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Cycle? An Investigation using Monthly  
Industrial Production Series**

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## ABSTRACT

This paper examines the proposition that the business cycle affects seasonality in industrial production, with output being switched to the traditionally low production summer months when recent (annual) growth has been strong. This is investigated through the use of a restricted threshold autoregressive model for the monthly growth rate in a total of 74 industries in 16 OECD countries. Approximately one third of the series exhibit significant nonlinearity, with this nonlinearity predominantly associated with changes in the seasonal pattern. Estimates show that the summer slowdown in many European countries is substantially reduced when recent growth has been high.

JEL Classifications: E32, C22

KEYWORDS: Business cycles, seasonality, TAR models, industrial production

## 1. INTRODUCTION

Economists and statisticians have traditionally viewed seasonal patterns as devoid of economic information, leading to the widespread use of seasonally adjusted data for the analysis of economic phenomena. Recently, however, this has been questioned by a number of authors. Studying various series, Ghysels (1994), Canova and Ghysels (1994), Cecchetti and Kashyap (1996), Miron and Beaulieu (1996), Cecchetti, Kashyap and Wilcox (1997), Carpenter and Levy (1998), Krane and Wascher (1999) have found evidence that seasonality changes over the business cycle. On the other hand, while agreeing that seasonality is not constant over time, van Dijk, Strikholm and Teräsvirta (2001) conclude that cyclical changes in seasonality for industrial production are relatively unimportant compared with changes in the seasonal pattern that depend on time alone.

It has been well documented that industrial production exhibits very strong seasonal movements, with developed countries in the Northern Hemisphere exhibiting marked declines in the summer and, to a lesser extent, around Christmas; see, for example, Miron and Beaulieu (1996). Presumably due to institutional and possibly climatic factors, the strength of this seasonality differs across countries. Nevertheless, the seasonal slowdown in production in certain months intuitively implies that, even at a business cycle peak, capital may not be fully utilised throughout the year. Therefore, with capacity given in the short-run, increased demand may be met by utilising spare capacity in low production months, leading to a reduction in seasonality during business cycle booms compared with recessions. More precisely, as examined in further detail by Cecchetti and Kashyap (1996), the effect of the business cycle on seasonality in

production will depend on the properties of the marginal cost function and the costs of holding inventories.

The seasonal behaviour of inventories over the business cycle has been examined by Carpenter and Levy (1998) and Cecchetti *et al.* (1997). However, at least to date, the study of inventories in this context does not appear to offer substantial new insights compared with examination of output series. Using monthly production data for 11 industries in 19 countries, Cecchetti and Kashyap (1996) examine the seasonality/business cycle interactions by measuring the extent of seasonal variation near business cycle peaks compared with troughs, concluding that seasonality is generally less marked at peaks. Nevertheless, although their model is nonlinear, their approach is not entirely satisfactory because they effectively eliminate the nonlinearity by using a second-order approximation. They also take the business cycle indicator as given by an economy-wide variable after the application of the filter proposed by Baxter and King (1999), thereby utilising future information not available when production decisions are taken and also not focusing on the position within a specific industry. The approach of van Dijk *et al.* (2001) is more coherent, since they explicitly examine a nonlinear model that allows seasonal dummy variable coefficients to change as a function of the (lagged) change in the annual growth of the variable of interest. In fact, however, the model allows all parameters to vary over time as well as over the business cycle, leading to a highly parameterised specification.

The results of van Dijk *et al.* (2001), implying that any interactions are relative minor, have (in effect) questioned the findings of Cecchetti and Kashyap (1996) about the interactions between seasonality and the business cycle. We agree with Cecchetti and Kashyap that the existence, or otherwise, of interactions is important from an economic perspective because of the additional information this may provide about the

nature of the cost function faced by producers. This paper sheds further light on the issue.

In terms of technique, our approach is fairly close to van Dijk *et al.* (2001). However, while they use quarterly aggregate industrial production for the G7 countries, we examine a potentially richer dataset of monthly industrial production series for 16 OECD countries, using data on major components as well as the aggregate. Thus we allow the possibility that different sectors may exhibit different business cycles. Further, the use of monthly data may be important because the effect of the dominant summer slowdown in production will be substantially masked at the quarterly level<sup>1</sup>. We also provide a direct overall test for nonlinearity over the business cycle in a common framework for all series, while also allowing for deterministic time varying effects. In contrast, the tests of van Dijk *et al.* are indirect, in that they are based on Taylor series approximations to their underlying nonlinear model.

The outline of the paper is as follows. After discussion of the models in Section 2, the characteristics of our data, including the evidence for business cycle nonlinearities, are considered in Section 3. Estimates of the business cycle/seasonal interactions are then discussed in Section 4. However, to minimise problems associated with spurious effects, these results are considered only for those series that exhibit (statistically) significant evidence of business cycle nonlinearity. Some conclusions (Section 5) complete the paper.

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<sup>1</sup> Carpenter and Levy (1998) note that monthly inventory data reveals a large amount of seasonal variation that is undetectable with quarterly data.

## 2. MODELLING SEASONAL/BUSINESS CYCLE INTERACTIONS

This study aims to test whether seasonality in production is associated with the business cycle and, where such interactions are found, to explicitly estimate the cyclical shifts in production over the twelve months of the year. In particular, we wish to examine the proposition of Cecchetti and Kashyap (1996) that summer declines in production are less marked during business cycle booms. The threshold autoregressive (TAR) approach is suitable for this purpose because it allows the parameters to change when growth exceeds some threshold. In order to focus explicitly on seasonality, we use a restricted form of the TAR model.

Our basic model is described in subsection 2.1, followed by a discussion of the practical issue of trending seasonality, which could be associated with technological and institutional changes, as considered by van Dijk *et al.* (2001). The section concludes with a discussion of estimation issues.

### 2.1 The Basic Model

The basic model we employ has the form

$$f(L)\Delta y_t = \mathbf{d}_0 + \mathbf{g}_0 I_t + \sum_{j=1}^{11} \mathbf{d}_j s_{jt} + \sum_{j=1}^{11} \mathbf{g}_j I_t s_{jt} + \mathbf{e}_t \quad (1)$$

where the disturbance process  $\mathbf{e}_t \sim NID(0, \mathbf{S}^2)$ . The autoregressive operator  $f(L)$ , defined in terms of the usual lag operator  $L$ , is assumed to have all roots strictly outside the unit circle. Seasonality is captured through the variables  $s_{jt}$  which are defined by  $s_{jt} = D_{jt} - D_{12t}$ ,  $j = 1, \dots, 11$  where  $D_{jt}$  are the conventional monthly seasonal dummy variables. This formulation is frequently used for seasonality because it allows the separation of the overall mean from the deterministic seasonal effects. More precisely,

$\mathbf{f}^{-1}(1)\mathbf{d}_0$  is the overall steady state mean for  $\Delta y_t$  corresponding to the lower regime (with  $I_t = I_{t-1} = \dots = 0$ ).

In the lower regime at time  $t$  ( $I_t = 0$ ), the coefficients  $\mathbf{d}_j$  ( $j = 1, \dots, 11$ ) measure the seasonal intercept shift in each of eleven months compared with the overall intercept  $\mathbf{d}_0$ , with the intercept shift for the final month computed as  $\mathbf{d}_{12} = -\sum_{j=1}^{11} \mathbf{d}_j$ . The monthly seasonal deviations in steady state from the overall mean can be calculated from the parameters of (1), as discussed in Appendix II.

We define the regime indicator  $I_t$  by:

$$I_t = \begin{cases} 1 & \text{if } (1 + L + L^2)\Delta_{12}y_{t-1} \geq r \\ 0 & \text{if } (1 + L + L^2)\Delta_{12}y_{t-1} < r \end{cases} \quad (2)$$

Equations (1) and (2) then define a restricted TAR model, where  $r$  is the (single) threshold parameter. The coefficients  $\mathbf{g}_j$  ( $j = 0, \dots, 12$ ) give the amount by which the overall intercept and seasonal intercept terms shift in the upper regime ( $I_t = 1$ ) compared with the lower, where the seasonal intercept shift omitted from (1) can be computed as  $\mathbf{g}_{12} = -\sum_{j=1}^{11} \mathbf{g}_j$ . Within the upper regime, the overall steady state mean is given by  $\mathbf{f}^{-1}(1)[\mathbf{d}_0 + \mathbf{g}_0]$ , with the calculation of seasonal deviations from this mean again outlined in Appendix II.

From a behavioural perspective, (2) has the interpretation that seasonality changes when, over the previous three months, production has increased by more than some threshold amount  $r$  compared with a year earlier. For many countries, production peaks during the spring and early summer, before falling (sometimes dramatically) during July or August; see, for example, the monthly growth rate patterns in Miron and Beaulieu (1996, Table 3). Therefore, it can be anticipated that capacity constraints will typically be more pressing in the spring and early summer than in other months of the



year. The use of (2) as the business cycle indicator allows the possibility that the seasonal pattern in July/August will reflect capacity constraints that have operated in these earlier months.

There are, of course, other possibilities than (2) for the definition of the regime. An obvious one is to use the lagged annual difference,  $\Delta_{12}y_t$ , without smoothing through the moving sum  $1 + L + L^2$ . However, some experiments with this specification indicated that it is too noisy as a business cycle regime indicator, implying relatively frequent regime changes. Another possibility is to define the regime in terms of differences over a period shorter than a year, allowing relatively quick reactions to business cycle regime changes. Seasonality in these shorter differences would imply the use of a seasonally varying threshold parameter, leading to a type of periodic TAR model. However, such models involve a large number of parameters and hence we prefer the more parsimonious specification of (2).

Note that, in contrast to the model used by van Dijk *et al.* (2001), (1) restricts changing seasonal behaviour to the seasonal intercepts, with no effect operating through the dynamics in  $f(L)$ . This restriction is adopted to keep the parameterisation as simple as possible, with the practical advantages that interpretation is straightforward and relatively few parameters need to be estimated. Nevertheless, we also believe that an examination of shifts in the seasonal intercepts captures the essential feature of the possible relationship between seasonality and the business cycle.

Another assumption implicit in this specification is that the series  $y_t$  is integrated of order 1, or  $I(1)$ , when due allowance is made for deterministic seasonal effects through  $\sum d_j s_{jt} + \sum g_j s_{jt} I_t$ . In particular, it is assumed that  $y_t$  contains no seasonal unit roots. Indeed, the presence of seasonal unit roots would obscure the meaning of interactions between seasonality and the business cycle, because such roots imply that the seasonal

pattern is subject to constant change and hence “summer can become winter”; see Ghysels and Osborn (2001). In any case, the existence of the full set of seasonal unit roots required for annual differencing appears to be relatively rare in practice; see, for example, Beaulieu and Miron (1993), Osborn, Heravi and Birchenhall (1999), van Dijk *et al.* (2001). Nevertheless, we acknowledge the potential importance of deterministic changes in the seasonal pattern over time, as discussed next.

## 2.2 Trending Seasonality

Equation (1) assumes that seasonality in  $\Delta y_t$  has constant mean over time, after allowing for cyclical changes. In practice, however, some of our industrial production series exhibit graphical evidence that the seasonal pattern is, at least for some months of the year, trending over time. Canova and Ghysels (1994) note the presence of such seasonal trends in M1, while they also appear to be a feature of inventory investment series examined by Carpenter and Levy (1998). Nevertheless, the models used in these and other studies (including Cecchetti and Kashyap, 1996) do not incorporate trending seasonality. An exception is van Dijk *et al.* (2001), who model changing seasonality as logistic time trends. They find that trending seasonal effects dominate those associated with the business cycle, calling into question other results that do not allow for possible trends in seasonality over time.

Our approach to trending seasonality is to add a set of linear seasonal trends to (1). We guard against the possibility of a trend for  $\Delta y_t$  by also including an overall (nonseasonal) trend<sup>2</sup>. Thus, the model becomes

$$\mathbf{f}(L)\Delta y_t = \mathbf{d}_0 + \mathbf{l}_0 t + \mathbf{g}_0 I_t + \sum_{j=1}^{11} \mathbf{d}_j S_{jt} + \sum_{j=1}^{11} \mathbf{l}_j S_{jt} t + \sum_{j=1}^{11} \mathbf{g}_j I_t S_{jt} + \mathbf{e}_t. \quad (3)$$

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<sup>2</sup> It might be noted that this allows the possibility that  $y_t$  exhibits an underlying nonlinear (quadratic) trend.

There is one further complication. If the overall (nonseasonal) trend is a characteristic of the series  $\Delta y_t$ , our threshold variable will also display such trending behaviour. To avoid this problem, we detrend the annual growth rate variable, used to define the threshold in (2), by a prior regression on a constant and a linear time trend.

### 2.3 Estimation

Estimation of (3) can be undertaken using the standard approaches developed for TAR models. The crucial parameter is the threshold  $r$ , since ordinary least squares (OLS) can be applied conditional on its value. Chan (1993) shows that, for a given order of  $f(L)$ , searching over all possible values of  $r$  to minimise the sum of squared residuals produces a super-consistent estimate of this threshold. To implement this search procedure, we chose an autoregressive order of 24, thereby allowing for dynamics of up to two years<sup>3</sup>.

Following conventional practice, for instance, Hansen (1996) or Tsay (1989), we apply the grid search for  $r$  over the empirical distribution of the threshold variable, excluding its extremes. Chan and Cheung (1994) argue that a natural way to robustify the estimate of the threshold parameter in TAR models is to restrict the interval upon which the grid search is conducted, and thereby avoid the problem that one “regime” may correspond to only a small number of observations. This is particularly important in our context, since a reasonable number of observations in each regime are required in order to obtain reliable estimates of the regime-dependent monthly seasonal coefficients  $\mathbf{d}_j$  and  $\mathbf{g}_j$  in (3). Our specific procedure is to obtain the empirical distribution function of the threshold variable and to ignore the extreme 20 percent in both tails. The estimated

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<sup>3</sup> This relatively conservative value is selected in preference to choosing the lag order by an information criterion as part of the model selection procedure. Given the large number of series in our study, we prefer this on grounds of practicality. It also avoids some of the potential pitfalls of using an information criterion for lag selection in the context of seasonal time series; for example, Hall (1994) finds that such procedures may not work well when the autoregressive operator has a seasonal form.

threshold is then obtained by searching over the central 60 percent of the empirical distribution function in 1 percent increments. Conditional on this  $r$ , we then estimate (3) by OLS.

### 3. DATA CHARACTERISTICS

We analyse seasonally unadjusted monthly indexes of industrial production for 16 OECD countries available from the OECD *Main Economic Indicators* database. The variables and countries selected are those classified as industrial production series for the specific country and available over a long period. In all, 74 series are analysed. Information about the sample period and some descriptive statistics for each series can be found in Appendix I. It is clear that these variables represent a variety of historical experiences, as captured by their overall means and standard deviations. Typically, our series commence in January 1960, with the latest starting date used being January 1971. The sample ends in December 1994 or during 1995. Prior to analysis, all series are transformed to monthly percentage growth rates by taking first differences of the (natural) logarithms and multiplying by 100.

Both total industrial production (this being the quarterly analogue of the series considered by van Dijk *et al*, 2001) and manufacturing output are available for all 16 countries studied here. In addition, for most countries, monthly industrial production data for the consumer goods (for either total or non-durable and durable separately), intermediate goods and investment goods sectors are also available. A small number of other series classified as industrial production are available for a few countries, the most common of these being the construction series included for four European countries. The countries covered include all G7 countries except Canada, which is omitted due to data availability considerations. European countries are covered particularly well, with

14 such countries (including the UK) represented. The two major non-European countries, namely the US and Japan, are included.

Virtually all series exhibit strong seasonality, as measured by the  $R^2$  from a simple linear regression of the growth rate on twelve seasonal dummy variables; see Appendix I. Indeed, with the single exception of consumer goods in Greece, the value of this  $R^2$  measure exceeds 0.5 for all 74 series. According to this measure, the extent of seasonality varies over countries, with the Scandinavian countries of Finland, Norway and Sweden having particularly marked patterns compared with, say, the US, UK or Germany. This finding is not new. Indeed, when they examine the relative importance of country and industry effects in seasonal patterns for production series, Cecchetti and Kashyap (1996) conclude that the former dominate the latter.

Our model for seasonal/business cycle has already been discussed in the previous section. However, inference in such models is not straightforward, because the key parameters capturing the interactions are not identified when the true process is linear. To guard against the possibility that the results obtained from the estimated models are spurious, prior testing for nonlinearity is undertaken. Other characteristics discussed in this section are outliers and the nature of the business cycle and trends captured by the estimated TAR models.

### 3.1 Nonlinearity

The problem of how to conduct hypothesis tests is sometimes solved by linearisation of the nonlinear model. This is the route favoured by van Dijk *et al.* (2001) in their study of the interactions between seasonality and the business cycle for total industrial production. We, however, favour more direct tests based explicitly on our nonlinear TAR specification.

In the context of (3), linearity is tested through the null hypothesis

$$H_0 : \mathbf{g}_0 = \mathbf{g}_1 = \mathbf{g}_2 = \dots = \mathbf{g}_{11} = 0. \quad (4)$$

A test of (4) involves non-standard inference, since the threshold parameter is not identified under this null hypothesis; see, for instance, Hansen (1996). Although Hansen develops an asymptotic theory for such cases, Potter (1995) finds that the size of his test is sometimes too conservative with finite sample sizes. Thus, we will follow an approach based on direct Monte Carlo simulation, as in Balke and Fomby (1997) or Obstfeld and Taylor (1997).

Using a grid search over  $r$ , as described above, (3) is estimated and the usual Wald  $F$ -test statistic for  $H_0$  is computed. Since  $r$  is chosen to minimise the residual sum of squares, this Wald statistic must be the maximal value over the values of  $r$  considered and hence it is often denoted as *sup-Wald*. Monte Carlo simulations are then used to generate data from the estimated null (linear) model, and the TAR model estimation (including the grid search) is repeated for each of 10,000 replications in order to generate the empirical distribution of the test statistic under the null hypothesis<sup>4</sup>. The reported  $p$ -value for the *sup-Wald* statistic is obtained using this empirical distribution.

The summary *sup-Wald* test results in Table 1 (detailed results are in Appendix D), points to our nonlinear model being appropriate in some countries to a greater extent than in others. In particular, four out of five series for both Finland and Spain yield significant statistics at the 5 percent level, with two out of three series for Luxembourg also indicating significant business cycle nonlinearity. At the other extreme, none of the eight US series yield a significant *sup-Wald* statistic at even the 10 percent level. In terms of industrial sectors, this table also indicates that the aggregate industrial

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<sup>4</sup> This number of replications should give a reasonable approximation of the true critical value for the *sup-Wald* statistic at significance levels of, say, 5% or greater. However, the smaller the empirical “ $p$ -value” reported, the less reliable is the approximation to the true  $p$ -value due to the smaller number of replications that allow estimation of the tail values of true *sup-Wald* distribution.

production and the manufacturing production series for around a quarter of the countries reject linearity. However, rejections are (proportionately) even more marked for the intermediate goods sector, with rejection for six of the nine series at 5 percent. Such nonlinearity is, however, apparently not an important general feature of the consumer goods and investment goods sectors.

#### TABLE 1 ABOUT HERE

In addition to the *sup-Wald* test, we also report results from the TAR nonlinearity test proposed by Tsay (1989); the implementation of this test is discussed in Appendix I. Overall, the Tsay test confirms the extent of nonlinearity found for our series, which is reassuring. There is some disagreement about the significance of nonlinearity for specific series, but this is not surprising given the different forms of the tests. For nonlinearity of the type we are seeking through the two-regime TAR model of (3), we anticipate the more specific *sup-Wald* statistic to have greater power and we pursue the analysis of this model on the basis of significance of the *sup-Wald* statistic. Estimation results are presented in Table 2 for the series which yield a significant statistic at the 10 percent level.

#### TABLE 2 ABOUT HERE

### 3.2 Outliers

Conventional residual diagnostic tests for autocorrelation, ARCH effects and non-normality were computed for all estimated models of Table 2. In practice, these revealed no major problems, with the exception of excess kurtosis. Such excess kurtosis

may be associated with outliers, which are known to be potentially important for the estimation of nonlinear models (see, in the context of smooth transition models, the discussions in Öcal and Osborn, 2000, or van Dijk, Franses and Lucas, 1999). For those series with significant excess kurtosis at the 1 percent level, dummy variables were introduced to handle outliers. Our procedure was to include a dummy for a specific observation when the largest residual (in absolute value) exceeded four times the overall residual standard deviation. The threshold variable (2) was corrected for the outlier by simple interpolation of the offending observation based on an AR(24) model for  $\Delta y_t$ , including seasonal dummy variables. The TAR model was then re-estimated and the procedure repeated until no further outliers were detected.

The detailed estimation results presented are computed with outlier dummies included and the number of such dummies is indicated<sup>5</sup>. As seen from Table 2, most series required none or only one outlier dummy.

### 3.3 Business cycle and trend characteristics

In addition to the *sup-Wald*  $p$ -value, Table 2 shows the  $p$ -value for a conventional  $F$ -test of the upper regime seasonal restrictions  $\mathbf{g}_j = 0, j = 1, \dots, 11$ , which we denote  $F_{11}$ . Significance for individual coefficients, including the overall intercept shift term  $\mathbf{g}_0$ , is indicated using conventional  $t$ -tests. Chan (1993) shows, in the context of a conventional TAR model, that estimation of the threshold parameter  $r$  is super-consistent and hence, conditional on the presence of TAR nonlinearity, standard distributional results apply for the coefficients. The threshold itself is shown as the proportion of observations estimated to fall in the lower regime, denoted  $r^*$ .

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<sup>5</sup> Our outlier procedure was invoked only for the series of Table 2, namely series that yielded a significant *sup-Wald* statistic (at 10 percent). Where relevant, the values presented for this statistic in Table 2 and the Appendix Table are computed for the model including outlier dummies.



One of the most striking results in Table 2 is that a shift in the overall (nonseasonal) intercept does not appear to be the dominant source of nonlinearity. Only four of the 26 series produce an estimate of  $g_0$  which is significant at 5 percent. In contrast, the  $F_{11}$ -test for constant seasonal dummy variable coefficients fails to reject the null hypothesis (at 5 percent) for only one series. Therefore, it appears that the nonlinearity detected by the *sup-Wald* statistic is associated primarily with a change in the seasonal pattern (rather than the overall intercept) over regimes.

The estimated values of  $g_0$  and  $r^*$  indicate that the model captures a variety of business cycle characteristics in our series. In approximately a third of the series in Table 2,  $g_0$  is positive and  $r^*$  less than 0.5. In such cases it is reasonable to associate the lower regime with recession and the upper one with expansion, even though this labelling may not be entirely accurate. Other cases (such as Finland Manufacturing or France Intermediate) have estimated  $r^*$  greater than 0.5 with positive  $g_0$ , so that the regimes appear to be associated with high growth versus low to moderate growth. However, yet other series (including Spain Intermediate and Sweden Manufacturing) produce negative estimates of  $g_0$ . Although this implies a lower overall growth rate in the upper regime compared with the lower one, it must be recalled that the regimes are defined in terms of the annual growth rate over the previous three months, and not the contemporaneous growth rate.

Significant (at the 5 percent level) overall trend effects are indicated by the estimated  $I_0$  in Table 2 for eight series. In all these cases, the trend is downward. Therefore, although the mean growth is almost always positive over the sample period, in many cases it has nevertheless declined significantly over time. Perhaps more remarkable, however, is that the  $F_{11}$ -statistic that tests the presence of seasonal trends through the null hypothesis  $I_j$  ( $j = 1, \dots, 11$ ) points to these being significant at 5

percent for 18 of the 26 series. This echoes the important role found by van Dijk *et al.* (2001) for seasonal trends in aggregate quarterly industrial production series.

The next section examines the nature of the changes in seasonality over the regimes of the TAR model.

#### 4. SEASONALITY OVER THE BUSINESS CYCLE

The results in Table 2 include the estimated seasonal dummy variable coefficients in the lower regime, the seasonal shift terms which apply for the upper regime and the estimated seasonal trend coefficients. That is, in terms of (3), we show the estimated values of  $\mathbf{d}_j$ ,  $\mathbf{g}_j$  and  $\mathbf{l}_j$  ( $j = 1, \dots, 11$ ), together with an indication of significance for each coefficient according to a conventional  $t$ -test. The twelfth seasonal coefficient and its significance is obtained in each case from the restriction that the corresponding terms must sum to zero over the year.

Our interest focuses on the seasonal/business cycle interactions and, in Figure 1, we also present the implied deviation in steady state for each month in relation to the overall mean. These mean deviations are shown for both the upper and lower regimes. The details of our computational method are presented in Appendix II, but it should be noted here that each seasonal mean deviation depends on all seasonal intercepts, with weights that are nonlinear functions of the autoregressive parameters. The series included in Figure 1 are identical to those in Table 2, namely those series that produce a significant *Sup-Wald* statistic at the 10 percent level. Figure 1 expresses the means as deviations from the overall mean in each regime, with the trend terms ignored, so that each set of monthly seasonal mean deviations sums to zero.

## FIGURE 1 ABOUT HERE

Both the coefficients of Table 2 and the seasonal means of Figure 1 indicate that industrial production series of European countries typically experience their largest seasonal change in July or August, depending on the country, with a large fall immediately followed by an increase of similar magnitude in the following month. It is precisely this summer slowdown that exhibits the greatest effect from cyclical influences. To be specific, ignoring German Construction (which does not exhibit a marked summer slowdown) and the three Japanese series, 16 out of the remaining 22 series in Table 2 show the estimated upper regime shift coefficient corresponding to the summer slowdown,  $g_7$  or  $g_8$  as appropriate, to be positive and significant at the 5 percent level. The size of this estimated seasonal intercept shift is not negligible, as its magnitude is typically equal to one or two times the residual standard deviation. Corresponding to the reduced summer slowdown, the mean growth in the following one or two months also tends to be lower in the upper regime, which is compatible with the summer reduction being less dramatic. These results are consistent with the hypothesis of Cecchetti and Kashyap (1996) that firms reallocate production to the usually slack summer months during business cycle expansions.

The one non-European country represented in Table 2 and Figure 1 is Japan. Although the summer slowdown is not as dominant as for European countries, the general pattern is again a summer decline that is tempered in the upper regime compared with the lower one.

There is also evidence in some series of a seasonal reallocation of production in the upper regime at the Christmas holiday period (December and/or January), although

it is not as large or widespread as the cyclical change observed during the summer. For example, Figure 1 shows that the mean for the Austrian Intermediate series exhibits a substantial seasonal fall in December, with this being less dramatic in the upper than the lower regime.

There are some notable exceptions to the comments just made. In particular, the construction industry in various countries exhibits a pattern where a December/January decline is exaggerated in the upper regime compared with the lower. This pattern is particularly notable in Figure 1 for German Construction. The four series for which we have data on the construction industry (France, Germany, Belgium and Luxembourg) relate to Northern European countries and have substantial winter seasonal effects. It is possible that the additional seasonality detected during winter for these series in the upper regime may relate to the weather, rather than a conscious choice to reallocate production over months. Consider the situation where output has been growing strongly in the autumn, leading to a high level in December say, but only a given level of construction activity is feasible in January because of the weather. In this case, a *greater* decline will result compared with the norm. Table 2 shows significant negative upper regime shift coefficients for the winter months (December, January and/or February, depending on the country) for all construction series analysed.

Because we have used significance of the business cycle effects as the criterion for inclusion in Figure 1 (and Table 2), not all countries are examined there. However, there do appear to be substantial country-specific influences that affect the extent of interaction between seasonal patterns and the business cycle. In particular, the reduction of the summer slowdown in the upper regime appears to be less pronounced for Austria, Germany, Japan and the UK than for other included countries. Nevertheless,

irrespective of the particular seasonal pattern over the months of the year, this pattern is typically less marked in the upper regime compared with the lower.

Although our interest focuses on the seasonal/business cycle interaction, it might be noted from Table 2 that significant seasonal trend terms are often observed in the summer months. Focusing again on July or August as appropriate, the associated trend coefficient is typically positive and significant (this is the case at 5 percent in 15 of the 26 cases). Therefore, over time the summer seasonal has generally increased in magnitude, although the opposite is true for the two UK series. A significant increase in the magnitude for the December seasonal slowdown is also noticeable in many cases. Since all series, with the single exception of Belgium Construction, have positive mean growth over our sample period (see the table of Appendix I), the phenomenon of reduced seasonality in the upper business cycle regime appears to be a short-term one when capacity constraints operate. The evidence suggests that, in the longer term, as production increases capacity generally expands to facilitate or even increase the seasonal pattern in production.

## **5. CONCLUSIONS**

We have found evidence of business cycle nonlinearity in around a third of the monthly industrial production series examined. To summarise our substantive finding, production is spread more smoothly over the months of the year when the series is growing strongly than when it is not. In particular, the summer slowdown is less marked when recent growth has been relatively strong. Although we use a different approach, our findings reinforce those of Cecchetti and Kashyap (1996).

Our results also raise some interesting issues. Although we define business cycle regimes in terms of the (lagged) annual growth rate, the regimes appear to exhibit greater effects on the seasonal pattern than on the overall series mean. In other words, at least for some series, the stage of the business cycle captured here has more impact on the organisation of production over the months of the year than on the overall growth rate of output. Building on the work of Cecchetti and Kashyap (1996), this may reflect the cost structures in different industries and countries. However, it also implies that the use of seasonally adjusted data will obliterate the information in seasonality itself about the stage of the business cycle.

Seasonal/business cycle interactions are apparently stronger in some countries than others. Indeed, although much earlier work focuses on the US, the interactions there appear to be substantially weaker than in European countries such as Finland, Germany and Spain. Further, the communality of some patterns across series within such countries in Figure 1 points to the potential value of a panel data approach. This is, however, beyond the scope of the present paper.

We have found than an important aspect of seasonality in industrial production is the trend-like changes in the pattern that have occurred over our sample period. If the explanation of changing seasonality over the business cycle lies in the nature of the cost function faced by producers, then we might anticipate that at least part of these seasonal trends may also be attributable to similar causes. Research may be warranted on whether long-run changes to the seasonal pattern in production shed further light on the nature of the cost function faced by producers.

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**Table 1. Summary Nonlinearity Test Results**

Country	Series #	Significant at		Classification	Series #	Significant at	
		5%	10%			5%	10%
Austria	5	1	1	Total industrial prod.	16	4	4
Belgium	7	0	1	Manufacturing	16	3	6
Finland	5	4	4	Consumer durables	3	0	1
France	7	1	2	Consumer non-durables	3	0	0
Germany	7	1	3	Consumer goods	9	1	2
Greece	4	0	0	Intermediate goods	9	6	6
Italy	4	1	2	Investment goods	11	2	2
Japan	6	1	3	Construction	4	2	4
Luxembourg	3	2	2	Other series	3	0	1
Nederlands	2	0	0				
Norway	2	1	1				
Portugal	2	0	0				
Spain	5	4	4				
Sweden	2	0	1				
United Kingdom	5	2	2				
United States	8	0	0				
Total	74	18	26		74	18	26
Percentage		24%	35%			24%	35%

**Table 2. Estimated Models**

	Estimated Coefficients												$g_0/I_0$	$F_{11}$	SupWald $r^*$	$s$ Outliers
	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec				
<u>Austria Intermediate</u>																
Lower regime	<b>-8.91</b> <sup>a</sup>	0.91	<b>5.60</b> <sup>a</sup>	<b>5.95</b> <sup>a</sup>	<b>3.41</b>	-0.06	<b>-9.60</b> <sup>a</sup>	<b>-8.26</b> <sup>a</sup>	<b>7.46</b> <sup>a</sup>	<b>6.70</b> <sup>a</sup>	<b>4.07</b> <sup>b</sup>	<b>-7.25</b> <sup>a</sup>				
Upper regime shift	<b>2.68</b> <sup>b</sup>	-1.47	<b>-3.51</b> <sup>a</sup>	-1.06	-0.47	-0.14	0.55	0.21	1.02	-0.93	-0.69	<b>3.83</b> <sup>a</sup>	0.53	0.003	0.023	3.18
Trend	<b>-0.018</b> <sup>a</sup>	-0.007	0.002	0.006	0.002	0.000	-0.008	<b>-0.035</b> <sup>a</sup>	<b>0.019</b> <sup>a</sup>	<b>0.021</b> <sup>a</sup>	0.002	<b>0.016</b> <sup>b</sup>	0.000	0.000	0.50	0
<u>Belgium Construction</u>																
Lower regime	14.57	-8.79	<b>16.22</b> <sup>b</sup>	8.18	1.26	<b>17.56</b> <sup>b</sup>	<b>-60.91</b> <sup>a</sup>	<b>35.03</b> <sup>a</sup>	11.15	-1.86	1.07	<b>-33.48</b> <sup>a</sup>				
Upper regime shift	<b>-22.87</b> <sup>a</sup>	1.87	<b>-13.71</b> <sup>b</sup>	1.38	4.03	1.04	<b>13.22</b> <sup>b</sup>	-6.86	3.27	3.16	4.85	<b>10.61</b>	-1.65	0.006	0.061	15.04
Trend	<b>0.046</b>	0.001	-0.010	-0.006	-0.008	0.041	<b>-0.102</b> <sup>a</sup>	<b>0.049</b>	0.028	-0.003	-0.001	-0.033	-0.006	0.070	0.28	0
<u>Finland Intermediate</u>																
Lower regime	1.85	0.24	<b>4.69</b>	0.30	-2.77	-2.18	<b>-21.15</b> <sup>a</sup>	<b>10.09</b> <sup>a</sup>	<b>7.52</b> <sup>a</sup>	<b>4.05</b>	0.58	-3.21				
Upper regime shift	1.50	-0.95	-1.92	-1.60	0.84	0.26	<b>7.72</b> <sup>a</sup>	<b>-3.51</b> <sup>b</sup>	-1.96	0.50	0.18	-1.06	0.39	0.001	0.021	3.84
Trend	-0.003	0.003	-0.005	0.011	-0.002	<b>0.019</b> <sup>a</sup>	<b>-0.017</b> <sup>a</sup>	-0.006	0.004	-0.009	0.006	-0.003	-0.002	0.069	0.21	3
<u>Finland Investment</u>																
Lower regime	0.42	4.83	3.13	-6.11	0.17	-3.63	<b>-41.93</b> <sup>a</sup>	<b>10.72</b>	<b>13.55</b> <sup>b</sup>	<b>19.01</b> <sup>a</sup>	0.83	-1.00				
Upper regime shift	-0.20	1.49	0.19	1.95	-2.29	1.17	<b>11.51</b> <sup>a</sup>	<b>-4.81</b> <sup>b</sup>	-2.20	<b>-4.03</b>	-1.42	-1.35	<b>2.41</b> <sup>b</sup>	0.000	0.000	6.29
Trend	<b>-0.030</b> <sup>a</sup>	-0.008	-0.008	<b>0.019</b>	<b>0.019</b>	0.016	-0.013	-0.004	-0.003	0.001	0.001	0.010	0.000	0.131	0.38	1
<u>Finland Manufacturing</u>																
Lower regime	1.58	-1.87	0.41	3.84	1.25	<b>-4.82</b>	<b>-18.47</b> <sup>a</sup>	<b>8.12</b> <sup>a</sup>	<b>4.84</b>	<b>5.89</b> <sup>b</sup>	-0.86	0.08				
Upper regime shift	-0.64	1.22	<b>3.50</b> <sup>b</sup>	-1.71	1.72	<b>-2.94</b> <sup>b</sup>	<b>6.35</b> <sup>a</sup>	<b>-3.58</b> <sup>b</sup>	-2.28	-1.99	-0.25	0.59	0.91	0.000	0.000	3.62
Trend	-0.005	-0.002	0.003	-0.003	0.006	0.005	-0.001	0.002	0.004	0.004	-0.004	-0.009	-0.002	0.818	0.80	3
<u>Finland Total</u>																
Lower regime	0.92	0.49	1.39	1.01	0.18	-3.51	<b>-12.32</b> <sup>a</sup>	<b>5.68</b> <sup>b</sup>	<b>4.44</b> <sup>b</sup>	3.57	-0.65	-1.21				
Upper regime shift	0.07	0.38	0.50	0.51	1.30	-1.88	<b>6.86</b> <sup>a</sup>	<b>-3.15</b> <sup>b</sup>	<b>-3.01</b> <sup>b</sup>	-1.67	-0.64	0.73	<b>0.88</b>	0.000	0.000	2.86
Trend	-0.003	-0.002	0.002	-0.001	0.004	0.006	-0.001	-0.001	0.002	0.002	-0.001	-0.007	-0.002	0.915	0.80	0
<u>France Construction</u>																
Lower regime	-2.70	-0.00	1.47	<b>9.39</b> <sup>b</sup>	<b>8.69</b> <sup>b</sup>	5.18	-4.86	<b>-31.04</b> <sup>a</sup>	<b>14.28</b> <sup>a</sup>	<b>6.63</b>	1.58	<b>-8.63</b> <sup>b</sup>				
Upper regime shift	<b>-4.40</b> <sup>a</sup>	<b>-3.88</b> <sup>b</sup>	-0.42	-1.21	-0.43	2.00	2.13	<b>2.47</b>	0.84	1.31	-0.11	1.72	<b>1.65</b>	0.025	0.087	5.31
Trend	<b>0.030</b> <sup>a</sup>	<b>0.018</b>	-0.005	-0.002	0.016	0.000	-0.017	<b>-0.052</b> <sup>a</sup>	0.013	0.004	0.010	-0.014	<b>-0.005</b>	0.001	0.40	0
<u>France Intermediate</u>																
Lower regime	<b>13.06</b>	-0.01	5.81	-7.14	<b>10.75</b> <sup>b</sup>	-2.14	<b>-13.23</b> <sup>a</sup>	<b>-43.60</b> <sup>a</sup>	<b>33.87</b> <sup>a</sup>	4.07	<b>9.25</b>	<b>-10.67</b> <sup>b</sup>				
Upper regime shift	-0.29	-0.51	-0.52	-0.18	-1.65	-1.32	-1.05	<b>4.75</b> <sup>a</sup>	<b>-3.43</b> <sup>b</sup>	0.56	1.45	<b>2.20</b>	<b>0.94</b>	0.032	0.000	3.33
Trend	<b>0.023</b> <sup>a</sup>	-0.001	<b>0.017</b> <sup>b</sup>	-0.006	<b>0.019</b> <sup>a</sup>	0.005	<b>0.027</b> <sup>a</sup>	<b>-0.021</b> <sup>a</sup>	0.004	<b>-0.017</b> <sup>b</sup>	-0.009	<b>-0.039</b> <sup>a</sup>	<b>-0.004</b> <sup>b</sup>	0.000	0.67	4

**Table 2 (continued)**

	Estimated Coefficients												$g_0/I_0$	$F_{11}$	SupWald $r^*$	$s$ Outliers
	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec				
<u>Germany Construction</u>																
Lower regime	<b>-11.17</b> <sup>a</sup>	<b>-7.86</b> <sup>b</sup>	<b>13.62</b> <sup>a</sup>	<b>8.46</b> <sup>b</sup>	4.63	<b>7.06</b>	2.71	-1.41	<b>7.34</b> <sup>b</sup>	0.57	-5.41	<b>-18.54</b> <sup>a</sup>				
Upper regime shift	<b>-11.30</b> <sup>a</sup>	<b>8.03</b> <sup>b</sup>	-4.34	-3.80	-1.76	-2.48	2.13	1.73	1.44	1.05	<b>6.54</b> <sup>b</sup>	2.77	1.30	0.005	0.048	8.64
Trend	<b>0.051</b> <sup>a</sup>	<b>-0.044</b> <sup>a</sup>	<b>0.043</b> <sup>a</sup>	-0.007	-0.017	0.000	0.017	-0.006	0.003	0.009	-0.014	<b>-0.034</b> <sup>b</sup>	0.000	0.001	0.58	2
<u>Germany Food</u>																
Lower regime	<b>-7.49</b> <sup>a</sup>	<b>-5.18</b> <sup>a</sup>	<b>2.93</b>	<b>5.37</b> <sup>a</sup>	1.10	0.85	<b>-5.37</b> <sup>a</sup>	<b>-3.68</b> <sup>b</sup>	1.93	<b>6.80</b> <sup>a</sup>	<b>6.88</b> <sup>a</sup>	<b>-4.16</b> <sup>a</sup>				
Upper regime shift	-0.32	-0.62	-1.32	-1.73	<b>2.18</b> <sup>b</sup>	-0.52	-0.58	1.11	-0.46	<b>2.62</b> <sup>b</sup>	0.81	-1.16	<b>-1.15</b> <sup>a</sup>	0.122	0.096	2.71
Trend	<b>0.031</b> <sup>a</sup>	-0.001	<b>0.026</b> <sup>a</sup>	<b>-0.031</b> <sup>a</sup>	<b>-0.016</b> <sup>b</sup>	<b>-0.019</b> <sup>a</sup>	<b>0.017</b> <sup>b</sup>	<b>0.031</b> <sup>a</sup>	0.006	0.008	-0.010	<b>-0.043</b> <sup>a</sup>	-0.002	0.000	0.26	1
<u>Germany Manufacturing</u>																
Lower regime	<b>-5.92</b> <sup>a</sup>	0.49	<b>6.14</b> <sup>a</sup>	1.28	<b>-3.31</b> <sup>b</sup>	0.20	-1.26	<b>-7.81</b> <sup>a</sup>	<b>6.01</b> <sup>a</sup>	<b>6.39</b> <sup>a</sup>	<b>2.42</b>	<b>-4.64</b> <sup>a</sup>				
Upper regime shift	<b>1.68</b>	-0.66	<b>-1.94</b> <sup>b</sup>	-0.56	0.16	<b>1.99</b> <sup>b</sup>	-1.29	-0.68	-1.32	0.11	0.34	<b>2.18</b> <sup>b</sup>	<b>1.05</b> <sup>b</sup>	0.045	0.091	2.65
Trend	<b>0.011</b> <sup>b</sup>	-0.001	<b>0.018</b> <sup>a</sup>	<b>-0.016</b> <sup>a</sup>	<b>-0.018</b> <sup>a</sup>	-0.007	<b>0.027</b> <sup>a</sup>	0.007	-0.007	<b>0.011</b>	-0.007	<b>-0.019</b> <sup>a</sup>	<b>-0.003</b> <sup>b</sup>	0.000	0.55	1
<u>Italy Consumer</u>																
Lower regime	<b>13.24</b> <sup>b</sup>	-2.33	-4.49	<b>18.78</b> <sup>a</sup>	7.07	<b>-11.65</b>	8.26	<b>-48.74</b> <sup>a</sup>	<b>21.49</b> <sup>a</sup>	-7.30	-9.19	0.44				
Upper regime shift	-1.14	0.47	0.48	-2.67	-1.87	1.09	-1.03	<b>5.43</b> <sup>a</sup>	-1.36	-2.31	0.20	<b>2.73</b>	<b>1.68</b>	0.030	0.094	4.15
Trend	0.021	0.008	0.014	0.000	0.009	0.012	-0.005	<b>-0.073</b> <sup>a</sup>	0.009	-0.005	-0.004	0.013	-0.004	0.031	0.62	0
<u>Italy Total</u>																
Lower regime	2.47	5.49	3.84	5.53	-1.09	-0.07	-3.95	<b>-38.58</b> <sup>a</sup>	<b>19.67</b> <sup>a</sup>	5.64	<b>7.33</b>	-6.29				
Upper regime shift	<b>-2.53</b> <sup>b</sup>	<b>-2.75</b> <sup>b</sup>	1.54	-0.14	-0.23	0.09	0.01	<b>4.69</b> <sup>a</sup>	<b>-3.09</b> <sup>b</sup>	-0.59	0.23	<b>2.75</b> <sup>b</sup>	<b>1.00</b>	0.000	0.006	3.34
Trend	0.004	0.014	0.014	0.010	-0.008	0.001	0.000	<b>-0.088</b> <sup>a</sup>	<b>0.039</b> <sup>a</sup>	0.013	0.019	-0.018	<b>-0.005</b> <sup>a</sup>	0.000	0.33	1
<u>Japan Consumer Durables</u>																
Lower regime	<b>-10.03</b> <sup>a</sup>	<b>6.01</b> <sup>b</sup>	<b>8.33</b> <sup>a</sup>	-1.75	<b>-6.15</b> <sup>a</sup>	<b>8.53</b> <sup>a</sup>	<b>6.59</b> <sup>a</sup>	<b>-19.31</b> <sup>a</sup>	<b>6.36</b> <sup>b</sup>	<b>9.11</b> <sup>a</sup>	-3.21	<b>-4.48</b>				
Upper regime shift	<b>-3.44</b> <sup>a</sup>	-0.12	-1.25	1.56	1.61	-1.49	<b>-2.27</b> <sup>b</sup>	0.78	-0.76	1.60	<b>2.70</b> <sup>b</sup>	1.06	-0.36	0.010	0.059	2.94
Trend	<b>0.023</b> <sup>a</sup>	0.010	<b>0.015</b> <sup>b</sup>	<b>-0.021</b> <sup>a</sup>	-0.012	0.006	<b>0.023</b> <sup>a</sup>	<b>-0.049</b> <sup>a</sup>	<b>0.020</b> <sup>a</sup>	<b>0.017</b> <sup>b</sup>	0.004	<b>-0.036</b> <sup>a</sup>	<b>-0.005</b> <sup>b</sup>	0.000	0.29	1
<u>Japan Intermediate</u>																
Lower regime	<b>-7.59</b> <sup>a</sup>	-0.05	<b>7.02</b> <sup>a</sup>	<b>2.13</b>	<b>-2.25</b> <sup>b</sup>	<b>3.70</b> <sup>a</sup>	1.21	<b>-7.98</b> <sup>a</sup>	<b>2.79</b> <sup>b</sup>	<b>3.91</b> <sup>a</sup>	<b>-1.96</b>	-0.94				
Upper regimeshift	0.77	-0.10	-0.64	<b>-3.12</b> <sup>a</sup>	-0.15	<b>-1.14</b>	0.27	<b>1.33</b> <sup>b</sup>	0.80	0.95	0.62	0.41	<b>-0.61</b> <sup>b</sup>	0.006	0.054	1.38
Trend	0.000	0.004	0.005	0.002	<b>-0.009</b> <sup>a</sup>	<b>0.007</b> <sup>a</sup>	<b>0.008</b> <sup>a</sup>	<b>-0.018</b> <sup>a</sup>	0.005	<b>0.007</b> <sup>b</sup>	-0.001	<b>-0.009</b> <sup>a</sup>	<b>-0.001</b>	0.000	0.21	3
<u>Japan Manufacturing</u>																
Lower regime	<b>-3.99</b> <sup>a</sup>	-0.19	<b>8.57</b> <sup>a</sup>	-1.79	<b>-5.37</b> <sup>a</sup>	<b>3.98</b> <sup>a</sup>	<b>4.16</b> <sup>a</sup>	<b>-8.82</b> <sup>a</sup>	<b>4.80</b>	<b>2.24</b>	<b>-5.13</b> <sup>a</sup>	1.55				
Upper regime shift	-0.68	<b>-0.99</b>	<b>-1.22</b> <sup>b</sup>	0.94	0.09	-0.80	-0.93	0.28	-0.23	<b>1.97</b> <sup>a</sup>	0.69	0.88	-0.07	0.001	0.030	1.44
Trend	0.000	-0.001	<b>0.006</b>	0.005	<b>-0.013</b> <sup>a</sup>	0.005	<b>0.016</b> <sup>a</sup>	<b>-0.021</b> <sup>a</sup>	<b>0.008</b> <sup>b</sup>	<b>0.006</b>	-0.004	<b>-0.008</b> <sup>b</sup>	<b>-0.002</b> <sup>b</sup>	0.000	0.27	2

**Table 2 (continued)**

	Estimated Coefficients													<i>SupWald</i>	<i>s</i>	
	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	$g_0/I_0$	$F_{11}$	$r^*$	Outliers
<u>Luxembourg Construction</u>																
Lower regime	-5.65	<b>-13.01</b>	7.04	3.53	<b>11.96</b>	3.52	4.63	<b>-25.30<sup>a</sup></b>	2.18	1.21	6.75	3.14				
Upper regime shift	<b>-21.08<sup>a</sup></b>	5.87	0.42	5.37	3.02	7.37	4.65	2.52	5.73	2.72	-3.88	<b>-12.72<sup>b</sup></b>	-2.38	0.005	0.050	12.31
Trend	0.022	0.030	0.022	-0.025	-0.008	-0.005	0.021	<b>-0.184<sup>a</sup></b>	<b>0.052</b>	0.012	0.046	0.017	0.004	0.000	0.23	1
<u>Luxembourg Total</u>																
Lower regime	<b>-3.78</b>	3.30	2.50	<b>4.73<sup>b</sup></b>	0.17	3.00	<b>-4.52<sup>b</sup></b>	<b>-18.00<sup>a</sup></b>	1.79	<b>7.69<sup>a</sup></b>	-3.22	0.23				
Upper regime shift	<b>2.99</b>	-0.02	-0.63	0.12	-1.63	<b>-3.50<sup>b</sup></b>	0.54	<b>5.71<sup>a</sup></b>	<b>-3.72<sup>b</sup></b>	1.63	-1.52	0.01	0.35	0.001	0.021	4.31
Trend	-0.004	0.008	0.003	0.008	<b>-0.015</b>	0.004	-0.006	<b>-0.046<sup>a</sup></b>	0.015	<b>0.017</b>	<b>0.024<sup>a</sup></b>	-0.009	0.000	0.000	0.45	3
<u>Norway Manufacturing</u>																
Lower regime	-3.65	6.27	6.12	<b>14.78<sup>a</sup></b>	-2.67	<b>12.00<sup>b</sup></b>	<b>-50.67<sup>a</sup></b>	-7.15	<b>9.01</b>	<b>10.99<sup>b</sup></b>	<b>12.00<sup>b</sup></b>	-7.04				
Upper regime shift	0.17	-0.39	-1.81	-2.02	0.24	-1.42	<b>6.46<sup>a</sup></b>	0.40	-2.33	-0.49	0.27	0.91	0.73	0.000	0.027	3.91
Trend	<b>0.009</b>	0.003	-0.004	-0.002	0.000	-0.001	<b>0.009<sup>b</sup></b>	0.003	-0.001	-0.007	-0.005	-0.007	<b>-0.003<sup>b</sup></b>	0.408	0.37	2
<u>Spain Consumer</u>																
Lower regime	1.98	<b>6.48</b>	5.52	3.45	2.67	-6.56	2.15	<b>-35.66<sup>a</sup></b>	<b>19.96<sup>a</sup></b>	<b>11.18<sup>a</sup></b>	-0.59	<b>-10.59<sup>b</sup></b>				
Upper regime shift	<b>3.99<sup>b</sup></b>	-0.14	-1.61	-2.02	0.17	1.90	-1.81	<b>5.40<sup>a</sup></b>	<b>-5.28<sup>a</sup></b>	<b>-4.03<sup>b</sup></b>	2.16	1.27	0.33	0.003	0.017	4.09
Trend	-0.001	0.007	0.003	-0.014	0.010	-0.005	0.020	<b>-0.063<sup>a</sup></b>	<b>0.049<sup>a</sup></b>	<b>0.029<sup>b</sup></b>	0.007	<b>-0.042<sup>a</sup></b>	<b>-0.008<sup>b</sup></b>	0.000	0.26	0
<u>Spain Intermediate</u>																
Lower regime	3.06	-1.45	1.29	-0.18	0.54	-1.79	-0.08	<b>-17.19<sup>a</sup></b>	<b>7.61<sup>a</sup></b>	<b>7.71<sup>a</sup></b>	1.87	-1.40				
Upper regime shift	-0.15	0.66	1.16	-0.08	-0.17	1.63	-1.10	<b>5.57<sup>a</sup></b>	<b>-2.91</b>	<b>-2.97<sup>b</sup></b>	-1.33	-0.32	<b>-1.07</b>	0.021	0.020	0
Trend	0.014	0.002	0.002	-0.007	0.011	-0.001	0.016	<b>-0.044<sup>a</sup></b>	0.017	0.002	0.002	-0.015	-0.004	0.116	0.75	3.28
<u>Spain Investment</u>																
Lower regime	<b>11.68</b>	-3.14	9.15	8.00	0.28	-7.46	-5.98	<b>-48.28<sup>a</sup></b>	10.66	<b>17.74<sup>a</sup></b>	1.87	-1.40				
Upper regime shift	-4.20	7.39	1.52	0.65	2.05	3.92	-5.87	<b>19.37<sup>a</sup></b>	<b>-11.30<sup>a</sup></b>	<b>-10.91<sup>b</sup></b>	-4.18	1.56	-1.29	0.000	0.025	9.21
Trend	0.043	0.008	0.027	0.010	-0.012	-0.037	-0.028	<b>-0.069<sup>b</sup></b>	0.004	0.033	0.012	0.009	0.001	0.196	0.68	3
<u>Spain Manufacturing</u>																
Lower regime	2.52	3.45	<b>6.12</b>	1.89	3.44	<b>-6.33</b>	0.49	<b>-32.41<sup>a</sup></b>	<b>15.21<sup>a</sup></b>	<b>14.13<sup>a</sup></b>	0.85	<b>-9.36<sup>b</sup></b>				
Upper regime shift	2.26	-0.64	-0.89	-1.81	0.55	1.47	-1.48	<b>6.48<sup>a</sup></b>	<b>-5.72<sup>a</sup></b>	<b>-2.89</b>	0.37	2.31	-0.10	0.002	0.045	3.66
Trend	0.015	0.007	0.012	-0.012	0.016	-0.011	0.020	<b>-0.083<sup>a</sup></b>	<b>0.034<sup>b</sup></b>	<b>0.038<sup>a</sup></b>	0.003	<b>-0.038<sup>a</sup></b>	<b>-0.006<sup>b</sup></b>	0.000	0.21	0
<u>Sweden Manufacturing</u>																
Lower regime	-1.67	-0.55	-3.07	-2.97	2.60	-3.55	<b>-13.56<sup>a</sup></b>	5.70	<b>5.97</b>	5.19	5.72	0.18				
Upper regime shift	<b>2.58</b>	1.46	2.65	0.81	<b>-3.56<sup>b</sup></b>	-0.06	-0.32	<b>-5.09<sup>a</sup></b>	-0.37	<b>-2.79</b>	1.88	<b>2.80</b>	-1.05	0.043	0.061	3.88
Trend	-0.009	-0.003	0.004	0.000	0.001	0.004	<b>0.022<sup>a</sup></b>	<b>-0.012<sup>b</sup></b>	0.000	-0.005	-0.001	-0.001	0.000	0.037	0.78	3

**Table 2 (continued)**

	Estimated Coefficients												$F_{11}$	<i>supWald</i> $r^*$	<b>S</b> Outliers	
	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec				$g_0/I_0$
<u>UK Intermediate</u>																
Lower regime	<b>3.81</b>	0.31	5.00	<b>-5.27</b> <sup>b</sup>	<b>-5.28</b> <sup>a</sup>	-2.47	<b>-5.22</b> <sup>a</sup>	<b>-10.60</b> <sup>a</sup>	<b>8.41</b> <sup>a</sup>	<b>9.23</b> <sup>a</sup>	<b>6.26</b> <sup>a</sup>	<b>-4.19</b> <sup>b</sup>				
Upper regime shift	-1.80	-2.37	-1.23	<b>-5.69</b> <sup>a</sup>	0.50	0.40	1.46	<b>3.44</b> <sup>b</sup>	1.76	0.23	1.50	1.80	<b>0.99</b>	0.004	0.029	2.97
Trend	0.009	-0.007	<b>0.015</b> <sup>b</sup>	-0.006	-0.008	-0.006	<b>0.019</b> <sup>b</sup>	<b>0.017</b> <sup>b</sup>	<b>-0.019</b> <sup>b</sup>	-0.008	0.004	-0.009	0.001	0.011	0.40	0
<u>UK Total</u>																
Lower regime	-0.19	-0.85	<b>5.24</b> <sup>a</sup>	-1.58	<b>-5.10</b> <sup>a</sup>	-1.18	-3.82	<b>-9.07</b> <sup>a</sup>	<b>7.96</b> <sup>a</sup>	<b>7.72</b> <sup>a</sup>	<b>4.14</b> <sup>b</sup>	<b>-3.27</b>				
Upper regime shift	-0.07	0.47	-0.86	<b>-2.45</b> <sup>a</sup>	-0.87	0.13	-0.22	<b>1.98</b> <sup>b</sup>	0.41	-0.46	1.21	0.73	-0.21	0.002	0.038	2.49
Trend	-0.002	<b>-0.011</b> <sup>b</sup>	<b>0.012</b> <sup>a</sup>	-0.005	-0.008	0.000	<b>0.015</b> <sup>a</sup>	<b>0.011</b> <sup>a</sup>	-0.007	-0.001	0.000	-0.004	-0.001	0.000	0.29	3

The columns labelled Jan-Dec show the estimated coefficients for the seasonal intercepts in the lower regime ( $d_j$ ), the shift terms corresponding to the upper regime ( $g_j$ ), and the coefficients for the seasonal trends ( $I_j$ );  $g_0$  is the coefficient for the shift in the nonseasonal intercept in the upper regime and  $I_0$  is the overall trend coefficient;  $F_{11}$  is the  $p$ -value for the conventional  $F$ -test of  $g_j = 0$  or  $I_j = 0$  ( $j = 1, \dots, 11$ ), as appropriate; *Sup-Wald* is the Sup-Wald test of section 4.1 (shown as a  $p$ -value) with  $r^*$  the estimated threshold, expressed as the proportion of observations below the threshold  $r$ ; Outliers reports the number of outliers removed while **S** is the residual standard deviation. Significance of coefficients is indicated by

Bold numbers :  $p$ -value  $\leq 0.10$

<sup>b</sup>  $0.01 < p$ -value  $\leq 0.05$

<sup>a</sup>  $p$ -value  $\leq 0.01$

## APPENDIX I

### Descriptive Statistics and Nonlinearity Test Results: All Series

The Appendix Table below reports descriptive statistics for all series: sample period, mean, standard deviation and  $R^2$  from a regression on twelve seasonal dummies. Also shown are the results (as  $p$ -values) for the *Sup-Wald* and Tsay nonlinearity tests discussed in Section 3.1. For the former, the threshold estimate is shown as  $r^*$ , which is the proportion of observations in the lower regime for the estimated TAR model of equation (3).

The Tsay (1989) test is general in that no specific number of thresholds is assumed under the alternative hypothesis. Also, no particular form of the nonlinearity is assumed in the sense that under the alternative all the coefficients (intercepts, trend and autoregressive) may change with the regime. The basis of Tsay's test in our context is a  $k$ -regime TAR model, with the order of each autoregression again taken to be 24. The test is based on a conventional  $F$ -test statistic computed using recursive residuals obtained after all variables are re-ordered according to the threshold variable of (2). Tsay shows that, for large samples, the associated  $F$ -statistic follows the conventional  $F$ -distribution.

In practice, the recursive estimation using the re-ordered data begins from a certain minimum number of observations. We choose 10 percent of the sample for this purpose and generate the test with both increasing and decreasing orderings of the data so that no threshold is missed within this initial portion. The Tsay test result shown in the table below is the more significant  $p$ -value of the two  $F$ -tests obtained in this fashion.

**Appendix Table. Descriptive Statistics and Nonlinearity Test Results for All Series**

Country	Series	Sample period		Mean	Std.dev.	$R^2$	<i>Sup-Wald</i>	Tsay	$r^*$
Austria	Consumer	1960:01	1995:07	0.27	9.72	0.83	0.844	0.006	0.64
	Intermediate	1960:01	1995:07	0.31	7.25	0.66	0.023	0.004	0.50
	Investment	1960:01	1995:07	0.41	17.42	0.84	0.241	0.271	0.75
	Manufacturing	1960:01	1995:05	0.32	9.15	0.78	0.660	0.337	0.79
	Total	1960:01	1995:07	0.32	7.59	0.78	0.831	0.052	0.33
Belgium	Construction	1960:01	1994:12	-0.06	39.60	0.76	0.061	0.947	0.28
	Consumer Dur.	1960:01	1994:12	0.23	17.33	0.89	0.886	0.524	0.65
	Consumer Non-Dur.	1960:01	1994:12	0.16	9.06	0.77	0.877	0.564	0.80
	Intermediate	1960:01	1994:12	0.16	11.84	0.88	1.000	0.665	0.50
	Investment	1960:01	1994:12	0.21	14.43	0.76	0.289	0.239	0.21
	Manufacturing	1960:01	1994:12	0.21	12.18	0.88	0.987	0.483	0.50
	Total	1960:01	1994:12	0.18	11.86	0.89	0.993	0.499	0.72
Finland	Consumer	1960:01	1995:07	0.22	19.27	0.91	0.914	0.564	0.80
	Intermediate	1960:01	1995:07	0.26	14.95	0.80	0.021	0.000	0.21
	Investment	1960:01	1995:07	0.38	31.24	0.91	0.000	0.421	0.38
	Manufacturing	1960:01	1995:07	0.26	19.48	0.92	0.000	0.000	0.80
	Total	1960:01	1995:07	0.27	17.38	0.93	0.000	0.000	0.80
France	Construction	1960:01	1995:06	0.15	19.54	0.85	0.087	0.000	0.40
	Consumer	1963:01	1995:06	0.30	25.69	0.93	0.794	0.337	0.73
	Energy	1963:01	1995:06	0.16	9.38	0.73	0.122	0.154	0.50
	Intermediate	1963:01	1995:06	0.18	21.75	0.97	0.000	0.030	0.67
	Investment	1963:01	1995:06	0.26	18.93	0.67	0.840	0.000	0.21
	Manufacturing	1960:01	1995:06	0.22	18.18	0.94	0.233	0.192	0.59
	Total	1960:01	1995:06	0.22	17.04	0.93	0.641	0.047	0.61

**Appendix Table (continued)**

		Sample period		Mean	Std.dev.	$R^2$	<i>Sup-Wald</i>	Tsay	$r^*$
Germany	Construction	1960:01	1994:12	0.13	19.28	0.68	0.048	0.694	0.58
	Consumer	1960:01	1994:12	0.14	9.27	0.69	0.436	0.010	0.21
	Food	1960:01	1994:12	0.22	6.87	0.54	0.096	0.209	0.26
	Intermediate	1960:01	1994:12	0.21	5.69	0.66	0.228	0.149	0.25
	Investment	1960:01	1994:12	0.25	10.75	0.77	0.248	0.000	0.20
	Manufacturing	1960:01	1994:12	0.22	7.57	0.72	0.091	0.116	0.55
	Total	1960:01	1994:12	0.22	7.10	0.71	0.339	0.147	0.24
Greece	Consumer	1961:01	1995:05	0.39	8.05	0.47	0.746	0.000	0.75
	Investment	1961:01	1995:05	0.42	9.99	0.55	0.215	0.255	0.61
	Manufacturing	1960:01	1995:05	0.39	7.44	0.54	0.668	0.124	0.38
	Total	1962:01	1995:05	0.39	6.41	0.54	0.668	0.160	0.24
Italy	Consumer	1971:01	1995:05	0.24	33.81	0.95	0.094	0.080	0.62
	Investment	1971:01	1995:05	0.35	41.96	0.93	0.629	0.897	0.58
	Manufacturing	1960:01	1995:07	0.27	30.61	0.86	0.274	0.565	0.28
	Total	1960:01	1995:07	0.27	27.03	0.89	0.006	0.071	0.33
Japan	Consumer Dur.	1960:01	1995:08	0.56	10.63	0.82	0.059	0.213	0.29
	Consumer Non-Dur.	1960:01	1995:08	0.29	9.23	0.77	0.330	0.003	0.57
	Intermediate	1960:01	1995:08	0.41	4.38	0.83	0.054	0.000	0.21
	Investment	1960:01	1995:08	0.54	11.28	0.86	0.395	0.113	0.28
	Manufacturing	1960:01	1995:08	0.44	6.62	0.87	0.030	0.005	0.27
	Total	1960:01	1995:08	0.44	6.34	0.87	0.370	0.000	0.22
Luxemb'g	Construction	1960:01	1994:12	0.00	35.38	0.53	0.050	0.167	0.23
	Manufacturing	1960:01	1995:06	0.22	11.37	0.65	0.275	0.092	0.47
	Total	1960:01	1995:06	0.20	10.98	0.65	0.021	0.004	0.45
Netherlands	Manufacturing	1960:01	1995:06	0.31	7.76	0.81	0.403	0.002	0.21
	Total	1960:01	1995:06	0.28	8.03	0.75	0.164	0.234	0.23
Norway	Consumer	1960:01	1991:12	0.16	18.62	0.93	0.368	0.052	0.41
	Export	1960:01	1991:12	0.58	14.65	0.71	0.216	0.091	0.54
	Intermediate	1961:01	1991:12	0.26	19.64	0.92	0.315	0.000	0.37
	Investment	1960:01	1991:12	0.30	34.60	0.87	0.150	0.000	0.78
	Manufacturing	1960:01	1995:07	0.09	25.44	0.94	0.027	0.148	0.37
	Total	1960:01	1995:07	0.39	18.21	0.80	0.678	0.679	0.74
Portugal	Manufacturing	1960:01	1995:06	0.35	14.03	0.61	0.346	0.140	0.80
	Total	1960:01	1995:06	0.37	12.64	0.61	0.821	0.487	0.32
Spain	Consumer	1965:01	1995:06	0.37	19.82	0.87	0.017	0.183	0.26
	Intermediate	1965:01	1995:06	0.30	14.86	0.79	0.020	0.074	0.75
	Investment	1965:01	1995:06	0.24	45.15	0.80	0.025	0.282	0.68
	Manufacturing	1961:01	1995:06	0.38	21.01	0.80	0.045	0.139	0.21
	Total	1961:01	1995:06	0.38	18.12	0.80	0.255	0.358	0.20
Sweden	Manufacturing	1960:01	1995:07	0.12	32.31	0.95	0.061	0.227	0.78
	Total	1960:01	1995:07	0.12	32.18	0.95	0.189	0.114	0.48
UK	Consumer	1968:01	1995:04	0.10	8.05	0.88	0.369	0.680	0.21
	Intermediate	1968:01	1995:07	0.11	7.36	0.75	0.029	0.115	0.40
	Investment	1968:01	1995:07	0.05	10.95	0.78	0.737	0.878	0.28
	Manufacturing	1960:01	1995:07	0.11	8.29	0.76	0.254	0.375	0.22
	Total	1960:01	1995:07	0.13	7.42	0.78	0.038	0.022	0.29
US	Consumer	1960:01	1995:08	0.23	3.68	0.77	0.989	0.034	0.80
	Durables	1960:01	1995:08	0.30	3.13	0.70	0.979	0.125	0.21
	Non-Durables	1960:01	1995:08	0.25	3.26	0.84	0.823	0.332	0.33
	Intermediate	1960:01	1995:08	0.25	2.77	0.70	0.648	0.166	0.34
	Investment	1960:01	1995:08	0.34	2.32	0.58	0.790	0.885	0.80
	Manufacturing	1960:01	1995:08	0.31	2.90	0.83	0.542	0.187	0.20
	Raw Materials	1960:01	1995:08	0.24	2.57	0.66	0.830	0.007	0.21
	Total	1960:01	1995:08	0.29	2.51	0.78	0.753	0.050	0.20



## APPENDIX II

### Calculation of Monthly Seasonal Mean Deviations

Consider, firstly, the linear autoregressive process for monthly data such that

$$\mathbf{f}(L)[y_t - \mathbf{m}_j] = \mathbf{e}_t \quad (\text{A.1})$$

where  $\mathbf{f}(L)$  is a  $p$ th order polynomial with all roots outside the unit circle,  $E[y_t] = \mathbf{m}_j$  and the means follow a twelve month cycle such that

$$\mathbf{m}_j = \mathbf{m}_{j \pm 12n}, \quad j = 1, \dots, 12; n = 1, 2, \dots \quad (\text{A.2})$$

Therefore, (A.1) and (A.2) define a process  $y_t$  which is stationary around the monthly seasonal means  $\mathbf{m}_1, \dots, \mathbf{m}_{12}$ .

This can be compared with the usual representation in terms of a seasonal intercept, namely

$$\mathbf{f}(L)y_t = \sum_{j=1}^{12} \mathbf{d}_j D_{jt} + \mathbf{e}_t \quad (\text{A.3})$$

in which  $D_{jt}$  is the conventional dummy variable for month  $j$ . Equation (A.3) defines a special case of a periodic process, which has a mean that is a nonlinear function of all seasonal intercepts and autoregressive coefficients; see Ghysels and Osborn (2001, pp.146/147). Equivalence of the two representations follows from stationarity, and hence

$$\mathbf{m}_j - \sum_{i=1}^p \mathbf{f}_i \mathbf{m}_{j-i} = \mathbf{d}_j, \quad j = 1, \dots, 12 \quad (\text{A.4})$$

Given values of the autoregressive coefficients and the seasonal dummy variable coefficients, (A.4) defines a set of linear equations that can be solved for the unknown seasonal means  $\mathbf{m}_1, \dots, \mathbf{m}_{12}$ .

In order to solve these equations, it is convenient to collect the coefficients on the left-hand side of (A.4) that refer to each specific monthly mean. This yields

$$\mathbf{f}_0(1)\mathbf{m}_j + \sum_{k=1}^{11} \mathbf{f}_k(1)\mathbf{m}_{j-k} = \mathbf{d}_j, \quad j = 1, \dots, 12 \quad (\text{A.5})$$

where

$$\begin{aligned} \mathbf{f}_0(1) &= 1 - \sum_{n=1}^{\lfloor p/12 \rfloor} \mathbf{f}_{12n} \\ \mathbf{f}_k(1) &= - \sum_{n=0}^{\lfloor (p-k)/12 \rfloor} \mathbf{f}_{12n+k}, \quad k = 1, \dots, 11 \end{aligned}$$

and  $\lfloor . \rfloor$  denotes the integer part of the expression in brackets. Therefore, (A.5) can be written as the conventional linear equation system

$$\mathbf{A}\mathbf{m} = \mathbf{d} \quad (\text{A.6})$$

in which the vectors  $\mathbf{m} = (\mathbf{m}_1, \dots, \mathbf{m}_{12})'$ ,  $\mathbf{d} = (\mathbf{d}_1, \dots, \mathbf{d}_{12})'$  and the elements of the matrix  $\mathbf{A}$  are given by

$$a_{ij} = \begin{cases} \mathbf{f}_0(1) & i = j \\ \mathbf{f}_{i-j}(1) & i > j \\ \mathbf{f}_{12+i-j}(1) & j > i \end{cases}$$

The monthly means can then be obtained as

$$\mathbf{m} = \mathbf{A}^{-1}\mathbf{d}. \quad (\text{A.7})$$

In the context of this paper, the  $\mathbf{d}_j$  in equation (3) of the text represent the monthly shift in the intercept in the lower regime, so that application of (A.7) to these values yields the monthly

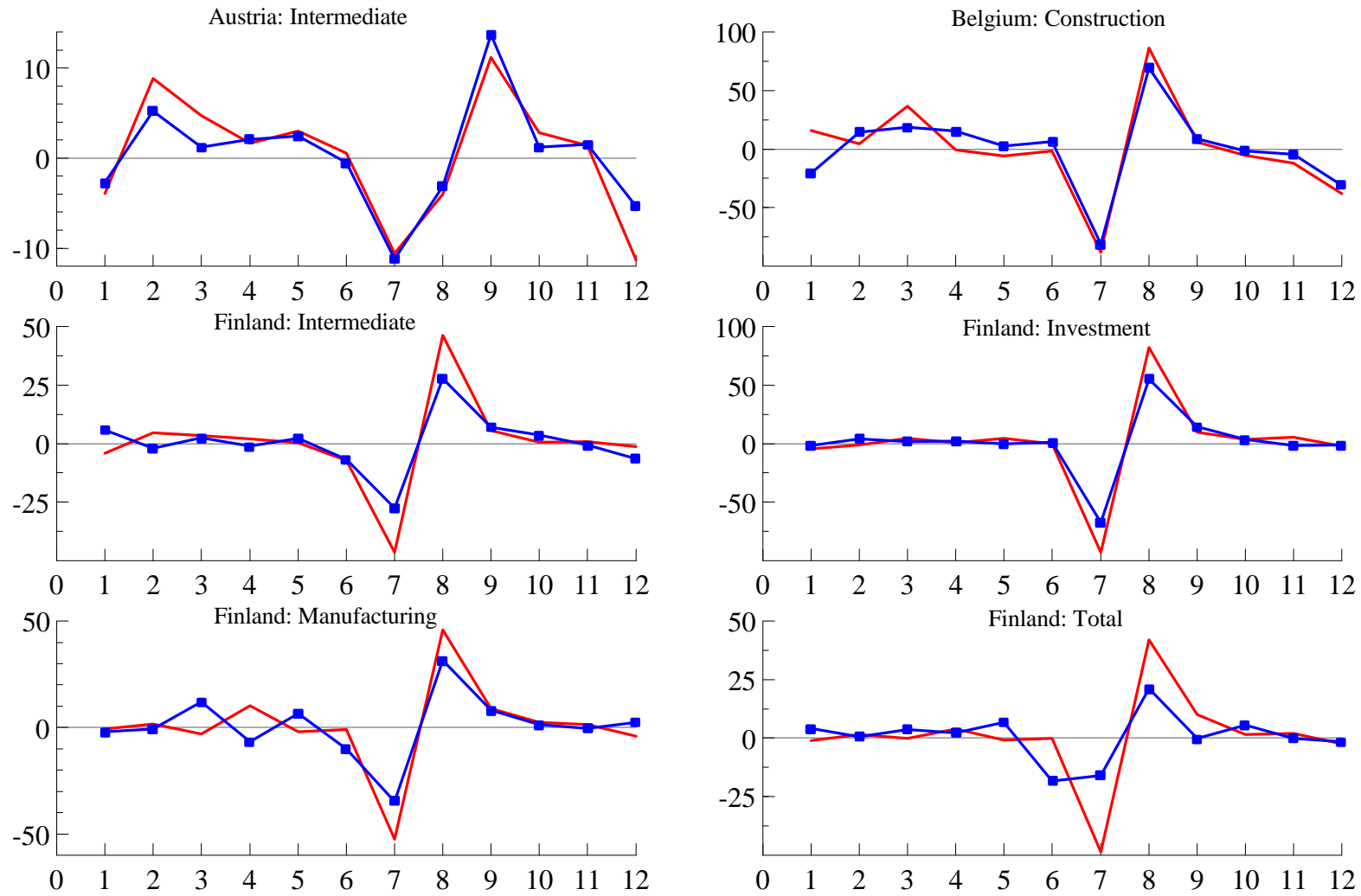
mean deviation from the overall mean in the lower regime. For the upper regime, the corresponding equation becomes

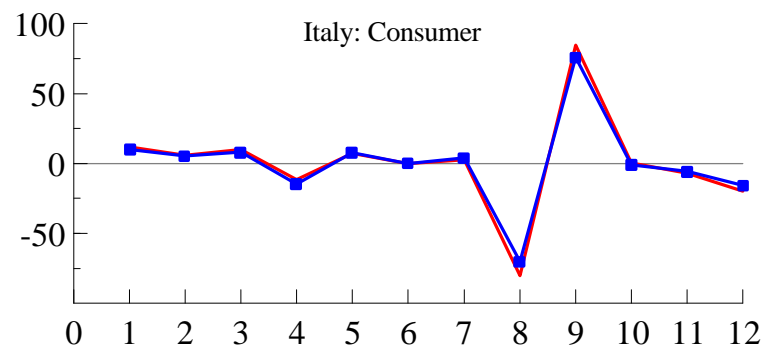
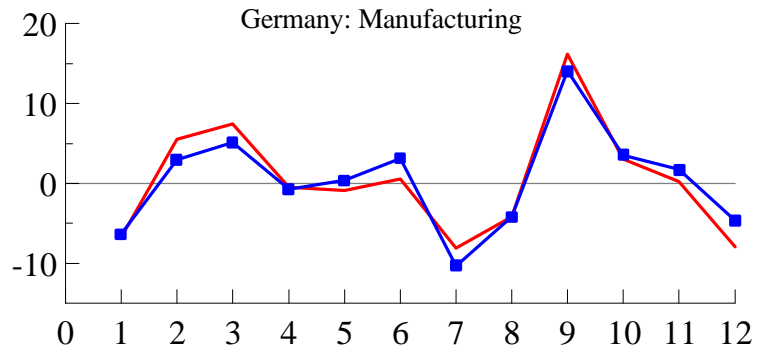
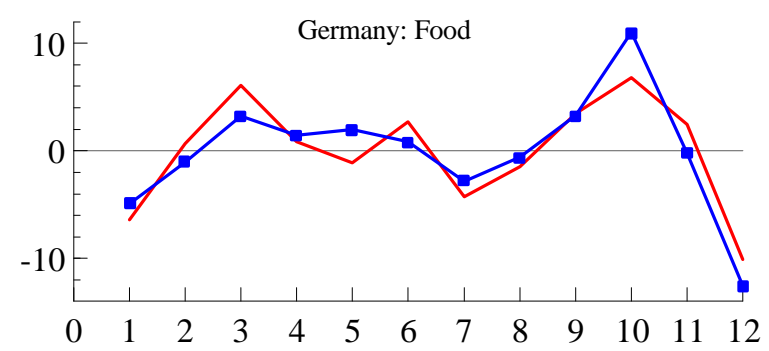
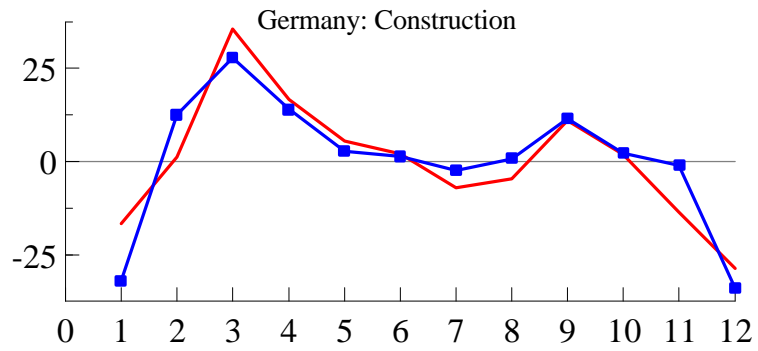
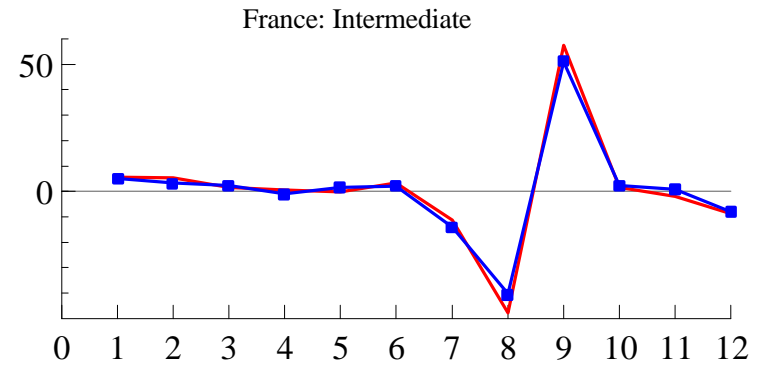
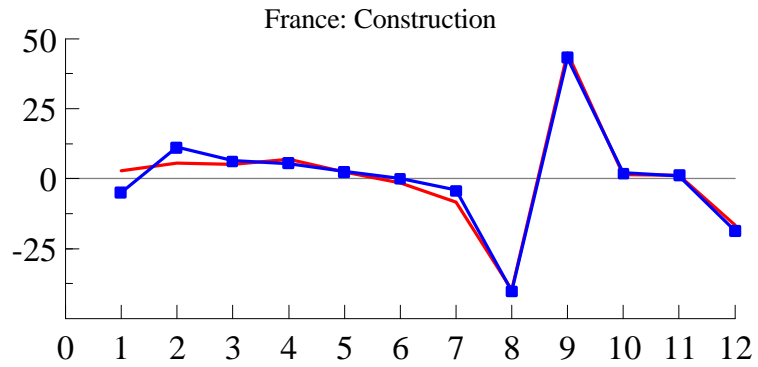
$$\mathbf{m} = A^{-1}(\mathbf{d} + \mathbf{g}). \quad (\text{A.8})$$

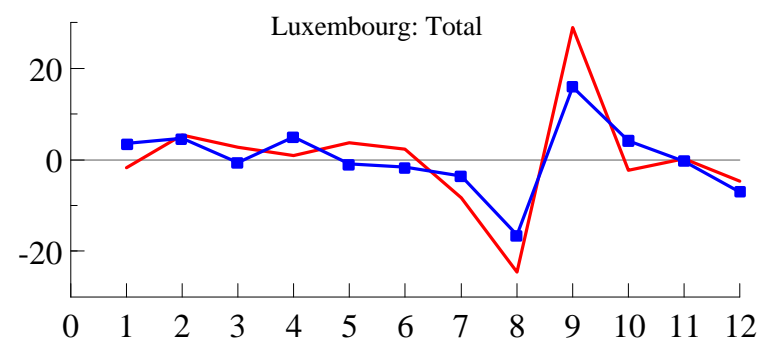
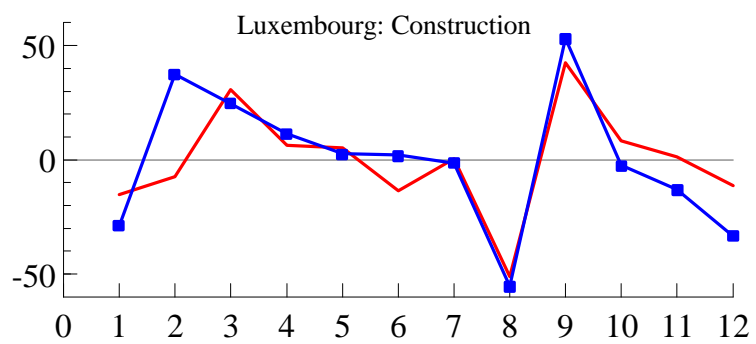
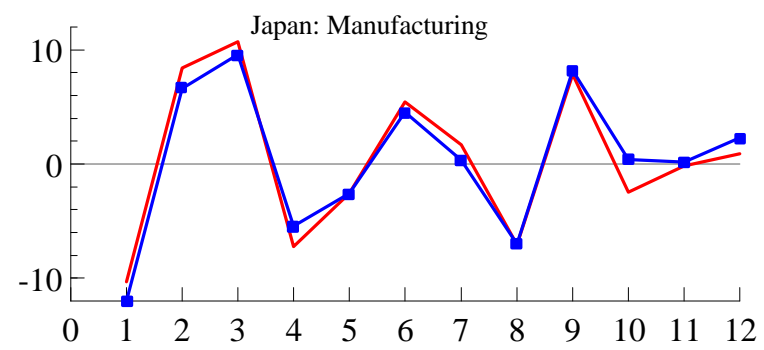
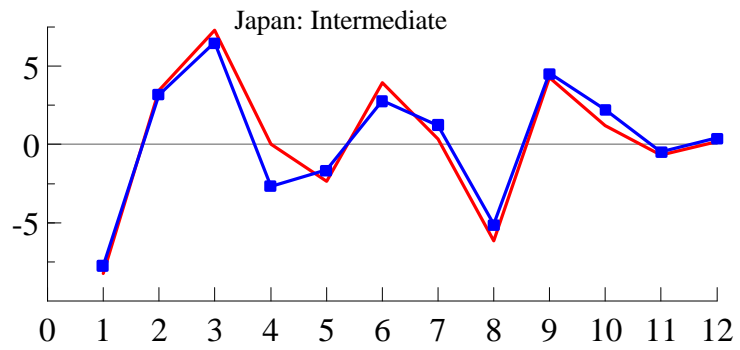
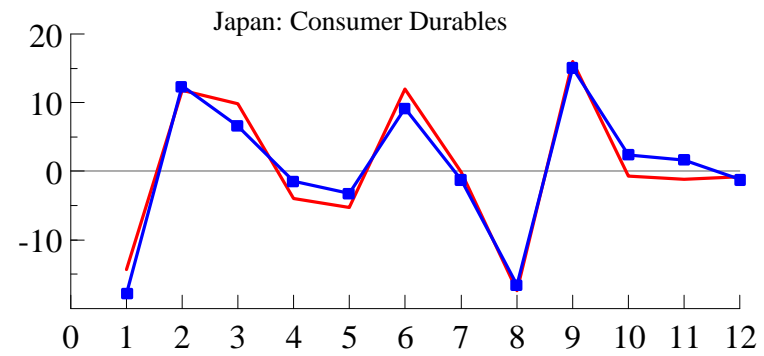
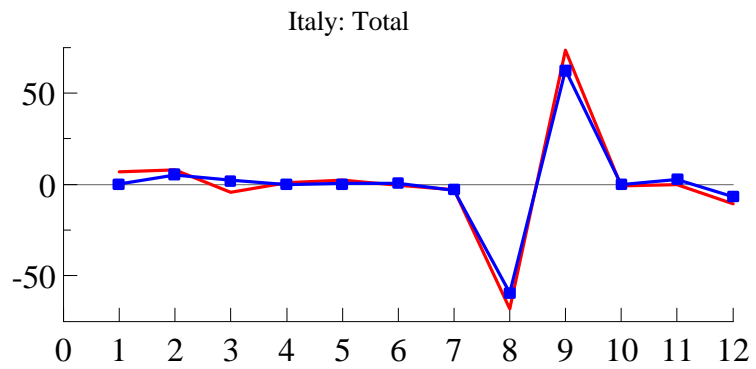
where  $\mathbf{g} = (\mathbf{g}_1, \dots, \mathbf{g}_{12})'$ , with the interpretation that the values represent the mean deviations for each month compared to the overall mean in the upper regime. The overall means for the lower and upper regimes are obtained as  $\mathbf{f}^{-1}(1)\mathbf{d}_0$  and  $\mathbf{f}^{-1}(1)[\mathbf{d}_0 + \mathbf{g}_0]$  respectively.

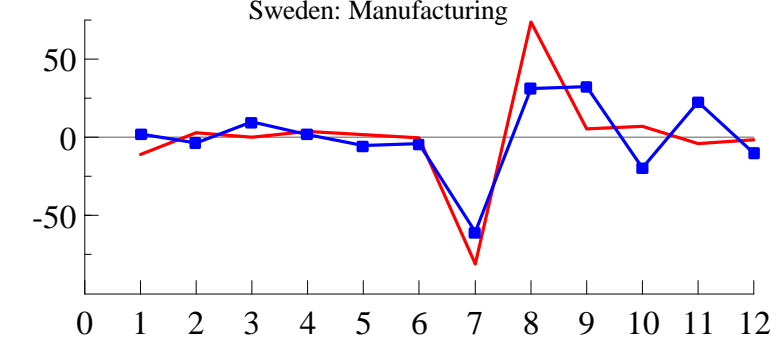
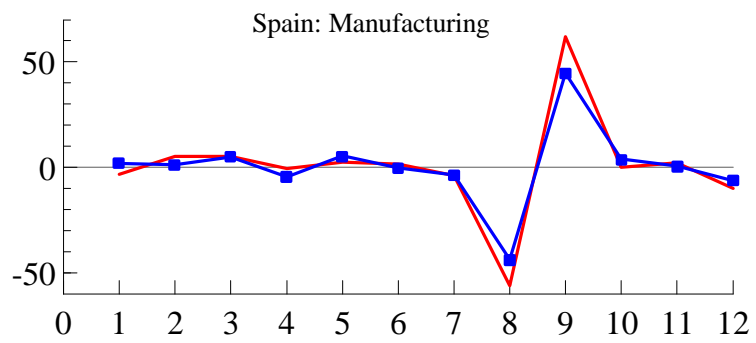
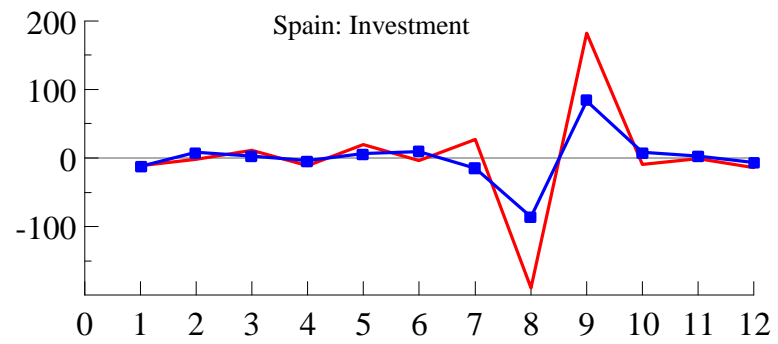
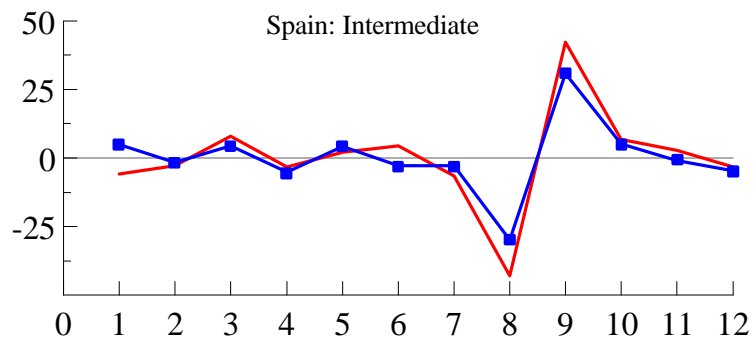
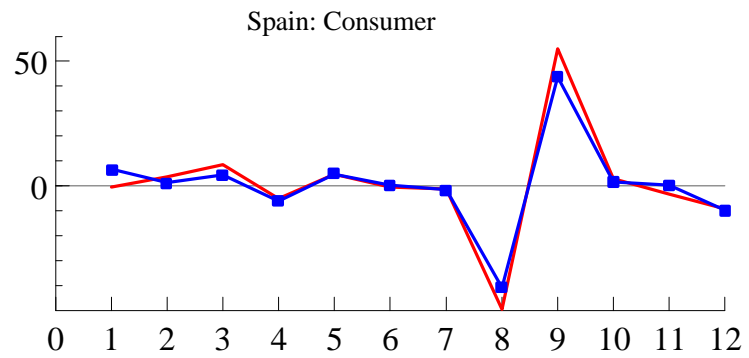
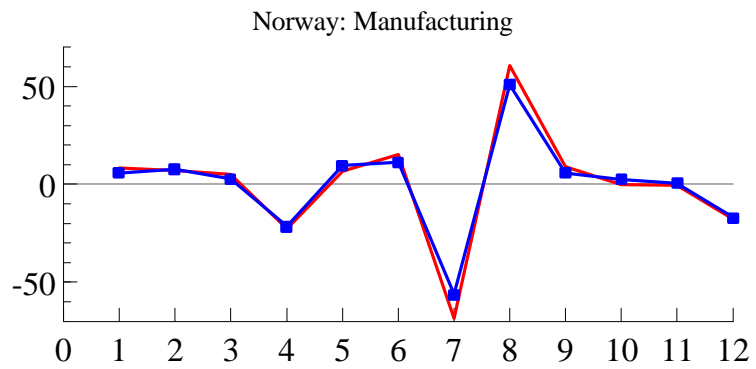
Finally, as mentioned in the text, it should be noted that the regime-dependent means and monthly mean shifts derived from equation (3) are steady state values. That is, their computation assumes that the regime does not change over time. When a regime switch occurs, the representations analogous to (A.1) and (A.3) are not equivalent; this is discussed by Hamilton (1993) in the context of a Markov-switching model.

**Figure 1. Estimated Monthly Means in the Upper (indicated by ■) and Lower Regimes**

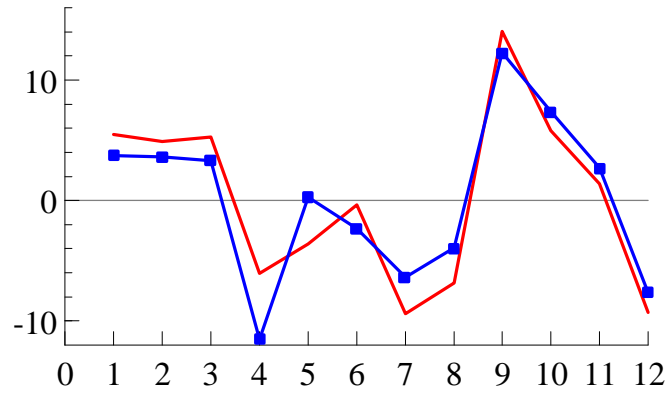








UK: Intermediate



UK: Total

