A Circuitist Model of Monetary Production

Steve Keen, University of Western Sydney Australia[1]

One of the key dilemmas in economic modelling has been incorporating money into models of production and exchange. The standard approach has been to specify one commodity as "the money commodity" and denominate all transactions in terms of it. In general, this approach has either failed to uncover anything distinctively different between a monetary and a barter economy, or resulted in findings of the super-neutrality of money.

The European "Circuitist" school of economists provides a very different basis for considering the role of money in a model of a production economy, by arguing from first principles that money *cannot* be a commodity. Their initial proposition is extremely simple: if a commodity acts as money, then the only difference between a barter and a money model is that one has *n* commodities and the other has *n+1*. As Graziani puts it:

> The starting point of the theory of the circuit, is that a true monetary economy is inconsistent with the presence of a commodity money. A commodity money is by definition a kind of money that any producer can produce for himself. But an economy using as money a commodity coming out of a regular process of production, cannot be distinguished from a *barter economy*. A true monetary economy *must therefore be using a token money*, which is nowadays a paper currency. (Graziani 1989: 3; emphases in original)

A token raises two additional problems which are addressed by a further two conditions that are needed 'In order for money to exist':

> b) money has to be accepted *as a means of final settlement* of the transaction (otherwise it would be credit and not money);
>
> c) money must not grant privileges of seignorage to any agent making a payment. (Graziani 1989: 3)

From this the Circuitists derive the insight that 'any monetary payment must therefore be a triangular transaction, involving at least three agents, the payer, the payee, and the bank'. This is because the only way these three conditions can be satisfied:

> is to have payments made by means of *promises of a third agent*, the typical third agent being nowadays a bank... Once the payment is made, no debt and credit relationships are left between the two agents. But one of them is now a creditor of the bank, while the second is a debtor of the same bank. (Graziani 1989: 3)

This perspective clearly delineates a monetary vision of capitalism from a barter paradigm. In a barter world, transactions are two sided, two commodity barter exchanges: person A gives person B units of commodity X in return for units of commodity Y (see Figure 1). Calling one of these 'the money commodity' does not alter the essentially barter personality of the transaction.

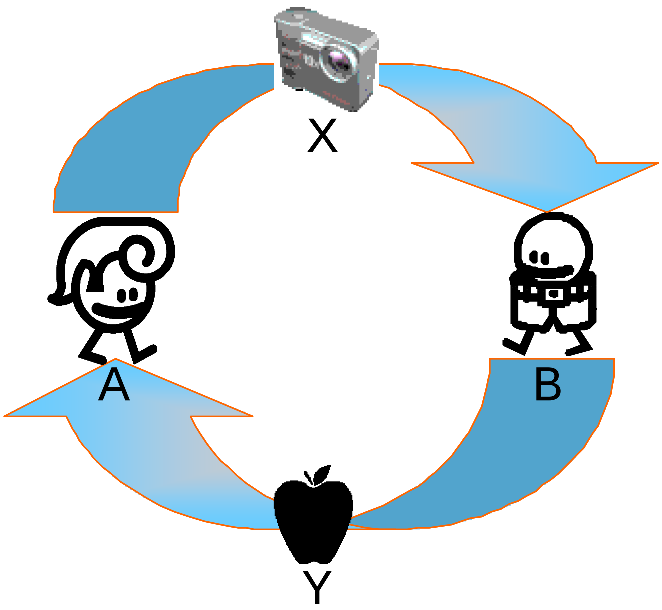


Figure 1. Barter Exchange

In a monetary world, transactions are three-sided, single commodity, financial exchanges: person A gives person B units of commodity X, in return for person B requesting the bank Z (and the bank agreeing) to debit Y currency units from B's account and credit A's account with the same amount (see Figure 2).[2]

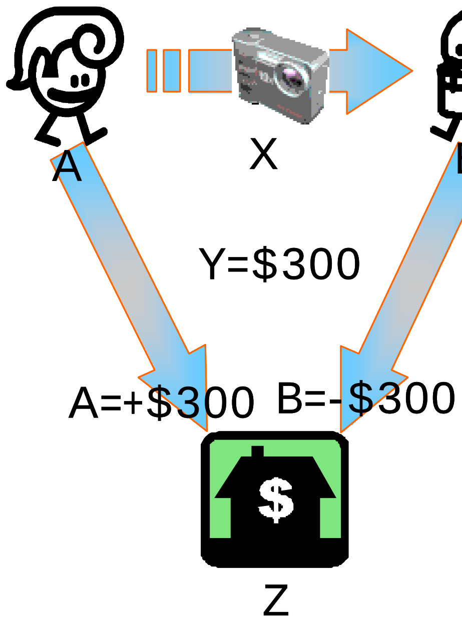


Figure 2. Monetary Exchange

The financial aspects of a credit system become integral to capitalism from this Circuitist perspective. Money is quintessentially credit money, with the bank in Figure 2 granting $300 of credit to agent B, and B in turn incurring a debt obligation to the bank of this amount with its concomitant interest payment commitments, etc. Banks are thus an essential component of capitalism, and cannot be treated simply as a particular type of firm, as Graziani emphasizes:

> Since in a monetary economy money payments necessarily go through a third agent, the third agent being one that specialises in the activity of producing means of payment (in modern times a bank), *banks and firms must be considered as two distinct kinds of agents*... In any model of a monetary economy, *banks and firms cannot be aggregated into one single sector*. (Graziani 1989: 4)

The minimum requirements for a model of a monetary production economy are thus (a) the existence of a token money or bank accounting system whose "production" is completely independent of the system of commodity production; (b) three-sided exchanges where the transfer of a commodity from one agent to another requires a monetary transfer recorded by a bank; and (c) the treatment of banks and bank income as completely different phenomena to firms and firm income. Money is used for all payments and the banks are the source of money, so that causation runs from the granting of a loan by banks to firms, and concludes with the repayment of the incurred debt from the proceeds of production.

1 Money as a disequilibrium phenomenon

Graziani's outline of this process (Graziani 1989: 4) starts with banks creating money by issuing a loan to firms, who in turn hire workers. Workers then deposit the money in their bank accounts, and either spend their wages upon products produced by the firm(s), or use it to purchase securities issued by the firms.

The latter two activities enable the firms to extinguish their obligations to the banking sector, and once they do so 'the money initially created is destroyed' (Graziani 1989: 5). Graziani then observes that if workers spent their salaries instantly, money would be destroyed virtually as soon as it was created and would therefore effectively not exist. Money is therefore essentially a disequilibrium phenomenon.

2 Not following through

Unfortunately Graziani did move on from these excellent foundations to build a disequilibrium model of the monetary circuit, and no subsequent writer in this tradition has produced an adequate dynamic rendition of the School's insights. Their analysis beyond these foundations has tended to be either verbal in nature, or to use simultaneous equations (Graziani 1989; Bellofiore et al. 2000; Bossone 2001), and they have universally failed to explain how capitalists can manage to repay the interest obligations they incur (see for example Bellofiore et al. 2000: footnotes 8, 9). Implicitly, the problem Circuitists seem to have encountered is explaining how capitalists could borrow money and manage to repay it, let alone do so at a profit. In this paper, starting from the first principles laid down by Graziani, I build a dynamic model of the circuit that explicitly accounts for all interest flows and shows how capitalists can repay their debt and make a profit.

This paper simply considers (a) whether the Circuitist vision is internally consistent, in that a model that implements its assumptions is internally coherent; (b) whether this system can "lift itself up by its own bootstraps", in that an initial injection of credit money starts the system off; (c) whether this system at the simplest level (abstracting from growth, profit-driven investment, moods, etc.) can continue without an additional injection of funds. Point (c) has been disputed by some previous attempts to model the Circuitist vision, including Fontana 2000, but it is possible that these papers have reached their conclusions via incomplete modelling of the monetary dynamics (see Andresen 2005).

Stage 1: Debt issuance

The initial step in the monetary production process is the granting of working capital credit to firms by banks. This creates both money and debt obligations between firms and banks. The initial loan *WK* simultaneously creates a positive entry in capitalists' credit accounts and a entry in their debit accounts. The deposit initiates the capitalists' credit account K_C which then earns the rate of interest r_c, while debit initiates the debit account K_D incurs the higher interest rate r_d. A debt necessarily incurs a repayment obligation, and the repayment amount is deducted from the credit account and paid to the bankers account (the corresponding entry in the capitalist debit account is an accounting entry only to record the repayment of debt).[3] In this model I presume that capitalists set a target date *T* by which time they intend to pay the debt incurred to the proportion *X* of its original level. Two equations specify the repayment factor *R* needed to achieve this:

$$
\begin{aligned}
&K_D(T) = K_D(0)\cdot X \\
&\frac{d}{dt}K_D = r_d\cdot K_D - R\cdot K_D
\end{aligned}
\tag{1}
$$

Solving for R yields $R = r_d - \frac{ln(X)}{T}$. The initial equations of this model are therefore:

$$
\begin{aligned}
&\frac{d}{dt}K_D = r_d\cdot K_D - R\cdot K_D = \frac{ln(X)}{T}\cdot K_D \\
&\frac{d}{dt}K_C = r_c\cdot K_C - R\cdot K_D \\
&\frac{d}{dt}B_Y = R\cdot K_D - r_c\cdot K_C
\end{aligned}
\tag{1}
$$

where B_Y is the bankers' account (this is later divided into an income and a principal account).

The incomplete model is simulated in Figure 3 with an initial loan of \$100, debit interest rate of 5%, credit interest rate of 3%, a repayment term of 1 year and a repayment target of 0.01: $WK = \$100, r_d = .05, r_c = .03, T = 1, X = .01$. At this stage, the net outcome is that after one year, capitalists are indebted to bankers for \$0.44, the net sum of the interest payments incurred over the time period.

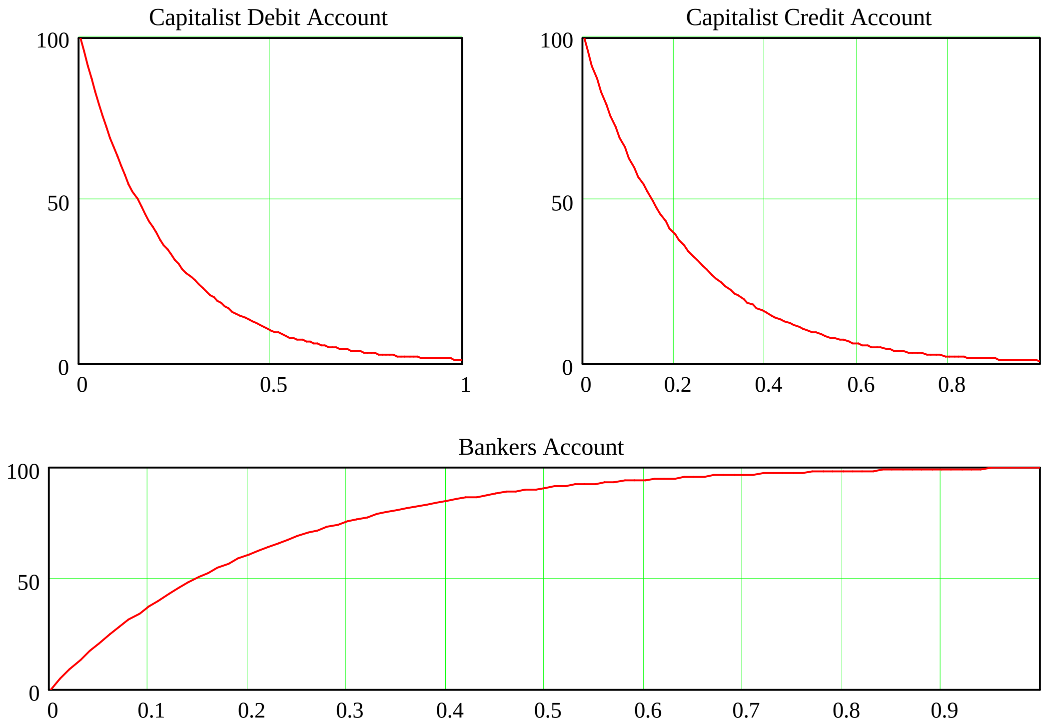


Figure 3. Basic capitalist-banker cash flows

Stage 2: "Closure" with workers accounts

Graziani "closed" his verbal model by assuming that the money borrowed by capitalists was *immediately* paid to workers. Workers then spent their wages gradually, thus generating a cash flow to capitalists that *partly* enabled them to repay the incurred debt (not including the interest bill accumulated during the loan period, though this was not acknowledged by Graziani: see Graziani 1989: 5).

As I note in Keen 2005, Graziani agonized over his "assumption" of gradual rather than instantaneous expenditure, despite his insight that money could only exist in this system if spending was gradual rather than immediate. Though his insights were essentially dynamic, he analyzed his system in terms of equilibrium and simultaneous equations, rather than dynamics and differential equations.

This agonizing was unnecessary. Once we are working in terms of a dynamic model, *every* process takes time. The truly unjustified assumption that Graziani made was not the realistic statement that 'wage-earners spend their money incomes gradually over time' (Graziani 1989: 6), but his unrealistic unstated assumption that capitalists *instantly* disperse all of the loan to workers in the form of wages. Instead, capitalists will also disperse their working capital gradually over time, and this additional flow out of their credit accounts then finances production.

Modeling this flow of working capital into the funding of production as a first-order time lag,[4] the modified expression for the capitalist credit account is:

$$\frac{d}{dt}K_C = r_c \cdot K_C - R \cdot K_D - \frac{1}{\tau_{Kp}} \cdot K_C \tag{3}$$

where τ_{Kp} is the time lag (in years) for the expenditure of working capital (hence the "*p*" for "production" in the subscript). In these simulations I set $\tau_{Kp} = \frac{1}{4}$ (3 months).

Capitalists divide this flow into two streams (π) for expenditure on commodities and $1-\pi$ for hiring workers. In the simulation below, I set $\pi = .3$.

Leaving aside employment and the production of output for the moment, the flow of working capital to workers is recorded in workers bank accounts, and attracts an interest payment at the rate of r_c.[5] Workers then spend this income with a time lag t_w that reflects their consumption requirements. The inflows into workers accounts W_c are thus the flow of capitalist working capital into wages ($(1-\pi)\cdot\frac{1}{\tau_{Kp}}\cdot K_C$) and interest on the existing balance ($r_c \cdot W_C$); this last term becomes an additional outflow from bankers' credit accounts. The outflow of expenditure by workers on commodities is proportional to the existing balance ($\frac{1}{\tau_W}\cdot W_C$); this in turn becomes an inflow into capitalist accounts. The expression for workers' accounts balance is thus:

$$\frac{d}{dt}W_C = (1-\pi)\cdot\frac{1}{\tau_{Kp}}\cdot K_C + r_c\cdot W_C - \frac{1}{\tau_W}\cdot W_C \tag{5}$$

Bankers also spend their credit balances on the output of capitalists with a time lag t_B: thus there is an outflow from bankers accounts and an inflow into capitalist accounts of $\frac{1}{\tau_B}\cdot B_C$. In the simulation below, I set $\tau_b = 1, \tau_W = \frac{1}{26}$.

The expressions for capitalist and banker accounts now need to be amended to include the new inflows and outflows that, prior to the consideration of production, close this Circuitist model. The equations prior to the consideration of production are:

$$\begin{aligned}
\frac{d}{dt}K_D &= \frac{ln(X)}{T}\cdot K_D \\
\frac{d}{dt}K_C &= r_c\cdot K_C - R\cdot K_D - \frac{1}{\tau_{Kp}}\cdot K_C + \left(\frac{\pi}{\tau_{Kp}}\cdot K_C + \frac{1}{\tau_B}\cdot B_Y + \frac{1}{\tau_W}\cdot W_C\right) \\
\frac{d}{dt}B_Y &= R\cdot K_D - r_c\cdot K_C - r_c\cdot W_C - \frac{1}{\tau_B}\cdot B_Y \\
\frac{d}{dt}W_C &= (1-\pi)\cdot\frac{1}{\tau_{Kp}}\cdot K_C + r_c\cdot W_C - \frac{1}{\tau_W}\cdot W_C
\end{aligned} \tag{6}$$

The model up to this point is shown in Figure 4. At this level of closure, all three classes are long term beneficiaries of the system, with each accumulating a positive bank balance; however the system is unsustainable since the dynamics of the bankers account will ultimately drive it into deficit.

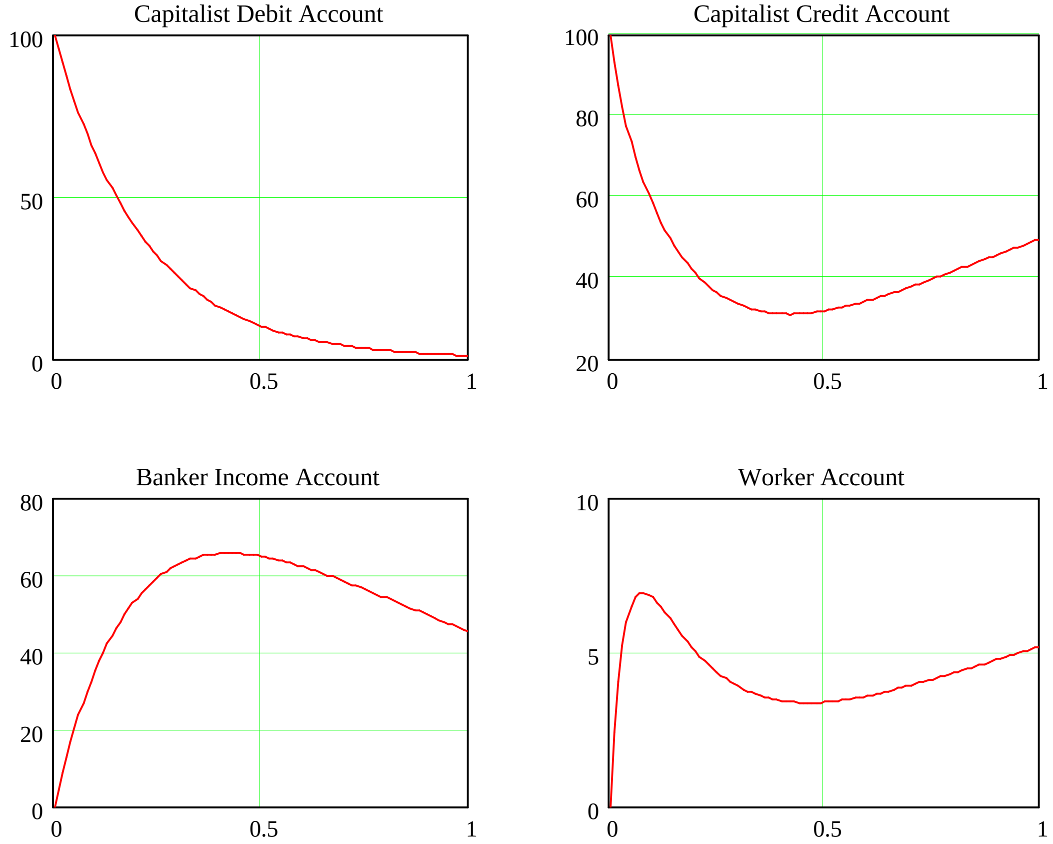


Figure 4. Model withour relending or production

Stage 3: Relending of banker's principal

Graziani's verbal analysis considered the path through the monetary circuit of a single injection of credit money, and concluded that "As soon as firms repay their debt to the banks, the money initially created is destroyed. With the destruction of money, the circuit is closed..." (Graziani 1989: 5). He also inferred that a "new production cycle" would require "the concession of a new credit" by the bank. However, a dynamic analysis of the rate of change of credit money shows that no new injection is necessary if bankers relend the principal repaid by capitalists.

To illustrate this, I now introduce a banker's principal account B_P and relending of this repaid principal with a time lag of τ_{Bp}. This turns up as a negative entry on the banker's principal account and a positive entry to the capitalist credit account (with a corresponding book-keeping entry on the debit account). The flow of funds from capitalists to bankers is now broken into two streams with interest on debt being paid into the banker's income account, and expenditure by bankers $\frac{1}{\tau_B} \cdot B_Y$ is now more realistically shown as based on the income earned from the spread between debit and credit payments. In the simulation below I set $\tau_{Bp} = \frac{1}{2}$; I also run the simulation for 4 years to illustrate that the system reaches an equilibrium.

$$
\begin{aligned}
\frac{d}{dt}K_D &= \frac{ln(X)}{T}\cdot K_D + \frac{1}{\tau_{Bp}}\cdot B_P \\
\frac{d}{dt}K_C &= r_c\cdot K_C - R\cdot K_D - \frac{1}{\tau_{Kp}}\cdot K_C + \left(\frac{\pi}{\tau_{Kp}}\cdot K_C + \frac{1}{\tau_B}\cdot B_Y + \frac{1}{\tau_W}\cdot W_C\right) + \frac{1}{\tau_{Bp}}\cdot B_P \\
\frac{d}{dt}B_P &= -\frac{ln(X)}{T}\cdot K_D - \frac{1}{\tau_{Bp}}\cdot B_P \\
\frac{d}{dt}B_Y &= r_d\cdot K_D - r_c\cdot K_C - r_c\cdot W_C - \frac{1}{\tau_B}\cdot B_Y \\
\frac{d}{dt}W_C &= (1-\pi)\cdot\frac{1}{\tau_{Kp}}\cdot K_C + r_c\cdot W_C - \frac{1}{\tau_W}\cdot W_C
\end{aligned}
\tag{7}
$$

The model now describes a sustainable system. Contrary to Graziani’s verbal surmise, the system can continue indefinitely without the need for any additional injection of funds (a similar point was made by Andresen 2005 in a systems engineering analysis of Fontana 2000).

This situation may and probably will change with the introduction of more dynamic elements (such as population growth, technical change, income distribution conflicts, etc.). However, the basic point remains that this model of the monetary circuit can be “kick-started” and sustained by a single initial issuance of credit: once in equilibrium, the initial injection of $100 finances $139 p.a. of new lending. The annual flow of funds into and out of capitalist accounts totals totals $109 p.a. (interest income and cash flows from other capitalists, workers and bankers constitute the inflows while interest payments on debt and expenditure on production constitute the outflows).

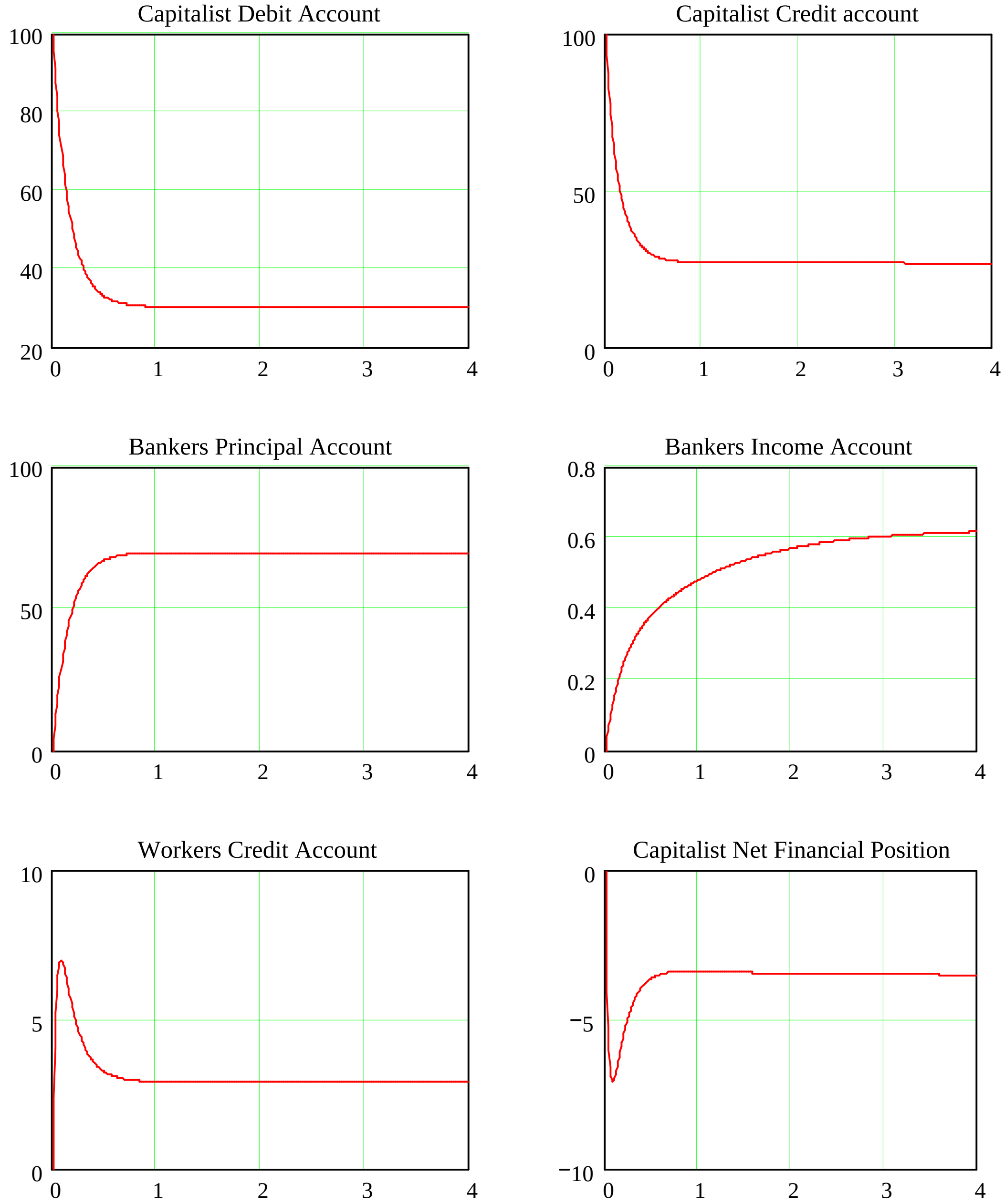


Figure 5. Model without production

Stage 4: Production

To fully close the model, we have to introduce production and the means by which capitalists can make a surplus and realize a profit from sales. In Graziani's verbal model, production was proportional to labour employed, and I will maintain that convention in this simple model. Given a fixed wage of w, employment is $L = \frac{1}{w} \cdot (1-\pi) \cdot \frac{1}{\tau_{Kp}} \cdot K_C$ and given fixed labour productivity of a units p.a. per worker, equilibrium output is given by $Q = L \cdot a$; to reflect the fact that production takes time, a production time lag t_Q is introduced so that the production relation is:

$$\frac{d}{dt}Q = \frac{-1}{\tau_Q} \cdot \left(Q - \frac{a}{w}\frac{1-\pi}{\tau_{Kp}} \cdot K_C\right) \tag{8}$$

A price mechanism is now needed. This is modelled as a reaction to excess demand pressure with a time lag, where d_s signifies the price pressure and t_{Inf} the time lag in price setting:

$$\frac{d}{dt}P = \frac{1}{\tau_{Inf}} \cdot \left(-P \cdot \delta_\sigma \cdot \frac{P \cdot Q - D}{D} \right) \tag{9}$$

where D is demand: $D = \frac{\pi}{\tau_{Kp}} \cdot K_C + \frac{1}{\tau_B} \cdot B_Y + \frac{1}{\tau_W} \cdot W_C$. In the simulation below, initial output is set at zero, initial price at $\frac{1}{1000}$, labour productivity at 1000 p.a., $\tau_{Inf} = \frac{1}{4}$ and $\delta_\sigma = 1$.

Capitalist income is now price times quantity rather than simply the time lagged flow of funds out of agent accounts. This raises the additional complication that the flow of expenditures may not match the flow of price times output, which is considered in the next extension. In this manifestation of the model, capitalists always receive price times quantity, and outflows from agents accounts are proportional to price times quantity:

$$\begin{aligned}
\frac{d}{dt}K_D &= \frac{ln(X)}{T} \cdot K_D + \frac{1}{\tau_{Bp}} \cdot B_P \\
\frac{d}{dt}K_C &= r_c \cdot K_C - R \cdot K_D - \frac{K_C}{\tau_{Kp}} + P \cdot Q + \frac{1}{\tau_{Bp}} \cdot B_P \\
\frac{d}{dt}B_P &= -\frac{ln(X)}{T} \cdot K_D - \frac{B_P}{\tau_{Bp}} \\
\frac{d}{dt}B_Y &= r_d \cdot K_D - r_c \cdot K_C - r_c \cdot W_C - \frac{B_Y}{\tau_B} \cdot \frac{P \cdot Q}{\frac{\pi \cdot K_C}{\tau_{Kp}} + \frac{B_Y}{\tau_B} + \frac{W_C}{\tau_W}} \\
\frac{d}{dt}W_C &= (1-\pi) \cdot \frac{K_C}{\tau_{Kp}} + r_c \cdot W_C - \frac{W_C}{\tau_W} \cdot \frac{P \cdot Q}{\frac{\pi \cdot K_C}{\tau_{Kp}} + \frac{B_Y}{\tau_B} + \frac{W_C}{\tau_W}} \\
\frac{d}{dt}Q &= \frac{-1}{\tau_Q} \cdot \left(Q - \frac{a}{w} \frac{1-\pi}{\tau_{Kp}} \cdot K_C \right) \\
\frac{d}{dt}P &= \frac{1}{\tau_{Inf}} \cdot \left(-P \cdot \delta_\sigma \cdot \left(\frac{P \cdot Q}{\frac{\pi \cdot K_C}{\tau_{Kp}} + \frac{B_Y}{\tau_B} + \frac{W_C}{\tau_W}} - 1 \right) \right)
\end{aligned} \tag{10}$$

This model also reaches an equilibrium, but with a substantially lower level of income for capitalists. Equilibrium capitalist income/expenditure is now \$82 p.a. versus \$109 p.a. when time lags alone determined the inflow of funds, though the annual generation of credit from the initial \$100 injection remains at \$139.

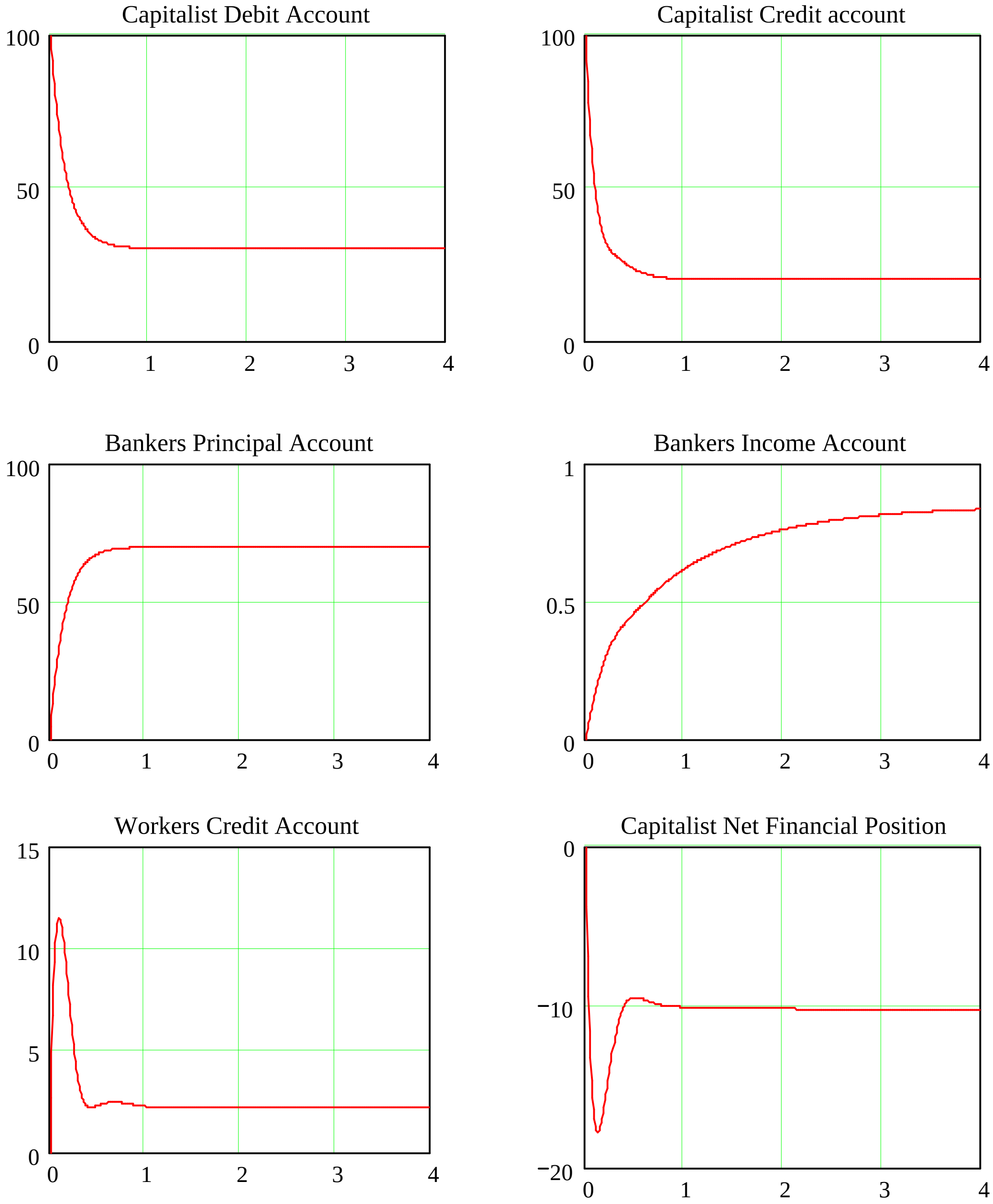


Figure 6. Financial dynamics of model without effective demand

Second-order dynamics also occur in prices, reflecting the overshooting sometimes of demand by supply and at other times of supply by demand:

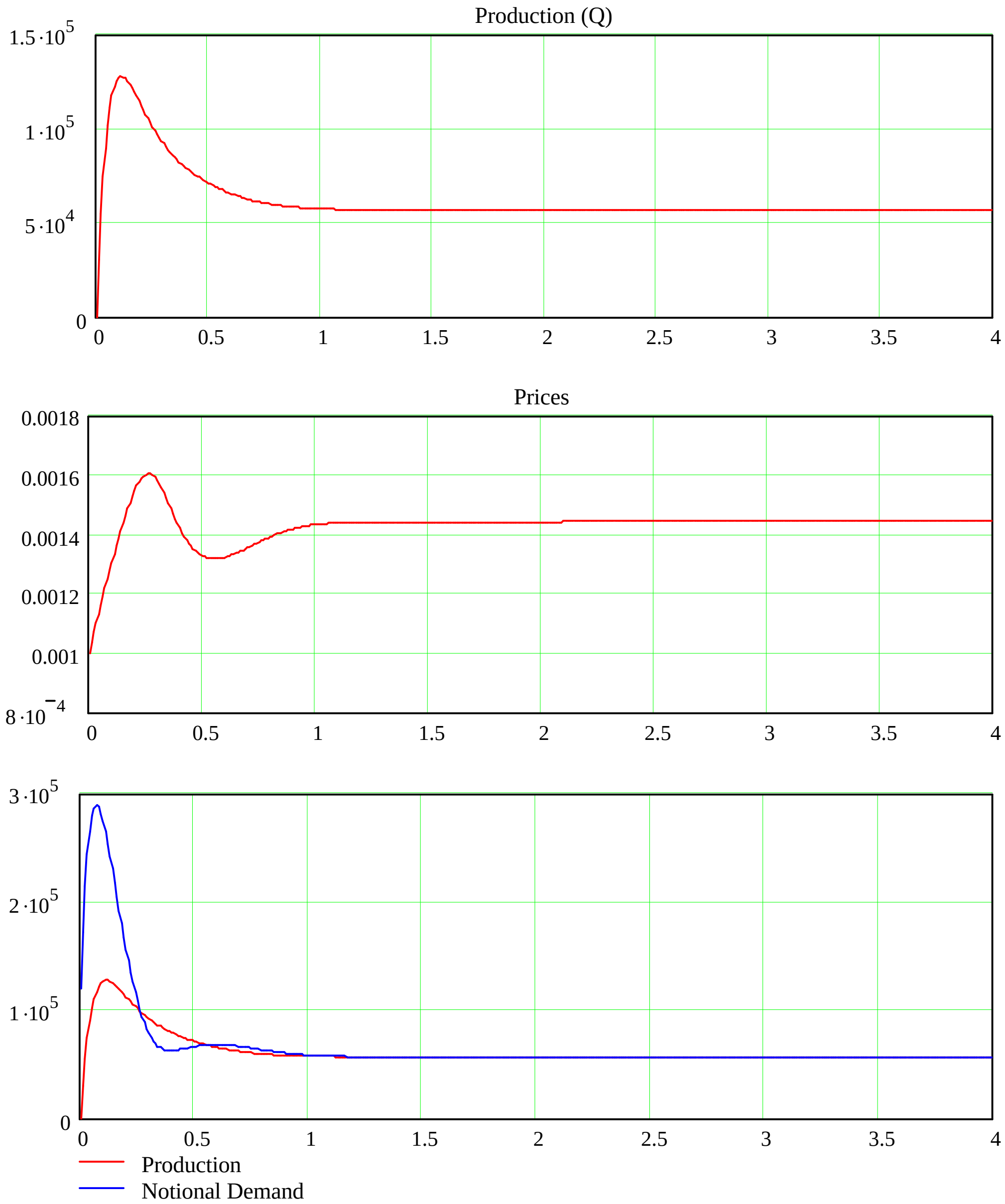


Figure 7. Output and price dynamics without effective demand

As can be seen from the bottom chart in Figure 7, production and notional demand take over a year to converge. Before they do, when there is an imbalance, the lower of the two should determine the outcome: in the case where demand exceeds supply, the outflow from accounts will therefore be less than the time lagged amount and proportional to the imbalance between supply and demand; in the case where supply exceeds demand, the outflow from accounts will equal the lagged amount but capitalist revenues will be less than price times quantity and unsold goods will accumulate.

To indicate the impact of these constraints of effective demand on the system, I introduce a *min* function: capitalists receive the minimum of price times quantity or the cash flow of expenditure from agent accounts, and the stock of commodities is augmented by unsold items when supply exceeds demand:[6]

$$
\begin{aligned}
&\frac{d}{dt}K_D = \frac{ln(X)}{T}\cdot K_D + \frac{1}{\tau_{Bp}}\cdot B_P \\
&\frac{d}{dt}K_C = r_c\cdot K_C - R\cdot K_D - \frac{K_C}{\tau_{Kp}} + \min\left(P\cdot Q, \frac{\pi\cdot K_C}{\tau_{Kp}} + \frac{B_Y}{\tau_B} + \frac{W_C}{\tau_W}\right) + \frac{1}{\tau_{Bp}}\cdot B_P \\
&\frac{d}{dt}B_P = -\frac{ln(X)}{T}\cdot K_D - \frac{B_P}{\tau_{Bp}} \\
&\frac{d}{dt}B_Y = r_d\cdot K_D - r_c\cdot K_C - r_c\cdot W_C - \frac{B_Y}{\tau_B}\cdot\min\left(\frac{P\cdot Q}{\frac{\pi\cdot K_C}{\tau_{Kp}} + \frac{B_Y}{\tau_B} + \frac{W_C}{\tau_W}}, 1\right) \\
&\frac{d}{dt}W_C = (1-\pi)\cdot\frac{K_C}{\tau_{Kp}} + r_c\cdot W_C - \frac{W_C}{\tau_W}\cdot\min\left(\frac{P\cdot Q}{\frac{\pi\cdot K_C}{\tau_{Kp}} + \frac{B_Y}{\tau_B} + \frac{W_C}{\tau_W}}, 1\right) \\
&\frac{d}{dt}Q = \frac{-1}{\tau_Q}\cdot\left(Q - \frac{a}{w}\frac{1-\pi}{\tau_{Kp}}\cdot K_C\right) + \min\left(0, Q - \frac{\frac{\pi\cdot K_C}{\tau_{Kp}} + \frac{B_Y}{\tau_B} + \frac{W_C}{\tau_W}}{P}\right) \\
&\frac{d}{dt}P = \frac{1}{\tau_{Inf}}\cdot\left(-P\cdot\delta_\sigma\cdot\left(\frac{P\cdot Q}{\frac{\pi\cdot K_C}{\tau_{Kp}} + \frac{B_Y}{\tau_B} + \frac{W_C}{\tau_W}} - 1\right)\right)
\end{aligned}
\tag{11}
$$

Now when production exceeds demand, demand determines capitalist revenue, and vice versa. As a consequence, equilibrium capitalist income falls further, to \$75 p.a. compared to the previous level of \$82 p.a. and the original pre-production level of \$109.

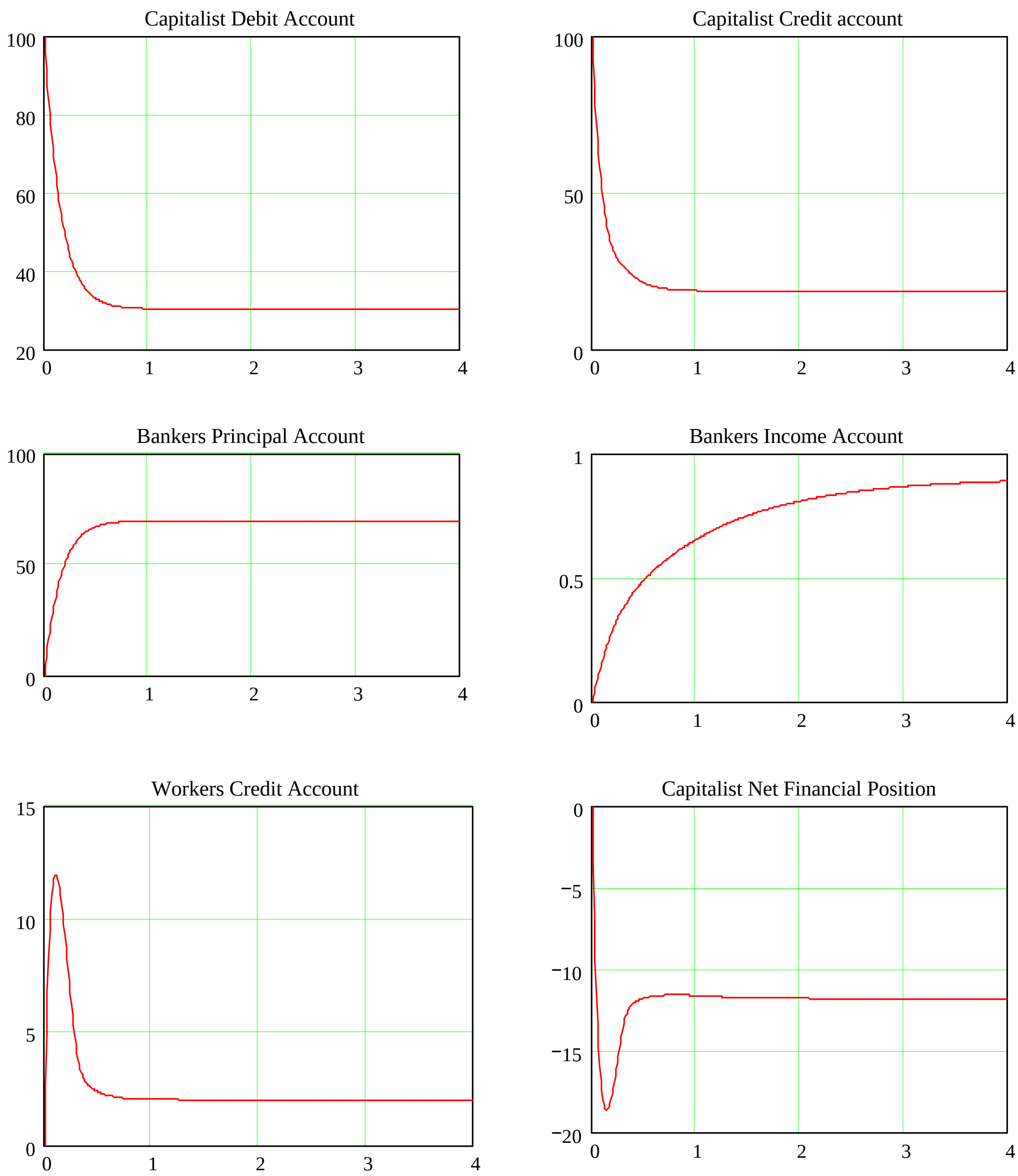


Figure 8. Financial dynamics with effective demand

The price dynamics are now muted compared to the model without effective demand constraints (of course, an actual system would start from an initial non-zero production level, which would further dampen the initial cyclical dynamics shown here).

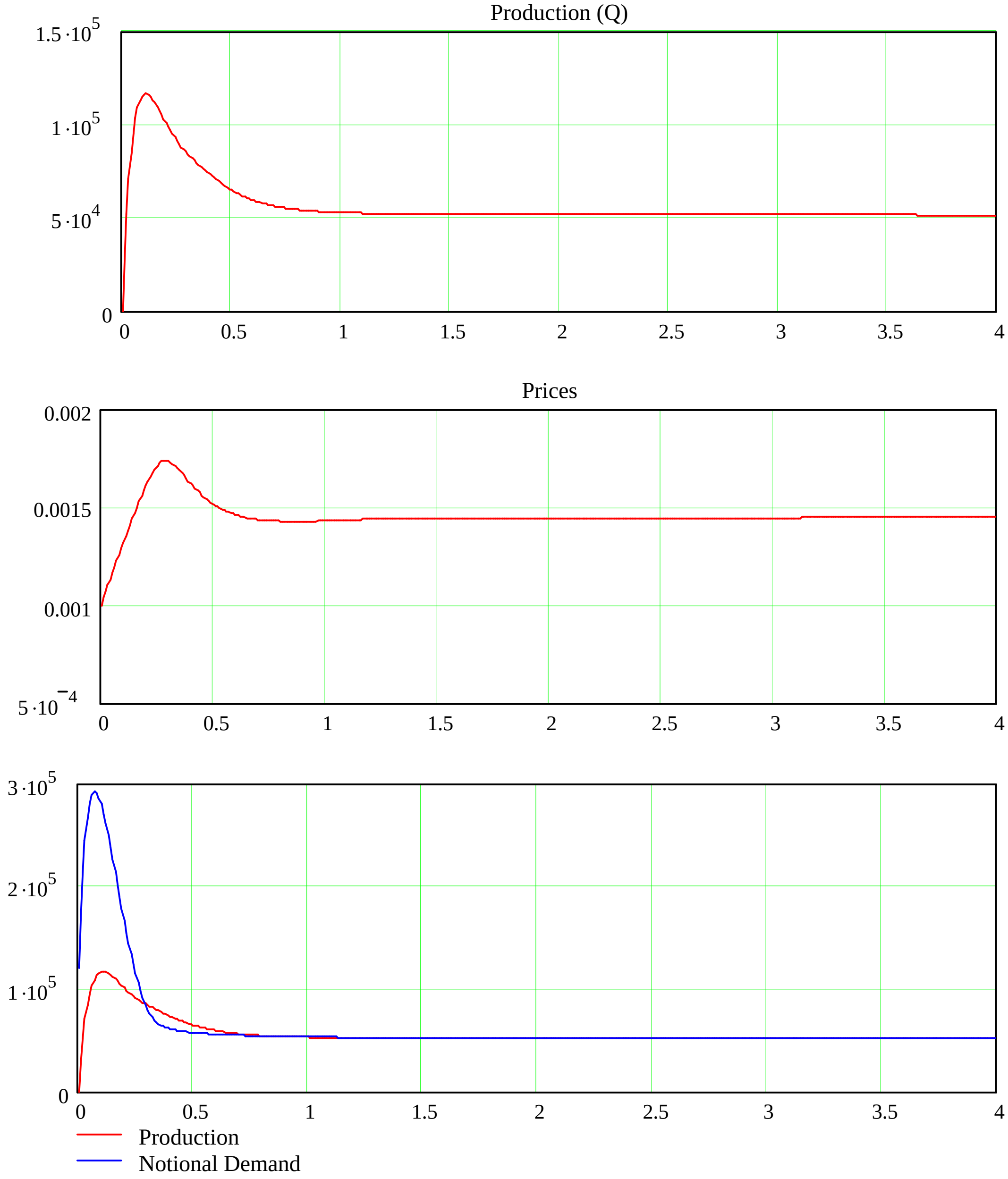


Figure 9. Production & Price Dynamics with Effective Demand

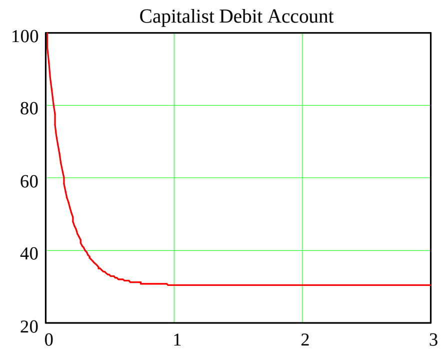
Stage 5: Stocks

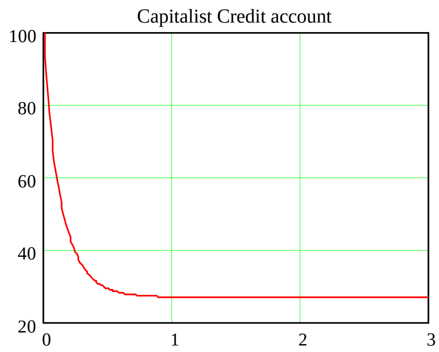
The previous extension was an incomplete and somewhat illicit modelling of the existence of demand and supply constraints: unsold goods are not really added to existing production but to inventory, which in turn must be the first destination of newly produced goods. The production equation thus returns to the previous lagged form, while a new equation for stocks Q has flows in of new production and flows out of aggregate nominal demand divided by the price level. Now when demand exceeds supply, capitalists can sell from existing stocks; as a result, inflows are always capable of meeting demand (so that there is never a problem of effective supply) but the rate of production at some point can exceed the flow of demand (so that there can be a problem of effective demand).

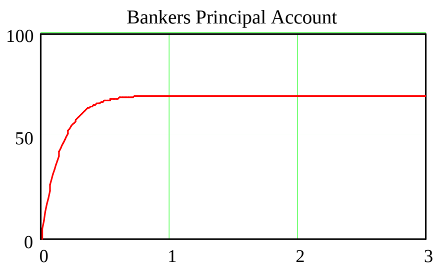
This realistic extension simplifies the model considerably. The cash flow into capitalist accounts now returns to the original formulation: it is whatever the three classes in the model wish to spend on their output. Production similarly returns to the original formulation without a minimum constraint. The stocks relation is also relatively simple: the rate of change of stocks is the flow in of new production Q minus the flow out of sales, which is equal to expenditure by the three classes divided by the price level. In the following simulations, the initial stock level is set to 20,000 units.

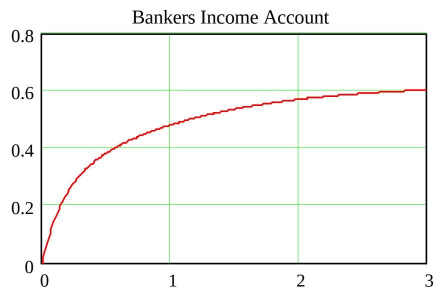
$$\begin{aligned}
\frac{d}{dt}K_D &= \frac{ln(X)}{T} \cdot K_D + \frac{1}{\tau_{Bp}} \cdot B_P \\
\frac{d}{dt}K_C &= r_c \cdot K_C - R \cdot K_D - \frac{K_C}{\tau_{Kp}} + \frac{\pi \cdot K_C}{\tau_{Kp}} + \frac{B_Y}{\tau_B} + \frac{W_C}{\tau_W} + \frac{1}{\tau_{Bp}} \cdot B_P \\
\frac{d}{dt}B_P &= -\frac{ln(X)}{T} \cdot K_D - \frac{B_P}{\tau_{Bp}} \\
\frac{d}{dt}B_Y &= r_d \cdot K_D - r_c \cdot K_C - r_c \cdot W_C - \frac{B_Y}{\tau_B} \\
\frac{d}{dt}W_C &= (1-\pi) \cdot \frac{K_C}{\tau_{Kp}} + r_c \cdot W_C - \frac{W_C}{\tau_W} \\
\frac{d}{dt}Q &= \frac{-1}{\tau_Q} \cdot \left(Q - \frac{a}{w} \frac{1-\pi}{\tau_{Kp}} \cdot K_C \right) \\
\frac{d}{dt}P &= \frac{1}{\tau_{Inf}} \cdot \left(-P \cdot \delta_\sigma \cdot \left(\frac{P \cdot Q}{\frac{\pi \cdot K_C}{\tau_{Kp}} + \frac{B_Y}{\tau_B} + \frac{W_C}{\tau_W}} - 1 \right) \right) \\
\frac{d}{dt}\Theta &= Q - \frac{1}{P} \cdot \left(\frac{\pi \cdot K_C}{\tau_{Kp}} + \frac{B_Y}{\tau_B} + \frac{W_C}{\tau_W} \right)
\end{aligned} \tag{12}$$

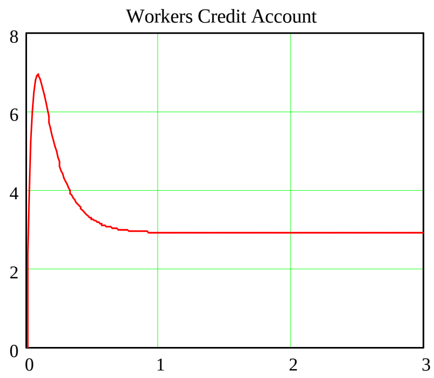
This extension completes the model prior to the replacement of simple time lags with behavioral relations. It shows that the Circuitist perspective on capitalism as a system of monetary circulation can be made into a coherent dynamic model. Capitalists are able to repay the debt they initially incur and still make a profit; bankers are able to convert their initial endogenous creation of credit into a relendable stock; a single injection of credit money can enable the system to continue indefinitely; all three classes (capitalists, bankers and workers) receive sustainable income flows; and price and production dynamics eventually stabilize.

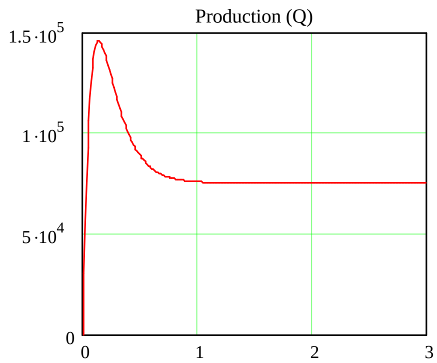

Capitalist Debit Account
100
80
60
40
20
0
1
2
3

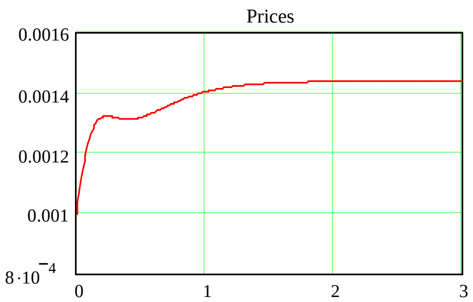

Capitalist Credit account
100
80
60
40
20
0
1
2
3


Bankers Principal Account
100
50
0
0
1
2
3

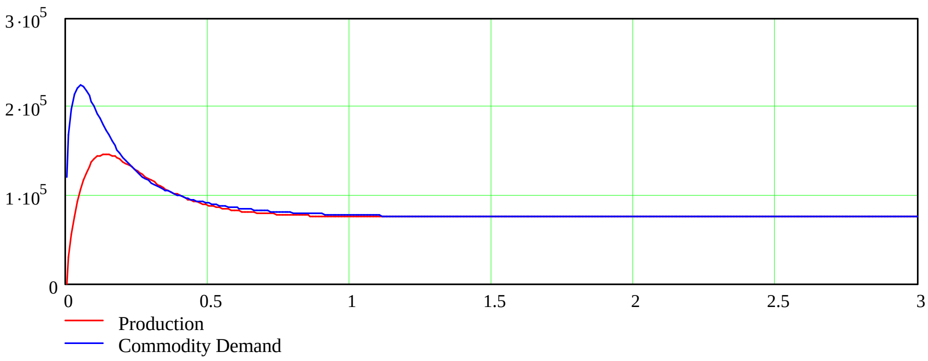

Bankers Income Account
0.8
0.6
0.4
0.2
0
0
1
2
3


Workers Credit Account
8
6
4
2
0
0
1
2
3


Production (Q)
$1.5 \cdot 10^5$
$1 \cdot 10^5$
$5 \cdot 10^4$
0
0
1
2
3


Prices
0.0016
0.0014
0.0012
0.001
$8 \cdot 10^{-4}$
0
1
2
3


Inventories
$2 \cdot 10^4$
$1.5 \cdot 10^4$
$1 \cdot 10^4$
5000
0
0
1
2
3


$3 \cdot 10^5$
$2 \cdot 10^5$
$1 \cdot 10^5$
0
0
0.5
1
1.5
2
2.5
3
Production
Commodity Demand

3 Conclusion

This simple model indicates that the Circuitist model provides a coherent picture of a pure credit capitalist economy. Economic activity is initiated by a loan from bankers to capitalists, and the monetary proceeds of production enable capitalists to repay the loan and finance ongoing production. It is possible for this system to “pull itself up by its own bootstraps”, starting from an initial injection of borrowed money with zero production.

It should be relatively straightforward to extend the model to include income distribution and investment dynamics: investment decisions by capitalists should determine the outflow from their account (and the demand, if any, for additional credit money), while employment and productivity dynamics should affect the level of wages, etc. The model should also be able to incorporate multi-sectoral dynamics, though this would necessitate introducing additional bank accounts for different sectors of capitalist production.

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[1] I would like to thank Trond Andresen of the Norewegian University of Technology for numerous discussions and valuable suggestions as to how this model might be developed and extended

[2] The use of a money token is equivalent but has confused economists, and the Circuitists emphasis upon a pure credit economy where all monetary exchanges are bank account transfers puts the focus upon the true nature of money.

[3] This is one of the times it helps to think of money as a physical token rather than a double-entry book-keeping phenomenon. The bank “produces” 100 tokens, gives them to the capitalist and records the action in the capitalist debit account; the capitalist deposits the tokens in its safety deposit box and the banker records their presence in the capitalist’s credit account. When the tokens are given back to the banker in the debt repayment process, the tokens are transferred from the capitalist’s to the bankers safety deposit box. The transfer is noted twice: once for the reduction in the physical tokens in the capitalist’s safety deposit box, and once for the reduction of the debt outstanding to the banker.

[4] A time lag is a decay function related to the value of the variable in question—in this sense the classic “radioactive decay” equation $\frac{d}{ct}U = -k \cdot U$ is a time lag. A discrete model of the form $x_{t+1} = k \cdot x_t$, which economists often describe as a time lag, is more correctly described as a time-delay.

[5] For the sake of simplicity I use only a credit account for workers, in effect presuming that

workers' accounts are always positive. This can be modified later.

[6] This should be modelled by introducing inventories, and will be in later versions of the model. A minimum function is being used here for the purposes of illustration only.