models from the perspectives of hidden cointegration. This line of research is reported in Yoon (2001).

3. Simulation results

In this section, we briefly examine the size of conventional cointegrating regression augmented Dickey-Fuller [CRADF] tests when data series are subject to hidden cointegration with an unknown threshold value *d*. The power of the test is rather well known, so we do not report it; see for instance Kremers *et al.* (1992). We generate 10,000 independent random numbers from a standard normal distribution $N(0,1)$ with a sample size of 100, 300, and 500, after discarding the initial 100 observations. We set a common threshold $d \in [-1.0,\ 1.1]$ with an increment of 0.1 and employ the CRADF(k) test, with k = 0, 1, and 4, for the null of no cointegration between X and Y.

3.1 Size of cointegration tests: case 2

We assume that $\left\{\sum_1^t \varepsilon_i^{\vee} \quad \sum_1^t \eta_i^{\vee}\right\}$ are cointegrated such that $\sum_1^t \varepsilon_i^{\vee} - \sum_1^t \eta_i^{\vee} = \varsigma_t$ ~ I(0).

We use the following data generating process:

$$X_i = \sum_1^t \eta_i^{\vee} + \sum_1^t \varepsilon_i^{\wedge} - dt + \varsigma_t$$

and

$$Y_t = \sum_1^t \eta_i^{\vee} + \sum_1^t \eta_i^{\wedge} - dt\ .$$

To test for cointegration, we use Y as a dependent variable and a constant and X as independent variables. X and Y are not cointegrated by design. As d is decreasing, the proportion of $\sum_1^t \eta_i^\vee$ becomes higher relative to that of $\sum_1^t \eta_i^\wedge$ or $\sum_1^t \varepsilon_i^\wedge$ and so does the importance of hidden cointegration between the $\left\{\sum_1^t \varepsilon_i^\vee \quad \sum_1^t \eta_i^\vee\right\}$ components in X and Y. The size of standard cointegration tests is summarized in figure 1 at a nominal size of 5% for the CRADF tests. Note that the scale of each plot is different. The empirical size is shown for a sample of 100, 300, and 500. The size of the tests is increasing as sample size increases. CRADF(4) test is undersized while CRADF(0) test is badly oversized. A CRADF(1) test becomes oversized as d is decreasing. Hence, a spurious cointegrating relationship would be found more often as d is decreasing, because the importance of hidden cointegration is increasing. We may avoid the spurious cointegration finding by including more lags in the CRADF test. However, the power of the test will be lower.

3.2 Size of cointegration tests: case 3

We assume that both $\left\{\sum_1^t \varepsilon_i^\vee \quad \sum_1^t \eta_i^\vee\right\}$ and $\left\{\sum_1^t \varepsilon_i^\wedge \quad \sum_1^t \eta_i^\wedge\right\}$ are cointegrated, but with different cointegrating vectors of (1, -1) and (1, -3), respectively such that $\sum_1^t \varepsilon_i^\vee$ – $\sum_1^t \eta_i^\vee = \varphi_{1t}$ and $\sum_1^t \varepsilon_i^\wedge - 3\sum_1^t \eta_i^\wedge = \varphi_{2t}$, where φ_{1t} and φ_{2t} are stationary error terms. X and Y are not cointegrated. The data generating process is

$$X_t = \sum_1^t \eta_i^{\vee} + \sum_1^t \eta_i^{\wedge} - dt + \varsigma_t \ ,$$

where ς_t is stationary and

$$Y_t = \sum_1^t \eta_i^{\vee} + 3\sum_1^t \eta_i^{\wedge} - dt \ .$$

As *d* is increasing, the importance of the cointegration between the negative components is increasing over that between the positive ones. Figure 2 shows the size of the CRADF tests. Note that the scales of the plots are not the same. The CRADF(4) tests is undersized, while the CRADF(0) test is oversized for all *d* and CRADF(1) is oversized for *d* is bigger than 0.7 or 0.8. Therefore, a spurious cointegrating relationship would be found more often as *d* is increasing. We may avoid spurious cointegration by including more lags in the cointegration tests at the loss of power of the tests.

4. Examples

In this section, we provide two examples of hidden cointegration. The first example is concerned with U.S. short-term and long-term interest rates. The second example is on the relationship between U.S. output and unemployment, known as Okun's law.

4.1 Short-term and long-term interest rates

We use two monthly interest rates on federal funds [FYFF] and ten-year T-bill [FYGT10]. The data series, in % per annum, are available at the DRI database from 1954:8 ~ 2001:3. The data are not seasonally adjusted. Figure 3 shows the original data series. We present empirical results for two different sample periods, before and after the Fed's monetary policy change in 1979:9.

4.1.1 The 1983:1 ~ 2001:3 sample period

The starting date is chosen to avoid more volatile episodes caused by the Fed's monetary policy changes that were effective until 1982:10. The total number of observations is 219. We choose the threshold d = 0 because it makes the interpretation of estimation results very easy and natural.[3] We calculate cumulative sums of $\sum_{i=1954:9}^{t} \Delta FYFF_i^+$ and $\sum_{i=1954:9}^{t} \Delta FYGT10_i^+$ to determine $FYFF_{1983:1}^+$ and $FYGT10_{1983:1}^+$ with $FYFF_{1983:1}^+ = \sum_{i=1954:9}^{1983:1} \Delta FYFF_i^+$, for instance. Note that earlier observations up to 1982:12 are used only to determine the initial values. Figure 4 shows the interest rates in first difference and figure 5 shows $\left(FYFF_t^+, FYGT10_t^+\right)$ and $\left(FYFF_t^-, FYGT10_t^-\right)$.

There is only weak evidence of cointegration between the two interest rates. Table 2 shows the cointegration test results. The lag order in the CRADF test is selected to be the first significant one, starting with 20 lags and reducing the order one by one. Enders and Siklos (2001) also find that they are not cointegrated using the data series in logs and suggest threshold adjustment toward long-run equilibrium relationship. In this paper, we

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[3] We may select thresholds based on certain criterion, for example, maximizing the sum of correlations between $\left(\Delta FYFF_t^{\vee} \quad \Delta FYGT10_t^{\vee}\right)$ and $\left(\Delta FYFF_t^{\wedge} \quad \Delta FYGT10_t^{\wedge}\right)$. We assume for convenience that the thresholds are the same for both interest rates. For $d \in [-0.2, 0.2]$ with an increment of 0.001, d = 0.021~0.029 is selected for the current sample period, with maximum correlations of 0.67. If we set d = 0 instead, the sum of correlations is lower only by 0.00038. For the earlier sample period of 1956:1 ~ 1979:3, which will be considered below, d = 0.070 is selected, with maximum correlations of 0.54. If we set d = 0 instead, the sum is lower by 0.01237.

use the data series in levels without taking log transformation. From the various combinations of positive or negative sums of interest rate changes, evidence of hidden cointegration is found for $\left(FYFF_t^+ \quad FYGT10_t^+\right)$ and $\left(FYFF_t^+ \quad FYGT10_t^-\right)$, as summarized in table 3. Figure 6 compares residuals from the two possible hidden cointegrating regressions, estimated with a constant and trend. The residual from the first regression seems to have a clearer appearance like a stationary series. On the contrary, the evidence for hidden cointegration between $\left(FYFF_t^+ \quad FYGT10_t^-\right)$ is somewhat weak; for instance, CRADF(15) is only -3.39, where 15 is the next significant lag length. We can infer indirectly that if $\left(FYFF_t^+ \quad FYGT10_t^-\right)$ are also cointegrated, $\left(FYGT10_t^+ \quad FYGT10_t^-\right)$ are cointegrated as well and there is some evidence for it; CRADF(10) = -3.91 with $FYGT10_t^+$ as a dependent variable with trend. However, the evidence is still not definite; for instance CRADF(7) becomes only -3.32. Keeping the results in mind, it will be assumed that only $\left(FYFF_t^+ \quad FYGT10_t^+\right)$ are cointegrated. The following hidden cointegrating regression is estimated with OLS;

$$FYGT10_t^+ = \underset{(19)}{-19.5} + \underset{(32)}{0.06} \times trend + \underset{(23)}{0.52} \times FYFF_t^+ + residual_t ,$$

where *t*-values are reported in the parentheses. The residuals are already shown at the upper panel in figure 6.

In sum, there is evidence of hidden long-run equilibrium relationship between cumulated positive changes in long-term and short-term interest rates, while the interest rates themselves are not cointegrated. We may provide the following observations. The increase in the long-term interest rate may be interpreted as signaling the increase in inflationary expectations. To be credible in its fight against inflation, the Fed has responded by increasing short-term interest rate under its control. The estimated hidden cointegrating regression indicates that the Fed has responded by increasing the federal funds rate more than the increases in the long-term interest rate. Meanwhile, the Fed is more tolerant to falling long-term interest rates. The asymmetric behavior of the Fed is responsible for the no-cointegration result for the short-term and long-term interest rates. See also Enders and Siklos (2001) for the discussion on the Fed's asymmetric response. Clarida *et al.* (2000) find that the Fed's interest rate policy becomes very sensitive to changes in inflation expectations during the sample period that we consider in this subsection.

Using the residuals from the above hidden cointegrating regression, we estimate the following crouching error correction models after eliminating insignificant terms;

$$\Delta FYFF_t^+ = \underset{(3.51)}{.04} + \underset{(2.26)}{.07} \times residual_{t-1} + \underset{(3.67)}{.29} \times \Delta FYGT10_{t-1}^+ - \underset{(2.24)}{.15} \times \Delta FYGT10_{t-2}^+$$
$$+ \underset{(2.18)}{.15} \times \Delta FYGT10_{t-3}^+ + \underset{(2.06)}{.17} \times \Delta FYFF_{t-1}^+$$

$$\bar{R}^2 = .227$$

and

$$\Delta FYGT10_t^+ = \underset{(1.71)}{.03} - \underset{(3.46)}{.14} \times residual_{t-1} + \underset{(6.33)}{.43} \times \Delta FYGT10_{t-1}^+ + \underset{(2.81)}{.22} \times \Delta FYGT10_{t-4}^+$$
$$+ \underset{(3.54)}{.27} \times \Delta FYGT10_{t-7}^+ - \underset{(2.24)}{.14} \times \Delta FYFF_{t-3}^+ + \underset{(1.99)}{.13} \times \Delta FYFF_{t-5}^+ - \underset{(2.72)}{.20} \times \Delta FYFF_{t-6}^+$$

$$\bar{R}^2 = .202.$$

Robust t-values are reported in the parentheses. Note that the residual term is significant in both equations.

4.1.2 The 1956:1 ~ 1979:3 and 1961:1 ~ 1979:3 sample periods

We repeat the same analysis for the earlier sample period of 1956:1 ~ 1979:3, a total of 279 observations. The ending date is chosen to avoid the more volatile episodes starting at 1979:9. We find quite different implications on the Fed's behavior. Still little evidence of cointegration is found for the two interest rates as summarized in table 4. From the various combinations of positive and negative components of interest rates, evidence of hidden cointegration is found for $\left(FYFF_t^+ \quad FYGT10_t^-\right)$; see table 5. The estimated hidden cointegrating relationship is

$$FYFF_t^+ = \underset{(5)}{-1.3} + \underset{(12)}{0.06} \times trend - \underset{(19)}{1.64} \times FYGT10_t^- + residual_t \, .$$

The results indicate that during the current sample period the Fed has responded only to falling long-term interest rates by lowering federal funds rate. If we take the falling long-term interest rate as signaling lower expected inflation induced by future economic slowdown, the Fed has tried to avoid slowdown by lowering short-term interest rate. Compared to the estimation results reported previously for the latter sample period, we may conclude at least that the Fed did not conduct systematic monetary policy to fight inflation before Volcker's appointment as its Chairman in 1979. Recall that during the late 1960s through the 1970s, U.S. experienced high and volatile inflation. Clarida *et al.* (2000) show that the Fed is more accommodating inflation rather than fighting it during the sample period considered here. The estimated crouching error correction models are,

$$\Delta FYFF_t^{+} = \underset{(6.74)}{.13} - \underset{(3.10)}{.03} \times residual_{t-1} + \underset{(1.96)}{.20} \times \Delta FYGT10_{t-1}^{-} + \underset{(3.80)}{.33} \times \Delta FYGT10_{t-3}^{-} + \underset{(2.89)}{.30} \times \Delta FYFF_{t-1}^{+}$$

$$\bar{R}^2 = .125,$$

and

$$\Delta FYGT10_t^{-} = -\underset{(6.01)}{.04} + \underset{(3.91)}{.31} \times \Delta FYGT10_{t-1}^{-} + \underset{(2.37)}{.03} \times \Delta FYFF_{t-1}^{+}$$

$$\bar{R}^2 = .102 .$$

Robust *t*-values are reported in the parentheses. We note that the $\bar{R}^2 s$ are only half of those previously found for latter sample period. Further, the error correcting term is significant only in federal funds rate. Therefore, $FYGT10_t^-$ is a common stochastic trend responsible for the long-run dynamic behavior of the two rates, following the discussion in Gonzalo and Granger (1995).

Somewhat weaker results are found for cointegrating relationship between $\left(FYFF_t^+ \quad FYGT10_t^-\right)$ for a sample of 1961:1 ~ 1979:3. The starting date is chosen to have the same sample size as used in subsection 4.1.1 and to avoid more volatile episodes as well. The two interest rates are still not cointegrated. Further, neither are any combinations of sums of interest rate changes, including $\left(FYFF_t^+ \quad FYGT10_t^-\right)$. For instance with $FYFF_t^+$ as a dependent variable, CRADF(12) is only -3.74, while at the 5% significance level the critical value is about -3.82. The finding reveals that during the sample period, the Fed's interest rate policy is driven separately from the shocks to inflationary expectations, whether they are increasing or decreasing.

4.2 Output and unemployment rate

In this subsection, we investigate possible long-run relationship between output and unemployment. First, a note on data availability. While unemployment data are available monthly, GDP, which is widely used as a measure of output, is not. Hence we decide to use industrial production in manufacturing [IP] as a measure of output instead, along with unemployment rate in manufacturing [LURM], both available monthly from the DRI database. Figure 7 shows the data series from 1948:1 ~ 2001:3 and they are seasonally adjusted. The output is set 100 at 1992 and transformed into log. The unemployment is in %. We will use *Y* and *U* to denote output and unemployment rate, respectively. The figure indicates that the series are not cointegrated and the results reported in table 6 concur. Using real GNP and unemployment, Altissimo and Violante (2001) also find no evidence of cointegration and non-linear cointegration relationship.[4]

We investigate the existence of hidden cointegration. We select thresholds that maximize the sum of correlations between $\left(\Delta Y_t^{\vee} \quad \Delta U_t^{\wedge}\right)$ and $\left(\Delta Y_t^{\wedge} \quad \Delta U_t^{\vee}\right)$. For output, a threshold of $d_1 = 0$ is selected from $d_1 \in [-0.1, 0.1]$ with an increment of 0.001. For unemployment rate, values between -0.09 and 0.09 do not make any difference out of $d_2 \in [-0.2, 0.2]$ with an increment of 0.01. Therefore, we set $d_2 = 0$ as well. Figure 8 shows the positive and

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[4] They use the procedures suggested by Balke and Fomby (1997) and Corradi *et al.* (2000) to find a non-linear cointegrating relationship.

negative components of *Y* and *U*. In figure 9, *Y* is superimposed on *U* with appropriate modifications so that they look similar in magnitudes. In contrast to the original data, their positive and negative components are moving very closely. We test for hidden cointegration between the components and find evidence of hidden cointegration between Y_t^- and U_t^+. The results are summarized in table 7. The following hidden cointegrating regression is estimated with OLS;

$$U_t^+ = \underset{(19)}{1.31} + \underset{(91)}{0.06} \times trend - \underset{(121)}{30.17} \times Y_t^- + residual_t .$$

t-values are reported in the parenthesis. Figure 10 shows the residual from the above regression. We may provide the following observations. Output and unemployment do not share a common stochastic trend. However, falling output would increase unemployment in the long-run. There is an asymmetric response in unemployment to increase in output; when output is increasing, it does not share a common stochastic trend with unemployment rate. This asymmetric behavior of unemployment to output changes is widely noted in literature as a hysteresis hypothesis; when output returns to a level where it was before the shock, unemployment fails to return to its original level. Various explanations are given for hysteresis, for instance, an unemployment model based on a human capital, an insider/outsider model, or an institutional model; see Schorderet (2001) for discussion.

Finally, after eliminating insignificant terms the following crouching error correction models are estimated;

$$\Delta U_t^+ = \underset{(9.0)}{0.08} - \underset{(3.7)}{0.07} \times residual_{t-1} + \underset{(3.1)}{0.18} \times \Delta U_{t-3}^+ - \underset{(4.9)}{15.9} \times \Delta Y_{t-1}^- + \underset{(2.8)}{2.94} \times \Delta Y_{t-14}^-$$

$\bar{R}^2 = 0.27,$

and

$$\Delta Y_t^- = -\underset{(4.0)}{0.00} - \underset{(3.2)}{0.005} \times \Delta U_{t-1}^+ - \underset{(2.7)}{0.005} \times \Delta U_{t-3}^+ + \underset{(2.2)}{0.002} \times \Delta U_{t-6}^+ + \underset{(4.3)}{0.37} \times \Delta Y_{t-1}^- - \underset{(2.4)}{0.13} \times \Delta Y_{t-4}^-$$

$\bar{R}^2 = 0.28.$

Robust *t*-values are reported in the parentheses. The error correcting term is significant only in ΔU_t^+. Therefore, Y_t^- is a common stochastic trend, responsible for the long-run dynamic behavior of U_t^+.

5. Conclusion

In this paper, as yet possibly unnoticed cointegrating relationships between nonstationary components of data series are identified. When their components are cointegrated, the data series have hidden cointegration. We also define crouching error correction models of cointegrated components. We show that hidden cointegration is a simple example of nonlinear cointegration. We show also that standard cointegration emerges as a special case of hidden cointegration. While the data series are not cointegrated in the conventional sense, it is still possible for them to have hidden cointegration, which would help better understand their dynamic relationships and produce improved forecasts. We investigate the size of conventional cointegration tests when the data series are subject to hidden cointegration.

We apply the hidden cointegration framework to two sets of U.S. data series, for which no standard cointegration is found. First, we use U.S. short-term and long-term interest rates and find quite different implications on the Fed's interest rate policy. If the Fed is more tolerant to falling interest rates, while fighting rising long-term interest rates by aggressively adjusting short-term interest rate in fear of inflation, its asymmetric responses would produce no-cointegration between the short-term and long-term interest rates but hidden cointegration among their components. We indeed find that federal funds rate and ten-year Treasury bill rate posses hidden cointegration during the sample period in which the Fed is known to become sensitive to changes in expected inflation. Second, for U.S. output and unemployment, we find that there is an asymmetric response in unemployment rate to changes in output so that increasing output does not share common stochastic trend with decreasing unemployment rate, while falling output is cointegrated with increasing unemployment rate.

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Table 1: Hidden cointegration

Case	+ components	- components	Cointegration between *X* and *Y*	VAR
1	N	N	N	In difference
2	Y	N	N	In difference
	N	Y	N	In difference
3	Y	Y	No, if cointegrating vectors are different.	In difference
4	Y	Y	Yes, if cointegrating vectors are the same.	ECM

N: No cointegration. Y: Cointegration. ECM: error correction model

"In difference" denotes VAR in first difference.

+ and - components indicate $\left\{ \sum_1^t \varepsilon_i^+ \quad \sum_1^t \eta_i^+ \right\}$ and $\left\{ \sum_1^t \varepsilon_i^- \quad \sum_1^t \eta_i^- \right\}$, respectively.

Table 2: Cointegration tests, 1983:1 ~ 2001:3

Dependent variable	With trend	Without trend
FYFF	-2.14 [14]	-2.00 [1]
FYGT10	-3.68 [7]	-1.85 [14]
Critical value (5%)	-3.82	-3.37

FYFF: Federal funds rate. *FYGT10*: Ten-year T-bill rate. Cointegrating regression augmented Dickey-Fuller [CRADF] tests are reported. Numbers in the square parentheses are lag orders used in the CRADF tests.

Table 3: Hidden cointegration tests, 1983:1 ~ 2001:3

Dependent variable	Independent variable	With trend	Without trend
$FYFF^+$	$FYGT10^+$	-3.72 [7]	-1.84 [0]
$FYGT10^+$	$FYFF^+$	-5.40 [11] -4.51 [7]	-1.70 [0]
$FYFF^-$	$FYGT10^-$	-1.87 [1]	-1.26 [1]
$FYGT10^-$	$FYFF^-$	-2.44 [3]	-1.04 [1]
$FYFF^+$	$FYGT10^-$	-2.83 [15]	-2.87 [15]
$FYGT10^-$	$FYFF^+$	-4.39 [19] -3.39 [15]	-2.81 [15]
$FYFF^-$	$FYGT10^+$	-1.64 [1]	-1.83 [1]
$FYGT10^+$	$FYFF^-$	-2.88 [7]	-1.88 [11]
Critical value (5%)		-3.82	-3.37

$FYFF_t^+ = \sum_{i=1954:9}^{t} \Delta FYFF_i^+$, where $\Delta FYFF_i^+ = max(\Delta FYFF_i, 0)$. See Table 2 for more details.

Table 4: Cointegration tests, 1956:1 ~ 1979:3

Dependent variable	With trend	Without trend
FYFF	-3.48 [3]	-4.35 [12]
FYGT10	-2.43 [17]	-2.75 [3]
Critical value (5%)	-3.81	-3.36

See table 2 for details.

Table 5: Hidden cointegration tests, 1956:1 ~ 1979:3

Dependent variable	Independent variable	With trend	Without trend
$FYFF^{+}$	$FYGT10^{+}$	-3.29 [16]	-3.20 [16]
$FYGT10^{+}$	$FYFF^{+}$	-2.92 [16]	-3.17 [16]
$FYFF^{-}$	$FYGT10^{-}$	-2.83 [5]	-2.82 [5]
$FYGT10^{-}$	$FYFF^{-}$	-2.55 [19]	-2.78 [5]
$FYFF^{+}$	$FYGT10^{-}$	-4.25 [12]	-4.19 [12]
$FYGT10^{-}$	$FYFF^{+}$	-4.08 [12]	-4.24 [12]
$FYFF^{-}$	$FYGT10^{+}$	-2.42 [14]	-2.43 [14]
$FYGT10^{+}$	$FYFF^{-}$	-2.16 [11]	-2.33 [14]
Critical value (5%)		-3.81	-3.36

See table 2 for details.

Table 6: Cointegration test: 1948:1 ~2001:3

Dependent variable	Independent variable	With trend	Without trend
U	***Y***	-2.55 [12]	-3.29 [12]
Y	***U***	-2.21 [15]	-0.93 [12]
Critical value (5%)		-3.80	-3.35

U and *Y* denote unemployment rate and output, respectively. See table 2 for details.

Table 7: Hidden cointegration tests: 1948:1 ~2001:3

Dependent variable	Independent variable	With trend	Without trend
Y^+	U^+	-3.02 [18] -2.86 [12]	-3.14 [18] -3.06 [12]
U^+	Y^+	-2.80 [18] -2.95 [12]	-3.04 [18] -3.00 [12]
Y^-	U^+	-4.48 [0]	-0.67 [16] -0.53 [4]
U^+	Y^-	-4.69 [0]	-0.44 [16] -0.31 [4]
Y^+	U^-	-1.28 [1]	-1.24 [1]
U^-	Y^+	-1.93 [19] -2.04 [3]	-1.16 [1]
Y^-	U^-	-2.53 [12] -3.41 [8]	-1.80 [12] -2.56 [4]
U^-	Y^-	-3.36 [12] -3.90 [8]	-1.64 [12] -2.40 [4]
Critical value (5%)		-3.80	-3.35

See table 2 for details.

Captions for figures

Figure 1: Size of cointegration tests

Figure 2: Size of cointegration tests

Figure 3: U.S. short-term and long-term interest rates, 1954:8 ~ 2001:3

Figure 4: First differences of interest rates

Figure 5: Sum of positive and negative changes in interest rates

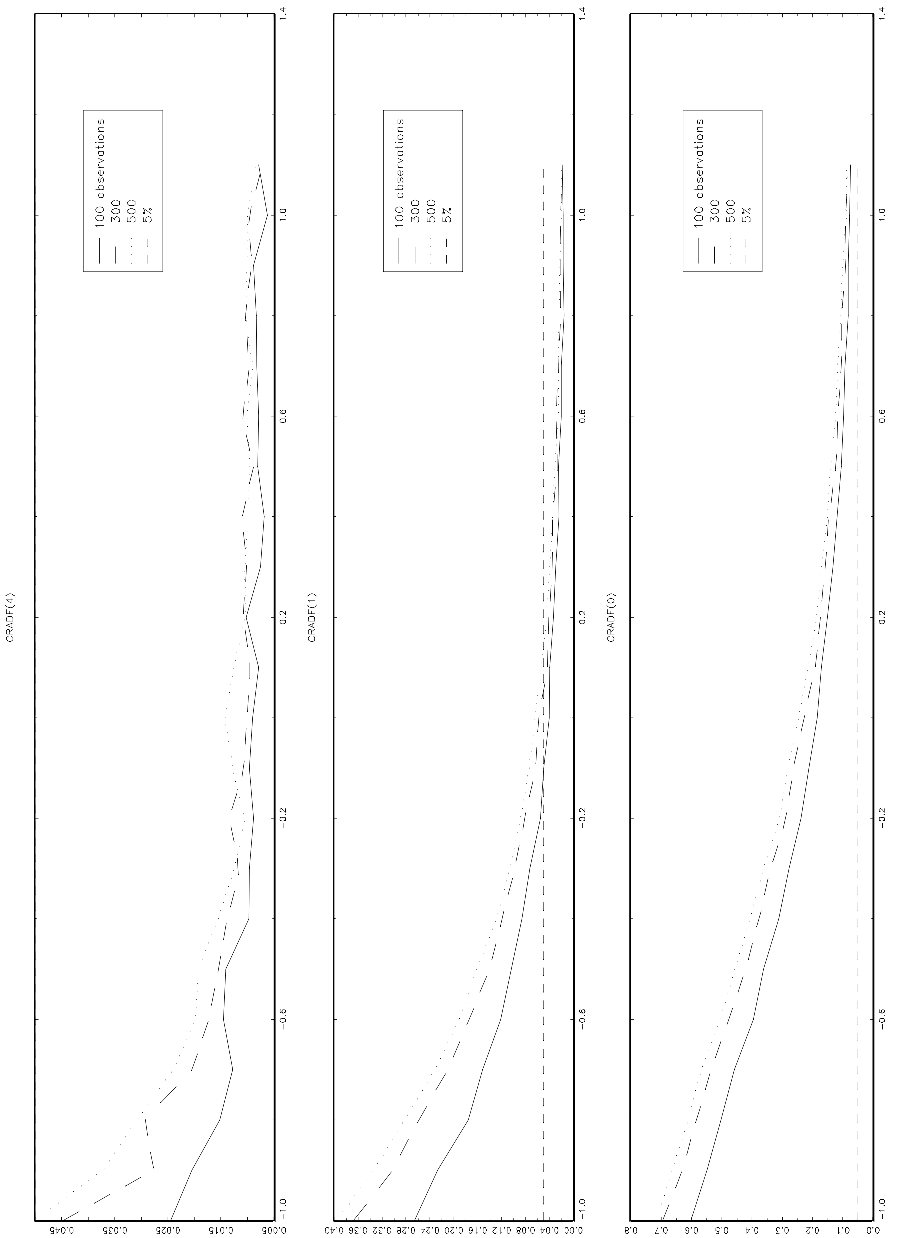
Figure 6: Comparison of two residuals, 1983:1 ~ 2001:3

Figure 7: U.S. industrial production and unemployment rate, 1948:1 ~ 2001:3

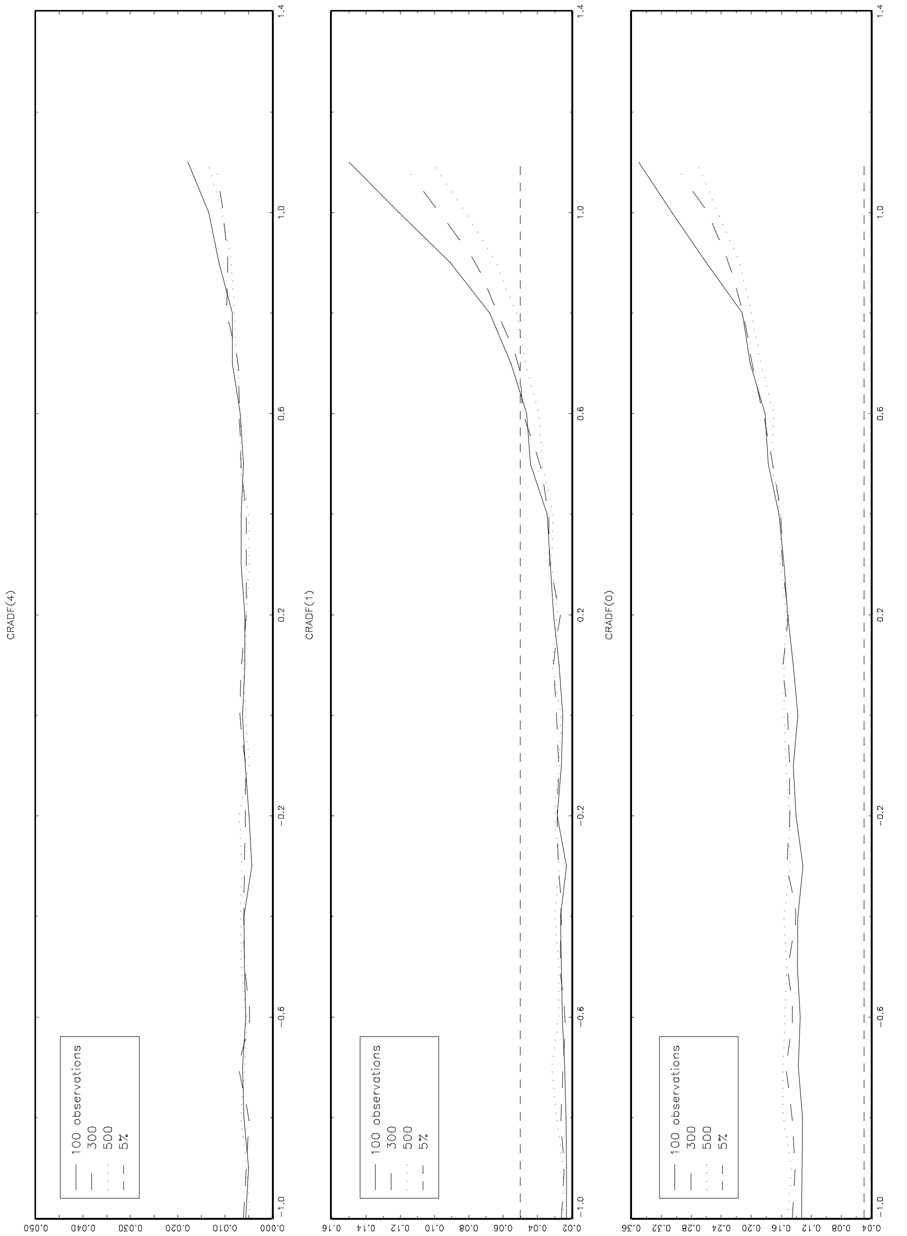
Figure 8: Transformed data series: 1948:2 ~ 2001:3

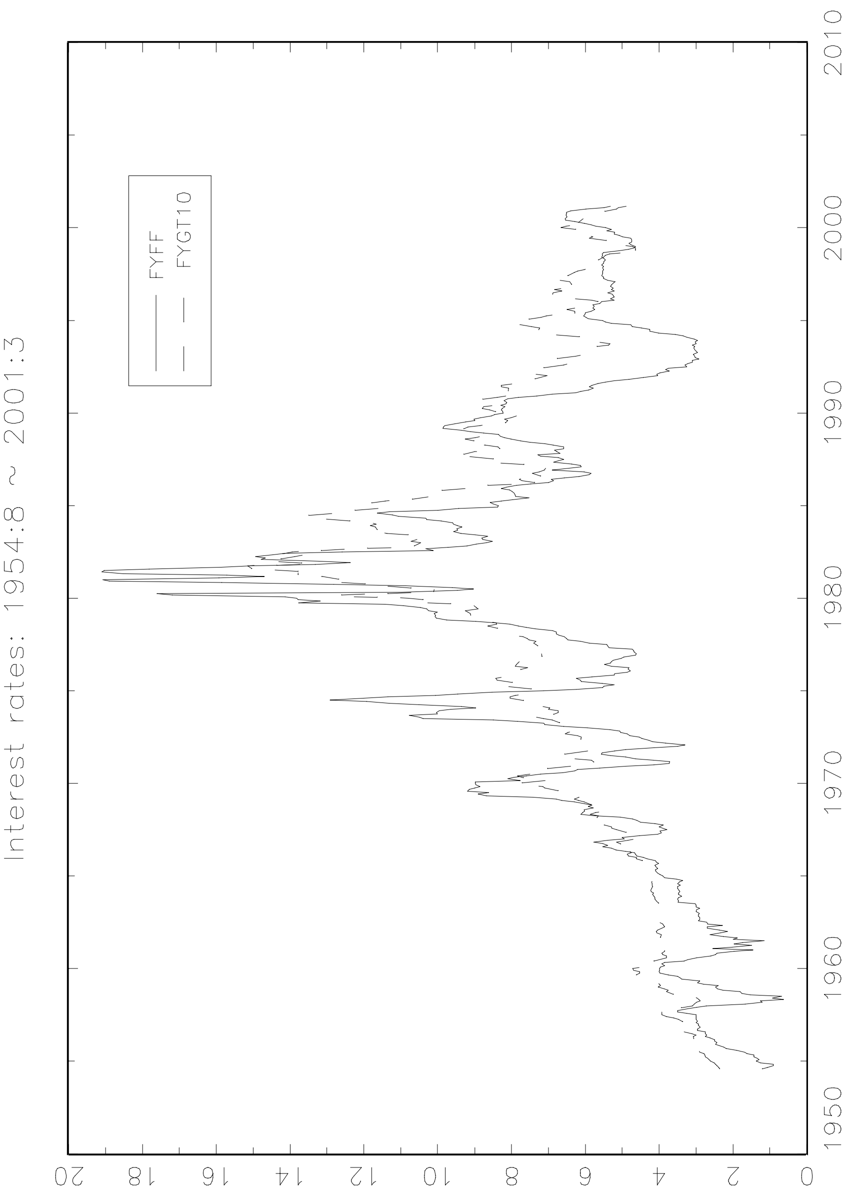
Figure 9: Comparison of transformed data series: 1948:2 ~ 2001:3

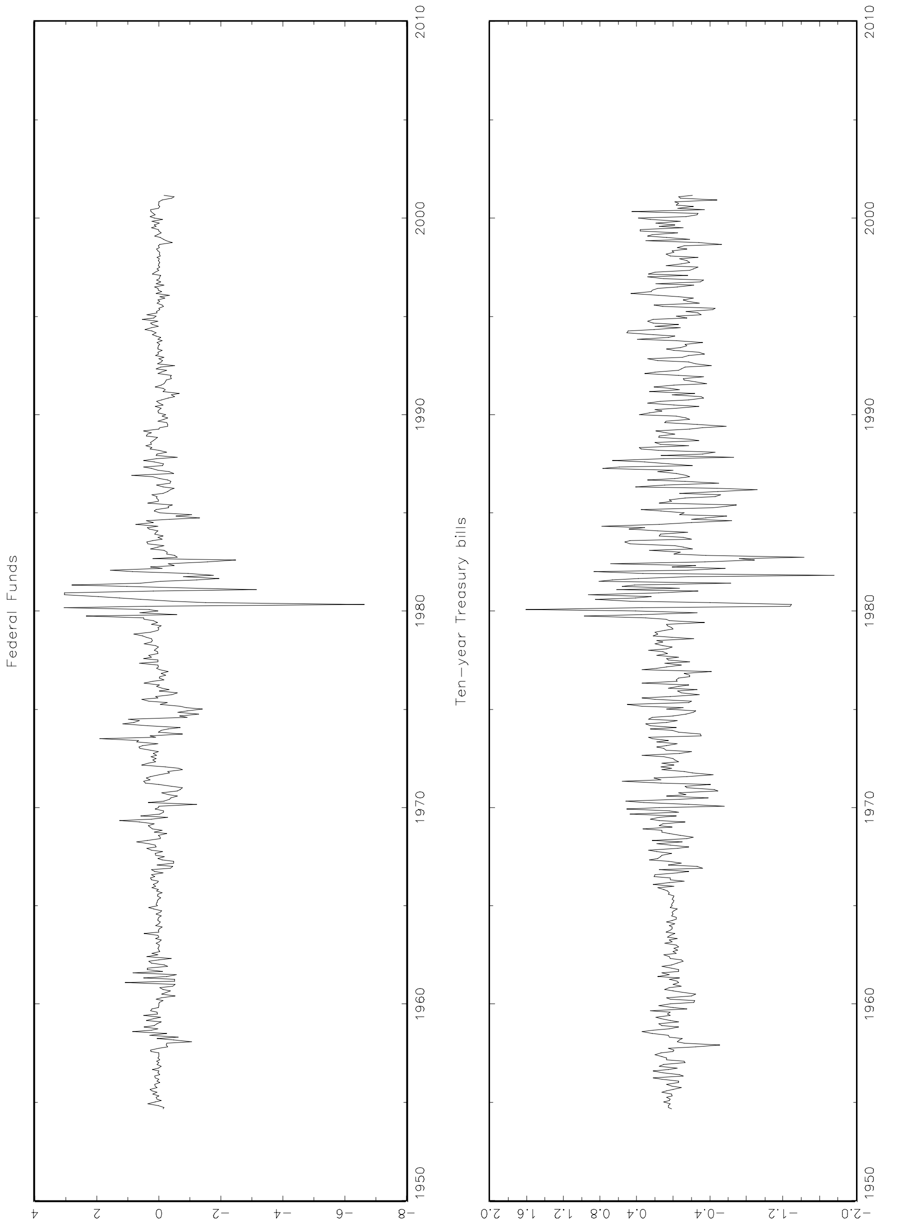
Figure 10: Estimated error correcting term between Y^- and U^+

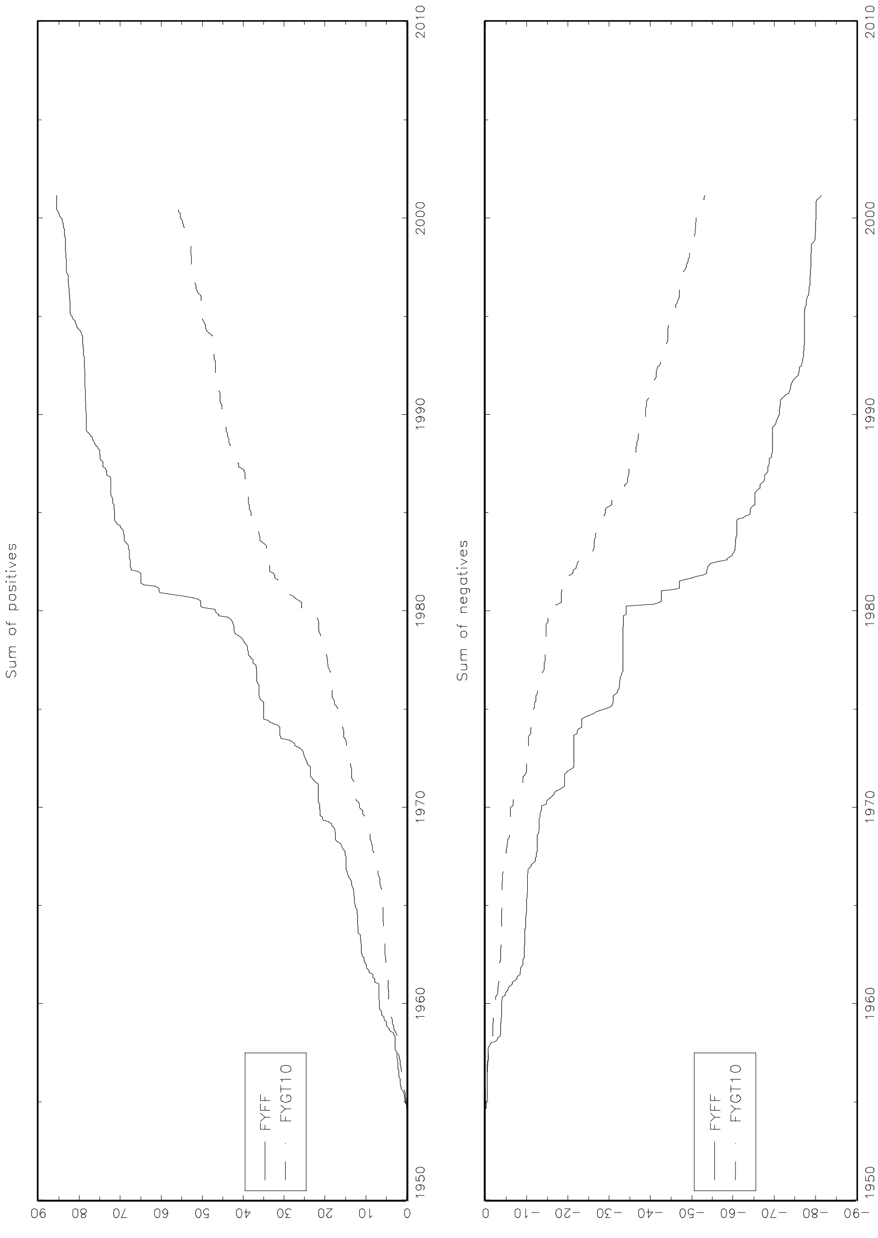


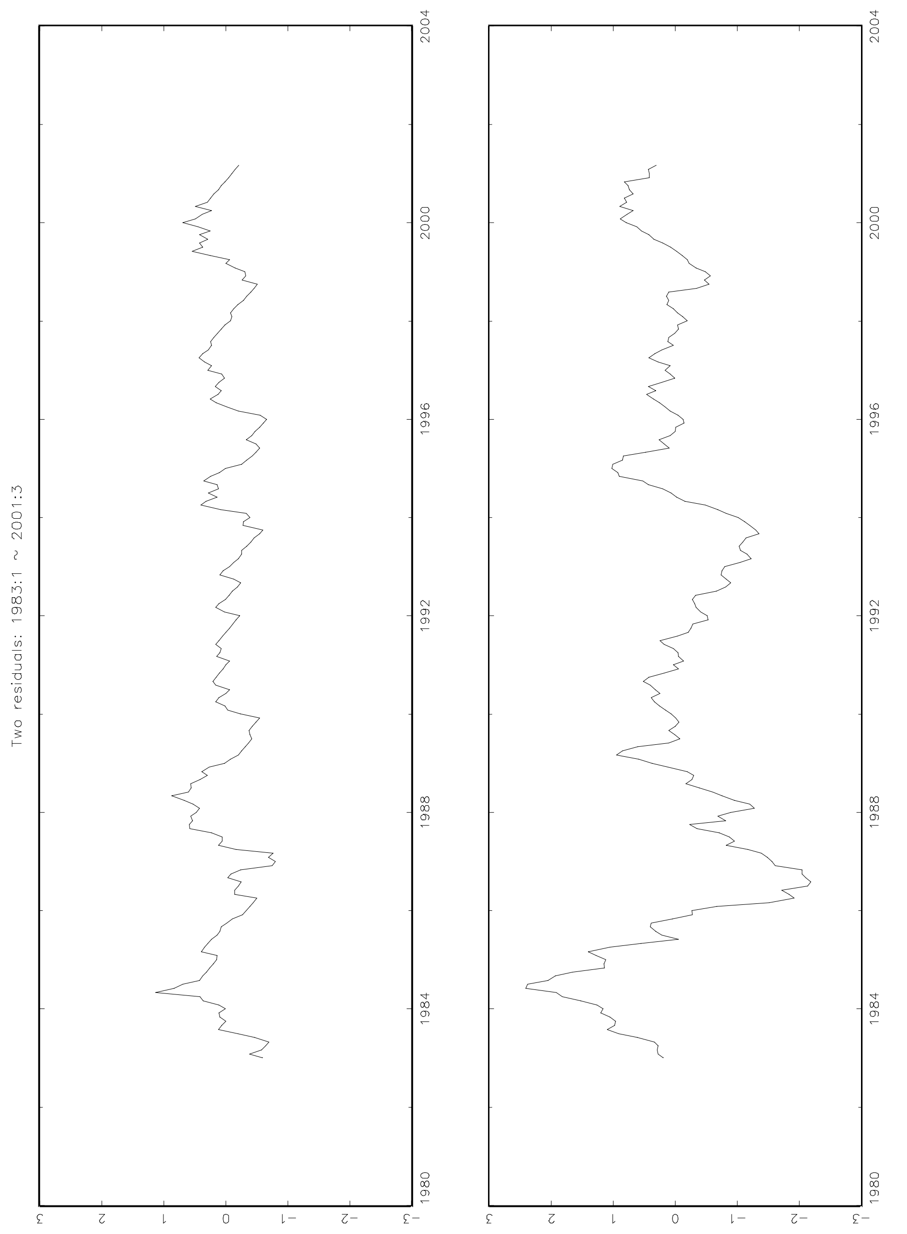
CRADF(4)
CRADF(1)
CRADF(0)
100 observations
300
500
5%

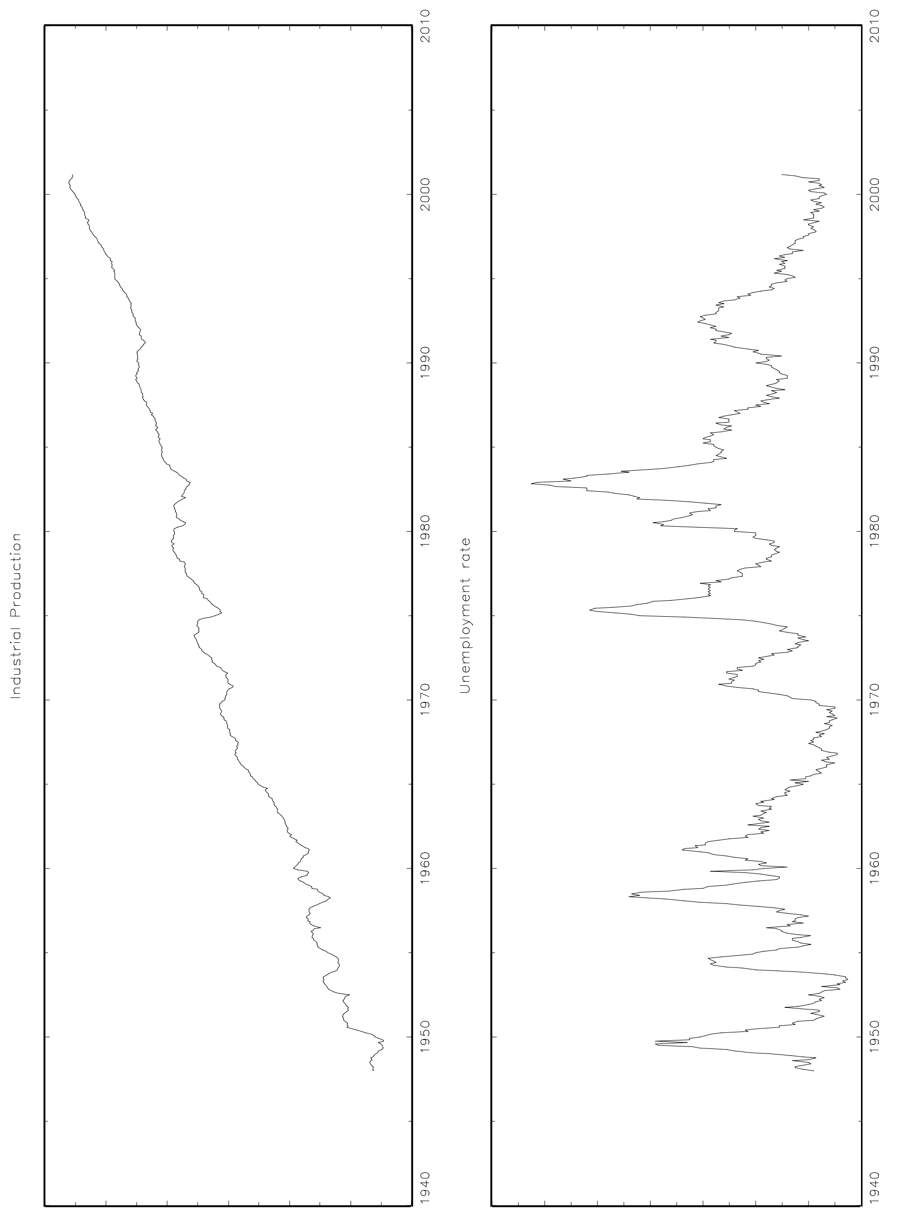

CRADF(4)
CRADF(1)
CRADF(0)
100 observations
300
500
5%


Interest rates: 1954:8 ~ 2001:3
FYFF
FYGT10
0
2
4
6
8
10
12
14
16
18
20
1950
1960
1970
1980
1990
2000
2010

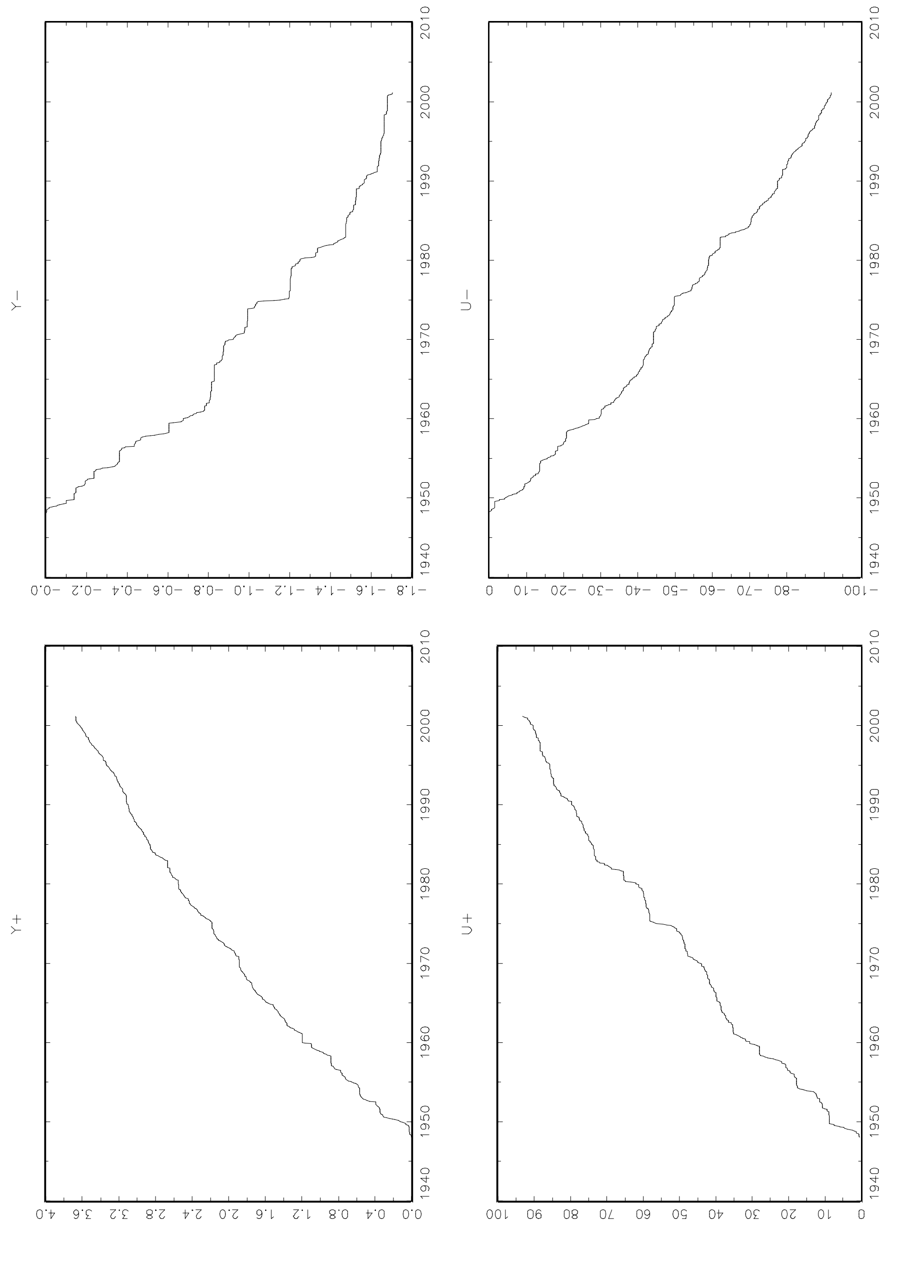

Federal Funds
Ten-year Treasury bills
1950
1960
1970
1980
1990
2000
2010
4
2
0
-2
-4
-6
-8
2.0
1.6
1.2
0.8
0.4
-0.4
-1.2
-2.0

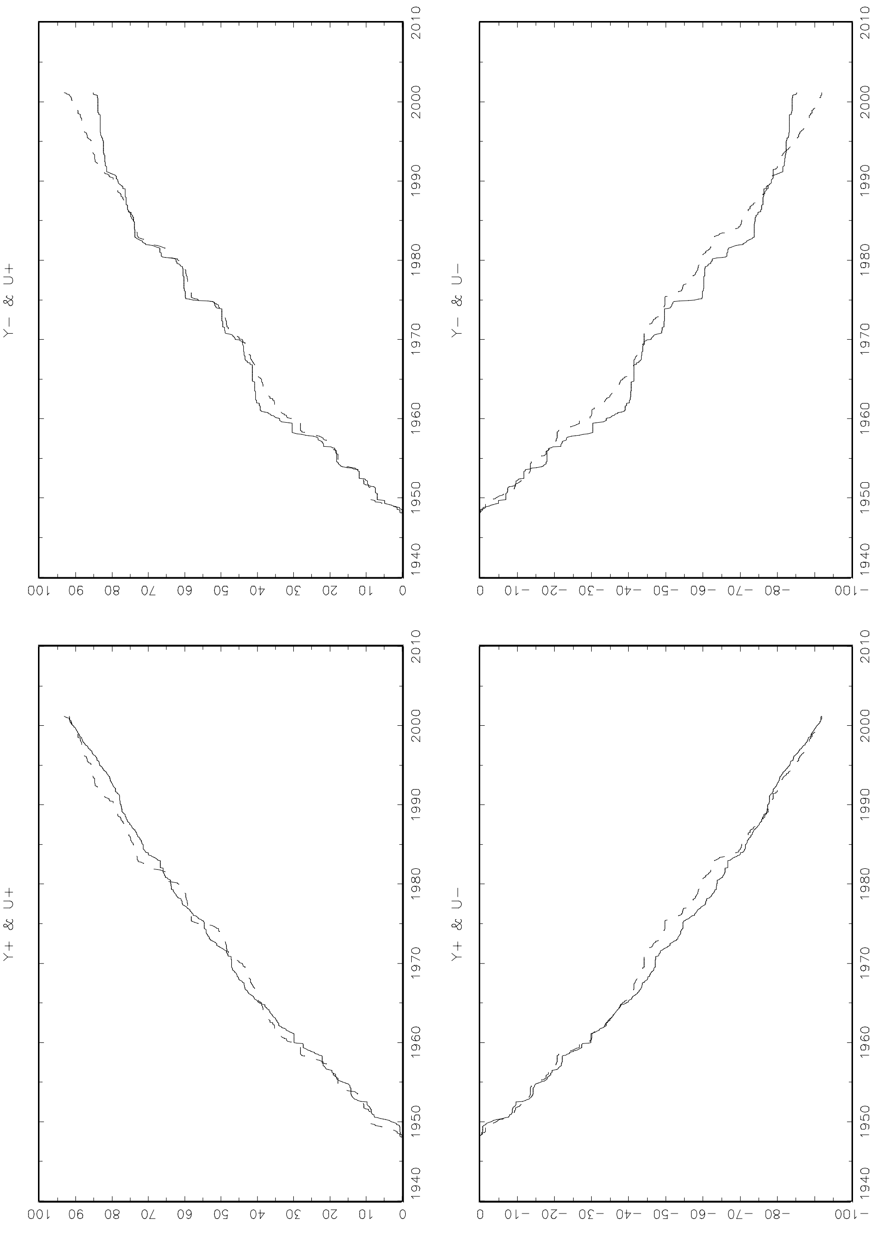

Sum of positives
Sum of negatives
FYFF
FYGT10
1950
1960
1970
1980
1990
2000
2010

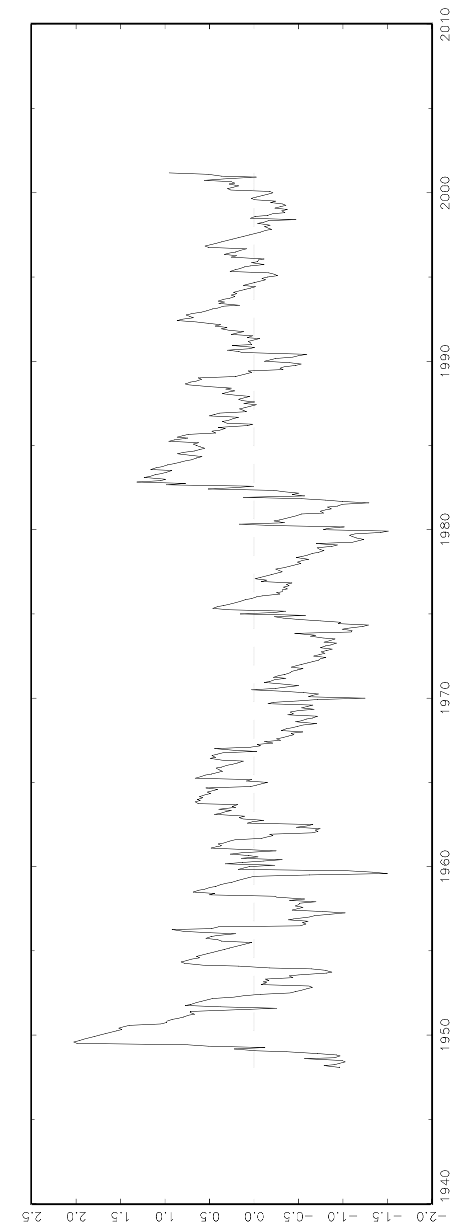

Two residuals: 1983:1 ~ 2001:3
1980
1984
1988
1992
1996
2000
2004
3
2
1
0
-1
-2
-3



Industrial Production
1940
1950
1960
1970
1980
1990
2000
2010
2.8
3.2
3.6
4.0
4.4
4.8
5.2
Unemployment rate
1940
1950
1960
1970
1980
1990
2000
2010
2
4
6
8
10
12
14
16


Y+
Y−
U+
U−


Y− & U+
Y+ & U+
Y− & U−
Y+ & U−


1940
1950
1960
1970
1980
1990
2000
2010
−2.0
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0
2.5