# Income Inequality, Monetary Policy, and the Business Cycle

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November 15, 2003

#### Abstract

The effects of changes in monetary policy are studied in a general equilibrium model where money facilitates transactions. Because there are two types of agents, workers and capitalists, different elasticities of money demand exist, implying that monetary policy influences the distribution of income. Only when earnings inequality is incorporated into monetary policy rule is the model able to replicate cyclical fluctuations of both real and nominal aggregates as well as the inequality measure. Additionally, monetary policy becomes more countercyclical when the fraction of transfers received by the workers increases. These results support a theory that the distribution of seigniorage revenues between the workers and capitalists changed in 1979.

**Key Words**: Inflation, Income Distribution, Heterogenous Agents, Perturbation. **JEL**: E32, E42, E50.

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## 1 Introduction

The dynamic effects of monetary policy are examined and documented in a model where inflation has differential impacts on segments of the population. There are several motivations. First, previous research (Al-Marhubi, 1997; Romer and Romer, 1999; Dolmas, Huffman, and Wynne, 2000; and Albanesi, 2000) has shown that inflation and income equality are positively correlated in a cross-section of countries. Second, there is a growing body of evidence suggesting that the poor tend to only hold money while the rich diversify (Guiso, Haliassos, and Jappelli, 2002). Therefore, it is hoped that by quantifying the effects of monetary policy in a model with heterogenous agents one can potentially show how policy affects the distribution of income and hence inequality. Third, Dolmas, Huffman, and Wynne (2000) and Albanesi (2000) suggest political conflicts are the reason for inflation. Thus, agent heterogeneity allows us to examine how changes in the distribution of the benefits to monetary policy (from possible changes in political-economic conditions) influence the cyclical properties of the economy's aggregates and thus facilitate the search for the correct political-economic theories that characterize dynamic monetary economies.

A key assumption of the analysis is that there are two types of agents in the model economy: capitalists and workers. Additionally, as in Kydland (1991) and Gavin and Kydland (1999), money offers a time-saving role, thereby altering the labor-leisure trade-off. As a result, the capitalists will hold both capital and fiat money even though capital's rate of return dominates money. The workers, who cannot enter the capital markets, save only through fiat money balances, implying that there exist two different elasticities for money demand. The main reason that this partition is relevant is that the concentration of capital is extreme for industrialized countries; the poor and the rich hold different portfolios. For example, Guiso, Haliassos, and Jappelli (2002) find that over the 1983-98 time period only 40.5 percent of all U.S. households held risky assets.<sup>1</sup>

In the first part of the paper, the empirical relationship between monetary policy and inequality is documented. The seigniorage tax rate and Gini coefficient are found to have a negative contemporaneous correlation over the business cycle. However, in a reduced form regression, the lagged Gini coefficient significantly and positively affects current monetary policy. The estimated impulse response functions also imply that positive innovations in inequality are associated with increases in the next period's money stock. Interestingly, the converse is not true – innovations in monetary policy have a negative but small effect

<sup>&</sup>lt;sup>1</sup>Similarly, only 17.0 percent of German and 17.5 percent of Italian households hold risky assets. But there appears to be a strong positive trend in the percentage of households that hold risky assets.

on inequality. The interpretation given is that monetary policy systematically responds to deviations in income with a lag. Finally, though the parameters of the regression are constant (stable) with respect to time, seigniorage has become more countercyclical. Because the seigniorage tax rate became more volatile after 1979, the regression stability implies that the correlation between the seigniorage tax rate and the lagged Gini must be lower after 1979.

In the second part of the paper, the quantitative results of the modelling are presented. In the first model, the relationship between income distribution and monetary policy is explored when the monetary policy rule is exogenous to inequality. Even though the relationship between income inequality and monetary policy does not resemble the observed pattern for the U.S. economy, this economy is to act as a benchmark against which to compare the results. Alternatively, when the monetary policy feedback rule is modelled as endogenous to earnings inequality, the relationship between the seigniorage tax rate and inequality can be replicated. That is, an increase in earnings inequality will increase the rate of money growth, thus increasing transfers. As transfers increase, income inequality falls, giving a negative correlation between the Gini coefficient and seigniorage. Because there is persistence in earnings inequality, an increase in income inequality will imply increases in the next period's seigniorage tax rate – the lagged Gini coefficient positively affects current monetary policy.

In their empirical exercise, Romer and Romer (1999) find that temporary expansionary monetary policy can increase the welfare of the poor through redistribution; the effects are temporary and small, however. In contrast, they conclude that monetary policy aimed at reducing inflation is the appropriate long-run policy for the poor; the negative effects on the poor from inflation are large. In my modelling, temporary and exogenous monetary expansions are found to increase the welfare of the poor when monetary policy is endogenous to inequality. Additionally, the effects are temporary and small. However, the welfare effects of long-run inflation can be positive or negative, depending on the cause of the inflation. For example, when the fraction of seigniorage-generated transfers received by the workers is low, a low-inflation equilibrium will result that is welfare-reducing for the workers. Alternatively, exogenous and permanent decreases in the seigniorage rate are welfare-increasing; simultaneously, the mean of lifetime utility increases and the variance of lifetime utility decreases for both the workers and capitalists. Interestingly, the level of income inequality is insensitive to exogenous changes in the seigniorage rate.

Monetary policies in representative-agent economies are well studied. Examples include Cooley and Hansen (1991, 1992) and Gavin and Kydland (1999), to name just two. Gavin and Kydland (1999) are able to replicate the cyclical properties of both real and nominal aggregates that characterize the U.S. economy after 1979 by making the monetary policy rule more countercyclical. In our heterogenous agent model, changes in the composition of the types of households receiving the seigniorage are found to alter the cyclical properties of nominal aggregates. Specifically, the rate of money creation becomes more procyclical when the fraction of seigniorage-generated transfers received by the workers falls. Whether the fraction of seigniorage revenues devoted to the working poor increased after 1979 – presumably from a shift in political power – is a direction for which future research is needed.

The rest of this paper is organized as follows. Section 2 examines the empirical relationship between income inequality and monetary policy. Section 3 defines the model economy. Section 4 characterizes the equilibrium. Section 5 contains details of the computational algorithm and the model calibration. The results and conclusion are presented in Sections 6 and 7, respectively.

## 2 Empirical Facts

### 2.1 The Data

The annual U.S. data are from three sources: the IMF's International Financial Statistics, the FRED database at the Federal Reserve Bank of St. Louis, and Deininger and Squire's (1996) inequality data set. The series for real GDP and population are identified from the IFS data set. An annual per capita monetary base (denoted m) is constructed by averaging the monthly per capita base. Then the seigniorage tax rate is computed from the following definition: the percent change in per capita monetary base:  $\theta_t = (m_t - m_{t-1})/m_{t-1}$ . Finally, the Gini coefficient from Deininger and Squire's (1996) database is computed by the U.S. Bureau of the Census<sup>2</sup> and defined formally as  $gini = 1 - 2 \int_0^1 L(y) dy$ , where L(y) is the Lorenz curve of the income distribution. The combined data give the sample years 1965-91.

Table 1 reports that the seigniorage tax rate and Gini coefficient averaged 6.37 and 35.5 percent, respectively, over the sample period. Additionally, their business-cycle components<sup>3</sup> are slightly procyclical. However, because there are indications that monetary policy was altered around 1979, the sample is divided into two sub-samples. Panels B and C of Table 1 show that, prior to 1979, the monetary policy variable is procyclical. Alternatively, the seigniorage rate is slightly countercyclical after 1979. The cyclical Gini shows no significant changes over the two samples. An equality of variance test indicates that only the seigniorage

<sup>&</sup>lt;sup>2</sup>Though the Bureau of the Census has updated the inequality series since 1992, its definition of income changed significantly. Thus, comparisons of coefficients computed before and after 1992 are difficult.

<sup>&</sup>lt;sup>3</sup>The business-cycle component is defined by the Hodrick-Prescott (HP) filter with the smoothing parameter set to  $\lambda = 6.25$  following the discussion in Ravn and Uhlig (1997).

rate is significantly different after the break; the variances of the Gini and GDP do not appear to change.

So far, the results are consistent with the break in covariance structure of monetary policy found by Gavin and Kydland (1999). Their story is that in 1979 monetary policy becomes systematically countercyclical. By making their theoretical monetary policy rule more countercyclical, Gavin and Kydland (1999) are able to replicate the covariance structure of most nominal aggregates. However, this view contradicts the empirical results of Sims (1980, 1999) and Bernanke and Mihov (1998), who find stability in the parameters for monetary policy rules (though possibly nonlinear) and shifting variances for the shocks to monetary policy. The suggestion of Sims (1999) is to estimate linear reaction functions only if they have been amended for heteroscedasticity.

### 2.2 Regression Results

The business-cycle components of the seigniorage tax rate and the Gini coefficient are negatively correlated at -7.07 percent; the first panel of Figure 1 illustrates the negative relationship. A negative correlation is expected since the Gini computed by the Bureau of the Census includes all government transfers. For example, an increase in the money supply used to finance government transfers to the poor would cause a fall in the inequality coefficient. This theory is viable because it is the current practice in the United States for the Federal Reserve to pay for its operating expenses and then rebate all other revenues to the Treasury. Then, to eliminate the effects of the implied simultaneity, the focus is now shifted to estimation of the reduced-form relationships between the contemporaneous and lagged variables.

The second and third panels of Figure 1 are the time series and scatter plots for the tax rate and the lag of the Gini coefficient. There appears to be a positive link between the two variables. In support, Panel A of Table 2 reports the reduced-form estimation results using the HP-filtered data and Generalized Method of Moments (GMM). The GMM estimation results<sup>4</sup> confirm that the lag of the Gini coefficient significantly affects the seigniorage tax rate. The estimated coefficient for the lagged Gini on the seigniorage tax rate is 2.525 with a standard error at 0.085. The point estimate implies that a one percentage point cyclical increase of the Gini coefficient (measured on a scale of 0 to 100) is associated with a 2.5 percentage increase in the next period's seigniorage tax rate. Additionally, the R-squared coefficient indicates that 24 percent of all fluctuations in the monetary policy variable are

<sup>&</sup>lt;sup>4</sup>The HAC covariances are estimated with a Parzen kernel of order one.

explained by the lagged values. The predictive power of the regressions significantly drops when the Gini variable is omitted. Panel B in Table 2 presents the estimation results with, for example, real per-capita GDP; the regression produces an extremely low R-squared.

Additional dummy slope coefficients for the two periods are added to the regressions. The test for instability around 1979 is rejected, indicating that the results of Table 2 are stable across the two periods. The results of monetary policy parameter stability of Sims (1980, 1999) and Bernanke and Mihov (1998) are thus reinforced. Note that the regression stability implies that the correlation between the seigniorage tax rate and the lagged Gini must have been lower after 1979 if the seigniorage tax rate became more volatile. Indeed, Table 1 is consistent with this fact.

The three panels of Figure 2 show, through the impulse response functions,<sup>5</sup> how the variables respond to a one-standard deviation shock associated with each vectored equation of Panel A in Table 2. In the first panel, a shock to the monetary equation is plotted.<sup>6</sup> Most of the response occurs in the seigniorage tax rate; the responses by the Gini and GDP are to slightly decrease and to slightly increase, respectively. The next panel indicates that both monetary policy and output respond, in a significant way, to changes in the shocks associated with the inequality equation. Together, shocks to the Gini variable are important for the dynamics of all variables, but shocks to the policy and output variable stay contained and not significantly transferred to the other variables.

In review of the literature, additional empirical results are found; Al-Marhubi (1997), Romer and Romer (1999), Dolmas, Huffman, and Wynne (2000), and Albanesi (2000) document significant cross-country correlations between inflation and the Gini coefficient. Romer and Romer (1999), and the papers cited within, also find a negative contemporaneous time series link between income inequality and monetary policy; they do not estimate in reduced form, however. In related empirical literature, Click (1998) and Han and Mulligan (2001) report that the use of seigniorage, and hence inflation, is not determined by the size of government. Rather, governments tend to use inflation to finance transitory government expenditures. In a political-economic study, Dolmas, Huffman, and Wynne (2000) find that the poor, their median voters, vote for greater inflation due to redistribution properties. Albanesi (2000) models inflation as the result of low bargaining power by the poor, who are more vulnerable to inflation. In both studies, the results are steady-state implications under

 $<sup>^{5}</sup>$ Computed by the generalized (non-orthogonalized) impulse response functions of Pesaran and Shin (1997)

<sup>&</sup>lt;sup>6</sup>The typical example of a shock to monetary policy, given by Christiano, Eichenbaum, and Evans (1999), is that of measurement error by the monetary authority; the monetary authority does not observe the true M, and thus the shock represents deviations of the monetary policy from the true value.

different political powers of the agents; they do not consider policy implications over the business cycle.

The presented results and cited literature suggest that monetary policy is a function of income inequality and that seigniorage is used for transitory government spending. The policy rule appears stable (though possibly nonlinear), but the covariance structure has changed, making it more countercyclical. Second, political-economic conditions appear important. Therefore, the model presented in the next section will allow for these general features.

## 3 The Model Economy

The model economy is populated by households who are separated into two occupations: workers and capitalists. Workers do not own capital – they merely supply labor to firms. Capitalists, by contrast, can rent capital to firms. This separation, used previously in the growth model by Judd (1985) and Krusell (2002), allows us to highlight how different government policies affect different groups.

For the agents, time evolves in discrete units called periods (which are specified to be one year long in the quantitative results reported later on). A period has two parts in which the economic agents make decisions: the beginning of the period and the end of the period. In the beginning of the period, the monetary authority injects money into the economy by transferring lump-sum cash to the households. At the end of the period, the households, who are in possession of the economy's entire stock of money, make decisions on labor, consumption, investment in capital, and money investment. Note that both groups can save by holding fiat money.

### 3.1 The Workers

The time spent on transactions-related activities is assumed to be given by the expression:

$$\omega_0 - \omega_1 \left(\frac{m_t^w}{p_t c_t^w}\right)^{\omega_2},$$

where  $m^w$  is the current nominal money balance,  $c_t^w$  is current real consumption, and  $p_t$  is the aggregate price level. Then leisure time in period t is

$$\ell_t^w \equiv T - n_t^w - \omega_0 + \omega_1 \left(\frac{m_t^w}{p_t c_t^w}\right)^{\omega_2}$$

where T is the total time endowment. To insure a solution, the parameters are restricted so that  $\omega_2 < 1$  and  $\omega_2$  and  $\omega_1$  have the same sign. This specification is by Kydland (1991) and Gavin and Kydland (1999).

Workers solve the dynamic programming problem

$$\mathcal{U}^{w}(s_{t}) = \max_{c_{t}^{w}, m_{t+1}^{w}, n_{t}^{w}} \left\{ u^{w}(c_{t}^{w}, \ell_{t}^{w}(c_{t}^{w}, m_{t}^{w}, n_{t}^{w})) + E_{t}\beta^{w}\mathcal{U}^{w}(s_{t+1}) \right\}$$

subject to the budget constraint

$$c_t^w + \frac{m_{t+1}^w}{p_t} \le \frac{m_t^w}{p_t} + (1 - \tau_t)w_t n_t^w + T_t^w,$$

where  $w_t$  is the wage rate,  $n_t^w$  is the labor supply,  $T_t^w$  is the lump-sum real transfer from the government,  $s_t$  is a vector of state variables, and  $E_t$  is the conditional expectation operator given the current full level of information at time t.

## 3.2 The Capitalists

Capitalists solve the dynamic programming problem

$$\mathcal{U}^{k}\left(s_{t}\right) = \max_{c_{t}^{k}, k_{t+1}, m_{t+1}^{k}} \left\{ u^{k}\left(c_{t}^{k}, \ell_{t}^{k}(c_{t}^{k}, m_{t}^{k})\right) + E_{t}\beta^{k}\mathcal{U}^{k}\left(s_{t+1}\right) \right\}$$

subject to the budget constraint

$$c_t^k + k_{t+1} + \frac{m_{t+1}^k}{p_t} \le \frac{m_t^k}{p_t} + (1 - \tau_t)w_t \,\bar{n} + \left[(1 - \tau_t)R_t^k + 1 - \delta\right]k_t + T_t^k,$$

where  $k_t$  is the amount of capital,  $R_t^k$  is the real return to capital, and  $\delta \in (0, 1]$  is the depreciation rate.

Leisure time, which is a function of a fixed labor supply choice for the capitalist,  $\bar{n}$ , is given by

$$\ell_t^k \equiv T - \bar{n} - \omega_0 + \omega_1 \left(\frac{m_t^k}{p_t c_t^k}\right)^{\omega_2}.$$

Note that the assumption that money facilitates transactions allows the capitalist to hold nonzero amounts of money even though capital rate-of-return dominates money.

### 3.3 The Firm

The representative firm rents capital and hires labor. The firm produces consumption goods via a neoclassical constant returns to scale production function and chooses  $\{k_t, n_t\}$  to maximize:

$$\pi_t = A_t F(k_t, n_t) - w_t n_t - R_t^k k_t.$$

The firm takes as given  $\{w_t, R_t^k\}$ . In equilibrium, the factors of production are paid their marginal products:

$$A_t F_1(k_t, n_t) = R_t^k, \quad A_t F_2(k_t, n_t) = w_t.$$

The Cobb-Douglas form is chosen for the production technology because it is consistent with the relative constancy of income shares:

$$Y_t = F\left(k_t, n_t\right) = k_t^{\alpha} n_t^{1-\alpha},$$

where  $\alpha \in (0,1)$  is the share of income that goes to capital. Finally, I assume the log of aggregate technology, denoted  $a = \ln(A)$ , follows the process:

$$a_{t+1} = \phi a_t + \sigma \varepsilon_{t+1},\tag{1}$$

where  $\varepsilon$  is independent and N(0, 1).

### **3.4** The Government and Monetary Authority

The government and monetary authority are assumed to pool all revenue for government consumption and transfers. The level of government consumption is assumed to be a fixed fraction  $\xi$  of total output. To keep the focus on monetary policy, the income tax rate is set at this fixed rate:  $\tau_t = \bar{\tau} = \xi$ . Thus, the government's budget is

$$T_{t}^{k} + T_{t}^{w} + \xi Y_{t} = \frac{m_{t+1} - m_{t}}{p_{t}} + \bar{\tau} \left( w_{t} n_{t} + R_{t}^{k} k_{t} \right).$$

Note that the pooling assumption follows current practice in the United States by which the Federal Reserve pays for its operating expenses and then rebates all other revenues to the Treasury. Then, using the law of motion  $m_{t+1} = (1 + \theta_t)m_t$ , the public budget constraint is rewritten as

$$T_t^k + T_t^w + \xi Y_t = \theta_t \frac{m_t}{p_t} + \bar{\tau} \left( w_t n_t + R_t^k k_t \right);$$

the term  $\theta_t$  is the seigniorage tax rate. Finally, transfers are assumed to be made to the workers as a fraction  $\lambda$  of the total. The parameter  $\lambda$  is the weight attached to workers by some political mechanism; it need not equal the share of workers in the economy.

The monetary policy feedback rule that the seigniorage tax rate is assumed to follow is

$$\theta_t = \theta_0 + E_{t-1}[\theta_1 \left( ineq_t - \theta_2 \right)] + \sigma_\theta \,\varepsilon_{\theta,t} \tag{2}$$

where  $ineq_t$  is a measure of inequality,  $\varepsilon_{\theta}$  is an exogenous shock that is independently distributed standard normal,  $\theta$ 's are parameters, and  $\sigma_{\theta}$  is a scale variable. Because of the assumption that the monetary authority sets transfers by its choice of seigniorage tax rate, the inequality index,  $ineq_t$ , is defined on pretax and pre-transfer income, or *earnings*. More specifically, the following definition for computation of  $ineq_t$  is used:

$$ineq_t = 1 - 2\left(\frac{w_t n_t^w}{F(k_t, n_t)}\right).$$

For computation of the Gini coefficient, the following definition is used:

$$gini_{t} = 1 - 2\left(\frac{w_{t}n_{t}^{w} + T_{t}^{w}}{F(k_{t}, n_{t}) + T_{t}^{w} + T_{t}^{k}}\right)$$

which includes pretax and post-transfer income; this is the *money income* definition used by the U.S. Bureau of the Census.

Why would monetary policy be endogenous to income inequality? In the model, an increase in the seigniorage rate is an increase in real transfers to the households and capitalists. This can be seen when transfers are written in stationary form<sup>7</sup> as:

$$T_t^w = \lambda \left[ \frac{\theta_t \hat{m}_t}{(1+\theta_t)\hat{p}_t} \right], \quad T_t^k = (1-\lambda) \left[ \frac{\theta_t \hat{m}_t}{(1+\theta_t)\hat{p}_t} \right].$$

Transfers are a direct non-decreasing function of  $\theta_t$ :  $\partial T_t^w / \partial \theta_t \ge 0$  and  $\partial T_t^k / \partial \theta_t \ge 0$ , all else being constant. However, as seigniorage increases, the value of agent time t money holdings – which is a factor of wealth – falls as individuals begin to economize on money holdings. It is straightforward to show a net increase in the worker's and capitalist's wealth from an increase in seigniorage-financed transfers when  $\lambda > (1 - \hat{m}_t^k)$  and  $\lambda < (1 - \hat{m}_t^k)$ , respectively. If, for example,  $\lambda$  represents the relative political power of workers, one would expect  $\lambda$  to

<sup>&</sup>lt;sup>7</sup>Since p and m will be growing over time, all nominal variables are deflated by the money stock to obtain a stationary environment. This results in the equilibrium conditions  $\hat{m}_{t+1} = \hat{m}_t = 1$ ,  $\hat{p}_t = p_t/m_{t+1}$ ,  $\hat{m}_t^k = m_t^k/m_t$ , and  $\hat{m}_t^w = 1 - \hat{m}_t^k$ .

be relatively high in industrialized countries since the median voter holds little capital and is thus poor. For example, Guvenen (2001) finds that stock market participation has been below 50 percent in the United States for the entire period during which data are reliable.<sup>8</sup> Then, if  $\lambda > (1 - \hat{m}_t^k)$ , an increase in the seigniorage tax rate can redistribute income and wealth from the rich to the poor.<sup>9</sup>

## 4 Characterization of the Equilibrium

### 4.1 The Stationary Recursive Problem

The budget constraints in stationary form are:

$$c_t^w + \frac{1 - \hat{m}_{t+1}^k}{\hat{p}_t} \le \frac{1 - \hat{m}_t^k}{(1 + \theta_t)\hat{p}_t} + (1 - \bar{\tau})w_t n_t + T_t^w$$
$$c_t^k + k_{t+1} + \frac{\hat{m}_{t+1}^k}{\hat{p}_t} \le \frac{\hat{m}_t^k}{(1 + \theta_t)\hat{p}_t} + (1 - \bar{\tau})w_t \bar{n} + [R_t^k(1 - \bar{\tau}) + 1 - \delta] k_t + T_t^k.$$

Given the setup, the worker's, the capitalist's, and the firm's optimal behavior define the Stochastic Euler Equations (SEEs):

$$1 = (1 - \tau_t) w_t \left\{ \frac{(u_1^w(t) + [u_2^w(t)\ell_1^w(t)])}{-u_2^w(t)\ell_3^w(t)} \right\}$$
(3a)  
$$\beta^w \hat{n}_t \qquad \left\{ u_1^w(t+1) + u_2^w(t+1) \left[ \ell^w(t+1) + (1 + \theta_{t+1}) \hat{n}_{t+1} \ell^w(t+1) \right] \right\}$$

$$1 = E_t \frac{\beta^w \hat{p}_t}{(1+\theta_{t+1})\hat{p}_{t+1}} \left\{ \frac{u_1^w(t+1) + u_2^w(t+1) \left[ \ell_1^w(t+1) + (1+\theta_{t+1})\hat{p}_{t+1}\ell_2^w(t+1) \right]}{u_1^w(t) + u_2^w(t)\ell_1^w(t)} \right\}$$
(3b)

$$1 = E_t \beta^k [R_{t+1}^k(1-\bar{\tau}) + 1 - \delta] \left\{ \frac{u_1^k(t+1) + u_2^k(t+1)\ell_1^k(t+1)}{u_1^k(t) + u_2^k(t)\ell_1^k(t)} \right\}$$
(3c)

$$1 = E_t \frac{\beta^k \hat{p}_t}{(1+\theta_{t+1})\hat{p}_{t+1}} \left\{ \frac{u_1^k(t+1) + u_2^k(t+1) \left[\ell_1^k(t+1) + (1+\theta_{t+1})\hat{p}_{t+1}\ell_2^k(t+1)\right]}{u_1^k(t) + u_2^k(t)\ell_1^k(t)} \right\}$$
(3d)

and the equilibrium functions of the current states:  $\hat{m}_{t+1}^k = \mathcal{M}(s_t), k_{t+1} = \mathcal{H}(s_t), n_t^w = \mathcal{N}(s_t), \text{ and } \hat{p}_t = \mathcal{P}(s_t), \text{ where } s_t \equiv \{k_t, \hat{m}_t^k, a_t\} \text{ and }$ 

$$u_1(t) \equiv \frac{\partial u(c_t, \ell_t)}{\partial c_t} \quad u_2(t) \equiv \frac{\partial u(c_t, \ell_t)}{\partial \ell_t} \\ \ell_1(t) \equiv \frac{\partial \ell(c_t, m_t)}{\partial c_t} \quad \ell_2(t) \equiv \frac{\partial \ell(c_t, m_t)}{\partial m_t}.$$

<sup>&</sup>lt;sup>8</sup>Market participation started very low and has risen over the past 20 years but appears to have settled into a new steady-state value below 50 percent.

<sup>&</sup>lt;sup>9</sup>Note that the standard representative agent model lacks this feature since  $1 = \lambda = (1 - \hat{m}_t^k)$ .

The equilibrium functions and constant returns to scale in production then define the worker's and the capitalist's consumptions by:

$$\mathcal{C}^{w}(s_{t}) = \frac{1 - \hat{m}_{t}^{k}}{(1 + \theta_{t})\mathcal{P}(s_{t})} + (1 - \bar{\tau})w_{t}\mathcal{N}(s_{t}) - \frac{1 - \mathcal{M}(s_{t})}{\mathcal{P}(s_{t})} + \lambda \left[\frac{\theta_{t}\hat{m}_{t}}{(1 + \theta_{t})\mathcal{P}(s_{t})} + (\bar{\tau} - \xi)Y_{t}\right];$$
$$\mathcal{C}^{k}(s_{t}) = \frac{\hat{m}_{t}^{k}}{(1 + \theta_{t})\mathcal{P}(s_{t})} + (1 - \bar{\tau})w_{t}\bar{n} + \left[R_{t}^{k}(1 - \bar{\tau}) + 1 - \delta\right]k_{t} - \frac{\mathcal{M}(s_{t})}{\mathcal{P}(s_{t})} - \mathcal{H}(s_{t}) + (1 - \lambda)\left[\frac{\theta_{t}\hat{m}_{t}}{(1 + \theta_{t})\mathcal{P}(s_{t})} + (\bar{\tau} - \xi)Y_{t}\right].$$

## 4.2 The Equilibrium

We are now in a position to define an equilibrium for our economy.

**Definition 1** A competitive equilibrium for this economy consists of a savings function  $\mathcal{H}(s)$ , a labor supply function  $\mathcal{N}(s) + \bar{n}$ , a money demand function  $\mathcal{M}(s)$ , consumption functions  $\mathcal{C}^w(s)$  and  $\mathcal{C}^k(s)$ , pricing functions w(s),  $R^k(s)$ , and  $\mathcal{P}(s)$ , and policy functions  $\theta(s)$ , and  $T^w + T^k = T(s)$  such that

- (i)  $C^w(s)$ ,  $\mathcal{N}(s)$ , and  $1 \mathcal{M}(s)$  solve the worker's intratemporal condition and the budget constraint for the given prices and policies;
- (ii)  $\mathcal{H}(s)$ ,  $\mathcal{C}^{k}(s)$ , and  $\mathcal{M}(s)$  solve the capitalist's Euler equation and the budget constraint for the given prices and policies;
- (iii) the firm's first-order conditions are satisfied, given prices;
- (v) the goods market clears:

$$\mathcal{C}^{w}\left(k,\hat{m}^{k},a\right) + \mathcal{C}^{k}\left(k,\hat{m}^{k},a\right) + \mathcal{H}\left(k,\hat{m}^{k},a\right) - (1-\delta)k = (1-\bar{\tau})e^{a}F\left(k,\mathcal{N}(k,\hat{m}^{k},a) + \bar{n}\right);$$

and

(vi) the government budget constraint holds:

$$T(s) = \frac{\theta(s)}{[1+\theta(s)]\mathcal{P}(s)} + (\bar{\tau} - \xi) e^a F\left(k, \mathcal{N}(k, \hat{m}^k, a) + \bar{n}\right)$$
$$= \frac{\theta(s)}{[1+\theta(s)]\mathcal{P}(s)}.$$

## 5 Solution and Calibration Methods

#### 5.1 Perturbation

For computation of the equilibria, the smooth approximation method of Judd (1998) and Gaspard and Judd (1997), which relies on a second-order Taylor series expansion, is applied to the functional equations, the SEEs, that jointly characterize the equilibrium. As in Schmitt-Grohé and Uribe (2002), the scale parameters for the variance of the exogenous shocks,  $\sigma$  and  $\sigma_{\theta}$ , are incorporated and used as an argument in differentiation for the expansion. The center for the expansion is the deterministic rest point of the economy. Letting the states of the economy be defined as  $s = \{k, \hat{m}^k, a, \sigma, \sigma_{\theta}\}$ , the private agent's policy functions are then approximated, for example, by the expansions:

$$\mathcal{H}(x) = \mathcal{H}^{(0)} + \mathcal{H}^{(1)}_{1}(k - \bar{k}) + \mathcal{H}^{(1)}_{2}(\hat{m}^{k} - \bar{\hat{m}}^{k}) + \mathcal{H}^{(1)}_{3}(a) + \mathcal{H}^{(1)}_{4}(\sigma) + \mathcal{H}^{(1)}_{5}(\sigma_{\theta}) + \frac{1}{2}\mathcal{H}^{(2)}_{1}(k - \bar{k})^{2} + \frac{1}{2}\mathcal{H}^{(2)}_{2}(\hat{m}^{k} - \bar{\hat{m}}^{k})^{2} + \dots + \mathcal{H}^{(2)}_{1,2}(k - \bar{k})(\hat{m}^{k} - \bar{\hat{m}}^{k}) + \mathcal{H}^{(2)}_{1,3}(k - \bar{k})(a) + \dots$$

where  $\bar{k} = \mathcal{H}^{(0)} = \mathcal{H}(\bar{k}, \bar{\hat{m}}^k, 0, 0, 0)$ , and

$$\mathcal{H}^{(1)} = \left. \frac{\partial \mathcal{H}}{\partial s} \right|_{s=\bar{s}}, \, \mathcal{H}^{(2)} = \left. \frac{\partial^2 \mathcal{H}}{\partial s \, \partial s^{\top}} \right|_{s=\bar{s}}, \dots,$$

with  $\mathcal{M}, \mathcal{N}, \text{ and } \mathcal{P}$  defined likewise.

The SEEs, equations (3a)-(3d), evaluated at  $\bar{s}$  are denoted, respectively, as

$$0 = SEE_i^{(0)}(\mathcal{H}^{(0)}, \mathcal{M}^{(0)}, \mathcal{N}^{(0)}, \mathcal{P}^{(0)})|_{i=a,...,d}$$

Taking the derivative n times of equations (3a)-(3d) and evaluating at  $\bar{s}$  produces additional equations that the competitive equilibrium must satisfy. These are denoted by

$$0 = SEE_i^{(n)}(\mathcal{H}^{(0)}, \mathcal{M}^{(0)}, \mathcal{N}^{(0)}, \mathcal{P}^{(0)}, \dots, \mathcal{H}^{(n)}, \mathcal{M}^{(n)}, \mathcal{N}^{(n)}, \mathcal{P}^{(n)})|_{i=a,\dots,d}$$

where  $i = a, \ldots, d$  identifies the equation. In practice, the expansion is terminated after the quadratic terms; higher-order coefficients become very small and do not justify the additional computational burden. Thus, the solution,  $\{\mathcal{H}^{(i)}, \mathcal{M}^{(i)}, \mathcal{N}^{(i)}, \mathcal{P}^{(i)}\}_{i=0}^{2}$ , can be derived by taking successive derivatives (up to n = 2) and setting any higher-order polynomial coefficients – for example,  $\mathcal{H}^{(3)}, \mathcal{M}^{(3)}, \mathcal{N}^{(3)}$ , and  $\mathcal{P}^{(3)}$  – to zero.

This problem has a recursive structure and can be solved by first finding the steady states from the first three equations:  $\{\mathcal{H}^{(0)}, \mathcal{M}^{(0)}, \mathcal{P}^{(0)}\}$ . Second, the higher-order coefficients,  $\{\mathcal{H}^{(i)}, \mathcal{M}^{(i)}, \mathcal{P}^{(i)}, \mathcal{P}^{(i)}\}_{i=1}^2$ , are found from the remaining equations. Because this second step might yield two solutions, an unstable and stable solution, the parameter search for  $\mathcal{H}_1^{(1)}$ and  $\mathcal{M}_2^{(1)}$  is restricted to be in absolute value less than one as suggested by Krusell (2002). This procedure is implemented and automated in Matlab via the symbolic toolbox. Finally, starting from  $\bar{s}$ , the time series for the allocations and prices are derived from 8,000 simulated technology shocks; the first 1,000 of each series are dropped to eliminate any transitional dynamics.

Note that the procedure does not impose certainty equivalence in the decision rules. However, it turns out that the first-order terms are zero in  $\sigma$  and  $\sigma_{\theta}$ ; as in Schmitt-Grohé and Uribe (2002), uncertainty is found to have at most second-order effects on decision rules. Furthermore, the cross terms are also zero. Even though the impact of uncertainty is second-order, it turns out to be quantitatively important.

### 5.2 Calibration

To conduct the quantitative analysis, the functional forms for utility must be selected. Since there is no trend in hours worked in the data, but there is a trend in wages, the momentary utility function is chosen for workers and capitalists from the family of constant relative risk aversion:

$$u^{w}(c_{t}^{w},\ell_{t}^{w}) = \frac{\left[(c_{t}^{w})^{\mu}(\ell_{t}^{w})^{1-\mu}\right]^{1-\gamma}}{1-\gamma}, \quad u^{k}(c_{t}^{k},\ell_{t}^{k}) = \frac{\left[\left(c_{t}^{k}\right)^{\mu}(\ell_{t}^{k})^{1-\mu}\right]^{1-\gamma}}{1-\gamma}$$

The parameter  $\gamma$  is the Arrow-Pratt coefficient of relative risk aversion; as a benchmark  $\gamma = 2$  is selected (baseline model). The parameter  $\mu$  is a relative share parameter selected to match aggregate labor supply equal to one-third of the time endowment. This results in a value of  $\mu = 0.33$ .

For the real economy, the following vector of parameters is to be calibrated:  $\Theta_1 \equiv [\alpha, \beta, \delta, \phi, \sigma, \xi, \lambda]$ . I choose  $\alpha = 0.36$ , which roughly matches the share of capital income in output for the United States since the Second World War. The average discount rate  $\beta$  is fixed at a value compatible with a yearly psychological rate of 3 percent. The depreciation rate is set at  $\delta = 0.0435$ , which obtains a steady-state investment/GDP ratio of 0.15. The government's share parameter  $\xi$  is set at 21.4 percent, the average U.S. share of total government consumption (18 percent) plus net interest expenses (3.4 percent) in output since 1960. The parameters for the technology process a are set by  $\phi = 0.85$  and  $\sigma = 0.0304$ , the annual values most commonly found in the literature. For now, the transfer weight  $\lambda$  is

arbitrarily set to 0.50; different values for this parameter are considered.

For the calibration of the transaction cost variables,  $\Theta_2 \equiv [\omega_0, \omega_1, \omega_2, \bar{n}]$ , Gavin and Kydland (1999) are followed by first setting  $\omega_2 = -1.0$  and  $\omega_1 = -0.0136$ . This sets the interest rate elasticity equal to -0.50 and the real interest rate equal to about 9 percent per year (or a net rate of 5.25 percent). Next, for capitalists, the labor supply parameter is fixed to equal the worker's average hours;  $\bar{n} = 0.33$ . The parameter  $\omega_0$  is set so that  $\bar{\ell}^w + \bar{n} = T = 1$  at the economy's deterministic rest point. This amounts to setting  $\omega_0 =$  $\omega_1[\bar{m}^w/(\bar{p}\cdot\bar{c}^w)]^{\omega_2}$ . The calibration of the monetary policy rule,  $\Theta_3 \equiv [\theta_0, \theta_1, \theta_2, \sigma_{\theta}, \epsilon]$ , follows the estimation results in Tables 1 and 2: specifically,  $\theta_0 = 0.06$ ,  $\theta_2 = \bar{ineq}$ , and  $\sigma_{\theta} = 0.00912$ . Finally,  $\theta_1 = 0.0$  for the initial baseline model. The results for the calibrations are in Table 3.

## 6 Results

A wide range of experiments are conducted by altering the values for the policy rule parameters and the distribution for the composition of agents receiving transfers. Changes in the policy rule are intended to capture deviations from the mean growth of money and the variance of the shock to the policy rule. Changes in the distribution of agents receiving transfers allows for the distribution of political power to be altered.

### 6.1 Baseline Economy

The simulation results for the baseline economy are presented in Table 4. As expected, the Gini coefficient is inversely related to  $\lambda$ . The workers are taxed relatively more when  $\lambda$  increases, causing them to economize their money holdings and increase work effort. The decreased volatility in their transfer income also decreases the variance of labor effort. As a result, output increases and becomes less volatile. In terms of welfare, the total effects of a fall in  $\lambda$  are that average utility for the worker falls and the average variance of lifetime utility for the worker increases. Thus, a decrease in the total fraction of seigniorage received by the worker represents a transfer of welfare from the worker to the capitalist.

The baseline model is unable to replicate the observed properties of the U.S. economy with respect to inequality and monetary policy. When  $\lambda = 0.50$ , the simulated contemporaneous correlation of the seigniorage tax rate and the Gini coefficient is -56.76 percent. An increase in transfers would, as previously described, increase the income of the worker, thereby decreasing income inequality. However, the correlation between the seigniorage tax rate and the lagged Gini is -0.76 percent. Because the seigniorage tax has zero persistence, lagged Gini coefficients will be uncorrelated with monetary policy. These results are robust to increases in  $\lambda$ . Specifically, the contemporaneous correlation becomes positive when  $\lambda$ falls to and below 0.25; this is still inconsistent with the observed facts.

### 6.2 Gini-Based Feedback Rule

The monetary policy rule will be a function of earnings inequality when  $\theta_1 \neq 0$ . To include this feature, equation (2) is calibrated to the estimation results of Table 2; this results in  $\theta_1 = 2.491$  and  $\theta_2 = 0.3754$ . Additionally, the expectation in (2) is computed by a 50-state Markov chain approximation of the autoregression (1) as described in Tauchen (1986). There are five main features of the model.

*First*, Figures 3 and 4 show that the Gini-based model is able to replicate several features of the economy when the monetary policy feedback rule is endogenous to earnings inequality. Specifically, Figure 3 indicates that an increase in earnings inequality will increase the rate of money growth, thus increasing transfers. As transfers increase, income inequality falls, giving a negative correlation between the Gini coefficient and seigniorage. Because there is persistence in earnings inequality, an increase in income inequality will imply increases in the next period's seigniorage tax rate – the lagged Gini coefficient positively affects current monetary policy.<sup>10</sup> As depicted in the first panel of Figure 4, the impulse response from a one-standard error shock to the monetary policy rule equation is contained mainly in the policy variable; there is little variation in output and inequality. Additionally, the second panel indicates that monetary policy and inequality are insensitive to shocks to technology. Because the dynamic responses implied by the model are similar to the data, the Gini-based feedback model appears to be consistent with the empirical facts presented in Section 2.

Second, the second panel of Table 4 indicates an inverse relationship between income inequality and the mean growth of the money supply with respect to changes in  $\lambda$ . Decreasing  $\lambda$  causes total income of the worker to fall. This encourages labor effort, thereby increasing labor earnings. Because earnings inequality falls (as opposed to income inequality), the mean seigniorage tax rate decreases. Though aggregate output increases, decreases in  $\lambda$  are unambiguously welfare-reducing for the worker; the worker's average mean and variance of utility decrease and increase, respectively.

*Third*, as evident in the third of panel Table 4, the Gini coefficient appears insensitive to exogenous changes in the seigniorage rate. However, labor choice and hence output are

<sup>&</sup>lt;sup>10</sup>This relationship is found to hold also for  $\lambda \ge 0.25$ .

sensitive to changes in the rate of growth of money. As the seigniorage rate is increased, the distortions to capital accumulation increase, causing the demand for investment and labor to fall. Additionally, workers attempt to economize on their money holdings by selling to the capitalist; the fraction of the worker's consumption to total consumption falls. Therefore, exogenous and permanent increases in the seigniorage rate are welfare-decreasing; simultaneously, the mean of lifetime utility and the variance of lifetime utility increase for both the workers and capitalists.

Fourth, the fourth panel of Table 4 indicates that the mean level for the economy's aggregates is insensitive to changes in the variance of the monetary policy shock  $\sigma_{\theta}$ . The only noticeable effect is an increase in the variance of the seigniorage rate and hence the variance of the inflation rate.

Fifth, as shown in Figure 5, increases in the seigniorage revenues devoted to the workers decrease the correlation between the seigniorage rate and output as well as the correlation between the seigniorage rate and the lag of the Gini coefficient. Because these correlations diverge in opposite directions from changes in  $\sigma_{\theta}$ , increases in  $\lambda$  followed by a fall in  $\theta_0$  can replicate the stylized facts of the U.S. economy since 1979. That is, increases in  $\lambda$  decrease both correlations of seigniorage but increase the mean rate of money creation. Simultaneously decreasing  $\theta_0$  preserves the correlations while decreasing the mean rate of money growth; Figure 5 indicates that the relevant correlations are insensitive to changes in  $\theta_0$ .

In order to provide further intuition of what could have happened to monetary policy after 1979, the last panel of Table 4 presents the simulation results of a Gini-based economy where  $\lambda = 1$  and  $\theta_0 = 0.03$ . In this economy, compared to the case where  $\lambda = 0.50$  and  $\theta_0 = 0.06$ , the relatively low rate of money creation combined with the increased  $\lambda$  combine to alter the covariance structure of monetary policy. The correlations for this economy are considerably lower:  $corr(\theta_t, y_t) = -30.056$  and  $corr(\theta_t, gini_{t-1}) = 16.652$ ; this is a direct result of the different elasticities of demand for the workers and capitalists. Therefore, the alteration in  $\lambda$ followed subsequently by a fall in  $\theta_0$  can replicate the increased countercyclicity of monetary policy while preserving the stability of the monetary policy rule

The change in policy appears to have benefitted the worker as indicated by their increased level utility as well as decreased variance in that utility. However, it is important to note that the effects of the policy change on the welfare of the capitalist is indeterminate. The capitalist receives lower levels of utility but decreased variance to utility. Though suggestive, whether the change of policy is the result of political power being shifted to the workers from the capitalists is unclear.

## 7 Conclusion

This paper has shown that incorporation of a Gini-based monetary feedback rule is generally compatible with several features of the U.S. economy. Specifically, Gini-based feedback rules replicate the relationship between inequality and the seigniorage rate; the lagged Gini coefficient positively affects the current level of monetary policy. Increases in the fraction of revenues received by the workers,  $\lambda$ , can also replicate the increased countercyclicity of monetary policy while preserving the stability of the monetary policy rule. Whether there is political-economic theory (or even demographic change) that supports changes in  $\lambda$  while preserving the monetary policy rule is a possible direction of future research.

The analysis could also be extended to allow for differences in risk aversions. Under this scenario, exogenous changes in the mean level of money growth may potentially alter the covariance structure of monetary policy while preserving the stability of the feedback rule. The parameter estimations of Guvenen (2001) would be the guide for calibration.

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	Trend	Std Dev	Corr with $y_t$	Corr with $\theta_{t-1}$				
<u>Panel A: 1965-1991</u>								
$\theta_t$	6.372	1.026	17.642	48.082				
$gini_t$	35.596	0.201	3.076	16.649				
$y_t$	_	1.502	—	42.776				
Panel	Panel B: 1965-1979							
$\overline{ heta_t}$	5.891	0.790	67.070	63.628				
$gini_t$	34.410	0.238	7.588	22.337				
$y_t$	—	2.164	—	47.339				
Panel C: 1980-1991								
$\theta_t$	6.973	1.266	-22.615	41.456				
$gini_t$	37.078	0.161	2.536	2.536				
$y_t$	—	1.939	—	33.565				
Panel D: Equality of Sub-Sample Variances (Folded-F)								
Variance(p-value)								
$\theta_t$	$\hat{F}_{(11,14)} = 2.56(0.0571)$							
$gini_t  \hat{F}_{(14,11)} = 1.96(0.2686)$								

 $\hat{F}_{(14,11)} = 1.61(0.4329)$ 

 $y_t$ 

 Table 1: Sample Statistics of HP-Filtered Series

Table 2. Givini Estimation Results	Table 2:	GMM	Estimation	$\operatorname{Results}^\dagger$
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<u>Panel A: 196</u>	<u>5-1991</u>						
	$\theta_t$	$gini_t$	$\underline{y_t}$				
$\theta_{t-1}$	$\underset{(0.131)}{0.057}$	-0.022 (0.030)	$\underset{(0.294)}{0.494}$				
$gini_{t-1}$	$2.525^{*}_{(0.817)}$	$\underset{(0.171)}{0.134}$	$4.419^{*}_{(1.727)}$				
$y_{t-1}$	$-0.033$ $_{(0.085)}$	$-0.026^{**}$ $_{(0.014)}$	$0.255^{**}$ $(0.146)$				
$R^2$	0.240	0.114	0.329				
$\mathrm{Chow}(1979)$	$\hat{F} = 0.54$	$\hat{F} = 0.10$	$\hat{F} = 0.90$				
Panel B: Single Equation Estimation							
	A.	A.	A.				

	$\theta_t$	$\overline{ heta_t}$	$\theta_t$
$\theta_{t-1}$	$\underset{(0.208)}{0.011}$	—	_
$gini_{t-1}$	_	$2.491^{*}_{(0.897)}$	_
$y_{t-1}$	_	_	$-0.028$ $_{(0.103)}$
$R^2$	0.001	0.235	0.003
Chow(1979)	$\hat{F} = 0.01$	$\hat{F} = 0.24$	$\hat{F} = 2.14^{**}$

 $^{\dagger}\mathrm{HP}\text{-filtered}$  values used.

\*Significant at 5 percent.

\*\*Significant at 10 percent.

Preferences:	$\beta = 0.97,  \gamma = 2.0,  \mu = 0.33$
Technology:	$\alpha = 0.36,  \phi = 0.85,  \sigma = 0.0304$
	$\delta = 0.0435$
Transaction Costs:	$\omega_0 = -0.020043,  \omega_1 = -0.0136$
	$\omega_2 = -1.0,  \bar{n} = 0.33$
Government:	$\xi = 0.214,  \lambda = 0.50$
Monetary Auth.:	$\theta_0 = 0.06,  \theta_1 = 0.0,  \theta_2 = 0.0$
	$\sigma_{\theta} = 0.00912$

 Table 3: Parameter Values for the Baseline Model

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	$ heta_t$	$gini_t$	$y_t$	$n_t$	$m_t^k$	$\frac{c_t^w}{C_t}$	$u^w(t)$	$u^k(t)$	$infla_t$
	Baseline: $\theta_0 = 0.06,  \theta_1 = 0.0,  \theta_2 = 0.0$								
$\lambda = 1.00$	$\begin{array}{c} 5.999 \\ (0.784) \end{array}$	$\underset{(0.425)}{35.206}$	$\underset{(2.049)}{30.221}$	$\underset{(0.147)}{30.708}$	$\underset{(0.382)}{59.172}$	$\underset{(52.807)}{41.247}$	$-1.818 \\ (4.528)$	-1.657 (2.862)	$\underset{(2.787)}{6.033}$
$\lambda = 0.75$	$\begin{array}{c} 5.999 \\ (0.784) \end{array}$	$\underset{(0.287)}{35.911}$	$\underset{(2.044)}{31.030}$	$\underset{(0.143)}{31.228}$	$\substack{59.737\\(0.382)}$	$\underset{(51.824)}{40.595}$	-1.832 (4.558)	-1.647 (2.839)	$\underset{(2.771)}{6.032}$
$\lambda = 0.50$	$\begin{array}{c} 5.999 \\ (0.784) \end{array}$	$\underset{(0.168)}{36.618}$	$\underset{(2.039)}{31.850}$	$\underset{(0.139)}{31.760}$	$\underset{(0.382)}{60.303}$	$\underset{(50.954)}{39.942}$	-1.846 $(4.599)$	-1.636 (2.822)	$\underset{(2.757)}{6.032}$
$\lambda=0.25$	$\underset{(0.784)}{5.999}$	$\underset{(0.140)}{37.326}$	$\underset{\left(2.033\right)}{32.681}$	$\underset{(0.135)}{32.303}$	$\underset{(0.381)}{60.869}$	$\underset{(50.204)}{39.286}$	-1.861 (4.653)	-1.626 (2.811)	$\underset{(2.742)}{6.031}$
$\lambda = 0.00$	$5.999 \\ (0.784)$	$\underset{(0.237)}{38.036}$	$\underset{(2.028)}{33.523}$	$\underset{(0.131)}{32.858}$	$\underset{(0.381)}{61.435}$	$\underset{(49.582)}{38.620}$	-1.877 (4.721)	-1.615 (2.805)	$\underset{(2.727)}{6.031}$
	Gini: $\theta_0 = 0.06, \theta_1 = 2.491, \theta_2 = 0.3754$								
$\lambda = 1.00$	$\underset{(0.857)}{12.417}$	$\underset{(0.358)}{34.484}$	$28.232 \\ (2.091)$	29.444 (0.186)	$\underset{(0.321)}{58.947}$	$\underset{(53.166)}{41.857}$	-1.810 (4.359)	-1.683 $(2.973)$	$\underset{(3.332)}{12.446}$
$\lambda = 0.75$	$\underset{(0.835)}{8.807}$	$\underset{(0.281)}{35.731}$	$\underset{(2.071)}{30.327}$	$\underset{(0.171)}{30.774}$	$\underset{(0.343)}{59.782}$	$\underset{(51.970)}{40.727}$	-1.831 (4.489)	-1.656 (2.887)	$8.884 \\ (3.109)$
$\lambda = 0.50$	$\underset{(0.823)}{6.229}$	$\underset{(0.188)}{36.588}$	$\underset{(2.057)}{31.848}$	$\underset{(0.159)}{31.757}$	$\underset{(0.370)}{60.338}$	$\underset{(51.018)}{39.957}$	-1.846 (4.584)	-1.636 (2.827)	$\substack{6.268 \\ (3.035)}$
$\lambda = 0.25$	4.488 (0.815)	$\underset{(0.154)}{37.142}$	$\underset{(2.047)}{32.886}$	$\underset{(0.150)}{32.436}$	$\underset{(0.404)}{60.690}$	$\underset{(50.321)}{39.465}$	-1.856 $(4.649)$	-1.623 $(2.793)$	4.527 (3.016)
$\lambda = 0.00$	$\underset{(0.810)}{3.343}$	$\underset{(0.285)}{37.486}$	$\underset{(2.038)}{33.572}$	$\underset{(0.141)}{32.888}$	$\underset{(0.440)}{60.911}$	$\underset{(49.941)}{39.165}$	-1.862 (4.703)	-1.615 (2.784)	$\underset{(3.039)}{3.383}$
	Gini: $\lambda = 0.50,  \theta_1 = 2.491,  \theta_2 = 0.3754$								
$\theta_0 = 0.08$	$\substack{9.127\\(0.812)}$	$\underset{(0.159)}{36.683}$	$\underset{(2.039)}{31.288}$	$\underset{(0.139)}{31.393}$	$\underset{(0.370)}{60.498}$	$\underset{(50.866)}{39.855}$	-1.850 $(4.588)$	-1.643 (2.849)	$\substack{9.166\\(3.056)}$
$\theta_0 = 0.07$	$\substack{7.699\\(0.816)}$	$\underset{(0.172)}{36.637}$	$\underset{(2.047)}{31.553}$	$\underset{(0.148)}{31.565}$	$\underset{(0.371)}{60.421}$	$\underset{(50.941)}{39.903}$	$-1.848 \\ (4.586)$	-1.640 (2.839)	$7.738 \\ (3.045)$
$\theta_0 = 0.05$	$\underset{(0.832)}{4.699}$	$\underset{(0.209)}{36.536}$	$\underset{(2.071)}{32.186}$	$\underset{(0.174)}{31.977}$	$\underset{(0.369)}{60.247}$	$\underset{(51.095)}{40.0179}$	-1.844 (4.580)	-1.632 (2.812)	$\underset{(3.021)}{4.738}$
$\theta_0 = 0.04$	$\underset{(0.848)}{3.079}$	$\underset{(0.240)}{36.478}$	$\underset{(2.090)}{32.594}$	$\underset{(0.195)}{32.244}$	$\underset{(0.364)}{60.147}$	$\underset{(51.167)}{40.090}$	-1.841 $(4.574)$	-1.627 (2.790)	$\underset{(2.993)}{3.117}$
	Gini: $\lambda = 0.50,  \theta_0 = 0.06,  \theta_1 = 2.491,  \theta_2 = 0.3754$								
$\sigma_{\theta}\% = 4(.919)$	$\substack{6.151 \\ (3.139)}$	$\underset{(0.474)}{36.585}$	$\underset{(2.057)}{31.895}$	$\underset{(0.159)}{31.788}$	$\underset{(0.371)}{60.267}$	$\underset{(51.310)}{39.964}$	-1.846 (4.599)	-1.636 (2.835)	$\substack{6.191 \\ (4.803)}$
$\sigma_{\theta}\% = 2(.912)$	$\underset{(1.582)}{6.213}$	$\underset{(0.269)}{36.587}$	$\underset{(2.057)}{31.858}$	$\underset{(0.159)}{31.763}$	$\underset{(0.370)}{60.324}$	$\underset{(51.091)}{39.957}$	-1.846 (4.588)	-1.636 (2.827)	$\substack{6.253 \\ (3.465)}$
$\sigma_{\theta}\% = \frac{1}{2}(.912)$	$\underset{(0.475)}{6.233}$	$\underset{(0.163)}{36.588}$	$\underset{(2.057)}{31.846}$	$\underset{(0.159)}{31.755}$	$\underset{(0.3707)}{60.342}$	$\underset{(50.990)}{39.956}$	-1.846 (4.582)	-1.636 (2.828)	$\substack{6.272 \\ (2.915)}$
$\sigma_{\theta}\% = \frac{1}{4}(.912)$	$\underset{(0.340)}{6.234}$	$\underset{(0.156)}{36.588}$	$\underset{(2.057)}{31.845}$	$\underset{(0.159)}{31.755}$	$\underset{(0.370)}{60.343}$	$\underset{(50.978)}{39.956}$	-1.846 $(4.581)$	-1.636 (2.828)	$\underset{(2.883)}{6.273}$
				Gini: $\theta_1$	= 2.491,	$\theta_2 = 0.37$	54		
$\lambda = 1,  \theta_0 = .03$	5.717 (0.9293)	$\underset{(0.562)}{35.012}$	$\underset{(2.268)}{29.817}$	30.911 (0.275)	59.364 (0.272)	41.352 (57.240)	-1.821 (3.326)	-1.658 $(2.067)$	$\underset{(3.162)}{5.763}$

Table 4: Simulation Results for Baseline and Gini-Based Feedback Rule<sup>†</sup>

 $^{\dagger}\mathrm{Standard}$  errors in parentheses.



Figure 1: Historical Comparison of Seigniorage Tax Rate and U.S. Gini Coefficient (Note: HP-filtered values).



Figure 2: Generalized Impulse Response Functions (Note: nonorthogonalized method of Pesaran and Shin, 1997)

Figure 3: Comparison of Seigniorage Rate and Gini for Gini-Based Monetary Economy (Note:  $\lambda=0.50)$ 



Figure 4: Impulse Response Functions for Gini-Based Monetary Economy (Note:  $\lambda=0.50)$ 





Figure 5: Simulated Correlations for Gini-Based Feedback Rule

## A Appendix

### A.1 Optimality Conditions

The optimality conditions for the worker's dynamic program are

$$\begin{aligned} 0 &= u_2^w(t)\ell_3^w(t) + \lambda_1(1-\tau_t)w_t \\ 0 &= u_1^w(t) + [u_2^w(t)\ell_1^w(t)] - \lambda_1 \\ 0 &= -\frac{1}{\hat{p}_t}\lambda_1 + E_t \frac{\beta^w}{(1+\theta_{t+1})\hat{p}_{t+1}} [\lambda_1'] + E_t\beta^w \left[u_2^w(t+1)\ell_2^w(t+1)\right], \end{aligned}$$

and for the capitalist:

$$0 = u_1^k(t) + \left[u_2^k(t)\ell_1^k(t)\right] - \eta_1$$
  

$$0 = -\eta_1 + E_t \,\beta^k [R_{t+1}^k(1 - \tau_{t+1}) + 1 - \delta]\eta_1'$$
  

$$0 = -\frac{1}{\hat{p}_t}\eta_1 + E_t \frac{\beta^k}{(1 + \theta_{t+1})\hat{p}_{t+1}}[\eta_1'] + E_t \beta^k \left[u_2^k(t+1)\ell_2^k(t+1)\right].$$

Combining these equations give (3a)-(3d):

$$\begin{split} 1 &= (1 - \tau_t) w_t \frac{(u_1^w(t) + [u_2^w(t)\ell_1^w(t)])}{-u_2^w(t)\ell_3^w(t)} \\ 1 &= E_t \frac{\beta^w \hat{p}_t}{(1 + \theta_{t+1})\hat{p}_{t+1}} \left\{ \frac{u_1^w(t+1) + u_2^w(t+1) \left[\ell_1^w(t+1) + (1 + \theta_{t+1})\hat{p}_{t+1}\ell_2^w(t+1)\right]}{u_1^w(t) + u_2^w(t)\ell_1^w(t)} \right\} \\ 1 &= E_t \beta^k [R_{t+1}^k(1 - \tau_{t+1}) + 1 - \delta] \left\{ \frac{u_1^k(t+1) + u_2^k(t+2)\ell_1^k(t+1)}{u_1^k(t) + u_2^k(t)\ell_1^k(t)} \right\} \\ 1 &= E_t \frac{\beta^k \hat{p}_t}{(1 + \theta_{t+1})\hat{p}_{t+1}} \left\{ \frac{u_1^k(t+1) + u_2^k(t+1) \left[\ell_1^k(t+1) + (1 + \theta_{t+1})\hat{p}_{t+1}\ell_2^k(t+1)\right]}{u_1^k(t) + u_2^k(t)\ell_1^k(t)} \right\}. \end{split}$$

### A.2 Gini Definition

U.S. Bureau of the Census data are collected for all people in the sample 15 years old and over. Money income includes earnings, unemployment compensation, workers' compensation, social security, supplemental security income, public assistance, veterans' payments, survivor benefits, pension or retirement income, interest, dividends, rents, royalties, estates, trusts, educational assistance, alimony, child support, assistance from outside the household, and other miscellaneous money income. It is income before deductions for taxes or other expenses and does not include lump-sum payments or capital gains.