A Panel Data Approach to Testing Anomaly Effects in Factor Pricing Models*

Laura Serlenga Yongcheol Shin

Andy Snell

Department of Economics, University of Edinburgh

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Abstract

There has been a large anomaly literature where firm specific characteristics such as leverage, past returns, dividend-yield, earnings-to-price ratios and book-to-market ratios as well as size help explain cross sectional returns. These anomalies that have been attributed to market inefficiency could be the result of a mis-specification of the underlying factor pricing model. The most popular approach to detecting these anomaly effects has been the two pass (TP) cross-sectional regression models, advanced by Black, Jensen and Scholes (1972) and Fama and MacBeth (1973). However, it is well-established that the TP method suffers from the errors in variables problem, because estimated betas are used in place of true betas in the second stage cross sectional regression. In this paper we address the issue of testing for factor price misspecification via the panel data approach. It is a salient fact that conventional approaches have completely ignored the benefits of using panel data techniques. Perhaps one of the main reasons for this neglect is that in factor pricing models, all betas are heterogeneous in the first pass time series regression. As a result there is no room for exploiting the panel dimension since there are no homogeneous coefficients to estimate. If our interest lies

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solely in testing the significance of these characteristics, we can show how to construct a theoretically coherent example to which panel data techniques dealing with both homogeneous and heterogeneous parameters can be applied. Panel-based anomaly tests have one clear advantage over TP-based tests; they are based on full information maximum likelihood estimates so that they do not suffer from the errors in variable problem and have all the usual asymptotic properties associated with likelihood tests. The empirical illustration shows the importance of market to book and market value in helping explain asset returns.

JEL Classification: C12, C13, G12.

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1 Introduction

The central prediction of the asset pricing models of Sharpe (1964), Lintner (1965), and Black (1972) is that the market portfolio of invested wealth is mean-variance efficient. This efficiency of the market portfolio implies that expected returns on securities are an exact positive linear function of their market betas. But, there have been several empirical findings which contradict the prediction of these models. The most prominent is the size effect of Banz (1981), who finds that the market value of equity adds to the explanation of the cross-section of average returns provided by market betas. More recently, there has been a large anomaly literature where firm specific characteristics such as leverage, past returns, dividend-yield, earnings-to-price ratios and book-to-market ratios as well as size help explain cross sectional returns. See for example Keim (1983), Fama and French (1992, 1996), Berk (1995) and Gauer (1999).

To accommodate these anomaly effects, a general procedure pursued in the literature is to find characteristics that are associated with average returns, sort portfolios based on those characteristics, compute betas for portfolios and check whether the average return spread is accounted for only by the spread in betas. Fama and French's (1993, 1996) model successfully explains the average returns of the 25 size and book-to-market sorted portfolios with three factors, namely, returns on the market, returns on a small minus big (SMB) portfolio and returns on a high minus low (HML) portfolio. Even though the choice of factors is motivated mostly by empirical experience and thus somewhat arbitrary, their three factor model has been widely used in evaluating various expected return puzzles.

One practically important issue is to check whether the factor pricing models need to be augmented by asset specific characteristics. For example, momentum effects, where a portfolio consists of short-term winners have been found to be important [see Jagadeesh and Titman (1993)], violating the Fama and French three factor model, see Fama and French (1996). In a similar vein, Daniel and Titman (1997), and Daniel, Titman and Wei (2001) have advanced ways to distinguish between factor models and characteristic models.

These anomalies that have been attributed to market inefficiency could be the result of a mis-specification of the underlying factor pricing model. The most popular approach to detecting these anomaly effects has been the two pass (TP) cross-sectional regression models, advanced by Black, Jensen and Scholes (1972) and Fama and MacBeth (1973), which have been widely used to evaluate linear factor pricing models, including the capital asset pricing model (CAPM), the arbitrage pricing theory (APT) and their variants [see Cochrane (2001) for an excellent survey]. In the first stage, the asset betas are estimated by time series linear regression of the asset's return on a set of common factors. Then, the cross sectional regression of mean returns on betas and characteristics are estimated, and the significance of asset specific regressors are evaluated along with factor risk premia estimation. The same approach could be applied to evaluating momentum anomaly effects using asset specific proxy variables for the past short term performance of portfolios (such as lagged portfolio returns).

However, it is well-established that the aforementioned TP method suffers from the errors in variables (EIV) problem, because estimated betas are used in place of true betas in the second stage cross sectional regression. In this regard, many econometricans have suggested several ways to derive the EIV corrected standard errors of the TP estimators under different set of assumptions. A detailed treatment of TP estimation and associated asymptotic theories can be found in Shanken (1985, 1992), Jagannathan and Wang (1998), and Ahn and Gadarowski (2001).

In this paper we address the issue of testing for factor price misspecification via the panel data approach. It is a salient fact that conventional approaches have completely ignored the benefits of using panel data techniques. Perhaps one of the main reasons for this neglect is that in factor pricing models, all betas are heterogeneous in the first pass time series regression. As a result there is no room for exploiting the panel dimension since there are no homogeneous coefficients to estimate. Instead, the validity of the null hypothesis that the time series factor pricing model is correctly specified is in fact tested in the second pass cross sectional regression, for example, of pricing errors on characteristics.

If our interest lies solely in testing the significance of these characteristics, we can show how to construct a panel data regression model with one set of variables varying over time such as common factors and another set of variables varying both over time and over assets. A statistical model where the parameters on factors are heterogenous and the parameters on characteristics are homogeneous is required to analyse the existence of anomalies in factor pricing models such as the CAPM or APT. This model is a special case of the econometric framework recently proposed by Pesaran, Shin and Smith (1999), who develop a dynamic heterogeneous panel estimation techniques that allows us to investigate both homogenous long-run relationship and heterogenous short-run dynamic adjustment towards equilibrium simultaneously.

The current paper provides a theoretically coherent example to which panel data techniques dealing with both homogeneous and heterogeneous parameters can be applied. Though similar, the required econometric methodology is somewhat different from that in Pesaran, Shin and Smith (1999) and is therefore developed separately here. Panel-based anomaly tests have one clear advantage over TP-based tests; they are based on full information maximum likelihood estimates so that they do not suffer from the EIV problem and have all the usual asymptotic properties associated with likelihood tests. In addition the panel technique adopted here yields parameter estimates of firm specific effects that (under the alternative) are fully efficient.

The empirical illustration shows the importance of market to book and market value in helping explain asset returns. When such terms are added to the simple CAPM version of the model their significance is enormous. This confirms results from similar studies done on both US and UK data. Interestingly enough, however, when we execute a test of significance for the average beta we find that it remains important despite the addition of size and market to book variates. By contrast the classic empirical result from TP studies is that the significance of betas are driven out by the addition of such firm specific factors. We argue that the power of our tests is higher than would be expected from a TP procedure and that this may be behind the result here.

The next section outlines a heterogenous panel model within which factor pricing anomalies can be analysed and Section 3 derives the econometric theory required for this analysis. Section 4 gives an empirical illustration of the techniques applied to the UK excess stock returns. Section 5 concludes and discusses possible future empirical work on anomalies using the panel data methodology advanced here.

2 Overview on Modelling Issues

It is nowadays standard to assume that the individual stock returns (or returns on the individual portfolio) are linearly generated by multiple common factors,

$$r_{it} = a_i + \boldsymbol{\beta}_i'\mathbf{f}_t + \varepsilon_{it},\ i = 1, ..., N,\ t = 1, ..., T, \tag{1}$$

where r_{it} is the excess return of asset i at time t, $\mathbf{f}_t$ is the k vector of factors, a_i is the asset-specific intercept term, $\boldsymbol{\beta}_i$ is the k vector of betas (factor loadings) of asset i corresponding to $\mathbf{f}_t$, and ε_{it} is assumed to be the zero mean idiosyncratic error for asset i at time t. This model includes the standard CAPM as a special case. The (linear) beta-pricing restrictions imposed on (1) is given by

$$H_0 : E\left(r_{it}\right) = \gamma_0 + \boldsymbol{\beta}_i'\boldsymbol{\gamma}_1,\ i = 1, ..., N, \tag{2}$$

where $E\left(r_{it}\right)$ is the expected return on asset i, γ_0 is an unknown constant (*e.g.* zero-beta expected return), $\boldsymbol{\gamma}_1$ is the k vector of associated factor risk premia. If (2) holds then asset markets are efficient in the sense that there are no (asymptotic) gains to arbitrage. However, there is mounting empirical evidence that asset specific factors are also priced. To the extent that these asset specific factors have idiosyncratic components (*i.e.* sources of risk that are diversifiable), then their pricing is incompatible with zero (asymptotic) arbitrage.[1] Specifically, previous studies, most famously that by Banz (1981) have added asset specific regressors to (2) and have estimated alternative models of the form

$$H_A : E\left(r_{it}\right) = \gamma_0 + \boldsymbol{\beta}_i'\boldsymbol{\gamma}_1 + \mathbf{s}_{it}'\boldsymbol{\gamma}_2,\ i = 1, ..., N, \tag{3}$$

where $\mathbf{s}_{it}$ is a q vector of asset specific variables such as size or book-to-market value for asset i at time t, $\boldsymbol{\gamma}_2$ is a $q \times 1$ vector of unknown parameters of return premiums associated with $\mathbf{s}_{it}$.

[1] Fama and French (1996) argue that most of the asset specific variables (particularly size and book to market) that generate anomalies in this way can be accounted for by additional pricing factors. We do not enter this debate here but focus on tests of pre-specified factor models against asset specific alternatives. Interestingly, Fama and French also admit that their factors cannot drive out the significance of own lagged returns in the cross section. The inclusion of variables such as own lagged returns makes the model a heterogenous dynamic panel but does not raise any problems for our approach as wiil be shown below.

To test the null model against the alternative model H_A, a traditional two pass (TP) regression method has been applied to (3). To estimate $\boldsymbol{\gamma} = (\gamma_0, \boldsymbol{\gamma}_1', \boldsymbol{\gamma}_2')'$, we run the second pass cross sectional regression (CSR),

$$\bar{r}_i = \gamma_0 + \hat{\boldsymbol{\beta}}_i'\boldsymbol{\gamma}_1 + \bar{\mathbf{s}}_i'\boldsymbol{\gamma}_2 + \eta_i,\ i = 1, ..., N, \tag{4}$$

where $\bar{r}_i = T^{-1}\sum_{t=1}^{T} r_{it}$, $\bar{\mathbf{s}}_i = T^{-1}\sum_{t=1}^{T} s_{it}$, and $\hat{\boldsymbol{\beta}}_i$ are the OLS estimates of $\boldsymbol{\beta}_i$ obtained from the first pass time series regression (1). Alternatively, Fama and MacBeth (1973) considered a rolling CSR in each time period t,

$$r_{it} = \gamma_{0t} + \hat{\boldsymbol{\beta}}_i'\boldsymbol{\gamma}_{1t} + \mathbf{s}_{it}'\boldsymbol{\gamma}_{2t} + \eta_{it},\ i = 1, ..., N. \tag{5}$$

where $\hat{\boldsymbol{\beta}}_i$ is estimated using time series observations 1 through $t-1$. Once the consistent TP estimator of $\boldsymbol{\gamma}$, denoted $\hat{\boldsymbol{\gamma}}_{TP}' = \left(\hat{\gamma}_{0,TP}, \hat{\boldsymbol{\gamma}}_{1,TP}', \hat{\boldsymbol{\gamma}}_{2,TP}'\right)'$, is obtained, the validity of the asset price restriction (2) can be evaluated by testing $H_0 : \boldsymbol{\gamma}_2 = 0$, using for example a Wald test statistic given by

$$Wald = \hat{\boldsymbol{\gamma}}_{2,TP}'\left[Var\left(\hat{\boldsymbol{\gamma}}_{2,TP}\right)\right]^{-1}\hat{\boldsymbol{\gamma}}_{2,TP}, \tag{6}$$

which is distributed as χ_q^2 under the null.

A well-established problem with this TP-based estimation is that the use of estimated betas in the second pass regression generates a well-known errors in variables (EIV) problem. There has been a large literature attempting to derive the EIV corrected standard errors of the TP estimators under different set of assumptions. In particular, with arbitrary positive definite weighting matrix, the TP estimator can be obtained by OLS, GLS, or GMM estimation. [For a treatment of TP estimation and associated asymptotic theories, see Shanken (1985, 1992) and Jagannathan and Wang (1998).]

An alternative method used to avoid the EIV problem is the ML estimation of Gibbons (1982) and Shanken (1986). These authors express the null model in (7) as

$$H_0^* : a_i = \lambda_0 + \boldsymbol{\beta}_i'\boldsymbol{\lambda}_1,\ i = 1, ..., N, \tag{7}$$

where α_i is the individual intercept in the first-pass regression (1), $\boldsymbol{\lambda}_1$ is a unknown $k \times 1$ vector, and the following relationships hold between the γ's and λ's (see Ahn and Gadarowski, 2001, p.6):[2]

$$\lambda_0 = \gamma_0, \quad \boldsymbol{\lambda}_1 + E(\mathbf{f}_t) = \boldsymbol{\gamma}_1.$$

Similarly, the alternative model in (3) can be equivalently written as

$$H_A^* : a_i = \lambda_0 + \boldsymbol{\beta}_i' \boldsymbol{\lambda}_1 + \mathbf{s}_{it}' \boldsymbol{\lambda}_2, \ i = 1, ..., N, \tag{8}$$

where

$$\lambda_0 = \gamma_0, \quad \boldsymbol{\lambda}_1 = \boldsymbol{\gamma}_1 - E(\mathbf{f}_t), \quad \boldsymbol{\lambda}_2 = \boldsymbol{\gamma}_2.$$

Thus, the validity of the null H_0^* can be checked now by testing restriction $\boldsymbol{\lambda}_2 (= \boldsymbol{\gamma}_2) = 0$. Applying the minimum distance approach to (7) and (8) in terms of the TP estimation, Ahn and Gadarowski (2001) have developed several robust methods to estimate $\lambda = (\lambda_0, \lambda_1', \lambda_2')'$, but also provide EIV corrected standard errors of the TP estimators, such that the validity of asset pricing models can be evaluated under a general set of assumptions.

Suppose now that we are interested solely in testing the significance of the asset characteristics, as envisaged either by (3) or (8). More specifically, under (8), the time series linear factor pricing regression can be extended to the following panel data regression:

$$r_{it} = \alpha_i + \boldsymbol{\delta}' \mathbf{s}_{it} + \boldsymbol{\beta}_i' \mathbf{f}_t + \varepsilon_{it}, \ i = 1, ..., N, \ t = 1, ..., T, \tag{9}$$

where $\alpha_i = \lambda_0 + \boldsymbol{\beta}_i' \boldsymbol{\lambda}_1$ and $\boldsymbol{\delta} = \boldsymbol{\lambda}_2$. If certain asset characteristics are statistically significant for explaining excess returns, then these anomaly effects can be regarded as an evidence against the underlying multi-factor models. The aim of this paper is to provide an alternative panel data-based test for the null model (7) against the alternative model (8) in the context of multi-beta pricing models. We propose this is done via a simple Wald test of $\boldsymbol{\delta} = \mathbf{0}$ in (9). Because this does not require second pass cross sectional estimation, the panel-based test will not suffer from the EIV problem discussed above. Further, the fact that we use a Wald test gives the procedure all of the desirable (asymptotic) inferential properties associated with likelihood based tests. Finally, a by product of the method is that it generates full

[2]Notice here that the factor risk premia $\boldsymbol{\gamma}_1$ are now decomposed into the population mean vector of the factors $E(\mathbf{f}_t)$, and the so-called lambda component $\lambda_1 = \gamma_1 - E(\mathbf{f}_t)$. This lambda component can be interpreted as the vector of factor mean adjusted risk premia, see Zhou (1998).

information ML estimates of all the model's parameters under the alternative and these estimates will be fully efficient.

In retrospect, it is somewhat puzzling that previous (TP) approaches have completely ignored the potential efficiency gains associated with ML panel data estimation. One possible reason is that under the null, all betas are heterogeneous so that there are no homogeneous coefficients to estimate and no efficiency gains to be made from system wide ML estimation at all. However under the alternative as (9) clearly shows, a panel-based analysis becomes not only natural but desirable from the point of view of efficient estimation and inference.

We close this section with some brief comments on the panel model. First, there are two different types of regressors: one (the asset pricing factors) which varies over time but is constant across assets and another (the asset specific characteristic variables) which vary over time and over assets. In addition, factor loadings $\boldsymbol{\beta}_i$, are heterogenous across assets but the parameters on characteristics, $\boldsymbol{\delta}$, are homogeneous across assets. Hence, the panel data model (9) shares common features with the econometric framework recently proposed by Pesaran, Shin and Smith (1999), who develop dynamic heterogeneous panel estimation techniques that allow the simultaneous investigation of both homogenous long-run relationships and heterogenous short-run dynamic adjustment towards that long run relationship. Though similar, the econometric methodology developed and used in this paper is somewhat different from that of Pesaran, Shin and Smith (1999). Hence we must develop the underlying econometric theory for estimation and inference using (9) anew. This is achieved in the next section.

3 Heterogeneous Panel Data Methodology

In this section we formally develop the underlying econometric theory. To this end it will be convenient to generalize notation. Explicitly, we consider the panel

$$y_{it} = \boldsymbol{\delta}'\mathbf{x}_{it} + \boldsymbol{\beta}_i'\mathbf{f}_t + u_{it},\ i = 1, ..., N,\ t = 1, ..., T, \tag{10}$$

with error components,

$$u_{it} = \alpha_i + \varepsilon_{it}, \tag{11}$$

where y_{it} is a scalar dependent variable, $\mathbf{x}_{it}$ is a q vector of explanatory variables, $\mathbf{f}_t$ is the k vector of common factors, α_i contains individual effects, and ε_{it}'s are independently distributed (over time and cross-section) with mean zero and heterogeneous variance, σ_i^2. We assume that α_i are identically and independently distributed with mean zero and variance σ_α^2, and that α_i are uncorrelated with ε_{jt} for all i, j and t.

This panel data model is a case where some parameters $\boldsymbol{\beta}_i$ are allowed to be heterogenous, but others $\boldsymbol{\delta}$ are homogenous. Under the assumption that ε_{it} are normally distributed with heterogenous variances, σ_i^2, we obtain the following (concentrated) log-likelihood function:[3]

$$\ell_T(\boldsymbol{\varphi}) = -\frac{T}{2}\sum_{i=1}^{N} \ln 2\pi\sigma_i^2 - \frac{1}{2}\sum_{i=1}^{N}\frac{1}{\sigma_i^2}\left(\mathbf{y}_i - \mathbf{x}_i\boldsymbol{\delta}\right)' \mathbf{H}_i \left(\mathbf{y}_i - \mathbf{x}_i\boldsymbol{\delta}\right), \tag{12}$$

where $\mathbf{y}_i = (y_{i1}, ..., y_{iT})'$, $\mathbf{x}_i = (\mathbf{x}_{i1}, ..., \mathbf{x}_{iT})'$, $\mathbf{H}_i = \mathbf{I}_T - \mathbf{W}_i(\mathbf{W}_i'\mathbf{W}_i)^{-1}\mathbf{W}_i'$, $\mathbf{I}_T$ is an identity matrix of order T, $\mathbf{W}_i = (i_T, \mathbf{f})$ with $i_T = (1, ..., 1)'$ and $\mathbf{f} = (\mathbf{f}_1, ..., f_T)'$, and $\boldsymbol{\varphi} = (\boldsymbol{\delta}', \sigma_i^2, ..., \sigma_i^2)'$.

The maximum likelihood estimator of the homogeneous parameters $\boldsymbol{\delta}$ can now be obtained by maximizing (12) with respect to $(\boldsymbol{\delta}, \sigma_1^2, ..., \sigma_N^2)$, respectively. It is then straightforward to obtain the following formula for $\hat{\boldsymbol{\gamma}}$, and $\hat{\sigma}_i^2$:

$$\hat{\boldsymbol{\delta}} = \left(\sum_{i=1}^{N}\frac{1}{\hat{\sigma}_i^2}\mathbf{x}_i'\mathbf{H}_i\mathbf{x}_i\right)^{-1}\left(\sum_{i=1}^{N}\frac{1}{\hat{\sigma}_i^2}\mathbf{x}_i'\mathbf{H}_i\mathbf{y}_i\right), \tag{13}$$

$$\hat{\sigma}_i^2 = T^{-1}\left(\mathbf{y}_i - \mathbf{x}_i\hat{\boldsymbol{\delta}}\right)'\mathbf{H}_i\left(\mathbf{y}_i - \mathbf{x}_i\hat{\boldsymbol{\delta}}\right),\ i = 1, ..., N. \tag{14}$$

These need to be solved iteratively. Starting with an initial estimate of $\boldsymbol{\delta}$, say $\hat{\boldsymbol{\delta}}^{(0)}$, estimates of σ_i^2 can be computed using (14), which can then be substituted in (13) to obtain new estimate of $\boldsymbol{\delta}$, say $\hat{\boldsymbol{\delta}}^{(1)}$, and so on until convergence is achieved. Alternatively, these estimators can be computed by the familiar Newton-Raphson algorithm which makes use of both first and the second derivatives.

In order to derive the asymptotic distribution of the pooled ML estimators of $\boldsymbol{\varphi}$, we assume that all the underlying variables are stationary, in which case under fairly standard conditions the consistency and the asymptotic normality of the pooled ML and mean group

[3] Normality can be relaxed. In that case the QML approach follows.

estimators of the parameters in (12) can be easily established. In particular, as both $T \to \infty$ and $N \to \infty$, the pooled ML estimator of $\boldsymbol{\delta}$ has the following asymptotic distribution:

$$\sqrt{NT}\left(\hat{\boldsymbol{\delta}} - \boldsymbol{\delta}\right) \overset{a}{\sim} N\left\{\mathbf{0}, \left[\frac{1}{N}\sum_{i=1}^{N}\frac{1}{\sigma_{i0}^{2}}\mathbf{Q}_{x_i x_i}\right]^{-1}\right\}, \tag{15}$$

where $\mathbf{Q}_{x_i x_i}$ is the probability limits of $T^{-1}\mathbf{x}_i'\mathbf{H}_i\mathbf{x}_i$.[4] The proof can be easily established using the results in Pesaran and Smith (1995) and Pesaran, Shin and Smith (1999).[5]

Using these results, the joint null hypothesis $\boldsymbol{\delta} = 0$ can be tested simply by a Wald statistic given by

$$Wald = \hat{\boldsymbol{\delta}}' \left[Var\left(\hat{\delta}\right)\right]^{-1}\hat{\boldsymbol{\delta}} = \left(\sum_{i=1}^{N}\frac{1}{\hat{\sigma}_i^2}\mathbf{x}_i'\mathbf{H}_i\mathbf{y}_i\right)'\left(\sum_{i=1}^{N}\frac{1}{\hat{\sigma}_i^2}\mathbf{x}_i'\mathbf{H}_i\mathbf{x}_i\right)^{-1}\left(\sum_{i=1}^{N}\frac{1}{\hat{\sigma}_i^2}\mathbf{x}_i'\mathbf{H}_i\mathbf{y}_i\right), \tag{16}$$

where $\hat{\sigma}_i^2$ is the final consistent estimate of σ_i^2. Then, under the null, we have

$$Wald \overset{a}{\sim} \chi_q^2,$$

where q is the dimension of $\mathbf{x}_{it}$. As a special case the single null of $\delta_i = 0$, $i = 1, 2, ..., q$, can be tested using the t-test given by

$$t = \frac{\hat{\delta}_i}{\sqrt{Var(\hat{\delta}_i)}}, \tag{17}$$

where $\hat{\delta}_i$ is an ith element of $\hat{\boldsymbol{\delta}}$, which converges to the standard normal distribution under the null.

4 Empirical Illustration

In this section we apply the panel data test to a sample of UK firms. We focus on the significance and importance of size and market to book effects in explaining returns within the context of a single factor simple CAPM model. Several other authors have already

[4] For this result to hold it is necessary that the limit of $N^{-1}\sum_{i=1}^{N}\frac{1}{\sigma_{i0}^2}\mathbf{Q}_{x_i x_i}$ as $N \to \infty$ is a positive definite matrix.

[5] In principle, this asymptotic discussion can be extended to the case where the underlying variables are $I(1)$, but is more complicated. See Pesaran, Shin and Smith (1999).

discovered "anomalous" size and book-to-market effects in UK data. For example, Strong and Xu (1997) assess the significance of the time average of the coefficients in the cross sectional regressions using Fama and French's method discussed above. See also Levis (1989) and Hussain, Diacon and Toms (2000).

4.1 Data

Stock market returns for the Financial Times All Share Index and for the individual companies are obtained from *Datastream's* returns index (RI). Variables r^m and r represent the percentage of growth of the excess returns on the market and on the individual assets, respectively. Returns on the risk-free asset are given by the monthly yield on the 3-Month UK Treasury Bills. Market-to-Book value (MTB) and Market value (MV) are as given by *Datastream,* and proxy the firm characteristics of financial distress and size, respectively.[6,7] The data are collected from 1972 to 2001 and consists of 355 monthly observations on 246 UK firms quoted in the LSE, that belong to nineteen different sectors.

4.2 Empirical results

Table 1 shows the test results for anomaly effects. Under the null hypothesis the Wald test examines the joint significance of the homogeneous coefficients on MV and MTB in explaining excess returns. We also re-estimate excluding each of the variables in turn and perform a t-test for the significance of the included variable's homogenous coefficient. Column 2 of Table 2 presents the results for the sample as a whole and columns 3 to 5 present the analogous results for the three selected subsamples. The subsample estimates indicate some variation in the magnitude of each coefficient across the subsamples but the coefficients' signs

[6]Book-to-Market (BTM) is equal to the ratio between book value and market value. In Datastream book value is defined as total assets, excluding intangible assets less total liabilities, minority interest and preference stock. Market value (MV) is the share price multiplied by the number of ordinary shares in issue. The data have been transformed in logarithm.

[7]Berk (1995) points out that even if size and book-to-market effects were not priced, by their definition and construction, these variates will always covary with market price and hence returns. This spurious source of correlation will generate significant anomaly effects even when none are present.

are always negative. It is interesting that the significance of MTB and MV varies over the subperiods. If these variates are potentially proxying macro distress factors as claimed by Fama and French (1996), then this variation could be because of business cycle effects in the three sub-periods. For example, the first sub-period, in which MV has a t-ratio exceeding 6, was dominated by supply shocks and economy wide contractions. We might well expect MV to be more correlated with an unobserved distress factor at such times.

Turning to the full sample estimates, we see that when the model includes both MV and MTB the Wald test value exceeds 21, which corresponds to a p value of around .001% indicating massive significance of these terms in the sample as a whole. The coefficient on MV in the model that excludes MTB is only a quarter of the value obtained by Strong and Xu (1997) in their monthly sample although their sample straddles a slightly earlier time span than does ours. However, the size effect is significant and negative, which is consistent with other estimates in the literature. The coefficient on MTB in the model that excludes MV is insignificant and small but increases in size and becomes significant when MV is also present. The negative coefficients found on MTB and MV are consistent with Fama and French's (1996) argument that MV and MTB proxy for a macro"distress" factor with low MTB/MV firms being more exposed to bankruptcy risk and therefore paying a higher return.

Table 1 about here

Finally, we carry out a mean group test advanced by Pesaran and Smith (1995) and assess the "average" significance of market betas in the panel as a whole.[8] In particular, we test the joint null, $H_0 : \beta_i = 0,\ i = 1, ..., N$ against the (one-sided) alternative hypotheses $H_1 : \beta_i > 0$ for $i = 1, ..., N$, and thus construct the mean group t statistic as

$$\bar{t}_{NT}(\boldsymbol{\beta}) = \frac{1}{\sqrt{N}} \sum_{i=1}^{N} t_T(\beta_i), \qquad (18)$$

where $t_T(\beta_i)$ is an individual t-test for $\beta_i = 0$. Under the null as $N, T \to \infty$ and $\frac{N}{T} \to 0$, it would be possible to show under certain additional assumptions that [see Shin and Snell

[8] In some ways, the current application is an ideal environment for mean group testing because the error terms are cross sectionally independent (*i.e.* idiosyncratic) under the null, an assumption which is required by this analysis but which for many other applications may be considered too strong.

(2001)]

$$\bar{t}_{NT}\left(\boldsymbol{\beta}\right) \rightarrow_d N(0,1).$$

The test results summarised in Table 2 indicate that beta's remain significant on average at least marginally despite the introduction of our two asset specific size and distress effects. This contradicts most of the empirical findings to date which tend to show that the inclusion of firm specific factors, particularly size proxies, causes the coefficient on asset betas to become insignificant. We tentatively suggest that the efficient estimation and corresponding high power that we would expect from our test procedure may be the reason for this.

Table 2 about here

5 Concluding Remarks

In this paper we have presented a logically natural and theoretically coherent panel data framework within which to analyse asset return anomalies. We have derived the appropriate estimation and inference techniques within this framework together with their relevant asymptotic properties.

In our closing comments we indicate future research within our framework. Fama and French (1996) suggest that firm specific characteristics such as size and distress proxies are really only picking up the effects of missing factors. They propose two additional factors that when used in TP regressions, destroy the significance of all of the usual asset characteristic variables apart from lagged returns (which proxy for momentum effects). Given that we believe our framework provides efficient estimates and powerful tests, it would be interesting to see if this empirical result is robust with respect to our framework and such an investigation is currently underway.

Table 1. Estimation and Test Results for Anomaly Effects

	72-01	72-81	82-91	92-01
Joint[1]				
δ_{MTB}	-0.0027[3] (-1.98)[4]	-0.0100 (-1.06)	-0.0543 (-2.28)	-0.0011 (-0.58)
δ_{MV}	-0.0796 (-4.21)	-0.3376 (-6.49)	-0.0094 (-0.13)	-0.4287 (-4.11)
$Wald$	21.54	14.32	5.78	17.35
Single[2]				
δ_{MTB}	-0.0013 (-0.75)	-0.0099 (-1.05)	-0.0502 (-2.21)	-0.0013 (-0.69)
δ_{MV}	-0.0754 (-3.54)	-0.3374 (-3.63)	-0.0087 (-0.13)	-0.4305 (-4.13)

Notes: [1] The estimation and tests results are obtained from the panel data regression of r on r^m, MTB, and MV. [2] The estimation and tests results are obtained from the panel data regression of r on r^m and MTB, and on r^m and MTB separately. [3] The pooled ML estimate. [4] The associated t-ratio.

Table 2. Mean Group Test Results for Average Significance of Market Betas

	Model 1	Model 2	Model 3	Model 4
$\bar{t}_{NT}(\boldsymbol{\beta})$	2.08	1.66	1.59	1.63

Notes: Model 1 runs the panel data regression of r on r^m, Model 2 the panel data regression of r on r^m and MTB, Model 3 the panel data regression of r on r^m and MV, and finally Model 4 the panel data regression of r on r^m, MTB, and MV. In Models 2, 3 and 4, the values of $\bar{t}_{NT}(\boldsymbol{\beta})$ are computed conditional on the pooled ML estimates of $\boldsymbol{\delta}$. All estimation and test are carried out using the full sample periods 72-01.

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