Asymmetric Information and Financing with Convertibles*

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Abstract

We analyze the problem of dilution leading to inefficient underinvestment caused by the adverse selection problem. We assume that the market obtains information about the firm over time, but that at each date the manager possesses better information about firm prospects than does the market. We show that issuing callable convertible securities with fixed conversion prices and restrictive call provisions is optimal. Such securities make the payoff to new claimholders independent of the private information of the manager. The restrictive call provision serves as a commitment device, enabling the manager to call only when the stock price rises in the future. This benefits the new as well as the existing claim–holders so that adverse selection problem is costlessly solved without any dissipation or underinvestment. Furthermore, we show that this efficient outcome can also be implemented by issuing optimally designed floating price convertibles.

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1 Introduction

We investigate the classic problem of inefficient under–investment caused by the presence of a manager with superior information about the firm's prospects relative to the market, as in Myers and Majluf (1984). Due to the asymmetry of information, the market values any securities issued by the firm at the fair or expected price. This leads to dilution in the claims of the existing equity holders of the firm, when the manager knows that firm prospects and asset values are better than average. Such dilution may in fact lead the manager to take the socially inefficient decision of not investing in positive net present value projects.

In this paper we start from the premise that the initial asymmetry of information about the firm's assets in place and investment opportunities is likely to be resolved over time, even though at each date the manager's information is superior to the market's. Analyst announcements, future earnings, R&D outcomes, M&A announcements or decisions by regulators are few of the events that may reveal valuable information to the public over time. Our main goal is to use the future (imperfect) resolution of the initial asymmetry of information and design a security whose value is independent of the initial private information of the manager. Such a security has the property that the expected value of the security given commonly available information is equal to the expected value given the private information of the manager. Since competitive uninformed investors will be willing to pay the expected value of the new issue, the high quality firm will not suffer from either dissipation or dilution. Consequently, such a security will solve the under-investment problem costlessly.

We present our results first in a simple binary state model and then in a general model with multiple states and endogenous public information. In both cases the basic message is that the manager can costlessly solve the underinvestment problem by issuing convertible securities, as long as the resolution of the initial asymmetry of information occurs with enough fidelity.

In our binary state model, we focus on callable convertible securities, providing a rationale to its common features and practices. In particular, we design an optimal callable convertible debt (or preferred stock) contract, with fixed conversion prices and restrictive call provisions that mitigates adverse selection completely. The convertibility feature allows investors to choose which kind of security they would like to hold ex-post — the senior debt claim or the junior common stock claim.[1] The callability feature keeps the investors honest, by forcing conversion

[1] In order to focus on inefficiencies arising out of asymmetric information we abstract away from considerations of tax or clientele effects as well as bankruptcy and financial distress costs. As a result, debt is equivalent to

into common stock following good news. Convertibility combined with callability ensures that different types of securities are held, depending on the nature of information that is publicly disclosed. This allows the manager to choose the conversion ratio and face value such that the expected value of the security is constant across all private signals the manager might receive.

The optimally designed callable convertible security has the property that the value of the debt claim is higher than the value of the common stock claim. Consequently, the manager seeks to force conversion to common stock whenever he is able to. Because of this last feature, the optimal callable convertible security also has a restrictive call provision that does not allow the manager to call unless good information has been revealed to the market and the share price is high enough. This commitment raises the value of the security for new claim-holders, ultimately benefitting the firm and existing claim-holders. The better the initial information of the manager the higher is the chance that good information will reach the market, raising the stock price and enabling the manager to force conversion. However, when such information does not reach the market, the manager is unable to force conversion.

In our general model with multiple states we endogenize the public disclosure of information that is provided by a self-motivated analyst. We show that one can still design convertible securities whose payoff is independent of the private information of the manager. We achieve this by allowing the conversion ratio to depend on share prices (or the market value of equity) which itself is a direct (verifiable) consequence of analyst disclosure. As in the simple model, the expected market value of the claims sold are lower the more favorable is the public information in the market.

The seminal work of Myers and Majluf (1984) has been followed by a large literature attempting to identify securities which mitigate the dilution and associated underinvestment problem. The paper by Brennan (1986), is perhaps the closest in spirit to our work. Brennan points out that a floating-priced convertible security can avoid the adverse selection problem if the conversion price depends on the market price. Such a security is automatically converted into n shares, where n is the inverse of market price at the time of conversion. Thus it pays a fixed dollar amount independent of the public information in the economy, as measured by the market price. If the private information of the manager is perfectly reflected in the market price at the time of conversion, then the adverse selection problem can be costlessly resolved with such a security.[2] Unfortunately, this result crucially depends on the perfect resolution

preferred stock in the context of our model. For simplicity we will refer to the senior claim as debt.

[2]It can also be costlessly solved by issuing short–term debt and refinancing. This strategy is however not

of asymmetric information over time. When the manager's private information is imperfectly incorporated into the market price, issuing such a security leads to dilution and may cause underinvestment. What is needed in such cases is a security whose value is independent of the private information of the manager. We show that commonly used securities such as callable convertible preferred stock or debt, with fixed conversion prices and restrictive call provisions, may produce the symmetric information outcome, even when the initial asymmetry of information is never completely resolved.

A significant portion of the ensuing literature focusses on modes of financing that allow the management to separate by signaling its type and thus solve the under-investment problem. Since separation by signaling quality is typically costly, it creates another source of inefficiency and dissipation in value that might even exceed the dissipation in value caused by dilution.[3] In fact, Nachman and Noe (1994) show that in the Myers and Majluf framework, non-dissipative signaling is not possible if the firm is limited to issuing securities with payoffs that are weakly increasing in the underlying cash flows. Complementing this result, Gibson and Singh (2001) show that costless separation can be achieved when the firm is allowed to issue put warrants, whose payoffs are non-increasing in cash flows. Our work shows that the first-best outcome can in fact be implemented in equilibrium, without any dissipation in value, using securities with payoffs that are non-decreasing in cash flows. The difference with Nachman and Noe's work is that the manager is able to use the future imperfect resolution of the current asymmetry of information.

A number of papers restrict their attention to securities with non-decreasing payoffs and yet manage to attain separation without dissipation in value. Their result are achieved by using signaling devices that are costly to mimic for the bad type but not costly in equilibrium for the good type. Among these, Stein (1992) demonstrates the value of callable convertible securities in avoiding costs of financial distress, in a model where the initial asymmetry of information is completely resolved over time. He shows that callable convertible debt can be used by good firms in order to signal their types and separate from bad firms. The bad firm does not mimic the good firm whenever the expected cost of financial distress from doing so is high enough to overcome the benefits of selling an overvalued claim. Good firms are necessarily able to call the bonds and force conversion in Stein's model, thereby avoiding the costs of financial distress. In

robust to imperfect disclosure of private information.

[3]In additon to costly signaling, there are papers analyzing how costly information acquisition might be used to mitigate adverse selection. See, e.g., Fulghieri and Lukin (2001).

contrast, the value of the optimal securities that we characterize is independent of the private information of the manager. As a result, there is no scope for mispricing whether or not the bad firm mimics the good firm, and even when no type of the manager can guarantee that he will be able to force conversion. Furthermore, as we mentioned above, we do not require the initial asymmetry of information to be completely resolved at any point in time. Our results thus complement those of Stein by pointing out that the "back-door equity" value of convertible securities results purely from considerations of asymmetric information, and not from considerations of financial distress costs.

Constantinides and Grundy (1989) show that securities similar to (noncallable) convertible bonds can costlessly solve the adverse selection problem by signaling information, provided the firm is also allowed to buy back shares. In the absence of the possibility to buy back shares, there is no fully revealing equilibrium involving securities whose value is increasing in cash flows. On the other hand, Brennan and Kraus (1987) show that the good type may separate from the bad type by retiring existing debt, which is too costly for the bad type to mimic. We also allow the manager to buy back previously issued securities in our model, but such strategies are not utilized in equilibrium. Brennan and Schwartz (1987) (and Brennan and Kraus) show that convertible bonds can solve the problem caused by asymmetric information about volatility. In our model, the asymmetric information is about the distribution of cash flows and so covers the possibility of asymmetric information about volatility.

Other explanations have been offered for the use of convertible securities. Harris and Raviv (1985) investigate the information content of calling convertible securities as opposed to the information content of *issuing* such securites (e.g., Stein (1992)). They show that risk-averse managers signal their quality by not callling immediately, whereas bad managers whose equity is expected to decline in the near future are forced to call.[4] In addition to papers focusing on signaling, Cornelli and Yosha (2003) analyze a problem in which a manager can manipulate the interim signal about the quality of a project. If the investment occurs in multiple stages, this possibility of "window dressing" results in a conflict of interest and thus inefficient investment. Convertible debt can be used to solve this problem. Convertibility features may also mitigate moral hazard problems. Green (1984) shows that the incentive problem caused by conflicts of interest among claim holders can be mitigated by convertible debt rather than straight debt.

[4]Nyborg (1995) integrates Harris and Raviv (1985) with Stein (1992) in a model in which new private information arrives in each date to a risk averse manager.

Finally, a growing literature providing explanations for the use of convertible securities solving moral hazard problem within staged venture capital financing. (See, e.g., Repullo and Suarez (1998))

In Section 2, we set up our basic model. In Section 3, we first consider the benchmark case where the asymmetry of information is perfectly resolved over time, and present our first set of results. We then discuss the case where the asymmetry of information is never fully resolved. In Section 4, we present our general model together with an endogenization of the public information disclosure, as well as a numerical example. Section 5 concludes.

2 The Basic Model

The basic structure of our model is essentially identical to that of Myers and Majluf (1984). We consider a firm which has both assets in place and a new investment opportunity. The values of both the new investment opportunity and assets in place are uncertain. The uncertainty is captured by the "type" of the firm $\theta \in \{\theta_1, \theta_2\}$.[5] Let $\Pr[\theta = \theta_i] = \lambda_i \in (0,1)$ with $\sum_i \lambda_i = 1$. We assume that the manager privately knows θ. Both the assets in place of the firm and the cash flows from the new investment opportunity depend on the type θ. Initially the firm is all equity with the number of shares outstanding given by $M = 1$. The manager makes his decisions to maximize the welfare of the existing shareholders. Furthermore, the riskless rate is 0 and that all agents are risk–neutral.

Let A_i stand for the expected value of the cash flows from the assets in place given $\theta = \theta_i$.[6] The new investment opportunity requires an investment of $C = \$1$. The new investment and assets in place *combined* produce a random cash flow of X. We assume that the random variable X takes values in some set $\mathbf{X}$, a subset of the non–negative real numbers. The probability distribution of X depends on the type of the firm θ. Let $G(.|\,\theta)$ denote the cumulative distribution function of X given θ. We assume that project cash flows for type $\theta = \theta_2$ first order stochastically dominate those for type $\theta = \theta_1$:

$$\text{For all } x \in \mathbf{X},\ G(x|\theta_i) \leq G(x|\theta_j) \text{ for } i > j. \tag{1}$$

[5] In Section 4 we consider the N type case for $N > 2$.

[6] As is well–known, for adverse selection to cause inefficient under–investment, it is necessary that there be type–dependent uncertainty about the value of assets in place from the perspective of outside investors.

Moreover, regardless of θ, it is socially efficient for the firm to invest:

$$\text{Projects have positive NPV: } E[X|\theta_i] - A_i > 1 \text{ for all } \theta_i. \tag{2}$$

In other words, we will analyze the under-investment problem where, due to costs arising out of asymmetric information, the manager might not find it profitable to invest in a positive NPV project.

If the cash flows from the project together with the assets in place of the firm is greater than or equal to the cost of the project with probability one, then the firm can always issue riskless secured debt at zero cost and the problem would be uninteresting. To rule out the possibility of riskless debt we assume that

$$G(1|\theta_1) > 0. \tag{3}$$

Thus, when $\theta = \theta_1$, with strictly positive probability the total cash flows X will fail to cover the cost of the project of \$1.

We define the expected value of the total cash flows for type θ_i of the firm, given that it invests, to be $V_i = E[X|\theta_i]$. From (1) V_i must be non–decreasing in i. To make matters interesting, and without loss of generality, we assume that

$$V_i > V_j \text{ if } i > j. \tag{4}$$

Note that the net present value of the firm of type θ_i is $V_i - 1$. In the first–best world with symmetric information, all types of the manager will invest in the project and $V_i - 1$ will be the the expected payoff of the existing shareholders.

Let $U(x)$ be the payoff from a security U when total cash flows are x. We will restrict attention to securities with payoffs $U(x)$ that are non–decreasing in x and that satisfy limited liability, i.e., $0 \leq U(x) \leq x$ for all x. Let $\mathbf{U}$ be the set of admissible securities. Note from (1) that for any admissible security $U \in \mathbf{U}$, $E[U(X)|\theta_i]$ is non–decreasing in i.

Equity and debt are admissible securities. An equity share will be denoted by $\alpha \in (0, 1)$, with the expected value of the cash flows from a share α, given $\theta = \theta_i$, equal to αV_i. For any bond with face value $F \geq 0$ let $D_i(F)$ be the expected value of the cash flows from the bond given $\theta = \theta_i$:

$$D_i(F) = E[\min(X, F) \,|\, \theta_i] \tag{5}$$

$D_i(F)$ is continuous in F and $D_i(F) \leq V_i$ for each i. In case of preferred stock, F should be thought of as the sum of promised dividends and liquidation value.

Our model has two (groups of) players, the manager (who maximizes the welfare of old shareholders) and the potential investors. The manager knows θ when he makes his investment and financing decisions. In contrast, we will assume that initially the investors are uninformed about θ, though later they will obtain information about the firm type. Further, we will assume that the these investors are competitive and efficient, so that at each date they value all securities at the expected value given publicly available information. We will refer to the set of potential investors collectively as the market. We specify our timing structure, strategies and available securities, and the resolution of informational asymmetries over time, in more detail now.

Our model has three dates 0, 1 and 2. At date 0, given his private information, the manager decides whether or not to invest and what securities to issue to finance the investment. The market is uninformed about θ at date 0 and competitively values the securities issued by the manager. The manager invests in date 0 if the issue succeeds.[7]

At date 1, some of the asymmetric information present at date 0 is resolved. Specifically, we assume that at date 1 the market publicly observes a signal $m \in \{m_1, m_2\}$ of θ, with

$$\Pr[m = m_i \,|\, \theta = \theta_i] = \beta \in (\frac{1}{2}, 1] \text{ for all } i. \tag{6}$$

The parameter β is a proxy for the degree to which the initial asymmetry of information between the manager and the market is ultimately resolved.[8] The case $\beta = 1$ corresponds to the case of perfect resolution. On the other hand, the case $\beta = \frac{1}{2}$ corresponds to the case where none of the asymmetry is ever resolved before project cash flows are realized. Though our results do not depend on a specific interpretation of the signal m, it might help the reader to think of it as an analyst announcement or the outcome of a patent application which may or may not be approved by the government. For the present moment we will assume that the signal m is exogenous. In Section 4 we will endogenize the signal so that its distribution will depend, in equilibrium, on the date 0 financing strategy of the manager.

We allow the manager to issue securities at date 0 whose payoffs depend on the endogenous date 1 *response* of the market (e.g., the market value of equity or the stock price) to the public

[7]Though this possibility never arises in equilibrium, we assume that if the manager fails to raise the required outlay for the project, he invests the amount raised in a riskless asset. On the other hand if he raises more than the required outlay he immediately distributes the excess as dividends.

[8]The symmetry in the conditional distribution of the signal is not needed for our results but simplifies the exposition.

signal m. For example, the manager is allowed to issue a callable bond that can be called only if the stock price exceeds a threshold value (or a floating price convertible whose conversion rate depends on the stock price). If such a bond is used in equilibrium, the threshold value may be such that the bond can be called only when the public signal is equals m_2 and the ensuing stock price is higher than the threshold. However, even though the outcome will be identical, we do not allow the manager to use a security to set a call restriction or a conversion provision directly in terms of the public signal m. Such a restriction is desirable as, in practice, securities whose payoff depend on the endogenous stock price in some manner are quite common, whereas securities whose payoffs depend directly on some public signal such as an analyst announcement or an earnings announcement are not so common.[9] We also allow the manager to also take actions at date 1 depending on the public signal m. For example, the manager may issue short–term bonds at date 0 and refinance the project by retiring them at date 1, but only if the public signal is m_1.

To complete our description of the timing structure, we assume that at date 2 the project cash flows are realized and distributed. The information and timing structure above implicitly defines a dynamic game of incomplete information between the manager and the market. Our notion of equilibrium will correspond to the perfect Bayesian equilibria of this game.

3 The Optimality of Callable Convertible Securities

In this section, we convey the basic idea of the paper using the simple binary model described above. To gain intuition we start by analyzing the benchmark case where the date 0 asymmetry of information is perfectly resolved at date 1. This corresponds to the case where $\beta = 1$. In Subsection 3.2, we will consider the case where $\beta < 1$, so that the date 0 asymmetry of information is never perfectly resolved.

3.1 Perfect Resolution of Asymmetric Information

We will show that there exists a callable convertible security that is used by both types of the manager in a pooling equilibrium that solves the adverse selection problem costlessly. That is, there will be no dilution in the claims of the existing equity holders so that the manager will

[9]Presumably because such public signals are frequently quite amorphous and contracts directly contingent on them are not enforceable in a court.

invest regardless of his private information. In other words, such a security and the associated equilibrium achieves the symmetric information outcome.

A callable convertible security is specified in our model by a tuple (F, α, k, T), where T is the (common) maturity date of the call option and the convertibility option, k is the call price, α is the share of the firm the bondholders will have, if they decide to convert into common stock and F denotes the face value of the bond. In line with common practice, we assume that if the firm calls the convertible security then the holders still retain the right to convert into equity and do not have to surrender the security as long as they convert. Notice that we do not specify any call restrictions on our security. This is because such call restrictions are not needed when $\beta = 1$. In the next section, we will see that such restrictions are needed when $\beta < 1$.

Consider the following callable convertible security $(F^*, \alpha^*, k^*, T^*)$. Suppose F^* is equal to the face value that would be chosen in a world where it is common knowledge that $\theta = \theta_1$. That is, F^* satisfies

$$D_1(F^*) = 1. \tag{7}$$

Suppose that the conversion rate α^* is chosen such that, if $\theta = \theta_2$ and the bondholders convert to equity, the expected payoff is equal to the cost of the project. That is, α^* solves

$$\alpha^* V_2 = 1. \tag{8}$$

Suppose that $T^* = 1$, so that all the options on the bond expire on date 1, when there is no asymmetry of information.

We show now that the call value k^* can be chosen suitably to finance the project regardless of θ, at zero cost to the existing shareholders. To do this we proceed backwards in time and analyze the optimality of the decision of the manager to call the bonds and the optimality of the decision of the bondholders to convert.

Suppose that we are in date 1 and $m = m_2$, so that it is common knowledge that $\theta = \theta_2$. We want the call value k^* to be such that the manager wants to call the bonds in this case. The optimality of the call decision depends in turn on the optimal conversion decision of the bondholders, both in the case where the bonds are called and in the case where the bonds are not. If the bonds are not called, the bondholders will not want to convert, as their payoff from not converting is greater than their payoff from converting:

$$D_2(F^*) \geq D_1(F^*) = 1 = \alpha^* V_2. \tag{9}$$

If the bond is called, then the bondholders will want to convert if their payoff from converting is at least as high as the payoff from holding the bond. That is, they will convert if and only if

$$k^* \leq \alpha^* V_2 = 1. \tag{10}$$

Suppose that k^* is such that (10) holds. Then, when $\theta = \theta_2$, the manager will want to call the bonds and force conversion, as the payoff of the old shareholders from doing so is greater than the payoffs from not doing so.

Suppose now that we are in date 1 and in $m = m_1$ so that it is common knowledge that $\theta = \theta_1$. We want the call value k^* to be such that the manager does not want to call the bonds and the shareholders do not want to convert the bonds if they are not called. If the bonds are not called, the bondholders do not want to convert as

$$\alpha^* V_1 < \alpha^* V_2 = 1 = D_1(F^*). \tag{11}$$

Therefore the manager will not want to call the bonds if

$$k^* \geq D_1(F^*) = 1. \tag{12}$$

From (10) and (12), if $k^* = 1$, the manager will call the bond to force conversion if $\theta = \theta_2$ and will not call the bond if $\theta = \theta_1$. In the latter case, bondholders will not convert. Thus we set the call value k^* of the bond equal to the market value of the security:

$$k^* = 1. \tag{13}$$

For such a bond and sequentially optimal call and conversion decisions, the payoff to the bondholders will be equal to 1 regardless of the private information of the manager, from (7) and (8). As a result, when the bond is issued at date 0, investors will not face any adverse selection, as the payoff from the bond is constant regardless of the private information of the manager. They will be willing to provide \$1 and subscribe to the issue.

Finally, we have to specify beliefs off the equilibrium path at date 0 to complete the characterization of this perfect Bayesian equilibrium. We suppose that the uniformed investors believe that $\theta = \theta_1$ whenever the manager issues any other security at date 0. Thus, neither type of the manager has an incentive to deviate. Therefore, the expected payoff for the old shareholders in type θ_i of the firm at date 0 is equal to $V_i - 1 > A_i$, the first best value of the firm given $\theta = \theta_i$. As a result, the manager will always invest.

Proposition 1 *Suppose $\beta = 1$. Then it is an equilibrium for both types of the manager to invest by issuing the callable convertible security $(F^*, \alpha^*, k^*, T^*)$ characterized by (7), (8) and (13). The manager will call to force conversion only when m_2 is observed. The security will not be called or converted when m_1 is observed. The expected payoffs of the new claim holders is equal to 1 regardless of θ, and the expected payoff to old shareholders of type θ_i of the firm is equal to $V_i - 1$, the first best value of the firm given $\theta = \theta_i$.*

Proof. Follows from the discussion above. □

The optimal security that we characterize above is independent of the private information of the manager. Thus, the security is correctly valued even though the bad type mimics the good type. This property of the optimal security will be seen to carry over to the case where the resolution of the asymmetry of information is imperfect.

Note however that the outcome implemented above can also be implemented simply by using short–term debt maturing in period 1 and then refinancing when there is no asymmetry of information. In other words, the convertibility feature of the security above is not really needed in order to achieve the first best. However, the equivalence between a callable convertible security and short term debt breaks down once we consider the case where the date 0 asymmetry of information is never perfectly resolved. The simple scenario of this section serves to bring out the intuition why callable convertible securities mitigate adverse selection problems.

3.2 Imperfect Resolution of Asymmetric Information

In this section we still consider the case where the manager can be of one of two types but where the date 0 asymmetry of information is only imperfectly resolved at date 1, i.e., $\beta \in (\frac{1}{2}, 1)$. Thus, even at date 1 the manager has superior information compared to the market. We will show that in this case, if β is high enough, there exists a callable convertible security such that if both types of the manager pool and finance the investment with this security then the first best will also be implemented without any dilution in the claims of the existing shareholders.

The security that we will characterize will be similar to the one in the previous section with one major difference — the call provision on the security will have a restriction that the security can be called only when the share price of the firm exceeds a certain threshold value. By specifying this restriction at date 0, the manager will be able to commit to not calling the bond and using his privileged information in the future at the expense of the new investors. This commitment ultimately benefits the existing claim holders.

A callable convertible security with a restrictive call provision consists of a tuple (F, α, k, p, T) where F is the face value, α is the fraction of equity obtained upon conversion, k is the call price and T is the (common) maturity date of the call and convertibility options, as before. The only difference from the security of the previous section is that the bond can be called only if the share price at date 1 exceeds a threshold value p. For brevity, we will refer to this threshold value as the call restriction.

In what follows, we first characterize the optimal such security and then show that it is an equilibrium for the manager to issue such a security regardless of his private information.

Consider the following callable convertible security with a restrictive call provision. Let the maturity date be $T = 1$. Suppose that F and α satisfy the following two equations:

$$\beta D_1(F) + (1 - \beta)\alpha V_1 = 1, \tag{14}$$

$$(1 - \beta)D_2(F) + \beta\alpha V_2 = 1. \tag{15}$$

The first equation states that the required outlay of 1 dollar is equal to the expected value of the security, conditional on $\theta = \theta_1$ *and* conditional on the fact that the security will be converted to equity when $m = m_2$ but not when $m = m_1$. The second equation has the same interpretation, but for $\theta = \theta_2$. We show in the Appendix that (14) and (15) possess a solution $\alpha \in (0, 1)$ and $F > 0$ with $D_1(F) < V_1$ when β is high enough. For now assume that such a solution exists.

If the market conjectures at date 0 that, regardless of the manager's private information, the security will be converted to equity at date 1 when $m = m_2$ and good news about the firm is disclosed, but not when $m = m_1$, then the expected value of the security will be equal to \$1 at date 0. In competitive markets then, such a security will trade at that price at date 0.

We show now that at date 1 the expected value of the equity claim, conditional on the *market's* information m, is strictly less than the expected value of the debt claim. In fact, at a solution to (14) and (15) we must have

$$\alpha V_i < D_i(F) \text{ for } i = 1, 2. \tag{16}$$

In other words, the expected value of the equity claim must in fact be less than the expected value of the debt claim conditional on the *manager's* information θ. Thus, from the manager's perspective, conversion to equity lowers the expected value of the claims sold to new claim-holders. This is the "back–door equity" value of the convertible security in the context of our model — when converted, the equity share of the new claim-holders will be lower than what they would obtain in the first–best world. To compensate the new claim-holders, the value of the debt claim is "sweetened" and is higher than what they would obtain in the first–best world.

The proof of (16) is immediate. Suppose it does not hold for $i = 2$ so that $\alpha V_2 \geq D_2(F)$. Then we must have

$$(1-\beta)D_2(F) + \beta\alpha V_2 \geq \beta D_2(F) + (1-\beta)\alpha V_2 > \beta D_1(F) + (1-\beta)\alpha V_1. \tag{17}$$

The first inequality follows from the fact that $\beta > 1 - \beta$. The second inequality follows from the fact that $D_2(F) \geq D_1(F)$ and the fact that $V_2 > V_1$. But (17) contradicts the fact that α and F satisfy (14) and (15). Analogously one can show that (16) holds for $i = 1$.

Let $\mu_i^1(m)$ be the posterior probability at date 1 attached by the market to the event that $\theta = \theta_i$ after observing m. Note that since $\beta > \frac{1}{2}$ we must have $\mu_2^1(m_2) > \mu_2^1(m_1)$. Since the expected value of the equity claim conditional on θ is less than the expected value of the debt claim, for all θ, it follows that the expected value of the equity claim conditional on m is less than the expected value of the debt claim, for all signals m :

$$\mu_2^1(m)\alpha V_2 + \mu_1^1(m)\alpha V_1 < \mu_2^1(m)D_2(F) + \mu_1^1(m)D_1(F). \tag{18}$$

Consequently, bond holders will not want to convert to equity unless forced to do so and the manager would prefer to force conversion by calling the bond whenever he can. However, if he always forces conversion, the expected value of the security to the new claim-holders will fall below \$1 and they will be unwilling to provide the funds for the project, hurting the existing claim-holders. This implies that a restriction on the call provision is needed in order for the manager to commit to not calling the bond regardless of the state of the world.

Choose the restrictive call provision p that is in between the market value old shareholders claims when $m = m_1$ and when $m = m_2$:

$$\mu_2^1(m_1)[V_2 - D_2(F)] + \mu_1^1(m_1)[V_1 - D_1(F)] < p < \mu_2^1(m_2)(1-\alpha)V_2 + \mu_1^1(m_1)(1-\alpha)V_1. \tag{19}$$

Such an interval for p exists from (16) as $\mu_2^1(m_2) > \mu_2^1(m_1)$. In equilibrium, the stock price will equal the right–hand side of (19) when $m = m_2$ so that the manager will be able to force conversion by calling; and when $m = m_1$ the stock price will equal the left–hand side of (19), so that the bond will not be converted.

Choose any call price k that is less than the expected value of the equity claim given $m = m_2$:

$$k \leq \mu_2^1(m_2)\alpha V_2 + \mu_1^1(m_2)\alpha V_1. \tag{20}$$

In equilibrium, when $m = m_2$, and the bond is called to force conversion, the right–hand side of (20) will be the market value of the security.

This completes our characterization of the optimal security. We show now that it is an equilibrium for all types of the manager to issue this security at date 0 and that in this equilibrium there will be no dilution. In other words, even though the asymmetry of information is never exactly resolved, the adverse selection problem is *exactly* solved when β is high enough.

Proposition 2 *There exists $\beta^* \in (\frac{1}{2}, 1)$ such that for $\beta > \beta^*$, it is an equilibrium for the manager to invest by issuing a callable convertible security (F, α, k, p, T) with F and α satisfying (14) and (15), k and p satisfying (20) and (19) and with $T = 1$, regardless of his private information θ. In this pooling equilibrium, the manager calls to force conversion only when $m = m_2$ is observed, regardless of θ. The security cannot be called and is not converted when $m = m_1$ is observed. The date 0 expected payoff of the new claim holders is equal to 1 and that for the old shareholders of type θ_i of the firm is equal to $V_i - 1$, the first best value of the firm.*

Proof. In the Appendix. ■

The convertible bond characterized above has payoffs that are non–decreasing in underlying cash flows. Nevertheless, to make the expected value of the security independent of the private information of the manager and prevent dilution, the value of the equity claim must thus be lower than the value of the debt claim. The manager will call the bond to force conversion to equity when good information is disclosed. The better is the initial information of the manager the higher is the chance that this occurs.

Since the value of the equity claim is lower than that of the debt claim, the manager would like to force conversion regardless of the state. But if the manager always forces conversion, the value of the security will fall below the cost of the project and as a result the new claim-holders will not provide the necessary funds for the project. This would hurt the existing shareholders. Thus the manager needs to issue a security with a restrictive call provision, in order to commit to not calling the bond in the low state in date 1. This in turn implies that the manager may not always be able to call and force conversion. The better the initial information of the manager the higher is the chance that good information will arrive in the market raising the stock price and enabling the manager to force conversion.[10] On the other hand, the worse the

[10]The reader is referred to the well–known case of MCI Communications Corp., for a particularly vivid example supporting our results. See Greenwald (1984).

initial information of the manager the greater is the chance that he will be unable to force conversion, so that the new claimholders will be left holding the more valuable debt. The expected value of the claims that the new claimholders will have is however independent of the private information of the manager.

Recall that for $\beta = 1$, financing with short term debt and refinancing at date 1 also implements the same outcome as the optimal callable convertible security. However, for $\beta < 1$, this is no longer true as there is still residual asymmetric information at date 1 so that short–term debt essentially postpones the adverse selection problem to date 1.

4 The General Case with an Endogenous Signal

We now extend the basic model in Section 2 by letting the manager's private information take more than two values. Accordingly, let θ take values in the set $\{\theta_1, ..., \theta_N\}$, $N \geq 2$, with $\Pr[\theta = \theta_i] = \lambda_i$. We assume that (1)—(4) hold for all $i = 1, ..., N$. Let $\overline{V} = \sum_i \lambda_i V_i$ be the ex–ante expected value of the cash flows.

When there are more than two types, one convertible bond with a fixed conversion ratio will not be able to implement the symmetric information outcome for all types. One solution is to allow the manager to issue multiple convertibles with differing face values, conversion ratios and call restrictions. Another approach is to consider floating-price convertibles with conversion ratios that depend on date 1 endogenous variables like the market value of equity or the stock price. We will take the latter approach in this section, in order to demonstrate in closed form the existence of an equilibrium that implements the symmetric information outcome.

In this section we will also provide a simple story to endogenize the date 1 public signal m. We suppose that at date 1, there is an analyst who is either an expert (i.e., informed) with probability γ, or a charlatan (i.e., uninformed) with probability $1 - \gamma$, with $\gamma \in (0, 1]$. The analyst's type is private information and he makes a public announcement $m \in \{m_1, ..., m_N\}$ given his type on date 1, after observing the date 0 decisions of the manager. The message m_i is to be interpreted as a statement by the analyst that the state of the world is θ_i.

We assume that when the analyst is an expert he discloses the true state, i.e., sends message m_i when the state is θ_i. When the analyst is a charlatan, he tries to maintain his reputation for being an expert, i.e., chooses his disclosure strategy in order to maximize the market's posterior probability given the message that he is informed.[11]

As before, we will look for perfect Bayesian equilibria of this game. Let $\mu_i^0(U)$ denote the uninformed analyst's (as well as the market's) date 0 beliefs that the type of the manager is θ_i given that a security $U \in \mathbf{U}$ has been issued by the manager. Let $\sigma_i(U)$ be the probability with which the uninformed analyst sends message m_i at date 1 given that U has been issued at date 0.[12] Let $\mu_i^1(m, U)$ denote the market's date 1 beliefs of the market that $\theta = \theta_i$ given a message m sent by the analyst and given U has been issued at date 0. Let $\nu^1(m, U)$ denote the date 1 beliefs of the market that the analyst is an expert given that the date 0 security is U and that he has sent a message m. Finally, let

$$\overline{V}(m, U) = E[X|m, U] = \sum_{i=1}^{N} \mu_i^1(m, U) V_i \tag{21}$$

be the date 1 expected market value of the total cash flows of the firm given that the analyst's message is m and that the security issued is U.

We will look for a pooling equilibrium where each type of the manager issues the same floating price convertible bond at date 0. Such a security, denoted by $U^* = (F^*, \alpha^*, V^*)$, consists of a face value of the bond F^*, a *vector* of equity shares $\alpha^* = (\alpha_1^*, ..., \alpha_N^*)$ together with a vector $V^* = (V_1^*, ..., V_N^*)$ of cut-off levels for the date 1 market value of the firm. The interpretation is that the security is convertible to α_i^* shares when the date 1 market value of the firm is V_i^*.[13]

In order to state our result, it will be convenient to define

$$\widehat{V} = \left[\sum_{i=1}^{N} \lambda_i \frac{1}{V_i} \right]^{-1} \tag{22}$$

$\widehat{V}$ is the inverse of the average equity shares sold in the symmetric information world. Let

$$\gamma^* = \max[1 - \frac{\widehat{V}}{V_N}, \frac{\widehat{V} - V_1}{V_1(\widehat{V} - 1)}]. \tag{23}$$

[11] The qualitative results that we present do not depend on the analyst being either perfectly informed or perfectly uninformed, or on the precise specification of the uninformed analyst's preferences, as long as γ is high enough.

[12] Note that in the two type model of the previous section, where the public signal was exogenous, we essentially fixed $\sigma_i = \frac{1}{2}$ for all $i = 1, 2$ and set $\beta = \gamma + (1 - \gamma)\frac{1}{2}$.

[13] We can equally let the conversion ratios depend on the date 1 stock price of the firm, instead of the total market value, without affecting anything.

Proposition 3 *For all $\gamma > \gamma^*$ there exists a pooling equilibrium where all types of the manager issue the same floating price convertible $U^* = (F^*, \alpha^*, V^*)$ satisfying*

$$E[\min(F^*, X)|\theta_N] < \min_i\{\alpha_i^* V_i^*\} \tag{24}$$

$$V_i^* = \gamma V_i + (1-\gamma)\overline{V}, \tag{25}$$

and

$$\alpha_i^* = \frac{1}{\gamma}[\frac{1}{V_i} - (1-\gamma)\frac{1}{\widehat{V}}] \in (0,1), \tag{26}$$

for all $i = 1, ..., N$. On the equilibrium path, $\sigma_i(U^*) = \lambda_i$ and $\overline{V}(m_i, U^*) = V_i^*$ for all i. The security is converted to equity regardless of m. The date 0 expected payoff of the new claim holders is equal to 1 and that for the old shareholders of type θ_i of the firm is equal to $V_i - 1$, the first best value of the firm.

Proof. In the Appendix. ■

In the pooling equilibrium neither the market nor the uninformed analyst will infer anything about θ from the date 0 choice of securities. Since the informed analyst always tells the truth, the uninformed analyst, in order to maximize the market's posterior probability of his expertise, announces m_i with probability λ_i, the probability he attaches to the informed analyst sending message m_i. As a result, the market will attach probability γ to the analyst being informed *after* any message m_i and so the market value of the firm $\overline{V}(m_i, U^*)$ will be equal to V_i^* for each m_i. The face value of the debt will be set low enough so that the new claimholders will always convert to equity and will obtain a share α_i^* when the date 1 market value of the firm equals V_i^*.

Given this equilibrium behavior, the conversion ratios α^* will be chosen in such a way that the expected value of the claims sold will equal \$1 regardless of the private information of the manager. Since the manager of type θ_i attaches probability $\gamma + (1-\gamma)\lambda_i$ to the message m_i and a probability $(1-\gamma)\lambda_j$ to a message $m_j \neq m_i$, we must have that α^* solves

$$[\gamma + (1-\gamma)\lambda_i]\alpha_i^* V_i + (1-\gamma)\sum_{j\neq i}\alpha_j^* V_i = 1,$$

or, equivalently,

$$\gamma\alpha_i^* + (1-\gamma)\sum_{j=1}^{N}\alpha_j^* = \frac{1}{V_i} \tag{27}$$

for all $i = 1, ..., N$. Equation (27) has a simple interpretation—the expected equity share sold by type θ_i must equal the share $\frac{1}{V_i}$ that would be sold by this type in the first–best world. The solution to the system (27) is given by (26). When γ is greater than its threshold value γ^*, the solution is admissible, i.e., $\alpha_i^* \in (0, 1)$ for all i. To support the pooling equilibrium, we assume that if any other security is issued at date 0 everyone attaches probability 1 to type θ_1.

Note that

$$\alpha_i^* - \alpha_j^* = \frac{1}{\gamma}[\frac{1}{V_i} - \frac{1}{V_j}] \tag{28}$$

Thus, α_i^* is decreasing in i—the more optimistic is the market the lower is the share sold. Furthermore, it is easily checked that the market value $\alpha_i^* V_i^*$ of the claims sold when $m = m_i$ is also decreasing in i. Intuitively, the higher the type of the manager the greater is the chance that a favorable m will be disclosed in date 1. To keep the expected value of the claims sold constant across manager types, the market value of the claims sold must be decreasing in the date 1 market value of the company.

Note also that for $i > j$ the difference $\alpha_i^* - \alpha_j^*$ (as well as $\alpha_i^* V_i^* - \alpha_j^* V_j^*$) is decreasing in γ. The less the probability that the analyst is informed, the more sensitive must be the (market value of) shares sold to the analyst's message, in order to keep the expected value constant. Finally, since the firm initially has one share outstanding, after the conversion the share price p_i^* will be given by $(1 - \alpha_i^*)V_i^*$ which is increasing in i. The more optimistic is the market at date 1 the higher will be V_i^*, the total value of the firm. Furthermore, the lower will be α_i^* the number of shares sold and so the total number of shares outstanding. For both these reasons the stock price will be higher the more optimistic the market.

In Proposition 3 we provide an example of one security and one equilibrium that implements the symmetric information outcome. That is, the lower bound γ^* is sufficient but not necessary for the existence of such an equilibrium. There might exist other equilibria, possibly involving similar securities, that will also achieve the same outcome. Similarly, there may also exist equilibria that are inefficient. The standard Myers and Majluf equilibrium where all types issue equity (with the higher types suffering dilution) may remain an equilibrium in this model.[14]

[14] So might equilibria involving under–investment. Furthermore, extensions of standard forward induction refinements such as the Intuitive Criterion of Cho and Kreps (1987) will fail to refine the equilibrium set. However, suitable extensions of 'mistaken theory' refinements of the sort proposed by Van Damme (1989) and Hillas (1994) will imply that every equilibrium will implement the symmetric information outcome when γ is high enough. On the other hand, if we allow securities whose payoffs are *directly* contingent on m, then every perfect Bayesian equilibrium must be efficient for γ high enough. In this paper we do not allow such securities

We do not seek to make an argument that the efficient equilibrium will necessarily be played. We simply seek to make the point that even when the initial asymmetry of information is imperfectly resolved, there exist equilibria which *exactly* achieve the symmetric information outcome. As the example below shows, the cutoff value γ^* that we establish may in fact be quite low even with significant initial adverse selection.

4.1 A Numerical Example

We suppose that $N = 3$ so that there are three types (that we will refer to as types 1, 2 and 3, for brevity). The following table provides the rest of the parameters.

Parameters	$\mathbf{V}_i$	$\boldsymbol{\lambda}_i$	$\mathbf{A}_i$
$\mathbf{i = 1}$	1.5	$\frac{1}{3}$	0.48
$\mathbf{i = 2}$	1.6	$\frac{1}{3}$	0.55
$\mathbf{i = 3}$	2	$\frac{1}{3}$	0.7

Note that $\overline{V} = 1.7$. In the symmetric information world, the expected payoffs are equal to .5, .6 and 1 for types 1, 2 and 3 respectively which can be achieved by issuing equity shares equal to 0.67, 0.625 and 0.5 respectively.

If the manager is only allowed to issue equity then, with asymmetric information, there exists an equilibrium where only type 1 invests by issuing equity and types 2 and 3 do not invest. In such an equilibrium, type 1 issues an equity share equal to 0.67. Type 2 does not want to invest by mimicking type 1 as the expected payoff from doing so is $\frac{1}{3} * 1.6 < .55 = A_2$, the expected payoff from foregoing the investment. Similarly, type 3 does not want to invest by mimicking type 1 as the expected payoff from doing so is $\frac{1}{3} * 2 < .7 = A_3$.

Now suppose that the manager is allowed to issue floating price convertibles of the type considered in Proposition 3. For the parameter values chosen, $\widehat{V} = \left[\sum_{i=1}^{3} \lambda_i V_i^{-1}\right]^{-1} = 1.6744$ and so

$$\gamma^* = \max[1 - \frac{\widehat{V}}{V_3}, \frac{\widehat{V} - V_1}{V_1(\widehat{V} - 1)}] = 0.1724$$

or consider refinements.

For γ greater than this cut–off, there exists one equilibrium characterized by Proposition 3, in which all types invest and achieve their symmetric information first best payoff. The following table characterizes the properties of the optimal security for the case $\gamma = 0.25$.[15]

Pooling	$\boldsymbol{\alpha}_i^*$	$\mathbf{V}_i^*$	$\boldsymbol{\alpha}_i^*\mathbf{V}_i^*$	$\mathbf{p}_i^*$
$\mathbf{m} = \mathbf{m}_1$	0.87	1.65	1.44	0.21
$\mathbf{m} = \mathbf{m}_2$	0.71	1.67	1.19	0.48
$\mathbf{m} = \mathbf{m}_3$	0.21	1.77	0.37	1.40

To see that such a security works, consider type 3 of the manager and the probability he assigns to different date 1 scenarios. Since the analyst is informed with a 25% chance and since the uninformed analyst sends each message with equal probability in equilibrium, such a manger assigns a probability of $25 + 75 * \frac{1}{3} = 50\%$ chance to the date 1 message being m_3 and the market value of the company being equal to $V_3^* = 1.77$, with a 25% chance to each of the other two possibilities. Given the security above, the manager then expects to sell an equity share equal to

$$.25 * (0.87) + .25 * (0.71) + .5 * (0.21) = 0.5,$$

equal to the share he would sell in the symmetric information world. As a result, he suffers no dilution and is willing to invest.

Similarly, type 2 of the manager assigns a 50% chance to the date 1 message being m_2 and the market value of the company being equal to $V_2^* = 1.67$, with a 25% chance to each of the other two possibilities. Given the security above, the manager then expects to sell an equity share equal to

$$.25 * (0.87) + .5 * (0.71) + .25 * (0.21) = 0.625,$$

equal to the share he would sell in the symmetric information world. As a result, he suffers no dilution and is willing to invest; and similarly for type 1 of the manager.

From the perspective of the uninformed investors at date 0, they assign a $\frac{1}{3}$ probability to each of the three date 1 scenarios. As a result, the expected value of the claims sold at date 0 equals

$$\frac{1}{3} * 1.44 + \frac{1}{3} * 1.19 + \frac{1}{3} * 0.37 = 1,$$

and they are willing to subscribe to the issue.

[15] All numbers are rounded to the second decimal place.

5 Conclusion

In this paper, we show that when the asymmetry of information is imperfectly resolved over time, commonly used securities such as callable convertible preferred stock or debt can perfectly solve the adverse selection problem. By conditioning call and conversion decisions on the future public resolution of the manager's current private information, such securities make the value of the claim insensitive to the private information of the manager. The manager prefers to force conversion whenever he is able to, but may not be able to force conversion due to the presence of call restrictions. Moreover, complete mitigation of adverse selection can also be achieved by a floating price convertible, even when the future information disclosure is endogenous.

In our model, the manager never obtains additional information over the course of time. However, it is easily seen that as long as the manager's current information is equal to the expected value of his future information, our results are robust to this possibility.

6 Appendix

6.1 Proof of Proposition 2

1. Existence of a Solution to (14) **and** (15)

We will use the Intermediate Value Theorem to demonstrate the existence of a solution to (14) and (15) when β is high enough.

From (15) we can write α in terms of F as

$$\alpha = \frac{1 - (1-\beta)D_2(F)}{\beta V_2} \tag{29}$$

Using this in (14) we obtain that F must satisfy

$$\beta D_1(F) + \frac{1 - (1-\beta)D_2(F)}{\beta V_2}(1-\beta)V_1 = 1 \tag{30}$$

Since $D_2(F) \leq V_2$, the second term left–hand side of (30) is positive if $\beta > 1 - \frac{1}{V_2}$. Since $D_1(F) \leq V_1$, the left–hand side of (30) is strictly greater than 1 if we replace $D_1(F)$ by V_1 in that expression, provided we also have $\beta > \frac{1}{V_1}$.

On the other hand, for $F = 0$, $D_1(F) = D_2(F) = 0$ and so the left–hand side of (30) is equal to $\frac{(1-\beta)V_1}{\beta V_2} < 1$.

Further, $D_i(F)$ is continuous in F for all θ_i. Thus, if $\beta > \max[1 - \frac{1}{V_2}, \frac{1}{V_1}]$, then by the Intermediate Value Theorem, there exists a solution $F > 0$ with $D_1(F) < V_1$ to (30).[16]

For such an F, the solution α to (29) is strictly positive, given $\beta > \max[1 - \frac{1}{V_2}, \frac{1}{V_1}]$. Further, α is less than 1 iff

$$1 < (1 - \beta)D_2(F) + \beta V_2$$

But this follows from the fact that F solves (30) so that

$$(1 - \beta)D_2(F) + \beta\alpha V_2 = 1.$$

This shows that for $\beta > \max[1 - \frac{1}{V_2}, \frac{1}{V_1}]$ there exists a solution $\alpha \in (0, 1)$ and $F > 0$ with $D_1(F) < V_1$ to (14) and (15).

2. Existence of a pooling equilibrium

To show that pooling with such a security is indeed an equilibrium, we proceed backwards in time.

Date 1, $m = m_1$

In this case if the market conjectures that the bond will not be converted, then the market value of the security will be given by the right–hand side of (18) for the case $m = m_1$, so that the share price will be given by the left–hand side of (19). As a result the manager will not be able to call the bond and so, by (18) it will not be converted.

We also allow the manager to buy back the security in the market by issuing some other security. For any security U that is issued to buy back the debt, we assume that the market puts probability 1 on the type θ_i for whom $D_i(F) - E[U|\theta_i]$ is the maximum. Given such beliefs, it is straightforward to check that both types of the manager will either not want to issue such a security to buy back the existing claims, or will not be able to do so.

Date 1, $m = m_2$

In this case if the market conjectures that the bond will be converted, then the market value of the security will be given by the left–hand side of (18) for the case $m = m_2$, so that the share price will be given by the right–hand side of (19). From (16) and (20), the manager will call to force conversion regardless of his private information and investors will convert when the security is called.

[16] One can also show the monotonicity of the left–hand side side of (A2) in F demontrating the uniqueness of the solution.

If instead the manager tries to buy back the security and issue other claims U then, as above, the market attaches beliefs putting probability 1 on the type θ_i for whom $D_i(F) - E[U|\theta_i]$ is the maximum. No type of the manager will find such a deviation profitable.

Date 0

At date 0, given the call and conversion decisions of date 1 above, the market will value the security at

$$\lambda_1[(1-\beta)D_1(F) + \beta\alpha V_1] + \lambda_2[\beta D_2(F) + (1-\beta)\alpha V_2].$$

The first–term in square brackets is the expected date 1 value of the security given $\theta = \theta_1$ and given the date 1 decisions of all parties. The second term in square brackets has the same interpretation, but for $\theta = \theta_2$. From (14) and (15) the market value of the security at date 1 will equal 1 dollar, the required outlay for the project.

As a result, the manager, regardless of his private information will be able to raise the required funds. The date 0 expected payoff of the existing shareholders will thus be equal to $V_i - 1 > A_i$ for each θ_i. Consequently, the manager will find it profitable to invest.

We suppose that at date 0, if any type of the manager deviates by issuing some other security then the market puts probability 1 on type $\theta = \theta_1$. As a result, no type of the manager will find such a deviation profitable.■

6.2 Proof of Proposition 3

We begin our construction of the pooling equilibrium by considering the strategy of the uninformed analyst at date 1, on the equilibrium path. Since all types of the manager pool by issuing the same convertible U^*, neither the analyst nor the market learns nothing about θ from the date 0 financing decision. As a result, $\mu_i^0(U^*) = \lambda_i$ for all i. Since the informed analyst discloses the truth, this implies that the posterior probability that the market attaches to the analyst being informed after a message m_i is

$$\nu^1(m_i, U^*) = \frac{\lambda_i\gamma}{\lambda_i\gamma + \sigma_i(U^*)(1-\gamma)} \tag{31}$$

Since the uninformed analyst wants to maximize the posterior probability that he is informed, it follows that

$$\sigma_i(U^*) = \lambda_i \text{ and } \nu^1(m_i, U^*) = \gamma \text{ for all } i = 1, ..., N, \tag{32}$$

in equilibrium. To see this, note first that $\nu^1(m_i, U^*)$ cannot vary across messages m_i— if there exist messages m_i and m_j such that $\nu^1(m_i, U^*) > \nu^1(m_j, U^*)$, the uninformed analyst will strictly prefer to send message m_i (i.e., $\sigma_i(U^*) = 1$) implying that $\nu^1(m_i, U^*) = \frac{\lambda_i\gamma}{\lambda_i\gamma+(1-\gamma)} < 1 = \nu^1(m_j, U^*)$, a contradiction. So, we must have $\nu^1(m_i, U^*) = k$ for some constant $k \in [0,1]$ for all $i = 1, ..., N$. From (31) we then obtain

$$\sigma_i(U^*)(1-\gamma)k = \lambda_i\gamma(1-k)$$

for all i. Since $\sum_i \sigma_i(U^*) = 1$, it follows that $k = \gamma$ and $\sigma_i(U^*) = \lambda_i$ for all i.

Having established the equilibrium behavior of the uninformed analyst, we now turn to the date 1 market value of the firm $\overline{V}(m_i, U^*)$ after a message m_i. Note that

$$\mu_i^1(m, U^*) = \begin{cases} \gamma + (1-\gamma)\lambda_i & \text{if } m = m_i \\ (1-\gamma)\lambda_i & \text{otherwise} \end{cases} \tag{33}$$

Thus,

$$\overline{V}(m_i, U^*) = \gamma V_i + (1-\gamma)\overline{V} \tag{34}$$

Since $\overline{V}(m_i, U^*) = V_i^*$ for all i, the security U^* entitles the new shareholders to convert to α_i^* shares when the market value of the security is $\overline{V}(m_i, U^*)$. Clearly, for F^* low enough (e.g., satisfying (24)), the new claimholders will always be willing to convert to equity.

Next, we turn to the choice of the equity shares α^*. Since $\sigma_i(U^*) = \lambda_i$ for all i, type θ_i of the manager knows that the analyst's message will be m_i with probability $\gamma + (1-\gamma)\lambda_i$ and will be equal to m_j with probability $(1-\gamma)\lambda_j$ for $j \neq i$. We want to choose α^* such that the expected value of the claims sold in equilibrium is equal to the outlay of 1, for each type of the manager. That is, α_i^* must solve:

$$[\gamma + (1-\gamma)\lambda_i]\alpha_i^* V_i + (1-\gamma)\sum_{j\neq i}\lambda_j\alpha_j^* V_i = 1,$$

for all $i = 1, ..., N$. Rewriting we obtain,

$$\gamma\alpha_i^* + (1-\gamma)\sum_{j=1}^{N}\lambda_j\alpha_j^* = \frac{1}{V_i}, \tag{35}$$

for all $i = 1, ..., N$. Multiplying by λ_i and summing over i we obtain

$$\sum_{j=1}^{N}\lambda_j\alpha_j^* = \frac{1}{\widehat{V}} \tag{36}$$

Using (36) in (35) we obtain (26). It is easy to check that if $\gamma > \max[1 - \frac{\widehat{V}}{V_N}, \frac{\widehat{V}-V_1}{V_1(\widehat{V}-1)}]$ then $\alpha_i^* \in (0,1)$ for all i.

Given the equilibrium behavior derived above, the date 0 expected value of the claims sold by type θ_i of the manager is seen to be equal to 1, by construction. Thus, the date 0 market value of the security will also equal 1 and the expected payoff to the old claimholders will equal $V_i - 1 > A_i$ for all $i = 1, ..., N$. This implies that no type of the manager will prefer to under–invest.

Note that the manager is allowed to buy back the security U^* in the market by issuing some other security after a message m_i. For any security U that is issued to buy back the convertible, we assume that the market puts probability 1 on the type θ_j for whom $\alpha_i^* V_j - E[U|\theta_j]$ is the maximum. Given such beliefs, it is straightforward to check that all types of the manager will either not want to issue such a security to buy back the existing claims, or will not be able to do so.

It remains to check that no type of the manager will want to deviate at date 0 by issuing a different security U'. As with U^*, we will allow the payoff from such a security to depend on the date 1 market value of the security given m. Let $U'_{\overline{V}(m,U')} \in \mathbf{U}$ be the actual security sold when the date 1 market value is $\overline{V}(m, U')$ after a message m.

We suppose that if any such security $\{U'_{\overline{V}(m,U')}\}_m$ is issued by any type of the manager then the market attaches probability 1 to type θ_1, i.e., $\mu_1^0(U') = 1$. It follows that $\mu_1^1(m, U') = 1$ for all m so that $\overline{V}(m, U') = V_1$ for all m. Thus, the date 0 market value of such a security will be equal to $E[U'_{V_1}(X)|\theta_1]$.

If type θ_i of the manager deviates by issuing such a security then he will raise the required outlay and invest only if $E[U'_{V_1}(X)|\theta_1] \geq 1$. In such a case, he distributes the excess cash (if any) as dividends at date 0. If he fails to raise the required amount he invests the proceeds in a riskless asset. The expected value of the claims he sells, given his private information, is $E[U'_{V_1}(X)|\theta_i]$. Thus, the expected payoff from the deviation is equal to

$$V_i - 1 + E[U'_{V_1}(X)|\theta_1] - E[U'_{V_1}(X)|\theta_i]$$

if $E[U'_{V_1}(X)|\theta_1] \geq 1$ and equal to

$$A_i + E[U'_{V_1}(X)|\theta_1] - E[U'_{V_1}(X)|\theta_i]$$

otherwise. Since $U'_{V_1}(X) \in \mathbf{U}$, it follows that $E[U'_{V_1}(X)|\theta_1] \leq E[U'_{V_1}(X)|\theta_i]$, so that no such deviation is profitable.[17] ■

[17]The off–the–equilibrium–path beliefs that we specify in this construction (as well as that in the proof of

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Proposition 2) are the unique supporting beliefs. These beliefs satisfy the (suitably extended) forward induction notion of Divinity due to Banks and Sobel (1987) and so, the well–known Intuitive Criterion. See also Cho and Kreps (1987).

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