THE OBSERVATIONAL EQUIVALENCE OF TAYLOR RULE AND TAYLOR ¡ TYPE RULES[z]

Naveen Srinivasan (YE)[y]

Cardi® Business School, UK

Patrick Minford

Cardi® Business School, UK and CEPR

Francesco Perugini

Cardi® Business School, UK

Abstract

In a variety of recent papers, researchers have found that interest rate behaviour approximately follows a Taylor rule. From this they have concluded that the central bank is following a Taylor rule as its monetary policy reaction function. We show that such interest rate behaviour results when the central bank may be following quite di®erent monetary policy rules from the one proposed by Taylor. In other words an interest rate relation with output and in°ation does not identify a central bank reaction function.

JEL classi¯cations: D5, E4, E52

Keywords: observational equivalence; monetary policy rules

[z]We are grateful to David Peel for insightful comments that greatly helped this work; also to other participants at Cardi® Business School lunchtime workshop, especially Laurence Copeland, Gerald Harbour and Kent Matthews. We also thank for helpful comments Mervyn King and James

Talbot at the Bank of England, participants at Scottish Economic Society Annual Conference 2001, and participants at the Fourth Annual Conference on Money and Finance December 2001, IGIDR, India.

[y]Corresponding Author: Naveen Srinivasan, Cardi® Business School, Colum Drive, Cardi® CF10 3EU, United Kingdom. Tel: (029) 20876545. Fax: (029) 20874419. Email: SrinivasanNK@cf.ac.uk

I Introduction

In the past few years the view has commonly been expressed that central banks follow `Taylor Rules', that is that they set interest rates in response to deviations of in°ation from target and of output from its natural rate, with or without a lagged interest rate term.[1] Such rules were ¯rst promulgated by Bryant et al. [1993], and in particular Henderson and McKibbin [1993] in a Brookings study of multi-lateral models and of the properties of di®erent monetary rules in the face of stochastic simulations; subsequently John Taylor [1993] gave them a wide airing as an appropriate way to formulate monetary policy in a world where the money demand relationships had become unstable. Since then he and other authors have ¯tted single equations of this type to explain the behaviour of interest rates in the US and a variety of other countries in recent years; the general suggestion has been that they explained this central bank behaviour in recent years well enough for us to believe that this in fact was what central banks were doing.[2; 3] In this paper we consider whether this evidence should be persuasive.

We show that the appearance of such an interest rate rule- a `pseudo-Taylor rule'- can be created by a standard macro model in which actually a money supply rule is operating with no interest rate feedback- i.e, where there is in fact no Taylor rule operating at all. The pseudo-rule is implied by the model under a money supply rule, as a correlative relation (not a reduced form as all the arguments are endogenous variables). Hence an interest equation does not identify a (structural) Taylor rule; a Taylor rule and a pseudo-rule are `observationally equivalent' to use the expression coined by Thomas Sargent (1976).[4] In otherwords just because it appears ex post that Central Banks are following a monetary policy rule of the type outlined by Taylor, does not necessarily mean that they are ex ante. This is not altogether surprising since Taylor originally suggested that he had justified his rule as an approximation to the behaviour of interest rates in a fixed money supply regime with stable money demand; hence it could be used to `replicate' that interest rate behaviour where the money supply could not be targeted. This might be sensible operational advice to central banks; but it does not follow that when interest rate behaviour of this sort is observed therefore central banks are actually following such rules. They could just as well be setting the money supply under conditions of stable money demand.

It might be said that it is commonly observed that central banks do actually set interest rates rather than fix the money supply (or the monetary base); they even announce interest rates they will adhere to over the coming month. However we would point out that while this is certainly true of many if not all central banks, these practices allow market rates some latitude within the month around the set level and over the period of a quarter, which is the usual unit of our macro observations, they may change the interest rate set frequently; if they do so in order to hit a money supply target, this behaviour could be close to setting the money supply in a quarterly framework.

It might be thought that perhaps it does not matter whether they follow a Taylor rule or a money supply rule that gives rise to a Taylor pseudo-rule. But we show in an appendix that, as a moment's re°ection would reveal and indeed the earlier Brookings study showed in detail, a money supply rule and a (structural) Taylor rule produce quite di®erent stochastic behaviour in the macro economy. Thus it does make a serious di®erence and does remain a matter still to be determined whether one or other type of behaviour does the better job in welfare terms- contrary to a recent study [Clarida et al. 1999] which called Taylor rules the modern `science of monetary policy', thereby suggesting that other rules are essentially inferior, even irrelevant. Thus it remains an open question whether such rules are appropriate from a welfare viewpoint; and in this note we conclude that the `evidence' of single equation studies is unpersuasive that central banks actually follow them.

Our benchmark framework, as in McCallum and Nelson [1998, 1999] and Clarida et al. [1999] is a dynamic general equilibrium model with money, as explained in section II; this can be recast approximately as an IS/LM/Phillips curve model. A key di®erence of such a model from the traditional IS-LM framework, is that, these equations are derived from optimising behaviour. It also embeds nominal overlapping wage contracts as pioneered by Phelps and Taylor [1977] for which a rationale can be found in insurance against shocks given indexation imperfections [e.g. Minford et al, 1999]. Section III highlights the problem of identification with interest rate rules. The final section summarises our main conclusions.

II Theoretical Structure

Consider an economy populated by identical infinitely lived agents that produce a single good as output which can be used both for consumption and investment.

The Representative Household

In a stochastic environment the consumer maximises his expected utility subject to his budget constraint. Each agent's preferences are given by

$$U = Max E_t \left[\sum_{i=0}^{\infty} \beta^i u \left(C_{t+i} ; \left(\frac{M_{t+i}}{P_{t+i}} \right) ; L_{t+i} \right) \right] ; \qquad 0 < \beta < 1 \qquad (1)$$

where β is the discount factor, C_t is consumption (composite good which includes foreign consumption) in period `t', $L_t$ is the amount of leisure time consumed in period `t', $\frac{M_t}{P_t}$ is real money balances held in period `t', and E_t is the mathematical expectations operator.[5] We assume that the utility function is well behaved and satisfies Inada-type conditions. The representative household's budget constraint is given by

$$(d_t + p_t) Sh_t + \frac{M_{t-1}}{P_t} + b_t^d + Q_t b_t^f + w_t N_t = C_t^d + Q_t C_t^f$$

$$\frac{M_t}{P_t} \left(1 + \pi_{t+1}^e \right) + \frac{b_{t+1}^d}{1 + r_t^d} + \frac{Q_t b_{t+1}^f}{\left(1 + r_t^f \right) (1 + \Psi)} + p_t Sh_{t+1}^d + T_t \qquad (2)$$

where w_tN_t is labour income, C_t^d and C_t^f are domestic and foreign consumption (of finished goods) respectively, b_t^d and b_t^f are domestic and foreign real bonds, P_t is the general price level, p_t is the price of shares, T_t denotes lump-sum taxes, r_t^d is the domestic real rate of interest, π_{t+1}^e is expected inflation, and $Q_t = \frac{S_tP_t^f}{P_t^d}$ is the real exchange rate, where S_t is the nominal exchange rate (a rise is depreciation) and P_t^f and P_t^d are foreign and domestic price level respectively. Sh_t and d_t are shares and dividend income and Ψ is a risk-premium term that reflects temporary departures from uncovered interest parity.[6] Furthermore, each agent is endowed with a fixed amount of time which can be spent for leisure L_t or work N_t. H_t (total endowment of time) is normalised to unity in what follows.

The Representative Household's Optimisation problem

We assume that the functional form of $u(\cdot)$ is separable and of the form[7]

$$u(\cdot) = \theta_0(1-\sigma_0)^{-1}C_t^{(d)1-\sigma_0} + \theta_1(1-\sigma_1)^{-1}C_t^{(f)1-\sigma_1} + \omega(1-\eta)^{-1}m_t^{1-\eta}$$
$$+ (1-\theta_0-\theta_1-\omega)(1-\rho)^{-1}L_t^{1-\rho}$$

where $\theta_0, \theta_1, \omega, \sigma_0, \sigma_1, \eta, \rho \succ 0$. The first-order conditions for the household's optimal choice problem for real money balances (after invoking the Fisher equation) provides justification for a money demand function of the form (in natural logarithms):[8]

$$\log M_t - \log P_t \simeq \gamma_0 + \gamma_1 E_t \log C_{t+1}^{(d)} + \gamma_2 R_t \quad (3)$$

where $\gamma_0 = -\frac{1}{\eta}\log\frac{\theta_0\beta}{\omega} < 0$; $\gamma_2 = -\frac{1}{\eta} < 0$ and $0 < \left(\gamma_1 = \frac{\sigma_0}{\eta}\right) < 1$. The analysis also provides justification for a consumption function of the form:[9; 10]

$$\log C_t^{(d)} \approx -\frac{1}{\sigma_0}\log\beta - \frac{1}{\sigma_0}\log\left[1 + r_t^d\right] + E_t \log C_{t+1}^{(d)} \tag{4}$$

where we have used the approximation $\log E_t C_{t+1}^{(d)} \approx E_t \log C_{t+1}^{(d)}$.[11, 12] Similarly the first-order condition for (foreign) consumption yields:

$$C_t^{(f)-\sigma_1} = \frac{\beta\theta_0}{\theta_1} E_t \left(C_{t+1}^{(d)-\sigma_0}\right)\left[1 + r_t^d\right] Q_t \tag{4.1}$$

Note that in an open economy, the household has an additional choice variable, C_t^f. Equation (4.1) implies the existence of a demand function for imports, whose arguments are the real exchange rate (Q_t), (future) consumption of domestic goods ($C_{t+1}^{(d)}$), and r_t^d.

$$C_t^f = h\left(Q_t, r_t^d, C_{t+1}^{(d)}\right); \qquad h_1(\cdot) < 0;\ h_2(\cdot) < 0;\ h_3(\cdot) > 0 \tag{4.2}$$

We assume that in the rest of the world a parallel maximisation exercise has taken place with a typical foreign households' preferences described by the utility function

$$u^*(\cdot) = \theta_0 (1-\sigma_0)^{-1} C_t^{(d^*)1-\sigma_0} + \theta_1 (1-\sigma_1)^{-1} C_t^{(f^*)1-\sigma_1} + \omega (1-\eta)^{-1} m_t^{(*)1-\eta}$$
$$+ (1-\theta_0-\theta_1-\omega)(1-\rho)^{-1} L_t^{(*)1-\rho}$$

where (*) denotes rest of the world. Since $C_t^{f^*} = X_t$ (home country exports), the foreign sector's maximising behaviour will have produced a decision rule analogous to

(4.2), which gives the rest of the world's demand function for the domestic households' exports:

$$X_t = j\left(Q_t, r_t^{d*}, C_{t+1}^{(d*)}\right); \qquad j_1(\cdot) < 0;\ j_2(\cdot) < 0;\ j_3(\cdot) > 0 \tag{4.3}$$

Equations (4.2) and (4.3) together determine the real exchange rate Q_t, given that r_t^{d*} and $C_{t+1}^{(d*)}$ are exogenous. The representative household's lifetime budget constraint (given that all output (GDP) except government expenditure and investment expenditure is consumed) yields:

$$\sum_{j=0}^{\infty}\left(\frac{1}{1+r_t^*}\right)^j C_{t+j} = \sum_{j=0}^{\infty}\left(\frac{1}{1+r_t^*}\right)^j (Y_{t+j} - G_{t+j} - I_{t+j} - NX_{t+j})$$

or

$$\sum_{j=0}^{\infty}\left(\frac{1}{1+r_t^*}\right)^j \{\tfrac{1}{\sigma}(\beta(1+r_t^*))\}^j C_t = \sum_{j=0}^{\infty}\left(\frac{1}{1+r_t^*}\right)^j (1+g)^j \left[\overline{Y}_t - \overline{G}_t - \overline{I}_t\right]$$

where `g' denotes steady state growth of consumption, r_t^* is long-run interest rate, and $\overline{Y}_t$; $\overline{G}_t$; and $\overline{I}_t$ denote steady state values for output, government expenditure, and investment expenditure respectively (we assume that net exports are zero in steady state). Leading the above equality one-period (expressing it in natural logarithms) and taking expectations at time `t' yields a variable made up of slow-moving steady state elements.

$$E_t \log C_{t+1} \equiv \log \hat{C}_{t+1} = \log(\sigma - \beta) - \log\sigma + \log\left[\overline{Y}_{t+1} - \overline{G}_{t+1} - \overline{I}_{t+1}\right] + \log\left[1 + r_{t+1}^*\right] - \log\left[r_{t+1}^* - g\right] \tag{5}$$

Finally using the (open) economy's overall resource constraint $Y_t = C^d_t + I_t + G_t + X_t$ (the government and household budget constraint together gives us the market clearing condition which is expressed in natural logarithms), we can recast the log-linearised consumption and money demand equation's as (open-economy) IS and LM curve respectively (see McCallum and Nelson, [1999]) which is used in the appendix.

Next we de¯ne the domestic and foreign nominal interest rates as $R^d_t = r^d_t + E_t ¢ P^d_{t+1}$ and $R^f_t = r^f_t + E_t ¢ P^f_{t+1}$, where $P^d_t = \log P^d_t$, $P^f_t = \log P^f_t$ and ¢ denotes the ¯rst di®erence operator. Then using the ¯rst-order conditions for the household's optimal choice problem for domestic and foreign bonds imply that, as a ¯rst-order approximation, the following uncovered interest parity condition holds:

$$R^d_t = R^f_t + E_t ¢ S_{t+1} + ª \qquad (6)$$

where $S_t = \log S_t$ and ª is the risk-premium from the variance terms in a Taylor series expansion of the Euler equations. For simplifying convenience we assume that the exchange rate is set at Purchasing Power Parity, and that home and foreign goods are identical. Hence the current account de¯cit is the excess of domestic demand over output supply; in the long term we assume this de¯cit to be closed by ¯scal policy.

The Representative Firm

The representative ¯rm has two types of buyer for its good: domestic residents and the rest of the world (to which it may export its good). The technology available to the economy is described by the production function

$Y_t = Z_t f\left(N_t; \overline{K}\right)$ where $0 \leq \alpha \leq 1$, Y_t is aggregate output, $\overline{K}$ is capital stock which is assumed to be fixed, N_t is labour supply and Z_t reflects the state of technology.[13,14] Here, firms operate in competitive markets and therefore take prices as given when solving their own constrained maximisation problem. Each firms objective in period `t' is to maximise profit subject to the constant-returns-to-scale production technology.

Introduction of overlapping non-contingent wage contracts[15]

In what follows we replace the standard spot labour market with a market characterised by imperfectly flexible wages. It is assumed that the nominal wage is set to try and maintain equilibrium real wage (i.e., where supply equals demand in expectation). Suppose we have a situation where all wage contracts are set for four periods and the contracts drawn in period `t' specifies nominal wages for periods t+1, t+2, t+3 and t+4.[16] The actual wage rate at any given point in time would be an average of the wages that have been set at various dates in the past. Hence, nominal wage at time `t' in natural logarithms would be

$$W_t = 0.25\left({}_{t-1}W_t + {}_{t-2}W_t + {}_{t-3}W_t + {}_{t-4}W_t\right)$$

or

$$\log(W_t) = \log(w^{\star}) + 0.25 \cdot \sum_{i=1} E_{t-i}\left[\log(P_t)\right]$$

where $w^{\star}$ denotes equilibrium real wage.[17] If we let output supply be a declining function of the real wage (from firms maximising profits subject to a production function with labour inputs and some fixed overheads) then one can derive the Phillips curve (generalised version of Fischer [1977]) which is expressed in natural logarithms as follows:

$$\log Y_t = \log Y^{\star} + q \left\{ \log P_t - \frac{1}{N} \sum_{i=1} E_{t-i}[\log(P_t)] \right\} + \lambda_0 (\log Y_{t-1} - \log Y^{*}) \qquad (7)$$

where $q \succ 0$, λ_0 denotes persistence, N is the contract length $Y^{\star}$ is potential output; we assume this to be growing at the steady rate `g', while the lagged term captures persistence due to the capital stock overshooting or undershooting the steady state growth path.

III Constructing the Taylor pseudo-rule

To construct a relationship that looks like a Taylor rule, or `pseudo-rule', we substitute for $\log P_t$ (from our Phillips curve) and substitute (5) for $E_t \log C_{t+1}$ in (3) the LM curve to get:

$$\begin{aligned} R_t = & -\frac{\gamma_0}{\gamma_2} - \frac{\gamma_1}{\gamma_2} \log C_{t+1} - \frac{1}{q\gamma_2}(\log Y_t - \log Y^{*}) \\ & - \frac{1}{\gamma_2} \{(\log P_t - \log P_{t-1}) - (\log M_t - \log P_{t-1})\} + \\ & \frac{1}{\gamma_2} \{\log P_t - \frac{1}{N} \sum_{i=1} E_{t-i}[\log(P_t)]\} + \frac{\lambda_0}{q\gamma_2} (\log Y_{t-1} - \log Y^{*}) \qquad (8) \end{aligned}$$

We can now consider how two different money supply reaction functions could be embedded in (8) to mimic a Taylor rule.[18]

Two examples: i) The Taylor pseudo-rule when a Friedman rule is operating

First, consider a Friedman money growth rule:[19]

$$\log M_t - \log M_{t-N} = \mu + \frac{\varepsilon_t}{(1-(1-\lambda_1)L)}; \qquad 0 < \lambda_1 < 1$$

where $\mu = \pi^\star + g_m$ (is the k in Friedman's k% rule) is assumed to be split between an inflation target, $\pi^\star$, and an allowance for growth in real money demand, g_m; λ_1 denotes speed of adjustment with which the rule comes back on track, and L is the lag operator. Substituting the Friedman money growth rule in (8) for $\log M_t$ yields:

$$R_t = \phi_1 + a_0(\log Y_t - \log Y^*) - b_0(\pi_t - \pi^\star) + u_{1t} \tag{8.1}$$

where

$$\phi_1 = -\frac{\gamma_0}{\gamma_2} + \frac{1}{\gamma_2} g_m,\ a_0 = -\frac{1}{q\gamma_2},\ b_0 = -\frac{1}{\gamma_2},$$

$$u_{1t} = -b_0 \left((\log M_{t-1} - \log P_{t-1}) + \frac{\varepsilon_t}{(1-(1-\lambda_1)L)} + \{\log P_t - \frac{1}{N}\sum_{i=1} E_{t-i}(\log(P_t))\} \right)$$

$$+ b_0\gamma_1 \log \Phi_{t+1} - a_0\lambda_0(\log Y_{t-1} - \log Y^*)$$

This gives us the pseudo-rule in a simple form. To create a more complex rule where the lagged interest rate enters we extract the term in lagged real money balance from the error term and replace it with its value from equation (3) to obtain:

$$R_t = \phi_{11} + R_{t-1} + a_0(\log Y_t - \log Y^*) - b_0(\pi_t - \pi^\star) + u_{2t}$$

where

$$\chi_{11} = \chi_1 - b_1\gamma_0,$$

$$u_{2t} = -b_0\left(\frac{\varepsilon_t}{(1-(1-\lambda_1)L)} + \left\{\log P_t - \frac{1}{N}\sum_{i=1} E_{t-i}(\log(P_t))\right\}\right)$$
$$+ b_0\gamma_1\left(\log \mathcal{E}_{t+1} - \log \mathcal{E}_t\right) - a_{0,\lambda 0}(\log Y_{t-1} - \log Y^*)$$

We can also write:

$$R_t = \chi_{11} + \rho R_{t-1} + a_0(\log Y_t - \log Y^*) - b_0(\pi_t - \pi^*) + u_{3t}$$

where

$$u_{3t} = -b_0\left(\frac{\varepsilon_t}{(1-(1-\lambda_1)L)} + \left\{\log P_t - \frac{1}{N}\sum_{i=1} E_{t-i}(\log(P_t))\right\}\right)$$
$$+ b_0\gamma_1\left(\log \mathcal{E}_{t+1} - \log \mathcal{E}_t\right) + (1-\rho)R_{t-1} - a_{0,\lambda 0}(\log Y_{t-1} - \log Y^*)$$

Thus we have a full set of dynamic pseudo-rules, with the error and constant term adjusting according to the dynamics.[20] Before proceeding, we briefly address several econometric issues. First, our analysis maintains the assumption that a monetary regime with an inflation target nominal interest rates will be stationary; hence the error term will also be stationary. Therefore this pseudo-rule will appear to have a stationary error term when it is estimated and thus will pass the usual time-series tests. In addition to its empirical plausibility we would point out that stationarity of both inflation and the nominal interest rate is also a property of many of the papers that rationalize the use of the kind of policy rule considered here. Second, in the Taylor pseudo-rule the "disturbance" term u_{1t} is not exogenous as it is a composite of many variables i.e., the orthogonality restrictions will be violated leading to a statistical rejection of the model. Instrumental variable estimation would however remedy this problem. Third, the pseudo-rule in complex form where the lagged interest rate term enters should help eliminate any serial correlation in the error term.

ii) The pseudo-rule when the money supply rule targets the exchange rate

Now let us consider a money supply rule which reacts to nominal exchange rate fluctuations. Exchange rate targeting forces a tightening of monetary policy when there is a tendency for the domestic currency to depreciate or a loosening of policy when there is a tendency for the domestic currency to appreciate. As Mishkin [1995] points out, in an open economy, the exchange rate channel is an essential part of the transmission mechanism for monetary policy; the exchange rate affects the target variables of monetary policy, inflation and output gap, in different subchannels, even though for simplicity we have not included them here. Suppose we specify a money supply rule of the form:

$$\log M_t - \log M_{t-N} = \mu + \tau E_t \Delta S_{t+1} + \frac{\varepsilon_t}{(1-(1-\lambda_1)L)}; \qquad \tau \succ 0$$

then substituting (6) above for $E_t \Delta S_{t+1}$ and the resulting expression in (8) for $\log M_t$ yields:

$$R_t = \phi_2 + a_1(\log Y_t - \log Y^{*}) - b_1(\pi_t - \pi^{\star}) + u_{4t} \qquad (8.2)$$

where

$$\phi_2 = \left(\frac{1}{\gamma_2 - \tau}\right)\left(-\gamma_0 + g_m - \tau\left(R_t^f + \iota\right)\right), \; a_1 = -\frac{1}{\sqrt{(\gamma_2 - \tau)}}, \; b_1 = -\frac{1}{\gamma_2 - \tau},$$

$$u_{4t} = -b_1\left((\log M_{t-1} - \log P_{t-1}) + \frac{\varepsilon_t}{(1-(1-\lambda_1)L)} + \{\log P_t - \frac{1}{N}\sum_{i=1} E_{t-i}(\log(P_t))\}\right)$$

$$+ b_1\gamma_1 \log \theta_{t+1} - a_1\lambda_0(\log Y_{t-1} - \log Y^*)$$

If we as above, remove lagged real money balances from the error term and substitute from equation (3), we obtain:

$$R_t = \phi_{22} + \left(\frac{\gamma_2}{\gamma_2 - \tau}\right)R_{t-1} + a_1(\log Y_t - \log Y^*) - b_1(\pi_t - \pi^\star) + u_{5t}$$

where

$$\phi_{22} = \phi_2 - b_1\gamma_0,$$

$$u_{5t} = -b_1\left(\frac{\varepsilon_t}{(1-(1-\lambda_1)L)} + \{\log P_t - \frac{1}{N}\sum_{i=1} E_{t-i}(\log(P_t))\}\right)$$

$$+ b_1\gamma_1\left(\log \theta_{t+1} - \log \theta_t\right) - a_1\lambda_0(\log Y_{t-1} - \log Y^*)$$

The coefficient on lagged interest rates is greater than unity. But again we can make it smaller or larger by adjusting the error term and constant term. Note that the `correlative relation' between the three endogenous variables, R_t, Y_t, and ΔP_t emerges from a macro model with a variety of monetary rules.[21] We could go on in this manner with other money supply rules. Essentially, as these two examples indicate, we could substitute the contents of a money supply rule for $\log M_t$ and then sweep the components that are not in the Taylor rule into the error term or the constant. What we have shown is that money supply rules give rise to interest rate behaviour that looks like a Taylor rule. Yet the behaviour of the economy will be different under these money supply rules from what it would be under the Taylor rule they resemble.We demonstrate this for two simple examples in the appendix.

In other words what we are seeing is that all these quite di®erent rules have a Taylor rule representation. Yet plainly a Taylor rule is operating only in the case where central bank behaviour is truly governed by such a rule in a structural sense. Thus the Taylor rule is not identi¯ed - it cannot be distinguished from other regimes masquerading as a Taylor rule. It follows that the empirical evidence from estimation of Taylor-style equations will not help discriminate between a Taylor rule and other sorts of monetary behaviour by the central bank.

IV Conclusion

In a variety of recent papers, researchers have found that interest rate behaviour approximately follows a Taylor rule. From this they have concluded that the central bank is following a Taylor rule as its monetary policy reaction function. However, we have shown in this paper that such interest rate behaviour results when the central bank may be following quite di®erent monetary policy rules from the one proposed by Taylor. They include a Friedman rule, an exchange rate rule for money supply, and a money rule with a feedback from the output gap. In other words an interest rate relation with output and in°ation does not identify a central bank reaction function. Other information about the model structure must be used for identi¯cation. In short, seeing a Taylor Rule in the data should not be believing. Taylor Rules may be there or may be not, and there may or may not be good theoretical grounds for believing in them, but the empirical work to date relating interest rates to output and in°ation provides no evidence for their existence.

This raises the question: is the Taylor Rule nevertheless a good way to represent the range of monetary policy behaviour whatever it may be? A number of papers discussing past monetary policy have proceeded as if this were the case; for example some have argued that the lax monetary control of in°ation by the US Fed and the UK government in the 1970s can be represented by a low coe±cient (i.e, well below unity) on in°ation in a Taylor Rule. However our appendix shows that there can be no exact equivalence between money supply rules and Taylor Rules; furthermore, the work reported in Bryant et al. [1993] shows that there are substantial stochastic di®erences in the behaviour of economies under such di®erent rules. It also shows that the welfare ranking of these rules is a di±cult matter, depending inter alia on the stochastic environment and the welfare measure used. One might add that the vulnerability of the rules to measurement error [eg Orphanides, 1998] on identifying potential output) and of course, as originally stressed by Taylor, money demand instability, remains to be properly assessed.

Appendix

This Appendix illustrates that economic behaviour under a Taylor rule is different from a feedback rule for money supply, although interest rate behaviour across these two rules are indistinguishable. To illustrate this we consider a closed-economy model (for convenience) which tries to preserve our original framework in the text. Following Taylor [2000] the modified model to be used in the present section can be expressed as (in natural logarithms)

$$y_t = \gamma E_t y_{t+1} - \alpha r_t + \varepsilon_t; \qquad \gamma;\ \alpha \succ 0 \qquad \text{(A1)}$$

$$y_t = \delta\left(p_t - 0.5\sum_{i=1}^{2} E_{t-i}p_t\right) + \lambda y_{t-1}; \qquad \delta;\ \lambda \succ 0$$

$$\equiv \delta\left(\pi_t - 0.5\{E_{t-1}\pi_t + E_{t-2}\pi_t\}\right) + \lambda y_{t-1} \qquad \text{(A2)}$$

$$r_t = b_1\pi_t + b_2 y_t; \qquad b_1;\ b_2 \succ 0 \qquad \text{(A3)}$$

where ε_t is a stochastic disturbance, y_t is output, and π_t is the inflation rate. Here (A1) is an expectational IS curve, (A3) is a Taylor rule and (A2) is a Phillips curve. Substituting (A3) in (A1) for r_t and substituting the resulting expression for y_t in (A2) yields (where the operator B is defined by $B^{-1}(Ex_{t+j} \mid \Omega_t) = Ex_{t+j+1} \mid \Omega_t$, where Ω_t is the information set at time ‘t’, in other words B instructs to lag the variable while leaving the date of expectations unchanged):

$$\left(1 - \left(\frac{\gamma}{1+\alpha b_2}\right) B^{-1}\right) \{\delta(\pi_t - 0.5\{E_{t-1}\pi_t + E_{t-2}\pi_t)\}$$

$$= (1-\lambda L)\left(-\frac{\alpha b_1}{1+\alpha b_2}\pi_t + \frac{1}{1+\alpha b_2}\varepsilon_t\right) \qquad \text{(A4)}$$

Note that in a stochastic linear economic system, such as the model we are currently working with, the solution for the endogenous variable can be written as an infinite moving-average process in a random error. Thus using the method of "undetermined coefficients" we can express the solution for π_t as:

$$\pi_t = \bar{\pi} + \sum_{i=0}^{\infty} s_i \varepsilon_{t-i} \qquad \text{(A5)}$$

where $\bar{\pi}$ = the mean of the series, s_i = constant parameters, and ε = a normally distributed error with a mean of 0, constant variance σ_ε^2 and zero covariance. Thus substituting (A5) in (A4) yields (where $E_t\varepsilon_{t+i} = 0;\ i \geq 1$):

$$\delta\left(s_0\left[1-\frac{\gamma\lambda}{1+\alpha b_2}\right]\varepsilon_t + 0.5 s_1\varepsilon_{t-1}\right) - \left(\frac{\gamma\delta}{1+\alpha b_2}\right)\{0.5 s_1\varepsilon_t\}$$
$$= \left(-\frac{\alpha b_1}{1+\alpha b_2}\right)\sum_{i=0}^{\infty} s_i\varepsilon_{t-i} + \left(\frac{\lambda\alpha b_1}{1+\alpha b_2}\right)\sum_{i=1}^{\infty} s_{i-1}\varepsilon_{t-i} + \left(\frac{1}{1+\alpha b_2}\right)(\varepsilon_t - \lambda\varepsilon_{t-1})$$

We need to now evaluate s_0 and s_1 i.e., the undetermined coefficients. For this we collect terms in ε_t, ε_{t-1}, ε_{t-2} etc. Ignoring the constants results in:

$$\varepsilon_t: \quad \delta s_0\left[1-\frac{\gamma\lambda}{1+\alpha b_2}\right] - \left(\frac{\gamma\delta(0.5)}{1+\alpha b_2}\right)s_1 = \left(-\frac{\alpha b_1}{1+\alpha b_2}\right)s_0 + \left(\frac{1}{1+\alpha b_2}\right)$$

$$\varepsilon_{t-1}: \quad 0.5\,\delta s_1 = \left(-\frac{\alpha b_1}{1+\alpha b_2}\right)s_1 + \left(\frac{\lambda\alpha b_1}{1+\alpha b_2}\right)s_0 - \left(\frac{\lambda}{1+\alpha b_2}\right)$$

$$\varepsilon_{t-i}\ (\text{for } i \geq 2): \quad 0 = \left(-\frac{\alpha b_1}{1+\alpha b_2}\right)s_i + \left(\frac{\lambda\alpha b_1}{1+\alpha b_2}\right)s_{i-1}$$

$$\Rightarrow \quad s_i = \lambda s_{i-1}$$

The solution for s_0 and s_1 can be substituted in (A5) and (A2) in order to get a solution for π_t and y_t. Denoting the above relationships in matrix form ($Ax = z$)

yields:

$$\begin{bmatrix} (1+\alpha b_2-\gamma\lambda)\delta+\alpha b_1 & -0.5\gamma\delta \\ -\lambda\alpha b_1 & (1+\alpha b_2)0.5\delta+\alpha b_1 \end{bmatrix}\begin{bmatrix} s_0 \\ s_1 \end{bmatrix}=\begin{bmatrix} 1 \\ -\lambda \end{bmatrix}$$

Now let us consider a model with a constant rate of growth of the money supply (where m_t denotes logarithm of money supply and for simplicity we set $\bar{m}$, the growth rate to zero):

$$y_t = \gamma E_t y_{t+1} - \alpha r_t + \varepsilon_t \qquad \text{(B1)}$$

$$y_t = \delta\left(p_t - 0.5\sum_{i=1} E_{t-i}p_t\right) + \lambda y_{t-1} \qquad \text{(B2)}$$

$$m_t = p_t - \beta_1 R_t + \beta_2 E_t y_{t+1}; \qquad \beta_1, \beta_2 \geq 0 \qquad \text{(B3)}$$

$$m_t = \bar{m} \qquad \text{(B4)}$$

$$R_t = r_t + E_t p_{t+1} - p_t \qquad \text{(B5)}$$

where (B3) is the money demand function, (B4) is the Friedman rule, and (B5) is the Fisher equation. By straightforward substitution we get (using the solution for p_t (the price level) which can be expressed as $p_t = \bar{p} + \sum_{i=0}^{\infty} q_i \varepsilon_{t-i}$)

$$\delta(q_0[1-\gamma\lambda]\varepsilon_t + 0.5q_1\varepsilon_{t-1}) - \left\{0.5\delta q_1\left(\gamma - \frac{\alpha\beta_2}{\beta_1}\right) - \frac{\alpha\beta_2\delta\lambda q_0}{\beta_1}\right\}\varepsilon_t$$

$$= -\alpha\left(1+\frac{1}{\beta_1}\right)\sum_{i=0}^{\infty} q_i\varepsilon_{t-i} + \lambda\alpha\left(1+\frac{1}{\beta_1}\right)\sum_{i=1}^{\infty} q_{i-1}\varepsilon_{t-i} +$$

$$\alpha\sum_{i=0}^{\infty} q_{i+1}\varepsilon_{t-i} - \alpha\lambda\sum_{i=1}^{\infty} q_i\varepsilon_{t-i} + \varepsilon_t - \lambda\varepsilon_{t-1}$$

We need to now evaluate q_0 and q_1 i.e., the undetermined coefficients. For this we collect terms in ε_t, ε_{t-1}, ε_{t-2} etc. Ignoring the constants results in:

$$\varepsilon_t : \delta q_0[1 - \gamma\lambda] - \delta 0.5\left(\gamma - \frac{\alpha\beta_2}{\beta_1}\right) q_1 + \frac{\alpha\beta_2\delta\lambda}{\beta_1} q_0 = -\alpha\left(1 + \frac{1}{\beta_1}\right) q_0 + \alpha q_1 + 1$$

$$\varepsilon_{t-1} : 0.5\delta q_1 = -\alpha\left(1 + \frac{1}{\beta_1}\right) q_1 + \lambda\alpha\left(1 + \frac{1}{\beta_1}\right) q_0 + \alpha(q_2 - \lambda q_1) - \lambda$$

$$\varepsilon_{t-i} \text{ (for } i \geq 2) : 0 = \left\{-\alpha\left(1 + \frac{1}{\beta_1}\right) - \alpha\lambda\right\} q_i + \lambda\alpha\left(1 + \frac{1}{\beta_1}\right) q_{i-1} + \alpha q_{i+1}$$

$$\Rightarrow 0 = q_{i+1}(1 - \lambda L)\left(1 - \left(1 + \frac{1}{\beta_1}\right) L\right)$$

where L is the lag operator and $\left(1 + \frac{1}{\beta_1}\right)$ is an unstable root. Imposing the transversality condition that the q_i process is stable yields:

$$\Rightarrow q_i = \lambda q_{i-1}$$

Denoting the above relationships in matrix form ($Ax = z$) yields:

$$\begin{bmatrix} \delta\left(1 - \gamma\lambda + \frac{\alpha\beta_2\lambda}{\beta_1}\right) + \alpha\left(1 + \frac{1}{\beta_1}\right) & -\left(\alpha + 0.5\delta\left\{\gamma - \frac{\alpha\beta_2}{\beta_1}\right\}\right) \\ -\frac{\alpha\lambda(1+\beta_1)}{\beta_1} & 0.5\delta + \frac{\alpha(1+\beta_1)}{\beta_1} \end{bmatrix} \begin{bmatrix} q_0 \\ q_1 \end{bmatrix} = \begin{bmatrix} 1 \\ -\lambda \end{bmatrix}$$

Clearly the form of the solution with a Taylor rule is the same as the Friedman rule. But it is impossible for $q_i = s_i$, as can be seen by comparing the expressions for s_0, s_1 with those for q_0, q_1.

For $\begin{bmatrix} s_0 \\ s_1 \end{bmatrix} = \begin{bmatrix} q_0 \\ q_1 \end{bmatrix}$ we require $A^{-1}_{Taylor} = A^{-1}_{Money}$.

Suppose we denote

$a_{11} : (1 - \gamma\lambda + \alpha b_2)\delta + \alpha b_1 = \delta\left(1 - \gamma\lambda + \frac{\alpha\beta_2\lambda}{\beta_1}\right) + \alpha\left(1 + \frac{1}{\beta_1}\right)$

$a_{12} : -0.5\gamma\delta = -\left(\alpha + 0.5\delta\left\{\gamma - \frac{\alpha\beta_2}{\beta_1}\right\}\right)$

$a_{21} : -\alpha\lambda b_1 = -\frac{\alpha\lambda(1+\beta_1)}{\beta_1}$

$a_{22} : (1 + \alpha b_2)\, 0.5\delta + \alpha b_1 = 0.5\delta + \frac{\alpha(1+\beta_1)}{\beta_1}$

Note that a_{21} and a_{22} imply $b_1 = \frac{1+\beta_1}{\beta_1}$ and $b_2 = 0$. But if so we have from a_{11};

$0 = \frac{\delta\alpha\beta_2\lambda}{\beta_1}$ and from a_{12} $0 = -\alpha\left(1 - \frac{0.5\delta\beta_2}{\beta_1}\right)$; these are restrictions on the structural coefficients which there is no reason in general to be satisfied. Hence the solutions are in general different under the Taylor rule and the money rule even though a money rule generates a `semi-reduced form' for R_t which resembles a Taylor rule.

References

Barro, R.J., and R.G. King. \Time-Separable Preferences and Intertemporal-Substitution Models of Business Cycles." The Quarterly Journal of Economics LXXXIV (1984), 817-840.

Bryant, R.C., P. Hopper, and C.L. Mann \Evaluating Policy Regimes- New Research in Empirical Macroeconomics", Brookings Institution, Washington D.C., 1993.

Clarida, R., J. Gali, and M. Gertler. \Monetary policy rules in practice: Some international evidence." European Economic Review 42 (1998), 1033-1067.

Clarida, R., J. Gali, and M. Gertler. \The Science of Monetary Policy: A New Keynesian Perspective." Journal of Economic Literature 37(4) (1999), 1661-1707.

Clarida, R., J. Gali, and M. Gertler. \Monetary Policy Rules and Macroeconomic Stability: Evidence and some theory." The Quarterly Journal of Economics CXV (2000), 147-180.

Fischer, S. \Long term contracts, rational expectations and the optimum money supply rule." Journal of Political Economy 85 (1977), 191-205.

Henderson, D. W. and W.J. McKibbin \An Assessment of Some Basic Monetary-Policy Regime Pairs: Analytical and Simulation Results from Simple Multiregion Macroeconomic Models", in Bryant et al. \Evaluating Policy Regimes- New Research in Empirical Macroeconomics", Brookings Institution, Washington D.C., 1993.

McCallum, B.T., and M.S. Goodfriend. \Demand for Money: Theoretical Studies." In The New Palgrave: A Dictionary of Economics, edited by J. Eatwell, M. Milgate, and P. Newman, 775-781. New York: Stockton Press, 1987.

McCallum, B.T., and E. Nelson. \Nominal Income Targeting in an open-economy optimizing model." NBER Working Paper 6675 August (1998).

McCallum, B.T., and E. Nelson. \An Optimizing IS-LM Speci¯cation for Monetary Policy and Business Cycle Analysis." Journal of Money Credit and Banking 31 (1999), 296-316.

Minford, A.P.L. Rational Expectations Macroeconomics: An Introductory Handbook, 2nd ed. Basil Blackwell, Oxford, 1992.

Minford, A.P.L., E. Nowell, and B.D. Webb. \Nominal contracting and monetary targets." CEPR Discussion Paper 2215 (1999).

Mishkin, F.S. \Symposium on the Monetary Transmission Mechanism." Journal of Economic Perspectives 9(4) (1995), 3-10.

Orphanides, A. \Monetary Policy Rules Based on Real-Time Data." Finance and Economics Discussion Paper no. 98-03. Washington D.C.: Board of Governors of the Federal Reserve System, 1998.

Phelps, E.S. and J.B. Taylor. \The stabilizing powers of monetary policy under rational expectations." Journal of Political Economy 85 (1977), 163-190.

Sargent, T.J. \The Observational Equivalence of Natural and Unnatural Rate Theories of Macroeconomics." Journal of Political Economy 84 (1976), 631-640.

Srinivasan, N., L. Lungu, and P. Minford. \Is the Taylor Rule the same as the Friedman Rule.?" manuscript, Cardi® Business School, 2000.

Taylor, J.B. \Discretion versus Policy Rules in Practice." Carnegie-Rochester Series on Public Policy 39 (1993), 195-214.

Taylor, J.B. \An Historical Analysis of Monetary Policy Rules", in Monetary Policy Rules, edited by John B. Taylor (Chicago: University of Chicago Press for NBER), 1999.

Taylor, J.B. \Teaching Modern Macroeconomics at the Principles Level." American Economic Review 90(2) (2000), 90-94.

Notes

1. For example, the St. Louis Federal Reserve Bank now publishes an interest rate rule that John Taylor proposed.

2. In the remainder of this paper, this class of similar rules will be referred to as Taylor-type rules to distinguish them from the original Taylor rule.

3. In this spirit, Clarida et al. [1998, 2000] estimate a forward looking rule for Bundesbank, Bank of Japan, and the Federal Reserve.

4. Sargent (1976) in an important paper demonstrated that the reduced form for output in a Keynesian model, in which systematic monetary policy will influence the variance of output, may be statistically indistinguishable from that of a classical model in which only unanticipated movements in the money stock impact on output.

5. The following analysis is based on McCallum and Goodfriend [1987] in which $\frac{M_t}{P_t}$ is interpreted as end-of-period real money balances. In addition we would point out that in several recent papers, it is assumed that end-of-period money balances are relevant for facilitating transactions. See, for example McCallum and Nelson [1999].

6. In order to rule out a strategy of infinite consumption supported by unbounded borrowing, we impose a restriction that, for $t \geq 0$

$C_t + \sum_{j=1}^{\infty} \left(\prod_{k=0}^{j-1} R_{t+k}^{-1} \right) C_{t+j} = Y_t + \sum_{j=1}^{\infty} \left(\prod_{k=0}^{j-1} R_{t+k}^{-1} \right) Y_{t+j} + A_t$, where R_t is the (gross) interest rate, A_t denotes financial wealth (bonds and shares in our set up) and Y_t denotes labour (and dividend) income. The above equation is a standard budget constraint for an infinitely-lived household in discrete time. It is motivated as an additional necessary restriction, in addition to the static household budget constraint in equation (2), that obviates Ponzi strategies.

7. As Barro and King [1984] point out, preference ordering (time-separable) of this form would not restrict the sizes of intertemporal substitution effects. We do not claim that separability is theoretically an appropriate assumption. However, we believe that for the present purpose such an approximation will be satisfactory. Such approximations are certainly quite common in the literature.

8. The first-order conditions w.r.t real money balances and (domestic) consumption can be expressed as:

$$\left(\frac{M_t}{P_t}\right)^{-\eta} = \frac{\theta\beta}{\omega} E_t \left(C_{t+1}^{(d)-\sigma_0}\right)(1 + R_t)$$

$$C_t^{(d)-\sigma_0} = \beta E_t \left(C_{t+1}^{(d)-\sigma_0}\right)\left(1 + r_t^d\right)$$

9. Equations (3) and (4) involve certain standard first-order approximations, arising from passing nonlinear functions through the linear expectations operator.

10. Where we use a common approximation ($\log(1 + x) \simeq x$ for `x' small relative to 1.0).

11. In this general equilibrium framework we introduce a government that spends current output according to a non negative stochastic process (G_t) that satisfies $G_t \leq Y_t$ for all t. The government budget constraint is $G_t + \frac{M_{t-1}}{P_t} + b_t^d = \frac{M_t}{P_t}\left(1 + \pi_{t+1}^e\right) + \frac{b_{t+1}^d}{1+r_t^d} + T_t$:

The variable G_t denotes government expenditure at time t. The government finances its expenditure by a stream of lump-sum taxes (T_t) and seigniorage revenue. The government also issues debt, bonds $\left(b_t^d\right)$ each of which pays a return next period given the state of the economy at t+1.

12. We thus have a relation expressing real money balances as a function of (future) consumption spending and the nominal rate of interest. Similarly the expression for consumption (5) is in line with developments in contemporary macroeconomic research which suggests the dependence of current consumption on expected future consumption and the short-term real rates.

13. We assume that f(N, K) is smooth and concave and it satisfies Inada-type conditions.

14. The firm optimally chooses capital and labour so that their marginal products are equal to the price per unit of input; that is,

$$r_t = Z_t f_K\left(N_t;\ \overline{K}\right) \text{ and } w_t = Z_t f_N\left(N_t;\ \overline{K}\right)$$

Capital stock is assumed to be fixed because we are interested in the derivation of short run aggregate supply. If we assume that $f(\cdot) = N_t^{\alpha}\overline{K}^{1-\alpha}$ i.e., a Cobb-Douglas function, then we can express the demands for the two factors as a function of the optimal output choice i.e.,

$$N_t = \frac{\alpha Y_t}{w_t} \text{ and } \overline{K} = \frac{(1-\alpha)Y_t}{r_t}$$

In order to solve for the optimal choice of output we substitute N_t into the Cobb-Douglas production function which essentially yields the supply function of the firm, where output supply is a declining function of the real wage.

15. For a lucid exposition of this topic see Minford (1992).

16. Note that at any given point in time three-fourth's of the labour force is covered by a pre-existing contract. The assumption of rational expectations here entails that the forecast of the next period wage decisions is an unbiased one, given that agents possess the necessary information set.

17. Note that;

$_{t-i}W_t = w^{\star} + E_{t-i}P_t$ where $i = 1; 4$: Ideally the equilibrium real wage is a time-varying constant i.e., it would move with taste and technology shocks. Treating $w^{\star}$ as a constant here does not in anyway a®ect our ¯nal conclusion.

18. If in principle separate estimates of each of these structural parameters can be disentangled from the corresponding economic data, we say that the model is statistically identi¯ed. When this is not possible, a given data sample is consistent with an in¯nity of di®erent structural models which are said to be observationally equivalent, since it is impossible to distinguish between them.

19. Srinivasan et al. [2000] show that the Taylor rule is an implication of Friedman rule and a representative agent model with overlapping wage contracts.

20. Note that if we replace our Fischer style Phillips curve by the widely-used Calvo-Rotemberg Phillips curve, none of our conclusion are a®ected. We would have expected future in°ation instead of expected current in°ation appearing in the error term.

21. As Taylor [1999] points out, a function relating the interest rate to the price level and real output will emerge under a variety of monetary regimes, however, the magnitude of the response coe±cients (a_i and b_i in our case) will di®er depending on how monetary policy is run. The size of these coe±cients makes a big di®erence for the e®ect of policy.