Selection Tournaments, Sabotage, and Participation

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Abstract

This paper studies sabotage in tournaments with at least three contestants, where the contestants know each other well. Here, every contestant has an incentive to direct sabotage specifically against his most dangerous rival. In equilibrium, contestants that choose higher productive effort are sabotaged more heavily. This might explain findings from psychology, where victims of mobbing are sometimes found to be overachieving. Further, sabotage equalises promotion chances: in an interior equilibrium it is a matter of chance who will win, even when contestants differ a lot in their abilities. This, in turn, has adverse consequences on who might want to participate in a tournament. Since better contestants anticipate that they will be sabotaged more heavily, it may happen that the most able stay out and the tournament selects one of the least able with probability one. Several extensions are studied, for example, easy victims, different prize structures, and handicaps. Further, implications for the optimal design of tournaments and, more generally, career tracks in organisations, are considered.

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1 Introduction

Labor market tournaments have the double role of selecting the most able individuals and supplying incentives. Although many economists have voiced the opinion that the selection aspect is at least as important as the incentives aspect (e.g. Rosen 1986, Schlicht 1988, Glazer and Hassin 1988), the focus of the bulk of research has clearly been on the latter. This paper explicitly addresses the selection aspect. The question is whether tournaments, and more generally relative comparison contests, tend to select the most able individuals. Since there is usually an irreducible component of luck in winning a tournament, the relevant question is not whether the most able will always win, but rather whether they have the greatest chances of winning.

In most tournament models, it is clearly the case that more able contestants have a greater chance of winning.[1] But the picture changes radically once we take into account that tournaments - like other relative comparison contests - give each contestant an incentive to sabotage his rivals (Lazear 1989). Here "sabotage" is a catchall term for different kinds of activities that are intended to hinder the productive efforts of other contestants. These range from strategic withholding of information, less mutual help, outright forms of mobbing, and actual physical sabotage.

There is one obvious problem in using a tournament for selection in the presence of sabotage. The result might be the promotion of the best saboteur who might be not very good at working productively - and promoting the best saboteur is not necessarily in the interest of the firm. This is a particularly striking example of the more general point that ability is a multidimensional property, and that the abilities and personality traits needed to win a tournament are not always the same as those needed at a higher level in a hierarchy.

This paper focusses on a more subtle point. It starts with two observations. In many real world tournaments, there are more than two contestants who compete for a single prize. And the contestants often know each other well, especially if they work together closely and regularly. Then each contestant knows who is his most dangerous rival. Intuitively, sabotaging a strong rival improves one's own chance of winning more than sabotaging a weaker rival does. Therefore, each contestant has an incentive to sabotage this rival

[1] Lazear and Rosen (1981), Nalebuff and Stiglitz (1983), Green and Stocky (1983), Rosen (1986).

most fiercely.

I show, in a tournament model with linear production functions which is very close to the classic model of Lazear and Rosen (1981), that the result of this effect is an equalization of winning probabilities. In an interior equilibrium of a tournament between at least three contestants, each contestant has the same chance of winning. In other words, who will win the tournament is a matter of pure chance, even if some contestants are much more able than other contestants.

In fact, the selection properties of tournaments may even be much worse. Since the most able individuals are sabotaged most, they may well have a lower expected utility from participating in the tournament. Once we take into account that participation in a tournament is endogenous, it turns out that only the least able individuals may want to participate. In that case, a tournament selects one of the least able with probability one.

Having derived these results, I go on to consider some extensions. As a robustness check, I consider risk aversion and more general production and cost functions. An especially interesting case is when some contestants are "easy victims" (easier to sabotage than others). I show that easy victims are sabotaged more heavily and have lower chances of winning the tournament.

Another set of extensions concerns the tournament design. I study a different prize structure, when there are $n-1$ equal winner prizes, and only the contestant with the lowest output gets loser prize which is strictly smaller. Here, the incentives to sabotage are very different than in the case of only one winner prize. In any pure strategy equilibrium, every contestant will sabotage only one of his rivals. Moreover, there is one contestant ("the victim") who is sabotaged by all his rivals, and has the least chance of getting one of the prizes. Obviously, if the contestants have identical cost functions, anyone might be the victim. By continuity, even if the contestants differ in their abilities, one of the more able ones might turn out to be the victim. Therefore, there is no guarantee that on average better contestants will be selected by this kind of tournament.

I also consider competitive handicaps (as proposed by Lazear and Rosen 1981). I show that contestants who benefit from the handicap are sabotaged more heavily, whereas the contestants who are disadvantaged by the handicap are sabotaged less. In an interior equilibrium, it is still a matter of pure chance who wins. However, now existence of interior equilibria depends on the size of the handicaps as well as on the differences in ability.

Some evidence on sabotage

Since sabotage is usually illegal, it is difficult to test the importance of sabotage empirically. Nevertheless, there is considerable evidence for the importance of sabotage. Some anecdotal evidence is given by the following quotation (cited from Murphy 1992, fn. 4):

> John Dvorak, PC Magazine editor (September 27, 1988) explains how a friend received his promotion: "He managed to crack the network messaging system so that he could monitor all the memos. He also sabotaged the workgroup software and set back the careers of a few computer naive souls who didn't realize that someone was manipulating their appointment calendars. They would miss important meetings and be sent on wild-goose chases, only to look like complete buffoons when they showed up for appointments that were never made."

More importantly, Drago and Garvey (1998) use survey data to test the basic model of sabotage in tournaments due to Lazear (1989)). They find that when promotion incentives are strong, contestants tend to help each other less, corroborating Lazear's model.[2] Recent evidence from experimental economics also underlines the importance of sabotage in tournaments. Harbring and Irlenbusch (2002, 2003) find that the contestants tend to sabotage each other even more than one would expect on the basis of the game theoretic analysis. However, none of these papers directly addresses the question who will be sabotaged most fiercely.

There is a huge literature from psychology on bullying and mobbing in the workplace (see Einarsen et al. (eds.) (2003) for a survey). It is generally agreed that bullying is a multifaceted fact. Part of the behavior under investigation seems to be related to sabotage in tournaments. For example, Zapf and Einarsen (2003, 172) write that "Bullying due to micropolitical behavior indicates harassment of another person in order to protect or improve one's own position in the organization." Vartia (1996) finds that competition for tasks and advancements and competition for the superior's favour and approval belong to the most often perceived reasons for bullying - which fits nicely to the tournament literature. See also and Björkvist et al. (1994) who

[2] There are also papers using data from sports that give some empirical support to Lazear's model, see Becker and Huselid (1992) (auto racing) and Garicano and Palaciios-Huerta (2000) (European soccer).

find that experienced reasons to harassment were predominantly envy and competition about jobs and status.

In some studies, victims of bullying in the workplace are found to be "overachieving": more achievement oriented, punctual, accurate and conscientious than the control group (Zapf and Einarsen 2003, 178). While this is often explained with regard to group norms, the present paper offers another explanation. As I show below, people that choose higher productive effort are sabotaged more heavily. The reason is that they are more dangerous rivals in a contest for promotion.

Another finding is that victims of bullying tend to be more vulnerable than the control groups, e.g. "low in social competencies, bad conflict managers, unassertive and weak personalities" (Zapf and Einarsen 2003, 174ff). I capture this in a stylized way by considering the role of easy victims.

Related literature

The incentives to sabotage were pointed out early in the tournament literature (Nalebuff and Stiglitz 1983, p. 40). The present paper is most closely related to Lazear (1989) and Chen (2003). Lazear considered the optimal tournament reward structure from the incentives aspect and showed that, in the presence of sabotage, the optimal prize structure is compressed. However, Lazear does not discuss the possibility of directing sabotage specifically against stronger rivals and the implications of this for the selection properties. Chen (2003) also studies sabotage in selection tournaments. However, he is mostly concerned with the fact that some contestants may have a comparative advantage in sabotaging. My paper complements Chen's in several ways. Since Chen only considers the case of decreasing returns to sabotage, he does not get the result that winning probabilities are equalized. Moreover, Chen does not consider the participation decision, does not discuss easy victims, and considers only the case of a single prize. On a more technical level, Chen's analysis assumes the existence of interior equilibria. An additional contribution of my paper is that I derive necessary and sufficient conditions for the existence of interior equilibria for the case of a quadratic cost function. Other papers that study sabotage include Drago and Turnbull (1991) (who study how bargaining between workers about effort and mutual help affects optimal incentive schemes), Chan (1996) (who studies external recruitment as a means of keeping sabotage incentives low) and Kräkel (2000) (who considers the effect of relative deprivation in tournaments with sabotage).

Sabotage-like activities have been studied in other contexts as well. Shubik's (1954) model of a "truel" (three person duel) is closely related to the present paper. Here the "truelist" with the lowest shooting ability may have the best chances of survival. The reason is that the contestants have an incentive to shoot at the truelist who is the best shot. Baumol (1992) considers sabotage in the process of innovation. Skaperdas and Grofman (1995) and Hess and Harrington (1996) model negative campaigning in election races. Konrad (2000) studies sabotage in rent seeking contests. Auriol et al. (2002) show that, when the principal cannot commit to long term contracts, career concerns in teams give the agents incentives to sabotage, even if they are not involved in a tournament scheme. The results of the present paper are also relevant to these other contests.

In addition,the paper also contributes to the small but growing literature on the selection properties of tournaments and other kinds of contests. One important paper in this literature is Rosen (1986), who studied a sequential elimination tournament. Meyer (1991) works out how to design a repeated contest between the same contestants in order to get the most information about the contestants. Clark and Riis (2001) study a selection tournament in the case where performance deterministic. They show that by making the winner prize depend on which of two test standards are passed, the tournament can be designed to select the most able contestant as a winner. However, they do not consider sabotage. Hvide and Kristiansen (2003) consider risk taking in a selection contest. This literature contrasts with the statistical theory of selection (e.g., Gibbons, Olkin and Sobel 1977) in that equilibrium effects are important and lead to new, and often surprising, conclusions.

The paper proceeds as follows. Section 2 sets out the model. Section 3 studies how sabotage equalizes promotion chances. Section 4 considers the decision whether to participate in a tournament. Section 5 discusses extensions. Section 6 considers implications for the design of tournaments, and section 7 concludes.

2 The model

There are n contestants. For simplicity, the contestants are assumed to be risk neutral but this is not crucial. Contestant i chooses his productive effort x_i and his sabotage efforts s_{i1}, ..., $s_{i(i-1)}$, $s_{i(i+1)}$, ..., s_{in}, where s_{ij} denotes the sabotage of contestant i against contestant j. He has a personal cost of doing so which is given by

$$c_i\left(x_i, \sum_{j\neq i} s_{ij}\right), \tag{1}$$

where $c_i : R^2 \to R$ is is a twice differentiable function that is increasing in both arguments and convex. Note that the function c_i can be different for each contestant i.

The output produced by contestant i is given by[3]

$$q_i = x_i - \sum_{j\neq i} s_{ji} + \varepsilon_i. \tag{2}$$

Here ε_i is an error term. The ε_i are identically and independently distributed with PDF f. Let F denote the CDF corresponding to f. I assume that f has full support and is strictly log-concave.[4]

The cost functions are known to all the contestants. This simplifying assumption captures the idea that work colleagues often know each other pretty well, while their superiors know considerably less about them.

The contestant with the highest output gets a winner prize w which represents the monetary equivalent of a promotion. All the other contestants get a strictly lower loser prize which is normalized to zero. Let p_i denote contestant i's probability of winning. Then his payoff is

$$u_i = p_i w - c_i\left(x_i, \sum_{j\neq i} s_{i,j}\right).$$

Contestant i maximizes u_i subject to the non-negativity constraints $x_i \geq 0$ and $s_{ij} \geq 0$ for all $j \neq i$.

[3]One could also model the heterogeneity of the contestants by using alternative production functions. For example, one could assume $q_i = \beta_i x_i - \gamma_i \sum_{j\neq i} s_{ji} + \varepsilon_i$, where β_i and γ_i are player i's ability in working and sabotaging, respectively, or the additive specification $q_i = \beta_i + x_i - \sum_{j\neq i} s_{ji} + \varepsilon_i$. This would not affect the main results of the paper.

[4]The assumption of log-concavity is fulfilled by most commonly studied distribution functions, see Bagnoli and Bergstrom 1989. Log-concavity of the PDF implies unimodality of the PDF; in fact, log-concavity is equivalent to strong unimodality, see Ibragimov (1956) and An (1998). It will become clear in the proofs of lemmas 1 and 2, that the assumption that f is everywhere log-concave is sufficient, but strictly speaking not necessary, for the results of the paper.

Let us briefly review the case two contestants. Contestant 1 wins if

$$x_1 + s_{12} - x_2 - s_{21} > \varepsilon_2 - \varepsilon_1.$$

Let G denote the CDF of the difference $\varepsilon_2 - \varepsilon_1$. Then

$$p_1 = G\left(x_1 + s_{12} - x_2 - s_{21}\right)$$

and $p_2 = 1 - p_1$. Hence the marginal benefit of working and sabotaging are identically the same for the two contestants:

$$\frac{\partial p_1}{\partial x_1} = \frac{\partial p_1}{\partial s_{12}} = \frac{\partial p_2}{\partial x_2} = \frac{\partial p_2}{\partial s_{21}} \text{ for all } x_1, x_2, s_{12}, s_{21}.$$

In equilibrium, a contestant that has both lower marginal cost of working and lower marginal cost of sabotaging will work harder and sabotage more. Consequently, he will have better chances of winning the tournament. Of course, this does not mean that someone who is better at working productively will win more often - his rival might be much better at sabotaging. Still, if these abilities are positively correlated, the tournament can be used to select better contestants.

This is not true if $n \geq 3$. This case, on which I will focus for the rest of the paper, is radically different, due to the fact that each contestant has several rivals and can choose which one to sabotage most fiercely.

To analyze the case $n \geq 3$, define

$$y_{ij} := x_i - \sum_{k \neq i} s_{ki} - \left(x_j - \sum_{k \neq j} s_{kj}\right).$$

Note that $y_{ij} = E\left(q_i\right) - E\left(q_j\right)$. Hence y_{ij} can be interpreted as the deterministic headstart of contestant i against contestant j. Using this notation, we have

$$p_i = \int_{-\infty}^{\infty} \left[\Pi_{j \neq i} F\left(y_{ij} + \varepsilon_i\right)\right] f\left(\varepsilon_i\right) d\varepsilon_i. \tag{3}$$

The following lemma makes precise the intuition that sabotaging a strong rival improves one's own chance of winning more than sabotaging a weaker rival does.

Lemma 1 *For all values of the decision variables, the following inequalities are equivalent:*
a) $x_j - \sum_{l \neq j} s_{lj} > x_k - \sum_{l \neq k} s_{lk}$
b) $p_j > p_k$
c) $\frac{\partial p_i}{\partial s_{ij}} > \frac{\partial p_i}{\partial s_{ik}}$

Proof. See appendix. ■

The main intuition behind lemma 1 is as follows. By sabotaging j, contestant i increases the probability that he will win against j. But winning against j is beneficial for i only if contestant i simultaneously wins against all other contestants, including k. But it is more likely that i wins against k when $x_k - \sum_{l \neq k} s_{lk}$ is small. The assumption that $f(z)$ is everywhere log-concave is sufficient, but not necessary, for this intuition to carry over to the formal model.[5]

3 Sabotage equalizes promotion chances

Lemma 1 has interesting implications for the question of whether the more able contestants have greater chances of winning in equilibrium. Define an interior equilibrium as a pure strategy equilibrium[6] in which every contestant sabotages all his rivals. Now we can state the first main result of the paper.

Proposition 1 *In every interior equilibrium, every contestant* $i = 1, ..., n$ *wins with the same probability* $p_i = \frac{1}{n}$.

Proof. Assume that in a pure strategy equilibrium i sabotages all his rivals. If $x_j - \sum_{l \neq j} s_{lj} > x_k - \sum_{l \neq k} s_{lk}$, it follows from lemma 1 that $\frac{\partial p_i}{\partial s_{ij}} > \frac{\partial p_i}{\partial s_{ik}}$. Now i can decrease s_{ik} by a small amount and, at the same time, increase s_{ij} by the same amount. His cost is unchanged, but his probability of winning is higher than before, so the initial situation cannot have been an equilibrium. Therefore, we must have $x_j - \sum_{l \neq j} s_{lj} = x_k - \sum_{l \neq k} s_{lk}$ for all $j, k \neq i$ in an equilibrium where i sabotages all his rivals.

[5] In fact, the weaker assumption that $F(z)$ is log concave would be sufficient. (If the PDF f is log-concave, so is the CDF F, but not vice versa - see An (1998).) Furthermore, $F(z)$ doesn't have to be log-concave everywhere.

[6] Henceforth, I sometimes write "equilibrium" for "pure strategy equilibrium". I do not consider mixed strategy equilibria in this paper.

If all contestants sabotage all their rivals, then it follows that

$$x_j - \sum_{l \neq j} s_{lj} = x_k - \sum_{l \neq k} s_{lk} \text{ for all contestants } j, k \tag{4}$$

and hence $p_1 = ... = p_n = \frac{1}{n}$. ■

Proposition 1 says that, in an interior equilibrium, who will win the tournament is a matter of pure chance. The intuition behind the proposition is simple: If (say) contestant 1 had a higher probability of winning than contestant 2, than it would be better for contestant 3 to increase s_{31} by a small amount and, at the same time, decrease s_{32} by the same amount. By lemma 1, this would increase his chance of getting the promotion without changing his costs.

As the following proposition shows, "overachievers" - contestants that choose higher productive effort - are sabotaged more heavily. As mentioned in the introduction, this is in line with some recent results from psychology on mobbing.

Proposition 2 *In every interior equilibrium, contestants that choose a higher productive effort are sabotaged more heavily.*

Proof. From equation (4) it follows immediately that in every interior equilibrium $x_j - x_k = \sum_{l \neq j} s_{lj} - \sum_{l \neq k} s_{lk}$ holds for all contestants j, k. Therefore, if $x_j > x_k$, we have $\sum_{l \neq j} s_{lj} > \sum_{l \neq k} s_{lk}$. ■

One can strengthen propositions 1 and 2, in that we do not have to restrict attention to interior equilibria where literally *all* contestants sabotage *all* their rivals. I show in the appendix that if at least one of the following conditions holds in an equilibrium, then $p_i = \frac{1}{n}$ for all $i = 1, ..., n$ in this equilibrium:

1. There are at least two contestants who sabotage all their rivals.

2. Each contestant is sabotaged by at least two rivals.

3. The contestants can be renumbered so that $s_{i(i+1)} > 0$ for $i = 1, .., n-1$ and $s_{n1} > 0$.

On the other hand, existence of equilibria with equal promotion chances is not automatically ensured. There can be two types of corner solutions. Firstly, there might be no sabotage at all in equilibrium. This is especially likely when the marginal cost of the first unit of sabotage is high, and if the number of contestants is high (see Konrad (2000)). The reason is that sabotage involves a positive externality to all the contestants except the one who is sabotaged. This externality is more important when there are many contestants, and sabotage is therefore less attractive. Since the focus of this paper is on tournaments where sabotage plays a role, I will assume that $\frac{\partial c_i(x_i,0)}{\partial s_{ij}} = 0$ holds for all $x_i \geq 0$ and all contestants i, j $(i \neq j)$. This ensures that there is some sabotage in equilibrium.

However, there can still be corner solutions of a second type. For example, if there is one contestant ("she") who is much better than all her rivals, she will have a higher chance of winning in the equilibrium even though only she is sabotaged by all the other contestants. In such a situation, it doesn't pay for the other contestants to sabotage anyone except her, so they direct all sabotage against her. Intuitively, one would expect corner solutions of this type if the contestants are very different. Given the complexity of the problem, it is very difficult to derive general conditions for existence of interior equilibria. However, some important lessons can be learned by considering the following example.

Example 1 *The cost functions are given by*

$$c_i\left(x_i, \sum_{k\neq i} s_{ik}\right) = \frac{\gamma_i}{2}\left(x_i^2 + \left(\sum_{k\neq i} s_{ik}\right)^2\right).$$

There are two types of contestants: l low cost contestants with $\gamma_i = 1$, and $h = n - l$ high cost contestants with $\gamma_i = \gamma > 1$.

In this example, the contestants differ only in one parameter. This parameter γ is a natural way to measure how different the contestants are, something which is considerably more complicated to do in the model with general cost functions. Higher values of γ imply greater differences between contestants. Further, there will always be some sabotage in equilibrium, since the first unit of sabotage has zero marginal cost.

Proposition 3 *Consider example 1.*
a) A necessary condition for the existence of a pure strategy equilibrium with

$p_i = \frac{1}{n}$ *is*

$$\gamma \leq \begin{cases} 1 + \frac{n}{l(n-2)}, & \text{if } l \geq 2, \\ 1 + \frac{n(n-2)}{n^2-2n+2}, & \text{if } l = 1. \end{cases} \tag{5}$$

b) If, in addition, the inequality

$$\max_z \left(\frac{\partial^2}{\partial z^2} \int_{-\infty}^{\infty} F(z+\varepsilon)^{n-1} f(\varepsilon)\, d\varepsilon \right) < \frac{(n-1)^2}{n^2 - 2n + 2} \frac{1}{w} \tag{6}$$

holds, then existence of interior equilibria is ensured.

Proof. See appendix. ■

Inequality (6) serves to rule out problems related to possible non - concavities of the objective function.[7] It ensures that the objective functions are concave enough. To give an example, if the error terms follow a Gumbel distribution $F(\varepsilon) = \exp\left(-\exp\left(-\frac{\varepsilon}{\sigma}\right)\right)$, it can be shown that inequality (6) holds if the variance of the error terms is high enough.

While proposition 3 confirms the intuition that there will not be an interior equilibrium if the contestants are very different, it also shows that contestants can differ substantially and nevertheless have the same chance of winning in equilibrium. For example, if there are two low cost and one high cost contestant, then interior equilibria exist if $\gamma \leq \frac{5}{2}$. The high cost contestant can have a cost which is more than twice the cost of the low cost contestants, and still have the same chance of winning in the equilibrium! The point that interior equilibria exist even if the contestants are quite different in their abilities can be expected to carry over to more general cost functions.

The equilibrium is not unique. In fact, there is a continuum of interior equilibria, where only the total amount of sabotage that contestant $i = 1, ..., n$ chooses ($\sum_{j \neq i} s_{ij}$), and the total amount of sabotage that contestant i suffers ($\sum_{j \neq i} s_{ji}$) is determined. This can be illustrated as follows. Suppose that every contestant $i = 1, ..., n-1$ sabotages contestant $i+1$ one unit more, and contestant n sabotages 1 more. In addition, $i = 2, ..., n$ sabotages $i-1$ one unit less and contestant 1 sabotages contestant n less. Then the total

[7] This problem is common in tournament models. See, among others, Lazear and Rosen (1981), p. 845 fn. 2; Nalebuff and Stiglitz (1983), p. 29; Lazear (1989), p. 565 fn. 3; Kräkel (2000), p. 398 fn. 17; McLaughlin (1988), p. 236 and p. 241.

amount of sabotage against any contestant is unchanged, and so are all the marginal benefits of working and sabotaging. Further, the total amount of sabotage chosen by a contestant is the same as before, and so are the marginal costs. Therefore, if the situation before was an equilibrium, then the new situation is an equilibrium, too. Basically, the game is a coordination game where there are many ways to coordinate.

Proposition 3a implies that if the number of contestants is large, corner solutions are more likely. This is as it should be expected. With many contestants sabotage is less attractive, and therefore plays a less important role. So the range of the parameter γ for which sabotage completely equalizes promotion chances gets smaller.

Proposition 3a also shows that the case of a single low cost contestant ($l = 1$) is different from the other cases ($l \geq 2$). The reason for this is as follows. If $l \geq 2$, and the contestants are very different, then there will be a corner solution where *no one* will sabotage a high cost contestant. On the other hand, if $l = 1$, the low cost contestant will always sabotage high cost contestants, because he has no other rivals. Here, in a corner solutions all the high cost contestants sabotage only the single low cost contestant.

4 Participation

An individual will participate in the tournament only if his utility from participating exceeds his reservation utility. Once we take this into account, it turns out that only less productive individuals may want to participate in the tournament. I illustrate this point with the quadratic cost function example.

Assume that the conditions given in proposition 3 are fulfilled. In an interior equilibrium the following first order conditions have to hold for all $i = 1, ..., n$:

$$w\frac{\partial p_i}{\partial x_i} = \gamma_i x_i, \tag{7}$$

$$w\frac{\partial p_i}{\partial s_{ij}} = \gamma_i \sum_{j \neq i} s_{ij} \text{ for all } j \neq i. \tag{8}$$

By proposition 1, we have $p_i = \frac{1}{n}$ for all contestants $i = 1, ..., n$. Therefore, $y_{ij} = 0$ for all $i, j = 1, ..., n$, and equations (7) and (8) reduce to

$$
\begin{aligned}
x_i &= \frac{wg}{\gamma_i}, \\
\sum_{k\neq i} s_{ik} &= \frac{wg}{\gamma_i\,(n-1)},
\end{aligned} \tag{9}
$$

where

$$
g := \frac{\partial p_i}{\partial x_i}\,|_{y_{ij}=0\forall j\neq i} = (n-1)\int_{-\infty}^{\infty} F\left(\varepsilon\right)^{n-2} f\left(\varepsilon\right)^2 d\varepsilon.
$$

The more productive contestants will work harder (choose higher x_i), and sabotage more (choose higher $\sum_{j\neq i} s_{ij}$), than the less productive ones. But they are also the victims of more sabotage, and therefore do not have a higher chance of winning. Utility in equilibrium is given by

$$
\frac{w}{n} - \frac{\gamma_i}{2}\left(\left(\frac{wg}{\gamma_i}\right)^2 + \left(\frac{wg}{\gamma_i\,(n-1)}\right)^2\right) =: u^*\left(\gamma_i\right).
$$

Note that $u^*\left(\gamma_i\right)$ *increases* in γ_i. That is, the contestants with the higher costs have a strictly *higher* utility than the more productive contestants. Further, in an interior equilibrium the utility of i does not depend on the type of his rivals.

Even if all potential contestants have the same reservation utility $\bar{u}$, it can happen that only the less productive ones will participate. This will happen whenever the outside utility is greater than the utility of a low cost type but less than the utility of a high cost type. To be more precise, consider the following two stage model. In the first stage potential contestants observe w and n and then decide whether they want to participate. For those who do not want to participate, the game ends and they get the outside utility $\bar{u}$. If fewer than n contestants want to participate, the game is over and everyone gets $\bar{u}$. If exactly n contestants want to participate, they play the tournament in stage 2. Finally, if more than n want to participate, n of them are chosen randomly and play the tournament, while the remaining get their outside utility $\bar{u}$.

Proposition 4 *If participation in a selection tournament is endogenous, there is a non-empty set of parameters such that the individuals with the highest abilities stay out, and the tournament selects one of the least able individuals with certainty.*

Proof. Consider example 1 and assume that the conditions given in proposition 2 hold. Then a non-empty interval $(u^*(1), u^*(\gamma))$ exists such that if $\bar{u} \in (u^*(1), u^*(\gamma))$, only contestants with high costs will participate in the tournament. ■

It could be argued that it is more likely for the high productive contestants to have better outside opportunities, so their reservation utility should be higher.[8] This would make the kind of adverse selection considered in proposition 4 even more likely.

Note that proposition 4 does not rely on a positive correlation between abilities in working productively and sabotaging. For example, if $c_i = \gamma_i x_i^2 + c^S\left(\sum_{j \neq i} s_{ij}\right)$ for all i, where c^S is an increasing convex function, the contestants are equally able in sabotaging, but differ in their productive ability. Here it can also happen that the most productive contestants stay out.

5 Extensions

This section considers some extensions to the basic model presented above.

Risk aversion. Suppose that the contestants are risk averse. Following Rosen (1986) this can be modelled as follows. Let $u(w)$ be some increasing concave function and suppose

$$u_i = p_i u(w) - c_i\left(x_i, \sum_{j \neq i} s_{i,j}\right).$$

With some minor modifications to the proofs one can show that the results derived above hold also with risk averse agents. Of course, in proposition 3 we have to replace w by $u(w)$.

Production functions. The linear production functions considered so far are a special case of the functions considered by Lazear (1989):

$$q_i = q\left(x_i, s_{1i}, ..., s_{(i-1)i}, s_{(i+1)i}, ..., s_{ni}\right) + \varepsilon_i$$

[8] This seems reasonable when the outside option is (e.g.) to become self employed, while it is more questionable if the outside option is to participate in another tournament, where sabotage might be important, too.

where $q : R^n \to R$ is a differentiable function that is increasing in its first argument and decreasing in the remaining arguments. Now the marginal benefit of sabotaging j is the product of two factors:

$$\frac{\partial p_i}{\partial s_{ij}} = \underbrace{\frac{\partial q\left(x_j, s_{1j}, ..., s_{(j-1)j}, s_{(j+1)j}, ..., s_{nj}\right)}{\partial s_{ij}}}_{A_{ij}} *$$
$$\underbrace{\int_{-\infty}^{\infty} f\left(y_{ij} + \varepsilon_i\right)\left[\Pi_{l \neq i,j} F\left(y_{il} + \varepsilon_i\right)\right] f\left(\varepsilon_i\right) d\varepsilon_i}_{B_{ij}}$$

The first factor (A_{ij}) describes how much the expected output of contestant j decreases, if i sabotages j more. The second factor (B_{ij}) describes how decreased expected output of j translates into a higher probability of winning for contestant i. If the production function is linear in sabotage, the first factor A_{ij} is equal to a constant, and lemma 1 and the propositions derived above hold.

On the other hand, lemma 1 does not hold if the production function is nonlinear in sabotage. Of course, if $p_j > p_k$, contestant i still improves his chance of winning more if he decreases the output of j than if he decreases the output of j by the same amount (that is, $B_{ij} > B_{ik}$). But A_{ij} can be greater or smaller than A_{ik}.

I find it hard to argue for specific assumptions on the production function. For example, sabotage may have decreasing returns to scale. On the other hand, there may be complementarities - it may more effective to sabotage k if k is sabotaged by other players as well. Without more specific assumptions, little can be said about the properties of equilibria. Nevertheless it is important to note that there is no guarantee that better contestants will have a better chance of winning in equilibrium.[9]

Easy victims. Some contestants may be easier to sabotage than others. This can be due to personal differences between the contestants. People differ in their ability to cope with a hostile environment. Or it may be due to different positions or experience within the firm. For example, workers that are relatively new depend more strongly on the help of other workers, if

[9] Chen (2003) works out an example with a specific nonlinear production function where the best contestant does not have the highest chance of winning.

only to get information about the job and the firm. They are therefore more vulnerable to sabotage.

To capture this is in the model, I assume that

$$q_i = x_i - a_i \sum_{j \neq i} s_{ji} + \varepsilon_i \tag{11}$$

where a_i is a parameter. A high value of a_i means that i is an easy victim.

Proposition 5 *Suppose production functions are given by (11). In an interior equilibrium, contestant i has a higher chance of winning than contestant j if and only if $a_i < a_j$.*

Proof. In an interior equilibrium it must be true that

$$\frac{\partial p_k}{\partial s_{ki}} = \frac{\partial p_k}{\partial s_{kj}}$$

or

$$\begin{aligned} & a_i \int_{-\infty}^{\infty} f(y_{ki} + \varepsilon) \left[\Pi_{l \neq k,i} F(y_{kl} + \varepsilon)\right] f(\varepsilon)\, d\varepsilon \\ = \; & a_j \int_{-\infty}^{\infty} f(y_{kj} + \varepsilon) \left[\Pi_{l \neq k,j} F(y_{kl} + \varepsilon)\right] f(\varepsilon)\, d\varepsilon. \end{aligned}$$

If $a_i < a_j$ this implies

$$\begin{aligned} & \int_{-\infty}^{\infty} f(y_{ki} + \varepsilon) \left[\Pi_{l \neq k,i} F(y_{kl} + \varepsilon)\right] f(\varepsilon)\, d\varepsilon \\ > \; & \int_{-\infty}^{\infty} f(y_{kj} + \varepsilon) \left[\Pi_{l \neq k,j} F(y_{kl} + \varepsilon)\right] f(\varepsilon)\, d\varepsilon \end{aligned}$$

which, by the same reasoning as in the proof of lemma 1, implies

$$x_i - a_i \sum_{l \neq i} s_{li} > x_j - a_j \sum_{l \neq j} s_{lj}$$

or $p_i > p_{j.}$ Conversely, if $a_i \geq a_j$, we get $p_i \leq p_{j.}$. ■

Proposition 5 says that people that are easy to sabotage will have lower chances of winning in an interior equilibrium.[10] The tournament will select

[10] Obviously, existence of interior equilibria will depend on the cost functions and on the a_i parameters.

only on the basis of the a_i parameters. If $a_1 < a_2 < ... < a_n$, then $p_1 > ... > p_n$. This may or may not be in the interest of the firm. In particular, there is no reason to assume that low vulnerability to sabotage on the one hand and ability to work productively on the other always go together.

By proposition 5, if two contestants behave equally in an equilibrium, the one who is an easier victim will be sabotaged more heavily. As mentioned in the introduction, this fits to some results from the psychological literature on mobbing or bullying. Basically, within the model there are two reason why one might become a victim: by being an overachiever and therefore a dangerous rival, and by being an easy victim. Both is in line with results from psychology.

Cost functions. The cost function considered above depended only on the sum of all sabotage activities. At a more general level, one could assume

$$c_i \left(x_i, s_{i1}, ..., s_{i(i-1)}, s_{i(i+1)}, ..., s_{in} \right) \tag{12}$$

where function c_i is increasing in each argument and convex.

One natural assumption seems that the cost functions satisfy the following symmetry property. A cost function c_i is *symmetric in the sabotage activities* if exchanging s_{ij} and s_{ik} while holding constant all other decision variables of i does not change c_i.[11] For example, the cost function considered above, $c_i \left(x_i, \sum_{j \neq i} s_{ij} \right)$, is symmetric in the sabotage activities, as is the following cost function, which is additively separable in the activities:

$$c_i = c_i^X (x_i) + \sum_{j \neq i} c_i^S (s_{ij}), \tag{13}$$

where $c_i^X : R_0^+ \rightarrow R_0^+$ and $c_i^S : R_0^+ \rightarrow R_0^+$ are differentiable, increasing, and strictly convex functions.

In general, winning probabilities will not be completely equalized.[12] However, some equalizing effect is still at work. Proposition 2 - "overachievers" are sabotaged more heavily - generalizes as follows.

[11] More formally, let π be any permutation of the opponents of player i (a bijection of $\{1, .., i-1, i+1, ..., n\}$ to $\{1, .., i-1, i+1, ..., n\}$). A cost function is symmetric in the sabotage activities if and only if $c_i \left(x_i, s_{i\pi(1)}, ..., s_{i\pi(i-1)}, s_{i\pi(i+1)}, ..., s_{i\pi(n)} \right) =$
$= c_i \left(x_i, s_{i1}, ..., s_{i(i-1)}, s_{i(i+1)}, ..., s_{in} \right)$ for all $\left(x_i, s_{i1}, ..., s_{i(i-1)}, s_{i(i+1)}, ..., s_{in} \right)$.

[12] This can easily be seen in the case that the cost function is given by (13) .

Suppose that in an interior equilibrium all contestants have the same chance of winning. Then sabotaging j has the same marginal benefit for i as sabotaging k. In an interior equilibrium, this implies that the marginal costs have to be the same as well. Hence, if

Proposition 6 *Suppose that production functions are linear, and the cost functions are as given in (12) and are symmetric in the sabotage activities.*
a) In any pure strategy equilibrium, if $x_i - s_{ji} \geq x_j - s_{ij}$*, then* $\sum_{k \neq i,j} s_{ki} \geq \sum_{k \neq i,j} s_{kj}$.
b) If the equilibrium is an interior equilibrium and $x_i - s_{ji} > x_j - s_{ij}$*, then* $\sum_{k \neq i,j} s_{ki} > \sum_{k \neq i,j} s_{kj}$.

Proof. a) Suppose $x_i - s_{ji} \geq x_j - s_{ij}$ in a pure strategy equilibrium. Towards a contradiction further suppose $\sum_{k \neq i,j} s_{ki} < \sum_{k \neq i,j} s_{kj}$. Then $E(q_i) > E(q_j)$ and by lemma 1 we have for all $k \neq i, j$

$$\frac{\partial p_k}{\partial s_{kj}} < \frac{\partial p_k}{\partial s_{ki}}. \tag{14}$$

In any pure strategy equilibrium, the following Kuhn Tucker conditions have to hold for all k and all $l \neq k$:

$$\frac{\partial p_k}{\partial s_{kl}} w \leq \frac{\partial c_k}{\partial s_{kl}},\ s_{kl} \geq 0,$$

where one of the inequalities holds strictly.

Next I will show this implies $s_{ki} \geq s_{kj}$. This is obvious if $s_{kj} = 0$. So suppose $s_{kj} > 0$. From the Kuhn Tucker conditions and equation (14),

$$\frac{\partial c_k}{\partial s_{kj}} = \frac{\partial p_k}{\partial s_{kj}} w < \frac{\partial p_k}{\partial s_{ki}} w \leq \frac{\partial c_k}{\partial s_{ki}}. \tag{15}$$

Cost functions that are convex and symmetric in the sabotage activities have the property that $s_{ki} < s_{kj}$ implies $\frac{\partial c_k}{\partial s_{ki}} \leq \frac{\partial c_k}{\partial s_{kj}}$.[13] Hence from inequality (15) we get $s_{ki} \geq s_{kj}$.

the cost function is given by (13) and c_i^S is strictly convex, it follows that $s_{ij} = s_{ik}$ for all i and $j, k \neq i$. Then $E(q_j) - E(q_k) = x_j - s_{kj} - (x_k - s_{jk})$ since all other contestants treat j and k equally. But in general $x_j - s_{kj}$ will not equal $x_k - s_{jk}$. Suppose for example that j and k have the same cost of sabotaging, that is, $c_j^S(\cdot) = c_k^S(\cdot)$. Then $s_{kj} = s_{jk}$. Since we assumed that all players have the same chance of winning, contestants j and k have the same marginal benefit from working productively. But if j has a lower marginal cost of productive effort , $\forall x : \frac{\partial}{\partial x} c_j^X(x) < \frac{\partial}{\partial x} c_k^X(x)$, it follows that $x_j > x_k$ and hence $E(q_j) > E(q_k)$, contradicting the assumption that all players have the same chance of winning.

[13] This holds for all convex functions that are symmetric in the sense discussed here. Let $f(x, y)$ be any convex function. Since a convex function is underestimated by a linear

Summing over all $k \neq i, j$, we get $\sum_{k \neq i,j} s_{ki} \geq \sum_{k \neq i,j} s_{kj}$, a contradiction. The proof of b) follows the same lines and is omitted. ■

Proposition 6 says the following. Suppose that - looking only at the decisions of contestants i and j - contestant i has a higher expected output and a hence better chance of winning than contestant j (disregarding the decisions of the other contestants). Then the other contestants will sabotage i more heavily than j. In this sense, the effect that sabotage tends to equalize promotion chances is robust.

Proposition 6 also generalizes proposition 2 by taking corner solutions into account. This is important since interior equilibria fail to exist if players are very different in their abilities. But as part a shows, the result that overachievers are sabotaged more heavily is robust. Of course, it may happen that both i and j are not sabotaged at all. Think of two players that have much higher costs of working and sabotaging than all their rivals. They are going to lose anyway - why should anyone bother sabotaging them?

Prize structure. In principle, in a rank order tournament there could be n different prizes corresponding to the n possible ranks. Dealing with a general prize structure is an important challenge for future research, but some important lesson can be learned by considering the case where there are $n-1$ equal winner prizes $w > 0$ and only one strictly lower loser prize of 0. In such a situation the probability that contestant i gets one of the winner prizes is equal to

$$\hat{p}_i = 1 - \int_{-\infty}^{\infty} \left(\prod_{j \neq i} \left(1 - F\left(y_{ij} + \varepsilon\right)\right) \right) f\left(\varepsilon\right) d\varepsilon \tag{16}$$

and his payoff is given by

$$u_i = \hat{p}_i w - c_i \left(x_i, \sum_{j \neq i} s_{ij} \right).$$

As the following lemma shows, in this situation the incentives to sabotage are very different from the case of one winner prize.

approximation, $f(y,x) \geq f(x,y) + f_1(x,y)(y-x) + f_2(x,y)(x-y)$, where f_i denotes the partial derivative with respect to the ith argument. If f satisfies the symmetry property $f(x,y) = f(y,x)$, we get $0 \geq (f_2(x,y) - f_1(x,y))(x-y)$. It follows that $y < x$ implies $f_2(x,y) \leq f_1(x,y)$

Lemma 2 *For all values of the decision variables,*

$$\frac{\partial \hat{p}_i}{\partial s_{ij}} > \frac{\partial \hat{p}_i}{\partial s_{ik}} \Leftrightarrow x_j - \sum_{l \neq j} s_{lj} < x_k - \sum_{l \neq k} s_{lk}$$

Proof. See appendix. ■

It is instructive to compare lemma 2 with lemma 1. If there is a single winner prize, then the biggest marginal benefit from sabotaging comes from sabotaging the opponent with the *highest* expected output. Here, with only one loser, the biggest marginal benefit of sabotaging comes from sabotaging the opponent with the *lowest* expected output.

The intuition behind lemma 2 is as follows. Suppose there are three contestants, and contestant 1 has the highest expected output, and contestant 3 the lowest. If contestant 2 sabotages 1, this increases his chance of winning against 1. But winning against 1 is beneficial only if player 2 does not win against player 3 and therefore gets a winner prize anyway. Since 2 is likely to win against 3 anyway, sabotaging 1 does not increase the chances of 2 to get a prize by a great amount. On the other hand, by sabotaging 3 contestant 2 can raise his probability of getting a prize by a greater amount, since 2 is likely to lose against 1, and if 2 loses against 1 winning against 3 is important. The assumption that f is log-concave is sufficient to guarantee that this intuition carries over to the formal model.

Sabotage in equilibrium will take a very different form with $n-1$ winner prizes, as the next lemma shows.

Lemma 3 *Suppose there are $n-1$ identical winner prizes and only one loser prize. In a pure strategy equilibrium, each contestant sabotages exactly one of his rivals. In equilibrium, $s_{ik} > 0$ holds only if $\hat{p}_k < \hat{p}_j$ and $x_k - \sum_{l \neq i,k} s_{lk} \leq x_j - \sum_{l \neq i,j} s_{lj}$ for all $j \neq i, k$.*

Proof. See Appendix. ■

According to this lemma, the behavior of the contestants in a pure strategy equilibrium can be described as follows: Every contestant i sabotages only the opponent who has the lowest chance of winning given the behavior of all the other contestants $j \neq i$. As the next proposition shows, $n-1$ of the contestants will choose to sabotage the same person. Call this contestant, who is sabotaged by all other contestants, “the victim”.

Proposition 7 *Suppose there are $n-1$ identical winner prizes and only one loser prize. In every pure strategy equilibrium, there exist one contestant a (the victim) such that $s_{ka} > 0$ and $s_{kl} = 0$ for all $k, l \neq a$. Further, there exists one contestant b such that $s_{ab} > 0$ and $s_{ak} = 0$ for all $k \neq b$. Moreover, $\hat{p}_a < \hat{p}_b < \hat{p}_k$ for all $k \neq a, b$.*

Proof. We know that every contestant sabotages exactly one opponent. For example, contestant 1 sabotages a contestant, say i_1, that is, $s_{1(i_1)} > 0$. From lemma 3, it follows that $\hat{p}_{i_1} < \hat{p}_j$ for all $j \neq 1, i_1$. Then we can show that either i_1 is the victim (see case 1 below), or 1 is the victim (case 2).

Case 1: $\hat{p}_{i_1} < \hat{p}_1$. Then i_1 has the lowest chance of getting a prize: $\hat{p}_{i_1} < \hat{p}_j$ for all $j \neq i_1$. Every $j \neq i_1$ sabotages i_1 (if there were $j, k \neq i_1$ such that $s_{jk} > 0$ then by lemma 3 $\hat{p}_k < \hat{p}_{i_1}$, a contradiction). In this case i_1 is the victim. Further, i_1 sabotages someone, say contestant b. That is, $s_{i_1 b} > 0$. Therefore, by lemma 3, $\hat{p}_b < \hat{p}_k$ for all $k \neq i_1, b$.

Case 2: $\hat{p}_{i_1} > \hat{p}_1$. By the same arguments as in case 1 above it follows that in case 2, $s_{k1} > 0$ for all $k \neq 1$, and $\hat{p}_1 < \hat{p}_{i_1} < p_k$ for all $k \neq 1, i_1$. In case 2 contestant 1 is the victim.

Case 3: $\hat{p}_{i_1} = \hat{p}_1$. Then $\hat{p}_k > \hat{p}_{i_1} = \hat{p}_1$ for all $k \neq 1, i_1$ by lemma 3 and $s_{1(i_1)} > 0$. Hence all the other players sabotage 1 or i_1 (if $s_{jl} > 0$ for some j and $l \neq 1, i_1$ then $\hat{p}_l < \hat{p}_{i_1}$, a contradiction). But then it by lemma 3 it cannot be true that $\hat{p}_{i_1} = \hat{p}_1$. So case 3 leads to a contradiction. ■

It remains to consider which contestant will be the victim. Typically, there will be an equilibrium where the least able contestant is the victim. But there can be multiple equilibria. This is very clear in the case of identical contestants. Obviously, if a pure strategy equilibrium exists at all, then there are multiple pure strategy equilibria, since everyone might be the victim. By continuity, in the case of heterogenous contestants equilibria where all contestants sabotage the most able one may exist. In such an equilibrium, the most able player has the *lowest* chance of winning the tournament. However, existence of pure strategy equilibria is not trivially ensured here.

Handicaps. Suppose that there are player specific handicaps as considered by Lazear and Rosen (1981). That is, player i wins the contest if $q_i - d_i > \max_j \{q_j - d_j\}_{j \neq i}$, where d_i is the handicap of player i. Returning to the case of linear production functions, cost function (1), and a single winner prize, the following generalization of lemma 1 can be shown by an argument paralleling the proof of lemma 1:

Lemma 4 *Suppose that i wins if and only if $q_i - d_i > \max_j \{q_j - d_j\}_{j \neq i}$. Then for all values of the decision variables, the following inequalities are equivalent:*
a) $x_j - \sum_{l \neq j} s_{lj} - d_j > x_k - \sum_{l \neq k} s_{lk} - d_k$
b) $p_j > p_k$
c) $\frac{\partial p_i}{\partial s_{ij}} > \frac{\partial p_i}{\partial s_{ik}}$

From lemma 4 it follows immediately that proposition 1 continues to hold with handicaps as well. Of course, the existence of interior equilibria now depends not only on the differences in ability, but on the handicaps as well. Moreover, now those players whose expected output minus handicap is highest are sabotaged most heavily. To sum up,

Proposition 8 *With handicaps, in an interior equilibrium $p_i = \frac{1}{n}$ and $x_i - \sum_{l \neq j} s_{li} - d_i = x_k - \sum_{l \neq k} s_{lk} - d_k$ for all i, k.*

6 Implications

If sabotage plays an important role, it is better to limit the tournament to only two contestants, since the equalizing effect of sabotage depends on there being at least three contestants.[14] This is surprising because intuition would suggest that more contestants should on average lead to higher quality of winners. But here the strategic possibilities of the contestants change if there more than two contestants, since then every contestant can direct sabotage specifically against his most dangerous opponent.

One might think that a sequential elimination tournament as studied in Rosen (1986) solves the problem that sabotage equalizes promotion chances. In such a tournament, contestants are paired in each round. One winner emerges from each pair and moves on to the next round. So in any given round, each contestant has only one rival, and it might be thought that the equalizing effect of sabotage is not at work in such a sequential elimination tournament. But contestants do not only care about moving on into the next round, they are also interested in meeting weak rivals in the coming

[14] Hvide and Kristiansen (2003) find a similar result in contests where the decision variable of the contestants is risk taking.

rounds. This gives them an incentive to interfere with the other paired contests in any given round. Consider, for example, the incentives of the semi-finalists. By helping the weaker contestant in the semi-final contest in which one is not directly involved, and by sabotaging the stronger ones, one increases the probability of meeting a weaker rival in the final round (if one gets there). Therefore, there is some equalizing effect of sabotage at work also in a sequential elimination tournament.

Lazear (1989) has argued that by the right design of promotion tracks contestants can be separated and thereby sabotage can be made more difficult and hence less important (p. 557). Lazear's point is that separating contestants is good for the firm because sabotage decreases the valuable output of the contestants. The results of this paper show that, in addition, separating contestants also helps to make better promotion decisions. People who do not work with each other closely and regularly are less likely to know each other well, and so they cannot direct sabotage against their strongest rivals. Therefore, the effect that sabotage equalizes promotion chances does not apply.

7 Conclusion

This paper studied sabotage in selection tournaments with heterogeneous contestants. Sabotage can lead to equalization of promotion chances, even if the contestants differ a lot in their abilities. Furthermore it may happen that only the least productive individuals participate. Therefore, using a tournament for selection can result in selecting (with probability one) someone who is among the least productive.

The results are relevant for other types of contest as well. For example, in rent seeking contests, yardstick competition between regulated firms, or political election contests, sabotage can equalize the probabilities of winning the contest. In rent-seeking contests, the heterogeneity between contestants often takes the form of different valuations of winning the contest. Applied to this setting, the results of the paper imply that there is no guarantee that the contestants with the highest valuations will win most often.

A related problem of tournaments in the presence of sabotage is that sabotage reduces the incentives for productivity enhancing investments in human capital. Since the contestants know that the better they are, the more they will be sabotaged, they have little incentive to invest in their human capital. This adds to the potential severity of the problems described.

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8 Appendix

8.1 Proof of lemma 1

The equivalence of a) and b) is obvious.

Differentiating equation (3), we get

$$\frac{\partial p_i}{\partial s_{ij}} = \int_{-\infty}^{\infty} f\left(y_{ij} + \varepsilon_i\right)\left[\Pi_{l \neq i,j} F\left(y_{il} + \varepsilon_i\right)\right] f\left(\varepsilon_i\right) d\varepsilon_i,$$

and

$$\frac{\partial p_i}{\partial s_{ik}} = \int_{-\infty}^{\infty} f\left(y_{ik} + \varepsilon_i\right)\left[\Pi_{l \neq i,k} F\left(y_{il} + \varepsilon_i\right)\right] f\left(\varepsilon_i\right) d\varepsilon_i.$$

Therefore,

$$\frac{\partial p_i}{\partial s_{ij}} - \frac{\partial p_i}{\partial s_{ik}} = \int_{-\infty}^{\infty} \left[f\left(y_{ij}+\varepsilon_i\right) F\left(y_{ik}+\varepsilon_i\right) - f\left(y_{ik}+\varepsilon_i\right) F\left(y_{ij}+\varepsilon_i\right) \right] \\ * \left[\Pi_{l\neq i,j,k} F\left(y_{il}+\varepsilon_i\right) \right] f\left(\varepsilon_i\right) d\varepsilon_i. \tag{17}$$

Now suppose $x_j - \sum_{l\neq j} s_{lj} > x_k - \sum_{l\neq k} s_{lk}$. This is equivalent to $y_{ij} < y_{ik}$. Since $f(z)$ is strictly log-concave, $f(z)/F(z)$ decreases strictly in z (see An 1998). It follows that

$$\frac{f\left(y_{ij}+\varepsilon_i\right)}{F\left(y_{ij}+\varepsilon_i\right)} > \frac{f\left(y_{ik}+\varepsilon_i\right)}{F\left(y_{ik}+\varepsilon_i\right)}$$

for all ε_i. Therefore the integrand in equation (17) is strictly positive, and hence $\frac{\partial p_i}{\partial s_{ij}} > \frac{\partial p_i}{\partial s_{ik}}$. This proves that

$$x_j - \sum_{l\neq j} s_{lj} > x_k - \sum_{l\neq k} s_{lk} \Rightarrow \frac{\partial p_i}{\partial s_{ij}} > \frac{\partial p_i}{\partial s_{ik}}.$$

The converse statement is proved similarly. Therefore, a) and c) are equivalent as well.

8.2 Proof of stronger version of propositions 1 and 2

As stated in the text following proposition 2, it is possible to strengthen propositions 1 and 2. We do not have to restrict attention to equilibria where literally *all* contestants sabotage *all* their rivals.

1) If there are two contestants i, j who sabotage all their rivals, then it follows that for all $k \neq i, j$: $p_k = p_i$ (since j sabotages both k and i) and $p_k = p_j$ (since i sabotages both k and j). Therefore we have $p_k = p_i = p_j = \frac{1}{n}$.

2) Suppose in an equilibrium every contestant is sabotaged by two rivals. That is, we have $\forall i \exists j_i, k_i : i \neq j_i \neq k_i \neq i, s_{(j_i)i} > 0$ and $s_{(k_i)i} > 0$. Then $p_i \geq p_l$ for all $l \neq j_i$ since i is sabotaged by j_i. Also, since i is sabotaged by k_i, we have $p_i \geq p_l$ for all $l \neq k_i$. Putting things together, $p_i \geq p_l$ for all $l \neq i$. Since this holds for all i, we have $p_i = \frac{1}{n}$ for all $i = 1, ..., n$.

3) If, in an equilibrium, we have $s_{(k-1)k} > 0$, then it follows that $p_k \geq p_{k+1}$, since $k-1$ sabotages k. Therefore, if the contestants can be renumbered so that $s_{i(i+1)} > 0$ for $i = 1, .., n-1$ and $s_{n1} > 0$, we have $p_2 \geq p_3 \geq ... \geq p_n \geq p_1 \geq p_2$ or $p_i = \frac{1}{n}$ for all $i = 1, ..., n$.

In all those three cases, equation (4) holds. Therefore, proposition 2 also generalizes: Contestants that choose higher productive effort are sabotaged more heavily.

8.3 Proof of proposition 3 part a)

The proof is by contradiction. Suppose there is a pure strategy equilibrium with $p_i = \frac{1}{n}$. Then $y_{ij} = 0$ for all i and $j \neq i$. Define

$$g := \frac{\partial p_i}{\partial x_i} \left|_{y_{ij}=0 \forall j \neq i} \right. = (n-1) \int_{-\infty}^{\infty} F(\varepsilon)^{n-2} f(\varepsilon)^2 \, d\varepsilon.$$

and denote the set of all low cost contestants by L, and the set of all high cost contestants by H. The following first order conditions have to hold in the supposed equilibrium:

$$\begin{aligned} x_i &= wg \text{ for all } i \in L, && (18) \\ x_i &= \frac{wg}{\gamma} \text{ for all } i \in H, && (19) \\ \sum_{k \neq i} s_{ik} &= \frac{wg}{n-1} \text{ for all } i \in L, && (20) \\ \sum_{k \neq i} s_{ik} &= \frac{wg}{(n-1)\gamma} \text{ for all } i \in H. && (21) \end{aligned}$$

Further, from the assumption that $p_i = \frac{1}{n}$ we get

$$x_i - \sum_{k \neq i} s_{ki} = x_j - \sum_{k \neq j} s_{kj} \text{ for all } i \text{ and } j. \quad (22)$$

Equations (18) and (22) imply that $\sum_{k \neq i} s_{ki} = \sum_{k \neq j} s_{kj}$ for all $i, j \in L$. That is, all low cost types endure the same amount of sabotage. Call this amount S_l :

$$\sum_{k \neq i} s_{ki} = \sum_{k \neq j} s_{kj} =: S_l \text{ for all } i, j \in L. \quad (23)$$

In the same way it follows from equations (19) and (22) that all high cost types endure the same amount of sabotage:

$$\sum_{k \neq i} s_{ki} = \sum_{k \neq j} s_{kj} =: S_h \text{ for } i, j \in H. \quad (24)$$

Now let us calculate S_l and S_h. Summing over equations (20) and (21) we find that the total amount of sabotage equals

$$\sum_{i=1}^{n}\sum_{j\neq i} s_{ji} = l\frac{wg}{n-1} + h\frac{wg}{\gamma(n-1)} = lS_l + hS_h, \tag{25}$$

where the second equality follows from equations (23) and (24). From equations (18) to (24) we get

$$wg - S_l = \frac{wg}{\gamma} - S_h. \tag{26}$$

Combining equations (25) and (26) we finally get

$$S_h = wg\frac{n-(\gamma-1)\,l\,(n-2)}{\gamma n\,(n-1)}, \tag{27}$$

$$S_l = wg\frac{n+(\gamma-1)\,(l+h\,(n-1))}{\gamma n\,(n-1)}. \tag{28}$$

If $\gamma > 1 + \frac{n}{l(n-2)}$, equation (27) implies $S_h < 0$, a contradiction. This completes the proof for the case $l > 1$.

Note that the low cost contestants will be sabotaged more heavily:

$$S_l - S_h = (\gamma-1)\frac{wg}{\gamma} > 0.$$

If there is only one low cost contestant, this contestant directs all his sabotage against high cost contestant. We can calculate the total amount of sabotage that high cost contestants inflict on high cost contestants as the difference of the total amount of sabotage suffered by high cost contestants, hS_h, and the amount of sabotage chosen by the low cost contestant, $wg/(n-1)$:

$$\sum_{i\in H}\sum_{j\in H, j\neq i} s_{ij} = hS_h - \frac{wg}{n-1} = wg\frac{n\,(n-2)-(\gamma-1)\,(n^2-2n+2)}{\gamma n\,(n-1)} \tag{29}$$

This is non-negative if and only if $\gamma \leq 1 + \frac{n(n-2)}{n^2-2n+2}$. This completes the proof.

8.4 Proof of proposition 3 part b)

This section develops the sufficient condition for the existence of interior pure strategy equilibria in example 1. Existence is proved by direct construction of such equilibria. I focus on symmetric equilibria, in which

$$s_{ik} = \left\{ \begin{array}{c} s_{ll} \\ s_{lh} \\ s_{hl} \\ s_{hh} \end{array} \right\}, \text{ if } i \in \left\{ \begin{array}{c} L \\ L \\ H \\ H \end{array} \right\} \text{ and } j \in \left\{ \begin{array}{c} L \\ H \\ L \\ H \end{array} \right\}.$$

Let us first derive candidates for symmetric interior equilibria, and check afterwards that they are really equilibria. In an interior equilibrium, the effort choices x_i have to be given by equations (18) and (19). Further, a contestant $i \in H$ sabotages l low cost contestants and $h-1$ high cost contestants. The total amount of sabotage that a contestant $i \in H$ chooses in a symmetric equilibrium is therefore

$$\sum_j s_{ij} = l s_{hl} + (h-1)\, s_{hh} = \frac{wg}{\gamma\,(n-1)} \text{ for all } i \in H. \tag{30}$$

The second equality follows from equation (21). Similarly

$$\sum_j s_{ij} = (l-1)\, s_{ll} + h s_{lh} = \frac{wg}{(n-1)} \text{ for all } i \in L.$$

The total amount of sabotage suffered by contestant i is

$$\begin{aligned} \sum_j s_{ji} &= l s_{lh} + (h-1)\, s_{hh} = S_h \text{ if } i \in H, \\ \sum_j s_{ji} &= (l-1)\, s_{ll} + h s_{hl} = S_l \text{ if } i \in L, \end{aligned} \tag{31}$$

where S_h and S_l are given by equations (27) and (28). Assuming $l \geq 2$ and $h \geq 2$ (the remaining cases will be considered later) and using s_{lh} as a free variable, the system of equations (30) to (31) can be solved to get

$$
\begin{aligned}
s_{ll} &= \frac{wg}{(n-1)(l-1)} - \frac{n-l}{(l-1)} s_{lh}, && (32)\\
s_{hh} &= wg\frac{n-(\gamma-1)\,l\,(n-2)}{\gamma n\,(n-1)\,(n-l-1)} - \frac{l s_{lh}}{(n-l-1)}, && (33)\\
s_{hl} &= wg\frac{(2l+n\,(n-l-2))\,(\gamma-1)}{\gamma n\,(n-1)\,(n-l)} + s_{lh}. && (34)
\end{aligned}
$$

If the condition for γ given in proposition 2a) is satisfied, and

$$
s_{lh} \in \left[0, \min\left\{\frac{wg}{(n-1)(n-l)}, wg\frac{n-(\gamma-1)(n-2)}{\gamma n\,(n-1)}\right\}\right], \tag{35}
$$

then all the variables given in equations (32) to (35) are non-negative.

In what follows, I show that, if the conditions given in proposition 2 are satisfied, then there exists a continuum of symmetric equilibria given by (18), (19), and (32) to (35). In all these equilibria, $p_i = \frac{1}{n}$ for all contestants i.

Consider the maximization problem of contestant i, given that the other contestants behave according to one of these candidate equilibria. Contestant i chooses $\left(x_i, s_{i1}, ..., s_{i(i-1)}, s_{i(i+1)}, ..., s_{in}\right)$ to maximize u_i, subject to the non-negativity constraints $x_i \geq 0$ and $s_{ij} \geq 0$ for all $j \neq i$. As a first step, I will ignore for the moment the constraints and solve the unconstrained problem. We will check afterwards whether the constraints are satisfied.

The unconstrained problem certainly has a solution. This can be seen as follows. It is never optimal to choose very high values of the decision variables. Therefore we can consider the problem

$$
\text{maximize } u_i \text{ s.t. } -k \leq x_i \leq k \text{ and } -k \leq s_{ij} \leq k \text{ for all } j \neq i \tag{36}
$$

for some sufficiently high $k \in R$. By the Weierstrass theorem, a solution to problem (36) exists. If k is high enough, the solution to problem (36) also solves the unconstrained problem.

A difficulty in finding the solution is that the objective function might be non-concave, and checking concavity for a n-dimensional optimization problem is quite cumbersome. The following lemma allows the n-dimensional optimization problem to be reduced to a one-dimensional one:

Lemma 5 *Let $l \geq 2$ and $h \geq 2$. Suppose all contestants except i behave symmetrically according to equations (18), (19), and (32) to (35). In the optimum of the unconstrained optimization problem of contestant i, the following conditions have to hold.*
a) Contestant i sabotages all his low cost rivals equally:

$$s_{ij} = s_{ik} =: s_{il} \text{ for all } j,k \in L, j,k \neq i,$$

and i also sabotages all his high cost rivals equally:

$$s_{ij} = s_{ik} =: s_{ih} \text{ for all } j,k \in H, j,k \neq i.$$

b) Contestant i sabotages his high cost and his low cost rivals so that they have the same chance of winning:
$wg - (l-2)\,s_{ll} - hs_{hl} - s_{il} = \frac{wg}{\gamma} - (l-1)\,s_{lh} - (h-1)\,s_{hh} - s_{ih}$, *if $i \in L$, and*
$wg - (l-1)\,s_{ll} - (h-1)\,s_{hl} - s_{il} = \frac{wg}{\gamma} - ls_{lh} - (h-2)\,s_{hh} - s_{ih}$, *if $i \in H$.*
c) All rivals of i have the same chance of winning: $y_{ij} = y_{ik}$ for all j,k.
d)

$$x_i = \begin{cases} (n-1)\left((l-1)\,s_{il} + hs_{ih}\right), & \text{if } i \in L, \\ (n-1)\left(ls_{il} + (h-1)\,s_{ih}\right), & \text{if } i \in H. \end{cases}$$

Proof. a) Suppose there are $j,k \in L$ $(j,k \neq i)$ such that $s_{ij} > s_{ik}$. Since j and k are treated in the same way by all other contestants, and choose the same $x_j = x_k = wg$, this implies $y_{ij} > y_{ik}$. Now contestant i could decrease s_{ij} a little and increase s_{ik} by the same amount. By lemma 1, this increases p_i, while the costs of contestant i are unchanged. Therefore, it cannot be optimal to choose $s_{ij} > s_{ik}$.

The case $j,k \in H$ and part b) are proved in the same way as a). Part c) is obvious from a) and b).

d) It follows from equation (3) that $\sum_{j\neq i} \frac{\partial p_i}{\partial s_{ij}} = \frac{\partial p_i}{\partial x_i}$. Using c) we have $\frac{\partial p_i}{\partial s_{ij}} = \frac{\partial p_i}{\partial s_{ik}}$ for all $j,k \neq i$ and hence

$$(n-1)\frac{\partial p_i}{\partial s_{ij}} = \frac{\partial p_i}{\partial x_i}.$$

In the optimum of the unconstrained problem, the first order conditions $w\frac{\partial p_i}{\partial x_i} = \gamma_i x_i$ and $w\frac{\partial p_i}{\partial s_{ij}} = \gamma_i \sum_{j\neq i} s_{ij}$ have to hold with equality. Putting things together, $x_i = (n-1)\sum_{j\neq i} s_{ij}$. Finally, using a) completes the proof. ■

This lemma establishes that, in the optimum of the unconstrained problem, certain relations between $s_{i1}, ..., s_{in}$ and x_i must hold. It allows the unconstrained problem to expressed as a one-dimensional problem, where contestant i maximizes only over x_i. Denote the objective function in this reduced problem by $\hat{u}_i(x_i)$.

Take the case $i \in L$. Straightforward but tedious omitted calculations show that

$$\hat{u}_i(x_i) = \int_{-\infty}^{\infty} F(b(x_i - wg) + \varepsilon)^{n-1} f(\varepsilon)\, d\varepsilon w - \frac{1}{2} b x_i^2, \tag{37}$$

where $b := 1 + \frac{1}{(n-1)^2}$.

Lemma 6 *If inequality (6) holds, $\hat{u}_i(x_i)$ is strictly concave.*

Proof. The objective function $\hat{u}_i(x_i)$ is strictly concave for all x_i if

$$\max_{x_i} \left(\frac{\partial^2}{\partial x_i^2} \int_{-\infty}^{\infty} F\left(b\left(x_i - \frac{wg}{\gamma}\right) + \varepsilon\right)^{n-1} f(\varepsilon)\, d\varepsilon w \right) < b. \tag{38}$$

Note that

$$\begin{aligned} & \frac{\partial^2}{\partial x_i^2} \int_{-\infty}^{\infty} F\left(b\left(x_i - \frac{wg}{\gamma}\right) + \varepsilon\right)^{n-1} f(\varepsilon)\, d\varepsilon = \\ = \; & b^2 \left(\frac{\partial^2}{\partial z^2} \int_{-\infty}^{\infty} F(z+\varepsilon)^{n-1} f(\varepsilon)\, d\varepsilon \right)\Bigg|_{z = b\left(x - \frac{wg}{\gamma}\right)}. \end{aligned}$$

Therefore, inequality (38) is equivalent to inequality (6). ■

By this lemma, the solution to $\max_{x_i} \hat{u}_i(x_i)$ can be found simply as the solution of the first order condition $\frac{d\hat{u}_i(x_i)}{dx_i} = 0$, which is, of course, unique and as given by equation (18): $x_i = wg$.

By using lemma 5 we can verify that $s_{ij} = s_{ll}$ for all $j \in L$ (where s_{ll} is given in (32)), and $s_{ij} = s_{lh}$ for all $j \in H$, solve the unconstrained maximization problem of contestant i. Again, these calculations are straightforward but tedious and hence omitted.

Finally, we have to check whether all decision variables satisfy the nonnegativity constraints. This is guaranteed by the condition $\gamma \leq 1 + \frac{n}{l(n-2)}$.

Therefore, we have shown that no $i \in L$ has an incentive to deviate from any of the symmetric candidate equilibria.

In the same way it can be shown that no $i \in H$ has an incentive to deviate. For an $i \in H$, the objective function of the reduced problem turns out to be

$$\hat{u}_i\left(x_i\right) = \int_{-\infty}^{\infty} F\left(b\left(x_i - \frac{wg}{\gamma}\right) + \varepsilon\right)^{n-1} f\left(\varepsilon\right) d\varepsilon w - \frac{1}{2}\gamma b x_i^2.$$

The condition ensuring concavity is

$$\max_z \left(\frac{\partial^2}{\partial z^2} \int_{-\infty}^{\infty} F\left(z+\varepsilon\right)^{n-1} f\left(\varepsilon\right) d\varepsilon\right) < \frac{\gamma}{bw}. \tag{39}$$

The only difference from inequality (6) is the γ on the right hand side. Since $\gamma > 1$, if (6) holds, so does (39). This completes the proof of proposition 2 for the case that $l \geq 2$ and $h \geq 2$.

The two remaining cases where there is only one low cost contestant or only one high cost contestant can be dealt with similarly. In these cases there is a unique symmetric equilibrium. If $l = 1$,

$$\begin{aligned} s_{lh} &= \frac{wg}{\left(n-1\right)^2}, \\ s_{hl} &= wg\frac{\left(2+n\left(n-2\right)\right)\left(\gamma-1\right)+n}{\gamma n\left(n-1\right)^2}, \\ s_{hh} &= wg\frac{n\left(n-2\right)-\left(\gamma-1\right)\left(n^2-2n+2\right)}{\gamma n\left(n-1\right)}\frac{1}{h\left(h-1\right)}. \end{aligned}$$

Of course, s_{hh} is non-negative if, and only if, the inequality given in proposition 2a for the case $l = 1$ holds. Also note that, as it must be the case, $h\left(h-1\right) s_{hh} = \sum_{i \in H} \sum_{j \in H, j \neq i} s_{ij}$ where the right hand side is given in equation (29) above.

Finally, if $h = 1$,

$$\begin{aligned} s_{hl} &= \frac{wg}{\gamma\left(n-1\right)^2}, \\ s_{ll} &= wg\frac{\left(2\gamma-1\right)\left(n-1\right)^2-1}{\gamma n\left(n-1\right)^2\left(n-2\right)}, \\ s_{lh} &= wg\frac{n-\left(\gamma-1\right)\left(n-1\right)\left(n-2\right)}{\gamma n\left(n-1\right)\left(n-1\right)}. \end{aligned}$$

8.5 Proof of lemma 2

Differentiating equation (16) we get

$$\frac{\partial \hat{p}_i}{\partial s_{ij}} = \int_{-\infty}^{\infty} f\left(y_{ij}+\varepsilon\right)\left(\prod_{l\neq i,j}\left(1-F\left(y_{il}+\varepsilon\right)\right)\right) f\left(\varepsilon\right) d\varepsilon$$

and

$$\frac{\partial \hat{p}_i}{\partial s_{ik}} = \int_{-\infty}^{\infty} f\left(y_{ik}+\varepsilon\right)\left(\prod_{l\neq i,k}\left(1-F\left(y_{il}+\varepsilon\right)\right)\right) f\left(\varepsilon\right) d\varepsilon.$$

Therefore,

$$\frac{\partial \hat{p}_i}{\partial s_{ij}} - \frac{\partial \hat{p}_i}{\partial s_{ik}} = \int_{-\infty}^{\infty} \left[f\left(y_{ij}+\varepsilon\right)\left(1-F\left(y_{ik}+\varepsilon\right)\right) - f\left(y_{ik}+\varepsilon\right)\left(1-F\left(y_{ij}+\varepsilon\right)\right)\right] * \left(\prod_{l\neq i,j,k}\left(1-F\left(y_{il}+\varepsilon\right)\right)\right) f\left(\varepsilon\right) d\varepsilon$$

Now suppose $x_j - \sum_{l\neq j} s_{lj} < x_k - \sum_{l\neq k} s_{lk}$. This is equivalent to $y_{ij} > y_{ik}$. If f is log-concave, the hazard rate function $f\left(z\right)/\left(1-F\left(z\right)\right)$ is monotonically increasing in z (see An 1998). Therefore

$$f\left(y_{ij}+\varepsilon\right)\left(1-F\left(y_{ik}+\varepsilon\right)\right) > f\left(y_{ik}+\varepsilon\right)\left(1-F\left(y_{ij}+\varepsilon\right)\right)$$

for all ε, and and hence $\frac{\partial \hat{p}_i}{\partial s_{ij}} > \frac{\partial \hat{p}_i}{\partial s_{ik}}$. This proves that

$$x_j - \sum_{l\neq j} s_{lj} < x_k - \sum_{l\neq k} s_{lk} \Rightarrow \frac{\partial \hat{p}_i}{\partial s_{ij}} > \frac{\partial \hat{p}_i}{\partial s_{ik}}.$$

The converse statement can be shown by a similar argument.

8.6 Proof of lemma 3

Lemma 3. Suppose there are $n-1$ identical winner prizes and only one loser prize. In a pure strategy equilibrium, each contestant sabotages exactly one of his rivals. In equilibrium, $s_{ik} > 0$ holds only if $\hat{p}_k < \hat{p}_j$ and $x_k - \sum_{l\neq i,k} s_{lk} \leq x_j - \sum_{l\neq i,j} s_{lj}$ for all $j \neq i,k$.

Proof. In a pure strategy equilibrium, the following Kuhn Tucker condition will have to hold for all i and all $j \neq i$:

$$\frac{\partial \hat{p}_i}{\partial s_{ij}} w - \frac{\partial c_i}{\partial s_{ij}} \leq 0, \quad s_{ij} \geq 0, \quad s_{ij} \frac{\partial u_i}{\partial s_{ij}} = 0$$

Since by assumption $\frac{\partial c_i(x_i,0)}{\partial s_{ij}} = 0$, every contestant will sabotage at least one opponent. For if $\sum_{j \neq i} s_{ij} = 0$, then the Kuhn Tucker conditions imply $\frac{\partial \hat{p}_i}{\partial s_{ij}} w - \frac{\partial c_i}{\partial s_{ij}} \leq 0$. But the marginal cost of sabotage are zero if $\sum_{j \neq i} s_{ij} = 0$, and $\frac{\partial \hat{p}_i}{\partial s_{ij}} > 0$. Therefore every contestant sabotages at least one opponent.

In what follows I show that no contestant sabotages *more* than one opponent. Suppose to the contrary that there are $j, k \neq i$ such that $s_{ij} > 0$ and $s_{ik} > 0$. Since $\frac{\partial c_i}{\partial s_{ij}} = \frac{\partial c_i}{\partial s_{ik}}$, it follows that $\frac{\partial \hat{p}_i}{\partial s_{ij}} = \frac{\partial \hat{p}_i}{\partial s_{ik}}$, and by lemma 2 $x_j - \sum_{l \neq j} s_{lj} = x_k - \sum_{l \neq k} s_{lk}$, hence $y_{ij} = y_{ik}$.

Now consider what happens if i decreases s_{ik} to zero and increases s_{ij} by the same amount. This wouldn't change $i's$ cost. But (I claim) it would increase his probability of getting a prize. Let

$$\hat{p}_i (s_{ij}, s_{ik}) = 1 - \int [1 - F(y_{ij} + \varepsilon)]^2 (\Pi_{l \neq i,j,k} [1 - F(y_{il} + \varepsilon)]) f(\varepsilon) d\varepsilon$$

denote $i's$ probability of getting a prize if i sabotages j by s_{ij} and k by s_{ik} (remember $y_{ij} = y_{ik}$), and let

$$\begin{aligned} \hat{p}_i (s_{ij} + s_{ik}, 0) &= 1 - \int [1 - F(y_{ij} + s_{ik} + \varepsilon)] [1 - F(y_{ij} - s_{ik} + \varepsilon)] * \\ & \quad (\Pi_{l \neq i,j,k} [1 - F(y_{il} + \varepsilon)]) f(\varepsilon) d\varepsilon \end{aligned}$$

denote $i's$ probability of getting a prize if he shifts all sabotage from k to j. We have to show that $\hat{p}_i (s_{ij} + s_{ik}, 0) > \hat{p}_i (s_{ij}, s_{ik})$. Define

$$h(s) = \left\{ [1 - F(y_{ij} + \varepsilon)]^2 - [1 - F(y_{ij} + s + \varepsilon)] [1 - F(y_{ij} - s + \varepsilon)] \right\} \quad (40)$$

Note

$$h'(s) = f(y_{ij} + s + \varepsilon) [1 - F(y_{ij} - s + \varepsilon)] - [1 - F(y_{ij} + s + \varepsilon)] f(y_{ij} - s + \varepsilon)$$

If f is log-concave, the hazard rate $\frac{f(z)}{1-F(z)}$ is increasing in z[15]. Hence for all $s > 0$,

$$\frac{f(y_{ij} + s + \varepsilon)}{[1 - F(y_{ij} + s + \varepsilon)]} > \frac{f(y_{ij} - s + \varepsilon)}{[1 - F(y_{ij} - s + \varepsilon)]}$$

[15] See An 1998

and therefore $h'(s) > 0$ for all $s > 0$. Since $h(0) = 0$ this implies that $h(s) > 0$ holds for all $s > 0$.

Now we can write

$$\hat{p}_i(s_{ij} + s_{ik}, 0) - \hat{p}_i(s_{ij}, s_{ik}) = \int h(s_{ik}) \left(\Pi_{l \neq i,j,k}\left[1 - F(y_{il} + \varepsilon)\right]\right) f(\varepsilon)\, d\varepsilon$$

and since $h(s_{ik}) > 0$ it follows that $\hat{p}_i(s_{ij} + s_{ik}, 0) > \hat{p}_i(s_{ij}, s_{ik})$. This completes the prove that every contestant sabotages at most one of his opponents.

Next I show that if $s_{ik} > 0$ in an equilibrium, then $\hat{p}_k < \hat{p}_j$ for all $j \neq i, k$ in this equilibrium. Towards a contradiction suppose that $s_{ik} > 0$ and $\hat{p}_k \geq \hat{p}_j$, or $y_{ik} \leq y_{ij}$. Consider what happens if i decreases s_{ik} to zero and increases s_{ij} by the same amount. This wouldn't change $i's$ cost. But (I claim) it would increase his probability of getting a prize. To see this, let

$$\begin{aligned}\hat{p}_i(s_{ij}, s_{ik}) &= 1 - \int \left[1 - F(y_{ij} + \varepsilon)\right]\left[1 - F(y_{ik} + \varepsilon)\right] * \\ &\quad \left(\Pi_{l \neq i,j,k}\left[1 - F(y_{il} + \varepsilon)\right]\right) f(\varepsilon)\, d\varepsilon\end{aligned}$$

denote $i's$ probability of getting a prize if i sabotages k by $s_{ik} > 0$ and j by s_{ik}, and let

$$\begin{aligned}\hat{p}_i(s_{ij} + s_{ik}, 0) &= 1 - \int \left[1 - F(y_{ij} + s_{ik} + \varepsilon)\right]\left[1 - F(y_{ik} - s_{ik} + \varepsilon)\right] * \\ &\quad \left(\Pi_{l \neq i,j,k}\left[1 - F(y_{il} + \varepsilon)\right]\right) f(\varepsilon)\, d\varepsilon\end{aligned}$$

denote $i's$ probability of getting a prize if he shifts all sabotage from k to j. We want to show that $\hat{p}_i(s_{ij} + s_{ik}, 0) > \hat{p}_i(s_{ij}, s_{ik})$. Define

$$\tilde{h}(s) = \{\left[1 - F(y_{ij} + \varepsilon)\right]\left[1 - F(y_{ik} + \varepsilon)\right] - \left[1 - F(y_{ij} + s + \varepsilon)\right]\left[1 - F(y_{ik} - s + \varepsilon)\right]\}$$

By essentially the same argument as the one following equation (40) above one can show that $\tilde{h}(s) > 0$ for all $s > 0$.

Now we can write

$$\hat{p}_i(s_{ij} + s_{ik}, 0) - \hat{p}_i(s_{ij}, s_{ik}) = \int \tilde{h}(s_{ik}) \left(\Pi_{l \neq i,j,k}\left[1 - F(y_{il} + \varepsilon)\right]\right) f(\varepsilon)\, d\varepsilon$$

and since $\tilde{h}(s_{ik}) > 0$ it follows that $\hat{p}_i(s_{ij} + s_{ik}, 0) > \hat{p}_i(s_{ij}, s_{ik})$. This shows that if $s_{ik} > 0$, then $\hat{p}_k < \hat{p}_j$ for all $j \neq i, k$.

Finally, I show that if $s_{ik} > 0$, then $x_k - \sum_{l \neq i,k} s_{lk} \leq x_j - \sum_{l \neq i,j} s_{lj}$ for all $j \neq i, k$. To see this, suppose that in an equilibrium we have $s_{ik} > 0$ and $x_k - \sum_{l \neq i,k} s_{lk} > x_j - \sum_{l \neq i,j} s_{lj}$. From the considerations above we know that we must have $s_{ij} = 0$ and $x_k - \sum_{l \neq i,k} s_{lk} - s_{ik} < x_j - \sum_{l \neq i,j} s_{lj}$. I claim that by decreasing s_{ik} to zero and increasing s_{ij} by the same amount, i can increase the probability that he will get a prize.

Suppose that i decreases s_{ik} in two steps. In the first step, s_{ik} is decreased to $\hat{s}_{ik} = x_k - \sum_{l \neq i,k} s_{lk} - \left(x_j - \sum_{l \neq i,j} s_{lj} \right) > 0$ and s_{ij} is increased to $\hat{s}_{ij} = s_{ik} - \hat{s}_{ik} > 0$. This does not change $\hat{p}_i$, since it only reverses the roles of j and k.

After this change, we have $x_k - \sum_{l \neq i,k} s_{lk} - \hat{s}_{ik} \geq x_j - \sum_{l \neq i,j} s_{lj} - \hat{s}_{ij}$ or $p_k \geq p_j$. Now suppose i now decreases $\hat{s}_{ik}$ to zero and increases $\hat{s}_{ij}$ by the same amount. By the considerations above, this raises $\hat{p}_i$.

9 Omitted calculations (not for publication)

9.1 With a Gumbel distribution, inequality (6) holds if the variance of the error terms is high enough

In this section I study the case where the error terms follow a Gumbel (or extreme value type I) distribution, $F(\varepsilon) = \exp\left(-\exp\left(-\frac{x}{\sigma}\right)\right)$, and show that inequality (6) holds if, and only if, the variance of the error terms is high enough.

Obviously F is log-concave. With this distribution[16]

$$\int_{-\infty}^{\infty} F(z+\varepsilon)^{n-1} f(\varepsilon)\, d\varepsilon = \frac{\exp\left(\frac{z}{\sigma}\right)}{\exp\left(\frac{z}{\sigma}\right) + (n-1)}.$$

Therefore,

$$\frac{\partial^2}{\partial z^2} \int_{-\infty}^{\infty} F(z+\varepsilon)^{n-1} f(\varepsilon)\, d\varepsilon = e^{\frac{z}{\sigma}} \frac{n-1-e^{\frac{z}{\sigma}}}{\left(e^{\frac{z}{\sigma}} + n - 1\right)^3} \frac{(n-1)}{\sigma^2}. \tag{41}$$

To maximize this expression over z, first note that we can, without loss of generality, restrict ourselves to values of z that satisfy

$$n - 1 - e^{\frac{z}{\sigma}} > 0. \tag{42}$$

[16] This and further results can be found in Anderson, dePalma and Thisse 1992.

Therefore, we can take logs and solve (ignoring the multiplicative constant $\frac{(n-1)}{\sigma^2}$)

$$\phi(z) := \frac{z}{\sigma} + \ln\left(n - 1 - e^{\frac{z}{\sigma}}\right) - 3\ln\left(e^{\frac{z}{\sigma}} + n - 1\right) \to \max_z$$

The function $\phi(z)$ is strictly concave. It is maximized by $z = \left(\ln\left(2 - \sqrt{3}\right)(n-1)\right)\sigma$. Plugging this back into (41) gives us

$$\max_z \frac{\partial^2}{\partial z^2} \int F(z+\varepsilon)^{n-1} f(\varepsilon)\, d\varepsilon = \frac{k}{\sigma^2},$$

where $k := \frac{\left(-1+\sqrt{3}\right)\left(-2+\sqrt{3}\right)}{\left(-3+\sqrt{3}\right)^3}$ is a constant somewhat smaller than 0.1. It follows that inequality (6) holds if and only if

$$\sigma^2 > \frac{n^2 - 2n + 2}{(n-1)^2} kw.$$

The variance of the Gumbel distribution is equal to $var(\varepsilon) = \frac{\sigma^2\pi^2}{6}$. Therefore, inequality (6) holds if and only if

$$var(\varepsilon) = \frac{\sigma^2\pi^2}{6} > \frac{n^2 - 2n + 2}{(n-1)^2} kw \frac{\pi^2}{6}.$$

9.2 Derivation of equation (37)

Here I give a sketch of the derivation of the reduced problem of a low cost player in the case $l \geq 2$, $h \geq 2$.

From part a) of lemma 2 we can immediately reduce the optimization problem of i to three dimensions:

$$u_i(x_i, s_{il}, s_{ih}) =$$
$$= \int_{-\infty}^{\infty} F(y_{il} + \varepsilon)^{l-1} F(y_{ih} + \varepsilon)^h f(\varepsilon)\, d\varepsilon w - \frac{1}{2}\left(x_i^2 + (hs_{ih} + (l-1)s_{il})^2\right)$$
$$\to \max_{x_i, s_{il}, s_{ih}}$$

where

$$y_{il} = x_i - S_l - (wg - (l-2)s_{ll} - s_{il} - hs_{hl}),$$

and

$$y_{ih} = x_i - S_l - \left(\frac{wg}{\gamma} - (l-1)\, s_{lh} - s_{il} - (h-1)\, s_{hl}\right).$$

Using lemma 4d) we get

$$\frac{1}{2}\left(x_i^2 + (hs_{ih} + (l-1)\, s_{il})^2\right) = \frac{1}{2}\left(x_i^2 + \left(\frac{1}{n-1}x_i\right)^2\right) = \frac{1}{2}bx_i^2.$$

By lemma 4c), in the optimum $p_i = \int_{-\infty}^{\infty} F\,(y_{il}+\varepsilon)^{n-1} f\,(\varepsilon)\, d\varepsilon$. We want to find an expression for y_{il} that depends only on x_i. Use lemma 4b) to express s_{ih} as a function of s_{il}. Then substitute this into lemma 4d) and solve for s_{il} to express s_{il} as a function of x_i. Finally, plug the result into y_{il} to get $y_{il} = b\,(x_i - wg)$.

This shows that the reduced problem of player i is indeed to maximize the function $\hat{u}_i\,(x_i)$ given in equation (37) above.

9.3 Proof of Proposition 6b

Suppose the costs functions are as given in (12) and symmetric in the sabotage activities. If the equilibrium is an interior equilibrium and $x_i - s_{ji} > x_j - s_{ij}$, then $\sum_{k\neq i,j} s_{ki} > \sum_{k\neq i,j} s_{kj}$.

Proof. Suppose that $x_i - s_{ji} > x_j - s_{ij}$ in an interior equilibrium. Towards a contradiction further suppose $\sum_{k\neq i,j} s_{ki} \leq \sum_{k\neq i,j} s_{kj}$. Then $E\,(q_i) > E\,(q_j)$ and by lemma 1 we have for all $k \neq i, j$

$$\frac{\partial p_k}{\partial s_{ki}} > \frac{\partial p_k}{\partial s_{kj}}.$$

In an interior equilibrium we have for all k and all $l \neq k$:

$$\frac{\partial p_k}{\partial s_{kl}} w = \frac{\partial c_k}{\partial s_{kl}}$$

and therefore

$$\frac{\partial c_k}{\partial s_{ki}} > \frac{\partial c_k}{\partial s_{kj}}. \tag{43}$$

The next step is to show that this implies $s_{ki} > s_{kj}$. To see this, write (as an abbreviation)

$$c_k\,(s_{ki}, s_{kj}) := c_k\,(x_k, s_{k1}, ..., s_{ki}, s_{kj}, ...s_{kn}).$$

Since a convex function is underestimated by a linear approximation,

$$c_k\left(s_{kj}, s_{ki}\right) \geq c_k\left(s_{ki}, s_{kj}\right) + \frac{\partial c_k\left(s_{ki}, s_{kj}\right)}{\partial s_{ki}}\left(s_{ki} - s_{kj}\right) + \frac{\partial c_k\left(s_{ki}, s_{kj}\right)}{\partial s_{kj}}\left(s_{kj} - s_{ki}\right)$$

By symmetry, $c_k\left(s_{kj}, s_{ki}\right) = c_k\left(s_{ki}, s_{kj}\right)$. Hence

$$0 \geq \left(s_{ki} - s_{kj}\right)\left(\frac{\partial c_k\left(s_{ki}, s_{kj}\right)}{\partial s_{ki}} - \frac{\partial c_k\left(s_{ki}, s_{kj}\right)}{\partial s_{kj}}\right)$$

Therefore $s_{kj} > s_{ki}$ implies $\frac{\partial c_k\left(s_{ki}, s_{kj}\right)}{\partial s_{ki}} \leq \frac{\partial c_k\left(s_{ki}, s_{kj}\right)}{\partial s_{kj}}$.

Now suppose $s_{ki} = s_{kj}$. By symmetry, $\frac{\partial c_k\left(s_{ki}, s_{kj}\right)}{\partial s_{ki}} = \frac{\partial c_k\left(s_{kj}, s_{ki}\right)}{\partial s_{kj}}$. Hence $s_{ki} = s_{kj}$ implies $\frac{\partial c_k\left(s_{ki}, s_{kj}\right)}{\partial s_{ki}} = \frac{\partial c_k\left(s_{kj}, s_{ki}\right)}{\partial s_{kj}}$.

Putting things together, we have shown that $s_{ki} \leq s_{kj}$ implies $\frac{\partial c_k\left(s_{ki}, s_{kj}\right)}{\partial s_{ki}} \leq \frac{\partial c_k\left(s_{ki}, s_{kj}\right)}{\partial s_{kj}}$. Of course this equivalent to

$$\text{if } \frac{\partial c_k}{\partial s_{ki}} > \frac{\partial c_k}{\partial s_{ki}}, \text{ then } s_{ki} > s_{kj}. \tag{44}$$

From equations (43) and (44), we have $s_{ki} > s_{kj}$. Summing over all k, we find that $\sum_{k \neq i,j} s_{ki} > \sum_{k \neq i,j} s_{kj}$, a contradiction. ■