Two-Stage Bargaining with Reversible Coalitions: the Case of Apex Games

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Abstract

This paper studies coalition formation and payoff division in a particular case of majority games (apex games) under the following assumptions: first, payoff division can only be agreed upon after the coalition has formed (two-stage bargaining); second, negotiations in the coalition can break down, in which case a new coalition may be formed (reversible coalitions). In contrast with the results of other two-stage models, all minimal winning coalitions may form and expected payoffs coincide with the per capita nucleolus. These results are robust to small changes in the bargaining procedure. Surprisingly, having a two-stage process (rather than a one-stage process with simultaneous coalition formation and payoff division) benefits the apex player.

Keywords: coalition formation, two-stage bargaining, reversible coalitions, apex games, per capita nucleolus.

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1 Introduction

Noncooperative models of coalition formation usually assume that players can agree on payoff division at the time they form a coalition (see e.g. Selten (1981), Baron and Ferejohn (1989), Chatterjee et al. (1993), Okada (1996)). However, there are situations in which players form coalitions before agreeing on payoff division: in coalition governments, parties cannot negotiate effectively on all issues before the formation of the government, partly because one does not know exactly which issues will come up (see Aumann and Myerson (1988)). A coalition is then understood as a *negotiation group*: if coalition S forms, it means that players in S bargain over the division of $v(S)$. In this context, it is natural to think that negotiations may break down, resulting in the dissolution of the coalition and possibly in the formation of a new one (see the discussion in Aumann and Drèze (1974)). We will refer to the first class of models (in which players agree on payoff division at the time the coalition is formed) as one-stage models, and to the second class (in which player agree on payoff division after the coalition has formed) as two-stage models. Coalition formation in two-stage models can be thought of as reversible or irreversible, depending on whether players can change coalitions. We will be interested in two-stage reversible processes.

The two-stage models studied in the literature are hybrid: the process of coalition formation is modeled as a noncooperative game, but payoff division is determined by a cooperative solution concept (Hart and Kurz (1983), Aumann and Myerson (1988)).[1] A fully noncooperative approach has been missing, with the important exception of the models of bargaining in markets, in which only two-player coalitions can form (see Osborne and Rubinstein (1990) and Muthoo (1999)).

In this paper, we study a fully noncooperative game of coalition formation and payoff division. The formation of a coalition means the beginning of negotiations between the players in the coalition over the division of the coalitional value. Negotiations can break down, in which case the coalition

[1] Aumann and Myerson (1988) clearly have irreversible coalitions in mind, since they assume that payoff division does not depend on the players' opportunities outside the coalition. In contrast, Hart and Kurz (1983) see coalitions as reversible.

is dissolved and a new coalition may be formed. Motivated by the case of government formation, we analyze in detail a very common type of majority games: apex games.

In apex games there is a major player and $n-1 \geq 3$ minor players. In order for a coalition to be winning, it must contain the major player together with a minor player, or all the minor players together. The major player seems to be stronger than the minor players. However, Hart and Kurz (1984) and Aumann and Myerson (1988) predict that the coalition of all minor players always forms, so that the *a priori* strongest player gets nothing.[2] The results of our model are quite robust to the details of the bargaining process and radically different from those of the hybrid models: expected payoffs coincide with the per capita nucleolus (see Maschler (1992)), which gives most of the payoff to the apex player. Payoff division conditional on the apex coalition being formed corresponds to the one often suggested as reasonable (see the discussion in Davis and Maschler (1967)).

Compared with one-stage models, two-stage models may seem to protect the minor players. After all, once a minor player forms a coalition with the apex player the two players are in a more symmetric position, and if coalitions were irreversible one would expect the two players to divide the payoff equally. However, we will see that, because coalitions are reversible, the minor players are actually worse-off in the two-stage model.

The remainder of the paper is organized as follows. Section 2 introduces the two-stage game and some general properties of the equilibria. Section 3 studies the case of apex games in detail. Section 4 concludes with some further remarks on the comparison with the literature and on possible extensions.

2 The game

Let $N = \{1, 2, ..., n\}$ be the set of players (parties) and (N, v) a proper simple characteristic function game ($v(S) = 0$ or 1 for all $S \subset N$, $v(N) = 1$,

[2] There are also variants of those models that predict that the grand coalition will always form (see Slikker (2000)).

$v(\varnothing) = 0$ and $v(S) + v(N \backslash S) \leq 1$). We assume that there is a budget of size 1 that has to be divided by majority rule. The characteristic function v indicates which coalitions of parties have a majority (and thus can divide the budget between themselves). We will assume that all players in N are risk neutral and discount future payoffs with a discount factor $\delta < 1$.

Given the underlying characteristic function game, bargaining proceeds as follows:

- Nature selects a proposer according to a probability distribution θ ($\theta_i \geq 0$ for all i and $\sum_{i \in N} \theta_i = 1$).
- The selected proposer i proposes a coalition S such that $S \ni i$.
- Players in S accept or reject the proposal sequentially. If one of them rejects, a period elapses and Nature selects a new proposer according to the probability distribution θ.
- If all players in S accept, coalition S is formed. If S is a losing coalition, the game ends and all players get 0. If S is a winning coalition, players in S bargain over the division of the budget. The "internal" game, played only by players in S, is a bargaining game with random proposers (Nature follows a probability distribution θ^S with $\theta_i^S \geq 0$ for all $i \in S$ and $\sum_{i \in S} \theta_i^S = 1$) and breakdown probability. A proposal x^S is a division of the budget between the players in S ($\sum_{i \in S} x_i^S = 1$). Every time a responder rejects a proposal, coalition S is dissolved with probability $1 - p$ ($0 < p < 1$).
- If coalition S is dissolved, Nature selects a proposer again according to the probability distribution θ.

We will be interested in stationary subgame perfect equilibria (SSPE).

Since coalition formation occurs before payoff division, we can think of the extensive game described above as a two-stage game. Of course, both the coalition formation "stage" and the payoff division "stage" are complicated objects and the play may bring the players back from the second to the first stage.

We will refer to the probability distribution θ as the *protocol*, and to θ^S as the *internal protocol*. Let y be the expected equilibrium payoff vector computed before Nature starts the game, and y^S be the expected equilibrium payoff vector computed after S has formed and before Nature starts the internal game. Let z^S be the vector of continuation values (i.e., expected payoffs after a proposal has been rejected) in the internal game. We start from the equilibrium of the internal game.

2.1 The equilibrium of the internal game

Suppose we have a SSPE of the game with associated expected equilibrium payoff y. We now show that the internal game has a unique stationary subgame perfect equilibrium payoff y^S.

If a player rejects a proposal in the internal game, with probability p Nature starts the internal game again (so that player i expects to get y_i^S) and with probability $1-p$ coalition S breaks apart and Nature starts the coalition formation game again (so that player i expects to get y_i). We have the following equation for the continuation value of player i

$$z_i^S = p y_i^S + (1-p) y_i$$

As for y_i^S, it is given by the probability i is selected to be a proposer in the internal game times his expected payoff as a proposer plus the probability that he is selected to be the responder (which is $1-\theta_i^S$ because bargaining is unanimous in the internal game) times his continuation value.[3]

$$y_i^S = \theta_i^S [1 - \sum_{j \in S \setminus \{i\}} z_j^S] + (1-\theta_i^S) z_i^S$$

From this system of equations (and taking into account that $\sum_{j \in S} y_j^S =$

[3] The payoff for player i as a proposer is $1-\sum_{j \in S \setminus \{i\}} z_j^S$ regardless of whether agreement is immediate. There are two possible cases:

1) If $\sum_{i \in S} z_i^S < 1$ the proposer strictly prefers making acceptable proposals; since in any SPE the proposer must offer exactly z_j to each responder j the results holds.

2) If $\sum_{i \in S} z_i^S = 1$ acceptable and unacceptable proposals give the same payoffs to the players. Notice that this second case is only possible if $\sum_{i \in S} y_i^S = 1$ and $\sum_{i \in S} y_i = 1$.

1) we see that

$$y_i^S = \theta_i^S(1 - \sum_{j \in S} y_j) + y_i. \tag{1}$$

This is a well-known result in bargaining games with breakdown probability: player i's expected payoff equals the breakdown payoff (in this case, y_i) plus a share of the surplus proportional to the probability of being proposer (cf. Binmore (1987) and Binmore et al. (1986)).

Equation (1) resembles an allocation rule in that each player receives a share of the available surplus and this share is determined by the internal protocol. However, there is an important difference: the payoff a player gets from being in a coalition is not fully determined by the rules of the internal game because the breakdown outcome is *endogenous*.

Let s denote $|S|$. The first possibility that comes to mind regarding the internal protocol θ^S is the *egalitarian protocol*, $\theta_i^S = \frac{1}{s}$ for all $i \in S$, which implies $y_i^S = \frac{1}{s}(1 - \sum_{j \in S} y_j) + y_i$.

2.2 The equilibrium of the game

If player i is selected to be a proposer at the coalition formation stage, he will choose one of the coalitions which maximize his expected payoff. Thus, he will choose S such that $\theta_i^S(1 - \sum_{j \in S} y_j)$ is maximized. If $\theta_i^S = \frac{1}{s}$ for all $i \in S$, player i will choose a coalition *with maximal per capita excess at* y.[4]

Lemma 1 *If for any player i there is a winning coalition $S \ni i$ such that $\theta_i^S > 0$, then in any SPE a winning coalition is formed without delay.*

Proof. By proposing coalition S, player i can get at least y_i; by making an unacceptable proposal he gets δy_i. Thus, he will strictly prefer to make acceptable proposals unless $y_i = 0$, which implies $\sum_{j \in S \setminus \{i\}} y_j = 1$. But if unacceptable proposals are made with positive probability, then $\sum_{j \in N \setminus \{i\}} y_j < 1$, which (since player i can get a positive share of the surplus in coalition S) contradicts $y_i = 0$. Thus, $y_i > 0$, which implies that a coalition is formed

[4] The per capita excess of coalition S at y is given by $\frac{1}{s}[v(S) - y(S)]$.

immediately. Since a losing coalition gives zero to all players, it will never be proposed or accepted. ■

Corollary 2 *Let λ_i^S be the probability that player i proposes coalition S. In an SPE the following conditions are satisfied for all i in N*

$$y_i = \theta_i \sum_{S \ni i} \lambda_i^S \left[\theta_i^S (1 - \sum_{k \in S} y_k) + y_i \right] + \sum_{j \in N \setminus \{i\}} \theta_j \sum_{S \supset \{i,j\}} \lambda_j^S \left[\theta_i^S (1 - \sum_{k \in S} y_k) + y_i \right]$$

$$\sum_{S \ni i} \lambda_i^S = 1$$

$$\lambda_i^S > 0 \textit{ implies } S \in \arg\max_{T : T \ni i} \theta_i^T (1 - \sum_{j \in T} y_j)$$

Remark 3 *If $\theta^S = \frac{1}{s}$ for all $S \subset N$, then only minimal winning coalitions form in an SPE.*

3 The case of apex games

Apex games are a special class of weighted majority games with one major player (the apex player) and $n - 1$ minor players (also called base players). If player 1 is the apex player, $v(S) = 1$ if either $\{1\} \subseteq S$ and $S \setminus \{1\} \neq \varnothing$, or $S = N \setminus \{1\}$. There are two types of minimal winning coalitions: the apex player together with one of the minor players, and all the minor players together. Apex games have received a lot of attention in the literature, both in theory (Davis and Maschler (1967), Horowitz (1973), Hart and Kurz (1984), Aumann and Myerson (1988), Bennett and van Damme (1991), Montero (2003)) and in experiments (Selten and Schuster (1968), Horowitz and Rapoport (1974), Albers (1978), Rapoport et al. (1978), Rapoport et al. (1979), Miller (1980), Komorita and Tumonis (1980)). They are also empirically common, especially in parliaments with a small number of parties. Below are two current examples.

	Total	*SPD*	*CDU*	*FDP*	*Grüne*
Seats	231	102	88	24	17

The *Landtag* in North Rhine-Westphalia

Assuming that decisions are taken by simple majority, the SPD can form a winning coalition with any of the other three parties. In Catalonia we also have an apex player (CiU) and three minor players (PSC, PP and ERC) plus a dummy player (IpC-V).

	Total	CiU	PSC	PP	ERC	IpC-V
Seats	135	56	52	12	12	3

The *Parlament* in Catalonia

3.1 Apex games when the internal game has an egalitarian protocol

Suppose $\theta_i^S = \frac{1}{s}$ for all $S \subseteq N$ and for all $i \in S$. As we have seen, this implies that in the internal game expected payoffs are given by $y_i^S = \frac{1}{s}(1 - \sum_{j \in S} y_j) + y_i$.

As for the probability vector θ, we will assume that all minor players are treated equally, i.e. $\theta_i = \theta_j$ for any minor players i and j, and that $\theta_i > 0$ for all i in N. Then the protocol θ is characterized by one parameter, the probability that the apex player is selected to be the proposer, which can take any value strictly between 0 and 1.

The first thing to notice is that, since the internal protocol is egalitarian, in an SPE each player will propose one of the coalitions containing him with maximal per capita excess at y. Thus, only minimal winning coalitions can form in equilibrium. There are two types of minimal winning coalitions: coalitions consisting of the apex player and one minor player, and the coalition of all minor players. In an SPE all minor players must have the same expected payoffs (see lemma 4), and thus the apex player must be indifferent between all the minimal winning coalitions containing him.

Lemma 4 *If $\theta_i = \theta_j$ for any minor players i and j, then in any SPE $y_i = y_j$.*

Proof. Suppose $y_i > y_j$. If i is selected to be the proposer, his payoff will be $y_i + \Delta$ (where $\Delta = \frac{1}{s}[v(S) - \sum_{k \in S} y_k]$ for some coalition S which is optimal for player i). If he is not selected to be the proposer but receives a proposal to be in a coalition, his average payoff will be $y_i + \Delta'$ (where $\Delta' \leq \Delta$). Let r_i the probability that player i receives a proposal from other players (computed before Nature starts the game). Then

$$y_i = \theta_i \left[y_i + \Delta\right] + r_i \left[y_i + \Delta'\right] \tag{2}$$

The payoff of player j as a proposer is at least $y_j + \Delta$ (if $j \in S$, then j can always propose coalition S; if $j \notin S$, then j can always propose coalition $\{S \backslash \{i\}\} \cup \{j\}$). Moreover, since $y_i > y_j$, whenever i receives a proposal j receives it as well. Thus

$$y_j \geq \theta_j \left[y_j + \Delta\right] + r_i \left[y_j + \Delta'\right] + (r_j - r_i)(y_j + \Delta'') \tag{3}$$

where $r_j \geq r_i$ and $\Delta'' \geq 0$. Substracting 3 from 2 and taking into account $\theta_i = \theta_j$ we obtain

$$(1 - \theta_i - r_i)(y_i - y_j) \leq -(r_j - r_i)(y_j + \Delta'')$$

Since the apex player will never propose to player i, $r_i < 1 - \theta_i$ and the left-hand side of the equation is strictly positive. Since the right-hand side is at most zero we have a contradiction. ■

We have shown that in any SPE only minimal winning coalitions may form and that the apex player must be indifferent between all the coalitions he can propose. As for the minor players, there are three possible cases: they may prefer a coalition with the apex player, the minor player coalition, or they may be indifferent. We will examine each possibility in turn.

Let y_a denote the expected payoff for the apex player and y_m the expected payoff for a minor player. We will also denote the probability that the apex player is selected to be the proposer as θ_a.

Lemma 5 *There is no SPE in which the apex player is always part of the coalition that forms.*

Proof. If all minor players propose to the apex player, the apex player is always part of a coalition and his expected payoff is given by the equation

$y_a = y_a + \frac{1}{2}(1 - y_a - y_m)$. This equation can only hold if $y_a + y_m = 1$, which (since $y_a + (n-1)y_m = 1$) implies $y_a = 1$. But then it would not be optimal for a minor player to propose to the apex player. ■

Proposition 6 *If $\theta_a \geq 1 - \frac{1}{n}$, there is an SPE in which all minor players propose the minor player coalition.*

Proof. If all minor players propose the minor player coalition, expected equilibrium payoffs are given by the following equations (taking into account that, in order for all minor players to have the same expected payoffs, the apex player must propose to each of them with equal probability)

$$y_a = \theta_a \left[y_a + \frac{1}{2}(1 - y_a - y_m) \right]$$
$$y_m = (1 - \theta_a)\frac{1}{n-1} + \theta_a \frac{1}{n-1}\left[y_m + \frac{1}{2}(1 - y_a - y_m) \right]$$

The solution to this system is $y_a = \frac{\theta_a(n-2)}{n(2-\theta_a)-2}$ and $y_m = \frac{2(1-\theta_a)}{n(2-\theta_a)-2}$. In order for this strategy combination to be an equilibrium we need $y_m + \frac{1}{2}(1 - y_a - y_m) \leq \frac{1}{n-1}$. This is the case if $\theta_a \geq 1 - \frac{1}{n}$. ■

Proposition 7 *If $\theta_a \leq 1 - \frac{1}{n}$, there is an SPE in which the minor players are indifferent between proposing to the apex player or proposing the minor player coalition.*

Proof. The indifference condition for the minor players, $y_m + \frac{1}{2}(1 - y_a - y_m) = \frac{1}{n-1}$, together with equation $y_a + (n-1)y_m = 1$ implies $y_a = \frac{n-2}{n}$ and $y_m = \frac{2}{n(n-1)}$. In order to construct an equilibrium, we need to find mixed strategies that yield those expected payoffs.

For any minor player i, let λ_i be the probability that i proposes to the apex player and $\lambda := \frac{\sum_{i \in N \setminus \{1\}} \lambda_i}{n-1}$. Expected equilibrium payoffs for the apex player are given by

$$y_a = [\theta_a + (1 - \theta_a)\lambda] \left[y_a + \frac{1}{2}(1 - y_a - y_m) \right].$$

Substituting for y_a and y_m, we find $\lambda = 1 - \frac{1}{n(1-\theta_a)}$. In order for λ to be nonnegative we need $\theta_a \leq 1 - \frac{1}{n}$.

There is a continuum of equilibria, all with the same value of λ.Let μ_i the probability that the apex player proposes to player i. Expected payoffs for player i are given by

$$y_m = \left[(1-\theta_a)(1-\lambda) + \frac{1-\theta_a}{n-1}\lambda_i + \theta_a \mu_i\right] \frac{1}{n-1}$$

Substituting for the equilibrium values of y_a, y_m and λ, we obtain $\mu_i = \frac{1}{n\theta_a} - \frac{\lambda_i(1-\theta_a)}{\theta_a(n-1)}$ (notice that $\sum_{i \in N\setminus\{1\}} \mu_i = 1$). Any collection of λ_i's with average $1 - \frac{1}{n(1-\theta_a)}$ and such that $0 \leq \mu_i \leq 1$ for all i is part of an SPE. The symmetric equilibrium has $\lambda_i = \lambda$ for any minor player i and $\mu_i = \frac{1}{n-1}$. ■

Remark 8 *For $\theta_a \leq 1 - \frac{1}{n}$, expected payoffs coincide with the per capita nucleolus.*

The per capita nucleolus is a solution concept introduced by Wallmeier (1983). Like the nucleolus (Schmeidler, 1969), it is usually thought of as a normative concept. Suppose we are looking for a fair payoff division for the value of the grand coalition. Given a possible division y, the difference between what a coalition can get by itself, $v(S)$, and what is getting at y, $y(S)$, can be thought of as a measure of the dissatisfaction of the coalition with S, or the temptation to defect from y. The nucleolus minimizes the largest excess.

Since the decision units are the players and not the coalitions, it makes sense to measure the temptation to defect by the *per capita* excess, $\overline{e}(S, y) := \frac{1}{s}(v(S) - y(S))$, instead of the excess. The per capita nucleolus minimizes the largest per capita excess.

When we constructed the SPE of the game we proceeded as follows. Given a candidate expected payoff vector (or a range of payoff vectors), we constructed the optimal proposing strategies of the players. In all cases, since the internal protocol is egalitarian, it is optimal for the players to propose one of the coalitions containing then with maximal per capita excess. Then we checked whether there were optimal strategy combinations consistent with the candidate payoff vector or range. This was not always possible: the expected payoff for the apex player could not be so low that the minor players would prefer to propose to him. What makes the per capita nucleolus special is that it makes the players indifferent between several coalitions, and this in turn means more degrees of freedom when constructing optimal strategies consistent with the payoff vector.

In two-stage games the payoff a player gets in a coalition does not depend on who proposed the coalition. In the apex game with an egalitarian internal protocol and $\theta_a \leq 1 - \frac{1}{n}$, we can say something stronger: the payoff a player gets when entering a coalition is always the same, i.e., in equilibrium players only enter coalitions that offer them the maximal possible payoff. In other words, the equilibrium payoffs conditional on being in a coalition correspond to an aspiration (see Bennett (1983)). The particular aspiration vector we obtain (the apex player gets $\frac{n-2}{n-1}$ if he enters a coalition, and a minor player gets $\frac{1}{n-1}$) is the one selected by refinements of the aspiration set.

3.2 Apex games with an internal protocol related to pivotal power

So far, we have assumed that all players have the same proposing power in the internal game: asymmetries in payoffs are due to different opportunities for the players if negotiations break down. This seems logical in minimal winning coalitions (all players are pivotal), but less so in larger coalitions. Changing the internal protocol so that the apex player is selected to be the proposer more often than a minor player when there are several minor players makes no difference to the results unless the probability that the apex player is chosen to be proposer is increasing (not only in comparison to that of a minor player, but also in absolute terms) in the number of minor players in the coalition. In that case coalitions larger than minimal winning may arise in equilibrium. For illustration, we consider the case in which the probability that the apex player is selected to be the proposer in the internal game is $\frac{s-1}{s}$, where s is the total number of players in the coalition.[5] The payoff the apex player gets as a proposer is

$$[1 - y_a - (s-1)y_m]\,\frac{s-1}{s} + y_a$$

[5] The number $\frac{s-1}{s}$ is also the coalition structure Shapley value for the apex player.

Taking into account that $y_a = 1 - (n-1)y_m$, we obtain $\frac{y_m(n-s)(s-1)}{s}$. This expression is maximized for $s = \sqrt{n}$. For $n \geq 6$, it is optimal for the apex player to propose coalitions that are not minimal winning.

More generally, we can study the equilibria of the game in which the share of the surplus a player gets in a coalition depends on his pivotal power. For minimal winning coalition the surplus will be divided equally since all players are equally pivotal. As for larger coalitions, we will assume that the fraction of the surplus the apex player gets is a function ϕ of the number of players in the coalition.[6] Suppose we are in equilibrium, and let s be the size of the optimal coalition for the apex player. Then the expected payoff for the apex player is

$$y_a = \theta_a \left[(1 - y_a - (s-1)y_m)\, \phi(s) + y_a \right] + (1-\theta_a)\lambda \left[\frac{1 - y_a - y_m}{2} + y_a \right]$$

It is easy to check that, in any SPE, all minor players propose the minor player coalition with positive probability. Thus, $\lambda < 1$. We look for two types of equilibria, depending on whether $\lambda = 0$ or $\lambda > 0$.

Set $\lambda = 0$. This is an equilibrium only if solving the equation above we obtain $y_a \geq \frac{n-2}{2}$. This is the case if $\theta_a \geq \frac{(n-1)(n-2)}{2\phi(s)(n-s)+(n-1)(n-2)}$.

A strictly positive λ implies $y_a = \frac{n-2}{2}$ and $y_m = \frac{2}{n(n-1)}$. Substituting for y_a and y_m into the equations indeed yields $\lambda = \frac{(n-1)(n-2)(1-\theta_a)-2\phi(s)\theta_a(n-s)}{n(n-2)(1-\theta_a)}$. In order for this value to be larger than 0, we need $\theta_a \leq \frac{(n-1)(n-2)}{2\phi(s)(n-s)+(n-1)(n-2)}$. Notice that the critical value of θ_a is large (we can find a lower bound for it by setting $s = 0$ and $\phi(s) = 1$) and converges to 1 as n grows. This means that, for most values of the parameter θ_a, expected payoffs coincide with the per capita nucleolus. Thus, allowing for general internal protocols based on pivotal power does not change the qualitative results concerning equilibrium payoffs. The only difference is that coalitions other than minimal winning may form, and that the equilibrium outcomes do not longer correspond to an

[6] The function ϕ could be interpreted as a reduced form of a more complicated internal game in which partial breakdown is possible, resulting in the formation of a smaller coalition. The possibility of partial breakdown makes the apex player profit from bargaining with several minor players rather than only one.

aspiration vector: the minor players may accept offers in which they receive less than their maximum possible payoff.

A very important assumption we have made is that coalitions cannot be enlarged once formed. If coalitions can be enlarged and the apex player profits from proposing coalitions larger than minimal winning, the coalition of the apex player with only one minor player is not possible: if formed, the apex player would invite new minor players and they would accept rather than get 0. Then the choice of the minor players is between the minor player coalition and a coalition of the optimal size for the apex player, and a mixed strategy equilibrium need not yield the per capita nucleolus. Interestingly, equilibrium payoffs can be *lower* for the apex player if coalitions can be enlarged. This result is in line with Aumann and Myerson (1988).

4 Concluding remarks

The results of two-stage bargaining for apex games are qualitatively similar to those of one-stage bargaining (see Montero (2003)) in terms of expected payoffs: there is a large region in which expected payoffs are constant, and the minor players randomize between proposing to the apex player and proposing the minor player coalition. With two-stage bargaining the division of payoffs inside a coalition does not depend on who proposed to form the coalition, whereas in one-stage bargaining the proposer always gets more than half of the total payoff. Perhaps surprisingly, the apex player has a larger expected payoff in the two-stage game, despite of the fact that the two-stage game might seem to protect the minor players by splitting the surplus equally. The two-stage model also provides some support for the formation of coalitions larger than minimal winning, something impossible in the one-stage model.

One-stage models with random proposers have provided noncooperative foundations for the nucleolus (see Montero (2001)). One may wonder whether two-stage models can provide noncooperative foundations for the per capita nucleolus. The answer is negative, at least for the present model. A difficulty is that players with zero payoffs in the per capita nucleolus may have a positive payoff in the two-stage game, even if they are never chosen to be proposers in the coalition formation stage.

The noncooperative approach to bargaining and coalition formation has often been critisized because of the sensitiveness of the results to the details of the extensive form game. However, this property may also be seen as a strenght: institutions presumably matter, and different extensive form games may reflect different institutional environments. In this sense, the two-stage model with reversible coalitions presented here is a first step towards modelling situations in which player's commitment to coalitions is imperfect.

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