International Risk Sharing and Bank Runs*

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Abstract

Banks act as maturity transformers, who take liquid deposit and invest in illiquid assets. In this classical framework, we introduce uncertainty in the asset returns. We show that banks can insure individuals against the risk of illiquidity at the cost of increasing the riskiness of their portfolios. In an open financial market, they can better diversify their portfolio and decrease its risk. In that way, they can also increase the level of insurance against the risk of illiquidity. This improves individual welfare, but the banks' short-term deposit-reserve ratio and the fragility of the financial system result higher in an open economy than in an autarchic regime. For this reason, the mechanism of deposit insurance against bank runs becomes more difficult to implement by each country's central bank.

Keywords: Bank run, international risk sharing, fragility of financial markets, deposit insurance.

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1 Introduction

The last 20 years have been characterized by a steady increase of cross-country capital mobility.[1] However, in the same period, a growing number of financial crises hit countries with open financial markets.[2] For this reason it has been argued that financial liberalization and opening the capital account increase the vulnerability of a country to a crisis.

However, this seems at odds with the argument that a more globalized financial market, raising the opportunity of risk sharing, should increase the stability of each country.

In this paper we show that this contradiction is only apparent, and the higher fragility of the financial system is a natural by-product of the better portfolio insurance allowed by international risk sharing.

We model the role of the bank, building heavily on the celebrated model of Diamond and Dybvig (1983) (henceforth DD), where banks invest in long run assets and insure individuals against the risk of illiquidity, giving them the possibility of liquidating their investments at a lower cost. We extend the DD model to cope with uncertainty of asset return.

We show that the bank faces the trade-off between insuring individuals against the risk of illiquidity and the portfolio risk. In an open economy, a bank can diversify away part of the portfolio risk and it can assure a higher level of consumption to the individuals hit by a shock of illiquidity than it can in a regime of autarchy. In practice a bank achieves this result by raising the short-term interest rate.

The welfare of each individual increases because the level of individuals' insurance is higher, but the financial system becomes more fragile in the sense that the deposit-reserve ratio of every bank becomes higher. As a result, the banking system in an open economy can be driven to a generalized bank run by an exogenous shock smaller in its magnitude than the one which can destabilize an

[1] See Obsfeld and Taylor (2001) for an article on Globalization and Capital markets in an historical perspective.

[2] Chile in the early 80s, Scandinavia in the early 90s, Mexico in 1995, Asia in 1998, Argentina in 2002.

autarchic economy.

This finding is consistent with Stiglitz's (1998) observation:

> "We argue that the evidence is consistent with the belief that large short-term debt exposure made the East Asian countries vulnerable to a sudden withdrawal of confidence".

Moreover, Demirguç-Kunt and Detragiache (1998) find that high interest rates are associated with banking crises.

This increased fragility has a direct consequence in terms of economic policy. It is well known that governments can avoid the bank run equilibrium with a deposit insurance, provided that it is free to impose a sufficiently high tax on the agents who withdraw "early". Alternatively, a central bank can have the same effect by creating money.

We show that, in an open economy, the government must be able to levy higher taxes or to create more inflation than in an autarchic economy in order to avoid the bank run. On the other hand, a high level of taxes or inflation can be very damaging for the economy and can reduce welfare even more than a financial crisis. If this is the case of an economy after opening to the foreign financial markets, the government's deposit insurance becomes not credible. In that case the intervention of a supranational agency that supplies additional insurance may be necessary.

In the second part of the paper we introduce the ex-ante shock expectations; we see that individuals in making their choice, besides the risk of illiquidity and the asset risk, face a third source of uncertainty: the risk of bank run. All above results are robust to this extension. Furthermore, we show that, besides the level of risk sharing, also the ex-ante shock expectations play an important role in raising the financial fragility; the higher the belief in the probability of a shock, the lower the fragility of the banking system. Moreover, we see that the effect of opening the financial market increases the fragility of the system more when the shock expectations are low.

This seems consistent with the fact that the last crises were all largely unexpected. The ex-ante probabilities of the crises in Thailand, Malaysia and Indonesia have been estimated between 3 and 6 percent (see Stiglitz 1998), and Sachs, Tornell and Velasco (1996) provide evidence that the Mexican crisis in 1994 was not anticipated.

Finally, we analyze the relation between risk aversion and fragility of the banking system. We show that individual risk aversion has a rather counterintuitive impact on the fragility of the banks. If ex-ante, individuals try to minimize the risk of a bank run when they are more risk averse, raising the soundness of the financial systems; after the shock they tend to run to the counter before, when risk aversion is high. For these reasons, in many cases, risk aversion actually increases the fragility of the system. Therefore, we argue that after the shock high risk aversion becomes "high propensity to panic".

The high number of crises from 1994 onwards, and in particular the ones in South East Asia, stimulated a new generation of models (the so-called 3rd generation). Aghion, Bachetta and Banerjee (1999) and Krugman (1999) emphasize the effect of a shock leading to an unexpected devaluation on a firm's profitability. Firms have many liabilities in foreign currency, therefore a sudden devaluation negatively affects their balance sheets. This effect propagates the initial shock throughout the economy.

There is no doubt that the above elements are all present in many of the recent crises, and especially in South East Asia, however, a comparison between Brazil and Argentina shows that this might not be the whole story. Brazil was even more exposed than Argentina to foreign nominated debt[3]; therefore, Brazil's decision to abandon the parity with the dollar should have brought the country to a financial crisis more so than in Argentina, where the central bank tried to defend the fixed peg with the dollar until much longer. Instead, a more serious crisis happened in Argentina, and many blame the decision to keep the fixed peg as the main cause of its financial distress.

The model which is closest to the present is that of Chang and Velasco (1998), who extend the DD framework to consider an exogenous foreign inflow of capitals. They show that this increases the fragility of the banking system because it becomes vulnerable to the refusal of foreign creditors to roll over the short-terms loans. In the present model, we introduce the asset risk and we derive endogenously the demand of foreign funds (to diversify the bank portfolio). We

[3] In the ranking of the ratio of liabilities to claims with respect to foreign banks (NBIS-IMF-OECD world Bank statistics) , Brasil appears two position head respect to Argentina.

show that the financial system is more fragile when the markets are open, independently from the foreign investors' decision. Furthermore, neither Chang and Velasco nor the original model of Diamond and Dybvig explicitly take into account the ex-ante shock expectations.

This paper is organized as follows: In the next section, we present the economic environment. In the third section, we analyze the different equilibria under the simplifying assumption that the shock is completely unexpected, and we compare the regime of autarchy and the regime of open economy. The fourth section is dedicated to the implication for the economic policy of opening the economy to the international financial markets. In the fifth section we extend the model to consider the shock expectations. The last section offers some concluding remarks.

2 The theoretical framework

In this section we present a model where agents use banks to transform the liquidity structure of investments along the line of the well known model of DD. In this framework, we introduce uncertainty in the asset returns which we suppose partially negatively correlated between countries. Therefore, in an open economy, a bank in one country has the option of buying assets in another country, in order insure its portfolio against the country idiosyncratic risk.

2.1 The basic model

There are 2 countries: Home and Foreign country. In both of them a one unit investment yields:

$$\tilde{R} = \begin{cases} R & \text{with probability } p \\ 1 & \text{with probability } 1-p \end{cases}, \tag{1}$$

with $R > 1$.

The final outcome in the 2 countries depends on a common shock and on a shock at country-level, which is inversely correlated in the two countries. Let the probability of the common shock be $p_{H,F}$, in that way we have, for the four resulting states of the world, the following probability distribution:

Table 1: Probability distribution of the states of the world

Countries	H		
	Revenues	R	1
F	R	$p_{H,F}$	$p - p_{H,F}$
	1	$p - p_{H,F}$	$1 - 2p + p_{H,F}$

with $2p - 1 \leq p_{H,F} \leq p$. We have 4 states of the world depending on the country's asset outcome. We will indicate these states of the world with the couple $(i, j) \in \{1, R\}$, where i represents the outcome in the home country and j in the foreign country.

We can distinguish 2 extreme cases:

- $p_{H,F} = Max[0, 1 - 2p]$; the level of negative correlation is maximal and the possibility of risk sharing is maximal.
- $p_{H,F} = p$; they are perfectly positively correlated, there is no possibility of risk sharing. In this case agents face exactly the same economic environment in an autarchy as in an open economy.

Every asset needs two periods to yield the expected revenue $\tilde{R}$, if they are liquidated after the first period they yield no revenue, and only the initial capital can be recovered.[4] Therefore, every asset has the following temporal revenue structure:

$$\begin{array}{ccc} t=0 & t=1 & t=2 \\ & \nearrow 0 & \longrightarrow \tilde{R} \\ -1 & & \\ & \searrow 1 & \longrightarrow 0 \end{array} \quad . \tag{2}$$

This structure captures the idea that long run investments are not liquid. Liquidating assets before the maturity implies the cost of loosing one period revenue. Finally, we suppose that the total investment can be arbitrarily divided and a portion of it can be liquidated at no cost.

[4]For simplicity we are not considering any transaction cost in selling assets before the maturity.

The rest of the model is similar to Diamond and Dybvig. We consider each country populated with a continuum of agents with mass 1. There is a single good that can be consumed and invested. Every agent owns a unit of endowment at $t = 0$ and lives for three periods. They can consume only in the second and third period.

If an individual is hit by an illiquidity shock in $t = 1$, she becomes "impatient", and wants to consume all her wealth by $t = 1$. Otherwise she is "patient" and prefers to consume in $t = 2$; since the money does not devaluate, a patient individual can withdraw at $t = 1$ and consume at $t = 2$. At time 0, every individual ignores her own type but she knows that she will be impatient with probability $\bar{\lambda}$.

Therefore, the expected utility function for all individuals at time 0 is:

$$E(U(c_1, c_2)) = \bar{\lambda}u(c_1) + (1 - \bar{\lambda})E(u(c_2) \mid p, p_{H,F}\ ), \tag{3}$$

with the usual assumptions $u'(c) > 0$ and $u''(c) < 0$.

In the absence of a bank, individuals invest at time 0 all their initial endowments independently from their preferences. Indeed, individuals always recover, at least, their initial capital: this happens either when they liquidate their investment at $t = 1$, given the payoffs structure (2), or when they do not liquidate the assets but they yield 1 at $t = 2$, given the asset revenue structure (1).

Accordingly, the choice between consuming in the short or in the long run (i.e. the choice between continuing or liquidating the investment) is made at time 1. Moreover, we can notice that there is not asset trade at time 1, given that patient individuals who would like to have more assets cannot buy them because they have already invested all their wealth at time 0.

Therefore, without a banking system, all individuals have the following pattern of consumption across states of nature:

$$\begin{aligned} c_1 &= 1 \text{ and } c_2 = 0 \text{ if impatient} \\ c_1 &= 0 \text{ and } c_2^e = \tilde{R}^{i,j} \text{ if patient} \end{aligned}$$

where $\tilde{R}^{i,j}$ represents the revenue of the portfolio determined by mixing Home and Foreign assets if the economy is open, while in autarchy it is simply $\tilde{R}$.

2.2 The bank

Following DD, a bank can increase the utility of individuals and achieve the first best equilibrium, because it can perfectly insure them against the illiquidity shock. In this section, we introduce the bank, which has the double task of managing the asset portfolio, besides insuring individuals against the illiquidity risk like in the DD model.

Each individual has a probability $\bar{\lambda}$ of being hit by the illiquidity shock. Consequently, there is a measure $\bar{\lambda}$ of impatient individuals at time 1. We recall that liquidating the asset before its maturity is costly, therefore individuals, who are risk averse, would benefit from an insurance against the illiquidity risk. Nevertheless, the type of individuals is not observable and a specific insurance contract contingent on this event cannot be written.

A bank with a deposit contract can still provide the first best level of insurance to individuals, by using the following contract: individuals accept to sign a contract with the bank, surrendering to it all their endowments; the bank invests all the wealth in a risky portfolio, and it commits to give back to the individuals an amount c_1 if they withdraw their deposit at time 1, and an expected amount c_2^e at time 2.

The bank can always observe whether the individuals withdrew at time 1 and in this case can refuse a second withdrawal. Moreover, if financial markets are open, the bank can select an optimal asset portfolio by part of its deposit, say an amount b, in foreign assets.

The objective of the bank is to maximize the welfare of every individual. Alternatively, we can imagine that a large number of banks are in perfect competition with 0 profit.[5]

A bank is subject to the following constraints:

[5] The 0 profit assumption simplifies the analysis. Even if banks could extract some profit the following results will not change as far customers can obtain part of the surplus.

$$\begin{aligned}
\rho c_1 &\leq 1 && (4)\\
(1-\rho)c_2^{R,R} &\leq (1-\rho c_1)R && (5)\\
(1-\rho)c_2^{R,1} &\leq (1-\rho c_1 - b)R \ + b && (6)\\
(1-\rho)c_2^{1,R} &\leq (1-\rho c_1 - b) \ + bR && (7)\\
(1-\rho)c_2^{1,1} &\leq 1-\rho c_1 && (8)\\
E(u(c_2)) &\geq u(c_1) && (9)
\end{aligned}$$

Where ρ is the number of individuals withdrawing their deposit at time 1, and the superscripts over c_2 indicate the different states of the world in table 1.

Constraint (4) is the budget constraint at period 1, whereas (5)-(8) represent the budget constraints in each period-2 states of the world.[6] Finally (9) is the usual IC constraint for patient individuals, which ensures that an individual is willing to reveal his type. When this constraint is violated, all patient individuals claim to be impatient at time 1, withdraw c_1, and consume it at time 2. Clearly, impatient types always have an incentive to declare their true type, given that they derive no utility from consuming at time 2.

Constraint (4) always holds strictly, otherwise all investments are liquidated at time 1, and $c_2 = 0$, this cannot be optimal given the concavity of the utility function (3). Similarly, constraints (5) - (8) must bind if the banks are in perfect competition and make 0 profit and each of them maximizes the individuals' utility. Accordingly, the consumption of a patient individual in each state of the world is given by:

$$\begin{aligned}
c_2^{R,R}(\rho) &= \frac{(1-\rho c_1)R}{1-\rho} && (10)\\
c_2^{R,1}(\rho) &= \frac{(1-\rho c_1)R+b}{1-\rho} && (11)\\
c_2^{1,R}(\rho) &= \frac{(1-\rho c_1-b)+bR}{1-\rho} && (12)\\
c_2^{1,1}(\rho) &= \frac{(1-\rho c_1)}{1-\rho} && (13)
\end{aligned}$$

[6] We implicitly supposed that banks invest in home assets the amount it has to disinvest in period 1. This is a totally neutral assumption given that all assets yield 1 in the first period.

For example constraint (6) refers to the state in which home asset yields $(1-\rho c_1 - b)R$ and foreign asset b. The amount ρc_1 is obtained by liquidating part of the home asset.

3 Equilibria

We will now proceed as it follows. We compute the first best efficient equilibrium, where the bank maximizes the individual utility function (3). After, we show that the efficient equilibrium is not the only one, but a generalized bank run, where everybody decides to withdraw at time 1 her deposit, is an equilibrium as well.

At this point, it is possible to object that in the bank optimization problem we made the implicit assumption that the bank run equilibrium is fully unexpected by agents at time 1. Accordingly, in the last section we extend the model to consider an expected shock with a given probability distribution on its intensity. In general, we will observe that the results of this section still hold when we consider the shock expectations.

3.1 Efficient Equilibrium

An equilibrium is defined by the mass of individuals who withdraw at time 1 ρ^* and by the bank's optimal choices c_1^* and b^*. In order to determine the efficient equilibrium, we begin with the conjecture that only impatient individuals withdraw at time 1, so that $\rho^* = \bar{\lambda}$, and we determine the optimal level of c_1 and b. Finally we show that this is an equilibrium, where the IC constraint (9) holds. Note, however, that such an equilibrium relies on the expectation by each depositor that this strategy is adopted by all patient individuals, we will relax this assumption in the last section.

In order to simplify the analysis and derive a close solution, we assume that $u(\cdot)$ is a CRRA utility function:

$$\frac{c^{1-\sigma}}{1-\sigma}. \tag{14}$$

Substituting ρ with $\bar{\lambda}$ and (10)-(13) into the utility (3) and using the functional form (14), we find that the resulting expression is maximized for:

$$b^* = \begin{cases} \frac{(1-\bar{\lambda}c_1^*)}{2} & \text{for } 2p-1 \leq p_{H,F} < p \\ \in (0,1) & p_{H,F} = p \end{cases}, \tag{15}$$

when $p_{H,F} = p$ there is not distinction between home and foreign investment, and

$$c_1^* = \frac{1}{\lambda + (1-\lambda)\psi(p_{H,F}, R, \sigma)^{\frac{1}{\sigma}}}. \tag{16}$$

with: $\psi(p_{H,F}, R, \sigma) \equiv 1 + p_{H,F}\left((1+R^{1-\sigma}) - 2\left(\frac{1+R}{2}\right)^{1-\sigma}\right) - 2\left(1 - \left(\frac{1+R}{2}\right)^{1-\sigma}\right)p.$

It is possible to show (see the Appendix) that expressions (15) and (16) are the optimal bank choice because the IC constraint $E(u(c_2^*)) > u(c_1^*)$ is always satisfied for these values. This shows that the deposit contract is always compatible with the truth-telling constraint if $\rho = \lambda$: a bank can stipulate a demand deposit contract by which individuals can withdraw at their discretion either c_1^* at time 1 or c_2^{e*} at time 2. On the other hand, the bank can commit to fulfill the requests of all depositors until all investments have been liquidated.

Accordingly, from (16) we observe that when the coefficient of risk aversion

$$\sigma > 1 \tag{17}$$

then $c_1^* \geq 1$.[7]

This implies that individuals want to insure themselves against the need of liquidity at time 1 only if they are sufficiently risk averse (recall that without banks we have $c_1 = 1$). Since we are interested in economies where banks play this role we focus, from now on, on the case where (17) is always true.

From expression (16), we have:

Proposition 1 *i) If* $p_{H,F} > \max[2p-1, 0]$ *then* $lim_{\sigma\to\infty} c_1^* = 1$*; ii) if* $p_{H,F} = \max[2p-1, 0]$ *then* $lim_{\sigma\to\infty} c_1^* > 1$.

Proof. *See the appendix.* ■

When $p_{H,F} > 2p - 1$ the state of world $(1,1)$, where there is no return on investment, is always possible. Therefore this proposition states that when there is a possibility that at time 2 there are no revenue, very risk averse individuals

[7] The higher expected revenue at time 2, has two opposite effects on the level of c_1: on one side, individuals want to save more at time 1 so that their revenue at time 2 will be higher; on the other side they do not want to bear the risk of low consumption at time 1. The prevailing effect depends on the level of risk aversion. In the case of a CRRA utility function the second effect prevail for when (17) is true.

prefer not to insure against the liquidity risk. If they are impatient they obtain a low level of wealth but if the state $(1,1)$ happens, they would obtain an even lower amount if they chose an higher level of illiquidity insurance. A highly risk averse individual does not want to bear this risk, even if the probability of a bad state of nature is low.

This is graphically illustrated in figure 1: the behavior of c_1^* when $p_{H,F} > \max[2p-1,0]$ (solid lines) underlines a trade-off between the two risks every individual faces: the risk of liquidity and the asset risk. With a sufficiently high level of risk aversion individuals do not want to bear the risk of a low level of consumption in the state $(1,1)$. For these reasons as σ grows, $c_1 \to 1$: individuals tend not to get insured against the risk of liquidity.

When $p_{H,F} = \max[2p-1,0]$ the international financial market allows a complete insurance against the asset risk. The level of c_1^*, in this case represented by the dashed line in figure 1, is always increasing in σ : in an open market with full insurance against the asset risk, individuals only face the risk of illiquidity (i.e. being an impatient type). Therefore, the more risk averse individuals are, the more they want to get insured against the event of being impatient, choosing a higher consumption in the first period.

In more general terms, the introduction of a risky asset in a DD setting reduces the possibility for the bank to insure individuals against illiquidity risk. However, the portfolio's riskiness can be lowered (or completely removed) by mixing foreign and internal assets (as far as $p_{H,F} < p$). As a result, banks have the second task of managing an asset portfolio. We saw that the bank implements this activity by choosing b, and from it derives:

Proposition 2 *The level of insurance against the risk of illiquidity is a positive function of the level of diversification allowed by the international financial markets:* $\frac{\partial c_1^*(p_{H,F})}{\partial p_{H,F}} < 0$.

Proof. In the appendix we prove that $\frac{\partial \psi(p_{H,F},R,\sigma)}{\partial p_{H,F}} > 0$. Given the sign of this derivative, and given the expression (16), it follows that $\frac{\partial c_1^*(p_{H,F})}{\partial p_{H,F}} < 0$. ■

We recall that $p_{H,F}$ is an index of positive correlation between home and foreign assets: the lower this probability, the more the foreign and home assets are negatively correlated between each other. This increases risk sharing and the advantage of the international asset market.

Therefore, international financial markets supply a further instrument to banks for insuring agents. In that way, financial markets allow to separate the risk of portfolio from the risk of illiquidity. This possibility is total if $p_{H,F} = Max[0, 2p - 1]$, given that in this case all the risk can be diversified away, it is partial if $p_{H,F} > Max[0, 2p - 1]$.

Moreover, as we argued before, when $p_{H,F} = p$ opening the financial markets to foreign assets is totally irrelevant with respect to the bank portfolio's choice, and the bank investment in foreign asset b can indifferently be a value between 0 and 1. As a result, Proposition 2 states that in an autarchy the level of c_1^* is always lower than in an open economy.

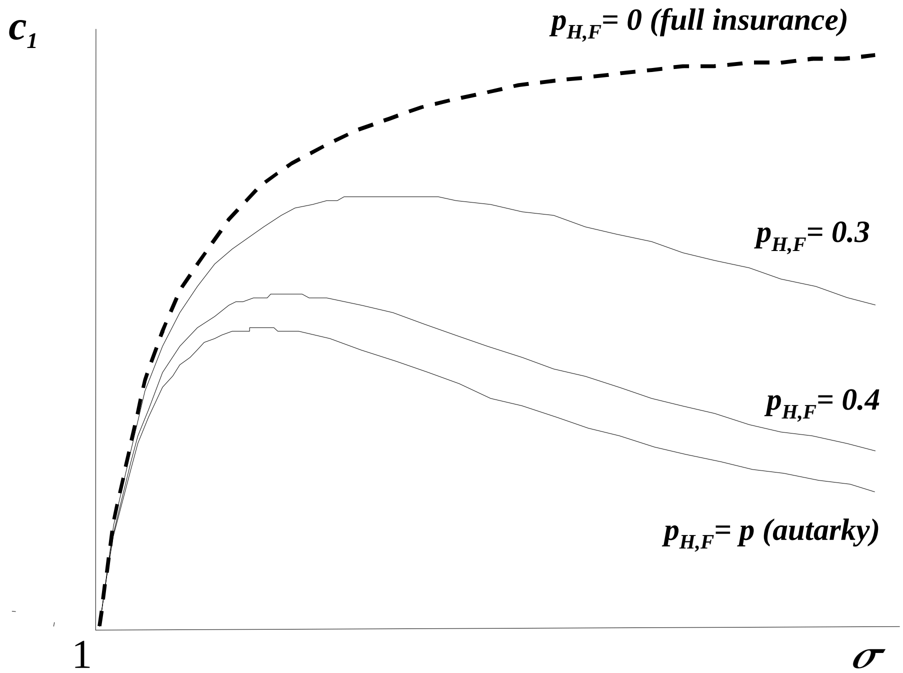

Figure 1: Consumption in relation to risk aversion for different degrees of international risk sharing (in this example $p = \frac{1}{2}$).

3.2 Bank runs

We saw that the optimal paths of consumption (c_1^*, c_2^{e*}) can be implemented via a demand deposit contract. Individuals at time 0 surrender all their respective endowments to a bank and they withdraw at their discretion either c_1 at $t = 1$ or c_2 at $t = 2$.

Yet for $\sigma > 1$, we have seen that $c_1 > 1$. Thus, banks cannot fulfill all demands if all individuals (patient and impatient) want to withdraw at time 1. In such a case, the bank would go bankrupt at time 1, and it is rational for every individual to run to the counter and try to get at least c_1. Therefore, there is another equilibrium, in which all individuals run to the bank at time 1.

Accordingly, like in the model of DD, also in our model an efficient equilibrium coexists with a bank run equilibrium. However, in this paper we are interested in the effect of risky assets and of portfolio diversification on the final equilibrium. Thus, we want to investigate how the possibility of investing in foreign assets affects the fragility of the system.

In order to achieve this result we will provide a definition of fragility; in particular, we will show that there is a critical measure of individuals, who are able to trigger a generalized bank run, when hit by a shock in their preferences or when they have pure shift in their expectation on the bank solvability. Therefore, we can argue that the lower this number, the more fragile the banking system is.

Let us suppose that a run has the following timing: i) between $t = 0$ and $t = 1$, a number $\lambda > \bar{\lambda}$ of individuals are hit by the liquidity shock and go to the bank, everybody can observe the number of individuals at the counter, ii) the other $1 - \lambda$ individuals decide whether to run to the bank or not, iii) at $t = 1$, if the mass of withdrawals is such that not all deposit demands can be fulfilled, the bank decides to liquidate all the assets and to distribute the same amount to all individuals at the counter.[8]

Since the number of individuals at the counter can be observed by everybody, if the bank cannot fullfil the demand of all customers at the counter, all 1 individuals run to the counter (i.e. there is a generalized bank run), and everybody obtains their initial capital 1.

3.3 The shock leading to a bank run

The necessary and sufficient condition for a generalized bank run is that the number of impatient individuals when there is a shock $\lambda > \bar{\lambda}$ is such that

[8]For simplicity we assumed this rule, another possibility is the "first come first serve" rule. In this section it is indifferent to assume one or the other rule. In the next the present rule simplifies the model.

$E(u(c_2(\lambda))) < u(c_1)$. In such a case, constraint 9 does not hold, and all individuals have an incentive to withdraw at time 1 ($\rho = 1$).

From expressions (10)-(13) and the utility function (3) we see that $E(u(c_2(\rho)))$ is strictly decreasing in ρ. Accordingly, there will be a $\hat{\rho} : E(u(c_2(\hat{\rho}))) = u(c_1)$, or:

$$\hat{\rho} = \frac{\lambda\psi(p_{H,F}, R, \sigma)^{\frac{1}{1-\sigma}} + (1-\lambda)\psi(p_{H,F}, R, \sigma)^{\frac{1}{\sigma(1-\sigma)}} - 1}{\psi(p_{H,F}, R, \sigma)^{\frac{1}{1-\sigma}} - 1}, \tag{18}$$

so that for all $\lambda \in [\hat{\rho}, 1)$, $E(u(c_2(\lambda))) < u(c_1)$: for all $\lambda > \hat{\rho}$ everybody will have an incentive to withdraw her deposit and $\rho = 1$.

The measure $\hat{\rho} - \lambda$ is a an index of fragility of the banking system with respect to unexpected negative shocks. The closer $\hat{\rho}$ is to λ the lower the number of individuals who, by withdrawing in $t = 1$, are able to trigger a generalized bank run.

We said that $p = p_{H,F}$ corresponds to the autarchic case, so let us define $\hat{\rho}^A \equiv \frac{\lambda\psi(p,R,\sigma)^{\frac{1}{1-\sigma}} + (1-\lambda)\psi(p,R,\sigma)^{\frac{1}{\sigma(1-\sigma)}}}{\psi(p,R,\sigma)^{\frac{1}{1-\sigma}} - 1}$. Accordingly, we state that:

Proposition 3 *The banking system in an open economy is more fragile than in an autarchy: $\hat{\rho}^A > \hat{\rho}$; moreover $\hat{\rho}$ is monotonically increasing in $p_{H,F}$: the fragility is lower the lower the level of international risk sharing ($p_{H,F}$ closer to p).*

Proof. Both statements follow directly from $\frac{\partial\hat{\rho}}{\partial p_{H,F}} > 0$ (this proof is in the appendix). ■

This proposition says that the banking system in an open economy is more sensitive to a negative liquidity shock, in the sense that a smaller shock can generate a generalized bank run. In other words, an autarchic economy has a better capacity of sustaining aggregate liquidity shocks.

In an open economy, the bank better insures individuals against the assets' riskiness, and, as we saw in the previous section, c_1 increases. This decreases the short-term deposit to liquidity ratio, which is equal to $\frac{1}{c_1^*}$. As a result, it will be more difficult to guarantee to patient individuals a level of expected utility $E(u(c_2(\rho))) \geq u(c_1)$, because, when c_1 is larger, the bank has to liquidate a higher number of assets for each customer withdrawing at $t = 1$.

In an economy where it is not possible to diversify the asset risks, c_1 is closer to one (as we discussed in the preceding section). Therefore, if a mass $\lambda > \bar{\lambda}$ of individuals decide to withdraw earlier, this will compromise less the capacity of the bank of fulfilling the demand of money at time 2.

4 Economic Policy

4.1 Deposit insurance

In order to avoid a bank run, governments or central banks can put in place a demand deposit insurance. The government has the possibility, after observing ρ, to impose an ex-post tax on the early withdrawals if $\rho > \hat{\rho}$. The revenue from this tax is finally given back to the banks. Accordingly, this kind of tax has two effects, to lower the utility of withdrawing early and to refund the banking system.

Let τ be a tax rate on consumptions. The tax structure able to avoid the bank run is:

$$\{\tau : u(c_1(1-\tau)) \geq E(c_2(\rho)) \ , \ \rho \in (\lambda, 1)\}$$

or, omitting for notational simplicity the arguments of function $\psi(p_{H,F}, R, \sigma)$, :

$$\tau(\rho, p_{H,F}) = \begin{cases} 0 & \rho \leq \hat{\rho} \\ \frac{1-\lambda\psi^{\frac{1}{1-\sigma}} - (1-\lambda)\psi^{\frac{1}{\sigma(1-\sigma)}} + \rho(\psi^{\frac{1}{1-\sigma}} - 1)}{1-\rho(\psi^{\frac{1}{1-\sigma}} - 1)} & \rho > \hat{\rho} \end{cases} \tag{19}$$

Moreover as already DD noticed, also the central bank can put in place such a mechanism of imposition by creating money. In this case, τ becomes the tax of inflation.

The taxation (19) guarantees the following level of utility to individuals withdrawing at time 1:

$$u(c_1(\rho)) = \begin{cases} u(c_1^*) & \rho \leq \hat{\rho} \\ E(u(c_2(\rho))) & \rho > \hat{\rho} \end{cases}$$

In this way, they will never obtain a utility larger than they would obtain at time 2. This rules out any generalized bank run equilibrium. As a result, the only possible equilibrium is $\rho = \lambda'$ if there is a real shock in the preferences and $\rho = \lambda$ in case of a mere shock in the expectations.

However, the feasibility of (19) is linked to its level. If this is very high, it may cause a loss of welfare for the whole economy, larger than the one caused by a bank run. For example, the central bank may have to print so much money that it creates hyperinflation. Given the devastating effect on the entire economy of such a policy, no government can credibly commit to impose (19). In this case, the deposit insurance cannot be implemented.

Similarly as before, let us define $\tau(\rho, p) \equiv \tau^A(\rho)$ as the taxation schedule in an autarchy. In order to determine the feasibility of (19) in the two different regimes we can prove the following:

Proposition 4 *In order to implement a mechanism of deposit insurance the government has to be able to impose a higher level of* τ *in an open economy than in an autarchy:* ${}^A\tau(\rho) < \tau(\rho, p_{H,F})$ *for all* $\rho \in (\lambda, 1)$. *Moreover* $\frac{\partial \tau(\rho, p_{H,F})}{\partial p_{H,F}} < 0$ *for all* $p_{H,F} \in (0, p)$

Proof. this proof is in appendix. ■

This result is directly linked to the preceding proposition. In order to increase the capacity of the bank of fulfilling the time 2 withdrawals, a higher tax is necessary when c_1 is higher.

5 An expected shock

In the past sections we analyze the model with the simplifying assumption that the shocks in the preferences before time 2 is entirely unexpected. In this section we will remove such assumption and analyze the same model when individuals expect the shock with a given probability distribution. Individuals are now aware of the risk of bank run. Therefore, besides the illiquidity and the asset risk, they will also consider a third source of uncertainty, namely the bank run risk.

Let us suppose that individuals at $t = 0$ know that there is a shock such that $Proba[\lambda > \bar{\lambda}] = q$. Moreover, when the economy is hit by this shock, the measure of impatient individuals is stochastic and uniformly distributed over the interval

$(\bar{\lambda}, 1)$. The distribution function of the impatient individuals is $\frac{q}{1-\bar{\lambda}}$, so that $\int_{\bar{\lambda}}^{1} \frac{q}{1-\bar{\lambda}} d\rho = q$.

Banks are in competition and they face the same budget constraints as before. Accordingly, the consumptions at time 2 across states of world are always determined by expressions (5)-(8).

They choose c_1^* and b^* at $t = 0$ and sign the contract with the individuals, who, at $t = 1$ after observing λ, decide to run to the bank if:

$$E(u(c_2) \mid b^*, p_{H,F}, p, \lambda) < u(c_1^*), \tag{20}$$

when this last inequality is true, $\rho = 1$ and $c_2 = 1$.

Given expressions (5)-(8), it is possible to show that the LHS of (20) is monotonically decreasing in λ; let us define the critical level or ρ, $\hat{\rho}$ in an implicit way as: $E(u(c_2) \mid b, p_{H,F}, p, \hat{\rho}(b, c_1)) = u(c_1)$. In that way, for any b, c_1, when $\lambda > \hat{\rho}(b, c_1)$ there is a bank run.

Accordingly, at $t = 0$ banks must solve the following problem:

$$\max_{c_1, b} \; (1-q)(\bar{\lambda} u(c_1) + (1-\bar{\lambda}) E(u(c_2) \mid b, p_{H,F}, p, \bar{\lambda})) + \int_{\bar{\lambda}}^{\hat{\rho}(b,c_1)} \rho u(c_1) + (1-\rho) E(u(c_2) \mid b, p, p_{H,F}, \rho) \frac{q}{1-\bar{\lambda}} d\rho + \int_{\hat{\rho}(b,c_1)}^{1} u(1) \frac{q}{1-\bar{\lambda}} d\rho \tag{21}$$

for $q = 0$, i.e. when the probability of a shock is 0 or the shock is completely unexpected, problem (21) corresponds to the problem in the preceding sections. While, when individuals expect a shock with a positive probability $q > 0$, two possibilities have to be taken into account:

- $\hat{\rho}(b, c_1) \geq \rho > \bar{\lambda}$: there is a higher number of individuals withdrawing and $c_2(\rho) < c_2(\bar{\lambda})$, but there is not a generalized bank run. The expected value of the utility in this case is represented by the first integral.

- $\rho > \hat{\rho}(b, c_1)$: There is a bank run, and the utility for every individual is: $u(1)$ (last term of (21)).

Problem (21) can be solved numerically for c_1, after substituting the optimal value of b, which in appendix we show to be the same as before for a given c_1^*, i.e. $b^* = \frac{1-c_1^*}{2}$.

5.1 Equilibrium

Figure 2 represents the level of c_1^*, resulting from the numerical optimization of (21), respect to different level of shock expectation q and the level of individual's risk aversion. In order to emphasize the role of the bank run risk, we plotted this figure for $p = 2p - 1$, i.e. when there is perfect insurance against the asset risk. Accordingly in this case, individuals face the risk of illiquidity and the risk of bank run.

When $q = 0$ there is not a bank run risk and this case is the same as the one we considered in figure 1, accordingly since $p_{H,F} = 2p - 1$ the dashed line in figure 2 corresponds to the dashed line in fugure1. In this case there is only the illiquidity shock to be insured. As a result, the more they are risk averse the more they choose a high level of c_1.

When $q > 0$, individuals know that when they choose a higher level of c_1 they increase the fragility of the bank to a shock, therefore the higher the probability of this shock q, the lower their insurance against the illiquidity shock c_1. This emphasizes the trade-off between illiquidity insurance and bank-run risk. Moreover it is important to notice that, like in figure 1, the level of insurance against the illiquidity risk tend to disappear for high level of σ. Highly risk averse individual are principally afraid of financial crises and prefer to avoid this to an higher c_1

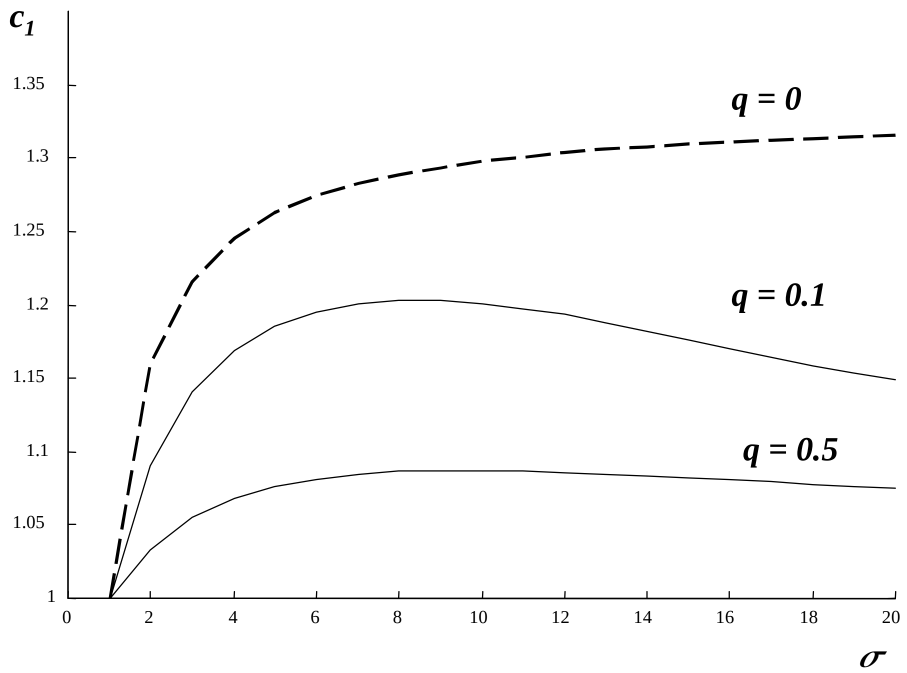

Figure 2: Illiquidity insurance (c_1) in relation to risk aversion for different probabilities of shock (q), when there is perfect insurance aganst the asset risk ($p = 0.5$, $R = 2$, $\bar{\lambda} = 0.25$, $p_{H,F} = 2p - 1 = 0$)

In figure 3 we can observe that the behavior of c_1^* for different values of the intercountry correlation $p_{H,F}$, is qualitatively the same as in the case with unexpected shock: the more the portfolio can be diversified ($p_{H,F}$ low) the more the bank insures individuals against illiquidity risk as well (in the appendix we report a table with a more complete grid of values).

Moreover, the ex-ante probability of shock q generates a downward shift of the curve. Like we observed in figure 2, the higher the shock expectation the lower the level of illiquidity insurance.

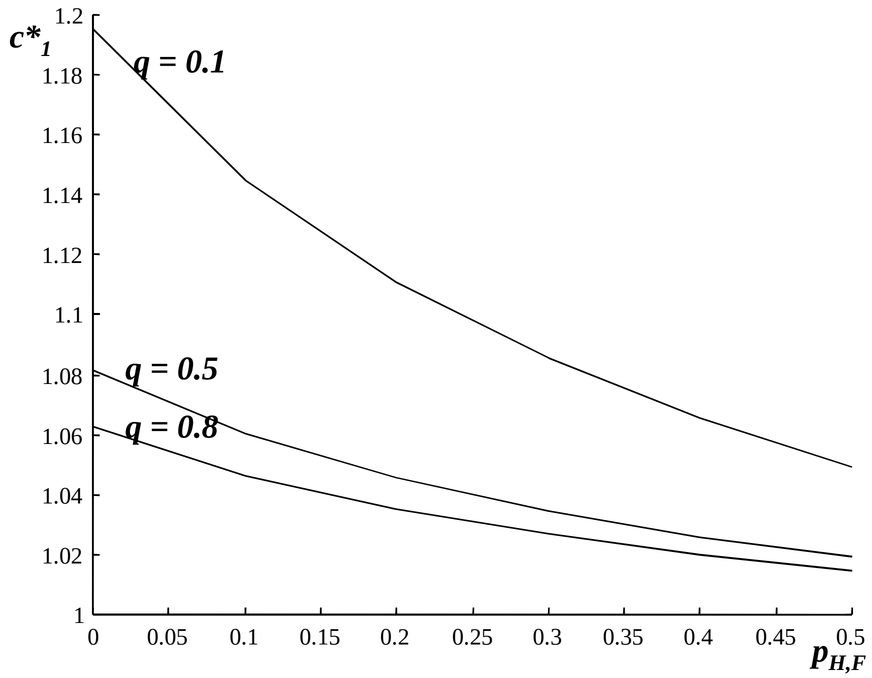

Figure 3: Optimal levels of c_1 for different level of ex-ante shock expectation q, with respect to $p_{H,F}$ (for values: $p = 0.5$, $R = 2$, $\sigma = 6$, $\lambda = 0.25$).

5.2 Fragility

The level of c_1^* and b^* determines the probability of a bank run that, given the probability distribution $\frac{q}{1-\bar{\lambda}}$, is:

$$\Pr[\lambda > \hat{\rho}] = q \left(\frac{1 - \hat{\rho}}{1 - \bar{\lambda}} \right),$$

again, the higher $\hat{\rho}$, the lower the fragility of the system.

The fragility with respect to $p_{H,F}$ is due to the fact that when there is the possibility of a higher level of international risk sharing the expected utility at time 2 is high. Therefore individuals are willing to bear more risk of bank run given the higher expected utility when there is not bank run.

In figure 4 we can observe this index for the different values of $p_{H,F}$ and q.

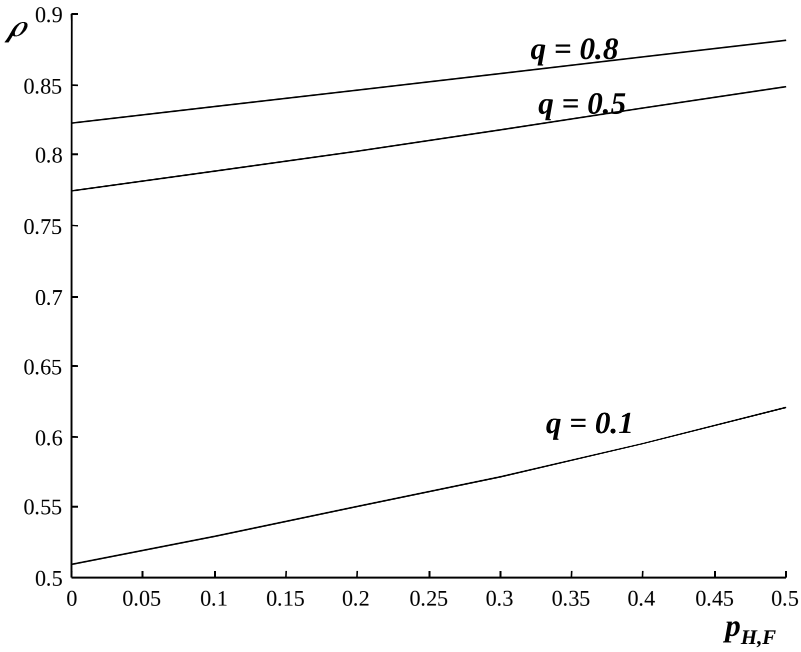

Figure 4: Index $\hat{\rho}$ in relation to the level of crisis expectation q and risk sharing (for values: $p = 0.5$, $R = 2$, $\sigma = 6$, $\lambda = 0.24$).

Like in the cases with a fully unexpected crisis, the fragility of the banking system increases with the level of risk sharing (it is higher the lower $p_{H,F}$ is). Moreover, we can notice that, as it is intuitive, the fragility of the system increases when the ex-ante expectation q of a shock increases.

It is interesting to notice that ρ is steeper when $q = 0.1$ than when $q = 0.5$ or $q = 0.8$. This means that opening the financial market increases the fragility of the system when the shock expectations are low. This seems to be consistent with the fact that past financial crises were rather unexpected (Stiglitz 1998, Sachs Tornell and Velasco (1996)).

5.3 Risk aversion and Propensity to Panic

Finally in figure 5 we can observe the relation between risk aversion and fragility. In our setting higher risk aversion is associated with higher fragility (lower $\hat{\rho}$). This relation emphasizes a rather counterintuitive effect; indeed a risk averse individual tries to minimize the probability of a bank run, and clearly this lowers the fragility of the system.

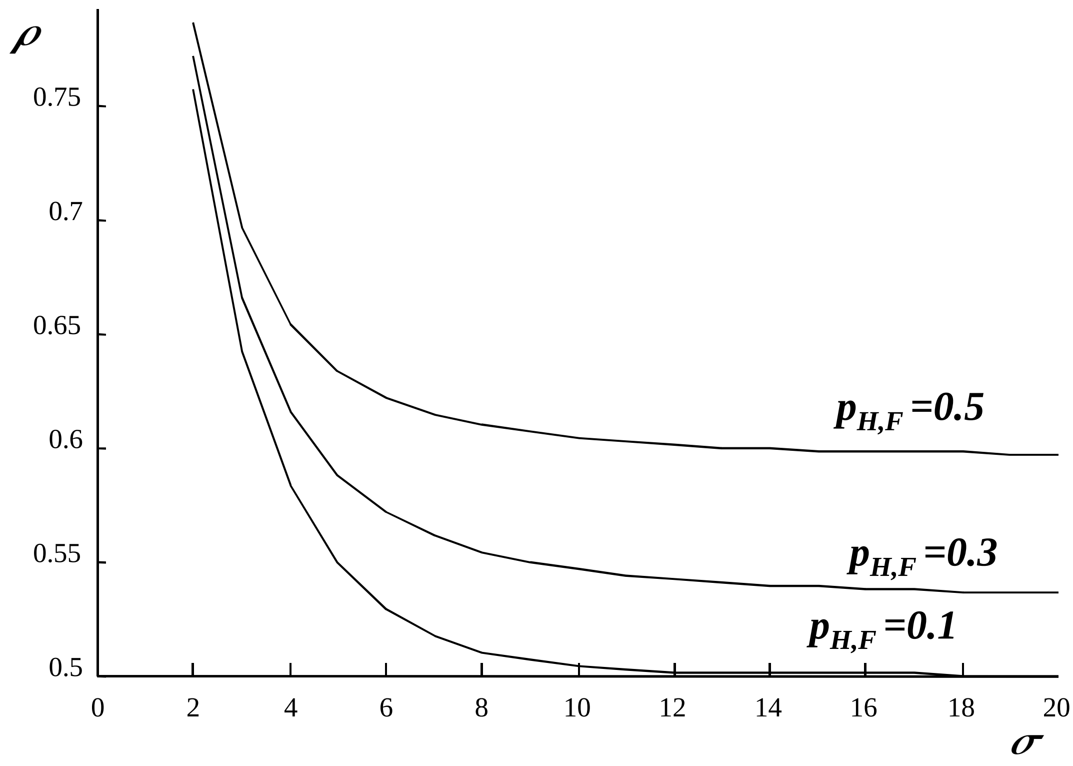

Figure 1: Figure 5: Relation between risk aversion and fragility ($p = 0.5$, $R = 2$, $\bar{\lambda} = 0.25$)

However, high risk aversion has another effect on individuals' behavior: after the shock, highly risk averse individuals are more sensitive to the risk of a low level of c_2 in the second period. As a result, once if there is a shock, they will prefer not to bear this risk and to run to the counter. In other words, high risk aversion becomes high *propensity to panic*.

In this setting, and for the range of parameters in figure 5 the high *propensity to panic* is stronger than the first effect.

6 Final remarks

We showed how in an open economy the banking system can be more fragile than in an autarchic one. It has been argued that this happens because an open economy is more exposed to the contagion from other countries' shocks, through the banks or the firms' balance sheets which often end up having excessively high levels of short-run foreign liabilities (Aghion, Bachetta and Banerjee 1999, Chang and Velasco 1998, Krugman 1999).

In this model we saw that the higher level of fragility can be seen as a collateral effect of the international portfolio diversification. Therefore it is linked to one of the main advantages of globalization.

An important implication is that the mechanisms of the regulation of a financial system has to take into account this increased fragility intrinsic in the financial market internationalization. The deposit insurance that governments or central banks can provide may not be sufficient anymore to avoid financial crises, this implies the necessity of some supranational agency that can supply a supplementary form of insurance.

A Appendix

A.1 Truth telling constraint

We show that for all p and $p_{H,F}$:

$$E(u(c_2)) \geq u(c_1). \tag{22}$$

With:

$$\begin{aligned} E(u(c_2)) &= p_{H,F}u(c_2^{R,R}(\rho)) + (p - p_{H,F})u(c_2^{R,1}(\rho)) + (p - p_{H,F})u(c_2^{1,R}(\rho)) + \\ &\quad (1 - 2p - p_{H,F})c_2^{1,1}(\rho) \end{aligned} \tag{23}$$

Using the functional form given by (14) and plugging expressions of c_2 derived from (5), (6), (7) and (8) in (9), we can observe that (22) is verified when:

$$c_1 \leq \frac{\psi(p_{H,F}, R, \sigma)^{\frac{1}{1-\sigma}}}{1 - \lambda(1 - \psi(p_{H,F}, R, \sigma)^{\frac{1}{1-\sigma}})} \tag{24}$$

where

$$\psi(p_{H,F},R,\sigma) \equiv 1+p_{H,F}* \left((1+R^{1-\sigma})-2\left(\frac{1+R}{2}\right)^{1-\sigma}\right)-2\left(1-\left(\frac{1+R}{2}\right)^{1-\sigma}\right)p. \tag{25}$$

From this last expression, we can observe that if $\sigma>1$ then $\frac{\partial\psi(p_{H,F},R,\sigma)}{\partial p_{H,F}}>0$ and $\frac{\partial\psi(p_{H,F},R,\sigma)}{\partial p}<0$. Moreover since $Max_{p_{h,f}}\psi(p_{H,F},R,\sigma)=\psi(p,p)=1+p((1+R^{1-\sigma})-2)$ and noticing that $\psi(p,p)<1$, we can argue that

$$\psi(p_{H,F},R,\sigma)<1 \tag{26}$$

Finally, using (26) we can see that (24) is always satisfied given that $\sigma>1$ ■

A.2 Proof of proposition 1

Part i)

Given the expression (16), part i) follows directly from the observation that:

$$\lim_{\sigma\to\infty}\psi(p_{H,F},R,\sigma)=1+p_{H,F}-2p.$$

Part ii)

If $1+p_{H,F}-2p=0$, we can rewrite $\psi=pR^{1-\sigma}+2\left(\frac{1+R}{2}\right)^{1-\sigma}(p-p_{H,F})$. Given that: For σ sufficiently big $pR^{1-\sigma}+2\left(\frac{1+R}{2}\right)^{1-\sigma}(p-p_{H,F})\backsim 2\left(\frac{1+R}{2}\right)^{1-\sigma}(p-p_{H,F})$ we have that $\lim_{\sigma\to\infty}\psi^{\frac{1}{\sigma}}=\lim_{\sigma\to\infty}\left(2\left(\frac{1+R}{2}\right)^{1-\sigma}(p-p_{H,F})\right)^{\frac{1}{\sigma}}=\lim_{\sigma}\left(\frac{1+R}{2}\right)^{\frac{1-\sigma}{\sigma}}=\frac{2}{1+R}<1$. Given the expression (16) $c_1>1$.

A.3 Function $\hat{\rho}(p_{H,F})$

The Derivative:

$$\frac{\partial\hat{\rho}(p_{H,F})}{\partial p_{H,F}}=\frac{(1-\lambda)\left(-\sigma\psi^{\frac{1}{1-\sigma}}+\psi^{\frac{1}{\sigma-\sigma^2}}+(\sigma-1)\,\psi^{\frac{1+\sigma}{\sigma-\sigma^2}}\right)\psi'_{P_{H,F}}}{(\sigma-1)\,\sigma\,\psi\left(-1+\psi^{\frac{1}{1-g}}\right)^2}$$

is always positive, given that:

$$\psi^{\frac{1}{\sigma}} + (\sigma - 1)\psi^{\frac{1}{\sigma-\sigma^2}} - \sigma > 0$$

for all $\sigma > 1$.

A.4 Function $\tau(p_{H,F})$

The derivative:

$$\tau'(p_{H,F}) = \left((1-\rho)\,\sigma\, p_{H,F}\,\psi^{\frac{1}{1-\sigma}} + (1-\rho)\,(1-p_{H,F})\,\psi^{\frac{1}{\sigma-\sigma^2}} - \rho\,(\sigma-1)\,(1-p_{H,F})\,\psi^{\frac{1+\sigma}{\sigma-\sigma^2}}\right) \frac{\psi'}{(\sigma-1)\,\sigma\,\psi\left(1-\rho+\rho\,\psi^{\frac{1}{1-\sigma}}\right)^2}$$

it is negative whenever:

$$\rho > \frac{\psi^{\frac{1}{\sigma}}\,(1-p_{H,F}) + \sigma\, p_{H,F}}{\psi^{\frac{1}{\sigma}}\,(1-p_{H,F}) + (\sigma-1)\,\psi^{\frac{1}{\sigma-\sigma^2}}\,(1-p_{H,F}) + \sigma\, p_{H,F}}$$

but this last condition is always true given that:

$$\frac{\psi^{\frac{1}{\sigma}}\,(1-p_{H,F}) + \sigma\, p_{H,F}}{\psi^{\frac{1}{\sigma}}\,(1-p_{H,F}) + (\sigma-1)\,\psi^{\frac{1}{\sigma-\sigma^2}}\,(1-p_{H,F}) + \sigma\, p_{H,F}} > \hat{\rho}(p_{H,F})$$

A.5 Maximization with shock expectations

Banks solve the following problem:

$$\max_{c_1,b}\ (1-q)(\bar{\lambda}u(c_1) + (1-\bar{\lambda})E(u(c_2)\ \mid\ b, p_{H,F}, p, \bar{\lambda})) + \int_{\bar{\lambda}}^{\hat{\rho}(b,c_1)} \rho u(c_1) + (1-\rho)E(u(c_2)\ \mid\ b, p, p_{H,F}, \rho)\frac{q}{1-\bar{\lambda}}d\rho + \int_{\hat{\rho}(b,c_1)}^{1} u(1)\frac{q}{1-\bar{\lambda}}d\rho \quad (27)$$

with

$$\hat{\rho}(c_1, b) : E(u(c_2) \mid b, p_{H,F}, p, \hat{\rho}(c_1, b)) = u(c_1) \tag{28}$$

.

In order to simplify, we can notice from (23) that $E(u(c_2) \mid b, p_{H,F}, p, \rho)$ is monotonically decreasing in ρ for any b and c_1. Therefore for any c_1, when $b = \arg\max E(u(c_2) \mid b, p_{H,F}, p, \rho)$, $\hat{\rho}$ must be maximal to fulfill (28). Finally we can notice that (23) is maximized when $b = \frac{1-c_1}{2}$.

Moreover, we can observe that (27) is increasing in $\hat{\rho}(c_1, b)$ and in $E(u(c_2) \mid b, p_{H,F}, p, \rho)$, and we saw that $b(c_1) = \frac{1-c_1}{2} = \arg\max \hat{\rho}(c_1, b) = \arg\max E(u(c_2) \mid b, p_{H,F}, p, \rho)$. Therefore, $b(c_1)$ maximizes (27) for any c_1. As a result, we can directly substitute $b(c_1)$ in (27) and maximize the resulting function only with respect to c_1^*:

$$\max_{c_1} \frac{(1-q)}{1-\sigma} \left(\lambda\, c_1^{1-\sigma} + \psi\, (1-\lambda) \left(\frac{1-\lambda\, c_1}{1-\lambda} \right)^{1-\sigma} \right) + \frac{q}{(1-\lambda)\,(1-\sigma)}$$
$$\left(\frac{\psi^{\frac{1}{1-\sigma}}\,(-1+c_1)}{\left(-1+\psi^{\frac{1}{1-\sigma}}\right) c_1} + \left(-\lambda^2 + \frac{\left(\lambda^{\frac{1}{1-\sigma}} - c_1\right)^2}{\left(-1+\psi^{\frac{1}{1-\sigma}}\right)^2 c_1^2} \right) \frac{c_1^{1-\sigma}}{2} + \psi \int_\lambda^{\frac{\psi^{\frac{1}{1-\sigma}} - c_1}{\left(-1+\psi^{\frac{1}{1-\sigma}}\right) c_1}} (1-\rho) \left(\frac{1-\rho\, c_1}{1-\rho} \right)^{1-\sigma} d\rho \right)$$

Table2: Values of $\hat{\rho}$

$q, p_{H,F}$	0	0.1	0.2	0.3	0.4	0.5
0.1	0.50	0.53	0.55	0.57	0.6	0.62
0.3	0.69	0.71	0.73	0.75	0.77	0.79
0.5	0.77	0.79	0.80	0.81	0.83	0.84
0.8	0.82	0.83	0.84	0.86	0.87	0.88
1	0.84	0.85	0.86	0.87	0.88	0.89

$R = 2$, $\sigma = 6$, $p = 0.5$, $\lambda = 0.25$

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