Downstream Investment in Oligopoly

Stefan Buehler

University of Zurich and University of St. Gallen

Armin Schmutzler

University of Zurich and CEPR*

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Abstract: We examine cost-reducing investment in vertically-related oligopolies, where firms may be vertically integrated or separated. Analyzing a standard linear Cournot model, we show that: (i) Integrated firms invest more than separated competitors. (ii) Vertical integration increases own investment and decreases competitor investment. (iii) Firms may integrate strategically so as to preempt investments by competitors. Adopting a reduced-form approach, we identify demand/mark-up complementarities in the product market as the driving force for these results. We show that our results generalize naturally beyond the Cournot example, and we discuss policy implications.

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*e-mail: `sbuehler@soi.unizh.ch`, `arminsch@soi.unizh.ch`.

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1 Introduction

In many industries, vertically integrated firms compete with separated firms. Well-documented examples include the oil industry (Bindemann 1999), the beer industry in the UK (Slade 1998a), and the gasoline retail market in Vancouver (1998b). These examples suggest that the coexistence of vertical integration and separation is an important aspect of product market competition in vertically related industries. Moreover, it is sometimes claimed that vertical integration and market share are closely related. For instance, the European Commission (1999) argued in the highly controversial *Airtours/First Choice* case that vertical integration was a prerequisite for significant increases of market shares and, conversely, that only firms of sufficient size were able to integrate vertically. While the theoretical basis of the argument was not entirely clear, the UK market for foreign package holidays under consideration appeared to match the Commission's description.[1]

Our main concern in this paper is to provide a theoretical explanation of such patterns. We start with a linear Cournot model adapted from Salinger (1988) to motivate our more general reduced-form analysis.[2] In this Cournot model, two upstream suppliers face two downstream firms. Downstream firms require an intermediate good produced by upstream firms. Downstream marginal costs consist of the costs of obtaining the intermediate good plus the costs of transforming the intermediate good into the final product. If a downstream firm integrates with one of the suppliers, its downstream marginal costs fall; this is the familiar *efficiency effect* of integration. However, integration also influences the marginal costs of the downstream competitor: As the integrated firm is no longer active in the wholesale market, both supply and demand on this market are affected, resulting in an ambiguous *wholesale price* effect of integration. In our example, the effect of decreasing demand dominates, and the wholesale price falls. Thus, from the integrating firm's point of view, vertical integration has an undesired side effect: the competi-

[1] We shall discuss this market in more detail in section 2.

[2] Our reduced-form approach is also consistent with other models of vertically-related industries that consider product market decisions explicitly. See, e.g., Ordover et al. (1990), Riordan (1998), and Linnemer (2003).

tor's marginal costs fall as well. Even so, the efficiency effect dominates in the sense that integration increases both the output and the mark-up of the integrating firm, and reduces the corresponding quantities for the competitor.[3] We further suppose that downstream competitors can invest into reducing their costs of transforming the intermediate good into the final product. Like vertical integration, higher downstream efficiency increases own equilibrium demand and mark-up and decreases the competitor's demand and mark-up.

For this setting, we provide an explanation of the coincidence of vertical integration and high market share: We show that, if competitors in the same market decide simultaneously about cost-reducing investments, integrated firms will invest more than separated firms. Thus efficiency and integration tend to go hand in hand, explaining why efficient firms have high market shares.

We then turn to a closely related set of issues. To analyze the impact of vertical integration on investment decisions, we compare the firms' cost-reducing investments in different vertical structures. First, we show that firm 1's vertical integration increases its own investment and decreases the investment of firm 2. Second, we consider the case where firms decide about vertical integration before carrying out cost-reducing investments. As an immediate consequence of the last result, firms may integrate strategically so as to avoid cost-reducing investments by competitors, i.e. vertical integration may serve as an instrument of preemption.[4]

To explore to what extent our results generalize beyond the linear Cournot model, we adopt a reduced-form analysis. This approach also allows us to understand the common intuition behind our results. We find that, for the above results to hold, it is crucial that the profit increase associated with cost-reducing investment is higher for integrated firms than for separated firms. Such a relation is a plausible consequence of *demand/mark-up complementarities*: Intuitively, any activity that increases equilibrium demand is

[3] A fortiori, integration must have a positive effect on equilibrium outputs and mark-ups in examples where integration increases the wholesale price and thus competitor marginal costs, i.e., if a *foreclosure effect* is present. See Rey and Tirole (forthcoming) for a survey of the foreclosure literature.

[4] This result is related to Colangelo (1995), who finds that vertical integration may pre-empt horizontal mergers.

more valuable the higher the equilibrium mark-up (price minus unit-costs), and any activity that increases the equilibrium mark-up is more valuable the higher equilibrium demand. As both integration and cost-reducing investment increase mark-up and demand, they are mutually complementary. Similar arguments show that cost-reducing investment is less valuable when the competitor is not integrated. Based on this notion of complementarity, we show that our above results generalize quite naturally beyond the linear Cournot model.

Our analysis suggests that evaluating vertical mergers may be even more subtle than was previously thought: Prohibiting vertical integration not only affects markets directly via the familiar efficiency and foreclosure effects. In addition, it has potential effects on investments. In the simple Cournot example, these effects of prohibiting investment are unambiguously negative, but this result is not necessarily very general.

This paper adds to the literature on the relation between vertical integration and investment incentives. A key results of this literature, which has typically adopted the incomplete contracts approach, is that underinvestment is likely to occur in bilateral monopoly.[5] That is, in the absence of suitable contractual arrangements, both upstream and downstream investments are usually too low for separated firms relative to the vertical integration case. The driving force of this finding is the hold-up problem: When contracts are incomplete, both firms face the risk of expropriation from the returns of their relation-specific investments. Anticipating the hold-up problem, both firms underinvest. Unlike this literature, we compare the investment incentives of integrated and separated *competitors* in any given industry. Accounting for the strategic interactions between them, we find that it is still true that integrated firms invest more than separated ones.[6] The driving force of our result is a double mark-up rather than a hold-up problem. In both cases, however, the presence of a vertical externality is crucial for the result.[7]

[5] The standard reference is Hart and Moore (1990). See Hart (1995) and Holmström and Roberts (1998) for surveys and further references.

[6] In a related paper (Buehler and Schmutzler 2003), we examine the endogenous vertical integration decisions of potentially asymmetric firms in more detail.

[7] Our analysis also differs from Banerjee and Lin (2003), who consider the investment incentives of downstream competitors facing a monopolistic upstream supplier. As all

The remainder of the paper is organized as follows. In section 2, we consider the UK tour operating business as a case study. Section 3 introduces our Cournot example. Section 4 presents our more general reduced form duopoly model. Section 5 discusses policy implications, and section 6 concludes.

2 A Case Study: The Market for Foreign Package Holidays in the UK

We consider the market for foreign package holidays in the UK to illustrate how asymmetric vertical market structures affect product market competition in vertically-related industries. The recent developments in this market are well documented, since it has twice been subject to antitrust proceedings during the 1990s.[8] The supply side of this industry consists of three vertical layers (Monopolies and Mergers Commission 1997):

(i) *Tour Operation*: Foreign holidays packages are assembled by tour operators contracting with the suppliers of transport services and accommodation. Tour operators sell their packages directly to the public or—more commonly—through travel agencies.

(ii) *Travel Agencies*: Travel agents act on behalf of tour operators to sell holidays packages to consumers; they are typically rewarded by a commission on the package price.

(iii) *Airlines*: Many foreign holiday packages require air transportation services. Airlines often provide seat capacities to more than one tour operator.[9]

downstream firms are assumed to be separated, these authors cannot compare the behavior of separated and integrated firms in the same industry. Instead, they are concerned with the effect of the number of downstream firms on investment.

[8] The first analysis was undertaken by the Monopolies and Mergers Commission (1997) in the light of increasing concentration and widespread vertical integration. The second investigation was carried out by the European Commission (1999) when *Airtours*, one of the most important players in the industry, was attempting to take over *First Choice*, another major player.

[9] We shall ignore a fourth layer, accomodation, because vertical integration of tour operators into accomodation plays a limited role.

Figure 1 reproduces the market shares of the largest tour operators as of 1998. Importantly, all of the four largest firms with market shares of 15% or higher are fully integrated, i.e. the dominant firms are both integrated backwards into the provision of air transportation and forward into the travel agency business. Most of the remaining firms are separated, the largest with a market share below 3%. The aggregate market share of all other firms with individual market shares below 1% is around 8%. That is, the industry consists of a small group of vertically integrated tour operators with high market shares, and a fringe, containing numerous firms with small market shares, most of which are vertically separated. The European Commission (1999, no. 73) was clearly aware of this asymmetric market structure when it pointed out that the

> "polarization of the market into large integrated companies and smaller non-integrated companies is a widely recognized trend in the industry."

Similarly, Damien Neven noted that there are essentially two ways of doing business in this industry: Either

> "stay small and buy inputs or produce large volumes and integrate vertically."[10]

<Figure 1 around here>

The market for foreign package holidays in the UK suggests that there are important differences in the conduct of vertically integrated and separated firms in a given industry. In particular, vertically integrated firms appear to be larger than separated firms. In the following, we shall investigate to what extent these differences can be attributed to general forces associated with product market competition. In doing so, it will be crucial to analyze the incentives of vertically integrated and separated firms to invest into cost reduction at the downstream level.

[10] Cited after the European Commission (1999, no. 73)

3 A Linear Cournot Example

To motivate our reduced-form analysis below, we consider a linear Cournot model adapted from Salinger (1988), where downstream firms can invest into reducing processing costs.

3.1 Assumptions

There are two upstream and two downstream firms. Each downstream firm $i = 1, 2$ may be integrated backwards with one of the upstream suppliers. We use V_i to denote firm i's state of vertical integration, i.e.

$$V_i = \begin{cases} 0, & \text{if } i \text{ is vertically separated,} \\ 1, & \text{if } i \text{ is vertically integrated,} \end{cases} \qquad i = 1, 2.$$

Further, let $\mathbf{V} = (V_1, V_2)$ denote the vertical structure of the industry. The inverse demand function for the final product is given by $P(Q) = a - Q$, with q_i denoting the quantity of firm i, $Q = q_1 + q_2$ and $a > 0$.

To produce one unit of the final product (the downstream good), firms require one unit of an intermediate product (the upstream good). Suppose that, for simplicity, the marginal cost of producing the intermediate product is constant and normalized to zero. The marginal cost of obtaining the intermediate product is given by w_i, which is the wholesale price faced by a separated firm i and the marginal cost of producing the intermediate product for an integrated firm i. Thus, we have $w_i = 0$ for an integrated firm. Further, we assume that transforming the intermediate product adds t_i to marginal costs, so that the marginal costs of downstream firm i are given by

$$c_i = w_i + t_i.$$

Firms may differ with respect to their initial transformation costs t_i^0. Define $\bar{t} \equiv \max(t_1^0, t_2^0)$ and let $Y_i^0 = \bar{t} - t_i^0$ denote the initial efficiency of firm i.

The sequence of events is as follows. In the first stage of the game, firms can invest into cost reduction. To reduce transformation costs by y_i, they have to invest $K(y_i) = ky_i^2$. After investment, the efficiency of firm i is

$Y_i = Y_i^0 + y_i$ and marginal costs are given by

$$c_i = w_i + \bar{t} - Y_i^0 - y_i. \qquad (1)$$

In the second stage of the game, upstream firms determine the wholesale price. In the third stage, Cournot competition takes place downstream, with cost structures determined in period 1 and 2 according to (1). Obviously, stage 2 is irrelevant if both firms are vertically integrated: If $\mathbf{V} = (1, 1)$, the costs of obtaining the input are given exogenously as $w_i = 0$ by assumption. However, if $\mathbf{V} = (1, 0)$ or $\mathbf{V} = (0, 1)$, the upstream firm sets the monopoly price for the separated downstream firm. If $\mathbf{V} = (0, 0)$, separated upstream firms compete à la Cournot.

3.2 Subgame Perfect Equilibrium

In the subgame perfect equilibrium, the investment levels are given as $y_i(\mathbf{V})$, and the resulting efficiency levels of downstream firms as $Y_i(\mathbf{V})$. Input costs are $w_i(\mathbf{V}, \mathbf{Y})$. Furthermore, we use the notation $Q_i(\mathbf{V}, \mathbf{Y})$ to denote downstream outputs for arbitrary integration vectors $\mathbf{V}$ and efficiency levels $\mathbf{Y}$, assuming that the input price is $w_i(\mathbf{V}, \mathbf{Y})$. Similarly, we write the equilibrium mark-ups and profits of downstream firms as $M_i(\mathbf{V}, \mathbf{Y})$ and $\Pi_i(\mathbf{V}, \mathbf{Y})$, respectively.

Table 1 summarizes the results for the Cournot example, assuming that firms are equally efficient initially ($Y_1^0 = Y_2^0 = 0$), which immediately implies $Y_i = y_i, i = 1, 2$. It describes the SPE for the three market configurations $\mathbf{V} = (0, 0)$, $\mathbf{V} = (1, 0)$ and $\mathbf{V} = (1, 1)$ and the associated reference configurations where firms are not allowed to invest (i.e. $\mathbf{y} = (y_1, y_2) = (0, 0)$ by assumption). Equilibrium wholesale prices, quantities, mark-ups and profits for each configuration are given as functions of the efficiency levels $\mathbf{y}$, and the efficiency levels $\mathbf{y}$ as functions of k. Throughout, $\alpha \equiv a - \bar{t}$ is a measure of market size. Table 2 summarizes the results for the special case $k = 1$.

<Tables 1 and 2 around here>

We now highlight the observations that are particularly important for our more general analysis below.

3.3 Comparing Investments of Competitors

First, we compare the investments of integrated and separated firms in the same market. Thus, we consider the asymmetric market structure $\mathbf{V} = (1, 0)$. Figure 2a) depicts the optimal investment levels y_1 and y_2 as functions of the cost parameter k, fixing $\alpha = 1$. Figure 2b) shows the resulting market shares s_1 and $s_2 = 1 - s_1$, respectively. Clearly, the integrated firm 1 invests more and has a higher market share than the separated firm 2.

<Figure 2 around here>

To put the result into perspective, consider output decisions when firms are unable to invest into cost reduction (or equivalently, $k \to \infty$). Figure 2b) indicates that even when firms cannot invest into cost reduction, the market share of the integrated firm is higher than that of the separated firm, i.e. $s_1(y_1 = 0, y_2 = 0) > 0.5$. This reflects the simple fact that the integrated firm has lower marginal costs than the separated firm.

However, this is not the end of the story: If firms can invest into cost reduction, the gap between the two firms widens, since the integrated firm invests more than the separated firm (see Figure 2b)). However, it is not quite as obvious *why* the integrated firm invests more than the separated firm. In section 4, we will show that the intuition for this result becomes clearer in a more general model. We summarize our results for $\mathbf{V} = (1, 0)$ as follows:

Observation 1 *In the linear Cournot example, the integrated firm has higher market share, output and mark-up than the separated firm, even if* $y_1 = y_2 = 0$*. If investment levels are endogenous, the integrated firm invests more and the differences in outputs and mark-ups increase.*

3.4 Comparing Investments for Different Vertical Structures

Observation 1 compared the investment behavior of integrated and separated firms in a given vertical structure. This comparison was natural to understand the relation between integration and market share. For policy discussions, it is also important to understand how changes in vertical structure affect the investment behavior of firms in the market. For instance, if, in a setting with two initially separated firms, a vertical merger is prohibited, does the prohibition affect the investment decisions of the firms, and ultimately social welfare? Table 2 indicates that starting from $\mathbf{V} = (0, 0)$, firm 1's vertical integration (i) increases own investment, (ii) decreases competitor investment, and (iii) increases welfare W.[11] Starting from $\mathbf{V}' = (1, 0)$, firm 2's integration has similar effects on investments and welfare. We summarize these results as follows:

Observation 2 *In the linear Cournot example, integration of one firm increases this firm's investments and decreases the competitor's investments. In addition, it increases welfare.*

In section 4.2, we argue that the effects of integration on investment are likely to be very general, whereas the impact on welfare will be ambiguous.

3.5 Strategic Integration Incentives

Next, we show that there are strategic incentives to integrate. To this end, reconsider Table 2. For $k = 1$, this table lists the equilibrium profits, net of investment costs, $\Pi_i^*(\mathbf{V}) = \Pi_i(\mathbf{V}, \mathbf{y}(\mathbf{V}))$. It is straightforward to calculate firm i's integration incentive $\Delta_i^*(V_j)$, which is defined as the profit differential resulting from integration if the competitor's integration status is V_j. The table shows that

$$\begin{aligned} \Delta_1^*(0) &\equiv \Pi_1^*(1,0) - \Pi_1^*(0,0) \approx 0.233\alpha^2; \\ \Delta_1^*(1) &\equiv \Pi_1^*(1,1) - \Pi_1^*(0,1) \approx 0.089\alpha^2. \end{aligned}$$

We now compare the profit differentials $\Delta_i^*(V_j)$ with the corresponding expressions when firms are not allowed to invest. Thus, define

$$\Delta_1^{NI}(V_2) \equiv \Pi_1(1, V_2; \mathbf{0}) - \Pi_1(0, V_2; \mathbf{0}),$$

[11] More specifically, $y_1(1,0) = 0.363\alpha > y_1(0,0) = 0.108\alpha$, $y_2(1,0) = 0.040\alpha < y_2(0,0) = 0.108\alpha$, and $W(1,0) = 0.564\alpha^2 > W(0,0) = 0.318\alpha^2$.

and similarly for $\Delta_2^{NI}(V_1)$, where the superscript indicates "No Investment". Straightforward calculations show that

$$\Delta_i^{NI}(0) \approx 0.146\alpha^2 < \Delta_1^*(0)\,; \quad \Delta_i^{NI}(1) \approx 0.083\alpha^2 < \Delta_1^*(1)\,.$$

Thus, compared with a game without investments, integration incentives are higher if integration is followed by investment. More in line with the strategic investment literature (e.g. Fudenberg and Tirole 1984), consider the game with investments, but suppose investments are fixed at pre-integration levels $\mathbf{y}(0, V_2)$. Define

$$\widetilde{\Delta}_1(V_2, \mathbf{y}(0, V_2)) \equiv \Pi_1(1, V_2; \mathbf{y}(0, V_2)) - \Pi_1^*(0, V_2; \mathbf{y}(0, V_2))\,.$$

Thus, $\widetilde{\Delta}_1$ gives direct integration incentives, assuming that efficiency levels are fixed at pre-integration levels. We have

$$\widetilde{\Delta}_1(0, \mathbf{y}(0, V_2)) \approx 0.179\alpha^2 < \Delta_1^*(0)\,; \quad \widetilde{\Delta}_1(1, \mathbf{y}(0, V_2)) \approx 0.042\alpha^2 < \Delta_1^*(1)\,.$$

Referring back to Observation 2, the intuition is simple: The strategic integration incentive comes from the negative effect of integration on the competitor's investment decision, which is desirable for the integrating firm. Thus, we obtain the following conclusion:

Observation 3 *In the linear Cournot example, there is a strategic incentive to integrate vertically.*

We now turn to our reduced-form analysis.

4 A More General Model

We now show how Observations 1-3 generalize beyond the linear Cournot model. This will also clarify the intuition behind these observations.

4.1 Assumptions and Preliminary Results

As in the Cournot example, we suppose that there are two downstream firms that may or may not be integrated backwards. There are three conceivable vertical industry structures:

(i) *Full separation* ($\mathbf{V} = (0,0)$)*:* Two vertically separated downstream firms are supplied by upstream firms.

(ii) *Asymmetric integration* ($\mathbf{V} = (1,0)$ *or* $\mathbf{V} = (0,1)$)*:* One of the downstream firms is integrated, facing a separated competitor. This competitor is supplied by at least one upstream firm.

(iii) *Full integration* ($\mathbf{V} = (1,1)$): Two vertically integrated firms compete in the downstream market.

We continue to assume that downstream marginal costs c_i consist of the costs of obtaining the input w_i and the transformation costs t_i that depend on the firms' efficiency levels $Y_i = Y_i^0 + y_i$, such that $t_i = \bar{t} - Y_i$ and

$$c_i(\mathbf{V}, \mathbf{Y}) = w_i(\mathbf{V}, \mathbf{Y}) + \bar{t} - Y_i.$$

However, rather than explicitly calculating w_i, we shall make the following assumption on the firms' cost functions, which is consistent with the linear Cournot example in section 3 (see Table 1):

Assumption 1 *For* $i = 1, 2$ *and* $j \neq i$*,* ***downstream costs*** *satisfy the following conditions:*

(i) $w_i = 0$ *if* $V_i = 1$; $w_i > 0$ *for* $V_i = 0$;

(ii) $\partial c_i / \partial Y_i < 0$;

(iii) $\partial c_j / \partial Y_i > 0$, *or, if* $\partial c_j / \partial Y_i < 0$, *then* $|\partial c_i / \partial Y_i|$ *is sufficiently large relative to* $|\partial c_j / \partial Y_i|$.

Intuitively, (i) states that if a firm is integrated, it obtains the intermediate good at marginal production costs, which are zero by assumption. Otherwise, it must buy the intermediate good from the wholesale market at a strictly positive price. (ii) follows directly from (i) for integrated firms. For separated firms, (ii) requires that the direct cost reduction effect of higher efficiency is not outweighed by a possible increase in wholesale prices. (iii) states that, as a result of an increase in transformation efficiency, the wholesale price must either increase or, as in the Cournot example, decrease not too strongly such that the cost reduction for the competitor is smaller than for the investing firm.[12]

For given marginal costs $c_i, i = 1, 2$, we model downstream competition in reduced form as follows:

Assumption 2 *: For every cost vector $\mathbf{c} = (c_1, c_2)$, there exists a unique* ***product market equilibrium*** *resulting in outputs $q_i(\mathbf{c})$, prices $p_i(\mathbf{c})$, mark-ups $m_i(\mathbf{c}) = p_i(\mathbf{c}) - c_i$, and profits $\pi_i(\mathbf{c})$, respectively, such that*

$$\pi_i(\mathbf{c}) = q_i(\mathbf{c}) \cdot m_i(\mathbf{c}),$$

where (i) q_i and m_i are decreasing in c_i, and (ii) q_i and m_i are increasing in c_j, with $|\partial q_i / \partial c_i| > \partial q_i / \partial c_j$.

Assumptions 1 and 2 hold in standard oligopoly models, including the linear Cournot example discussed above.

As in section 3, we write profits, outputs and mark-ups directly as a function $\Pi_i(\mathbf{V}, \mathbf{Y})$ of the vertical structure and cost reducing investments, i.e.

$$\Pi_i(\mathbf{V}, \mathbf{Y}) = \pi_i(c_1(\mathbf{V}, \mathbf{Y}), c_2(\mathbf{V}, \mathbf{Y})), \tag{2}$$

and similarly for Q_i and M_i.

In addition, we require that profits satisfy the following symmetry condition, which obviously holds in the Cournot case.

Assumption 3 *Product market profits are* ***exchangeable****, i.e. for all $V', V'' \in \{0, 1\}$ and $Y', Y'' \in [0, \infty)$,*

$$\Pi_1(V', V'', Y', Y'') = \Pi_2(V'', V', Y'', Y').$$

In the next two lemmas, we show that our assumptions imply conditions on product market profits that turn out to be essential for the investment game.

[12] The precise meaning of "sufficiently large" in (iii) will become clear in the proof of Lemma 1 below.

Lemma 1 *: Suppose Assumptions 1 and 2 hold. Then*

(i) $M_1(1,0;\mathbf{Y}) > M_1(0,1;\mathbf{Y})\,; Q_1(1,0;\mathbf{Y}) > Q_1(0,1;\mathbf{Y})\,;$

(ii) M_i *and* Q_i *are both increasing in* Y_i *and decreasing in* Y_j*.*

Proof. See appendix. ■

Result (i) compares mark-up and demand of an integrated firm facing a separated competitor with those of a separated firm facing an integrated competitor.[13] Intuitively, result (i) follows in two steps: First, starting from $\mathbf{V} = (0,1)$, suppose firm 1 integrates, resulting in $\mathbf{V} = (1,1)$. This reduces own costs and leaves competitor costs unaffected; thus own mark-up and demand increase. Next, compare $\mathbf{V} = (1,1)$ and $\mathbf{V} = (1,0)$: own costs are unaffected and the competitor's costs increase. Again, own mark-up and demand increase. Thus, combining the two steps, the result follows.

Result (ii) states that an increase in own efficiency increases own demand and mark-up, and conversely for an increase in competitor efficiency. The first part of the statement is obvious if efficiency increases not only decrease own costs, but also increase the wholesale price, so that competitor costs increase. If higher own efficiency decreases the wholesale price, competitor cost reductions could, in principle, outweigh the positive effect of lower own costs. Assumptions 1 (iii) and 2 (ii) guarantee that own effects dominate over cross effects. The second part of the statement uses similar arguments.

We now obtain our central lemma.

Lemma 2 *: Suppose Assumptions 1 and 2 hold.*

(i) Suppose that

$$\frac{\partial Q_1}{\partial Y_1}(1,0;\mathbf{Y}) \geq \frac{\partial Q_1}{\partial Y_1}(0,1;\mathbf{Y})\,; \quad \frac{\partial M_1}{\partial Y_1}(1,0;\mathbf{Y}) \geq \frac{\partial M_1}{\partial Y_1}(0,1;\mathbf{Y})\,. \quad (3)$$

Then

$$\frac{\partial \Pi_1}{\partial Y_1}(1,0;\mathbf{Y}) > \frac{\partial \Pi_1}{\partial Y_1}(0,1;\mathbf{Y})\,. \quad (4)$$

[13] Obviously, an analogous statement holds for M_2 and Q_2.

(ii) Suppose that

$$\frac{\partial^2 Q_1}{\partial Y_1 \partial Y_2} \leq 0; \frac{\partial^2 M_1}{\partial Y_1 \partial Y_2} \leq 0. \quad (5)$$

Then

$$\partial \Pi_i / \partial Y_i \text{ is decreasing in } Y_j \text{ for } j \neq i, \ i = 1, 2 \quad (6)$$

(iii) Suppose that

$$\frac{\partial^2 Q_1}{\partial Y_1^2} \geq 0, \frac{\partial^2 M_1}{\partial Y_1^2} \geq 0. \quad (7)$$

Then

$$\partial \Pi_i / \partial Y_i \text{ is increasing in } Y_i \text{ for } i = 1, 2. \quad (8)$$

Proof. See appendix. ■

Statement (i) gives conditions for investment incentives to be higher for an integrated firm facing a separated competitor than for a separated firm facing an integrated competitor. (ii) gives conditions for investment incentives to decrease when a competitor becomes more efficient. (iii) gives conditions for more efficient firms to have higher investment incentives than less efficient firms.

The intuition for statement (i) relies on Lemma 1. This result says that mark-up and demand are higher for an integrated firm facing a separated competitor than for a separated firm facing an integrated competitor. Since higher mark-up means that demand increases resulting from greater efficiency are more valuable, and higher demand means that mark-up increases are more valuable, the benefits from integration and cost reduction tend to reinforce each other. Put differently, there are *demand/mark-up complementarities.* As a result, an integrated firm with high mark-up typically finds it more beneficial to invest into cost reduction than a separated competitor. There is, however, a potential countervailing effect: The size of the demand and mark-up increases associated with higher efficiency could, in principle, decrease with vertical integration. The additional condition on $\partial Q_i / \partial Y_i$ and $\partial M_i / \partial Y_i$ excludes this possibility. This condition is fairly natural: higher efficiency increases the wholesale price. This tends to reduce the output and mark-up increases resulting from higher transformation efficiency for a separated firm—an effect that is absent for integrated firms.

In addition, this condition is stronger than necessary. All we require is that the demand/mark-up complementarities dominate over any negative effect of integration on $\partial Q_i/\partial Y_i$ and $\partial M_i/\partial Y_i$.

The arguments for statements (ii) and (iii) are analogous. In each case, the changes of variables under consideration (increasing in Y_i and decreasing in Y_j, respectively) lead to increases in demand and mark-up which are mutually reinforcing. Again, there might be countereffects of Y_i and Y_j on $\partial Q_i/\partial Y_i$ and $\partial M_i/\partial Y_j$ that could upset the results. The additional conditions on the second partial derivatives of Q_i and M_i exclude this possibility.[14]

4.2 The Cost Reduction Game

We now analyze how the cost reduction decisions of integrated firms differ from those of separated competitors. Consider a *cost-reduction game* such that, for given vectors $\mathbf{V}$ and $\mathbf{Y}^0$, firms simultaneously choose the extent to which they want to reduce their transformation costs. Reducing costs by y_i involves investment costs of $K_i\left(y_i, Y_i^0\right)$, with K_i increasing in y_i.[15] With $\mathbf{y} = (y_1, y_2)$, the firms' objective functions are given by

$$\Pi_i\left(\mathbf{V}, \mathbf{Y}^0 + \mathbf{y}\right) - K_i\left(y_i, Y_i^0\right).$$

The next result generalizes Observation 1. As a preliminary remark, recall what our analysis has achieved so far: In particular, we have shown that the primitive assumptions of our model on outputs, mark-ups and wholesale prices imply (4) if supplemented by (3), and (8) if supplemented by (5). We can thus formulate our next results either in terms of the more general conditions (4) and (8), or in terms of the more primitive conditions (3) and (5).

Proposition 1 *Consider the* ***cost reduction game*** *and assume it has a unique equilibrium. Suppose that assumptions 1 and 2 and (3) and (5), or, more generally, conditions (4) and (6) are satisfied. Then*

[14] In the linear Cournot example, these additional conditions are satisfied (see table 1).

[15] Though we make no additional assumptions on K_i, it is useful to think of $\partial K_i/\partial y_i$ as being increasing in Y_i^0 and y_i to capture decreasing returns to investment.

(i) If $V_k = 1, V_\ell = 0$ *and* $Y_k^0 = Y_\ell^0 = 0$*, then* $y_k > y_\ell$*.*

(ii) Suppose $\partial^2 \Pi_i / \partial Y_i \partial y_i \geq \partial^2 K_i / \partial Y_i \partial y_i$*. If* $V_k = 1, V_\ell = 0$ *and* $Y_k^0 \geq Y_\ell^0$*, then* $y_k > y_\ell$*.*

Proof. See Appendix. ■

Result (i) states that, if the firms differ only with respect to their vertical integration status, the integrated firm will invest more into cost reduction than the separated firm. Intuitively, by Lemma 1, the integrated firm has higher equilibrium demand and mark-up than its competitor. The demand/mark-up complementarity therefore implies that the integrated firm has higher incentives to invest than the competitor, as reflected in (4). In addition, (6) implies that the higher investment of the integrated firm and the lower investment of the competitor are mutually reinforcing. Thus, integrated firms should invest more than separated firms.

Result (ii) generalizes (i) to the case where integrated firms have higher initial efficiency levels. The generalization comes at the cost of the additional condition $\partial^2 \Pi_i / \partial Y_i \partial y_i \geq \partial^2 K_i / \partial Y_i \partial y_i$, which ensures that the greater initial efficiency of integrated firms reinforces the higher cost-reduction incentive coming from integration itself. This condition is more restrictive than (8), which merely guarantees that the product market profit effects of further investment (gross of investment costs) are greater for firms that are already more efficient. The more restrictive condition in (ii) makes sure that the same is true for net product market profits.[16]

To sum up, at least in a set-up where integrated downstream firms are not active as suppliers on the upstream market, the observation that integrated firms tend to invest more into cost reduction than separated firms should be expected to hold quite generally, as it only requires fairly natural assumptions on product market competition. A countervailing force might arise if there are strongly decreasing returns to investments, and integrated firms have greater initial efficiency. Proposition 1 is consistent with the observation that

[16] Proposition 1 says nothing about why one firm is integrated and the other one is not, as assumed in part (i) of Proposition 1. However, in Buehler and Schmutzler (2003), we show that such asymmetries are likely to arise even when otherwise identical firms decide whether to integrate.

integrated firms tend to have high market shares: Not only does integration have a direct efficiency effect which works towards higher market shares, but integrated firms also tend to invest more into cost reduction.[17]

We now move towards issues that are potentially relevant for policy considerations. When examining a vertical merger, antitrust authorities should evaluate the effect of a firm's vertical integration on both firms' investments and, ultimately, on welfare. This evaluation involves a comparison of the firms' investments under *different* market structures, whereas Proposition 1 was based on a comparison of investments within a *given* asymmetric vertical market structure. For example, antitrust authorities will be interested in the effect on the firms' investments when vertical structure changes from $\mathbf{V} = (0, 0)$ to $\mathbf{V}' = (1, 0)$, whereas our above comparison involved the investments of firm 1 and 2 for $\mathbf{V}' = (1, 0)$. We now discuss the effect of vertical integration on the firms' investment in some more detail.

The next result, which is in the spirit of Observation 2, gives the conditions under which a firm's vertical integration increases own investment and decreases competitor investment.

Proposition 2 *Suppose that condition (6) holds. In addition, suppose*

$$\frac{\partial \Pi_i}{\partial Y_i} \text{ is non-decreasing in } V_i \text{ and non-increasing in } V_j. \tag{9}$$

Then the equilibrium level of Y_i is non-decreasing in V_i and non-increasing in V_j.

The proof is a simple application of Milgrom and Roberts (1990, Th. 5). Intuitively, by (9), if one firm integrates, this increases its incentive to invest, and it decreases the competitor's investment incentive. By (6), these effects are mutually reinforcing.

Proposition 2 is useful, because the underlying condition (9) can be seen to be very plausible, using a similar intuition as in the justification of (4): In most conceivable examples, own integration increases equilibrium output and

[17] It should be noted that Proposition 1 can be generalized to more than two firms. The proof uses similar techniques as the special case of two firms.

mark-up, which, in turn, increases the benefits of the mark-up and output increases resulting from higher efficiency. Similarly, integration of the competitor tends to reduce own output and mark-up which tends to decrease the benefits of higher efficiency. As argued in the justification of (4), there may be countereffects arising because the size of the output and mark-up increases from integration may depend on the efficiency levels as well. Also, clearly, the required condition (9) is less general than the corresponding condition (4) for Proposition 1: (9) implies (4), but the reverse implication is not true.[18] Nevertheless, Proposition 2 indicates that under fairly natural assumptions that are based on demand/mark-up complementarities, the finding that a firm's vertical integration increases own investment and reduces competitor investment generalizes beyond the linear Cournot model.

Proposition 2 suggests that antitrust policy towards vertical mergers is likely to be tricky even when foreclosure is not an issue, that is, when integration does not raise rivals' costs: Prohibiting vertical mergers may help avoiding polarized market structures and strategic vertical integration, but the effects on investment are less obvious. Typically, prohibiting vertical integration reduces investments of those firms that would have integrated, whereas it increases the investments of those firms that would have not integrated. In the simple Cournot example, the net welfare effect of a prohibition was negative—but this is not necessarily true in general.

4.3 Strategic Vertical Integration

Generalizing Observation 3, we now analyze strategic incentives for vertical integration in a setting where integration decisions precede cost-reducing investments. We assume that initial states are given by $\mathbf{V}^0 = \mathbf{Y}^0 = \mathbf{0}$. In stage 1, firms make integration decisions v_i, leading to $\mathbf{V}^1 = \mathbf{v}$. In stage 2, they choose cost-reducing investments as in the cost reduction game analyzed in section 4.2. For simplicity, we suppose that, for each $\mathbf{V}^1$, the cost-reduction game has a unique equilibrium $\mathbf{y}(\mathbf{V}^1) = \mathbf{y}(\mathbf{v})$. Further, we generalize the notation introduced in the example:

[18] Also, generalization of Proposition 2 to more than two firms requires additional conditions.

Notation 1 *Firm i's* ***profit*** *as a function of integration decisions, evaluated at the equilibrium choices of cost reduction, is given by*

$$\Pi_i^* (\mathbf{v}) \equiv \Pi_i (\mathbf{v}, \mathbf{y} (\mathbf{v})) - K_{_i} (y_i (\mathbf{v}), 0), \quad i = 1, 2.$$

The ***profit differentials*** *associated with integration, accounting for the induced changes in cost reduction, are denoted by*

$$\Delta_1^* (v_2) \equiv \Pi_1^* (1, v_2) - \Pi_1^* (0, v_2); \quad \Delta_2^* (v_1) \equiv \Pi_2^* (v_1, 1) - \Pi_2^* (v_1, 0).$$

In this setting, the following observations are straightforward.

Proposition 3 *Consider the* ***two-stage game*** *where (4) and (6) are satisfied.*

(i) If $V_k^1 = 1$, $V_\ell^1 = 0$, *then* $y_k (\mathbf{V}) > y_\ell (\mathbf{V})$.

(ii) $\Delta_1^* (v_2) \geq \Delta_1 (v_2; \mathbf{y} (0, v_2)); \Delta_2^* (v_1) \geq \Delta_2 (v_1; \mathbf{y} (v_1, 0))$.

Part (i) of Proposition 3 is a direct application of Proposition 1 (i) to the second-period subgames of the two-stage game. Part (ii) states that there is a strategic incentive to integrate vertically: The left-hand side of each inequality in (ii) is the profit differential associated with integration, taking into account the induced change of second-period investment behavior. The right-hand side is the profit differential associated with integration, fixing the second period equilibrium at the pre-integration level. The difference is the strategic effect of vertical integration. Intuitively, because of demand/mark-up complementarities, integration makes cost-reducing investments for the competitor less attractive, which increases profits of the integrating firm. Thus, a firm might integrate to reduce the incentives for a competitor to cut costs.[19]

[19] In the terminology of Fudenberg and Tirole (1984), integration corresponds to a top dog strategy.

5 Conclusions

This paper considers investment incentives in vertically-related oligopolies, where firms may be vertically integrated or separated. We show that integrated firms invest more into cost reduction than separated competitors, which fits nicely with the observation that integrated firms have higher market share. Further, we demonstrate that a firm's vertical integration typically increases its own investment and decreases competitor investment. Finally, when firms decide sequentially about vertical integration and cost reduction, they may use vertical integration so as to strategically reduce investments by competitors.

Our analysis sheds new light on the competitive effects of vertical integration and possible antitrust policies towards them. For instance, we saw that the complementarity of vertical integration and cost-reducing investment tends to give rise to polarized market structure with large integrated and small separated firms. That is, dominance of a small number of firms is a more serious concern than the purely horizontal setting or the standard vertical setting without investment suggest. Furthermore, we found that firms may use vertical integration strategically so as to preempt cost-reducing investment by competitors. Antitrust authorities may be tempted to deal with these problems by prohibiting vertical mergers. However, doing so is likely to prevent cost-reducing investment on the part of the integrating firm, and may well reduce social welfare. That is, antitrust policy towards vertical mergers may be even more subtle than was previously thought.

6 Appendix 1: Proofs

6.1 Proof of Lemma 1

(i) By Assumption 1, we have $c_1(1,0;\mathbf{Y}) = 0 = c_1(1,1;\mathbf{Y}) < c_1(0,1;\mathbf{Y})$ and $c_2(0,1;\mathbf{Y}) = 0 = c_2(1,1;\mathbf{Y}) < c_2(1,0;\mathbf{Y})$. The claim follows from Assumption 2 (i) and (ii).

(ii) First consider

$$\frac{\partial Q_i}{\partial Y_i} = \frac{\partial q_i}{\partial c_i}\frac{\partial c_i}{\partial Y_i} + \frac{\partial q_i}{\partial c_j}\frac{\partial c_j}{\partial Y_i} \quad \text{for } j \neq i.$$

By (ii) in Assumption 1 and Assumption 2, $\partial Q_i/\partial Y_i > 0$ if $\partial c_j/\partial Y_i > 0$. If $\partial c_j/\partial Y_i < 0$, part (ii) of Assumption 2 guarantees the result provided that $|\partial c_i/\partial Y_i| > |\partial c_j/\partial Y_i|$. This is implied by (iii) of Assumption 1. Now, consider

$$\frac{\partial Q_i}{\partial Y_j} = \frac{\partial q_i}{\partial c_i}\frac{\partial c_i}{\partial Y_j} + \frac{\partial q_i}{\partial c_j}\frac{\partial c_j}{\partial Y_j} \quad \text{for } j \neq i.$$

By (ii) in Assumption 1 and Assumption 2, $\partial Q_i/\partial Y_j < 0$ if $\partial c_i/\partial Y_j > 0$. If $\partial c_i/\partial Y_j < 0$, part (iii) of Assumption 1 again guarantees the result.[20]

6.2 Proof of Lemma 2

(i) Differentiating firm 1's profit function yields

$$\frac{\partial \Pi_1}{\partial Y_1} = \frac{\partial Q_1}{\partial Y_1} M_1 + \frac{\partial M_1}{\partial Y_1} Q_1.$$

Using Lemma 1, all terms on the r.h.s. of this equation are positive, and both $M_1(1,0;\mathbf{Y}) > M_1(0,1;\mathbf{Y})$ and $Q_1(1,0;\mathbf{Y}) > Q_1(0,1;\mathbf{Y})$. (4) thus follows immediately from

$$\frac{\partial Q_1}{\partial Y_1}(1,0;\mathbf{Y}) \geq \frac{\partial Q_1}{\partial Y_1}(0,1;\mathbf{Y}) \text{ and } \frac{\partial M_1}{\partial Y_1}(1,0;(\mathbf{Y})) \geq \frac{\partial M_1}{\partial Y_1}(0,1;\mathbf{Y}).$$

(ii) Using

$$\frac{\partial^2 Q_1}{\partial Y_1 \partial Y_2} \leq 0, \frac{\partial^2 M_1}{\partial Y_1 \partial Y_2} \leq 0,$$

arguments similar to those used in the proof of (i) show that $\partial^2 \Pi_1/\partial Y_1 \partial Y_2 \leq 0$ and $\partial^2 \Pi_1/\partial Y_1^2 \geq 0$. (6) thus follows immediately.

(iii) The proof is analogous to the proof of (ii).

6.3 Proof of Proposition 1

(i) By (4) and exchangeability (Assumption 3), we have

$$\frac{\partial \Pi_2}{\partial Y_2}(0,1;Y) = \frac{\partial \Pi_1}{\partial Y_1}(1,0;\mathbf{Y}) > \frac{\partial \Pi_1}{\partial Y_1}(0,1;\mathbf{Y}) = \frac{\partial \Pi_2}{\partial Y_2}(1,0;\mathbf{Y}). \quad (10)$$

[20] In this case, however, $|\partial c_j/\partial Y_j| > |\partial c_i/\partial Y_j|$ is not sufficent to guarantee the result; it is necessary that the the left-hand side is sufficiently large.

Now, define $\theta = (V_1, V_2)$ and $x_1 = y_1, x_2 = -y_2$. By Assumption, $Y_1^0 = Y_2^0 = 0$. Thus, consider the game with objective function

$$f_i(x_1, x_2; \theta) = \Pi_i(\theta; x_1, -x_2) - K_i(|x_i|, 0).$$

By (6), this game is supermodular.[21] By (4), changing θ from $(0,1)$ to $(1,0)$ increases $\partial\Pi_1/\partial Y_1$ and reduces $\partial\Pi_2/\partial Y_2$. In other words, Π_i has increasing differences in (θ, x). Thus, the result follows from Theorem 5 in Milgrom and Roberts (1990).

(ii) Suppose $H, L \in [0, \infty), H > L$. Define $\theta_1 = (1, 0, H, L), \theta_2 = (0, 1, L, H)$ and an order on $\{\theta_1, \theta_2\}$ by $\theta_1 > \theta_2$. Furthermore, define $x_1 = y_1, x_2 = -y_2$ and

$$f_i(x_1, x_2; \theta) = \Pi_i(\theta; x_1, -x_2) - K_i(|x_i|, 0).$$

Then f is supermodular in x_1, x_2. It satisfies increasing differences in (x, θ) if

$$\frac{\partial\Pi_1}{\partial y_1}(1, 0; H + y_1, L + y_2) - \frac{\partial K_1}{\partial y_1}(y_1, H) > \frac{\partial\Pi_1}{\partial y_1}(0, 1, L + y_1, H + y_2) - \frac{\partial K_1}{\partial y_1}(y_1, L).$$

Using (4) and the condition $\partial^2\Pi_i/\partial Y_i\partial y_i \geq \partial^2 K_i/\partial Y_i\partial y_i$, it suffices to show

$$\frac{\partial\Pi_1}{\partial y_1}(1, 0; L + y_1, L + y_2) - \frac{\partial K_1}{\partial y_1}(y_1, L) > \frac{\partial\Pi_1}{\partial y_1}(0, 1, L + y_1, L + y_2) - \frac{\partial k_1}{\partial y_1}(y_1, L).$$

This inequality was shown to hold in the proof of (i).

6.4 Proof of Proposition 3

We need to show (ii). First note that by definition of $y_1(0, v_2)$ as firm 1's strategy in the equilibrium of the subgame $(0, v_2)$,

$$\begin{aligned}\Pi_1^*(1, v_2) &= \Pi_1(1, v_2; y_1(1, v_2), y_2(1, v_2)) - K_1(y_1(1, v_2), 0) \geq \\ &\quad \Pi_1(1, v_2; y_1(0, v_2), y_2(1, v_2)) - K_1(y_1(0, v_2), 0).\end{aligned}$$

Next, using Proposition 2, $y_2(0, v_2) \geq y_2(1, v_2)$. Thus, by Assumption 2,

$$\Pi_1(1, v_2; y_1(0, v_2), y_2(1, v_2)) \geq \Pi_1(1, v_2; y_1(0, v_2), y_2(0, v_2)).$$

[21] See Milgrom and Roberts (1990) for the concept of supermodular games.

Therefore,

$$\Pi_1^*(1, v_2) \geq \Pi_1(1, v_2; y_1(0, v_2), y_2(0, v_2)) - K_1(y_1(0, v_2), 0).$$

Subtracting $\Pi_1^*(0, v_2)$ on both sides gives $\Delta_1^*(v_2) \geq \Delta_1(v_2; \mathbf{y}(0, v_2))$. The proof of the second statement is analogous.

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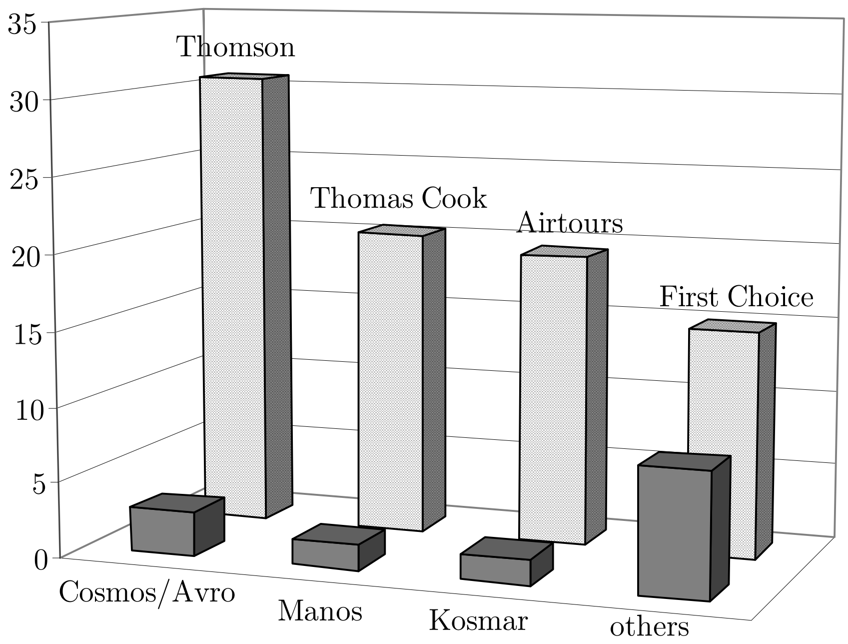

Figure 1: The largest tour operators as of 1998 (Source: European Commission 1999)

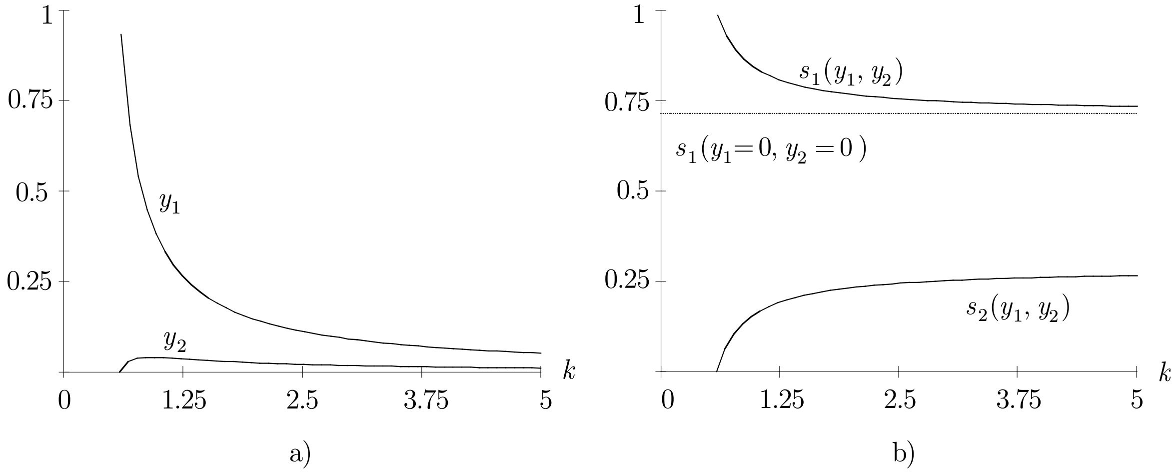

Figure 2: Investments and market shares in the linear Cournot model ($\alpha = 1; Y_1^0 = Y_2^0 = 0$).

Table 1: The linear Cournot example

	$\mathbf{V} = (0,0)$		$\mathbf{V} = (1,0)$		$\mathbf{V} = (1,1)$	
	$\mathbf{y} = (y_1(\mathbf{V}), y_2(\mathbf{V}))$	$\mathbf{y} = (0,0)$	$\mathbf{y} = (y_1(\mathbf{V}), y_2(\mathbf{V}))$	$\mathbf{y} = (0,0)$	$\mathbf{y} = (y_1(\mathbf{V}), y_2(\mathbf{V}))$	$\mathbf{y} = (0,0)$
$w_1(\mathbf{V}, \mathbf{y})$	$\frac{2\alpha+y_1+y_2}{4}$	$\frac{\alpha}{2}$	0	0	0	0
$w_2(\mathbf{V}, \mathbf{y})$	$\frac{2\alpha+y_1+y_2}{4}$	$\frac{\alpha}{2}$	$\frac{\alpha-y_1+2y_2}{4}$	$\frac{\alpha}{4}$	0	0
$M_1(\cdot) = Q_1(\cdot)$	$\frac{2\alpha+7y_1-5y_2}{12}$	$\frac{\alpha}{6}$	$\frac{5\alpha+7y_1-2y_2}{12}$	$\frac{5\alpha}{12}$	$\frac{4\alpha+8y_1-4y_2}{12}$	$\frac{\alpha}{3}$
$M_2(\cdot) = Q_2(\cdot)$	$\frac{2\alpha-5y_1+7y_2}{12}$	$\frac{\alpha}{6}$	$\frac{2\alpha-2y_1+4y_2}{12}$	$\frac{\alpha}{6}$	$\frac{4\alpha-4y_1+8y_2}{12}$	$\frac{\alpha}{3}$
$\Pi_1(\mathbf{V}, \mathbf{y})$	$\frac{(2\alpha+7y_1-5y_2)^2}{144} - ky_1^2$	$\frac{\alpha^2}{36}$	$\frac{(5\alpha+7y_1-2y_2)^2}{144} - ky_1^2$	$\frac{25\alpha^2}{144}$	$\frac{(4\alpha+8y_1-4y_2)^2}{144} - ky_1^2$	$\frac{\alpha^2}{9}$
$\Pi_2(\mathbf{V}, \mathbf{y})$	$\frac{(2\alpha-5y_1+7y_2)^2}{144} - ky_2^2$	$\frac{\alpha^2}{36}$	$\frac{(2\alpha-2y_1+4y_2)^2}{144} - ky_2^2$	$\frac{4\alpha^2}{144}$	$\frac{(4\alpha-4y_1+8y_2)^2}{144} - ky_2^2$	$\frac{\alpha^2}{9}$
$y_1(\mathbf{V})$	$\frac{7\alpha}{72k-7}$	0	$\frac{-14\alpha+105\alpha k}{432k^2-195k+14}$	0	$\frac{2\alpha}{9k-2}$	0
$y_2(\mathbf{V})$	$\frac{7\alpha}{72k-7}$	0	$\frac{-14\alpha+24\alpha k}{432k^2-195k+14}$	0	$\frac{2\alpha}{9k-2}$	0

Table 2: The linear Cournot example with $k = 1$

	$\mathbf{V} = (0,0)$		$\mathbf{V} = (1,0)$		$\mathbf{V} = (1,1)$	
	$\mathbf{y} = (y_1(\mathbf{V}), y_2(\mathbf{V}))$	$\mathbf{y} = (0,0)$	$\mathbf{y} = (y_1(\mathbf{V}), y_2(\mathbf{V}))$	$\mathbf{y} = (0,0)$	$\mathbf{y} = (y_1(\mathbf{V}), y_2(\mathbf{V}))$	$\mathbf{y} = (0,0)$
$w_1(\mathbf{V}, \mathbf{y})$	0.554α	$\frac{\alpha}{2}$	0	0	0	0
$w_2(\mathbf{V}, \mathbf{y})$	0.554α	$\frac{\alpha}{2}$	0.179α	$\frac{\alpha}{4}$	0	0
$M_1(\cdot) = Q_1(\cdot)$	0.185α	$\frac{\alpha}{6}$	0.622α	$\frac{5\alpha}{12}$	0.429α	$\frac{\alpha}{3}$
$M_2(\cdot) = Q_3(\cdot)$	0.185α	$\frac{\alpha}{6}$	0.120α	$\frac{\alpha}{6}$	0.429α	$\frac{\alpha}{3}$
$\Pi_1(\mathbf{V}, \mathbf{y})$	$0.022\alpha^2$	$0.028\alpha^2$	$0.255\alpha^2$	$0.174\alpha^2$	$0.102\alpha^2$	$\frac{\alpha^2}{9}$
$\Pi_2(\mathbf{V}, \mathbf{y})$	$0.022\alpha^2$	$0.028\alpha^2$	$0.013\alpha^2$	$0.028\alpha^2$	$0.102\alpha^2$	$\frac{\alpha^2}{9}$
$y_1(\mathbf{V})$	0.108α	0	0.363α	0	0.286α	0
$y_2(\mathbf{V})$	0.108α	0	0.040α	0	0.286α	0
$W(\mathbf{V}, \mathbf{y})$	$0.318\alpha^2$	$0.278\alpha^2$	$0.564\alpha^2$	$0.413\alpha^2$	$0.571\alpha^2$	$0.444\alpha^2$