Does Central Bank Transparency Matter for Economic Stability ? *

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Abstract

This paper studies the impact of monetary policy transparency on economic stability, when economic agents are boundedly rational. I first consider a simple class of microfunded general equilibrium models with nominal rigidities and learning. Under a transparent monetary regime, market participants have information about how monetary policy is conducted and use it when forming their forecasts. The paper shows that under plausible assumptions about the model environment, a transparent implementation of simple policy rules improves stability under learning dynamics. It is also shown that, independently of the degree of central bank transparency, the Taylor Principle is generally not sufficient to guarantee robustness of the rational expectations equilibrium to expectational mistakes by the central bank or the private sector.

The paper also attempts an evaluation of the benefits of transparency using a calibrated model of US data.

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1 Introduction

Monetary policy design is a difficult task, given the lack of consensus about a 'correct' model of the economy. Two main sources of model uncertainty are the monetary transmission mechanism and the market expectations' formation process. Recent monetary theory[1] has proposed the adoption of simple instrument rules, dictating that the interest rate should respond to deviations of inflation and output gap from their targets. Simple rules are claimed to be 'robust' to uncertainty about the true model.

On one side, many studies have attempted to design an optimal simple rule that is robust to different hypothesis about the impact of monetary policy on the economy[2]. On the other side, a growing literature on bounded rationality has focused on the robustness of simple policy rules to small expectations mistakes[3] on the part of central banks and the private sector.

A given policy rule is robust to expectational mistakes if it gives a satisfactory performance also when expectations are out of the (rational expectations) equilibrium, as a result of a change in policy or a structural change in the economy. The criterion used to evaluate the performance of a rule is whether it induces stability under learning. Thus we are interested in whether economic agents who correct their expectations over time, as new data are available, will converge to the rational expectation equilibrium (REE).

This paper explores the effects of central bank transparency on the performance of simple policy rules. I consider the hypothesis that the way a given policy rule is *implemented* might affect its robustness to expectational mistakes. In particular, central banks' transparency might affect economic stability: if market participants have access to some information about central bank actions, this can improve their predictions and stabilize the economic system. Some degree of *transparency and credibility might improve the private sector's learning process, affecting the stability under learning.*

Nevertheless, knowing the central bank policy actions does not mean that market participants fully understand the 'true' model of the economy, especially in the case of decentralized markets, where agents ignore each others' tastes, production possibilities and expectations. That implies that even a well understood policy rule might lead to economic instability, where the agents forecasting process does not converge to the REE.

Following Faust and Svensson (2000), I define transparency as the degree to which the central bank's intentions can be inferred by market participants. For example, a transparent central bank should provide monetary policy reports that explain and motivate its policy choices and should publish inflation and output forecasts, used for policy decisions.

[1] For example, see Woodford (2003).

[2] Among the others, Hansen and Sargent (2000) or Onatski and Stock (2000).

[3] See for example Bullard and Mitra (2002), Evans and Honkapohja (2002), Howitt (1992) and Preston (2002), Sargent and Williams (2002), Sargent (1999).

In the context of simple policy rules, I model transparency as the public's knowledge about the policy rule. In particular, knowledge about the *form* of the policy rule and about the main variables to which the central bank is responding when the interest rate is set. Still, I allow for the possibility that the central bank might not be fully transparent. This is captured by imperfect knowledge about the *coefficients* of the policy rule, i.e. how the interest rate reacts to the economic variables. A discretionary element of policy (modelled as white noise) makes more difficult for the public to infer the policy coefficients.

Current contribution to policy design in a bounded rationality framework assume that economic agents learn by recursive updating of an unrestricted VAR. This approach is not suitable for the analysis of transparency because it does not allow the agents to use prior information about the monetary policy rule. In the paper, learning with unrestricted VAR corresponds to the case of no transparency or secrecy on the part of the central bank. The agents have no information about the central bank decision process.

I propose a framework where market participants have a model of the economy that includes the monetary policy rule and its effects on output and inflation. The agents estimate recursively their model using recursive instrumental variable estimators. In order to study the conditions for stability under learning in this framework, I apply the results from stochastic approximation theory, elaborated by Marcet and Sargent (1989) and Evans and Honkapohja (2001), to study the convergence properties of recursive simultaneous equation estimation.

In this paper monetary policy is conducted by setting the interest rate according to a Taylor rule. The economy is described by a simple forward looking microfunded model with nominal rigidities, on the lines of Woodford (2002) and Benhabib et al. (2001). Given the uncertainty about the impact of monetary policy on the economy, I consider different hypotheses. I first analyze a version of the model where monetary policy has immediate effects on output and inflation. This is the most common assumption in the literature, but empirical evidence shows that monetary policy has little immediate effect on real activity and inflation. For this reason, following Woodford (2002) I consider a more general version of the model where there are delays in the effects of monetary policy. I assume that expenditures and pricing decisions are made *in advance*, and thus depend on old information about the economic conditions.

I show that in the case of a monetary policy rule that reacts to *current* inflation and output gap or in the case that expectations about current and future interest rates do not affect output gap and inflation, stability under learning is not affected by transparency. This seems to imply that some rules are destabilizing *per se*, independently of the way they are implemented. In other words, instability occurs even if every agent in the economy understands how monetary policy is conducted. In this case, even *expected and fully credible* changes in the policy rule lead the economy to instability induced by self-fulfilling expectations. I also show that the cost channel of monetary policy modifies the stability properties of Taylor rule. For a Taylor rule to induce a learnable equilibrium, the interest rate should react to some degree to the output gap, even in the case of perfect transparency of the central bank. This implies that the Taylor Principle, stating that a Taylor rule is stabilizing if it reacts aggressively to deviations of the inflation rate from target, is not a *sufficient condition* to guarantee stability. It also implies that determinacy of the mean state variable solution of a model under rational expectations is not enough to guarantee the stability of the equilibrium under learning, in contrast with the findings of McCallum (2002). A similar result is found in Preston (2003), in a different model environment.

Conversely, under plausible assumptions about the monetary transmission mechanism, *I show that lack of transparency can induce instability even if desirable policies are adopted.*

In the second part of the paper I attempt an evaluation of the effects of central bank transparency on the volatility and persistence of inflation output and the interest rate. I also propose an estimation approach for the monetary model with learning. The implementation is left for further research.

The results of the paper might suggest an alternative explanation for the observed response of the economy to a monetary policy shock observed in the US data. The response is more dramatic in the 70's and it is extremely reduced in the 90's. This might be due to changes in the public's understanding of monetary policy, rather than changes in the policy rule.

The paper is organized as follows. The first section introduces the model and discusses its stability under Rational Expectations. The second and third sections describe the model and its solution. In sections 4-6 I discuss the stability result under different hypothesis about the model environment. Finally, in section 7 I describe some simulation results and the plans for future research.

2 A Simple Model

I consider a simple model of the economy, on the lines of Woodford (2003) and Benhabib et al. (2001). The model is fully forward-looking, explicitly microfunded and displays sticky prices. In order to keep the analysis simple I abstract from capital accumulation. The economy is populated by a continuum of identical consumers/producers.

Each agent j produces a differentiated good (Y^j) in a monopolistically competitive market. Assuming a fixed capital stock, labor and money services are the production inputs. Therefore, output is produced according to

$$Y_t^j = f\left(h_t^j, \frac{M_t^j}{P_t}\right) \tag{1}$$

where M_t^j denotes nominal money balances and P_t is an index of the price level. In the paper I also consider the case where money gives direct utility to the consumer, because it facilitates transactions. The production function f satisfies the standard conditions. Each agent consumes a composite good C_t^j, obtained by some aggregation of each single differentiated goods produced. The aggregate demand for each good depends on the aggregate income and the relative price of the good

$$Y_t^j = D_t\left(\frac{P_t^j}{P_t}\right)Y_t \tag{2}$$

where Y_t is aggregate output, P_t^j is the price of good j and $t$he function D_t is assumed to be decreasing in the price and satisfies the following two conditions: $D_t(1) = 1$ and $\theta_t = D_t'/D_t < -1$ for every t. The parameter θ_t, measuring the elasticity of demand is assumed to vary over time according to an exogenous process. This implies time-varying mark-ups for the producers and introduces a source for supply shocks in the economy.

The economy is represented by a continuum of consumers-producers that seek to maximize the value of the sum of future expected utilities of the form

$$E_{t-1}^j\left\{\sum_{s=t}^{\infty}\beta^{s-t}U\left(C_s^j,\frac{M_s^j}{P_s}\right)-V\left(h_s^j\right)+\frac{\gamma_s}{2}\left(\frac{P_s^j}{P_{s-1}^j}-\Pi^*\right)^2\right\} \tag{3}$$

where M_t^j denotes nominal money balances held by agent j and Π^* is the steady state gross inflation rate. Also, $U(.,.)$ is the utility function from consumption and money balances and $V(.)$ denotes the disutility from labor. Moreover, $U(.,.)$ is assumed to be increasing, twice differentiable and strictly concave and $V(.)$ is increasing, twice differentiable and strictly convex. In particular, I consider the case in which $U(.,.)$ is non-separable in consumption and real money balances[4]. This might be considered the most empirically relevant case, given that the marginal benefit of additional real balances increases as consumption (i.e. total transactions) increases[5].

The last term in (3) denotes the cost of changing the current price. Assuming $\gamma_t = \bar{\gamma}(1+\theta_t) < 0$ for every t implies price stickiness (but not sticky inflation). The choice of convex adjustment costs follows Rotemberg (1982) and it is dictated by the necessity to keep the non-linear model as simple as possible. The linearized solution of the model takes the same form that would be obtained if a Calvo pricing scheme is used (even if the parameters have a different interpretation). Notice that the adjustment cost term has a plausible behavioral interpretation in an economic environment where gross inflation is close to Π^*, which is the case I consider in the paper.

The expectation operator $\hat{E}_{t-1}^i$ denotes the subjective beliefs of agent j about the probability distribution of the model's state variables. Given the assumption that the agents do not know the true model of the economy, the "^" denotes non-rational expectations. Notice that agents take decisions for time t consumption and production, on the basis of $t-1$ information. This can be interpreted in two ways: either the agents plan their consumption, production and asset holding in advance or they act on the basis of old information.

[4]The case where the utility function is separable is the most analyzed in the literature about policy rule.

[5]For details, see Woodford (2002).

I assume that financial markets are incomplete, and the only non-monetary asset that is possible to trade is a one period riskless bond. The agents' flow budget constraint is

$$M_t^j + B_t^j \leq \left(1 + i_{t-1}^m\right) M_{t-1}^j + \left(1 + i_{t-1}\right) B_{t-1}^j + P_t^j f\left(h_t^j, M_t^j\right) - T_t - P_t C_t^j \tag{4}$$

where B_t^jdenotes the riskless bond, i_t^m denotes the interest paid on money balances and i_t denotes the interest paid on the bond. Also T_t denotes lump-sum taxers from the government.

2.1 Monetary and Fiscal Policy

I assume that the government is committed to a zero debt fiscal policy. As a consequence, taxes evolve as follows

$$T_t = \left(1 + i_{t-1}^m\right) M_{t-1} + P_t G_t - M_t$$

where G_t defines government expenditures. I further assume that G_t is an exogenous AR(1) process.

Monetary policy is conducted according to a *simple* interest rate rule. I consider two possible ways of implementing the interest rule.

1. Quantity adjustments. The Central Bank decides the target interest rate and then implement it trough quantity adjustment in the money supply. In this case, i_t^m is fixed to zero.

2. Adjustments of the interest paid on the monetary base. In this case I assume that the central bank changes i_t^m as the target interest rate is modified, in order to keep a constant spread: i.e. $\hat{\imath}_t = \hat{\imath}_t^m$.

The policy rule determines the interest rate as a function of the current state of the economy, or estimates of it. In the course of the paper I consider different rules that are commonly considered in the literature. In order to keep the analysis simple, this paper does not consider *inertial* rules[6]. This case would require a separate study. In its general form the policy rule can be expressed as

$$i_t = \bar{\imath} + \phi_\pi E_{t-1}^{CB}(\pi_t - \pi^*) + \phi_x E_{t-1}^{CB}\left(x_t - x^*\right) + \epsilon_t \tag{5}$$

where x_t denotes the output gap, as defined below. In order to keep the analysis simple, I do not explicitly consider what decision process leads to a specific choice of the policy coefficients[7]. Forecasts by the central bank, E_{t-1}^{CB}, might be different from private sector's. Notice that I consider the hypothesis that the bank's rule is 'operational' in the sense of McCallum (1999). Also, ϵ_t can be seen as a control error or some discretionary component, modelled as white noise.

[6] See Woodford (2002).

[7] For example, the optimal non-inertial rules, under both commitment and discretion, in Giannoni and Woodford (2002).

3 Model Solution

The solution of the model gives a sequence of consumption, labor, and money balances that maximizes (3) subject to (4) and a market clearing condition. In equilibrium, agents will take identical production, consumption and saving decisions. I further assume that they have the same beliefs about the economy (even though they might not be aware of that). Appendix A describes the model solution in both cases where money enters in the utility function or in the production function. Assuming that consumption, money and pricing decisions are predetermined $t-1$ periods in advance, the log-linearized model is described by the following equations. The demand side of the economy is

$$x_t = -\tilde{\sigma} E_{t-1}^{PS} \left[\eta_1 \hat{\imath}_t - \eta_2 \hat{\imath}_t^m - \pi_{t+1} - \eta_2 \left(\hat{\imath}_{t+1} - \hat{\imath}_{t+1}^m \right) - \hat{r}_t^n \right] + E_{t-1}^{PS} x_{t+1} \tag{6}$$

where $\tilde{\sigma}, \eta_1$ and $\eta_2 = \eta_1 - 1$ depend on the coefficients of the demand for money and the utility function. E_{t-1}^{PS} denotes the expectation operator for the private sector. Also, $x_t = \hat{Y}_t - \hat{Y}_t^e$, is the output gap, defined as output deviations from the *efficient level* of output, obtained in the case of fully flexible prices and in the absence of mark-up shocks. Finally, $\hat{r}_t^n$ is the *natural* rate of interest, assumed to be evolving as an AR(1) process.

If monetary frictions are modeled by having money in the utility function, real money balances affect intertemporal consumption decisions. As shown in (6), expected high interest rates on nonmonetary assets, and therefore expected low money balances, stimulate consumption today relatively to next period's. Notice that this holds only in the case where the interest differential between monetary and nonmonetary assets is allowed to vary. This is the case where the policy rule is implemented through adjustment in the supply of money (in this case $\hat{\imath}_t^m = 0$). If the central bank keeps a fixed spread between monetary and nonmonetary asset, then the IS equation is equivalent to (6) with $\eta_1 = 1$ and $\eta_2 = 0$. Only the expected current interest rate on nonmonetary assets affects the evolution of the output gap. In the case of money in the production the IS equation is equivalent to (6) with $\eta_1 = 1$ and $\eta_2 = 0$, independently of how monetary policy is implemented.

Independently of how money enters in the model, the key parameter that describes the demand effects of monetary policy is $\tilde{\sigma}$: it is proportional to the intertemporal marginal rate of substitution of consumption. As shown in the Appendix, the value of $\tilde{\sigma}$ is higher in the case of monetary frictions, thus magnifying the effects of interest rate changes on the output gap.

The supply side of the economy is described by the following Phillips curve

$$\pi_t = E_{t-1}^{PS} \beta \pi_{t+1} + \kappa E_{t-1}^{PS} \left[\hat{x}_t + \eta_3 \left(\hat{\imath}_t - \hat{\imath}_t^m \right) \right] + u_t \tag{7}$$

where κ measures the inflation response to the output gap and η_3 measures the supply-side effects of monetary policy. An increase in interest rates differentials increases the opportunity cost of holding money and thus increases the marginal cost of production. Equation (7) with $\hat{\imath}_t^m = 0$ has the same functional form of the supply curve obtained from a simple model including the cost channel of monetary policy[8]. The parameter that captures the size of the cost channel is η_3: it is different from zero in both cases of money in the production function and money in the utility function[9].

The equation includes a cost-push shock u_t, that depends on shocks to the mark-up of firms. The shock is assumed to be generated by an AR(1) process.

The model of the economy is given by equations (6), (7), and the policy rule (5). It can be written in matrix term as follows

$$V_t = A_0 + A_1^{PS} E_{t-1}^{PS} V_t + A_1^{CB} E_{t-1}^{CB} V_t + A_2^{PS} E_{t-1}^{PS} V_{t+1} + A_3 X_t \tag{8}$$

$$X_t = H X_{t-1} + \zeta_t$$

$$H = \begin{bmatrix} \rho_r & 0 \\ 0 & \rho_u \end{bmatrix}$$

where $V_t = \begin{pmatrix} x_t & \pi_t & i_t \end{pmatrix}'$, $X_t = \begin{pmatrix} \hat{r}_t^n & u_t \end{pmatrix}'$ and ζ_t is a vector of i.i.d. shocks. In order to close the model I need to specify how expectations are formed.

3.1 The Expectation Formation Mechanism: Methodology

In this paper I follow the "Euler Approach" to econometric learning, as defined in Evans, Honkapohja and Mitra (2002) and widely used and discussed in Evans and Honkapohja (2001) and Marcet and Sargent (1989). It predicts that agents' behavior is based on equations (6) and (7), derived from the Euler equations, under the assumption of possibly non fully rational expectations[10]. The agents are modelled as econometricians. They are endowed with beliefs about the law of motion of the main economic variables. Their Perceived Law of Motion (PLM) includes all the relevant variables and is asymptotically correctly specified. As a result, the agents will eventually learn to make rational predictions, if the learning process converges to the Rational Expectations Equilibrium.

As noted in Preston (2002), the Euler equations are not the optimal decision rules given the assumed beliefs and microfundations. In fact,

the agents should take into account not only the flow budget constraint but also their intertemporal budget constraint. This results on decision rules that depend on infinite horizon forecasts. My choice is based on the analytical simplicity of the Euler approach. Nevertheless, the decision rules of the agents *converge asymptotically to the optimal decision rule*, under the assumption that their initial wealth is zero.

[8] See for example Christiano and Eichenbaum (1992) and Ravenna and Walsh (2003).

[9] Assuming money as a productive asset is a simple and coherent way to evaluate the cost channel of monetary policy, given the evidence that the highest fraction of money demand comes from firms.

[10] A recent contribution by Preston (2002) proposes a different approach where the agents decision rules depend on long horizon forecasts. It would be possible to extend my analysis in that framework. This is left for future research.

I assume that market participants are atomistic and they are *not coordinated* on some shared belief on the model of the economy. Also, I assume that they cannot observe aggregate expectations about the macroeconomic variables. As a consequence, their *model* of the economy cannot be the true aggregate model represented by (6) and (7). Their Perceived Law of Motion (PLM) of the economy might include contemporaneous variables, like the output gap and the interest rate, but does not include aggregate expectations.

This implies that, even if the model is asymptotically correct; a) during the learning process the agents' model is misspecified; b) its coefficients *will not be policy invariant*. Adopting a new policy will require the agents to adjust their model, irrespective to their knowledge of how monetary policy is conducted. No matter how precise it is the knowledge about the policy rule, the agents are still uncertain about the economic environment and thus they cannot properly calculate the effects of the monetary policy on the main economic variables such as output, inflation and the interest rate[11]. Still, once the learning process has converged the model delivers the same forecasts as the true model.

In conclusion, knowledge of the policy rule does not eliminate the problem of stability under learning. It is then possible to evaluate whether transparency has effects on the stability under learning of a given policy rule.

4 The Model With Current Information

I consider first the version of the model which is mostly used for policy analyses, including stability under learning. Under the assumption of no delays, the model can be expressed in matrix notation as

$$V_t = \hat{A}_1 + \hat{A}_2 E_t^{PS} V_{t+1} + \hat{A}_3 X_t \tag{9}$$

for suitable matrices.

4.1 VAR Learning: The Case of No Transparency

As I mentioned in the section above, in the case of no transparency, the public is not given enough reliable information to use the policy rule to predict interest rate movements and its impact on output gap and inflation. In this case, I model the agents' prediction process following Evans and Honkapohja (2001) and Marcet and Sargent (1989). I assume that each agent has the same PLM

$$V_t = \Omega_{0,t-1} + \Omega_{1,t-1} X_t + e_t \tag{10}$$

[11] This is because the information available to the agents is not enough to recover all the policy-invariant parameters that define the economy. In other words, the model that they estimate is still subject to the Lucas critique, since the parameters change with the monetary policy rule.

where output gap, inflation and the interest rate depend on exogenous shocks. Also, e_t denotes a vector of perceived i.i.d. shocks. The PLM is linear and include all the variables that are included in the MSV solution of the model. Thus the model is consistent with the REE. Nevertheless, it is misspecified during the learning process. The agents estimate recursively the coefficients of their linear model using recursive least squares (RLS). They assume the model to have fixed coefficients. The coefficients are updated according to the following algorithm

$$\Omega_t = \Omega_{t-1} + \delta_t R_{t-1}^{-1} X_t \left(V_t - X_t' \Omega_{t-1} + o_t \right)' \tag{11}$$

$$R_t = R_{t-1} + \delta_t \left(X_t X_t' - R_{t-1} \right)$$

where $\Omega_t = \begin{pmatrix} \Omega_{0,t} & \Omega_{1,t} \end{pmatrix}'$, R_t is the precision matrix and δ_t is a decreasing sequence of gains, satisfying certain properties[12]. The updating equation includes an observational i.i.d. error,o_t that makes the learning process non trivial. As it is well known from Evans and Honakapohja (2001), stability of the REE under learning obtains if the E-Stability conditions are met. Inserting (10) into (9) gives the Actual Law of Motion (ALM) of the economic system

$$V_t = T' \left(\Omega_{0,t-1}, \Omega_{1,t-1} \right) W_t$$

where $W_t = \begin{pmatrix} 1 & X_t \end{pmatrix}'$. The E-Stability condition requires that the mapping between PLM and ALM to be locally stable at the REE, where $T(\Omega^*) = \Omega^*$. It is apparent why during the learning process the agents' model is misspecified. The ALM implies a model with time-varyng coefficients. The PLM is a correctly specified model of the economy only asymptotically, if the learning process converges to the REE.

The following proposition defines the conditions for learnability under reduced-form learning. In order to obtain clear analytical results I impose assumptions on some of the parameters, that are not contradicted by standard calibrations, as showed in Table I.

Table I

Woodford (2003)	**Calibration**				
$\sigma = 6.3$	$\kappa = 0.024$	$\beta = 0.99$	$\eta_1 = 1.56$	$\eta_3 = 0.89$	$\chi = 0.02$
Clarida et al. (1999)	**Calibration**				
$\sigma = 1$	$\kappa = 0.3$	$\beta = 0.99$	-	-	-

Proposition 1 *Assume that* $\frac{\eta_3}{\sigma} \leq 1$. *Assume that the private sector's learning process is described by (10) and (11).*

[12] See Evans and Honkapohja (2001).

(i) Then the REE is stable under learning if and only if

$$\kappa\left(\phi_{\pi}-1\right)+\phi_{x}\left(1-\beta\right)-\eta_{3}\kappa\phi_{x}>0 \tag{12}$$

Proof. see Appendix B. ■

The proposition shows a 'qualified' version of the results obtained by Howitt (1992) and Bullard and Mitra (2002) and Preston (2002), for the case of monetary frictions. If the policy rule is *too passive* or it prescribes an *excessive reaction* to the output gap, then the REE is unstable under learning. If the REE is unstable under learning, there will be self-fulfilling expectations leading to potentially explosive behavior of output and inflation.

In the analysis above I have assumed that the agents do not have information concerning the monetary policy rule. With knowledge about future policy actions the agents can improve their forecasts and this might affect the stability properties of a given policy rule. This amounts to asking the following question: is a policy rule violating (12) *inherently* destabilizing or is it the way the policy is *implemented* that affects its performance?

4.2 A Transparent Central Banker

When agents have information about how monetary policy is conducted and they are willing to use it to improve their forecasts. Consider the most plausible case where market participants know the *form* of the policy rule but not the exact value of the parameters. The policy rule that they estimate is

$$i_{t}=\psi_{0,t-1}^{j}+\psi_{\pi,t-1}^{j}\pi_{t}+\psi_{x,t-1}^{j}x_{t}+e_{t}^{3} \tag{13}$$

where the constant captures the long-run objectives of the central bank, i.e. the inflation target, and the coefficients describe how aggressive the policy is in responding to inflation and output gap deviations from target. The initial parameters $\begin{pmatrix}\psi_{0,0} & \psi_{\pi,0} & \psi_{x,0}\end{pmatrix}$ can be interpreted as the *initial level of credibility* of the central bank, depending on how close they are to the true parameter values. The agents might use the information about the policy rule to improve their forecast of inflation and output.

In order for this information to be useful for prediction, the agents need a model to identify the effects of monetary policy on output and inflation. Since, in a decentralized market, agents do not have specific information about other market participants tastes and expectations, their model does not include the average opinion and does not correspond to the true model. On the other hand, it explicitly includes the effects of monetary policy on output and inflation. The agents PLM for output will then be

$$x_{t}=b_{01,t-1}^{j}+\gamma_{1,t-1}^{j}i_{t}+b_{11,t-1}^{j}r_{t}^{n}+e_{t}^{1} \tag{14}$$

where each agent j can observe the current interest rate and the demand shock r_t^n. The equation for output gap takes into account the possible effects of monetary policy on the current output gap, and movements in the natural rate of interest. The inflation equation can take the form

$$\pi_t = b^j_{02,t-1} + \gamma^j_{2,t-1} x_t + b^j_{11,t-1} u_t + e^2_t \tag{15}$$

where the agents take into account that monetary policy has its effects on the inflation process because it affects the output gap. The equation includes also the cost-push shock. Aggregating across different agents, and using the fact that expectations are identical, it is possible to write the PLM in a more compact notation

$$\Gamma_{t-1} V_t = B_{0,t-1} + B_{1,t-1} X_t + e_t \tag{16}$$

which gives a *system of simultaneous equations*. Notice that the last equation of the system, corresponding to the interest rate, depends on the form of the policy rule.

Given that the agents estimate a model with the unique purpose of prediction, I should discuss what are the incentives to use information about the policy rule. Assuming that the agents do not take the into account the effects of their learning process on the aggregate variables, if the model is exactly identified, they should be indifferent between reduced form and structural estimation[13].

Nevertheless, the recursive updating of the estimator allows them to use *prior information* about the coefficients of the estimated policy rule, thus making structural estimation more suitable. In the case of perfect transparency and credibility, the agents know the value of the policy coefficients, and the PLM (16) allows them to use this information for prediction. Notice also that by estimating the policy rule (13) *they actually estimate an equation that is well specified at any point in time*, and not only asymptotically, as it is the case for the other equations[14].

The agents are assumed to estimate recursively the system (16). Least squares estimation would lead to inconsistency. Hence, I assume that they update the coefficients of their model by using Recursive Instrumental Variables (RIV). In order for the model (16) to be estimated, it needs to be *identified*, i.e. we need as many instruments as many endogenous variables. Recall that an exogenous variable of the model can be used as an instrument if it is not included in the equation. In (14) we have one endogenous variable, the interest rate, and one instrument available, the cost-push shock u_t. In the inflation equation there is also one endogenous variable, and the instrument available is the demand shock $\hat{r}^n_t$. Finally, estimation of the Taylor rule (13) requires two instruments, since both x_t and π_t enter the equation. Both the demand and supply shocks can be used as instruments, because they do not appear in the equation. Given the instruments, the recursive version of the estimator can be showed to be[15]

$$\theta_t = \theta_{t-1} + \delta_t \overline{R}^{-1}_{t-1} Q_t \left(V_t - \left(\theta'_{t-1} Z_t\right)' + o_t \right) \tag{17}$$

[13] For example, Dhrymes (1978).

[14] In fact, it is well known that during the learning process the agents' model is misspecified because the ALM has time varying coefficients. This does not not hold for the estimated Taylor rule.

[15] In order to simplify the convergence analysis I assume that the gain matrix $\bar{R}$ appears lagged in the updating equation.

$$\overline{R}_t = \overline{R}_{t-1} + \delta_t \left(Q_t Z_t' - \overline{R}_{t-1} \right)$$

where

$$\theta_t = \begin{pmatrix} b_{01,t} & b_{11,t} & \gamma_{11,t} & b_{02,t} & \gamma_{21,t} & b_{22,t} & \psi_{0,t} & \psi_{\pi,t} & \psi_{x,t} \end{pmatrix}'$$

Consider the matrices $\overline{R}_t$, Q_t and Z_t. The first matrix is

$$\overline{R}_t = \left(I_3 \otimes R_t^h \right)$$

where R_t^h is the matrix gain associated to each single equation (with $h = y, \pi, i$ respectively). Also

$$Q_t = (I_3 \otimes W_t)$$

is the matrix of instruments and $W_t = \begin{pmatrix} 1 & X_t \end{pmatrix}'$. Finally

$$Z_t = \begin{bmatrix} Z_t^x & \mathbf{0}_{3\times 1} & \mathbf{0}_{3\times 1} \\ \mathbf{0}_{3\times 1} & Z_t^\pi & \mathbf{0}_{3\times 1} \\ \mathbf{0}_{3\times 1} & \mathbf{0}_{3\times 1} & Z_t^i \end{bmatrix}$$

is the matrix of regressors, where

$$Z_t^x = \begin{pmatrix} 1 & r_t^n & i_t \end{pmatrix}'; \; Z_t^\pi = \begin{pmatrix} 1 & x_t & u_t \end{pmatrix}'; \; Z_t^i = \begin{pmatrix} 1 & x_t & \pi_t \end{pmatrix}'.$$

Using the PLM (16), the aggregate expectation is

$$E_t V_{t+1} = \Gamma_{t-1}^{-1} B_{0,t-1} + \Gamma_{t-1}^{-1} B_{1,t-1} H X_t$$

where I assume, without loss of generality, that the agents know the matrix H. Inserting the PLM in (9), we get the Actual Law of Motion

$$V_t = A_1 + A_2 \left(\Gamma_{t-1}^{-1} B_{0,t-1} + \Gamma_{t-1}^{-1} B_{1,t-1} H X_t \right) + A_3 X_t + A_4 \epsilon_t \tag{18}$$

that can be re-written

$$V_t = \tilde{T}' \left(\theta_{t-1} \right) W_t + A_4 \epsilon_t. \tag{19}$$

Given (19), the REE equilibrium is defined as the fixed point of the map $\tilde{T}(.)$, such that[16]

$$\tilde{T}(\theta^*) = \theta^*.$$

Inserting (19) in (17) I obtain the following stochastic dynamical system

$$\theta_t = \theta_{t-1} + \delta_t \overline{R}_{t-1}^{-1} Q_t \left(\tilde{T}' \left(\theta_{t-1} \right) W_t - \left(\theta_{t-1}' Z_t \right)' \right) + \delta_t \overline{R}_{t-1}^{-1} Q A_4 \epsilon_t \tag{20}$$

[16] Notice that the REE can be expressed in the form of (16). In fact, $\Omega_1^* = \Gamma^{-1^*} B_1^*$ and $\Omega_0^* = \Gamma^{-1^*} B_0^*$

and

$$\overline{R}_t = \overline{R}_{t-1} + \delta_t \left(Q_t Z_t' - \overline{R}_{t-1} \right).$$

The following Proposition describes the conditions for stability under learning.

Proposition 2 *Assume that* $\frac{\eta_3}{\widetilde{\sigma}} \leq 1$*. Assume that the private sector's learning process is described by (16) and (17).*

(i) In the case where $U_{cm} = 0$*. The REE is stable under learning if and only if (12) is Satisfied. Monetary policy transparency does not affect local stability under learning.*

(ii) In the case with $U_{cm} > 0$*, (or money enters in the production function), under Woodford (2002) and Clarida et al. (1999) calibration, the stability condition is not affected by policy transparency.*

(iii) Assume full transparency, i.e. each agents knows the coefficients of the policy rule. The stability conditions are unchanged.

Proof. see Appendix B. ■

The Proposition shows that under the current assumptions about the model of the economy being transparent does not help. Notice that if $U_{cm} = 0$ expectations about the future interest rate are not used for prediction. This does not mean that information about the policy rule is not useful to forecast future output and inflation, since they depend on the future interest rate. As stated in (ii), transparency does not help even if the agents need to predict the future interest rate. This leads to the following conclusions. Condition (12) does not depend on the way the monetary authority implements the policy rule. Even if *every agent* understands how monetary policy is conducted, a policy rule that violates (12) would lead to economic instability. Instability is determined by the fact that the agents in the economy are not coordinated on the REE, and do not fully understand the true model of the economy.

More generally, this result seems to suggest that improved predictability does not necessarily improve stability. Even full knowledge about the policy rule does not improve stability under learning. Even if a *fully credible* central bank *announces* a policy rule that violates (12), the outcome will be destabilizing. This conclusion suggests that some policy options are destabilizing per se, without the possibility for the central bank to improve on their performance.

5 The general case with delays

As mentioned in the introduction, there is evidence from empirical studies and central bank practices that the assumptions made in the previous section about the monetary transmission mechanism and the policy rule are at odds with the facts. Considering the monetary transmission mechanism, VAR evidence from Rotemberg and Woodford (1997) and Boivin and Giannoni (2002) shows that output and inflation respond to a monetary shock with lags. Concerning monetary policy rules, Orphanides (2003) shows that the Federal Reserve makes active use of forecasts about current and future values of inflation and output gap. The next section investigates the implications of these assumptions on the effects of central bank transparency.

5.1 The Case of a Non-Transparent Central Bank

As above, the agents only know the sets of variables that appear in the MSV reduced form solution of the model under rational expectations and they conjecture a linear relationship between output, inflation, interest rate and these variables. The central bank does not disclose information about the relevant variables to which reacts, its forecasts or the policy rule coefficients. Therefore, the agents forecast current and future interest rates by using an unrestricted VAR, which the interest rate to the $t-1$ observations of the exogenous processes. The PLM becomes

$$V_t = \Omega^h_{0,t-1} + \Omega^h_{1,t-1} X_{t-1} \tag{21}$$

where $h = PS$ denotes the aggregate PLM of the private sector and $h = CB$ denotes the forecast of the central bank. The following proposition describes the stability conditions under learning.

Proposition 3 *Assume* $\frac{\eta_3}{\tilde{\sigma}} \leq 1$. *Given the model (8) and the PLM (21).*

(i) The REE is locally stable under learning if the following conditions are satisfied:

$$\kappa\left(\phi_\pi - 1\right) + \left(1-\beta\right)\phi_x - \kappa\eta_3\phi_x > 0 \tag{22}$$

and

$$\phi_x > \hat{\phi}_x = \frac{\tilde{\sigma}\kappa\left[\phi_\pi\left(\frac{\tilde{\sigma}+\eta_3(2-\beta)}{\tilde{\sigma}}\right) - 1\right] + \left[\kappa\tilde{\sigma} - (1-\beta)\right](2-\beta)}{\left[1+\kappa\eta_3\right]\tilde{\sigma}} \tag{23}$$

(ii) there exist learning equilibria, where inflation fluctuates around the inflation target, even in the case (12) is satisfied and the REE is locally determinate and unique.

Proof. see Appendix C. ■

Predetermined economic decisions and less information about the state of the economy affect the stability conditions under learning. The choice of ϕ_x becomes crucial for stability. If the policy rule reacts too much to the output gap (22) is violated and instability occurs. But (23) states that the policy rule should react to some degree to the output gap. The stability condition is modified if we consider the model with small real balances effects (i.e. η_3 small). In this case, (22) approximates the Taylor principle but (23) requires a positive value for $\hat{\phi}_x$, even if $\eta_3 = 0$. Using the Woodford calibration with $\phi_\pi = 2$, $\hat{\phi}_x \simeq 0.05$. Using the Clarida et al. calibration $\hat{\phi}_x \simeq 0.9$. Figure 1 shows the case for this latter calibration.

Condition (22) is represented by the solid line, while condition (23) is represented by the dotted line. In the area below the solid line the equilibrium is determinate, under rational expectations. The shaded area shows the combination of policy parameters for which we obtain stability, if the central bank is not transparent. It is immediate to see that determinacy is obtained for a wider set of parameters.

Also, from (ii) in the proposition we know that for values of ϕ_x that is close to the bifurcation value, the l*earning process induces additional fluctuations in inflation, output and interest rate, even in the case of local learnability of the REE.* Notice that under rational expectations the unique equilibrium is the inflation target. It is the learning behavior that generates other equilibria where inflation fluctuates. In the next section I consider whether some degree of transparency can avoid these outcomes or if they inherently depend on the learning process.

Summing up, in order to get stability (at least locally) the central bank would need to increase its response to the output gap. But we know that central banks do not have accurate data about this variable, and that excessive response to this variable might lead to destabilizing policies[17]. Hence, on one side the central bank has to respond aggressively to the output gap, in order to coordinate expectations. On the other side this might not be an option, because of the scarce reliability of output gap estimates. In the next section, I consider whether improved transparency can promote economic stability.

5.2 The Case of a Transparent Central Banker

In this case, I assume that the agents make use of information about the central bank decision making. The central bank is assumed to be more transparent about its policy decisions. For example, it is clear about its goals and how to achieve them and it publishes its forecasts about the current inflation rate and output gap. This means that these forecasts are in the agents' information set, at the time they form their expectations. On the other side, I allow for the possibility that the bank is not fully transparent, i.e. it has an incentive to misrepresent the economic conditions. Hence, I consider the possibility that the central bank reports its forecast with an i.i.d. error, i.e. $\hat{E}^{CB}_{t-1}\pi_t = E^{CB}_{t-1}\pi_t + e_{\pi,t-1}$ and $\hat{E}^{CB}_{t-1}x_t = E^{CB}_{t-1}x_t + e_{x,t-1}$. The agents are assumed to estimate over time the following policy rule

$$i_t = \psi_{0,t-1} + \psi_{\pi,t-d}\hat{E}^{CB}_{t-1}\pi_t + \psi_{x,t-1}\hat{E}^{CB}_{t-1}x_t + e_t \tag{24}$$

which, assuming a potential i.i.d. observation errors, can be estimated consistently using recursive instrumental variables, by regressing i_{t-1} on $\hat{E}^{CB}_{t-2d}\pi_{t-1}$ and $\hat{E}^{CB}_{t-2d}x_{t-1}$. At the end of each period the agents update their estimate of the Taylor rule according to

$$\psi_t = \psi_{t-1} + \delta_t R^{-1}_{\psi,t-1} W_{t-1}\left(i_t - \psi'_{t-1}Z_{t-1}\right) \tag{25}$$

[17] See, Orphanides (2001) and Bullard and Eusepi (2003).

$$R_{\psi,t} = R_{\psi,t-1} + \delta_t \left(W_{t-1} Z'_{t-1} - R_{\psi,t-1} \right)$$

where

$$\psi_t = \begin{bmatrix} \psi_{0,t-1} & \psi_{x,t} & \psi_{\pi,t} \end{bmatrix}'; \quad Z_{t-1} = \begin{bmatrix} 1 & \hat{E}^{CB}_{t-1} x_t & \hat{E}^{CB}_{t-1} \pi_t \end{bmatrix}'.$$

Notice that, in order to form expectations at time t, the agents are assumed to use $t-1$ estimates of the coefficients. I also allow the agents to use a less efficient but simpler and more robust learning rule, i.e. the Stochastic Gradient Algorithm. The updating equation is defined as follows

$$\psi_t = \psi_{t-1} + \delta_t W_{t-1} \left(i_t - \psi'_{t-1} Z_{t-1} \right) \tag{26}$$

The model (8) can be re-expressed as follows

$$\begin{aligned} \tilde{V}_t &= B_0 + B_1 \psi'_{t-1} E^{CB}_{t-1} \tilde{V}_t + B_2 E^{PS}_{t-1} \tilde{V}_t + B_3 \psi'_{t-1} E^{CB}_{t-1} \tilde{V}_{t+1} + B_4 E^{PS}_{t-1} \tilde{V}_{t+1} + B_5 X_{t-1} + \tilde{\zeta}_t \quad (27) \\ i_t &= \bar{\imath} + \phi_\pi E^{CB}_{t-1} (\pi_t - \pi^*) + \phi_x E^{CB}_{t-1} (x_t - x^*) + \epsilon_t \end{aligned}$$

where $\tilde{V}_t = \begin{bmatrix} x_t & \pi_t \end{bmatrix}'$. Notice that I use the fact that the private sector can observe the central bank forecast, i.e. $E^{PS}_{t-1}\left(E^{CB}_{t-1}\right) = \hat{E}^{CB}_{t-1}$ and $E^{PS}_{t-1}\left(E^{CB}_{t}\right) = E^{PS}_{t-1}\left(E^{CB}_{t-1}\left(E^{CB}_{t}\right)\right) = \hat{E}^{CB}_{t-1}$. The latter equality can be justified in two ways; a) the central bank publishes forecasts of current and future inflation, b) the central bank makes available its forecasting procedures. In both ways, the bank makes it easier for the public to predict its future policy moves.

Since contemporaneous variables dot not enter in the true model, the agents estimate the behavior of output gap and inflation by using unrestricted VAR estimation. Their PLM is therefore

$$\tilde{V}_t = \tilde{\Omega}^h_{0,t-1} + \tilde{\Omega}^h_{1,t-1} X_{t-1} + e_t \tag{28}$$

where, again, $h = PS, CB$. The coefficients are updated using either RLS or SG. The following Proposition describes local stability under learning, under RLS and SG learning. To simplify the analysis I assume that the central bank and the private sector use the same learning rule.

Proposition 4 *Consider the case $U_{cm} = 0$.*

(i) In the case of Recursive Instrumental Variable or RLS learning, some degree of transparency implies REE stability, provided that the Taylor Principle is satisfied: full transparency is NOT needed for stability. The REE is locally unique;

(ii) In the case of SG learning, perfect transparency implies REE stability, provided that the Taylor Principle is satisfied. In the case of partial transparency stability is achieved provided the Taylor principle is satisfied and the eigenvalues of $M_x \tilde{\Omega}^{*\prime}$ are positive, where $M_x = E_{\lim t \to \infty} X_t X'_t$.

Consider the case $U_{cm} > 0$ (or money in the production function).

(iii) Transparency implies that the set of locally learnable Taylor rules is larger than in the case of no transparency; Nevertheless, pure inflation targeting leads to instability and learning equilibria. A sufficient condition for uniqueness and stability under learning, other than $\phi_\pi > 1$, is

$$\frac{\eta_3 + 1}{\phi_x} < \frac{\phi_\pi}{\phi_x} < \frac{\tilde{\sigma}}{\kappa \eta_3} \tag{29}$$

As $\beta \to 1$ condition (29) becomes also necessary.

Proof. see Appendix C. ■

The result shows that under a more plausible model environment transparency matters. It is important to remark that full transparency is not needed for stability. That is what makes the result appealing, given that full transparency (i.e. ϵ_t is observable by the public and no errors in expectations) is not observed in reality and it is not advocated by monetary theory, as showed in Faust and Svensson (2002).

Nevertheless, including expectations in the policy rule and having the agents taking decisions based on older information restricts the set of stabilizing policy rules, as showed in (29). The central bank needs to set $\phi_x > 0$ to guarantee stability. Even a *fully transparent policy violating (29) would be destabilizing*, as long as the agents have imperfect information about the economic environment. Lack of transparency has the effect of increasing the policy rules that do not induce instability under learning. Condition (29) is expressed in terms of the two key structural parameters of the model: $\tilde{\sigma}$ and η_3. If the demand channel of monetary policy is much stronger than the supply channel, stability can be achieved by choosing relatively low values of ϕ_x, which might be desirable, given the poor information available about the output gap. Estimates of η_3 in Ravenna and Walsh (2003) vary from 1.3 to around 5, indicating that a too low response to the output gap might lead to instability for plausible parameter values. Consider the following example. Assume $\phi_\pi = 2$, $\eta_3 = 0.89$ (or 2) and the other parameters at benchmark values. Then, for $\phi_x > 0.004$ (0.015) the REE is locally stable under learning. The Taylor rule prescribes a coefficient much higher than this value. Nevertheless, the central bank should choose ϕ_x much higher than 0.004, given that for values close to this, the learning behavior generates extra volatility in the main variables. Numerical simulations show that choosing $\phi_x > 0.13$ guarantees a monotone response of the economy to economic disturbances. This is very close to the coefficient of the Taylor, which is $\phi_x = 0.5/4$. Figures 2 and 3 show the response of output and inflation after a positive demand shock. In figure 2, which shows the case of $\phi_x = 0.13$, inflation increases as the output gap increases and it is promptly reduced. In figure 3, with $\phi_x = 0.05$, the increase in the output gap is followed by an initial increase in inflation and an oscillatory adjustment of the two variables. Notice that in the latter case the fluctuations in inflation and output gap are more pronounced[18]. Notice that

[18]The Figures show simulation of the ODE described in the text and in the appendix.

if (29) is satisfied the only equilibrium under learning is the REE, by the same argument used for the previous proposition.

Finally, notice that again *determinacy of the MSV under rational expectations is not sufficient to induce stability under learning*, even in the case where agents are informed about the policy rule. This results is in line with Preston (2003) and in contrast with McCallum (2002).

The results also show that if the public and the central bank update their information less efficiently, than even partial degrees of transparency can affect economic stability. Simulations show that SG learning does not induce instability, for plausible parameter values, but it slows down considerably the learning process, thus increasing substantially the volatility of the economic variables.

Concluding, the way a policy is implemented has effects on its stability under learning. A policy rule might be under performing not because the rule is inherently destabilizing, but because of the way it is implemented.

For completeness, the following Proposition discusses the performance of other policy rules currently analyzed in the literature. In particular, a policy rule responding to private sector expectations might not solve the problem of instability, at least for this class of policy rules.

Proposition 5 *Assume that private sector's decisions are taken with* $t-1$ *information and* $U_{cm} = 0$.

(i) if the central bank reacts to current inflation and output gap, transparency does not affect local stability under learning;

Consider the case of no transparency.

(ii) A policy rule of the form: $i_t = \phi_\pi E^h_{t-1}\pi_t + \sigma^{-1} r^n_t$, *with* $h = PS, CB$ *leads to a non-learnable REE;*

(iii) Consider the policy rule: $i_t = \phi_\pi E^h_{t-1}\pi_{t+1} + \phi_x E^h_{t-1}x_{t+1} + \phi_r E^h_{t-1}r^n_t + \phi_u E^h_{t-1}u_t$ *and assume, without loss of generality that* $\rho_r = \rho_u = 0$ *. Then the stability conditions under learning are the same as in Proposition (3). In particular, if* $\phi_x = 0$ *the REE is non-learnable for any parameter value.*

Consider the case of some degree of transparency;

(iv) the rules above are stable under learning, provided the Taylor Principle is satisfied.

Proof. see Appendix C. ■

6 When transparency matters. Comments.

The analysis in the previous sections shows that two conditions need to be verified in order for transparency to affect local stability under learning. First, agents need to forecast the current or the future interest rate, in order to take consumption or production decisions. Even though knowledge of the policy rule helps predicting future output and inflation, this does not alter the stability conditions under learning. Second, the central bank also needs to use forecast in order to take interest rate decisions. In the case where the bank reacts to current output and inflation, knowledge about the policy rule doe not change local stability under learning. This is because the learning process of the agents does not affect directly the interest rate dynamics. Hence, the unrestricted VAR coefficients corresponding to the interest rate equation converge for any parameter value. In other words, from the proofs in the Appendix, the characteristic equation of the Jacobian can be written as

$$P(\lambda) = (1+\lambda)\left(\lambda^2 + a_1\lambda + a_0\right)$$

where the eigenvalue corresponding to the interest equation is equal to-1 for any parameter value. It is obvious that under these conditions knowing the policy rule does not affect local stability after learning, even though it might have welfare improving effects in reducing overall volatility. Concluding, necessary and sufficient condition for transparency to matter is a) the central bank responding to forecasts (possibly different from the private sector's and b) the market forecasting the interest rate.

It is also worth mentioning that the response of the economy to a shock would be different, for empirically plausible parameter values, whether the market understands the policy rule or not. In the case of transparency the economy converges back to the steady state monotonically, even though the effects on inflation are likely to be persistent. In the case of no transparency the convergence is oscillatory, with possible negative effects on welfare. I leave a more complete analysis of these effects to further research. The following section attempts a numerical evaluation of the effects of lack of transparency with a model estimated on US data. Simulations show that under the Woodford calibration and a standard Taylor rule, the behavior of the economy under full transparency can hardly be distinguished by the REE. Lack of transparency instead lead to undesired fluctuations.

7 Monetary Policy Transparency and Macroeconomic Performance

7.1 Monetary Shocks and the Benefits of Transparency: Preliminary Simulation Results

As mentioned above, lack of transparency affects economic stability also in the case where the REE is locally stable under learning. If the monetary authority responds with sufficient aggressiveness to the output gap, it can prevent self-fulfilling inflation or deflation, but lack of transparency still affects the way the economy responds to shocks. In order to give a quantitative impact of these effects, I consider an artificial economy, calibrated using Woodford's estimates. In order to evaluate quantitatively the effects of transparency for a particular economies, we would need an estimated of the structural parameters and the shock disturbances affecting the economy. This analysis is performed in the sections below. This preliminary simulations are conducted including only the policy shock.

I consider a more general Taylor tule with interest smoothing. The rule can be written as

$$i_t = \bar{\imath} + \rho i_{t-1} + (1-\rho)\left[\phi_\pi E^{CB}_{t-1}(\pi_t - \pi^*) + \phi_x E^{CB}_{t-1}(x_t - x^*)\right] + \epsilon_t \tag{30}$$

In fact, Bullard and Mitra (2002) show that interest smoothing has a stabilizing effect on the economy. The simulations below show that even in this case lack of transparency has a considerable effect on the dynamic of output, inflation and interest rate. In the simulations I assume $\rho = 0.7$. The following figures show that also in the case of local stability under learning, lack of transparency have important consequences of the dynamics of inflation. The only sizable shock that I consider is the monetary shock ϵ_t. I also include i.i.d. r^n and u_t shocks with negligible standard deviation, in order to make the learning process nontrivial. I assume that $\sigma_\epsilon = 0.0025, \sigma_r = 0.0001, \sigma_u = 0.0001$. I further assume that the agents use the Stochastic Gradient algorithm to update their coefficients. The simulations start with the economy at its REE. I do not assume structural changes in the parameters. Fluctuations from learning depend on the constant revision of the agents' estimates. It is assumed that market participants expect changes in policy and other structural parameters so that they update their estimates using constant gain algorithms, where $\delta_t = \delta$.

7.1.1 Learning Dynamics

Assuming a fixed gain algorithm has the implication that the matrix of coefficients Ω_t described above does not converge to the REE Ω^*. This because learning becomes a persistent process The agents expect structural changes in their model and therefore discount past observations and keep updating their estimates as new observations are available. Nevertheless, provided the gain $\delta_t = \delta$ is 'small', results from Benviste, Metivier and Priouret (1993) show[19] that the matrix of coefficient converges to a time-invariant Gaussian distribution, centered in Ω^*. The variance of the distribution tends to zero as the gain tends to zero. Hence the the stochastic process generated by the model is asymptotically stationary, for large t. This allows us to use the estimation method described below. Also, the asymptotic behavior of the estimated coefficients is a source of extra volatility and fluctuations in the economic variables, with respect to the case of RE.

7.1.2 Simulation Results

In the simulations I use a benchmark case where $\phi_\pi = 1.5$, $\phi_x = 0.5/4$ and $\delta = 0.05$. I then consider the effects of, a) increasing the gain to 0.1 and b) decreasing the coefficient of the output gap to 0.2/4.

Simulation results show that, under the benchmark calibration, if the central bank is fully transparent and credible the dynamics of inflation under learning is virtually identical to the REE. This result resembles the finding of Williams (2002). Instead, lack of transparency implies more

[19] See also Evans and Honkapohja (2001).

volatility and persistence in the inflation process. The tables below reports more specific results on the effects of transparency.

Table II

$\phi_x = 0.5/4; \delta = 0.05$	**REE**	**Transparency**	**No Transparency**
$\sigma(x)$	1.7109	1.7118	2.1272
$\sigma(\pi)$	0.2451	0.2456	0.3825
$\sigma(i)$	1.0707	1.07118	1.1595
$corr(x_t, x_{t-1})$	0.3548	0.3574	0.5193
$corr(\pi_t, \pi_{t-1})$	0.3455	0.3516	0.7022
$corr(i_t, i_{t-1})$	0.3553	0.3580	0.4518

The results are mean values, obtained by simulating the economy for 2000 times. Each time the length of the simulation is 2000 periods. Notice how both standard deviation and autocorrelation of the variables increase, with respect of the REE. On the other hand, an higher gain also increases both standard deviation and volatility.

Table III

$\phi_x = 0.5/4; \delta = 0.1$	**REE**	**Transparency**	**No Transparency**
$\sigma(x)$	1.7109	1.7122	2.5005
$\sigma(\pi)$	0.2451	0.2463	0.5157
$\sigma(i)$	1.0707	1.0714	1.2570
$corr(x_t, x_{t-1})$	0.3548	0.3574	0.5995
$corr(\pi_t, \pi_{t-1})$	0.3455	0.3559	0.8277
$corr(i_t, i_{t-1})$	0.3553	0.3581	0.5307

Table IV

$\phi_x = 0.2/4; \delta = 0.05$	**REE**	**Transparency**	**No Transparency**
$\sigma(x)$	2.3728	2.3727	3.0792
$\sigma(\pi)$	0.3741	0.3741	0.6039
$\sigma(i)$	1.1000	1.0999	1.2869
$corr(x_t, x_{t-1})$	0.4165	0.4163	0.6010
$corr(\pi_t, \pi_{t-1})$	0.4119	0.4124	0.7479
$corr(i_t, i_{t-1})$	0.4167	0.4165	0.5707

This might suggest an alternative explanation for the higher volatility and persistence of inflation, at business cycle frequencies, during the 70's. Clarida, et al. (1999) and Boivin and Giannoni (2002) among the others claim that high fluctuations in the pre-Volker were due to a too passive Taylor rule. This implies an indeterminate REE equilibrium and thus undesired fluctuations. This view could be questioned on two grounds.

First, the analysis above seems to suggest that even allowing for perfect knowledge of the policy rule, the indeterminate REE associated to a passive policy rule is not going to be learnable. This rises the question of the plausibility of this equilibrium (unless one would regard the rising inflation in the 70s as a the non-stationary outcome of a non learnable REE).

Second, Orphanides (2003), using real time data, shows that the Taylor rule has been active in the post war period. If this is a plausible description of US monetary policy, then no indeterminate equilibrium would exist.

Assume that in the pre-Volker era the market did not properly understand monetary policy decisions, while from the 80' the public have spent more resources to analyze Fed behavior and Fed have become increasingly transparent about its decisions (especially since the 90s'). Then the results shown above seem to suggest an alternative explanation for undesired fluctuations. A proper evaluation of this hypothesis should be left to further research[20].

7.2 Research in Progress. What are the Benefits from Transparency: an Application to US data

In this section I propose a method to estimate a monetary model of the US economy and attempt an evaluation of the benefits from central bank transparency. It has been argued that in the last 10 years the Federal Reserve has increased its degree of transparency towards the market. It would be of interest to evaluate whether central bank's transparency played a role in the good performance of the US economy over the last decade. In the sequel, I estimate a model of the US economy, assuming that agents learn about the economic environment using the recursive learning algorithms described above.

I estimate the model using quarterly data for US inflation, output and interest rate[21] for the 1984-2002, assuming perfect transparency of the Federal Reserve. In other words, I assume that market participants incorporate the Fed's reaction function in their econometric models. Subsequently, I run counterfactual simulations of the estimated model, assuming that the bank is not transparent about its policy decisions. I then analyze its effects on the volatility and persistence of inflation, output and interest rate. The aim of the simulation exercise is to quantify the effects of a non-transparent monetary policy, leaving unchanged the other structural characteristics of the economy, that is: 1) the policy rule, 2) the structural disturbances, 3) the parameters of demand and supply curves.

[20] For example, the model should be evaluated including realistic assumptions on the shock processes.

[21] To be added.

As a last step, I estimate the model using the sample from 1964-1979. Since in that period the Fed was secretive about its policy decisions, I estimate the model under the assumption of no transparency. By comparing the estimates with the ones previously obtained with the sample 1984-2002, I attempt an evaluation of the factors that most contributed to the increased volatility and persistence in the macrovariables in that sample, with respect to the most recent years. The next section describes briefly the model used and the estimation method.

7.3 A model of the US economy

In order to estimated the model, I consider a more detailed model of the economy. I use a version of the model with habit formation and inflation indexation, as in Boivin and Giannoni (2002), in order to account for the observe inertia in output and inflation. More precisely, with habit formation the IS equation becomes

$$\hat{y}_{t+1} = -\bar{\sigma} E_{t-1} \left(i_{t+1} - \pi_{t+2} \right) + \nu_y \hat{y}_t - E_{t-1} \hat{y}_{t+1} + \left(1 + \beta \nu_y\right) E_{t-1} \hat{y}_{t+2} - \beta \nu_y E_{t-1} \hat{y}_{t+3} + d_t \quad (31)$$

where $\hat{y}_t$ denotes log deviation of output from its steady state value and d_t is the "demand" shock. The reduced-form parameters $\bar{\sigma}$ and ν_y denote interest rate sensitivity to the expected interest rate and the degree of inertia in output dynamics. They are function of the deep parameters of the model; details can be found in Boivin and Giannoni (2002). Notice also that the timing assumptions are different from the simplest version of the model. The information set of the agents is discussed more precisely below. The Phillips curve can be expressed as

$$\pi_{t+1} = \nu_\pi \pi_t + \kappa \omega E_t \hat{y}_{t+1} - \kappa E_t \hat{y}_{t+2} - \kappa E_t i_{t+1} + \bar{c} E_t \pi_{t+2} - \nu_\pi \left(1 + \beta\right) E_t \pi_{t+1} - \beta E_t \pi_{t+1} + s_t \quad (32)$$

where $\bar{c} = [\beta(1 + \nu_\pi) + \kappa + 1]$. Again, a precise definition of the parameters ω and κ can be found in Boivin and Giannoni. The parameter ν_π measures in this case the degree of inflation inertia. The latter depends on what proportion of past inflation in incorporated in current prices by non-optimizing firms. Notice also that the two shocks s_t and d_t are *predetermined* at time t and follow independent AR(1) processes. Finally, the Taylor rule is

$$\hat{\imath}_t = \phi_\pi E_t \pi_{t+2} + \phi_y E_t \hat{y}_{t+1} + \rho_1 \hat{\imath}_{t-1} + \rho_2 \hat{\imath}_{t-2} + \epsilon_t \quad (33)$$

where the central bank reacts to expected future inflation and output.

Define the vector $Z_{t+1} = \begin{bmatrix} \pi_{t+1} & \hat{y}_{t+1} & d_t & s_t & \hat{\imath}_t \end{bmatrix}'$. The model can be expressed as

$$\bar{Z}_{t+1} = \sum_{j=0}^{2} \sum_{i=1}^{3} C_{ij} E_{t-j} \bar{Z}_{t+i} + D_1 \bar{Z}_t + D_2 \zeta_t \quad (34)$$

where $\bar{Z}_{t+1} = \begin{bmatrix} Z_{t+1} & Z_t \end{bmatrix}'$ and ζ_t denotes a vector including the monetary policy shock and the i.i.d shocks of the demand and supply shocks. As above the agents learn about the economy over time. Their model is consistent with the MSV solution of the model under rational expectations.

7.3.1 Information Set and Timing

In order to keep the analysis simple and minimize departures from the RE benchmark, I make convenient timing assumptions so that every agents in the economy estimate the *same* model, independently of the information set used to take decisions and form expectations. In fact, consumer and producers are assumed to have different information sets, as it is clear from (31) and (32). The latter assumption is made to reconcile the model's impulse responses to a monetary shock with the estimated VAR impulse responses for US data[22]. More precisely I assume that consumers and producers update the estimates of their model *at the end of period* $t + 1$ with the following information set

$$S^{t+1} = \{\pi_{t-j+1}, \hat{y}_{t-j+1}, \hat{\imath}_{t-j}, d_{t-1-j}, s_{t-1-j}, \epsilon_{t-j}\}_{j\geq 0}$$

Notice that also the consumers observe the monetary policy shock ϵ_t and $\hat{\imath}_t$ at the end of period t+1, but only *after* forming expectations and choosing their consumption levels. Consumers and producers take their decisions at the beginning of each period. Consumers are assumed not to observe ϵ_t and $\hat{\imath}_t$, while producers do. That explains the different timing in the expectations operator. Furthermore, I assume that neither consumers nor producers observe d_t and s_t at the end of period $t + 1$. This hypothesis makes the learning procedure non-trivial, without the need to assume observational shocks in the equations. Finally, the central bank is assumed to estimate its model *before* observing d_t and s_t, while it sets the interest rate at time t with *all* the available information. The informational advantage attributed to the central bank is consistent with the VAR identification and can be defended in the light of recent evidence of the better performance of the Fed forecast with respect to those of the private sector.

7.3.2 The Actual Law of Motion

Under the above timing assumptions all the agents estimate the *same* model (which at the REE coincides with the MSV solution). Each period the agents run the following regressions[23] using a *fixed gain* stochastic gradient algorithm

$$\hat{y}_{t+1} = \omega_{11,t+1}\pi_t + \omega_{12,t+1}\hat{y}_t + \omega_{13,t+1}\hat{\imath}_{t-1} + \omega_{14,t+1}\hat{\imath}_{t-2} + \omega_{15,t+1}d_{t-1} + \omega_{16,t+1}s_{t-1}$$

$$\pi_{t+1} = \omega_{21,t+1}\pi_t + \omega_{22,t+1}\hat{y}_t + \omega_{23,t+1}\hat{\imath}_{t-1} + \omega_{24,t+1}\hat{\imath}_{t-2} + \omega_{25,t+1}d_{t-1} + \omega_{26,t+1}s_{t-1} + \omega_{27,t+1}\epsilon_t$$

[22] See Rotemberg and Woodford (1997) and Boivin and Giannoni (2002) for details.

[23] I assume implicitely that the agents know the long run values of the main variables.

and use them, together with the policy rule (33) to form expectations. In the case of a non transparent policy regime, consumers and producers estimate an additional equation for the interest rate

$$\hat{\imath}_t = \omega_{31,t+1}\pi_t + \omega_{32,t+1}\hat{y}_t + \omega_{33,t+1}\hat{\imath}_{t-1} + \omega_{34,t+1}\hat{\imath}_{t-2} + \omega_{35,t+1}d_{t-1} + \omega_{36,t+1}s_{t-1} + \omega_{37,t+1}\epsilon_t.$$

Substituting the expectations back in (34) I obtain the ALM of the system, which gives

$$\bar{Z}_{t+1} = T^1\left(\Omega_t\right)\bar{Z}_t + T^2(\Omega_t)\zeta_t \tag{35}$$

where T^1, T^2 define the mapping between PLM and ALM. This gives a model with *time-varying* coefficients. The dynamics under learning is described in the next section. In order to analyze the behavior of the model under different assumptions about central bank transparency I first estimate the model, under the assumption of perfect credibility. The next section describes the detail of the estimation procedure.

7.4 Estimating the Model

I estimate the model using simulated quasi-maximum likelihood[24] (SQML). I estimate a reduced-form VAR with US data on inflation, deviations of output from a linear trend and interest rate. I use the same dataset[25] as in Rotemberg and Woodford (1997). The estimated VAR provides information about the lag structure that gives the best fit to the data. I also calculate the likelihood function at the estimated coefficients. This latter informations is not used for the model's estimation, but it is needed in order to perform tests.

This method, relying on numerical simulations of the model's dynamic behavior, allows to conduct formal statistical inference in nonlinear dynamic economic models as that one described in this paper. As such, the first step consists in generating a simulated time series $\left\{\tilde{Z}_s(\beta)\right\}_{s=-(p-1)}^{S}$ of length $S+p$ for a given set of the model's structural parameter β. The vector $\tilde{Z}_s(\beta)$ includes output, inflation and the interest rate generated by the model. In the second step, using the data $\left\{\tilde{Z}_s(\beta)\right\}_{s=-(p-1)}^{S}$, the VAR coefficients θ are estimated by quasi-maximum likelihood, that is:

$$\widehat{\theta}_S^{\beta} \equiv \arg\max_{\theta\in\Theta} L_S\left(\left\{\tilde{Z}_s(\beta)\right\};\theta\right),$$

where L_S is the quasi-log-likelihood function. Notice that the structure of the VAR is the same as the one used for the US data. Finally, at the third step, given the actually observed time series $\{x_t\}_{t=-(p-1)}^{T}$, the SQML estimator $\widehat{\beta}_T$ is obtained solving the following problem:

[24] See, among the others, Smith A.A.(1993).

[25] Note to be added.

$$\widehat{\beta}_T \equiv \arg\max_{\beta} L_S \left(\{x_t\} ; \widehat{\theta}_S^{\beta} \right).$$

In particular, the conditional density at the basis of the quasi-log-likelihood function is given by:

$$f(\bar{Z}_t, \bar{Z}_{t-1}, ; \theta) \equiv -1/2 \log(\det(DD')) - 1/2 \varepsilon_t' (DD')^{-1} \varepsilon_t$$

where $\tilde{Z}_t = C\tilde{Z}_{t-1} + \tilde{\varepsilon}_t$, the vector of innovations $\tilde{\varepsilon}_t$ is assumed to be i.i.d. normally distributed with covariance matrix DD' and D is lower triangular. The validity of this approach is based on the following two assumptions: first, the processes x_t and $\tilde{Z}_s(\beta)$ are stationary and ergodic, and second, there exists a unique set of structural parameters β_0 such that the actual data x_t and the simulated process $\tilde{Z}_s(\beta_0)$ are drawn from the same distribution. The first assumption implies that, before performing the second step, it is necessary to ensure the stationarity of $\tilde{Z}_s(\beta)$ by simulating the data for a large number of periods and then employ only the last part of the process for the estimation. The asymptotic stationarity of the model-generated data is guaranteed by the fact that the coefficients of the learning model with fixed gain algorithm converge to an invariant distribution, as mentioned in the section above.

8 Conclusions

The paper shows that transparency matter for monetary policy design. I consider the case where a class of policy rules are evaluated for their robustness to forecasting mistakes of the market participants. I show that in a model where monetary policy has immediate effects on aggregate activity and inflation, knowledge about the policy rule does not enhance the stability of the economic system.

In the more empirically plausible case where expenditure and pricing decisions are predetermined, and therefore monetary policy affects the economy with delays, a transparent implementation of policy rule is crucial for the stability of the economic system. Lack of transparency might generate instability to forecasting mistakes and other equilibria generated by the learning behavior. A more transparent implementation of the rule instead guarantees stability of the unique REE, provided the central bank reacts to some degree to the output gap.

Finally, the paper proposes a method to estimate the learning model with simulated quasi-maximum likelihood methods.

Future research should address the case of inertial policy rule and the role of transparency in this case. Also, in this paper I model delays in the effects of monetary policy with lagged expectations. The role of transparency on learnability should be investigated in the case where delays are captured by lagged variables.

9 Appendix A. The Model Solution.

The budget constraint (4) can be rewritten as

$$A_t^j + C_t^j = \frac{1+i_{t-1}}{\Pi_t} A_{t-1}^j - \frac{i_{t-1} - i_{t-1}^m}{\Pi_t} m_{t-1}^j + \frac{P_t^j}{P_t} f\left(h_t^j, \frac{M_t}{P_t}\right) - T_t \tag{36}$$

where A_t^j $=(B_t + M_t)/P_t$ denotes real wealth and the term $\frac{i_{t-1}-i_{t-1}^m}{\Pi_t}$ is the opportunity cost of holding money, expressed in real terms. Substituting the budget constraint (36) in (3) the maximization problem becomes

$$\max_{A_t^j, m_t^j, h_t^j, P_t^j}$$

$$E_{t-1}^j \sum_{t=0}^{\infty} \beta^t \left[\begin{array}{c} U\left(\frac{1+i_{t-1}}{\Pi_t} A_{t-1}^j + \frac{i_{t-1}-i_{t-1}^m}{\Pi_t} m_{t-1}^j + \frac{P_t^j}{P_t} Y_t D(\frac{P_t^j}{P_t}) - A_t^j - T_t, m_t\right) \\ +V\left(h_t^j\right) - \frac{\gamma}{2}\left(\frac{P_t^j}{P_{t-1}^j} - \Pi^*\right)^2 \end{array} \right]$$

$$+\lambda_t \left[f\left(h_t^j, m_t^j\right) - Y_t D(\frac{P_t^j}{P_t}) \right]$$

where λ_t is the Lagrangian multiplier. Differentiating with respect to P_t^j, imposing a *symmetric* equilibrium and substituting for the equilibrium condition $Y_t = C_t + G_t = f\left(h_t, m_t\right)$, the expression is simplified to

$$E_{t-1}^j \left[\begin{array}{c} U_c\left(Y_t - G_t, m_t^j\right)\left(Y_t - G_t\right)\left(1+\theta_t\right) - \lambda_t \theta_t + \\ -\gamma_t\left(\Pi_t - \Pi^*\right)\Pi_t + \beta\gamma_t\left(\Pi_{t+1} - \Pi^*\right)\Pi_{t+1} \end{array} \right] = 0 \tag{37}$$

where $\theta_t = D_t'/D_t < -1$ for every t. The f.o.c. for the labor supply is

$$E_{t-1}^j \left[\lambda_t - \frac{V'(h_t^j)}{f_h\left(h_t^j, m_t^j\right)} \right] = 0 \tag{38}$$

Substituting for λ_t in (37) gives

$$\begin{aligned} \left(\Pi_t - \Pi^*\right)\Pi_t &= \beta E_{t-1}^j \left(\Pi_{t+1} - \Pi^*\right)\Pi_{t+1} - E_{t-1}^j \left[U_c\left(Y_t, m_t\right)\left(Y_t - G_t\right) \frac{\theta_t}{\gamma_t}\left(s_t - \mu_t^{-1}\right) \right] \\ &= \beta E_{t-1}^j \left(\Pi_{t+1} - \Pi^*\right)\Pi_{t+1} - E_{t-1}^j \left[\frac{U_c\left(Y_t - G_t, m_t\right)\left(Y_t - G_t\right)}{\bar{\gamma}}\left(\mu_t s_t - 1\right) \right] \end{aligned} \tag{39}$$

where

$$s_t = \frac{V'(h_t)}{f'\left(h_t, m_t\right) U_c\left(Y_t - G_t, m_t\right)} \tag{40}$$

is the average real marginal cost and $\mu_t = \theta_t / (1 + \theta_t)$ is the desired mark up. Last, the f.o.c. with respect to assets and money balances are

$$E_{t-1}^{j}\left[U_c\left(C_t, m_t\right)\right] = \beta E_{t-1}^{j}\left[\frac{U_c\left(C_{t+1}, m_{t+1}\right)\left(1 + i_t\right)}{\Pi_{t+1}}\right] \tag{41}$$

which gives the consumption Euler equation, and

$$\begin{aligned} E_{t-1}^{j}\left[U_m\left(C_t, m_t\right)\right] &= \beta E_{t-1}^{j}\left[\frac{U_c\left(C_{t+1}, m_{t+1}\right)\left(i_t - i_t^m\right)}{\Pi_{t+1}}\right] \\ &= E_{t-1}^{j}\left[\frac{U_c\left(C_t, m_t\right)\left(i_t - i_t^m\right)}{\left(1 + i_t\right)}\right] \end{aligned} \tag{42}$$

which gives (implicitly) a money demand function for the consumer. There is also an implicit demand function for the producer, which can be shown to be

$$E_{t-1}\left[U_m\left(C_t, m_t\right)\left(\frac{i_t - i_t^m}{1 + i_t}\right)\right] = E_{t-1}\left[\frac{V'\left(h_t\right)}{f_h\left(h_t, m_t\right)} f_m\left(h_t, m_t\right)\right] \tag{43}$$

9.1 The Linearized Model

9.1.1 Money in the Utility Function

As mentioned in the introduction, I consider separately the case of money in the utility function and money in the production function. In both cases the pricing equation () can be linearized to get

$$\pi_t = E_{t-1}\beta\pi_{t+1} + \xi E_{t-1}\left(\hat{s}_t + \hat{\mu}_t\right) \tag{44}$$

where

$$\xi = \frac{U_c \bar{Y}}{\bar{\gamma}}$$

is a measure of the degree of price stickiness. Notice that the linearization is a good approximation of the non-linear model only for small values of inflation.

In the case of money in the utility function, the real marginal cost is

$$s_t = \frac{V'(f^{-1}\left(Y_t\right))}{f'\left(f^{-1}\left(Y_t\right)\right) U_c\left(Y_t - G_t, m_t\right)} \tag{45}$$

Linearizing (45) I obtain

$$\hat{s}_t = \left(\sigma^{-1} + \omega\right)\hat{Y}_t - \sigma^{-1} g_t - \chi \hat{m}_t^j \tag{46}$$

where

$$\sigma = -\frac{U_c\left(\bar{Y}-\bar{G},\bar{m}\right)}{U_{cc}\left(\bar{Y}-\bar{G},\bar{m}\right)\bar{Y}} > 0$$

is the intertenporal elasticity of substitution of consumption

$$\chi = \frac{U_{cm}\left(\bar{Y}-\bar{G},\bar{m}\right)\bar{m}}{U_c\left(\bar{Y}-\bar{G},\bar{m}\right)} > 0$$

which measures the marginal utility of extra consumption, as real balances change

$$\omega_w = \frac{V''\left(\bar{Y}\right)\bar{Y}}{f'\left(\bar{Y}\right)V'\left(\bar{Y}\right)} > 0$$

defines the elasticity of the marginal disutility of work with respect to output

$$\omega_f = -\frac{f''\left(\bar{Y}\right)\bar{Y}}{f'\left(\bar{Y}\right)} > 0$$

defines the elasticity of the marginal product of labor with respect to output. Finally, $\omega = \omega_w + \omega_p$.

Linearization of the Euler equation (41) gives

$$\hat{c}_t^j = -\sigma E_{t-1}^j\left(\hat{\imath}_t - \pi_{t+1}\right) + E_{t-1}^j\hat{c}_{t+1}^j - \chi\sigma E_{t-1}^j\left(\hat{m}_{t+1}^j - \hat{m}_t^j\right) \tag{47}$$

Linearizing the money demand gives

$$\hat{m}_t^j = \eta_y\hat{c}_t^j - \eta_i E_{t-1}^j\left(\hat{\imath}_t - \hat{\imath}_t^m\right) \tag{48}$$

where I use the fact that each agent knows his consumption at time t, when deciding the amount of money. where $\eta_y > 0, \eta_i > 0$ denote the elasticity of money demand with respect to income and the nominal interest rate. The coefficients can be shown to be

$$\eta_y = \left(\frac{\chi_c + \bar{\Delta}\sigma^{-1}}{\bar{\Delta}\chi + \epsilon_{mm}}\right)$$

$$\eta_i = \left(\frac{1-\bar{\Delta}}{\bar{\Delta}\chi + \epsilon_{mm}}\right)$$

and

$$\chi_c = \frac{U_{mc}\left(\bar{Y}-\bar{G},\bar{m}\right)\bar{Y}}{U_c\left(\bar{Y}-\bar{G},\bar{m}\right)}$$

$$\epsilon_{mm} = \frac{U_{mm}\left(\bar{Y}-\bar{G},\bar{m}\right)\bar{m}}{U_c\left(\bar{Y}-\bar{G},\bar{m}\right)}$$

Substituting (48) in (47), I get

$$\hat{c}_t^j = -\tilde{\sigma} E_{t-1}^j \left[\eta_1 \left(\hat{\imath}_t - \hat{\imath}_t^m \right) - \pi_{t+1} - \eta_2 \left(\hat{\imath}_{t+1} - \hat{\imath}_{t+1}^m \right) \right] + E_{t-1}^j \hat{c}_{t+1}^j$$

where $\eta_1 = (1 + \chi \eta_i) > 1$, $\eta_2 = \eta_1 - 1 > 0$, and $\tilde{\sigma} = \frac{\sigma}{(1-\chi\sigma\eta_y)} > \sigma$. Assuming that the agents understand that their future consumption depend on aggregate output[26], i.e. that $C_{t+1}^j = Y_{t+1} - G_{t+1}$, then their consumption decisions depend on their expectations about future output, inflation, interest rate and government expenditures. Imposing equilibrium in the goods market and aggregating over the agents I obtain the IS equation with real money balances effects

$$x_t = -\tilde{\sigma} E_{t-1} \left[\eta_1 \hat{\imath}_t - \eta_2 \hat{\imath}_t^m - \pi_{t+1} - \eta_2 \left(\hat{\imath}_t - \hat{\imath}_t^m \right) - \hat{r}_t^n \right] + E_{t-1} x_{t+1} \tag{49}$$

where $x_t = \hat{Y}_t - \hat{Y}_t^e$, is the output gap, expressed as output deviations from the *efficient* level $\hat{Y}_t^e$. Following Woodford (2003), the efficient level of output can be defined by

$$s\left(Y_t^e, m_t, G_t\right) = \bar{\mu}^{-1} \tag{50}$$

Notice that markup shocks do not affect the efficient level of output. Using the money demand function and linearizing (**??**) I obtain

$$Y_t^e = -\frac{\left(\sigma^{-1} + \chi\eta_y\right)}{\epsilon_{mc}} \hat{g}_t$$

where $\epsilon_{mc} = \left(\sigma^{-1} + \omega - \chi\eta_y\right)$ denotes the elasticity of the real marginal cost and

$$\hat{Y}_t^e = \hat{Y}_t^n + \frac{1}{\epsilon_{mc}} \hat{\mu}_t$$

where $\hat{Y}_t^n$ is the equilibrium level of output if prices are fully flexible, given a monetary policy that maintains a constant interest rate spread between monetary and nonmonetary assets. The (exogenous) process $\hat{r}_t^n$ is defined as

$$\hat{r}_t^n = \left(\sigma^{-1} - \eta_y \chi\right) \left(E_{t-1} \hat{Y}_{t+1}^e - \hat{Y}_t^e \right) - \sigma^{-1} \left(E_{t-1} g_{t+1} - g_t \right)$$

In order to simplify the analysis and consistently with the previous literature I assume that $\hat{r}_t^n$ is observable and evolves as an AR(1) process. All variables are in log-levels. This is consistent with the assumption that the agents do not know the long run equilibrium of the variables, and in particular the inflation target. Notice that current expenditures depend on expectations about the current and the future interest rate.

[26] This is not a strong asumption. As mentioned in Evans, Honkapohja and Mitra (2002), running a regression of consumption on the output gap and government expenditures would reveal what I am assuming.

Finally, combining (46) with the money demand curve, and using the definition of efficient output I obtain the following Phillips curve

$$\pi_t = E_{t-1}\beta\pi_{t+1} + \kappa E_{t-1}\left[\hat{x}_t + \eta_3\left(\hat{\imath}_t - \hat{\imath}_t^m\right)\right] + u_t$$

where

$$\begin{aligned}\kappa &= \xi\epsilon_{mc} \\ \eta_3 &= \frac{\eta_i\chi}{\epsilon_{mc}}\end{aligned}$$

and $u_t = \left(\frac{\kappa}{\epsilon_{mc}}\right) E_{t-1}\hat{\mu}_t$. Notice that the shock is pre-determined at time t.

9.1.2 Money in the Production Function

In this case he real marginal cost is

$$s_t = \frac{V'(h_t)}{f'\left(h_t, m_t\right) U_c\left(Y_t - G_t\right)} \tag{51}$$

Defining

$$h_t = H\left(Y_t, m_t\right)$$

with $H_y > 0$ and $H_m < 0$, it is possible to rewrite (51) as a function of output and money balances only. Hence,

$$s_t = \frac{V'(H\left(Y_t, m_t\right))}{f'\left(H\left(Y_t, m_t\right)\right) U_c\left(Y_t - G_t\right)}.$$

Log-linearization leads to the following expression

$$\hat{s}_t = \left(\sigma^{-1} + \tilde{\omega}\right)\hat{Y}_t + \sigma^{-1}\hat{g}_t - \tilde{\chi}\hat{m}_t$$

where

$$\begin{aligned}\tilde{\omega} &= \tilde{\omega}_w + \omega_{hh} \\ \tilde{\chi} &= \chi_{vv} + \chi_{hh} + \chi_{hm}\end{aligned}$$

$$\tilde{\omega}_w = \frac{V''\left(H\left(\bar{Y}, \bar{m}\right)\right) H_y\left(\bar{Y}, \bar{m}\right)\bar{Y}}{V'\left(H\left(\bar{Y}, \bar{m}\right)\right)} > 0$$

$$\omega_{hh} = -\frac{f_{hh}\left(\bar{Y}, \bar{m}\right) H_y\left(\bar{Y}, \bar{m}\right)\bar{Y}}{f_h\left(\bar{Y}, \bar{m}\right)} > 0$$

$$\chi_{vv} = -\frac{V''\left(H\left(\bar{Y},\bar{m}\right)\right) H_m\left(\bar{Y},\bar{m}\right)\bar{m}}{V'\left(H\left(\bar{Y},\bar{m}\right)\right)} > 0$$

$$\chi_{hh} = \frac{f_{hh}\left(\bar{Y},\bar{m}\right) H_m\left(\bar{Y},\bar{m}\right)\bar{m}}{f_h\left(\bar{Y},\bar{m}\right)} > 0$$

$$\chi_{hm} = \frac{f_{hm}\left(\bar{Y},\bar{m}\right)\bar{m}}{f_h\left(\bar{Y},\bar{m}\right)} > 0$$

The Euler equation for consumption and thus the IS equation do not depend on real money balances. The IS is equivalent to (49) with $\eta_1 = 1$ and $\eta_2 = 0$.

The money demand function can be linearized to yield

$$\hat{m}_t = \tilde{\eta}_y \hat{c}_t - \tilde{\eta}_i E_{t-1}\left(\hat{\imath}_t - \hat{\imath}_t^m\right) \quad (52)$$

where

$$\tilde{\eta}_y = \left(\frac{\tilde{\omega}_w + \omega_{hm} + \omega_{hh} + \sigma^{-1}}{\chi_{vv} + \chi_{hm} + \chi_{mm} + \chi_{hh} + \chi_{hm}}\right)$$

where

$$\omega_{mh} = \frac{f_{mh}\left(\bar{Y},\bar{m}\right) H_c\left(\bar{Y},\bar{m}\right)\bar{Y}}{f_m\left(\bar{Y},\bar{m}\right)} > 0$$

$$\chi_{mh} = -\frac{f_{mh}\left(\bar{Y},\bar{m}\right) H_m\left(\bar{Y},\bar{m}\right)\bar{m}}{f_m\left(\bar{Y},\bar{m}\right)} > 0$$

$$\chi_{mm} = -\frac{f_{mm}\left(\bar{Y},\bar{m}\right)\bar{m}}{f_m\left(\bar{Y},\bar{m}\right)} > 0$$

and

$$\tilde{\eta}_i = \left(\frac{\Delta^{-1}}{\chi_{vv} + \chi_{hm} + \chi_{mm} + \chi_{hh} + \chi_{hm}}\right)$$

Inserting the demand for money (52) in the real marginal cost equation, I obtain

$$\hat{s}_t = \left(\sigma^{-1} + \tilde{\omega} - \tilde{\chi}\tilde{\eta}_y\right)\hat{Y}_t + \tilde{\chi}\tilde{\eta}_i E_{t-1}\left(\hat{\imath}_t - \hat{\imath}_t^m\right) + \left(\sigma^{-1} + \tilde{\chi}\tilde{\eta}_y\right)\hat{g}_t$$

Proceeding as for the case of money in the utility function, it is possible to express the real marginal cost in terms of deviations of current output from its efficient level

$$\hat{s}_t = \tilde{\epsilon}_{mc}\hat{Y}_t^e + \tilde{\chi}\tilde{\eta}_i E_{t-1}\left(\hat{\imath}_t - \hat{\imath}_t^m\right)$$

where, again, $\tilde{\epsilon}_{mc} = \left(\sigma^{-1} + \tilde{\omega} - \tilde{\chi}\tilde{\eta}_y\right)$. The log-linear Phillips curve is again

$$\pi_t = E_{t-1}^j \beta\pi_{t+1} + \xi E_{t-1}^j \hat{s}_t$$

Substituting the real marginal cost in the Phillips curve we get

$$\pi_t = E_{t-1}\beta\pi_{t+1} + \tilde{\kappa}E_{t-1}\left[\hat{x}_t + \tilde{\eta}_3\left(\hat{\imath}_t - \hat{\imath}_t^m\right)\right] + \tilde{u}_t$$

where

$$\begin{aligned}\tilde{\kappa} &= \tilde{\xi}\tilde{\epsilon}_{mc} \\ \tilde{\eta}_3 &= \frac{\tilde{\eta}_i\tilde{\chi}}{\tilde{\epsilon}_{mc}}\end{aligned}$$

and $\tilde{u}_t$ is defined as above.

10 Appendix B. Stability With t Expectations.

Proof. Proposition (1)

(i) Local stability is determined by the stability of the following differential in notional time

$$\frac{d\Omega}{d\tau} = T(\Omega) - \Omega$$

where

$$T(\Omega) = \left(\hat{A}_1 + \hat{A}_2\Omega_0,\ \hat{A}_2\Omega_1 H + \hat{A}_3\right)$$

Following Evans and Honkapohja (2001), E-Stability is obtained, provided the following matrices have eigenvalues with real parts less than one.

$$\hat{A}_2 - I_3 \tag{53}$$

and

$$H \otimes \hat{A}_2 - I_3 \tag{54}$$

It is enough to verify the condition for (53), given that ρ_r, $\rho_u < 1$. The characteristic equation of (53) can be factorized such that

$$P(\lambda) = -(1+\lambda)\left(\lambda^2 + a_1\lambda + a_0\right)$$

where

$$a_1 = \frac{\begin{array}{c}\tilde{\sigma}\kappa\left[\phi_\pi - 1\right] + \tilde{\sigma}\phi_x(1-\beta)\left[1 + (1-\beta)\eta_2\right] + \\ + \tilde{\sigma}\kappa\phi_\pi\left(1 - \frac{\eta_3}{\tilde{\sigma}}\right) - \tilde{\sigma}\kappa\eta_3\phi_x + (1-\beta)\end{array}}{\left(\eta_1 - \frac{\eta_3}{\tilde{\sigma}}\right)\tilde{\sigma}\kappa\phi_\pi + \tilde{\sigma}\phi_x\eta_1 + 1}$$

and

$$a_0 = \frac{\tilde{\sigma}\kappa\left(\phi_\pi - 1\right) + \tilde{\sigma}\phi_x\left(1-\beta\right) + \tilde{\sigma}\kappa - \tilde{\sigma}\kappa\eta_3\phi_x}{\left(\eta_1 - \frac{\eta_3}{\tilde{\sigma}}\right)\tilde{\sigma}\kappa\phi_\pi + \tilde{\sigma}\phi_x\eta_1 + 1}$$

The conditions to be satisfied to obtain stability are

$$a_0 > 0, \quad a_1 > 0$$

Given that $\eta_1 - \eta_3 > 0$ and $\sigma \geq 1$, we have that $a_1 > a_0$. Hence, stability obtains if $a_0 > 0$. This gives condition (12).

Also, when the change of stability occurs, a_0 is equal to zero. That means that the eigenvalues are real. Hence, no local Hopf bifurcation occurs and the inflation target is the only equilibrium. ■

Proof. Proposition (2)

I study the convergence properties of the algorithm by using stochastic approximation theory, using results by Evans and Honkapohja (2001) and Marcet and Sargent (1989). In order to apply those results, I need to put the system (20) in the following form:

$$\xi_t = \xi_{t-1} + \delta_t \Phi(\xi_{t-1}, S_t) \tag{55}$$

$$S_t = G(\xi_{t-1}) S_{t-1} + C\nu_t$$

which is achieved by setting:

$$\xi_t = \begin{bmatrix} \theta_t & vec\left(\bar{R}_t\right) \end{bmatrix}$$

$$S_t = \begin{bmatrix} Y_t & X_t \end{bmatrix}$$

$$G = \begin{bmatrix} 0_{3\times 3} & \tilde{T}(\xi_{t-1}) \\ 0_{2\times 3} & H \end{bmatrix}$$

where θ is the d-dimensional vector of the estimates, S represents the state vector, ν is the disturbance term and C its coefficients. The latter two are trivially identifiable. The local dynamics of this system (local convergence), i.e. the stability of the RE equilibrium depend on the associated ODE:

$$d\theta/d\tau = h(\theta) \tag{56}$$

where $h(\theta) = \lim_{t\to\infty} E\Phi(\theta, S_t(\theta))$. An exhaustive survey of this approach with analytical proofs can be found in Evans and Honkapohja (2001). Given that the system can be put in the in form

(55), it is easy to verify that it satisfies the properties, A.1, A.2, B.1, B.2 in Evans and Honkapohja (2001).

First, I can rewrite the matrices of regressors as:

$$Z_t(\theta_{t-1}) = K'(\theta_{t-1})\, Q_t \tag{57}$$

where

$$K'(\theta_{t-1}) = \begin{bmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \multicolumn{3}{c}{\tilde{T}^{i\prime}(\theta_{t-1})} \end{pmatrix} & \mathbf{0}_{3,3} & \mathbf{0}_{3,3} \\ \mathbf{0}_{3,3} & \begin{pmatrix} 1 & 0 & 0 \\ \multicolumn{3}{c}{\tilde{T}^{x\prime}(\theta_{t-1})} \\ 0 & 0 & 1 \end{pmatrix} & \mathbf{0}_{3,3} \\ \mathbf{0}_{3,3} & \mathbf{0}_{3,3} & \begin{pmatrix} 1 & 0 & 0 \\ \multicolumn{3}{c}{\tilde{T}^{x\prime}(\theta_{t-1})} \\ \multicolumn{3}{c}{\tilde{T}^{\pi\prime}(\theta_{t-1})} \end{pmatrix} \end{bmatrix}$$

Substituting (57) in (20) I get

$$\begin{aligned} \theta_t &= \theta_{t-1} + \delta_t \overline{R}_{t-1}^{-1} Q_t \left(\bar{T}(\theta_{t-1})\, Q_t - \left(\theta_{t-1}' K'(\theta_{t-1})\, Q_t\right)' \right) \\ &\quad + i.i.d. \text{ errors} \end{aligned}$$

$$\overline{R}_t = \overline{R}_{t-1} + \delta_t \left(Q_t \left(K'(\theta_{t-1})\, Q_t\right)' - \overline{R}_{t-1} \right)$$

where

$$\bar{T}(\theta_{t-1}) = \begin{pmatrix} \tilde{T}^{x\prime}(\theta_{t-1}) & \tilde{T}^{\pi\prime}(\theta_{t-1}) & \tilde{T}^{i\prime}(\theta_{t-1}) \end{pmatrix}$$

Rearranging, the system becomes

$$\begin{aligned} \theta_t &= \theta_{t-1} + \delta_t \overline{R}_{t-1}^{-1} Q_t Q_t' \left(\bar{T}(\theta_{t-1}) - K(\theta_{t-1})\, \theta_{t-1} \right) \\ &\quad + i.i.d. \text{ errors} \end{aligned}$$

$$\overline{R}_t = \overline{R}_{t-1} + \delta_t \left(Q_t Q_t' K(\theta_{t-1}) - \overline{R}_{t-1} \right)$$

By taking the asymptotic mean, the convergence properties can be studied by checking the stability of the following ODE

$$\frac{d\overline{R}}{d\tau} = M_Q K(\theta) - \overline{R} \tag{58}$$

$$\frac{d\theta}{d\tau} = \overline{R}^{-1} M_Q \left(\bar{T}(\theta) - K(\theta)\theta \right) \tag{59}$$

where $M_Q = E_{\lim t \to \infty}(Q_t Q_t')$. Given that, from the first equation

$$\overline{R} \to M_Q K(\theta)$$

the stability analysis reduces to:

$$\begin{aligned} \frac{d\theta}{d\tau} &= K(\theta)^{-1} M_Q^{-1} M_Q \left(\bar{T}(\theta) - K(\theta)\theta \right) \\ &= K(\theta)^{-1} \bar{T}(\theta) - \theta \end{aligned}$$

The resulting system is a non-linear function of Γ and B and a complicated expression of the single parameters. Using Matlab Symbolic Toolbox it is possible to show that the linearized system is indeed composed of .three independent subsystems

$$\begin{pmatrix} \dot{b}_{01} \\ \dot{b}_{02} \end{pmatrix} = G_0 \begin{pmatrix} b_{01} \\ b_{02} \end{pmatrix} \tag{60}$$

$$\begin{pmatrix} \dot{\psi}_0 \\ \dot{\psi}_x \\ \dot{\psi}_\pi \end{pmatrix} = \begin{pmatrix} i - \phi_\pi \pi^* - \phi_y x^* \\ \phi_x \\ \phi_\pi \end{pmatrix} - \begin{pmatrix} \psi_0 \\ \psi_x \\ \psi_\pi \end{pmatrix} \tag{61}$$

and

$$\begin{pmatrix} \dot{\gamma}_{11} \\ \dot{\gamma}_{21} \\ \dot{b}_{11} \\ \dot{b}_{22} \end{pmatrix} = G_1 \begin{pmatrix} \gamma_{11} \\ \gamma_{21} \\ b_{11} \\ b_{22} \end{pmatrix} \tag{62}$$

First, (61) shows that the estimates of the Taylor rule converge for any parameter value. This is expected, because as I mentioned in the main text, the agents' equation is correctly specified at any point in time. Hence, provided that output and inflation stay bounded, the estimates will converge. Also, it is possible to show that the characteristic equation of (60) is the same as for $\hat{A}_2 - I_3$ (even if $G_0 \neq \hat{A}_2 - I_3$, so that the matrices are similar).

(ii) The system (62) is more complicated and thus is not possible find an analytical solution. Imposing $U_{cm} = 0$ it is possible to show that the eigenvalues of (54) and G_1 are the same so that the matrixes are similar. For the general case, I had to resort to simulation, as mentioned in the Proposition. Repeated simulations suggest that G_1 and (54) are similar for every parameter value.

(iii) The result trivially depends on the fact that the stability of (60) and (62) do not depend on the estimates of the policy parameters. ■

11 Appendix C. Stability With t-1 Expectations.

Proof. Proposition (3)

The model can be written in matrix form as

$$V_t = A_0 + A_1^{PS} E_{t-1}^{PS} V_t + A_1^{CB} E_{t-1}^{CB} V_t + A_2 E_{t-1}^{PS} V_{t+1} + A_3 X_t + \epsilon_t \tag{63}$$

Consider the E-Stability conditions. Inserting the PLMs I get

$$V_t = T\left(\Omega_0^{PS}, \Omega_0^{CB}, \Omega_1^{PS}, \Omega_1^{CB}\right) \begin{bmatrix} 1 \\ X_{t-d} \end{bmatrix} + \epsilon_t \tag{64}$$

the mapping between the PLM and ALM is described by the following ODE

$$\frac{d\Omega_0^{PS}}{d\tau} = A_0 + \left(A_1^{PS} - I_3\right)\Omega_0^{PS} + A_1^{CB}\Omega_0^{CB} + A_2\Omega_0^{PS} \tag{65}$$

$$\frac{d\Omega_0^{CB}}{d\tau} = A_0 + \left(A_1^{CB} - I_3\right)\Omega_0^{CB} + A_1^{PS}\Omega_0^{PS} + A_2\Omega_0^{PS} \tag{66}$$

$$\frac{d\Omega_1^{PS}}{d\tau} = A_1^{PS}\Omega_1^{PS} + A_1^{CB}\Omega_1^{CB} + A_2\Omega_1^{PS} + A_3 H - \Omega_1^{PS} \tag{67}$$

$$\frac{d\Omega_1^{PS}}{d\tau} = A_1^{PS}\Omega_1^{PS} + A_1^{CB}\Omega_1^{CB} + A_2\Omega_1^{PS} + A_3 H - \Omega_2^{PS} \tag{68}$$

Stability under learning is determined by the following independent subsystems

$$\begin{bmatrix} \dot{\Omega}_0^{PS} \\ \dot{\Omega}_0^{CB} \end{bmatrix} = F_1 \begin{bmatrix} \Omega_0^{PS} \\ \Omega_0^{CB} \end{bmatrix} \tag{69}$$

$$\begin{bmatrix} vec\dot{\Omega}_1^{PS} \\ vec\dot{\Omega}_1^{CB} \end{bmatrix} = F_2 \begin{bmatrix} vec\Omega_1^{PS} \\ vec\Omega_1^{CB} \end{bmatrix} \tag{70}$$

where

$$F_1 = \begin{pmatrix} A_1^{PS} - I_3 + A_2 & A_1^{CB} \\ A_1^{PS} + A_2 & A_1^{CB} - I_3 \end{pmatrix} \tag{71}$$

$$F_2 = \begin{pmatrix} I_2 \otimes A_1^{PS} + H \otimes A_2 - I_6 & I_2 \otimes A_1^{CB} \\ I_2 \otimes A_1^{PS} + H \otimes A_2 & I_2 \otimes A_1^{CB} - I_6 \end{pmatrix} \tag{72}$$

In order to extract the stability conditions I follow Honkapohja and Mitra (2002). Stability under learning is obtained if the eigenvalues of F_1 and F_2 have negative real parts. The characteristic equations of associated to the two matrices can be simplified to

$$\begin{aligned} |F_1 - \lambda I_6| &= \begin{vmatrix} A_1^{PS} - I_3(1+\lambda) + A_2 & A_1^{CB} \\ A_1^{PS} + A_2 & A_1^{CB} - I_3(1+\lambda) \end{vmatrix} \\ &= (-(1+\lambda))^2 \left| A_1^{PS} + A_1^{CB} + A_2 - I_3(1+\lambda) \right| \end{aligned} \tag{73}$$

and

$$\begin{aligned} |F_2 - \lambda I_{12}| &= \begin{vmatrix} -(1+\lambda) I_6 & (1+\lambda) I_6 \\ I_2 \otimes A_1^{PS} + H \otimes A_2 & I_2 \otimes A_1^{CB} - I_6(1+\lambda) \end{vmatrix} \\ &= (-(1+\lambda))^6 \left| I_2 \otimes A_1^{PS} + H \otimes A_2 + I_2 \otimes A_1^{CB} - (1+\lambda) I_6 \right| \end{aligned} \tag{74}$$

So, determining stability boils down to determinate whether the eigenvalues of the following matrices have negative real part

$$\tilde{A}_1 = A_1^{PS} + A_1^{CB} + A_2 - I_3 = \begin{pmatrix} 0 & \tilde{\sigma} & -\tilde{\sigma} \\ \kappa & \beta - 1 & \kappa\eta_3 \\ \phi_y & \phi_\pi & -1 \end{pmatrix} \tag{75}$$

and

$$\tilde{A}_2 = A_1^{PS} + A_1^{CB} + \rho_i A_2 \tag{76}$$

for $i = r, u$.

Let us consider first (75). According to the Routh's Theorem, the number of roots of (75) with positive real parts is equal to the number of variations of sign in the following scheme

$$-1 \quad \text{Trace } (\tilde{A}_1) \quad -B_1 + \frac{\text{Det}(\tilde{A}_1)}{\text{Trace } (\tilde{A}_1)} \quad \text{Det}(\tilde{A}_1) \tag{77}$$

where

$$\text{Trace } (\tilde{A}_1) = \beta - 2 < 0 \tag{78}$$

$$\text{Det}(\tilde{A}_1) = -\left[\kappa(\phi_\pi - 1) + (1-\beta)\phi_x\right] + \eta_3\kappa\phi_x < 0 \tag{79}$$

$$\text{provided (22) holds} \tag{80}$$

$$B_1 = -\kappa\sigma + \sigma\phi_x + (1-\beta) - \eta_3\kappa\phi_\pi \tag{81}$$

where B is the sum of all second order principle minors of $\tilde{A}_1$. A pattern of $----$ corresponds to all eigenvalues having negative real part. In order to obtain that we need

$$-B_1 \cdot \text{Trace }(\tilde{A}_1) + \text{Det}(\tilde{A}_1) > 0 \tag{82}$$

Algebraic manipulations show that (82) is positive if $\phi_x > \hat{\phi}_x$ in the Proposition is verified. Consider the matrix $\tilde{A}_2$. It is easy to show that

$$\text{Trace }(\tilde{A}_2) = \rho_i (1+\beta) - 3 < 0 \tag{83}$$

$$\begin{aligned} \text{Det}(\tilde{A}_2) &= -\tilde{\sigma}\left[\kappa\left(\phi_\pi - \rho_i\right) + \phi_x\left(1-\beta\rho_i\right)\left(\eta_1 - \rho_i\eta_2\right)\right] + & (84) \\ -\kappa\left(1-\rho_i\right)\left(\tilde{\sigma}\eta_1 - \eta_3\right) - \left(1-\rho_i\right)\left(2-\beta\rho_i\right) &< 0 & (85) \\ & \text{provided (22) holds} \end{aligned}$$

$$\begin{aligned} B_2 &= -\kappa\rho_i\sigma + \left(\eta_1 - \rho_i\eta_2\right)\sigma\phi_y + 2 - \beta\left(1+\rho_i\right) + \\ & \left(1-\rho_i\right)\left(1-\beta\rho_i\right) - \eta_3\kappa\phi_\pi \end{aligned} \tag{86}$$

Assume (22) holds. Since$B_2 \geq B_1$, Det$(\tilde{A}_2)\leq$Det$(\tilde{A}_1)$ and Trace $(\tilde{A}_2)\leq$Trace $(\tilde{A}_1)$, $-B_2\cdot$Trace $(\tilde{A}_2)$+Det$(\tilde{A}_2)>0$, if (82) is satisfied. So that provided that $\phi_x > \hat{\phi}_x$, the REE is stable under learning.

(ii) Notice that if (82) is not verified the sign pattern becomes $--+-$, which indicates two eigenvalues with positive real parts. Since, the determinant of $\tilde{A}_1$ does not vanish at $\hat{\phi}_x$, we know that the eigenvalues are complex.

(ii) Determinacy obtains if $\left(I_3 - A_1^{PS} + A_1^{CB}\right)^{-1} A_2^{PS}$has eigenvalues inside the unit circle. The characteristic equation can be factorized to give

$$P(\lambda) = -\lambda\left(\lambda^2 + a_1\lambda + a_0\right)$$

where

$$a_1 = -\frac{\left(\eta_2\kappa\phi_\pi + \kappa\eta_3\phi_x + \kappa + \eta_2\phi_y + \beta\phi_x\eta_1\right)\tilde{\sigma} - \kappa\eta_3\phi_\pi + 1 + \beta}{\left(\kappa\eta_1\phi_\pi + \phi_x\eta_1\right)\tilde{\sigma} + 1 - \kappa\eta_3\phi_\pi}$$

and

$$a_0 = \frac{\beta(1+\phi_x\eta_2\tilde{\sigma})}{\left(\kappa\eta_1\phi_\pi + \phi_x\eta_1\right)\tilde{\sigma} + 1 - \kappa\eta_3\phi_\pi}$$

The conditions for determinacy are

$$|a_0| < 1, \quad |a_1| < 1 + a_0$$

The first condition is verified, given that $\eta_1\tilde{\sigma} > \eta_3$. Notice that the denominator is positive, provided $\eta_1 > \eta_3$ and $\sigma \geq 1$. Also, imposing the condition mentioned in the main text we have that $a_1 < 0$, so that the condition for determinacy becomes

$$-a_1 < 1 + a_0$$

which gives (12).

Finally, local unicity of the equilibrium comes from the fact that indeterminacy occurs as $-a_1 = 1 + a_0$. Hence the eigenvalues are real at the bifurcation point so that there is no Hopf bifurcation. Moreover, the maximum eigenvalue is positive, so that we can exclude a flip bifurcation. This implies that no other equilibria exist close to the inflation target. ■

Proof. Proposition (4)

(i) Consider first the convergence properties of the policy rule estimates. Assume that the central bank and the private sector have the same expectations. This is without loss of generality, from the results above. Then substituting the expectations in the Taylor rule we get

Proof.

$$\psi_t = \psi_{t-1} + \delta_t R^{-1}_{\psi,t-1} X_{t-1} \left[\phi' \left(\tilde{\Omega}_{t-1} W_{t-1}\right) + \epsilon_t - \psi'_{t-1} \left(\tilde{\Omega}_{t-1} W_{t-1}\right)\right]$$

$$R_{\psi,t} = R_{\psi,t-1} + \delta_t \left[X_{t-1} \left(\tilde{\Omega}_{t-1} W_{t-1}\right)' - R_{\psi,t-1}\right]$$

where ϕ denotes the true policy coefficients. This can be rearranged to yield

$$\psi_t = \psi_{t-1} + \delta_t R^{-1}_{\psi,t-1} W_{t-1} W'_{t-1} \tilde{\Omega}'_{t-1} \left(\phi - \psi_{t-1} + \epsilon_t\right)$$

$$R_{\psi,t} = R_{\psi,t-1} + \delta_t \left(W_{t-1} W'_{t-1} \tilde{\Omega}'_{t-1} - R_{\psi,t-1}\right)$$

The corresponding ODE is

$$\dot{\psi} = R^{-1}_{\psi} M \tilde{\Omega}' \left(\phi - \psi\right)$$

$$\dot{R}_{\psi} = \left(M \tilde{\Omega}' - R_{\psi}\right)$$

where $M = E_{t\to\infty} W_t W'_t$. Hence, we have that $R_{\psi} \to M\tilde{\Omega}'$. Substituting in the above we obtain $\phi \to \psi$. Consider the other coefficients. The updating mechanism is

$$\tilde{\Omega}'_t = \tilde{\Omega}'_{t-1} + \delta_t R^{-1}_{t-1} W_{t-1} \left[\tilde{V}_t - \tilde{\Omega}_{t-1} W_{t-1}\right]' \tag{87}$$

$$R_t = R_{t-1} + \delta_t \left(W_{t-1} W'_{t-1} - R_{t-1}\right)$$

It is possible to express (87) as ■

$$\tilde{\Omega}'_t = \tilde{\Omega}'_{t-1} + \delta_t R^{-1}_{t-1} W_{t-1} \left[T' \left(\tilde{\Omega}_{t-1}, \psi_{t-1}\right) W_{t-1} - \tilde{\Omega}_{t-1} W_{t-1} + \zeta_t\right]' \tag{88}$$

where

$$T'\left(\Omega_{t-1},\psi_{t-1}\right)=\left(B_1\psi'_{t-1}\tilde{\Omega}_{t-1}+B_2\tilde{\Omega}_{t-1}+B_3\psi'_{t-1}\tilde{\Omega}_{t-1}H+B_4\tilde{\Omega}_{t-1}H+\bar{B}_5\right)$$

$$\begin{aligned}\tilde{V}_t &= B_0+B_1\psi'_{t-1}E^{CB}_{t-1}\tilde{V}_t+B_2E^{PS}_{t-1}\tilde{V}_t+B_3\psi'_{t-1}E^{CB}_{t-1}\tilde{V}_{t+1}+B_4E^{PS}_{t-1}\tilde{V}_{t+1}+B_5X_{t-1}+\zeta_t \quad (89)\\ i_t &= \bar{\imath}+\phi_\pi E^{CB}_{t-d}(\pi_t-\pi^*)+\phi_x E^{CB}_{t-d}\left(x_t-x^*\right)+\epsilon_t\end{aligned}$$

It is then possible to rearrange (88) into

$$\tilde{\Omega}'_t=\tilde{\Omega}'_{t-1}+\delta_t R^{-1}_{t-1}W_{t-1}W'_{t-1}\left[T\left(\tilde{\Omega}_{t-1},\psi_{t-1}\right)-\tilde{\Omega}'_{t-1}+\zeta'_t\right]$$

The associated ODE can be calculated as

$$\frac{d\tilde{\Omega}'}{d\tau}=\left[T\left(\tilde{\Omega},\phi\right)-\tilde{\Omega}'\right]$$

where I use the fact that, a) $R\rightarrow M$; b) $\phi\rightarrow\psi$. It is straightforward to show that stability under learning depends on the eigenvalues of the following matrix

$$\tilde{B}_1+\tilde{B}_2-I_2=\begin{bmatrix}-\phi_x\tilde{\sigma} & \tilde{\sigma}\left(1-\phi_\pi\right)\\ \kappa\left(1+\eta_3\phi_y\right) & \beta+\kappa\eta_3\phi_\pi-1\end{bmatrix} \quad (90)$$

and

$$\tilde{B}_1+\rho_i\tilde{B}_2-I_2 \quad (91)$$

where

$$\tilde{B}_1=B_1\phi'+B_2;\ \tilde{B}_2=B_3\phi'+B_4$$

In order to have negative eigenvalues, I need both the trace and the determinant to be negative. Consider the case $U_{cm}=0$. It is straightforward to show that the eigenvalues of the matrix (90) are negative provided the Taylor Principle holds. Also it is possible to show that if the matrix (91) satisfies this property, then also matrix (90) will satisfy it. Furthermore, the determinant of $\tilde{B}_1+\tilde{B}_2-(1+\lambda)\,I_2$ vanishes if the Taylor condition holds with equality. Hence the eigenvalues are real and no Hobf bifurcation occur.

(ii) .In the case of perfect transparency and SG learning it is easy to verify that the associated ODE becomes

$$\frac{d\tilde{\Omega}'}{d\tau}=M\left[\tilde{\Omega}'\tilde{B}'_1+H\tilde{\Omega}'\tilde{B}'_2-\tilde{\Omega}'\right]$$

by vectorizing and transposing we obtain

$$vec\left(\dot{\Omega}\right)=\left(\left(M\otimes I_3\right)\right)\left[\left(I\otimes\tilde{B}_1\right)+\left(H\otimes\tilde{B}_2\right)-I_9\right]vec\left(\Omega\right)$$

Using the fact that M is diagonal I can re-express the matrix as

$$\left(\tilde{B}_1 + \tilde{B}_2 - I_3\right)$$
$$m_1 \left(\tilde{B}_1 + \rho_r \tilde{B}_2 - I_3\right)$$
$$m_2 \left(\tilde{B}_1 + \rho_u \tilde{B}_2 - I_3\right)$$

where I need to adjust notation for the constant and m_1,m_2 are the elements of M on the diagonal. Given the fact that by definition m_1,m_2 are positive, the stability condition is identical to the case with RLS.

consider the case of imperfect transparency. With SG, it is possible to show that the linearized ODE becomes

$$\begin{bmatrix} \dot{\psi} \\ vec(\dot{\Omega}') \end{bmatrix} = \begin{bmatrix} -M\tilde{\Omega}' & (\phi - \psi)' \otimes M \\ B_1 \otimes M\tilde{\Omega}' + B_3 \otimes MH\tilde{\Omega}' & CC \end{bmatrix} \begin{bmatrix} \psi \\ vec\left(\tilde{\Omega}'\right) \end{bmatrix}$$

$$CC = B_1\psi' \otimes M + B_2 \otimes M + B_3\psi' \otimes MH + B_4 \otimes MH - I \otimes M$$

evaluating the equation at the REE coefficients we get

$$\begin{bmatrix} \dot{\psi} \\ vec(\dot{\Omega}') \end{bmatrix} = \begin{bmatrix} -M\tilde{\Omega}^{*\prime} & \mathbf{0} \\ B_1 \otimes M\tilde{\Omega}^{*\prime} + B_3 \otimes MH\tilde{\Omega}^{*\prime} & \tilde{B}_1 \otimes M + \tilde{B}_2 \otimes HM - I \otimes M \end{bmatrix} \begin{bmatrix} \psi \\ vec\left(\Omega'\right) \end{bmatrix}$$

Stability conditions depend on the eigenvalues of the matrices $-M\tilde{\Omega}^{*\prime}$ and $\tilde{B}_1 \otimes M + \tilde{B}_2 \otimes HM - I \otimes M$. Hence, from the results in (ii), the Taylor principle it is not sufficient for stability of the REE.

(iii) Consider the case where $U_{cm} > 0$. In this case the trace is negative if $\phi_x\tilde{\sigma}+1-\beta-\kappa\eta_3\phi_\pi > 0$. This implies that also in the case of full transparency a policy rule that does not react to the output gap is destabilizing. Nevertheless transparency increases the set of rules that are robust to expectational mistakes. In order to show this, notice that (82) implies

$$\phi_x\tilde{\sigma} + 1 - \beta - \kappa\eta_3\phi_\pi > \tilde{\sigma}\frac{\kappa + \kappa\left(\phi_\pi - 1\right) + \phi_x\left(1-\beta\right) - \kappa\eta_3\phi_x}{2-\beta} > 0$$

which is a more stringent condition for stability than in the case of full transparency. Combining the stability conditions for the case of transparency, I obtain the condition (29) ■

Proof. Proposition (5)

(i) the characteristic equation in the case of no transparency can be written as

$$P\left(\lambda\right) = \left(1+\lambda\right)\left(\lambda^2 + a_1 + a_0\right)$$

where

$$\begin{aligned} a_1 &= \phi_x\tilde{\sigma} + (1-\beta) - \kappa\eta_3\phi_\pi \\ a_0 &= \kappa\left(\phi_\pi - 1\right) + \phi_x\left(1-\beta\right) - \kappa\eta_3\phi_x \end{aligned}$$

It can be easily verified that the same condition can be found imposing full transparency, following the same step as in the proof above.

(ii) this is equivalent to set $\phi_x = 0$ in (5).

(iii) I assume $\rho_r = \rho_u = 0$, in order to simplify the proof. Consistently with the findings in the previous proofs, I expect the result would not change for positive autoregressive components. Under the current assumptions, the stability under learning depends on the eigenvalues of (75).

(iv) Under the current assumptions, the stability under learning depends on the eigenvalues of (90). ■

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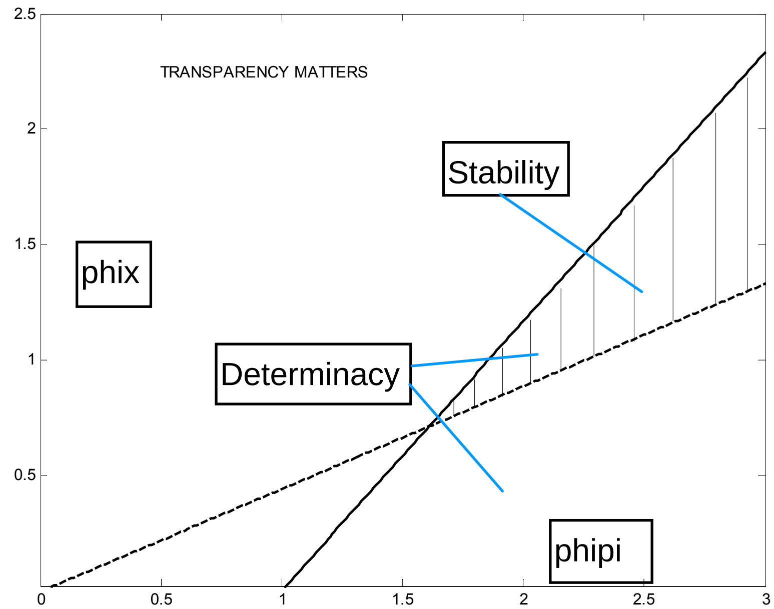

Figure 1.

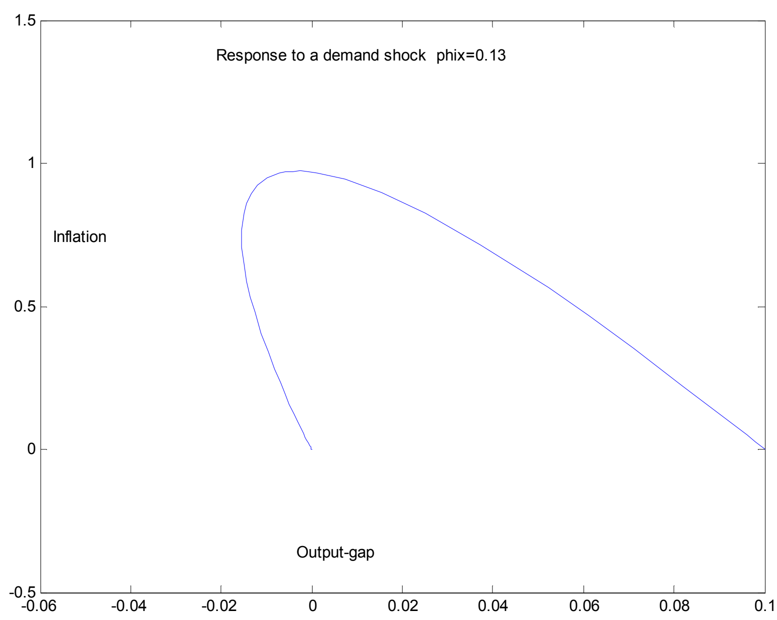

Figure 2.

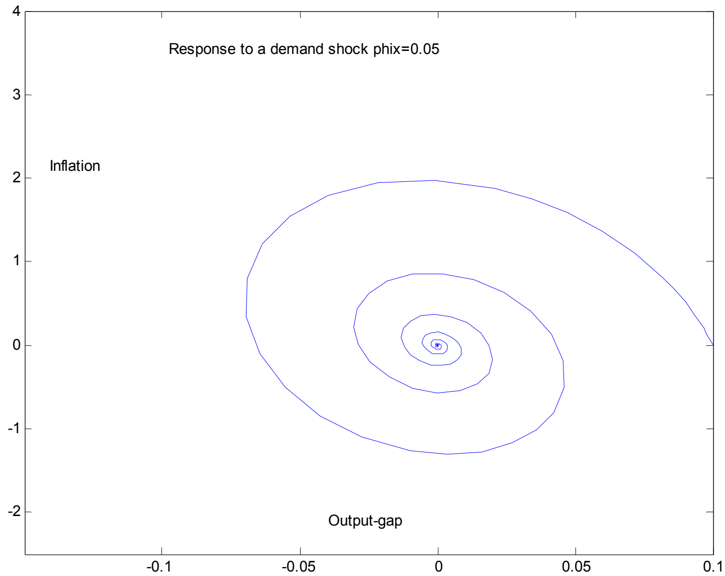
Response to a demand shock phix=0.05
Inflation
Output-gap
4
3
2
1
0
-1
-2
-0.1
-0.05
0
0.05
0.1

Figure 3.