Strategic vertical integration without foreclosure

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January 8, 2003

Abstract : We determine the endogenous degree of vertical integration in a model of successive oligopoly that captures both efficiency gains and strategic effects. We show that vertical merger waves can be expected to stop by themselves before integration is complete. Consequently, vertical foreclosure plays no significant role in this paper that claims for a soft approach of vertical integration by antitrust authorities.

JEL Classification numbers : L22, L40.

Key Words : Merger waves, vertical integration, vertical foreclosure.

1 Introduction

It is well known that there are three main motivations to vertically integrate for an upstream firm and a downstream firm. The first motivation documented in the economic literature is the minimization of transaction costs by the choice of an optimal governance structure for contractual relationships. In this strand of literature, vertical integration is viewed as one possible governance structure. The second motivation is described by the property rights literature that argues that the optimal allocation of property rights on assets allows firms to reduce to the minimum the problem of the ex ante nonoptimality of investment levels. Vertical integration is just one possible allocation of property rights and, in fact, one has to distinguish between different types of vertical integration, depending

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on who owns the assets (the downstream or the upstream firm). There are substantial differences between these two ways to envision vertical integration (see Whinston (2001) for more details), but both claim that vertical integration may allow firms to achieve efficiency gains that nonintegrated firms could not achieve. The third motivation for vertical integration is quite different. It relies on the analysis of vertical integration in a strategic context and claims that vertical integration may be profitable because it allows firms to reduce their costs through the elimination of double marginalization (this is in fact a story about efficiency gains resulting from vertical integration) and thus to be in a better position when competing with their rivals on the final market. At this point, the story goes on with the "vertical foreclosure" argument. Indeed, it is claimed that vertical integration is also a *rising rivals' costs strategy* because the integrated firm leaves the intermediate market, nonintegrated upstream firms enjoy more market power and rise the intermediate price that they charge to independent downstream firms. So, according to this foreclosure story, vertical integration is profitable both because it reduces the double margin for the merging firms and because it increases the double margin for those who remain independent. There is a quite hot debate on foreclosure since the Chicago school economists criticized the first, unformal version of the foreclosure theory. The seminal paper by Ordover, Saloner and Salop (1990) layed the foundations for a new foreclosure theory based on more convincing game theoretic arguments. More recently, Avenel (2000), Choi and Yi (2000) and Church and Gandal (2000), relying on the idea that firms can make strategic technological choices that commit them to foreclose their rivals, have proved that vertical foreclosure can emerge in models in which vertical integration is endogenous, technological choices are endogenous and there is no assumption on the ability of integrated firms to commit to a price.

Although the debate on vertical foreclosure is extremely interesting, it is quite unfortunate that the analysis of vertical integration in a strategic context has focused exclusively on this issue. Indeed, the previously quoted models assume that the upstream industry is a duopoly, so that under partial vertical integration, the independent upstream firm enjoys monopoly power on the residual demand and the rising rivals' costs is quite strong. But what about industries with less concentrated upstream markets ? Clearly, if competition is quite strong between upstream firms, the vertical merger of one upstream firm with a downstream firm will not allow its upstream competitors to substantially rise their price and the rising rivals' costs effect will be very small. It is in fact absent as soon as there are three (identical) upstream firms, constant returns to scale and Bertrand competition. This is precisely the framework that we use in this paper. This prediction that vertical foreclosure is not an issue as long as there is a substantial degree of competition on the residual intermediate market may well explain why it is so difficult to find empirical evidence of foreclosure (Rosengren and Meehan (1994), for example, find no empirical support for the foreclosure theory). Indeed, duopoly competition is not the general rule on intermediate markets. We think that it is time to develop a theory of endogenous vertical integration in strategic contexts that (i) takes into account the efficiency gains resulting from vertical integration as described in the transaction costs and property rights literatures and (ii) takes into account the possibility of vertical foreclosure without putting an exagerate emphasis on it. The model that we describe below is an attempt to contribute to such a theory of vertical integration.

This model is closely related to McLaren (2000), since we also determine the degree of vertical integration resulting from simultaneous integration choices in a industry composed of the same number of upstream and downstream firms. There is however a difference between the two models that happens to be critical. In McLaren (2000), there is no competition between downstream firms on the final market or, equivalently, the efficiency gains associated to vertical integration take the form of lower fixed costs for downstream firms. They thus have no impact on prices and outputs on the final market. In my model, the efficiency gains take the form of lower marginal costs for upstream firms, this reduction leading to lower marginal costs for downstream firms. Prices and outputs then depend on the vertical structure of the industry. This effect is strong enough to reverse the main result of McLaren (2000). There is no longer strategic complementarity in vertical integration, but rather strategic substituability, with profoundly different implications in terms of how antitrust policy should deal with vertical integration.

The structure of the article is as follows. In the next section, we present the model and determine equilibrium prices and outputs in the various possible industrial structures. In section 3, we examine how the technological choice of firms is related to their decision regarding vertical integration. In section 4, we establish our main result on the structural and technological features of the industry in equilibrium. Section 5 is devoted to the analysis of the implications for antitrust policy. In section 6, we discuss welfare. Section 7 concludes.

2 The model

In this section, we describe the model, beginning with the technologies available to firms.

2.1 Technologies

We assume that there exists a generic technology that any firm in the industry (upstream, downstream or integrated) can adopt. Furthermore, all the firms using this technology are equally efficient, with constant marginal costs of production taken equal to $c > 0$ for upstream firms and normalized to 0 for downstream firms. Alternatively, each pair of firms (either an integrated firm or a pair of independent upstream and downstream firms) can adopt a specific technology that is only available to this pair of firms and that is more efficient than the generic technology. More precisely, the upstream marginal cost is equal to $c-\varepsilon > 0$ and the downstream cost is equal to 0. The specificity of the technology implies that a specific intermediate good is less efficient when used with another technology. We denote by δ the cost, assumed to be constant, of adapting one unit of specific input to the generic technology or another specific technology. This is also the cost of adapting a unit of generic intermediate good to a specific technology. It may be convenient to think of the adoption of a specific technology as the decision to build two new production units located at the same place, wide away from other production units. Transportation costs are reduced between the two plants (this is ε), but increased between each of the two plants and any plant located elsewhere (this is δ). We assume that the adoption of a specific technology requires investment in specific assets both upstream and downstream. As a consequence, a pair of independent firms can adopt the specific technology available to it only if both firms agree on this decision. Given the incomplete nature of contracts in this model, vertical integration may be profitable to a pair of firms because it allows them to adopt the more efficient, specific technology. Given these premisses, we now can describe the model.

2.2 Set-up

We consider an industry composed of $n \geq 2$ downstream firms $(D_i)_{i=1,...,n}$ and the same number n of upstream firms $(U_i)_{i=1,...,n}$ that supply them with an intermediate good that they transform into a final good on a one for one basis. We assume that final goods are horizontally differentiated and, more specifically, that the demand for good i is given by

$$q_i = \alpha - \beta p_i + \gamma \sum_{j \neq i} p_j \tag{1}$$

In (1), p_i is the price charged by D_i, p_j the price charged by D_j and q_i the quantity sold by D_i for the vector of prices. We denote by P the vector of prices on the final market.

For this demand function to make sense, we must assume that condition (2) below holds. If it would not, a general increase in prices would increase the output of each firm and the total output.

$$\gamma < \beta / (n-1) \tag{2}$$

As regards competition on the intermediate market, we assume that upstream firms are engaged in price competition and denote by $W = (w_i)_{i=1,...,n}$ the vector of prices charged by upstream firms. We further assume that the upstream firms using the generic technology produce an homogeneous intermediate good, whereas each usptream firm using a specific technology produces an intermediate good different from every other variety of the intermediate good. It is possible to transform intermediate goods to adapt them to a technology they were not designed for at a cost δ. We assume, without loss of generality, that this cost is supported by upstream firms.

The model is built on a three stage game. In the first stage, each pair $U_i - D_i$ has to decide on two points, integration and technology. At the end of stage 1, there are thus four types of pairs of firms. We denote by n_{SG} the number of pairs of firms that are not integrated (i.e., separated) and use the generic technology, n_{IG} the number of integrated firms using the generic technology, n_{SS} the number of non-integrated pairs of firms using a specific technology and n_{IS} the number of integrated firms using a specific technology.

In the second stage, upstream firms simultaneously make offers on the intermediate market, which determines W. In the third stage, downstream firms put prices, observe the demand, purchase the needed quantity of intermediate good and transform it into the final good. We solve the game backward, thus determining subgame perfect equilibria.

Because of Bertrand competition on the intermediate market, no downstream firm will pay more than $c + \delta$ for its input, since this is the highest possible cost for an upstream firm to supply the downstream firm. Since we don't want to consider situations where downstream firms are driven out of the market, we assume that any firm can make positive profit on the final market with a cost equal to $c + \delta$, which is the case under condition (3).

$$\alpha - \beta (c + \delta) > 0 \tag{3}$$

The profit of downstream firm i is given by

$$\Pi_{D_i} (w_i, P) = (p_i - w_i) \left(\alpha - \beta p_i + \gamma \sum_{j \neq i} p_j \right) \tag{4}$$

This is a strictly concave function with a positive maximal value determined by the first-order condition below.

$$\frac{\partial}{\partial p_i} \Pi_{D_i} (w_i, P) = 0 \Longleftrightarrow p_i = \frac{1}{2\beta} \left(\alpha + \beta w_i + \gamma \sum_{j \neq i} p_j \right) \tag{5}$$

Solving the n first-order conditions determines $P^* (W)$, the vector of equilibrium downstream prices conditional on W. The next step in the resolution is thus to determine W^*, the vector of input prices in equilibrium (conditional on the structure of the industry determined in stage 1).

The internal transfer price within integrated firms is just the marginal cost of producing the intermediate good. We thus have n_{IG} firms with a marginal cost equal to c and n_{IS} firms with a marginal cost equal to $c-\varepsilon$ on the final market. Integrated firms have no interest in purchasing input on the market. To the contrary, they have an interest in selling input on the market and competing with non-integrated upstream firms. As a result, if there are at least two upstream firms (integrated or not) that use the generic technology, they charge a price equal to the marginal cost c. This is the usual Bertrand result. Upstream specific firms cannot match such a price. In this case, thus, the n_{SG} generic non-integrated downstream firms have a cost equal to c. If there is only one upstream firm using the generic technology, it can rise the price above c, because its competitors will not propose less than $c+\delta$. It may not be optimal for the upstream firm to rise the price up to $c+\delta$, but we assume that δ is small enough for the upstream firm to charge $c+\delta$ to the (unique) downstream firm using the generic technology. Let's finally, consider the price charged by specific upstream firms. Because the specific technologies are different, each specific upstream firm can rise the price it charges to its natural client up to $c+\delta$, the price at which other upstream firms are ready to supply the downstream firm. We assume that ε and δ are small enough for the upstream firm to reach this upper bound and charge $c+\delta$ for its variety of the intermediate good. Table 1 summarizes these results.

	$n_{IG} \geq 1$ or $n_{SG} \geq 2$	$(n_{IG}; n_{SG}) = (0; 1)$	$(n_{IG}; n_{SG}) = (0; 0)$
w_{IG}	c	$-$	$-$
w_{IS}	$c-\varepsilon$	$c-\varepsilon$	$c-\varepsilon$
w_{SG}	c	$c+\delta$	$-$
w_{SS}	$c+\delta$	$c+\delta$	$c+\delta$

In table 1, w_{SG} is the price paid by a non-integrated downstream generic firm for its input. w_{IS}, w_{IG} and w_{SS} are defined in a similar way, so that

$$W = \left(\underbrace{w_{IG}, ..., w_{IG}}_{n_{IG}}, \underbrace{w_{IS}, ..., w_{IS}}_{n_{IS}}, \underbrace{w_{SG}, ..., w_{SG}}_{n_{SG}}, \underbrace{w_{SS}, ..., w_{SS}}_{n_{SS}} \right) \tag{6}$$

The ordering of downstream firms implicit in (6) is evident and will be used in what follows.

An important implication of the results summarized in table 1 is that vertical integration doesn't rise the costs of non-integrated rivals in this model, unless the merger leaves only one non-integrated generic downstream firm and no integrated firm uses the generic technology ($n_{IG} = 0$ and $n_{SG} = 1$). This is precisely the case in the successive duopoly models discussed in the introduction. In this sense, this model is a generalization of these previous models

Replacing w_i by its expression in (5) and solving the system of first-order conditions leads to the expression of equilibrium downstream prices (conditional on the industrial structure resulting from stage 1). Due to the rising rivals' costs effect discussed above, the expression of prices is particular when $n_{IG} = 0$ and $n_{SG} = 1$. We thus obtain two sets of expressions for prices.

As soon as $n_{IG} \neq 0$ or $n_{SG} \neq 1$, the prices are given by equations (7) to (10).

$$p_{SG} = \frac{1}{2\beta + \gamma - \gamma n} \left(\alpha + \beta c + \frac{\gamma\beta}{2\beta + \gamma} \left(n_{SS}\delta - n_{IS}\varepsilon \right) \right) \tag{7}$$

$$p_{SS} = \frac{1}{2\beta + \gamma - \gamma n} \left(\alpha + \beta c + \beta\delta + \frac{\gamma\beta}{2\beta + \gamma} \left(\left(n_{SS} - n \right) \delta - n_{IS}\varepsilon \right) \right) \tag{8}$$

$$p_{IG} = p_{SG} \tag{9}$$

$$p_{IS} = \frac{1}{2\beta + \gamma - \gamma n} \left(\alpha + \beta c - \beta\varepsilon + \frac{\gamma\beta}{2\beta + \gamma} \left(n_{SS}\delta + \left(n - n_{IS} \right) \varepsilon \right) \right) \tag{10}$$

Each of the previous expressions has three terms : a constant reflecting the level of demand, a term determined by the level of the firm's cost that corresponds to the direct impact of costs on price and a term determined by the difference between the firm's cost and its rivals' costs that corresponds to the strategic effect of costs on prices.

For $n_{IG} = 0$ and $n_{SG} = 1$, the expressions of prices are equations (11) to (13).

$$\tilde{p}_{SG} = \frac{1}{2\beta + \gamma - \gamma n} \left(\alpha + \beta c + \beta \delta - \frac{\gamma \beta}{2\beta + \gamma} n_{IS} \left(\varepsilon + \delta \right) \right) \tag{11}$$

$$\tilde{p}_{SS} = p_{SG} \tag{12}$$

$$\tilde{p}_{IS} = \frac{1}{2\beta + \gamma - \gamma n} \left(\alpha + \beta c - \beta \varepsilon + \frac{\gamma \beta}{2\beta + \gamma} \left(n - n_{IS} \right) \left(\varepsilon + \delta \right) \right) \tag{13}$$

Again, we find the constant, the direct effect and the strategic effect described above.

From the vectors of prices W and P, we can deduce firms' outputs and profits, both upstream and downstream. It is not necessary to give these expressions to determine the industrial structure resulting from stage 1, which is the next (and final) step of the resolution of our model.

3 The link between integration and technology

The essence of the efficiency defense is to claim that vertical integration allows firms to achieve efficiency gains. In this model, this is justified only if non-integrated firms cannot adopt a specific technology due to the incomplete nature of contracts. Contracts are indeed incomplete, since it just specifies a price and the possibility for the downstream firm to get any quantity of input for that price. However, it is not necessary that this indeed prevents non-integrated firms from adopting a specific technology. In the first part of this section, we examine this point and prove that, in this model, non-integrated firms adopt a generic technology (in equilibrium) because the adoption of a specific technology reduces the profit of the downstream firm that will thus refuse it. In the second part of the section, we examine the related question of whether integrated firms adopt a specific technology and we show that, except for one possible equilibrium, it is the case, so that there is coincidence between integration and the use of a specific technology.

3.1 The technological choice of non-integrated firms

We show that, regardless of the type of the other pairs of firms in the industry, a non-integrated downstream firm using a specific technology would be better off if it used the generic technology. For this reason, it will refuse to adopt a specific technology and, since we assume that the adoption of a specific technology is possible only if the upstream and the downstream firms agree on it, we conclude that non-integrated firms adopt the generic technology.

We proceed by first showing that the variation in gross profit associated to this deviation is positive and then introducing the fixed costs associated to the different technologies to assert the net profitability of the deviation.

3.1.1 Gross incentives to deviate from SS to SG

Evaluating the sign of the variation in gross profit associated to the deviation from the specific to the generic technology for a non-integrated downstream firm leads to the following result.

Lemma 1 $\forall\left(n_{SS}, n_{SG}, n_{IS}, n_{IG}\right) \in \{1; ...; \} \times \{0; ...; n-1\} \times \{0; ...; n\}^2$ *such that* $n_{SS} + n_{SG} + n_{IS} + n_{IG} = n$,

$$\Pi_{SG}\left(n_{SS}-1, n_{SG}+1, n_{IS}, n_{IG}\right) \geq \Pi_{SS}\left(n_{SS}, n_{SG}, n_{IS}, n_{IG}\right)$$

Sketch of the proof (mathematical details in appendix A)

The gross profit of a non-integrated specific downstream firm is given by equation (14).

$$\Pi_{SS}\left(n_{SS}, n_{SG}\right) = \left(p_{SS}\left(n_{SS}, n_{SG}\right) - c - \delta\right) q_{SS}\left(n_{SS}, n_{SG}\right) \tag{14}$$

For notational simplicity, we skip the variables n_{IS} and n_{IG} that will be constant. We also denote by $m_{SS}(n_{SS}, n_{SG})$ the price cost margin and write the firm's profit as follows

$$\Pi_{SS}(n_{SS}, n_{SG}) = m_{SS}(n_{SS}, n_{SG})\, q_{SS}(n_{SS}, n_{SG}) \tag{15}$$

If the pair of firms switches to the generic technology, the downstream firm's profit becomes

$$\Pi_{SG}(n_{SS}-1, n_{SG}+1) = m_{SG}(n_{SS}-1, n_{SG}+1)\, q_{SG}(n_{SS}-1, n_{SG}+1) \tag{16}$$

We can now compute the variation in the downstream firm's profit as

$$\Delta\Pi(n_{SS}, n_{SG}) = \Pi_{SG}(n_{SS}-1, n_{SG}+1) - \Pi_{SS}(n_{SS}, n_{SG}) \tag{17}$$

Replacing the profits by their expressions and rearranging the terms leads to

$$\begin{aligned} \Delta\Pi(n_{SS}, n_{SG}) &= \begin{pmatrix} m_{SG}(n_{SS}-1, n_{SG}+1) \\ -m_{SS}(n_{SS}, n_{SG}) \end{pmatrix} \begin{pmatrix} q_{SG}(n_{SS}-1, n_{SG}+1) \\ -q_{SS}(n_{SS}, n_{SG}) \end{pmatrix} \\ &\quad + \left(m_{SG}(n_{SS}-1, n_{SG}+1) - m_{SS}(n_{SS}, n_{SG})\right) q_{SS}(n_{SS}, n_{SG}) \\ &\quad + \left(q_{SG}(n_{SS}-1, n_{SG}+1) - q_{SS}(n_{SS}, n_{SG})\right) m_{SS}(n_{SS}, n_{SG}) \end{aligned} \tag{18}$$

We now show that both the variation in margin and in output are positive. Since, $q_{SS}(n_{SS}, n_{SG})$ and $m_{SS}(n_{SS}, n_{SG})$ are positive, this implies that $\Delta\Pi(n_{SS}, n_{SG})$ is positive : any non-integrated specific downstream firm would like to switch to the generic technology. Conversely, no non-integrated generic downstream firm will accept to switch to a specific technology. Since the expression of prices and profits depends on n_{IG} and n_{SG}, we distinguish three cases.

Case 1 : $n_{IG} \geq 1$ or $n_{SG} \geq 2$

It is straightforward to calculate the price-cost margin and show that

$$m_{SS}(n_{SS},n_{SG})=\frac{1}{2\beta+\gamma-\gamma n}\left(\alpha-(\beta+\gamma-\gamma n)(c+\delta)+\frac{\gamma\beta}{2\beta+\gamma}((n_{SS}-n)\delta-n_{IS}\varepsilon)\right) \tag{19}$$

We then calculate the variation in margin as

$$\begin{aligned} m_{SG}(n_{SS}-1,n_{SG}+1) &= m_{SS}(n_{SS},n_{SG}) \\ &+\frac{1}{2\beta+\gamma-\gamma n}\left[\begin{array}{c}(2\beta+\gamma)(\beta+\gamma-\gamma n)\\ +\gamma\beta(n-1)\end{array}\right]\left(\frac{\delta}{2\beta+\gamma}\right) \end{aligned} \tag{20}$$

Since both terms in the bracketted expression are positive, this proves that the downstream firm increases its margin when the pair of firms switches to the generic technology. This is simply because it reduces its cost by δ, which provides it with a cost advantage over its competitors and allows it to rise its price-cost margin.

Calculating the variation in output is longer, but not more difficult. We establish equation (21) in appendix A.

$$\begin{aligned} q_{SG}(n_{SS}-1,n_{SG}+1) &= q_{SS}(n_{SS},n_{SG})+\beta\left[p_{SS}(n_{SS},n_{SG})-p_{SG}(n_{SS},n_{SG})\right] \\ &+\frac{1}{2\beta+\gamma-\gamma n}\left(\frac{\beta\gamma}{2\beta+\gamma}\right)(\beta+\gamma-\gamma n)\delta \end{aligned} \tag{21}$$

The first term in (21) is positive, as well as the third one. As regards the second term, it is easy to verify that

$$p_{SS}(n_{SS},n_{SG})-p_{SG}(n_{SS},n_{SG})=\frac{\beta\delta}{2\beta+\gamma}>0 \tag{22}$$

We thus conclude that the downstream firm increases its output after the switch to the generic technology. Since the margin is also increasing, the firm increases its gross profit.

Case 2 : $n_{IG}=0$ and $n_{SG}=1$

The difference with the previous case is that when switching to the generic technology, the downstream firm lowers its cost by δ, but it also lowers the cost of the other non-integrated generic downstream firm by δ. There is thus a lowering rivals' costs effect here that reduces the profitability of the generic technology. However, we can show (see appendix) that the margin increases, since

$$\begin{aligned}\left(m_{SG}\left(n_{SS}-1,n_{SG}+1\right)-m_{SS}\left(n_{SS},n_{SG}\right)\right)\left(2\beta+\gamma-\gamma n\right) &= \left(\beta+\gamma-\gamma n\right)\delta \\ +\frac{\gamma\beta}{2\beta+\gamma}\left(n_{SS}+n_{IS}-1\right)\delta &> 0 \qquad (23)\end{aligned}$$

As regards the level of output, we find

$$\begin{aligned}q_{SG}\left(n_{SS}-1,n_{SG}+1\right) &= q_{SS}\left(n_{SS},n_{SG}\right) \qquad (24)\\ +\frac{1}{2\beta+\gamma-\gamma n}\left(\frac{\beta\delta}{2\beta+\gamma}\right)\left[\gamma\left(\beta+\gamma-\gamma n\right)+\beta\left(2\beta-\gamma n\right)\right] &> 0\end{aligned}$$

This concludes the study of the second case. As in the first case, any non-integrated downstream firm prefers the generic technology to the specific one and, under our assumption regarding fixed costs (see below), no non-integrated pair of firms adopts a specific technology.

Case 3 : $n_{IG}=0$ and $n_{SG}=0$

In this case, the fact that a non-integrated specific pair of firms switches to the generic technology doesn't change the marginal cost of any downstream firm in the industry, not even the marginal cost of the switching firm, which is equal to $c+\delta$ in both cases.

3.1.2 Net incentives to deviate from SS to SG

To determine the profitability of the deviation, we have to specify assumptions on the fixed costs associated to the different technologies. We normalize the fixed cost associated to the generic technology to 0 and denote by E the fixed cost associated to specific technologies.

Assumption $E>0$

Under this assumption, any non-integrated downstream firm prefers the generic technology to the specific technology. Since a specific technology cannot be adopted by a pair of firms without the consent of the downstream firm, no pair of non-integrated firms adopts a specific technology. In other words, it is necessary to integrate in order to achieve the efficiency gains (in the sense of lower marginal costs) associated with a specific technology. This is the sense of the proposition below.

Proposition 2 *In equilibrium,* $n_{SS} = 0$.

It can be noted that this result holds as long as specific technologies are not associated with significantly lower downstream fixed costs than the generic technology. It is only in this case, that we don't consider here, that efficiency gains can be achieved without vertical integration.

3.2 The technological choice of integrated firms

We examine the incentives of integrated firms using the generic technology to switch to a non-integrated structure (while keeping the generic technology). The structure of marginal costs presented in table I shows that, in general, generic firms are indifferent between integration and separation, since their profit (gross of any cost/benefit of vertical integration not captured by the model) is the same in both cases. To avoid this indeterminacy, we assume that there is a strictly positive cost of integration, so that firms strictly prefer separation to integration. Since we don't want to focus on these integration costs that are not endogenous, we assume that they are very small.

There is one case, however, in which it is not indifferent in terms of gross profits for generic firms to be integrated or not. It is when $n_{IS} = n - 1$. Then, an integrated generic firm has a downstream cost equal to c, whereas a separated downstream generic firm has a cost equal to $c + \delta$. If we consider the joint profits of the pair of firms, the comparison of integration and separation is ambiguous. This is because there is a strategic positive effect on profits when the downstream firm is committed to a higher cost. There is thus not necessarily in this case a supplementary profit to be shared between the upstream and the downstream firms after a merger. We skip the analytical resolution that could allow us to alleviate the indeterminacy, both because it is quite ackward and, more importantly, because, as we said in the introduction, our focus in this paper is not on the strategic effects that happen on the market (here, it is a residual market) when it is supplied by a duopoly.

Proposition 3 *In equilibrium,* $n_{IG} = 0$ *or* $(n_{IS}, n_{IG}, n_{SG}, n_{SS}) = (n-1, 1, 0, 0)$.

Proof Since we know that in equilibrium $n_{SS} = 0$, we take this value as given. We then have to show that, for any $(n_{IS}, n_{IG}, n_{SG}) \neq (n-1, 1, 0)$ such that $n_{IG} \geq 1$, $\Pi_{IG}(n_{IS}, n_{IG}, n_{SG}) < \Pi_{SG}(n_{IS}, n_{IG} - 1, n_{SG} + 1)$. This is straightforward, given the values of costs given in table I and our assumptions concerning the costs of integration.

This proposition a partial result. It doesn't fully describe the technological choice of integrated firms in equilibrium. However, it is enough for our purpose in the present section. In particular, it implies that, as well as there is no non-integrated pair of firms using a specific technology, there is no integrated firm using the generic technology in an equilibrium such that $n_{SG} \geq 2$, that is equilibria in which vertical foreclosure plays no role. This is the type of equilibria that we focus on in the next section. We will thus focus on situations in which it is equivalent to be integrated and to use a specific technology. Since specific technologies are associated with lower marginal costs, vertical integration increases the social surplus, as long as E is not to large.

4 Structural and technological choices in equilibrium

In this section, we provide a characterization of the equilibrium of the game defined in section 2. Our main result concerns those equilibria that are such that $n_{SG} \geq 3$. It is established in the first part of the section. In the second part of the section, we briefly discuss other equilibria and the possibility of multiple equilibria.

4.0.1 Main result

Proposition 4 *For any* $k \in \{0; ...; n-3\}$*, there is a set of values of* E *such that*

$$(n_{IS}, n_{IG}, n_{SS}, n_{SG}) = (k, 0, 0, n-k)$$

is an equilibrium and there is no other equilibrium satisfying $n_{SG} \geq 3$.

Proof

We of course have to examine the incentives of firms to deviate from our equilibrium candidate. Some of the results established in the previous section will prove helpfull.

We first examine the incentives of firms to deviate from IS to IG and establish the following lemma.

Lemma 5 $\Pi_{IS}\left(n_{IG}-1;n_{IS}+1\right)>\Pi_{IG}\left(n_{IG};n_{IS}\right).$

Proof (details in appendix B)

Since we assume $n_{SG}\geq 3$, the technological choice of the integrated firm i will not change the cost of non-integrated generic downstream firms. We proceed by calculating the variations in margin and output for an integrated firm switching from the generic to a specific technology.

We show (see appendix B) that

$$m_{IS}\left(n_{IG}-1,n_{IS}+1\right)-m_{IG}\left(n_{IG},n_{IS}\right)=\frac{1}{2\beta+\gamma-\gamma n}\begin{pmatrix}\gamma+\beta-\gamma n\\ +\frac{\gamma\beta}{2\beta+\gamma}\left(n-1\right)\end{pmatrix}\varepsilon>0 \quad (25)$$

The variation in output is

$$q_{IS}\left(n_{IG}-1,n_{IS}+1\right)-q_{IG}\left(n_{IG},n_{IS}\right)=\frac{\beta^{3}+\beta\left(\beta+\gamma\right)\left(\beta+\gamma-\gamma n\right)}{\left(2\beta+\gamma-\gamma n\right)\left(2\beta+\gamma\right)}\varepsilon>0 \quad (26)$$

Since both the margin and the output increase, the (gross) profit also increases.

We now consider the net incentive to deviate from IS to IG. No matter what the structural and technological choices of rivals are, an integrated firm has an incentive to adopt a specific technology. However, to do so, it must pay the fixed cost E. Suppose that the incentive of an integrated firm to adopt a specific technology increases with the number of integrated firms that already use a specific technology. Then, if it is profitable for one integrated firm to adopt a specific technology, it will be profitable for every integrated firm. However, we show that (at least for $n_{SG}\geq 3$) it is the contrary that holds : the incentive of an integrated firm to switch to a specific technology is a decreasing function of the number of integrated firms using a specific technology. In other words, the more efficient competitors you have, the less profitable it is for you to become efficient as well. We establish this result below.

Definition 6 *For any* $(n_{IG}; n_{IS}; n_{SG}) \in \{1; ...; n\} \times \{0; ...; n\}^2$ *such that* $n_{IG} + n_{IS} + n_{SG} = n$*, we define* $\Delta\Pi(n_{IG}, n_{IS})$ *as the difference in an integrated firm's gross profit with a specific and with a generic technology, that is,*

$$\Delta\Pi(n_{IG}, n_{IS}) = \Pi_{IS}(n_{IG} - 1, n_{IS} + 1) - \Pi_{IG}(n_{IG}, n_{IS}).$$

Lemma 7 *For* $n_{SG} \geq 3$*,* $\Delta\Pi(n_{IG}, n_{IS}) < \Delta\Pi(n_{IG} + 1, n_{IS} - 1)$*.*

Proof (details in appendix C)

We rewrite $\Delta\Pi(n_{IG}, n_{IS})$ as

$$\begin{aligned} \Delta\Pi(n_{IG}, n_{IS}) &= -\begin{pmatrix} m_{IS}(n_{IG} - 1, n_{IS} + 1) \\ -m_{IG}(n_{IG}, n_{IS}) \end{pmatrix} \begin{pmatrix} q_{IS}(n_{IG} - 1, n_{IS} + 1) \\ -q_{IG}(n_{IG}, n_{IS}) \end{pmatrix} \\ &\quad + \left(m_{IS}(n_{IG} - 1, n_{IS} + 1) - m_{IG}(n_{IG}, n_{IS})\right) q_{IS}(n_{IG} - 1, n_{IS} + 1) \\ &\quad + \left(q_{IS}(n_{IG} - 1, n_{IS} + 1) - q_{IG}(n_{IG}, n_{IS})\right) m_{IS}(n_{IG} - 1, n_{IS} + 1) \end{aligned} \tag{27}$$

Since the variations in margin and output calculated above don't depend on $(n_{IG}; n_{IS}; n_{SG})$, we just have to examine how these parameters influence $q_{IS}(n_{IG} - 1, n_{IS} + 1)$ and $m_{IS}(n_{IG} - 1, n_{IS} + 1)$.

We show (see appendix C) that

$$\begin{aligned} q_{IS}(n_{IG} - 1, n_{IS} + 1) &= \alpha + \frac{1}{2\beta + \gamma - \gamma n} \begin{bmatrix} (\alpha + \beta c)(\gamma n - \beta - \gamma) \\ +\frac{\beta}{2\beta+\gamma} \begin{pmatrix} \beta(2\beta + \gamma) \\ +\gamma(\beta + \gamma)(1 - n) \end{pmatrix} \varepsilon \end{bmatrix} \\ &\quad - \frac{\gamma\beta^2}{(2\beta + \gamma - \gamma n)(2\beta + \gamma)} n_{IS}\varepsilon \end{aligned} \tag{28}$$

It is just the number n_{IS} of integrated specific firms that influences $q_{IS}(n_{IG} - 1, n_{IS} + 1)$ and as it increases, $q_{IS}(n_{IG} - 1, n_{IS} + 1)$ decreases.

We also show that

$$m_{IS}\left(n_{IG}-1,n_{IS}+1\right)=\frac{1}{2\beta+\gamma-\gamma n}\left[\alpha+\left(\gamma n-\beta-\gamma\right)\left(c-\varepsilon\right)+\frac{\beta\gamma}{2\beta+\gamma}\left(n-1-n_{IS}\right)\varepsilon\right] \tag{29}$$

As for the output, the margin decreases when the number of integrated, specific firms increases.

From (27), we conclude that the incentive of an integrated firm to adopt a specific technology decreases with the number of firms that already use this technology as long as it holds that $n_{SG}\geq 3$.

It is important to note that, for $n_{SG}\geq 3$, the incentive of firms to deviate from IS to SG is equal to the incentive of firms to deviate from IS to IG, so that the previous lemma also apply to this case.

To conclude our examination of deviations for IS firms, we have to consider the deviation to the SS strategy. We showed in section 3.1 that SG is always prefered to SS, so that if IS firms don't find profitable to deviate to SG, they also don't find profitable to switch to SS.

To sum up, the equilibrium candidate will not be destroyed by a deviation by one of the IS firms if and only if

$$\Delta\Pi\left(n_{IG}+1,n_{IS}-1\right)=\Pi_{IS}\left(n_{IG},n_{IS}\right)-\Pi_{IG}\left(n_{IG}+1,n_{IS}-1\right)>E \tag{30}$$

where $\Delta\Pi\left(n_{IG}+1,n_{IS}-1\right)>\Delta\Pi\left(n_{IG},n_{IS}\right)$.

This implies that, for any $n_{SG}\geq 3$ and any $n_{IS}=n-n_{SG}$, there exists an interval of values of E such that no IS firms can profitably deviate.

We now turn to the examination of the deviations of SG (pairs of) firms. We know that no SG firms will deviate to SS. Switching to IG is also not profitable, since there is no effect on the marginal cost of any firm, but there is a strictly positive integration cost. The only deviation that remains to examine is thus from SG to IS and the incentive for a SG firm to deviate to IS is exactly the opposite of the incentive for a IS firm to deviate to SG discussed above. This means that there will be no profitable deviation from the candidate equilibrium for any SG firm if and only if condition (30) is satisfied. This completes the proof.

This result shows that (quite) any degree of vertical integration can be an equilibrium in our model. This is not solely due to our assumption of a strictly positive fixed cost associated with specific technologies, but more importantly to the fact that we do not have the strategic complementarity found by McLaren (2000), but rather a strategic substituability. When a pair of firms integrates, it lowers the profitability of integration for other firms. In McLaren (2000), it rises this profitability. Strategic substituability implies that when we observe a (vertical) merger wave in an industry, we have no reason to expect that this merger wave will go on until vertical integration is generalized in the industry. This result has important implications for antitrust policy that are discussed in the next section.

4.0.2 Other equilibria

We discuss briefly here the other possible equilibria. Since the equilibrium $(n_{IS}, n_{IG}, n_{SG}) = (n-1, 1, 0)$ was discussed above, we focus on equilibria for which $n_{IG} = 0$ and thus consider a pair $(n_{IS}, n_{SG}) \in \{0; ...; n\} \times \{0; 1\}$ such that $n_{IS} + n_{SG} = n$.

First take $n_{SG} = 0$. Vertical integration is complete in the industry. Depending on E, it may or may not be profitable for one of the pair of firms to deviate to SG. The particularity of this case is that the pair of firms may also find profitable to switch to IG. However, if E is sufficiently low, $n_{IS} = n$ is an equilibrium. Essentially, as the number of IS firms increases, it gets less and less profitable to be an IS firm, but it remains profitable (even taking into account the fixed cost E) and full integration with every integrated firm using a specific technology is an equilibrium.

Now take $n_{SG} = 1$. Then, it may happen that the profitability of switching from IS to SG first decreases when n_{IS} increases from 0 to $n-2$, but is higher when $n_{IS} = n-2$ than when $n_{IS} = n-3$. Because of this, there may be a value of E such that none of the IS firms wants to switch to SG, but if one of them would do it, others may well follow it and switch. If this non-monotonicity is not present, there may also exist a value of E

such that $n_{IS} = n - 1$ is an equilibrium, because no IS firm wants to switch, but then, even if one does switch, no other will follow it.

Finally, for $n_{SG} = 2$, the logic is the same as for $n_{SG} = 1$, with the difference that these are only the SG firms that can rise rivals' costs by deviating to IS, thus leaving the remaining SG firm alone with the generic technology. A deviation by IS firms would not have the same effect, because it would leave two pairs of SG firms on the market.

4.0.3 Multiple equilibria

Note that for a given value of E, there may be two equilibria for the game. Indeed, when the profitability of IS rather than SG in not monotonic for $n_{SG} = 1$, the same E may be consistant with an equilibrium caracterized by $n_{IS} \geq n - 1$ and another equilibrium caracterized by $n_{IS} < n - 1$. This multiplicity of equilibria is induced by the existence of a rising rivals' costs effect in the industry when vertical integration is quite general. This is not the point that we want to point at in this article.

5 Implications for antitrust policy

In the famous Brown Shoe case, the US Supreme Court banned a benign vertical merger with the argument that the objective was to stop a merger wave that would end in full integration. This may be a very good argument if a "snowball effect" is present, that is, under strategic complementarity. However, under strategic substituability, there is no point stopping a merger wave at a very early stage instead of just waiting to see if the merger wave will not stop by itself at an acceptable level of vertical integration. The definition of what an acceptable level of vertical integration is leads us to discuss the effect of vertical integration on welfare in our model.

6 Welfare

The impact of vertical integration on welfare is ambiguous in this model. Indeed, vertical integration impacts on welfare in three different ways : (i) it allows merging firms to achieve efficiency gains, which rises the welfare, (ii) it is associated with a fixed cost, which reduces the welfare and (iii) it may be a rising rivals' costs strategy, which reduces the welfare. Even if we abstract from the third effect by focusing on merger waves that stop before the number of separated firms is low enough for rising rivals' costs effect to play, one can see that there is no clearcut result on welfare. This indeterminacy is difficult to avoid once we take into account both the efficiency gains associated to integration and the costs associated to it.

7 Conclusion

Within the framework of a successive Bertrand oligopoly model, we establish a result of strategic substituability that has strong implications both on the predictions of the model and on the implications of the analysis for the antitrust policy. Concerning the first point, we show that the degree of vertical integration in the model need not be either 0 or 100 percent, but can take quite any value, depending on the fixed cost associated to specific technologies. In particular, it means that vertical merger waves may stop before the industry is fully integrated. If this is the case, there is no foreclosure and no rising rivals' costs effect associated with vertical integration. Generalising the successive duopoly models that are the basis of the literature on strategic vertical integration thus leads to the conclusion that these models may well have exagerated the importance of the foreclosure issue. Insofar as we can base antitrust recommendations on our model, these would be in favor of a less severe control of vertical integration.

8 References

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9 Appendix A : Proof of lemma 1

Case 1 : $n_{IG} \geq 1$ or $n_{SG} \geq 2$

We can write the profit of a non-integrated specific downstream firm as follows :

$\Pi_{SS}(n_{SS}, n_{SG}) = m_{SS}(n_{SS}, n_{SG})\, q_{SS}(n_{SS}, n_{SG})$,

with

$m_{SS}(n_{SS}, n_{SG}) = p_{SS}(n_{SS}, n_{SG}) - c - \delta$

$= \frac{1}{2\beta+\gamma-\gamma n}\left(\alpha - (\beta+\gamma-\gamma n)(c+\delta) + \frac{\gamma\beta}{2\beta+\gamma}\left((n_{SS}-n)\delta - n_{IS}\varepsilon\right)\right)$

If the pair of firms switches to the generic technology, the downstream firm's profit becomes

$\Pi_{SG}(n_{SS}-1, n_{SG}+1) = m_{SG}(n_{SS}-1, n_{SG}+1)\, q_{SG}(n_{SS}-1, n_{SG}+1)$,

with $m_{SG}(n_{SS}-1, n_{SG}+1) = p_{SG}(n_{SS}-1, n_{SG}+1) - c$

$= \frac{1}{2\beta+\gamma-\gamma n}\left(\alpha - (\beta+\gamma-\gamma n)c + \frac{\gamma\beta}{2\beta+\gamma}\left((n_{SS}-1)\delta - n_{IS}\varepsilon\right)\right)$

We can compute the variation in D_i's profit (gross of any difference in the fixed cost associated to the different technologies) as

$\Delta\Pi(n_{SS}, n_{SG}) = \Pi_{SG}(n_{SS}-1, n_{SG}+1) - \Pi_{SS}(n_{SS}, n_{SG})$

$= m_{SG}(n_{SS}-1, n_{SG}+1)\, q_{SG}(n_{SS}-1, n_{SG}+1)$

$-m_{SS}(n_{SS}, n_{SG})\, q_{SS}(n_{SS}, n_{SG})$

$$= \begin{pmatrix} m_{SG}(n_{SS}-1, n_{SG}+1) \\ -m_{SS}(n_{SS}, n_{SG}) + m_{SS}(n_{SS}, n_{SG}) \end{pmatrix} \begin{pmatrix} q_{SG}(n_{SS}-1, n_{SG}+1) \\ -q_{SS}(n_{SS}, n_{SG}) + q_{SS}(n_{SS}, n_{SG}) \end{pmatrix}$$
$-m_{SS}(n_{SS}, n_{SG})\, q_{SS}(n_{SS}, n_{SG})$
$= (m_{SG}(n_{SS}-1, n_{SG}+1) - m_{SS}(n_{SS}, n_{SG}))\,(q_{SG}(n_{SS}-1, n_{SG}+1) - q_{SS}(n_{SS}, n_{SG}))$
$+ (m_{SG}(n_{SS}-1, n_{SG}+1) - m_{SS}(n_{SS}, n_{SG}))\, q_{SS}(n_{SS}, n_{SG})$
$+ (q_{SG}(n_{SS}-1, n_{SG}+1) - q_{SS}(n_{SS}, n_{SG}))\, m_{SS}(n_{SS}, n_{SG})$

Since quantities and margins are positive,

$\Delta\Pi(n_{SS}, n_{SG}) > 0$ if and only if $m_{SG}(n_{SS}-1, n_{SG}+1) - m_{SS}(n_{SS}, n_{SG}) > 0$ and $q_{SG}(n_{SS}-1, n_{SG}+1) - q_{SS}(n_{SS}, n_{SG}) > 0$.

We now show that both the variation in margin and in output are positive.

$m_{SG}(n_{SS}-1, n_{SG}+1) - m_{SS}(n_{SS}, n_{SG})$
$= \frac{1}{2\beta+\gamma-\gamma n}\left(\alpha - (\beta+\gamma-\gamma n)\, c + \frac{\gamma\beta}{2\beta+\gamma}\left((n_{SS}-1)\,\delta - n_{IS}\varepsilon\right)\right)$
$- \frac{1}{2\beta+\gamma-\gamma n}\left(\alpha - (\beta+\gamma-\gamma n)(c+\delta) + \frac{\gamma\beta}{2\beta+\gamma}\left((n_{SS}-n)\,\delta - n_{IS}\varepsilon\right)\right)$
$= \left(\frac{\beta+\gamma-\gamma n}{2\beta+\gamma-\gamma n} + \frac{\gamma\beta(n-1)}{(2\beta+\gamma-\gamma n)(2\beta+\gamma)}\right)\delta > 0$

$q_{SG}(n_{SS}-1, n_{SG}+1) = \alpha + (\gamma n_{SG} - \beta)\, p_{SG}(n_{SS}-1, n_{SG}+1)$
$+\gamma n_{IG} p_{IG}(n_{SS}-1, n_{SG}+1) + \gamma(n_{SS}-1)\, p_{SS}(n_{SS}-1, n_{SG}+1)$
$+\gamma n_{IS} p_{IS}(n_{SS}-1, n_{SG}+1)$

Since $p_{IG}(n_{SS}-1, n_{SG}+1) = p_{SG}(n_{SS}-1, n_{SG}+1)$,

$q_{SG}(n_{SS}-1, n_{SG}+1) = \alpha + (\gamma n_{SG} + \gamma n_{IG} - \beta)\, p_{SG}(n_{SS}-1, n_{SG}+1)$
$+\gamma(n_{SS}-1)\, p_{SS}(n_{SS}-1, n_{SG}+1) + \gamma n_{IS} p_{IS}(n_{SS}-1, n_{SG}+1)$

We now use the fact that

$p_{IG}(n_{SS}-1, n_{SG}+1) = p_{IG}(n_{SS}, n_{SG}) - \frac{\gamma\beta}{(2\beta+\gamma-\gamma n)(2\beta+\gamma)}\delta,$
$p_{SS}(n_{SS}-1, n_{SG}+1) = p_{SS}(n_{SS}, n_{SG}) - \frac{\gamma\beta}{(2\beta+\gamma-\gamma n)(2\beta+\gamma)}\delta$

and

$p_{IS}(n_{SS}-1, n_{SG}+1) = p_{IS}(n_{SS}, n_{SG}) - \frac{\gamma\beta}{(2\beta+\gamma-\gamma n)(2\beta+\gamma)}\delta$

to rewrite $q_{SG}(n_{SS}-1, n_{SG}+1)$ as

$q_{SG}(n_{SS}-1, n_{SG}+1) = \alpha + (\gamma n_{SG} + \gamma n_{IG} - \beta)\, p_{SG}(n_{SS}, n_{SG})$
$+\gamma(n_{SS}-1)\, p_{SS}(n_{SS}, n_{SG}) + \gamma n_{IS} p_{IS}(n_{SS}, n_{SG}) + \frac{\gamma\beta(\beta+\gamma-\gamma n)}{(2\beta+\gamma-\gamma n)(2\beta+\gamma)}\delta$

Rearranging the terms leads to

$q_{SG}(n_{SS}-1, n_{SG}+1) = \alpha + (\gamma n_{SG} + \gamma n_{IG})\, p_{SG}(n_{SS}, n_{SG})$

$+ (\gamma n_{SS} - \gamma - \beta) p_{SS} (n_{SS}, n_{SG}) + \gamma n_{IS} p_{IS} (n_{SS}, n_{SG}) - \beta p_{SG} (n_{SS}, n_{SG})$

$+\beta p_{SS} (n_{SS}, n_{SG}) + \frac{\gamma\beta(\beta+\gamma-\gamma n)}{(2\beta+\gamma-\gamma n)(2\beta+\gamma)}\delta$

and finally to

$q_{SG} (n_{SS} - 1, n_{SG} + 1) = q_{SS} (n_{SS}, n_{SG}) + \beta [p_{SS} (n_{SS}, n_{SG}) - p_{SG} (n_{SS}, n_{SG})]$

$+\frac{1}{2\beta+\gamma-\gamma n} \left(\frac{\beta\gamma}{2\beta+\gamma}\right) (\beta + \gamma - \gamma n) \delta$

Since

$p_{SS} (n_{SS}, n_{SG}) - p_{SG} (n_{SS}, n_{SG}) = \frac{\beta\delta}{2\beta+\gamma} > 0,$

every term on the right-hand side is positive and

$q_{SG} (n_{SS} - 1, n_{SG} + 1) - q_{SS} (n_{SS}, n_{SG}) > 0.$

Case 2 : $n_{IG} = 0$ and $n_{SG} = 1$

We proceed as in the previous case to establish that

$\Delta\Pi (n_{SS}, n_{SG}) > 0$ if and only if $m_{SG} (n_{SS} - 1, n_{SG} + 1) - m_{SS} (n_{SS}, n_{SG}) > 0$ and $q_{SG} (n_{SS} - 1, n_{SG} + 1) - q_{SS} (n_{SS}, n_{SG}) > 0.$

$m_{SG} (n_{SS} - 1, n_{SG} + 1) - m_{SS} (n_{SS}, n_{SG})$

$= \frac{1}{2\beta+\gamma-\gamma n} \left(\alpha - (\beta + \gamma - \gamma n) c + \frac{\gamma\beta}{2\beta+\gamma} ((n_{SS} - 1) \delta - n_{IS}\varepsilon)\right)$

$-\frac{1}{2\beta+\gamma-\gamma n} \left(\alpha - (\beta + \gamma - \gamma n) (c + \delta) - \frac{\gamma\beta}{2\beta+\gamma} n_{IS} (\delta + \varepsilon)\right)$

$= \left(\frac{\beta+\gamma-\gamma n}{2\beta+\gamma-\gamma n} + \frac{\gamma\beta(n_{SS}+n_{IS}-1)}{(2\beta+\gamma-\gamma n)(2\beta+\gamma)}\right) \delta$

$= \left(\frac{\beta+\gamma-\gamma n}{2\beta+\gamma-\gamma n} + \frac{\gamma\beta(n-2)}{(2\beta+\gamma-\gamma n)(2\beta+\gamma)}\right) \delta > 0$

$q_{SS} (n_{SS}, n_{SG}) = \alpha - \beta p_{SS} (n_{SS}, n_{SG}) + \gamma (n_{SS} - 1) p_{SS} (n_{SS}, n_{SG})$

$+\gamma n_{IS} p_{IS} (n_{SS}, n_{SG}) + \gamma n_{SG} p_{SG} (n_{SS}, n_{SG})$

$= \alpha + (\gamma n_{SS} - \gamma - \beta) p_{SS} (n_{SS}, n_{SG}) + \gamma n_{IS} p_{IS} (n_{SS}, n_{SG})$

$+\gamma n_{SG} p_{SG} (n_{SS}, n_{SG})$

$q_{SG} (n_{SS} - 1, n_{SG} + 1) = \alpha - \beta p_{SG} (n_{SS} - 1, n_{SG} + 1)$

$+\gamma (n_{SS} - 1) p_{SS} (n_{SS} - 1, n_{SG} + 1) + \gamma n_{IS} p_{IS} (n_{SS} - 1, n_{SG} + 1)$

$+\gamma n_{SG} p_{SG} (n_{SS} - 1, n_{SG} + 1)$

$= \alpha + (\gamma n_{SG} - \beta) p_{SG} (n_{SS} - 1, n_{SG} + 1) + \gamma (n_{SS} - 1) p_{SS} (n_{SS} - 1, n_{SG} + 1)$

$+\gamma n_{IS} p_{IS} (n_{SS} - 1, n_{SG} + 1)$

$= \alpha + \gamma n_{SG} p_{SG} (n_{SS} - 1, n_{SG} + 1) + (\gamma n_{SS} - \gamma - \beta) p_{SS} (n_{SS} - 1, n_{SG} + 1)$

$+\gamma n_{IS} p_{IS} (n_{SS} - 1, n_{SG} + 1)$

$+\beta\left(p_{SS}\left(n_{SS}-1,n_{SG}+1\right)-p_{SG}\left(n_{SS}-1,n_{SG}+1\right)\right)$
$=q_{SS}\left(n_{SS},n_{SG}\right)+\left(\gamma n_{SS}-\gamma-\beta\right)\left[p_{SS}\left(n_{SS}-1,n_{SG}+1\right)-p_{SS}\left(n_{SS},n_{SG}\right)\right]$
$+\gamma n_{IS}\left[p_{IS}\left(n_{SS}-1,n_{SG}+1\right)-p_{IS}\left(n_{SS},n_{SG}\right)\right]$
$+\gamma n_{SG}\left[p_{SG}\left(n_{SS}-1,n_{SG}+1\right)-p_{SG}\left(n_{SS},n_{SG}\right)\right]$
$+\beta\left[p_{SS}\left(n_{SS}-1,n_{SG}+1\right)-p_{SG}\left(n_{SS}-1,n_{SG}+1\right)\right]$

We now calculate the terms in square brackets.

$p_{SS}\left(n_{SS}-1,n_{SG}+1\right)-p_{SS}\left(n_{SS},n_{SG}\right)$
$=\frac{1}{2\beta+\gamma-\gamma n}\left(\alpha+\beta\left(c+\delta\right)+\frac{\gamma\beta}{2\beta+\gamma}\left(-\left(n_{IS}+n_{SG}+1\right)\delta-n_{IS}\varepsilon\right)\right)$
$-\frac{1}{2\beta+\gamma-\gamma n}\left(\alpha+\beta\left(c+\delta\right)-\frac{\gamma\beta}{2\beta+\gamma}n_{IS}\left(\varepsilon+\delta\right)\right)$
$=-\frac{\gamma\beta}{(2\beta+\gamma)(2\beta+\gamma-\gamma n)}\left(n_{SG}+1\right)\delta$

$p_{IS}\left(n_{SS}-1,n_{SG}+1\right)-p_{IS}\left(n_{SS},n_{SG}\right)$
$=\frac{1}{2\beta+\gamma-\gamma n}\left(\alpha+\beta\left(c-\varepsilon\right)+\frac{\gamma\beta}{2\beta+\gamma}\left(\left(n_{SS}-1\right)\delta+\left(n_{SS}+n_{SG}\right)\varepsilon\right)\right)$
$-\frac{1}{2\beta+\gamma-\gamma n}\left(\alpha+\beta\left(c-\varepsilon\right)+\frac{\gamma\beta}{2\beta+\gamma}\left(n_{SS}+n_{SG}\right)\left(\varepsilon+\delta\right)\right)$
$=-\frac{\gamma\beta}{(2\beta+\gamma)(2\beta+\gamma-\gamma n)}\left(n_{SG}+1\right)\delta$

$p_{SG}\left(n_{SS}-1,n_{SG}+1\right)-p_{SG}\left(n_{SS},n_{SG}\right)$
$=\frac{1}{2\beta+\gamma-\gamma n}\left(\alpha+\beta c+\frac{\gamma\beta}{2\beta+\gamma}\left(\left(n_{SS}-1\right)\delta-n_{IS}\varepsilon\right)\right)$
$-\frac{1}{2\beta+\gamma-\gamma n}\left(\alpha+\beta\left(c+\delta\right)-\frac{\gamma\beta}{2\beta+\gamma}n_{IS}\left(\delta+\varepsilon\right)\right)$
$=-\frac{\beta}{2\beta+\gamma-\gamma n}\delta+\frac{\gamma\beta}{(2\beta+\gamma)(2\beta+\gamma-\gamma n)}\left(n_{SS}+n_{IS}-1\right)\delta$
$=-\frac{\beta}{2\beta+\gamma-\gamma n}\delta+\frac{\gamma\beta}{(2\beta+\gamma)(2\beta+\gamma-\gamma n)}\left(n-2\right)\delta$

$p_{SS}\left(n_{SS}-1,n_{SG}+1\right)-p_{SG}\left(n_{SS}-1,n_{SG}+1\right)$
$=\frac{1}{2\beta+\gamma-\gamma n}\left(\alpha+\beta\left(c+\delta\right)+\frac{\gamma\beta}{2\beta+\gamma}\left(-\left(n_{IS}+n_{SS}+1\right)\delta-n_{IS}\varepsilon\right)\right)$
$-\frac{1}{2\beta+\gamma-\gamma n}\left(\alpha+\beta c+\frac{\gamma\beta}{2\beta+\gamma}\left(\left(n_{SS}-1\right)\delta-n_{IS}\varepsilon\right)\right)$
$=\frac{\beta\delta}{2\beta+\gamma-\gamma n}-\frac{\gamma\beta}{(2\beta+\gamma)(2\beta+\gamma-\gamma n)}n\delta$

Using these expressions, we calculate the variation in output as

$q_{SG}\left(n_{SS}-1,n_{SG}+1\right)-q_{SS}\left(n_{SS},n_{SG}\right)$
$$=\frac{1}{2\beta+\gamma-\gamma n}\left\{\begin{array}{c}\left(\gamma n_{SS}-\gamma-\beta\right)\frac{\gamma\beta}{(2\beta+\gamma)}\left(-n_{SG}-1\right)\delta\\+\gamma n_{IS}\frac{\gamma\beta}{(2\beta+\gamma)}\left(-n_{SG}-1\right)\delta+\gamma n_{SG}\left[-\beta\delta+\frac{\gamma\beta}{(2\beta+\gamma)}\left(n-2\right)\delta\right]\\+\beta^2\delta-\frac{\gamma\beta^2}{(2\beta+\gamma)}n\delta\end{array}\right\}$$

Simplifying this expression, taking into account the fact that $n_{SG}=1$, leads to

$q_{SG}(n_{SS}-1, n_{SG}+1) - q_{SS}(n_{SS}, n_{SG})$
$= \frac{1}{2\beta+\gamma-\gamma n}\left[-\frac{\gamma\beta}{(2\beta+\gamma)}(n-2)(\gamma+\beta)+\beta(\beta-\gamma)\right]\delta$

Since the sign of the term in square brackets is not a priori evident, we rewrite it as follows :

$-\frac{\gamma\beta}{(2\beta+\gamma)}(n-2)(\gamma+\beta)+\beta(\beta-\gamma)$
$= \frac{\beta}{(2\beta+\gamma)}\left[\gamma^2(1-n)+\beta\gamma(1-n)+2\beta^2\right]$
$= \frac{\beta}{(2\beta+\gamma)}\left[\underbrace{\gamma(\beta+\gamma-\gamma n)}_{>0}+\underbrace{\beta(\beta+\gamma-\gamma n)}_{>0}+\underbrace{\beta(\beta-\gamma)}_{>0}\right]$

We thus proved that

$q_{SG}(n_{SS}-1, n_{SG}+1) - q_{SS}(n_{SS}, n_{SG}) > 0.$

This completes the proof.

10 Appendix B : Proof of lemma 6

We proceed as for proposition 1, so that proving proposition 2 is just the same as proving that

$$\begin{cases} m_{IS}(n_{IG}-1, n_{IS}+1) - m_{IG}(n_{IG}, n_{IS}) > 0 \\ q_{IS}(n_{IG}-1, n_{IS}+1) - q_{IG}(n_{IG}, n_{IS}) > 0 \end{cases}$$

Case 1 : $n_{IG} \neq 1; n_{SG} \neq 1$

$m_{IS}(n_{IG}-1, n_{IS}+1) - m_{IG}(n_{IG}, n_{IS})$
$= p_{IS}(n_{IG}-1, n_{IS}+1) - (c-\varepsilon) - (p_{IG}(n_{IG}, n_{IS}) - c)$
$= \frac{1}{2\beta+\gamma-\gamma n}\left(\alpha-(\beta+\gamma-\gamma n)(c-\varepsilon)+\frac{\gamma\beta}{2\beta+\gamma}(n_{IG}+n_{SG}-1)\varepsilon\right)$
$-\frac{1}{2\beta+\gamma-\gamma n}\left(\alpha-(\beta+\gamma-\gamma n)c-\frac{\gamma\beta}{2\beta+\gamma}n_{IS}\varepsilon\right)$
$= \frac{1}{2\beta+\gamma-\gamma n}\left((\beta+\gamma-\gamma n)\varepsilon+\frac{\gamma\beta}{2\beta+\gamma}(n_{IG}+n_{SG}+n_{IS}-1)\varepsilon\right)$
$= \frac{1}{2\beta+\gamma-\gamma n}\left(\beta+\gamma-\gamma n+\frac{\gamma\beta}{2\beta+\gamma}(n-1)\right)\varepsilon > 0$

$q_{IG}(n_{IG}, n_{IS}) = \alpha + (\gamma n_{IG}-\gamma-\beta)p_{IG}(n_{IG}, n_{IS})$
$+\gamma n_{IS}p_{IS}(n_{IG}, n_{IS}) + \gamma n_{SG}p_{SG}(n_{IG}, n_{IS})$

$q_{IS}(n_{IG}-1, n_{IS}+1) = \alpha + (\gamma n_{IS}-\beta)p_{IS}(n_{IG}-1, n_{IS}+1)$
$+(\gamma n_{IG}-\gamma)p_{IG}(n_{IG}-1, n_{IS}+1) + \gamma n_{SG}p_{SG}(n_{IG}-1, n_{IS}+1)$

$q_{IS}(n_{IG}-1, n_{IS}+1) - q_{IG}(n_{IG}, n_{IS})$

$$= \gamma n_{SG} \left[\begin{array}{c} p_{SG}(n_{IG}-1, n_{IS}+1) \\ -p_{SG}(n_{IG}, n_{IS}) \end{array} \right] + \gamma n_{IS} \left[\begin{array}{c} p_{IS}(n_{IG}-1, n_{IS}+1) \\ -p_{IS}(n_{IG}, n_{IS}) \end{array} \right]$$
$$+ (\gamma n_{IG} - \gamma - \beta) [p_{IG}(n_{IG}-1, n_{IS}+1) - p_{IG}(n_{IG}, n_{IS})]$$
$$+\beta [p_{IS}(n_{IG}-1, n_{IS}+1) - p_{IG}(n_{IG}-1, n_{IS}+1)]$$

Replacing the terms in square brackets with their expression, we show that :

$$q_{IS}(n_{IG}-1, n_{IS}+1) - q_{IG}(n_{IG}, n_{IS})$$
$$= \tfrac{1}{2\beta+\gamma-\gamma n} \left\{ (\beta+\gamma-\gamma n) \tfrac{\gamma\beta}{2\beta+\gamma}\varepsilon + \beta^2\varepsilon - \tfrac{\gamma\beta^2}{2\beta+\gamma} n\varepsilon \right\}$$
$$q_{IS}(n_{IG}-1, n_{IS}+1) - q_{IG}(n_{IG}, n_{IS}) = \tfrac{\beta(\beta+\gamma-\gamma n)(\beta+\gamma)+\beta^3}{(2\beta+\gamma-\gamma n)(2\beta+\gamma)}\varepsilon > 0$$

This completes the proof.

11 Appendix C : Proof of lemma 8

$$m_{IS}(n_{IG}-1, n_{IS}+1) = p_{IS}(n_{IG}-1, n_{IS}+1) - (c-\varepsilon)$$
$$= \tfrac{1}{2\beta+\gamma-\gamma n} \left(\alpha - (\beta+\gamma-\gamma n)(c-\varepsilon) + \tfrac{\gamma\beta}{2\beta+\gamma}(n_{IG}+n_{SG}-1)\varepsilon \right)$$
$$= \tfrac{1}{2\beta+\gamma-\gamma n} \left(\alpha - (\beta+\gamma-\gamma n)(c-\varepsilon) + \tfrac{\gamma\beta}{2\beta+\gamma}(n-1)\varepsilon - \tfrac{\gamma\beta}{2\beta+\gamma} n_{IS}\varepsilon \right)$$
$$\Longrightarrow \tfrac{\partial}{\partial n_{IS}} m_{IS}(n_{IG}-1, n_{IS}+1) < 0$$
$$q_{IS}(n_{IG}-1, n_{IS}+1) = \alpha + (\gamma n_{IS} - \beta) p_{IS}(n_{IG}-1, n_{IS}+1)$$
$$+ (\gamma n_{IG} - \gamma) p_{IG}(n_{IG}-1, n_{IS}+1) + \gamma n_{SG} p_{SG}(n_{IG}-1, n_{IS}+1)$$
$$= \alpha + (\gamma n_{IS} - \beta) p_{IS}(n_{IG}-1, n_{IS}+1)$$
$$+ (\gamma n_{IG} + \gamma n_{SG} - \gamma) p_{IG}(n_{IG}-1, n_{IS}+1)$$
$$= \alpha + \tfrac{(\gamma n_{IS}-\beta)(\alpha+\beta c-\beta\varepsilon)}{2\beta+\gamma-\gamma n} + \tfrac{(\gamma n_{IG}+\gamma n_{SG}-\gamma)(\alpha+\beta c)}{2\beta+\gamma-\gamma n}$$
$$+ \tfrac{\gamma n_{IS}-\beta}{2\beta+\gamma-\gamma n} \tfrac{\gamma\beta}{2\beta+\gamma}(n_{IG}+n_{SG}-1)\varepsilon - \tfrac{(\gamma n_{IG}+\gamma n_{SG}-\gamma)}{2\beta+\gamma-\gamma n} \tfrac{\gamma\beta}{2\beta+\gamma}(n_{IS}+1)\varepsilon$$
$$= \alpha + \tfrac{1}{2\beta+\gamma-\gamma n} \left[(\gamma n - \beta - \gamma)(\alpha+\beta c) + \beta^2\varepsilon - \gamma n_{IS}\beta\varepsilon \right]$$
$$+ \tfrac{1}{2\beta+\gamma-\gamma n} \tfrac{\gamma\beta}{2\beta+\gamma} [(\gamma n_{IS} - \beta) - \gamma(n_{IS}+1)](n - n_{IS} - 1)\varepsilon$$
$$= \alpha + \tfrac{1}{2\beta+\gamma-\gamma n} \left[(\gamma n - \beta - \gamma)(\alpha+\beta c) + \beta^2\varepsilon - \gamma n_{IS}\beta\varepsilon \right]$$
$$- \tfrac{1}{2\beta+\gamma-\gamma n} \tfrac{\gamma\beta}{2\beta+\gamma}(\beta+\gamma)(n - n_{IS} - 1)\varepsilon$$
$$= \alpha + \tfrac{1}{2\beta+\gamma-\gamma n} \left[(\gamma n - \beta - \gamma)(\alpha+\beta c) + \beta^2\varepsilon - \tfrac{\gamma\beta}{2\beta+\gamma}(\beta+\gamma)(n-1)\varepsilon \right]$$
$$+ \tfrac{1}{2\beta+\gamma-\gamma n} \left[-\gamma\beta + \tfrac{\gamma\beta}{2\beta+\gamma}(\beta+\gamma) \right] n_{IS}\varepsilon$$

$$= \alpha + \frac{1}{2\beta+\gamma-\gamma n} \left\{ \begin{array}{c} (\gamma n - \beta - \gamma)(\alpha + \beta c) \\ +\frac{\beta}{2\beta+\gamma} \left[\beta (2\beta + \gamma) - \gamma (\beta + \gamma)(n-1)\right] \varepsilon \end{array} \right\}$$

$$-\frac{\gamma\beta^2}{(2\beta+\gamma-\gamma n)(2\beta+\gamma)} n_{IS} \varepsilon$$

$$\Longrightarrow \frac{\partial}{\partial n_{IS}} q_{IS} (n_{IG} - 1, n_{IS} + 1) < 0$$

This completes the proof.