Wage and Price Phillips Curves

An empirical analysis of destabilizing wage-price spirals

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Abstract

In this paper we introduce a small Keynesian model of economic growth which is centered around two advanced types of Phillips curves, one for money wages and one for prices, both being augmented by perfect myopic foresight and supplemented by a measure of the medium-term inflationary climate updated in an adaptive fashion. The model contains two potentially destabilizing feedback chains, the so-called Mundell and Rose-effects. We estimate parsimonious and congruent Phillips curves for money wages and prices in the US over the past five decades. Using the parameters of the empirical Phillips curves, we show that the growth path of the private sector of the model economy is likely to be surrounded by centrifugal forces. Convergence to this growth path can be generated in two ways: a Blanchard-Katz-type error-correction mechanism in the money-wage Phillips curve or a modified Taylor rule that is augmented by a term, which transmits increases in the wage share (real unit labor costs) to increases in the nominal rate of interest. Thus the model is characterized by local instability of the wage-price spiral, which however can be tamed by appropriate wage or monetary policies. Our empirical analysis finds the error-correction mechanism being ineffective in both Phillips curves suggesting that the stability of the post-war US macroeconomy originates from the stabilizing role of monetary policy.

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KEYWORDS: Phillips curves; Mundell effect; Rose effect; Monetary policy; Taylor Rule; Inflation; Unemployment; Instability.

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1 Introduction

1.1 The Phillips curve(s)

Following the seminal work in Phillips (1958) on the relation between unemployment and the rate of change of money wage rates in the UK, the 'Phillips curve' was to play an important role in macroeconomics during the 1960s and 1970s, and modified so as to incorporate inflation expectations, survived for much longer. The discussion on the proper type and the functional shape of the Phillips curve has never come to a real end and is indeed now at least as lively as it has been at any other time after the appearance of Phillips (1958) seminal paper. Recent examples for this observation are provided by the paper of Gali, Gertler and Lopez-Salido (2001), where again a new type of Phillips curve is investigated, and the paper by Laxton, Rose and Tambakis (1999) on the typical shape of the expectations augmented price inflation Phillips curve. Blanchard and Katz (1999) investigate the role of an error-correction wage share influence theoretically as well as empirically and Plasmans, Meersman, van Poeck and Merlevede (1999) investigate on this basis the impact of the generosity of the unemployment benefit system on the adjustment speed of money wages with regard to demand pressure in the market for labor.

Much of the literature has converged on the so-called *'New Keynesian Phillips curve'*, based on Taylor (1980) and Calvo (1983). Indeed, McCallum (1997) has called it the *"closest thing there is to a standard formulation"*. Clarida, Gali and Gertler (1999) have used a version of it as the basis for deriving some general principles about monetary policy. However, as has been recently pointed out by Mankiw (2001): *"Although the new Keynesian Phillips curves has many virtues, it also has one striking vice: It is completely at odd with the facts"*. The problems arise from the fact that although the price level is sticky in this model, the inflation rate can change quickly. By contrast, empirical analyses of the inflation process (see, *inter alia*, Gordon, 1997) typically give a large role to 'inflation inertia'.

Rarely, however, at least on the theoretical level, is note taken of the fact that there are in principle two relationships of the Phillips curve type involved in the interaction of unemployment and inflation, namely one on the labor market, the Phillips (1958) curve, and one on the market for goods, normally not considered a separate Phillips curve, but merged with the other one by assuming that prices are a constant mark-up on wages or the like, an extreme case of the price Phillips curve that we shall consider in this paper.

For researchers with a background in structural macroeconometric model building it is, however, not at all astonishing to use two Phillips curves in the place of only one in order to model the interacting dynamics of labor and goods market adjustment processes or the wage-price spiral for simplicity. Thus, for example, Fair (2000) has recently reconsidered the debate on the NAIRU from this perspective, though he still uses demand pressure on the market for labor as proxy for that on the market for goods (see Chiarella and Flaschel, 2000 for a discussion of his approach).

In this paper we, by contrast, start from a traditional approach to the discussion of the wage-price spiral which uses different measures for demand and cost pressure on the market for labor and on the market for goods and which distinguishes between temporary and permanent cost pressure changes. Despite its traditional background – not unrelated however to modern theories of wage and price setting, see appendices A.2 and A.3 – we are able to show that an important macrodynamic feedback mechanism can be detected in this type of wage – price spiral

that has rarely been investigated in the theoretical as well as in the applied macroeconomic literature with respect to its implications for macroeconomic stability. For the US economy we then show by detailed estimation, using the software package PcGets of Hendry and Krolzig (2001), that this feedback mechanism tends to be a destabilizing one. We finally demonstrate on this basis that a certain error correction term in the money-wage Phillips curve or a Taylor interest rate policy rules that is augmented by a wage gap term can dominate such instabilities when operated with sufficient strength.

1.2 Basic macro feedback chains. A reconsideration

The Mundell effect

The investigation of destabilizing macrodynamic feedback chains has indeed never been at the center of interest of mainstream macroeconomic analysis, though knowledge about these feedback chains dates back to the beginning of dynamic Keynesian analysis. Tobin has presented summaries and modeling of such feedback chains on various occasions (see in particular Tobin, 1975, 1989 and 1993). The well-known Keynes effect as well as Pigou effect are however often present in macrodynamic analysis, since they have the generally appreciated property of being stabilizing with respect to wage inflation as well as wage deflation. Also well-known, but rarely taken serious, is the so-called Mundell effect based the impact of inflationary expectations on investment as well as consumption demand. Tobin (1975) was the first who modeled this effect in a 3D dynamic framework (see Scarth, 1996 for a textbook treatment of Tobin's approach). Yet, though an integral part of traditional Keynesian IS-LM-PC analysis, the role of the Mundell is generally played down as for example in Romer (1996, p.237) where it only appears in the list of problems, but not as part of his presentation of traditional Keynesian theories of fluctuations in his chapter 5.

Figure 1 provides a brief characterization of the destabilizing feedback chain underlying the Mundell effect. We consider here the case of wage and price inflation (though deflation may be the more problematic case, since there is an obvious downward floor to the evolution of the nominal rate of interest (and the working of the well-known Keynes effect) which, however, in the partial reasoning that follows is kept constant by assumption).

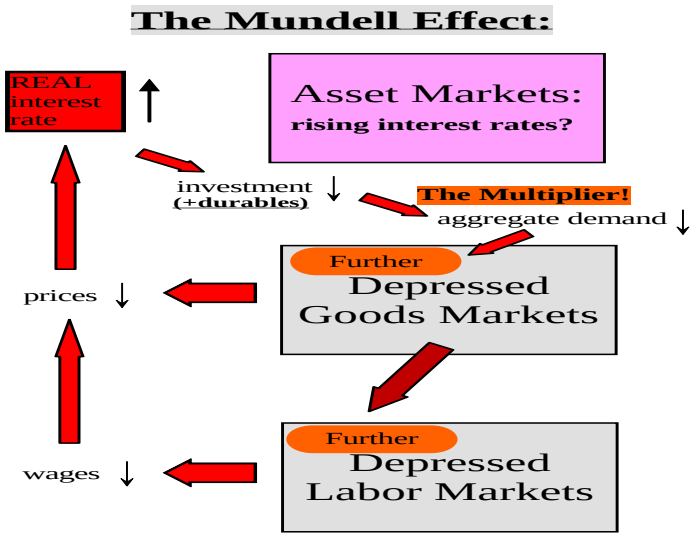

Figure 1 Destabilizing Mundell effects.

For a given nominal rate of interest, increasing inflation (caused by an increasing activity level of the economy) by definition leads to a decrease of the real rate of interest. This stimulates demand for investment and consumer durables even further and thus leads, via the multiplier process to further increasing economic activity in both the goods and the labor markets, adding further momentum to the ongoing inflationary process. In the absence of ceilings to such an inflationary spiral, economic activity will increase to its limits and generate an ever accelerating inflationary spiral eventually. This standard feedback chain of traditional Keynesian IS-LM-PC analysis is however generally neglected and has thus not really been considered in its interaction with the stabilizing Keynes- and Pigou effect, with works based on the seminal paper of Tobin (1975) being the exception (see Groth, 1993, for a brief survey on this type of literature).

Far more neglected is however an – in principle – fairly obvious real wage adjustment mechanism that was first investigated analytically in Rose (1967) with respect to its local and global stability implications (see also Rose, 1990). Due to this heritage, this type of effect has been called Rose effect in Chiarella and Flaschel (2000), there investigated in its interaction with the Keynes- and the Mundell effect, and the Metzler inventory accelerator, in a 6D Keynesian model of goods and labor market disequilibrium. In the present paper we intend to present and analyze the working of this effect in a very simple IS growth model – without the LM curve as in Romer (2000) – and thus with a direct interest rate policy in the place of indirect money supply targeting and its use of the Keynes effect (based on stabilizing shifts of the conventional LM-curve). We classify theoretically and estimate empirically the types of Rose effects that are at work, the latter for the case of the US economy.

Stabilizing or destabilizing Rose effects?

Rose effects are present if the income distribution is allowed to enter the formation of Keynesian effective demand and if wage dynamics is distinguished from price dynamics, both aspects of macrodynamics that are generally neglected at least in the theoretical macroeconomic literature. This may explain why Rose effects are rarely present in the models used for policy analysis and policy discussions.

Rose effects are however of great interest and have been present since long – though unnoticed and not in full generality – in macroeconometric model building, where wage and price inflation on the one hand and consumption and investment behavior on the other hand are generally distinguished from each other. Rose effects allow for at least four different cases depending on whether consumption demand responds stronger than investment demand to real wage changes (or vice versa) and whether – broadly speaking – wages are more flexible than prices with respect to the demand pressures on the market for labor and for goods, respectively. The figures 2 and 3 present two out of the four possible cases, all based on the assumption that consumption demand depends positively and investment demand negatively on the real wage (or the wage share if technological change is present).

In figure 2 we consider first the case where the real wage dynamics taken by itself is stabilizing. Here we present the case where wages are more flexible with respect to demand pressure (in the market for labor) than prices (with respect to demand pressure in the market for goods) and where investment responds stronger than consumption to changes in the real wage. We consider again the case of inflation. The case of deflation is of course of the same type with all shown arrows simply being reversed. Nominal wages rising faster than prices means that real wages are increasing when activity levels are high. Therefore, investment is depressed more

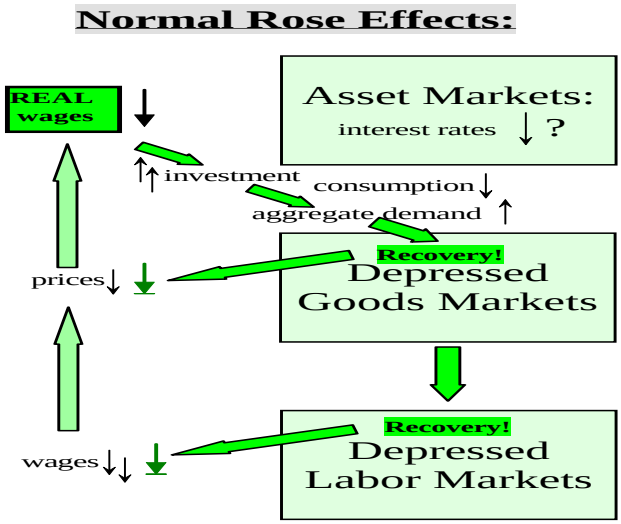

Figure 2 Normal Rose effects.

than consumption is increased, giving rise to a decrease in aggregate as well as effective demand. The situation on the market for goods – and on this basis also on the market for labor – is therefore deteriorating, implying that forces come into being that stop the rise in wages and prices eventually and that may – if investigated formally – lead the economy back to the position of normal employment and stable wages and prices.

The stabilizing forces just discussed however become destabilizing if price adjustment speeds are reversed and thus prices rising faster than nominal wages, see figure 3. In this case, we get falling real wages and thus – on the basis of the considered propensities to consume and invest with respect to real wage changes – further increasing aggregate and effective demand on the goods market which is transmitted into further rising employment on the market for labor and thus into even faster rising prices and (in weaker form) rising wages. This adverse type of real wage adjustment or simply adverse Rose effect can go on for ever if there is no nonlinearity present that modifies either investment or consumption behavior or wage and price adjustment speeds such that normal Rose effects are established again, though of course supply bottlenecks may modify this simple positive feedback chain considerably.[1]

Since the type of Rose effect depends on the relative size of marginal propensities to consume and to invest and on the flexibility of wages vs. that of prices we are confronted with a question that demands for empirical estimation. Furthermore, Phillips curves for wages and prices have to be specified in more detail than discussed so far, in particular due to the fact that also cost pressure and expected cost pressure do matter in them, not only demand pressure on the market for goods and for labor. These specifications will lead to the result that also the degree of short-sightedness of wage earners and of firms will matter in the following discussion of Rose effects. Our empirical findings in this regard will be that wages are considerably more flexible than prices with respect to demand pressure, and workers roughly equally short-sighted as

[1]The type of Rose effect shown in figure 3 may be considered as the one that characterizes practical macro-wisdom which generally presumes that prices are more flexible than wages and that IS goods market equilibrium – if at all – depends negatively on real wages. Our empirical findings show that both assumptions are not confirmed, but indeed both reversed by data of the US economy, which taken together however continues to imply that empirical Rose effects are adverse in nature.

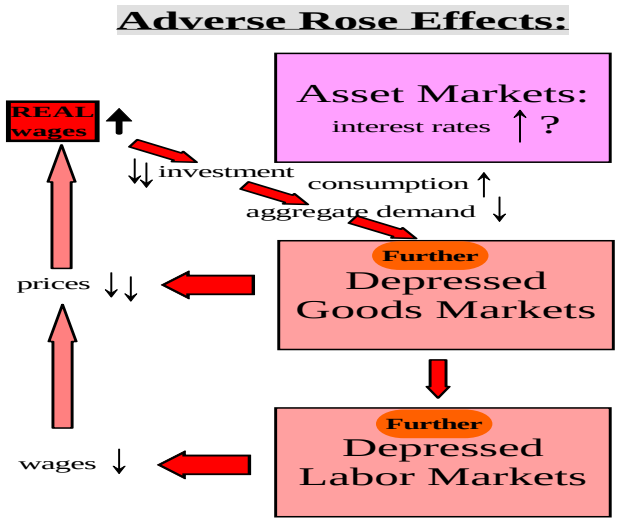

Figure 3 Adverse Rose effects..

firms with respect to cost pressure. On the basis of the assumption that consumption is more responsive than investment to temporary real wage changes, we then get that all arrows and hierarchies shown in figure 3 will be reversed. We thus get by this twofold change in the figure 3 again *an adverse Rose effect* in the interaction of income distribution dependent changes in goods demand with wage and price adjustment speeds on the market for labor and for goods.

1.3 Outline of the paper

In view of the above hypothesis, the paper is organized as follows. Section 2 presents a simple Keynesian macrodynamic model where advanced wage and price adjustment rules are introduced and in the center of the considered model and where – in addition – income distribution and real rates of interest matter in the formation of effective goods demand. We then investigate some stability implications of this macrodynamic model, there for the case where Rose effects are stabilizing, as in figure 2, due to an assumed dominance of investment behavior in effective demand and to sluggish price dynamics as well as sluggish inflationary expectations, concerning what we will call the inflationary climate surrounding the perfectly foreseen current inflation rate. We thus consider the joint occurrence of stable Rose and weak Mundell effects, but still do not find stability of the steady growth path in such a situation. A standard type of interest rate policy rule[2]is therefore subsequently introduced to enforce convergence to the steady state, indeed also for fast revisions of inflationary expectations and thus stronger destabilizing Mundell effects. Section 3 investigates empirically whether the type of Rose effect assumed in section 2 is really the typical one. We find evidence (in the case of the US economy) that wages are indeed more flexible than prices. Increasing wage flexibility is thus bad for economic stability (while price flexibility is not) when coupled with the observation that consumption demand responds stronger than investment demand to temporary real wage changes.

In section 4, this type of destabilizing Rose effect is then incorporated into our small macro-

[2]The discussion of such interest rate or Taylor policy rules originates from Taylor (1993), see Taylor (1999a), for a recent debate of such monetary policy rules and Clarida and Gertler (1998) for an empirical study of Taylor feedback rules in selected OECD countries.

dynamic model and the question of whether and which type of interest rate policy can stabilize the economy in such a situation is reconsidered. We find that a standard Taylor interest rate rule is not sufficient due to its specific tailoring that only allows to combat the Mundell type feedback chain – which it indeed can fight successfully. In case of a destabilizing Rose or real wage effect the tailoring of such a Taylor rule must be reflected again in order to find out what type of rule can fight such Rose effects. We here first reintroduce wage share effects considered by Blanchard and Katz (1999) into the money-wage Phillips curve which – when sufficiently strong – will stabilize a system operating under standard Taylor rule. Alternatively, however, the Taylor rule can be modified to include an income distribution term, which enforces convergence in the case where the wage share effect in the money wage Phillips curve is too weak to guarantee this.

We conclude that the role of income distribution in properly formulated wage-price spirals represents an important topic that is very much neglected in the modern discussion of inflation, disinflation and deflation.

2 A model of the wage–price spiral

This section briefly presents an elaborate form of the wage-price dynamics or the wage-price spiral and a simple theory of effective goods demand, which however gives income distribution a role in the growth dynamics derived from these building blocks. The presentation of this model is completed with respect to the budget equations for the four sectors of the model in the appendix A.1 to this paper. The wage-price spiral will be estimated, using US data, in section 3 of the paper.

2.1 The wage-price spiral

At the core of the dynamics to be modeled, estimated and analyzed in this and the following sections is the description of the money wage and price adjustment processes. They are provided by the following equations (1) and (2):

$$\hat{w} = \beta_{w_1}(\bar{U}^l - U^l) - \beta_{w_2}(u - u_o) + \kappa_w(\hat{p} + n_x) + (1 - \kappa_w)(\pi + n_x), \quad (1)$$

$$\hat{p} = \beta_{p_1}(\bar{U}^c - U^c) + \beta_{p_2}(u - u_o) + \kappa_p(\hat{w} - n_x) + (1 - \kappa_p)\pi. \quad (2)$$

In these equations for wage inflation $\hat{w} = \dot{w}/w$ and price inflation $\hat{p} = \dot{p}/p$ we denote by U^l and U^c the rate of unemployment of labor and capital, respectively, and by n_x the rate of Harrod–neutral technological change. u is the wage share, $u = wL^d/pY$.

Demand pressure in the market for labor is characterized by deviations of the rate of unemployment U^l from its NAIRU level $\bar{U}^l$. Similarly demand pressure in the market for goods is represented by deviations of the rate of underemployment U^c of the capital stock K from its normal underemployment level $\bar{U}^c$, assumed to be fixed by firms. Wage and price inflation are therefore first of all driven by their corresponding demand pressure terms.

With respect to the role of the wage share u, which augments the Phillips curves by the terms $\beta_{w_2}(u - u_o)$ and $\beta_{p_2}(u - u_o)$, we assume that increasing shares will dampen the evolution of wage inflation and give further momentum to price inflation (see Franke, 2001, for details of the effects of a changing income distribution on demand driven wage and price inflation).

As far as the money-wage Phillips curve is concerned, this corresponds to the error-correction mechanism in Blanchard and Katz (1999). In appendix A.2, we motivate this assumption within a wage-bargaining model. A similar, though less strong formulation has been proposed by Ball and Mofitt (2001), who – based on fairness considerations – integrate the difference between productivity growth and an average of past real-wage growth in a wage-inflation Phillips curve.

In addition to demand pressure we have also cost–pressure terms in the laws of motions for nominal wages and prices, of crossover type and augmented by productivity change in the case of wages and diminished by productivity change in the case of prices. As the wage–price dynamics are formulated we assume that myopic perfect foresight prevails, of workers with respect to their measure of cost pressure, $\hat{p}$, and of firms with respect to wage pressure, $\hat{w}$. In this respect we follow the rational expectations school and disregard model–inconsistent expectations with respect to short-run inflation rates. Yet, in the present framework, current inflation rates are not the only measuring root for cost pressure, so they enter wage and price inflation only with weight $\kappa_w \in [0,1]$ and $\kappa_p \in [0,1]$, respectively, and $\kappa_w \kappa_p < 1$. In addition, both workers and firms (or at least one of them) look at the inflationary climate surrounding current inflation rates.

A novel element in such cost-pressure terms is here given by the term π, representing the inflationary climate in which current inflation is embedded. Since the inflationary climate envisaged by economic agents changes sluggishly, information about macroeconomic conditions diffuses slowly through the economy (see Mankiw and Reis, 2001), wage and price are set staggered (see Taylor, 1999b), it is not unnatural to assume that agents, in the light of past inflationary experience, update π by an adaptive rule. In the theoretical model,[3] we assume that the medium-run inflation beliefs are updated adaptively in the standard way:

$$\dot{\pi} = \beta_\pi(\hat{p} - \pi). \tag{3}$$

In two appendices A.2, A.3 we provide some further justifications for the two Phillips curves here assumed to characterize the dynamics of the wage and the price level. Note that the inflationary climate expression has often been employed in applied work by including lagged inflation rates in price Phillips curves, see Fair (2000) for example. Here however it is justified from the theoretical perspective, separating temporary from permanent effects, where temporary changes in both price and wage inflation are even perfectly foreseen. We show in this respect in section 2.4 that the interdependent wage and price Phillips curves can however be solved for wage and price inflation explicitly, giving rise to two reduced form expressions where the assumed perfect foresight expressions do not demand for forward induction.

For the theoretical investigation, the dynamical equations (1)-(3) representing the laws of motion of w, p and π are part of a complete growth model to be supplemented by simple expressions for production, consumption and investment demand and – due to the latter – also by a law of motion for the capital stock. These equations will allow the discussion of so–called Mundell and Rose effects in the simplest way possible and are thus very helpful in isolating these effects from other important macrodynamic feedback chains which are not the subject of this paper. The econometric analysis to be presented in the following section will focus on the empirical counterparts of the Phillips curves (1) and (2) while conditioning on the other macroeconomic variables which enter these equations.

[3] In the empirical part of the paper we will simplify these calculations further by measuring the inflationary climate variable π as a 12 quarter moving average of $\hat{p}$.

2.2 Technology

In this and the next subsection we complete our model of the wage price spiral in the simples way possible to allow for the joint occurrence of Mundell and Rose effects in the considered economy.

For the sake of simplicity we employ in this paper a fixed proportions technology:[4]

$$y^p = Y^p/K = \text{ const. }, x = Y/L^d, \hat{x} = \dot{x}/x = n_x = \text{ const.}$$

On the basis of this, the rates of unemployment of labor and capital can be defined as follows:

$$\begin{aligned} U^l &= \frac{L - L^d}{L} = 1 - \frac{Y}{xL} = 1 - yk \\ U^c &= \frac{Y^p - Y}{Y^p} = 1 - \frac{Y}{Y^p} = 1 - y/y^p \end{aligned}$$

where y denotes the output–capital ratio Y/K and $k = K/(xL)$ a specific measure of capital–intensity or the full employment capital - output ratio. We assume Harrod–neutral technological change: $\hat{y}^p = 0, \hat{x} = n_x =$ const., with a given potential output-capital ratio y^p and labor productivity $x = Y/L^d$ growing at a constant rate. We have to use k in the place of K/L, the actual full employment capital intensity, in order to obtain state variables that allow for a steady state later on.

2.3 Aggregate goods demand

As far as consumption is concerned we assume Kaldorian differentiated saving habits of the classical type ($s_w = 1 - c_w = 1 - c \geq 0, s_c = 1$), i.e., real consumption is given by:

$$C = cuY = c\omega L^d, \quad u = \omega/x, \omega = w/p \quad \text{the real wage} \tag{4}$$

and thus solely dependent on the wage share u and economic activity Y. For the investment behavior of firms we assume

$$\frac{I}{K} = i\left((1-u)y - (r-\pi)\right) + n, \quad y = \frac{Y}{K}, n = \hat{L} + \hat{x} = n + n_x \quad \text{trend growth} \tag{5}$$

The rate of investment is therefore basically driven by the return differential between $\rho = (1-u)y$, the rate of profit of firms and $r - \pi$, the real rate of interest on long–term bonds (consols), only considered in its relation to the budget restrictions of the four sectors of the model (workers, asset-holders, firms and the government) in appendix A.1 to this paper.[5]

This financial asset is needed for the generation of Mundell (or real rate of interest) effects in the model, which as we will show later can be neutralized by a Taylor-rule.

Besides consumption and investment demand we also consider the goods demand G of the government where we however for simplicity assume $g = G/K =$const., since fiscal policy is not a topic of the present paper.

[4] We neglect capital stock depreciation in this paper.

[5] We consider the long-term rate r as determinant of investment behavior in this paper, but neglect here the short-term rate and its interaction with the long-term rate – as it is for example considered in Blanchard and Fisher (1999, 10.4) – in order to keep the model concentrated on the discussion of Mundell and Rose effects. We thus abstract from dynamical complexities caused by the term structure of interest rates. Furthermore, we do not consider a climate expression for the evolution of nominal interest, in contrast to our treatment of inflation, in order to restrict the dynamics to dimension 3.

2.4 The laws of motion

Due to the assumed demand behavior of households, firms and the government we have as representation of goods–market equilibrium in per unit of capital form ($y = Y/K$):

$$cuy + i((1-u)y - (r-\pi)) + n + g = y \tag{6}$$

and as law of motion for the full–employment capital–output ratio $k = K/(xL)$:

$$\hat{k} = i((1-u)y - (r-\pi)). \tag{7}$$

Equations (1), (2) furthermore give in reduced form the two laws of motion (8), (9), with $\kappa = (1-\kappa_w\kappa_p)^{-1}$:

$$\hat{u} = \kappa\left[(1-\kappa_p)\left\{\beta_{w_1}(\bar{U}^l - U^l) - \beta_{w_2}(u-u_o)\right\} - (1-\kappa_w)\left\{\beta_{p_1}(\bar{U}^c - U^c) + \beta_{p_2}(u-u_o)\right\}\right] \tag{8}$$

$$\hat{p} = \pi + \kappa\left[\beta_{p_1}(\bar{U}^c - U^c) + \beta_{p_2}(u-u_o) + \kappa_p\left\{\beta_{w_1}(\bar{U}^l - U^l) - \beta_{w_2}(u-u_o)\right\}\right] \tag{9}$$

The first equation describes the law of motion for the wage share u which depends positively on the demand pressure items on the market for labor (for $\kappa_p < 1$) and negatively on those of the market for goods (for $\kappa_w < 1$).[6] The second equation is a reduced form price Phillips curve which combines all demand pressure related items on labor and goods market in a positive fashion (for $\kappa_p > 0$). This equation is far more advanced than the usual price Phillips curve of the literature.[7] Inserted into the adaptive revision rule for the inflationary climate variable it provides as further law of motion the dynamic equation

$$\dot{\pi} = \beta_\pi\kappa\left[\beta_{p_1}(\bar{U}^c - U^c) + \beta_{p_2}(u-u_o) + \kappa_p\left\{\beta_{w_1}(\bar{U}^l - U^l) - \beta_{w_2}(u-u_o)\right\}\right] \tag{10}$$

We assume for the time being that the interest rate r on long-term bonds is kept fixed at its steady-state value r^o and then get that equations (7), (8) and (10), supplemented by the static goods market equilibrium equation (5), provide an autonomous system of differential equations in the state variables u, k and π.

It is obvious from equation (8) that the error correction terms β_{w_2}, β_{p_2} exercise a stabilizing influence on the adjustment of the wage share (when this dynamic is considered in isolation). The other two $\beta-$terms (the demand pressure terms), however, do not give rise to a clear-cut result for the wage share subdynamic. In fact, they can be reduced to the following expression as far as the influence of economic activity, as measured by y, is concerned (neglecting irrelevant constants):

$$\kappa\left[(1-\kappa_p)\beta_{w_1}k - (1-\kappa_w)\beta_{p_1}/y^p\right]\cdot y$$

In the case where output y depends negatively on the wage share u we thus get partial stability for the wage share adjustment (as in the case of the error correction terms) if and only if the term in square brackets is negative (which is the case for β_{w_1} sufficiently large). We have called this a normal Rose effect in section 1, which in the present case derives – broadly speaking – from investment sensitivity being sufficiently high and wage flexibility dominance.

[6] The law of motion (8) for the wage share u is obtained by making use in addition of the following reduced form equation for $\hat{w}$ which is obtained simultaneously with the one for $\hat{p}$ and of a very similar type:
$\hat{w} = \pi + \kappa\left[\beta_{w_1}(\bar{U}^l - U^l) - \beta_{w_2}(u-u_o) + \kappa_w\left\{\beta_{p_1}(\bar{U}^c - U^c) + \beta_{p_2}(u-u_o)\right\}\right]$.

[7] Note however that this reduced form Phillips curve becomes formally identical to the one normally investigated empirically, see Fair (2000) for example, if $\beta_{w_2}, \beta_{p_2} = 0$ holds and if Okun's law is assumed to hold (i.e. the utilization rates of labor and capital are perfectly correlated). However, even then the estimated coefficients are far away from representing labor market characteristics solely.

In the case where output y depends positively on u, where therefore consumption is dominating investment with respect to the influence of real wage changes, we need a large β_{p_1}, and thus a sufficient degree of price flexibility relative to the degree of wage flexibility, to guarantee stability from the partial perspective of real wage adjustments. For these reasons we will therefore call the condition

$$\alpha \;=\; (1-\kappa_p)\beta_{w_1}k_o - (1-\kappa_w)\beta_{p_1}/y^p \left\{ \begin{array}{c} < \\ > \end{array} \right\} 0 \iff \left\{ \begin{array}{c} \text{normal} \\ \text{adverse} \end{array} \right\} \text{Rose effects} \qquad (\alpha)$$

the critical or α condition for the occurrence of *normal (adverse) Rose effects*, in the case where the flexibility of wages (of prices) with respect to demand pressure is dominating the wage-price spiral (including the weights concerning the relevance of myopic perfect foresight). In the next section we will provide estimates for this critical condition in order to see which type of Rose effect might have been the one involved in the business fluctuations of the US economy in the post-war period.

Note finally with respect to equation (9) and (10) that $\dot{\pi}$ always depends positively on y and thus on π, since y always depends positively on π. This latter dependence of accelerator type as well as the role of wage share adjustments will be further clarified in the next subsection.

2.5 The effective demand function

The goods–market equilibrium condition (6) can be solved for y and gives

$$y = \frac{n+g-i(r_o-\pi)}{(1-u)(1-i)+(1-c)u}. \qquad (11)$$

We assume $i \in (0,1), c \in (0,1]$ and consider only cases where $u < 1$ is fulfilled which, in particular, is true close to the steady state. This implies that the output–capital ratio y depends positively on π.

Whether y is increasing or decreasing in the labor share u depends on the relative size of c and i.
In the case of $c = 1$, we get the following dependencies:

$$\begin{aligned} y_u &= \frac{(n+g-i(r_o-\pi))(i-1)}{[(1-u)(1-i)]^2} = \frac{y}{1-u}, \\ \rho_u &= -y-(1-u)y_u = 0. \end{aligned}$$

As long as y is positive and u smaller than one, we get a positive dependence of y on u. The rate of profit ρ is independent of the wage share u due to a balance between the negative cost and the positive demand effect of the wage share u.[8]

Otherwise, i.e. if the consumption propensity out of wage income is strictly less than one, $c < 1$, we have that

$$y_u = \frac{(c-i)y}{(1-i)(1-u)+(1-c)u} \geq 0 \quad \text{iff} \quad c \geq i, \qquad (12)$$

$$\rho_u = -y+(1-u)y_u < 0, \qquad (13)$$

[8]We note that the investment function can be modified in various ways, for example by inserting the normal-capacity-utilization rate of profit $\rho^n = (1-u)(1-\bar{U}^c)y^p$ into it in the place of the actual rate ρ, which then always gives rise to a negative effect of u on this rate ρ^n and also makes subsequent calculations simpler. Note here also that we only pursue local stability analysis in this paper and thus work for reasons of simplicity with linear functions throughout.

where the result for the rate of profit $\rho = (1-u)y$ of firms follows from the fact that y_u clearly is smaller than $y/(1-u)$.

Therefore, if a negative relationship between the rate of return and the wage share is desirable (given the investment function defined in equation 7), then for the workers consumption function, the assumption $c < 1$ is required: $C/K = cuy, c \in (0,1)$.

2.6 Stability issues

We consider in this subsection the fully interacting, but somewhat simplified 3D growth dynamics of the model which consist the following three laws of motion (14) – (16) for the wage share u, the full employment capital-output ratio k and the inflationary climate π:[9]

$$\begin{aligned}
\hat{u} &= \kappa[(1-\kappa_p)(\beta_{w_1}(\bar{U}^l - U^l) - \beta_{w_2}(u-u_o)) - (1-\kappa_w)\beta_{p_1}(\bar{U}^c - U^c)], && (14)\\
\hat{k} &= i((1-u)y - (r-\pi)), && (15)\\
\dot{\pi} &= \beta_\pi \kappa[\beta_{p_1}(\bar{U}^c - U^c) + \kappa_p(\beta_{w_1}(\bar{U}^l - U^l) - \beta_{w_2}(u-u_o))], && (16)
\end{aligned}$$

where $U^l = 1 - yk$ and $U^c = 1 - y/y^p$.

During this section, we will impose the following set of assumptions:[10]

(A.1) The marginal propensity to consume is strictly less than the one to invest: $0 < c < i$.
(A.2) The money-wage Phillips curve is not error-correcting w.r.t. the wage share: $\beta_{w_2} = 0$.
(A.3) The parameters satisfy that $u_o \in (0,1)$ and $\pi_o \geq 0$ hold in the steady state.
(A.4a) The nominal interest rate r is constant: $r = r_o$.
(A.4b) There is an interest rate policy rule in operation which is of the type:
$r = \rho_o + \pi + \beta_r(\pi - \bar{\pi})$
with $\beta_r > 0$, ρ_o the steady-state real rate of interest, and $\bar{\pi}$ the inflation target.

Assumption (A.1) implies that (i) $y_u < 0$ as in (12), (ii) $U^l_u > 0$ and $U^c_u > 0$ since the negative effect of real wage increases on investment outweighs the positive effect on consumption, and (iii) $\rho_u < 0$ with $\rho = (1-u)y$ (the alternative scenario with $c > i$ is considered in section 4). (A.2) excludes the potentially stabilizing effects of the Blanchard-Katz-type error-correction mechanism (will be discussed in section 4.2 for the money-wage Phillips curve). (A.3) ensures the existence of an interior steady state. Assumptions (A.4a) and (A.4b) stand for different monetary regimes and determine the nominal interest rate in (15) and the algebraic equation for the effective demand which supplements the 3D dynamics.

For the neutral monetary policy defined in (A.4a), we have that output y is an increasing function of the inflationary climate π:

$$y = \frac{n + g + i(\pi - r_o)}{(1-u)(1-i) + (1-c)u}. \qquad (17)$$

[9] We therefore now assume – for reasons of simplicity – that $\beta_{p_2} = 0$ holds throughout, a not very restrictive assumption in the light of what is shown in the remainder of this paper. Note here that two of the three laws of motion (for the wage share and the inflationary climate) are originating from the wage-price spiral considered in this paper, while the third one (for the capital output ratio) represents by and large the simplest addition possible to arrive at a model on the macro level that can be considered complete.

[10] In section 4 we will relax these assumptions in various ways.

By contrast, assumption (A.4b), the adoption of a Taylor interest rate policy rule, implies that the static equilibrium condition is given by

$$y = \frac{n+g-i(\rho_0+\beta_r(\pi-\bar{\pi}))}{(1-i)(1-u)+(1-c)u}. \tag{18}$$

which implies a negative dependence of output y on the inflationary climate π.[11]

Proposition 1. (The Unique Interior Steady State Position)
Under assumptions (A.1) - (A.4a), the interior steady state of the dynamics (14) – (16) is uniquely determined and given by

$$y_0 = (1-\bar{U}^c)y^p, \quad k_0 = (1-\bar{U}^l)/y_o, \quad u_0 = 1/c+(n+g)/y_0, \quad \rho_o = (1-u_o)y_o.$$

Steady-state inflation in the constant nominal interest regime (A.4a) is given by:

$$\pi_o = r_o - (1-u_o)y_o,$$

and under the interest rule (A.4b) we have that:

$$\pi_o = \bar{\pi}, \quad r_o = \rho_o + \bar{\pi}$$

holds true.

The proof of proposition 1 is straightforward. The proofs of the following propositions are in the mathematical appendix A.5.

■

The steady state solution with constant nominal interest rate (A.4a) shows that the demand side has no influence on the long-run output-capital ratio, but influences the income distribution and the long-run rate of inflation. In the case of an adjusting nominal rate of interest (A.4b), the steady state rate of inflation is determined by the monetary authority and its steering of the nominal rate of interest, while the steady-state rate of interest is obtained from the steady rate of return of firms and the inflationary target of the central bank.

Proposition 2. (Private Sector Instability)
Under assumptions (A.1) - (A.4a), the interior steady state of the dynamics (14) – (16) is essentially repelling (exhibits at least one positive root), even for small parameters β_{p_1}, β_π.

A normal Rose effect (stability by wage flexibility and instability by price flexibility in the considered case $c < i$) and a weak Mundell effect (sluggish adjustment of prices and of the inflationary climate variable) are thus not sufficient to generate convergence to the steady state.[12]

[11] Note that our formulation of a Taylor rule ignores the influence of a variable representing the output gap. Including the capacity utilization gap of firms would however only add a positive constant to the denominator of the fraction just considered and would therefore not alter our results in a significant way. Allowing for the output gap in addition to the inflation gap may also be considered as some sort of double counting.

[12] In the mathematical appendix A.5, it is shown that the carrier of the Mundell effect, $\dot{\pi}_\pi$, will always give the wrong sign to the determinant of the Jacobian of the dynamics at the steady state.

Proposition 3. (Interest Rate Policy and Stability)
Under assumptions (A.1) - (A.3), the interest rule in (A.4b) implies asymptotic stability of the steady state for any given adjustment speeds $\beta_\pi > 0$ if the price flexibility parameter β_{p_1} is sufficiently small.

As long as price flexibility does not give rise to an adverse Rose effect (dominating the trace of the Jacobian of the dynamics at the steady state), we get convergence to the steady state by monetary policy and the implied adjustments of the long-term real rate of interest $r - \pi$ which increase r beyond its steady state value whenever the inflationary climate exceeds the target value $\bar{\pi}$ and *vice versa*. The present stage of the investigation therefore suggests that wage flexibility (relative to price flexibility), coupled with the assumption $c > i$ and an active interest rate policy rule is supporting macroeconomic stability. The question however is whether this is the situation that characterizes factual macroeconomic behavior.

An adverse Rose effect (due to price flexibility and $c < i$) would dominate the stability implications of the considered dynamics: the system would then lose its stability by way of a Hopf–bifurcation when the reaction parameter β_r of the interest rate rule is made sufficiently small. However, we will find in the next section that wages are more flexible than prices with respect to demand pressure on their respective markets. We thus have in the here considered case $c < i$ that the Rose effect can be neglected (as not endangering economic stability), while the destabilizing Mundell effect can indeed be tamed by an appropriate monetary policy rule.

3 Estimating the US wage-price spiral

In this section we analyze US post-war data to provide an estimate of the two Phillips curves that form the core of the dynamical model introduced in section 2. Using *PcGets* (see Hendry and Krolzig, 2001), we start with a general, dynamic, unrestricted, linear model of $\hat{w} - \pi_t$ and $\hat{p} - \pi_t$ which is conditioned on the explanatory variables predicted by the theory and use the general-to-specific approach to find an undominated parsimonious representation of the structure of the data. From these estimates, the long-run Phillips curves can be obtained which describe the total effects of variables and allow a comparison to the reduced form of the wage-price spiral in (1) and (2).

3.1 Data

The data are taken from the Federal Reserve Bank of St. Louis (see http://www.stls.frb.org/fred). The data are quarterly, seasonally adjusted and are all available from 1948:1 to 2001:2. Except for the unemployment rates of the factors labor, U^l, and capital, U^c, the log of the series are used (see table 1).

For reasons of simplicity as well as empirical reasons, we measure the inflationary climate surrounding the current working of the wage-price spiral by an unweighted 12-month moving average:

$$\pi_t = \frac{1}{12} \sum_{j=1}^{12} \Delta p_{t-j}.$$

This moving average provides a simple approximation of the adaptive expectations mechanism (3) considered in section 2, which defines the inflation climate as an infinite, weighted moving average of past inflation rates with declining weights. The assumption here is that people

Table 1 Data.

Variable	Transformation	Mnemonic	Description of the untransformed series
U^l	UNRATE/100	UNRATE	Unemployment Rate
U^c	1−CUMFG/100	CUMFG	Capacity Utilization: Manufacturing. Percent of Capacity
w	log(COMPNFB)	COMPNFB	Nonfarm Business Sector: Compensation Per Hour, 1992=100
p	log(GNPDEF)	GNPDEF	Gross National Product: Implicit Price Deflator, 1992=100
$y - l^d$	log(OPHNFB)	OPHNFB	Nonfarm Business Sector: Output Per Hour of All Persons, 1992=100
u	$\log\left(\frac{\text{COMPRNFB}}{\text{OPHNFB}}\right)$	COMPRNFB	Nonfarm Business Sector: Real Compensation Per Hour, 1992=100

Note that w, p, l^d, y, u now denote the logs of wages, prices, employed labor, output and the wage share (1992=1) so that first differences can be used to denote their rates of growth. Similar results are obtained when measuring the wage share as unit labor costs (nonfarm business sector) adjusted by the GNP deflator.

apply a certain window (three years) to past observations, here of size, without significantly discounting.

The data to be modeled are plotted in figure 4. The estimation sample is 1955:1 - 2001:2 which excludes the Korean war. The number of observations used for the estimation is 186.

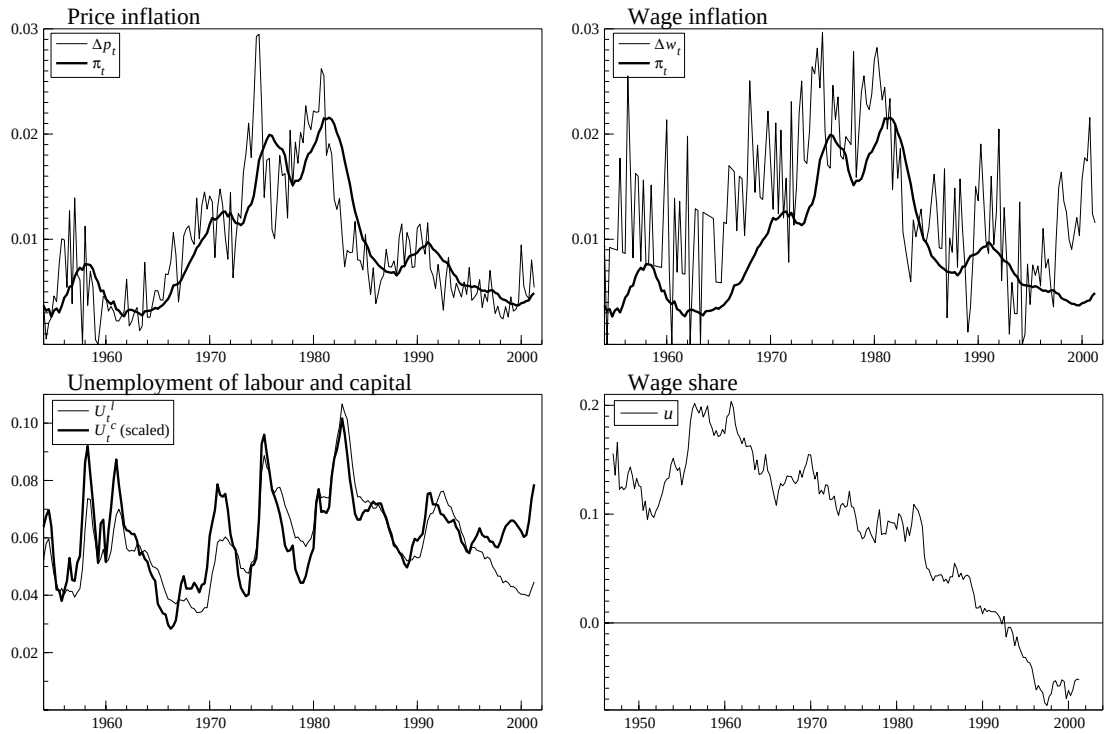

Figure 4 Price and Wage Inflation, Unemployment and the Wage Share.

3.2 The money-wage Phillips curve

Let us first provide an estimate of the wage Phillips curve (1) of this paper: We model wage inflation in deviation from the inflation climate, $\Delta w - \pi$, conditional on its own past, the history of price inflation, $\Delta p - \pi$, measured by the same type of deviations, overall labor productivity growth, $\Delta y - \Delta l^d$, the unemployment rate, U^l, and the log of the labor share, $u = w + l^d - p - y$,

by means of the equation (19):

$$\begin{aligned}\Delta w_t - \pi_t &= \nu_w + \sum_{j=1}^{5} \gamma_{wwj} \left(\Delta w_{t-j} - \pi_{t-j}\right) + \sum_{j=1}^{5} \gamma_{wpj} \left(\Delta p_{t-j} - \pi_{t-j}\right) \\ &\quad + \sum_{j=1}^{5} \gamma_{wxj} \left(\Delta y_{t-j} - \Delta l^d_{t-j}\right) + \sum_{j=1}^{5} \gamma_{wuj} U^l_{t-j} + \alpha_w u_{t-1} + \varepsilon_{wt}, \qquad (19)\end{aligned}$$

where ε_{wt} is a white noise process. The general model explains 43.7% of the variation of $\Delta w_t - \pi_t$ reducing the standard error in the prediction of quarterly changes of the wage level to 0.467%:

RSS	0.003551	$\hat{\sigma}$	0.004653	R^2	0.4373	$\bar{R}^2$	0.3653
$\ln L$	1011	AIC	−10.6298	HQ	−10.4752	SC	−10.2482

Almost all of the estimated coefficients of (19) are statistically insignificant and therefore not reported here. This highlights the idea of the general-to-specific (*Gets*) approach (see Hendry, 1995, for an overview of the underlying methodology) of selecting a more compact model, which is nested in the general but provides an improved statistical description of the economic reality by reducing the complexity of the model and checking the contained information. The *PcGets* reduction process is designed to ensure that the reduced model will convey all the information embodied in the unrestricted model (which is here provided by equation 19). This is achieved by a joint selection and diagnostic testing process: starting from the unrestricted, congruent general model, standard testing procedures are used to eliminate statistically-insignificant variables, with diagnostic tests checking the validity of reductions, ensuring a congruent final selection.

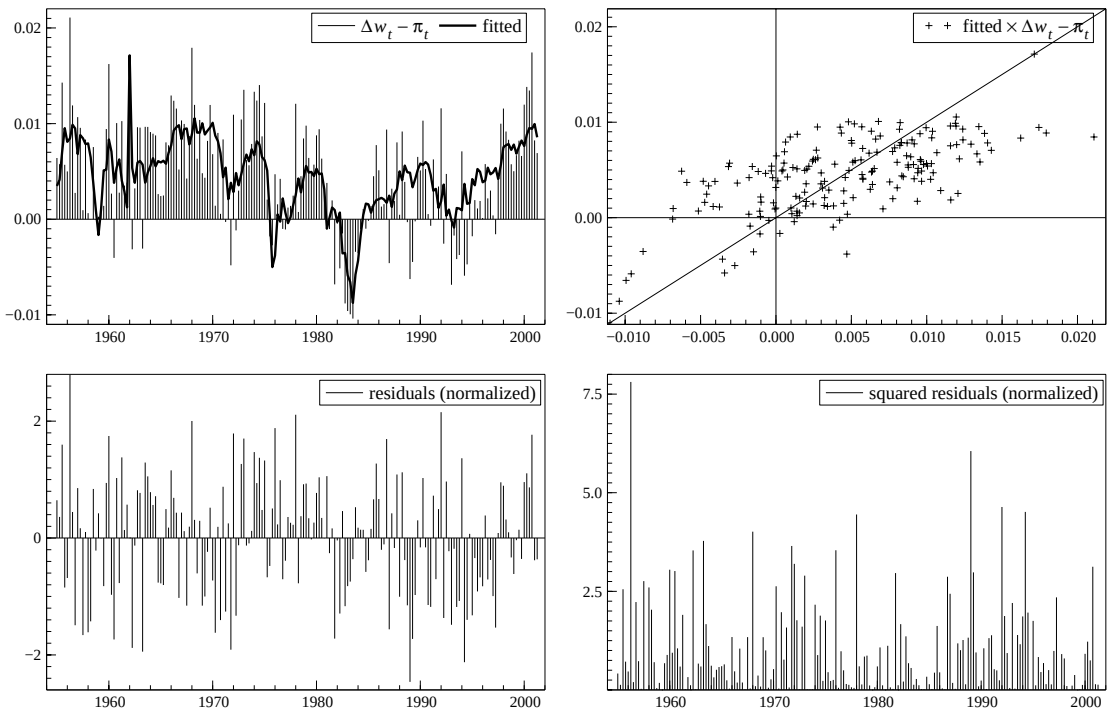

Figure 5 Money Wage Phillips curve.

In the case of the general wage Phillips curve in (19), *PcGets* reduces the number of coefficients from 22 to only 3, resulting in a parsimonious money-wage Phillips curve, which just

consists of the demand pressure U^l_{t-1}, the cost pressure $\Delta p_{t-1} - \pi_{t-1}$ and a constant (representing the integrated effect of labor productivity and the NAIRU on the deviation of nominal wage growth from the inflationary climate)[13],

$$\Delta w_t - \pi_t = \underset{(0.00163)}{0.0158} + \underset{(0.101)}{0.266}\ (\Delta p_{t-1} - \pi_{t-1}) - \underset{(0.0271)}{0.193}\ U^l_{t-1}, \tag{20}$$

without losing any relevant information:

RSS	0.003941	$\hat{\sigma}$	0.004641	R^2	0.3755	$\bar{R}^2$	0.3686
$\ln L$	1001	AIC	−10.7297	HQ	−10.7087	SC	−10.6777

An F test of the specific against the general rejects only at a marginal rejection probability of 0.5238. The properties of the estimated model (20) are illustrated in figure 5. The first graph (upper LHS) shows the fit of the model over time; the second graph (upper RHS) plots the fit against the actual values of $\Delta w_t - \pi_t$; the second graph (lower LHS) plots the residuals and the last graph (lower RHS) the squared residuals. The diagnostic test results shown in table 2 confirm that (20) is a valid congruent reduction of the general model in (19).

Table 2 Diagnostics.

	Wage Phillips curve		Price Phillips curve	
Diagnostic test	(19)	(20)	(21)	(22)
$F_{\text{Chow}(1978:2)}$	0.993 [0.5161]	0.866 [0.7529]	0.431 [0.9999]	0.421 [1.0000]
$F_{\text{Chow}(1996:4)}$	0.983 [0.4829]	0.771 [0.7315]	0.635 [0.8672]	0.551 [0.9288]
$\chi^2_{\text{normality}}$	0.710 [0.7012]	0.361 [0.8347]	0.141 [0.9322]	0.483 [0.7856]
$F_{\text{AR}(1-4)}$	1.915 [0.1105]	1.276 [0.2810]	2.426 [0.0503]	1.561 [0.1869]
$F_{\text{ARCH}(1-4)}$	1.506 [0.2030]	0.940 [0.4421]	1.472 [0.2133]	3.391 [0.0107]
F_{hetero}	0.615 [0.9634]	1.136 [0.3411]	0.928 [0.6072]	1.829 [0.0346]

Reported are the test statistic and the marginal rejection probability.

With respect to the theoretical wage Phillips curve (1)

$$\hat{w} = \beta_{w_1}(\bar{U}^l - U^l) - \beta_{w_2}(u - u_o) + \kappa_w(\hat{p} + n_x) + (1 - \kappa_w)(\pi + n_x)$$

we therefore obtain the quantitative expression

$$\hat{w} = 0.0158 - 0.193U^l + 0.266\hat{p} + 0.734\pi$$

We notice that the wage share and labor productivity do play no role in this specification of the money-wage Phillips curve. The result on the influence of the wage share is in line with the result obtained by Blanchard and Katz (1999) for the US economy.

3.3 The price Phillips curve

Let us next provide an estimate of the price Phillips curve (2) for the US economy. We now model price inflation in deviation from the inflation climate, $\Delta p - \pi$, conditional on its own past, the history of wage inflation, $\Delta w - \pi$, overall labor productivity growth, $\Delta y - \Delta l^d$, the

[13] We have $E(\hat{p} - \pi) = 0, E(\hat{w} - \pi) = 0.0045$ and $\bar{U}^l = E(U^l) = 0.058$.

degree of capital under-utilization, U^c by means of the equation (21), and the error correction term, u:

$$\begin{aligned}\Delta p_t - \pi_t &= \nu_p + \sum_{j=1}^{5} \gamma_{ppj} \left(\Delta p_{t-j} - \pi_{t-j}\right) + \sum_{j=1}^{5} \gamma_{pwj} \left(\Delta w_{t-j} - \pi_{t-j}\right) \\ &\quad + \sum_{j=1}^{5} \gamma_{pyj} \left(\Delta y_{t-j} - \Delta l^d_{t-j}\right) + \sum_{j=1}^{5} \gamma_{puj} U^c_t + \alpha_p u_{t-1} + \varepsilon_{pt}, \end{aligned} \tag{21}$$

where ε_{pt} is a white noise process. The general unrestricted model shows no indication of misspecification (see table 2) and explains a substantial fraction (63.8%) of inflation variability. Also note that the standard error of the price Phillips curve is just half the standard error in the prediction of changes in the wage level, namely 0.259%:

RSS	0.001072	$\hat{\sigma}$	0.002589	R^2	0.6376	$\bar{R}^2$	0.5810
$\ln L$	1122	AIC	−11.7843	HQ	−11.6015	SC	−11.3334

There is however a huge outlier ($\hat{\varepsilon}_{pt} > 3\hat{\sigma}$) associated with the oil price shock in 1974 (3) so a centered impulse dummy, I(1974:3), was included.

Here, the model reduction process undertaken by *PcGets* limits the number of coefficients to 9 (while starting again with 22) and results in the following price Phillips curve:

$$\begin{aligned}\Delta p_t - \pi_t &= \underset{(0.0011)}{0.00463} + \underset{(0.0413)}{0.12} \left(\Delta w_{t-1} - \pi_{t-1}\right) + \underset{(0.0397)}{0.0896} \left(\Delta w_{t-3} - \pi_{t-3}\right) \\ &\quad + \underset{(0.0691)}{0.254} \left(\Delta p_{t-1} - \pi_{t-1}\right) + \underset{(0.0653)}{0.196} \left(\Delta p_{t-4} - \pi_{t-4}\right) - \underset{(0.0634)}{0.18} \left(\Delta p_{t-5} - \pi_{t-5}\right) \\ &\quad - \underset{(0.0232)}{0.0467} \left(\Delta y_{t-1} - \Delta l^d_{t-1}\right) - \underset{(0.00551)}{0.0287}\, U^c_{t-1} + \underset{(0.00262)}{0.00988}\, \mathrm{I}(1974{:}3)_t \end{aligned} \tag{22}$$

RSS	0.001161	$\hat{\sigma}$	0.002562	R^2	0.6074	$\bar{R}^2$	0.5897
$\ln L$	1114	AIC	−11.8870	HQ	−11.8238	SC	−11.7309

The reduction is accepted at a marginal rejection probability of 0.7093. The fit of the model and the plot of the estimation errors are displayed in figure 6.

The long-run price Phillips curve implied by (22) is given by:

$$\begin{aligned}\Delta p - \pi &= \underset{(0.0016)}{0.00634} + \underset{(0.0608)}{0.286} \left(\Delta w - \pi\right) - \underset{(0.0305)}{0.064} \left(\Delta y - \Delta l^d\right) - \underset{(0.00795)}{0.0393}\, U^c \\ &\quad + \underset{(0.00366)}{0.0135}\, \mathrm{I}(1974{:}3) \end{aligned} \tag{23}$$

With respect to the theoretical price Phillips curve

$$\hat{p} = \beta_{p_1}(\bar{U}^c - U^c) + \beta_{p_2}(u - u_o) + \kappa_p(\hat{w} - n_x) + (1 - \kappa_p)\pi,$$

we therefore obtain the quantitative expression

$$\hat{p} = 0.006 - 0.039U^c + 0.286\hat{w} + 0.714\pi,$$

where we ignore the dummy and the productivity term in the long-run Phillips curve.[14] We notice that the wage share and labor productivity do again play no role in this specification

[14] From the perspective of the theoretical equation just shown this gives by calculating the mean of U^c the values $\bar{U}^c = 0.18, n_x = 0.004$.

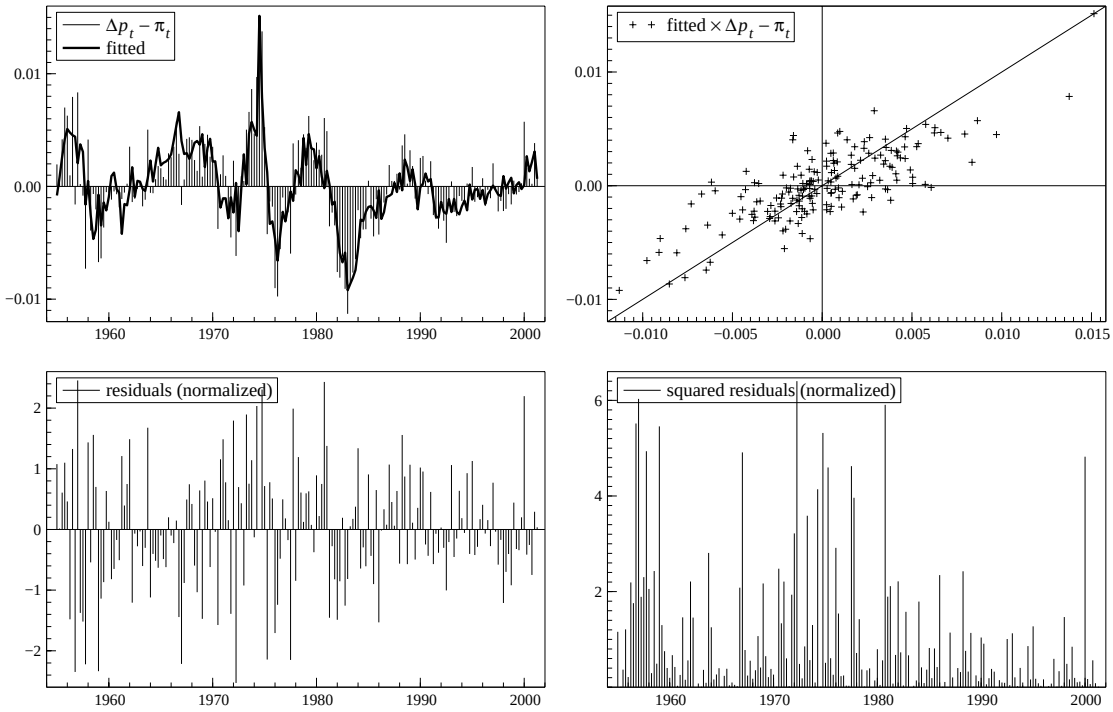

Figure 6 Price Phillips curve.

of the money-wage Phillips curve. The result that demand pressure matters more in the labor market than in the goods market is in line with what is observed in Carlin and Soskice (1990, section 18.3.1), and the result that firms are (slightly) more short-sighted than workers may be due to the smaller importance firms attach to past observations of wage inflation.

3.4 System results

So far we have modeled the wage and price dynamics of the system by analyzing one equation at a time. In the following we check for the simultaneity of the innovations to the price and wage inflation equations. The efficiency of a single-equation model reduction approach as applied in the previous subsection depends on the absence of instantaneous causality between $\Delta p_t - \pi_t$ and $\Delta w_t - \pi_t$ (see Krolzig, 2001). This requires the diagonality of the variance-covariance matrix Σ when the two Phillips curves are collected to the system

$$z_t = \sum_{j=1}^{5} A_j z_{t-j} + B q_t + \epsilon_t, \tag{24}$$

which represents $z_t = (\Delta p_t - \pi_t, \Delta w_t - \pi_t)'$ as a fifth-order vector autoregressive (VAR) process with the vector of the exogenous variables $q_t = (1, U^c_{t-1}, U^l_{t-1}, \Delta y_{t-1} - \Delta l^d_{t-1}, \text{I}(1974{:}3))'$ and the null-restrictions found by *PcGets* being imposed. Also, ϵ_t is a vector white noise process with $\mathsf{E}[\epsilon_t \epsilon_t'] = \Sigma$.

Estimating the system by FIML using PcGive10 (see Hendry and Doornik, 2001) gives almost identical parameter estimates (not reported here) and a log-likelihood of the system of 1589.34. The correlation of structural residuals in the $\Delta w - \pi$ and $\Delta p - \pi$ equation is just

0.00467, which is clearly insignificant.[15] Further support for the empirical Phillips curves (20) and (22) comes from a likelihood ratio (LR) test of the over-identifying restrictions imposed by *PcGets*. With $\chi^2(44) = 46.793[0.3585]$, we can accept the reduction. The presence of instantaneous non-causality justifies the model reduction procedure employed here, which was based on applying *PcGets* to each single equation in a turn.

The infinite-order vector moving average representation of the system corresponding to the system in (24) is given by

$$z_t = \sum_{j=0}^{\infty} \Psi_j B q_{t-j} + \sum_{j=0}^{\infty} \Psi_j \epsilon_{t-j} \tag{25}$$

where $\Psi(L) = A(L)^{-1}$ and L is the lag operator. By accumulating all effects, $z = A(1)^{-1}Bq$, we get the results in table 3.

Table 3 Static long run solution.

	Constant	U^c	U^l	$\Delta y - \Delta l^d$	I(1974:3)
$\Delta w - \pi$	0.0189	−0.0113	−0.2093	−0.0184	+0.0039
	(0.1109)	(0.0090)	(0.0300)	(0.0028)	(0.0011)
$\Delta p - \pi$	0.0118	−0.0426	−0.0600	−0.0693	+0.0146
	(0.0680)	(0.0340)	(0.0228)	(0.0105)	(0.0040)
$\Delta w - \Delta p$	0.0071	+0.0313	−0.1493	+0.0509	−0.0107

Derived from the FIML estimates of the system in (24).

Note here that all signs are again as expected, but that the estimated parameters are now certain compositions of the β, κ terms and are in line with the values of these parameters reported earlier. Taking into account all dynamics effects of U^l and U^c on wage and price inflation, real wage growth reacts stronger on the under-utilization of the factor labor U^l than of the factor capital U^c.

3.5 Are there adverse Rose effects?

The wage Phillips curve in (20) and the price Phillips curve in (22) can be solved for the two endogenous variables $\hat{w}$ and $\hat{p}$. The resulting reduced form representation of these equations is similar to equations (8) and (9), but for wages and prices and simplified due to the eliminated Blanchard-Katz-type error correction terms (*i.e.*, $\beta_{w_2} = \beta_{p_2} = 0$):

$$\hat{w} - \pi = \kappa \left[\beta_{w_1} (\bar{U}^l - U^l) + \kappa_w \beta_{p_1} (\bar{U}^c - U^c) \right] \tag{26}$$

$$\hat{p} - \pi = \kappa \left[\beta_{p_1} (\bar{U}^c - U^c) + \kappa_p \beta_{w_1} (\bar{U}^l - U^l) \right] \tag{27}$$

with $\kappa = (1 - \kappa_{w_1} \kappa_{p_1})^{-1}$.

For the US economy, we found that wages reacted stronger to demand pressure than prices ($\beta_{w_1} > \beta_{p_1}$), that β_{w_2}, β_{p_2} and wage share influences as demand pressure corrections could be ignored (as assumed in section 2) and that wage–earners are roughly equally short–sighted as firms ($\kappa_w \approx \kappa_p$). Furthermore, using the FIML estimates of the static long run solution of

[15] Note that under the null hypothesis, the FIML estimator of the system is given by OLS. So we can easily construct an LR test of the hypothesis $\Sigma_{12} = \Sigma_{21} = 0$. As the log-likelihood of the system under the restriction is 1587.51. Thus the LR test of the restriction can be accepted with $\chi^2(1) = 3.6554[0.0559]$.

system $\hat{w}-\pi$, $\hat{p}-\pi$ reported in table 3, we have the following empirical equivalents of (26) and (27):

$$\hat{w}-\pi \approx 0.019-0.209U^l-0.011U^c \quad (28)$$

$$\hat{p}-\pi \approx 0.012-0.060U^l-0.043U^c \quad (29)$$

where we abstract from the dummy and productivity term.

These calculations imply with respect to the critical condition (α) derived in section 2,

$$\alpha=(1-\kappa_p)\beta_{w_1}k_o-(1-\kappa_w)\beta_{p_1}/y^p \approx 0.714\cdot 0.209-0.734\cdot 0.043 \approx 0.118>0,$$

if we assume that $k=K/(xL)$ and $1/y^p=K/Y^p$ are ratios of roughly similar size, which is likely since full-employment output should be not too different from full-capacity output at the steady state.

Hence, the Rose effect will be of adverse nature if the side-condition $i<c$ is met. For the US, this condition has been investigated in Flaschel, Gong and Semmler (2001) in a somewhat different framework (see Flaschel, Gong and Semmler, 2002a for the European evidence). Their estimated investment parameters i is 0.136, which should be definitely lower than the marginal propensity to consume out of wages.[16] Thus the real wage or Rose effect is likely to be adverse. In addition to what is known for the real rate of interest rate channel and the Mundell effect, increasing wage flexibility might add further instability to the economy. Advocating more wage flexibility may thus not be as unproblematic as it is generally believed.

Given the indication that the US wage-price spiral is characterized by adverse Rose effects, the question arises which mechanisms stabilized the US economy over the post-war period by taming this adverse real wage feedback mechanism. Some aspects of this issue will be theoretically investigated in the remainder of the paper. But a thorough analysis from a global point of view must be left for future theoretical and empirical research on core nonlinearities possibly characterizing the evolution of market economies.

The results obtained show that (as long as goods demand depends positively on the wage share) the wage-price spiral in its estimated form is unstable as the critical condition (α) creates a positive feedback of the wage share on its rate of change. We stress again that the innovations for obtaining such a result are the use of two measures of demand pressure and the distinction between temporary and permanent cost pressure changes (in a cross-over fashion) for the wage and price Phillips curves employed in this paper.

4 Wage flexibility, instability and an extended interest rate rule

In section 2, we found that a sufficient wage flexibility supports economic stability. The imposed assumption $c<i$ ensured that the effective demand and thus output are decreased by a rising wage share; thus deviations from the steady-state equilibrium, are corrected by the normal reaction of the real wage to activity changes. In contrast, sufficiently flexible price levels (for given wage flexibility) result in an adverse reaction of the wage share, since a rising wage share stimulate further increases via output contraction and deflation.

[16] In the context of our model, one might want to estimate the effective demand function $y=[(n+g-i(r_o-\pi)]/[(1-u)(1-i)+(1-c)u]$. In view of the local approach chosen, it would in fact suffice to estimate a linear approximation of the form $y=a_0+a_1u+a_2(r-\pi)$, where $sign(a_1)=sign(c-i)$ and $a_2<0$ holds. However, in preliminary econometric investigations, we found a_1 being statistically insignificant so that no conclusions could be drawn regarding the sign of $c-i$.

Motivated by the estimation results presented in the preceding section, we now consider the situation where $c > i$ and $\alpha > 0$ holds true with respect to the critical Rose condition (α). The violation of the critical condition implies that $\hat{u}$ depends positively on y. In connection with $c > i$, i.e., $y_u > 0$ it generates a positive feedback from the wage share u onto its rate of change $\hat{u}$. Thus sufficiently strong wage flexibility (relative to price flexibility) is now destabilizing. This is the adverse type of Rose effect.

4.1 Instability due to an unmatched Rose effect

Here we consider the simplified wage-price dynamics (14) – (16) under the assumption $i < c$ instead of (A.1). If, in the now considered situation, monetary policy is still inactive (A.4a), the Rose effect and the Mundell effect are both destabilizing the private sector of the economy:

Proposition 4. (Private Sector Instability)
Assume $i < c$, i.e., $y_u > 0$, $\alpha > 0$ and $\kappa_p < 1$. Then, under the assumptions (A.2) - (A.4a) introduced earlier, the interior steady-state solution of the dynamics (14) – (16) is essentially repelling (exhibits at least one positive root).

Let us consider again to what extent the interest rate policy (A.4b) can stabilize the economy and in particular enforce the inflationary target $\bar{\pi}$. We state here without proof that rule (A.4b) can stabilize the previously considered situation if the adjustment speed of wages with respect to demand pressure in the labor market is sufficiently low. However, this stability gets lost if wage flexibility is made sufficiently large as is asserted by the following proposition, where we assume $\kappa_p = 0$ for the sake of simplicity.

Proposition 5. (Instability by an Adverse Rose Effect)
We assume (in the case $i < c$) an attracting steady-state situation due to the working of the monetary policy rule (A.4b). Then: Increasing the parameter β_{w_1} that characterizes wage adjustment speed will eventually lead to instability of the steady state by way of a Hopf bifurcation (if the parameters κ_p, i, β_r are jointly chosen sufficiently small). There is no reswitching to stability possible, once stability has been lost in this way.

Note that the proposition does not claim that there is a wage adjustment speed which implies instability for any parameter value β_r in the interest rate policy rule. It is also worth noting that the instability result is less clear-cut when for example $\kappa_p > 0$ is considered. Furthermore, increasing the adjustment speed β_r may reduce the dynamic instability in the case $\kappa_p = 0$ (as the trace of the Jacobian is made less positive thereby). In the next subsection we will however make use of another stabilizing feature which we so far neglected in the considered dynamics due to assumption (A.2): the Blanchard and Katz (1999) error correction term $\beta_{w_2}(u - u_o)$ in the money-wage Phillips curve.

4.2 Stability from Blanchard–Katz type 'error correction'

We now analyze dynamics under the assumption $\beta_{w_2} > 0$. Thus money wages react to deviations of the wage share from its steady-state value. In this situation the following proposition holds true:

Proposition 6. (Blanchard-Katz Wage Share Correction)
Assume $i < c$, i.e., $y_u > 0$, $\alpha > 0$ and $\kappa_p < 1$. Then, under the interest rate

policy rule (A.4b), a sufficiently large error correction parameter β_{w_2} implies an attracting steady state for any given adjustment speed $\beta_\pi > 0$ and all price flexibility parameters $\beta_{p_1} > 0$. This stability is established by way of a Hopf bifurcation which in a unique way separates unstable from stable steady-state solutions.

We thus have the result that the Blanchard–Katz error correction term if sufficiently strong overcomes the destabilizing forces of the adverse Rose effect in proposition 5.

Blanchard and Katz (1999) find that the error correction term is higher in European countries than in the US, where it is also in our estimates insignificant. So the empirical size of the parameter β_{w_2} may be too small to achieve the stability result of proposition 6. Therefore, we will again disregard the error correction term in the money-wage Phillips curve (A.2) in the following, and instead focus on the role of monetary policy in stabilizing the wage-price spiral.

4.3 Stability from an augmented Taylor rule

The question arises whether monetary policy can be of help to avoid the problematic features of the adverse Rose effect. Assume now that there interest rates are determined by an augmented Taylor rule of the form,

$$r \;=\; \rho_o + \pi + \beta_{r_1}(\pi - \bar{\pi}) + \beta_{r_2}(u - u_o), \quad \beta_{r_1}, \beta_{r_2} > 0, \tag{30}$$

where the monetary authority responds to rising wage shares by interest rate increases in order to cool down the economy, counter-balancing the initial increase in the wage share.

The static equilibrium condition is now given the

$$y = \frac{n + g - i(\rho_0 + \beta_{r_1}(\pi - \bar{\pi}) + \beta_{r_2}(u - u_o))}{(1 - i)(1 - u) + (1 - c)u}.$$

Thus the augmented Taylor rule (30) gives rise to a negative dependence of output y on the inflationary climate π as well as the wage share u.

We now consider the implications for the stability of the steady state:

Proposition 7. (Wage Gap Augmented Taylor Rule)
Assume $i < c$, $\alpha > 0$ and $\kappa_p < 1$. Then: A sufficiently large wage-share correction parameter β_{r_2} in the augmented Taylor rule (30) implies an attracting steady state for any given adjustment speed $\beta_\pi > 0$ and all price flexibility parameters $\beta_{p_1} > 0$. This stability is established by way of a Hopf bifurcation which in a unique way separates unstable from stable steady-state solutions.

Thus, convergence to the balanced growth path of private sector of the considered economy is generated by a modified Taylor rule that is augmented by a term that transmits increases in the wage share to increases in the nominal rate of interest. To our knowledge such an interest rate policy rule that gives income policy a role to play in the adjustment of interest rates by the central bank has not yet been considered in the literature. This is due to the general neglect of adverse real wage or Rose effects which induce an inflationary spiral independently from the one generated by the real rate of interest or Mundell effect, though both of these mechanisms derive from the fact that real magnitudes always allow for two interacting channels by their very definition, wages versus prices in the case of Rose effects and nominal interest versus expected inflation in the case of Mundell effects.

5 Conclusions

In context of the 'Goldilocks economy' of the late 1990s, Gordon (1998) stressed the need for explaining the contrast between decelerating prices and accelerating wages as well as the much stronger fall of the rate of unemployment than the rise of the rate of capital utilization. The coincidence of the two events is exactly what our approach to the wage-price spiral would predict: wage inflation is driven by demand and cost pressures on the labor market and price inflation is formed by the corresponding pressures on the goods markets.

Based on the two Phillips curves, we investigated two important macrodynamic feedback chains in a simple growth framework: (i) the conventional destabilizing Mundell effect and (ii) the less conventional Rose effect, which has been fairly neglected in the literature on demand and supply driven macrodynamics. We showed that the Mundell effect can be tamed by a standard Taylor rule. In contrast, the Rose effect can assume four different types depending on wage and price flexibilities, short-sightedness of workers and firms with respect to their cost-pressure measures and marginal propensities to consume c and invest i in particular (where we argued for $i < c$). Empirical estimates for the US-economy then suggested the presence of adverse Rose effects: the wage level is more flexible than the price level with respect to demand pressure (and workers roughly equally short-sighted as firms with respect to cost pressure). We showed that this particular Rose effect can cause macroeconomic instabilities which can not be tamed by a conventional Taylor rule. But the paper also demonstrated means by which adverse real interest rate and real wage rate effects may be modified or dominated in such a way that convergence back to the interior steady state is again achieved. We proved that stability can be re-established by (i) an error-correction term in the money-wage Phillips curve (as in Blanchard and Katz, 1999),[17] working with sufficient strength, or (ii) a modified Taylor rule with monetary policy monitoring the labor share (or real unit labor costs) and reacting in response to changes in the income distribution.

In this paper, we showed that adverse Rose effects are of empirical importance, and indicated ways of how to deal with them by wage or interest rate policies. In future research, we intend to discuss the role of Rose effects for high and low growth phases separately, taken account of the observation that money wages may be more rigid in the latter phases than in the former ones (see Hoogenveen and Kuipers, 2000, for a recent empirical confirmation of such differences and Flaschel, Gong and Semmler, 2002b, for its application to a 6D Keynesian macrodynamics). The existence of a 'kink' in the money-wage Phillips curve should in fact increase the estimated) wage flexibility parameter further (in the case where the kink is not in operation). Furthermore, the robustness of the empirical results should be investigated (say, by analyzing the wage-price spiral in other OECD countries). Finally, more elaborate models have to be considered to understand the feedback mechanisms from a broader perspective (see Flaschel *et al.*, 2001, 2002a, for first attempts of the dynamic AS-AD variety).[18]

[17] The related error correction in the price Phillips curve should allow for the same conclusion, but has been left aside here due to space limitations.

[18] Concerning the validity of Okun's law and the degree of correlation of labor and capital under-utilization we should then also distinguish between the unemployment rate on the external labor market and the under- or over-employment of the employed, which may in particular explain the difference in volatility in the under-utilization of labor and capital. This, however, introduces to further parameters into the model, the impact of employment within firms on money-wage inflation and the speed with which the labor force of firms is adjusted to the observed under or over utilization of labor within firms. Here again, significant differences may be expected regarding the situation in the United state and Europe.

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A Appendices

A.1 The sectoral budget equations of the model

For reasons of completeness, we here briefly present the budget equations of our four types of economic agents (see Sargent, 1987, ch.1, for a closely related presentation of such budget equations, there for the sectors of the conventional AS-AD growth model). Consider the following scenario for the allocation of labor, goods and assets:

$$\begin{aligned} cupY + \dot{B}^d &= upY + \bar{r}B \quad \text{(workers: consumption out of wage income and saving deposits)} \\ p_b\dot{B}^d + p_e\dot{E}^d &= B + (1-u)pY \quad \text{(asset–holders: bond and equity holdings)} \\ pI &= p_e\dot{E} \quad \text{(firms: equity financed investment)} \\ \bar{r}B + B + pG &= \dot{B} + p_b\dot{B} \quad \text{(government: debt financed consumption).} \end{aligned}$$

where $g = G/K$ =const. In these budget equations we use a fixed interest rate $\bar{r}$ for the saving deposits of workers and use – besides equities – perpetuities (with price $p_b = 1/r$) for the characterization of the financial assets held by asset-holders. Due to this choice, and due to the fact that investment was assumed to depend on the long-term expected real rate of interest, we had to specify the Taylor rule in terms of r in the body of the paper. These assumptions allow to avoid the treatment of the term structure of interest rate which would make the model considerably more difficult and thus the analysis of Mundell or Rose effects more advanced, but also less transparent. For our purposes the above scenario is however fully adequate and very simple to implement.

Furthermore, we denote in these equations the amount of saving deposits of workers by B (and assume a fixed interest rate $\bar{r}$ on these saving deposits). Outstanding bonds (consols or perpetuities) are denoted by B and have as their price the usual expression $p_b = 1/r$. We finally use p_e for the price of shares or equities E. These equations are only presented for consistency reasons here and they immediately imply

$$p(Y - C - I - G) = (\dot{B}^d - \dot{B}) + p_b(\dot{B}^d - \dot{B}) + p_e(\dot{E}^d - \dot{E}) = 0.$$

We have assumed goods–market equilibrium in this paper and assume in addition that all saving deposits of workers are channeled into the government sector ($\dot{B}^d = \dot{B}$). We thus can also assume equilibrium in asset market flows via a perfect substitute assumption (which determines p_e, while p_b is determined by an appropriate interest rate policy rule in this paper). Note that firms are purely equity financed and pay out all profits as dividends to the sector of asset holders. Note also that long-term bonds per unit of capital $b = B/(pK)$ will follow the law of motion

$$\dot{b} = r(b + g - s_w uy) - (\hat{p} + \hat{K})b$$

which – when considered in isolation (all other variables kept at their steady-state values). – implies a stable evolution of such government debt b towards a steady-state value for this ratio if $r^o - \hat{p}_o = \rho_o < n$ holds true. Since fiscal policy is not our concern in this paper we only briefly remark that this is the case for government expenditure per unit of capital that is chosen sufficiently small:

$$g < \frac{nu_o(1-c)}{1-u_o}$$

Similarly, we have for the evolution of savings per unit of capital $b = B/(pK)$ the law of motion

$$\dot{b} = s_w uy + (\bar{r} - (\hat{p} + \hat{K}))b$$

which – when considered in isolation – implies convergence to some finite steady-state value if $\bar{r} < \hat{p}_o + n$ holds true. Again, since the Government Budget Restraint is not our concern in this paper, we have ignored this aspect of our model of wage–price and growth dynamics.

A.2 Wage dynamics: theoretical foundation

This subsection builds on the paper by Blanchard and Katz (1999) and briefly summarizes their theoretical motivation of a money-wage Phillips curve which is closely related to our dynamic equation (1).[19] Blanchard and Katz assume – following the suggestions of standard models of wage setting – that real wage expectations of workers, $\omega^e = w_t - p_t^e$, are basically determined by the reservation wage, $\bar{\omega}_t$, current labor productivity, $y_t - l_t^d$, and the rate of unemployment, U_t^l:

$$\omega_t^e = \theta\bar{\omega}_t + (1-\theta)(y_t - l_t^d) - \beta_w U_t^l.$$

Expected real wages are thus a Cobb-Douglas average of the reservation wage and output per worker, but are departing from this normal level of expectations by the state of the demand pressure on the labor market. The reservation wage in turn is determined as a Cobb-Douglas average of past real wages, $\omega_{t-1} = w_{t-1} - p_{t-1}$, and current labor productivity, augmented by a factor $a < 0$:

$$\bar{\omega}_t = a + \lambda\omega_{t-1} + (1-\lambda)(y_t - l_t^d)$$

Inserting the second into the first equation results in

$$\omega_t^e = \theta a + \theta\lambda\omega_{t-1} + (1-\theta\lambda)(y_t - l_t^d) - \beta_w U_t^l,$$

which gives after some rearrangements

$$\begin{aligned} \Delta w_t &= p_t^e - p_{t-1} + \theta a - (1-\theta\lambda)[(w_{t-1} - p_{t-1}) - (y_t - l_t^d)] - \beta_w U_t^l \\ &= \Delta p_t^e + \theta a - (1-\theta\lambda)u_{t-1} + (1-\theta\lambda)(\Delta y_t - \Delta l_t^d) - \beta_w U_t^l \end{aligned}$$

where Δp_t^e denotes the expected rate of inflation, u_{t-1} the past (log) wage share and $\Delta y_t - \Delta l_t^d$ the current growth rate of labor productivity. This is the growth law for nominal wages that flows from the theoretical models referred to in Blanchard and Katz (1999, p.70).

In this paper, we proposed to operationalize this theoretical approach to money-wage inflation by replacing the short-run cost push term Δp_t^e by the weighted average $\kappa_w \Delta p_t^e + (1-\kappa_w)\pi_t$, where Δp_t^e is determined by myopic perfect foresight. Thus, temporary changes in the correctly anticipated rate of inflation do not have full impact on temporary wage inflation, which is also driven by lagged inflation rates via the inflationary climate variable π_t. Adding inertia to the theory of wage inflation introduced a distinction between the temporary and persistent cost effects to this equation. Furthermore we have that $\Delta y_t - \Delta l_t^d = n_x$ due to the assumed fixed proportions technology. Altogether, we end up with am equation for wage inflation of the type presented in section 2.1, though now with a specific interpretation of the model's parameters from the perspective of efficiency wage or bargaining models.[20]

[19] In this section, lower case letters (including w and p) indicate logarithms.

[20] Note that the parameter in front of u_{t-1} can now not be interpreted as a speed of adjustment coefficient.

A.3 Price dynamics: theoretical foundation

We here follow again Blanchard and Katz (1999, IV.), see also Carlin and Soskice (1990, ch.18), and start from the assumption of normal cost pricing, here under the additional assumption of our paper of fixed proportions in production and Harrod neutral technological change. We therefore consider as rule for normal prices

$$p_t = \mu_t + w_t + l_t^d - y_t, \quad \text{i.e.,} \quad \Delta p_t = \Delta\mu_t + \Delta w_t - n_x,$$

where μ_t represents a markup on the unit wage costs of firms and where again myopic perfect foresight, here with respect to wage setting is assumed. We assume furthermore that the markup is variable and responding to the demand pressure in the market for goods $\bar{U}^c - U_t^c$, depending in addition negatively on the current level of the markup μ_t in its deviation from the normal level $\bar{\mu}$. Firms therefore depart from their normal cost pricing rule according to the state of demand on the market for goods, and this the stronger the lower the level of the currently prevailing markup has been (markup smoothing). For sake of concreteness let us here assume that the following behavioral relationship holds:

$$\Delta\mu_t = \beta_p(\bar{U}^c - U_{t-1}^c) + \gamma(\bar{\mu} - \mu_{t-1}),$$

where $\gamma > 0$. Inserted into the formula for price inflation this in sum gives:

$$\Delta p_t = \beta_p(\bar{U}^c - U_{t-1}^c) + \gamma(\bar{\mu} - \mu_{t-1}) + (\Delta w_t - n_x)$$

In terms of the logged wage share $u_t = -\mu_t$ we get

$$\Delta p_t = \beta_p(\bar{U}^c - U_{t-1}^c) + \gamma(u_{t-1} - \bar{u}) + (\Delta w_t - n_x).$$

As in the preceding subsection of the paper, we again add persistence the cost pressure term $\Delta w_t - n_x$ now in the price Phillips curve in the form of the inflationary climate expression π and thereby obtain in sum the equation (2) of section 2.1.

A.4 Routh-Hurwitz stability conditions and Hopf bifurcations

We consider the matrix of partial derivatives at the steady state of the 3D dynamical systems of this paper in (u, k, π), the so-called Jacobian J, in detail represented by:

$$J = \begin{pmatrix} J_{11} & J_{12} & J_{13} \\ J_{21} & J_{22} & J_{23} \\ J_{31} & J_{32} & J_{33} \end{pmatrix}.$$

We define the principal minors of order 2 of this matrix by the following three determinants:

$$J_1 = \begin{vmatrix} J_{22} & J_{23} \\ J_{32} & J_{33} \end{vmatrix}, \quad J_2 = \begin{vmatrix} J_{11} & J_{13} \\ J_{31} & J_{33} \end{vmatrix}, \quad J_3 = \begin{vmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{vmatrix}$$

Note furthermore that Blanchard and Katz (1999) assume that, in the steady state, the wage share is determined by the firms' markup $u = -\mu$ (both in logs) to be discussed in the next subsection. Therefore the NAIRU can be determined endogenously on the labor market by $\bar{U}^l = \beta_w^{-1}\left[\theta a - (1-\theta\lambda)\bar{\mu} - \theta\lambda(\Delta y_t - \Delta l_t^d)\right]$. The NAIRU of their model therefore depends on both labor and goods market characteristics in contrast to the NAIRU levels for labor and capital employed in our approach.

We furthermore denote by a_1 the negative of the trace of the Jacobian $-\text{trace}J$, by a_2 the sum of the above three principal minors, and by a_3 the negative of the determinant $|J|$ of the Jacobian J. We note that the coefficients $a_i, i = 1, 2, 3$ are the coefficients of the characteristic polynomial of the matrix J.

The Routh Hurwitz conditions (see Lorenz, 1993) then state that the eigenvalues of the matrix J all have negative real parts if and only if

$$a_i > 0, i = 1, 2, 3 \quad \text{and} \quad a_1 a_2 - a_3 > 0.$$

These conditions therefore exactly characterize the case where local asymptotic stability of the considered steady state is given.

Supercritical Hopf bifurcations (the birth of a stable limit cycle) or subcritical Hopf bifurcations (the death of an unstable limit cycle) occur (if asymptotic stability prevailed below this parameter value) when the following conditions hold simultaneously for an increase of a parameter β of the model (see Wiggins, 1993, ch.3):

$$a_3(\beta) > 0, (a_1 a_2 - a_3)(\beta) = 0, (a_1 a_2 - a_3)'(\beta) > 0.$$

We note here that the dynamics considered below indeed generally fulfill the condition $a_3 > 0$ and also $J_2 = 0$, the latter up to proposition 6 and due to the proportionality that exists between the laws of motion (14), (16) with respect to the state variables u, π.

A.5 Proofs of propositions

In the following we present the mathematical proofs of the propositions 2 – 7 of the paper. The proofs involve the stability analysis of the 3D dynamics in (14) to (16) under certain parametric assumptions and different monetary regimes and are based on the Routh-Hurwitz conditions just considered.

Proof of Proposition 2: Choosing β_{p_1} or β_π sufficiently large will make the trace of J, the Jacobian of the dynamics (14) – (16) at the steady state, unambiguously positive and thus definitely lead to local instability.

Yet, even if β_{p_1} and β_π are sufficiently small, we get by appropriate row operations in the considered determinant the following sequence of result for the sign of $\det J$:

$$\begin{aligned}
|J| &\hat{=} \begin{vmatrix} 0 & + & 0 \\ - & 0 & + \\ - & 0 & + \end{vmatrix} \hat{=} -(+) \begin{vmatrix} - & + \\ - & + \end{vmatrix} \hat{=} \begin{vmatrix} -y_0 & +1 \\ y_u & y_\pi \end{vmatrix} \\
&= y_0 \begin{vmatrix} -1 & +1 \\ \frac{c-i}{(1-u)(1-i)+(1-c)u} & \frac{i}{(1-u)(1-i)+(1-c)u} \end{vmatrix} \\
&= \frac{y_0}{(1-u)(1-i)+(1-c)u} \begin{vmatrix} -1 & +1 \\ c-i & i \end{vmatrix} = \frac{cy_0}{(1-u)(1-i)+(1-c)u} > 0
\end{aligned}$$

One of the necessary and sufficient Routh-Hurwitz conditions for local asymptotic stability is therefore always violated, independently of the sizes of the considered speeds of adjustment. ■

Proof of Proposition 3: Inserting the interest rule in (A.4b) into the $y(u,\pi)$ and $i(\rho-(r-\pi))$ functions gives rise to the functional dependencies

$$\begin{aligned} y &= y(u,\pi) = \frac{n+g-i[\rho_0+\beta_r(\pi-\bar{\pi})]}{(1-u)(1-i)+(1-c)u}, \quad y_u<0, y_\pi<0, \\ i &= i(\rho-(r-\pi)) = i(u,\pi), \quad i_u<0, i_\pi<0. \end{aligned}$$

The signs in the considered Jacobian are therefore here given by

$$J = \begin{pmatrix} - & + & - \\ - & 0 & - \\ - & + & - \end{pmatrix}$$

if β_{p_1} is chosen sufficiently small (and thus dominated by wage flexibility β_{w_1}). We thus then have trace $J<0$ ($a_1 = -$ trace $J>0$) and

$$J_3 = \begin{vmatrix} - & + \\ - & 0 \end{vmatrix} > 0, \quad J_1 = \begin{vmatrix} 0 & - \\ + & - \end{vmatrix} > 0, \text{ i.e. },$$

$a_2 = J_1 + J_2 + J_3 > 0$ for β_{p_1} sufficiently small. Next, we get for $|J|$ with respect to signs:

$$\begin{aligned} |J| &\hat{=} \begin{vmatrix} 0 & + & 0 \\ - & 0 & - \\ - & 0 & - \end{vmatrix} \hat{=} -(+) \begin{vmatrix} - & - \\ - & - \end{vmatrix} \hat{=} \begin{vmatrix} -y_0 & -\beta_r \\ y_u & y_\pi \end{vmatrix} = - \begin{vmatrix} -y_0 & -\beta_r \\ \frac{c-i}{N} y_0 & \frac{-i\beta_r}{N} \end{vmatrix} \\ &= -\beta_r (y_o/N) \begin{vmatrix} -1 & -1 \\ c-i & -i \end{vmatrix} = -\beta_r \frac{y_o}{N} c < 0. \end{aligned}$$

since $N = (1-u)(1-i)+(1-c)u > 0$ at the steady state. Therefore: $a_1, a_2, a_3 = -|J|$ are all positive.

It remains to be shown that also $a_1 a_2 - a_3 > 0$ can be fulfilled. Here it suffices to observe that a_1, a_2 stay positive when $\beta_{p_1} = 0$ is assumed, while a_3 becomes zero then. Therefore $a_1 a_2 - a_3 > 0$ for all adjustment parameters β_{p_1} chosen sufficiently small. These qualitative results hold independently of the size of β_π and β_r (with an adjusting size of β_{p_1} however). ■

Note in addition that the trace of J is given by

$$\kappa \beta_{p_1} / y^p [(1-\kappa_w)(i-c)y - \beta_\pi \beta_r i] / ((1-i)(1-u)+(1-c)u)$$

as far as its dependence on the parameter β_{p_1} is concerned. Choosing β_π or β_r, for given β_{p_1}, sufficiently small will make the trace of J positive and thus make the steady state of the considered dynamics locally unstable.

Proof of Proposition 4: With $r \equiv r_o$, we have for the Jacobian J of the dynamics at the steady state:

$$J = \begin{pmatrix} + & + & + \\ - & 0 & + \\ + & + & + \end{pmatrix}$$

and thus in particular trace $J > 0$ and

$$|J| \hat{=} \begin{vmatrix} 0 & + & 0 \\ - & 0 & + \\ + & 0 & + \end{vmatrix} = -(+) \begin{vmatrix} - & + \\ + & + \end{vmatrix} > 0.$$

Thus there is at least one positive real root, which establishes the local instability of the investigated interior steady state solution.

■

Proof of Proposition 5: For the considered parameter constellations, the Jacobian J is given by

$$J = \begin{pmatrix} + & + & - \\ - & 0 & - \\ + & + & - \end{pmatrix}.$$

This Jacobian first of all implies

$$|J| \hat{=} \begin{vmatrix} 0 & + & 0 \\ - & 0 & - \\ + & 0 & - \end{vmatrix} = -(+) \begin{vmatrix} - & - \\ + & - \end{vmatrix} < 0$$

and thus for the Routh-Hurwitz condition $a_3 = -|J| > 0$ as necessary condition for local asymptotic stability. We assert here without detailed proof that local stability will indeed prevail if β_{w_1} is chosen sufficiently close to zero, since $|J|$ will be close to zero then too and since the Routh-Hurwitz coefficients a_1, a_2 are both positive and bounded away from zero. Wages that react sluggishly with respect to demand pressure therefore produce local stability in the case $c > i$.

This is indeed achieved for example by the assumption $\kappa_p = 0$: Obviously, trace of J is then an increasing linear function of the speed parameter β_{w_1} in the considered situation, since this parameter is then only present in J_{11} and not in J_{33}. This proves the first part of the assertion, if note is taken of the fact that $|J|$ does not change its sign. Eigenvalues therefore cannot pass through zero (and the speed condition for them is also easily verified). The second part follows from the fact that $a_1 a_2 - a_3$ becomes zero before trace $J = -a_1$ passes through zero, but cannot become positive again before this trace has become zero (since $a_1 a_2 - a_3$ is a quadratic function of the parameter β_{w_1} with a positive parameter before the quadratic term and since this function is negative at the value β_{w_1} where trace J has become zero).

■

Proof of Proposition 6: The signs in the Jacobian of the dynamics at the steady state are given by

$$J = \begin{pmatrix} - & + & - \\ - & 0 & - \\ - & + & - \end{pmatrix}$$

if β_{w_2} is chosen sufficiently large (and thus dominating the wage flexibility β_{w_1} term). We thus have trace $J < 0$ ($a_1 = -\text{trace}J > 0$) and

$$J_3 = \begin{vmatrix} - & + \\ - & 0 \end{vmatrix} > 0, \quad J_1 = \begin{vmatrix} 0 & - \\ + & - \end{vmatrix} > 0, \quad \text{sign}J_2 = \text{sign} \begin{vmatrix} - & - \\ + & - \end{vmatrix} > 0, \text{ i.e. },$$

$a_2 = J_1 + J_2 + J_3 > 0$, in particular due to the fact that the $\beta_{w_i}, i = 1, 2$-expressions can be removed from the second row of J_2 without altering the size of this determinant.

Next, we get for $|J|$ with respect to signs:

$$|J| \;\hat{=}\; \begin{vmatrix} - & + & - \\ - & 0 & - \\ + & 0 & - \end{vmatrix} \hat{=} -(+) \begin{vmatrix} - & - \\ + & - \end{vmatrix} < 0.$$

since the $\beta_{w_i}, i = 1, 2$-expressions can again be removed now from the third row of $|J|$ without altering the size of this determinant.

Therefore: a_1, a_2, and $a_3 = -|J|$ are all positive as demanded by the Routh-Hurwitz conditions for local asymptotic stability. There remains to be shown that also $a_1 a_2 - a_3 > 0$ can be fulfilled. In the present situation this however is an easy task, since – as just shown – $|J|$ does not depend on the parameter β_{w_2}, while $a_1 a_2$ depends positively on it (in the usual quadratic way). Finally, the statement on the Hopf bifurcation can be proved in a similar way as the one in proposition 5. ∎

Proof of Proposition 7: Inserting the Taylor rule

$$r \;=\; \rho_o + \pi + \beta_{r_1}(\pi - \bar{\pi}) + \beta_{r_2}(u - u_o), \quad \beta_{r_1}, \beta_{r_2} > 0$$

into the effective demand equation

$$y = \frac{n + g - i(r_o - \pi)}{(1-u)(1-i) + (1-c)u}$$

adds the term

$$\tilde{y} = -\frac{i\beta_{r_2}(u - u_o)}{(1-u)(1-i) + (1-c)u}$$

to our former calculations – in the place of the β_{w_2} term now. This term gives rise to the following additional partial derivative

$$\tilde{y}_u = -\frac{i\beta_{r_2}}{(1-u_o)(1-i) + (1-c)u_o}$$

at the steady state of the economy. This addition can be exploited as the β_{w_2} expression in the previous subsection used there to prove proposition 7. ∎