A multiple matching model with endogenous participation : something new about the supply-side policies. (Very preliminary version)

Etienne Campens[1]
EUREQua University Paris-I
Cepremap
Dares Ministère du Travail et de la Solidarité

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[1] etienne.campens@cepremap.cnrs.fr

Abstract

This paper aims at modelize a multiple matching model, in which the decision of activity on the labour market is endogenous. Most of works on the labour market concern the only demand-side. These works do not consider the supply-side as endogenous. Thus, no policy can be explorated to improve the labour market situation.

The supply policies seem to be the new way of action of the different governments among the European Union. In particular, in France, a policy which increases the level of earned incomes on the low wages is led since 2002, as the Earned Income Tax Credit (EITC) in the United States. This principle, called "Prime Pour l'Emploi" (PPE), is supposed to incite unskilled unemployed to take a job.

In this paper, I explain that, with an endogenous activity choice, such policies may have ambiguous effects on unemployment and social accounts. Indeed, new unemployed workers are attracted on the effective labour market by these job-taking incentives. The unemployement duration increase and the consequences on the social accounts are not obvious. Furthermore, inactivity traps phenomena can appear.

I modelize the effects of an alternative policy: tax cuts on low wage associated with minimum wage increase. I show that it improves employment and production at a lower cost for the government.

Introduction.

It appears today that the size of the active population matters, because of the debate over the sustainability of many welfare state programs. The recent demographic trends seem to show that only economies with high employment and participation rates can maintain high *levels of social security. But most economic analysis considers only the labor demand, or uses some very simple and mechanic effects (neoclassical labor supply function in particular). Event in the Handbook of Labor Economics devoted to labor supply, the mechanism are very simple and not very interesting.

Then, if different and varied tools are now available about the demand side policies and the consecutive dynamic adjustments, it's not the same concerning the supply side. Indeed, most of the possible policies on the demand side of the labor market seems to be considered in these models, eventually via some adaptations and improvements. Now, concerning the supply side, more developments and considering are possible ; and this all the more seriously that these questions seem to be one of the big challenges for the politics and people working on the labor market.

Indeed, the inactivities traps, incentive to take back a job, etc seem to be among the important questions on policies, but there also among the less clearly modelized in the labor market economics. Furthermore, in these times of retirement system financing, it's evidently important to know how incite people to go to job activity, and to take a job.

It' obvious that participation decisions in real labor markets are not fully understood. Furthermore, what do we kown about the interactions between participation decisions and job creations? Very few rigorous works been led concerning a global equilibrium with endogenous participation and unemployment.

Developping in particular the works of [Pietro Garibaldi, 2003] this work attempts to build a multi-state model of the labor market, in which the decisions of job destructions/creations, entry/exit in the labor market, employment/unemployment.

This theoritical consideration is based on the fact that people spend simultaneously a their time allowance in both market and home production. With a choice between leisure, home production and market work, the business cycle literature has improved the calibration of various aspects of the data, such as output volatility, the correlation between hours and productivity and the correlation between investments in home and market capital ([Rios-Rull, 1993], [Paul Gomme, 2001]). But the existing business cycle literature studies home production within frictionless labor markets. I try, as [Pietro Garibaldi, 2003] to study the border between market and home production in an imperfect labor market. It's supposed that heterogenous workers face idiosyncratic shocks to domestic production ability, but market frictions impose a cost to labor market participation. Since we work with a technologically fixed number of hours, our analysis abstracts from the intensive margin of labor supply, and concentrates to the extensive margin. In the paper, we explore in details the e.ects of time consuming search, a market friction into employment that has attracted a great deal of attention in the macro literature ([Hall,] and [Dale T. Mortensen,]).

The model here considered shows that the decisions to participate or not participate are different, then it will be more margins than worker categories, because of the existence of an entry cost on the labor market. The several labor supply margins readily rationalize a labor market with employment, unemployment and full time home production.

Furthermore, much of the developped matching models, except maybe [Campens *et al.*, 2002], [Campens, 2003], [Arnaud Chéron, 2003], don't use microeconomic fundations to explain differences between skills. Now wage distribution does not necesserarily explain skill differences, and it's important to know the reasons of transitions between skills. In particular, we have shown a real increase in the number of worker paid on the minimum wage in the last years, and it's important to have some elements to explain such transitions.

This work attemps to develop endogenous market participation in multiple a matching model, with endogenous human capital acquisition, real and nominal rigidities (minimum wage, frictions, wage bargaining, unemployment benefits, minimum income, etc.). Earlier, Burdett, have already examinated the relations between search frictions and labor supply, but with a fixed supply of jobs. The theoretical distinction between inactivity and unemployment, is inspired by [Burdett and Mortensen, 1998]. [Pissarides, 2000], have introduced a labor demand side and endogenous participation, and bring few new elements compared to the standard model of matching.

This model, building on both macroeconomic factors, individual (household) shocks, and microeconomic fundations about skills can show structural flows between activity and inactivity, and level of skill, even when macro-conditions are unchanged.

1 Methodology.

This model proposes to represent an endogenous labor supply comportment, in a multiple matching model. Most of the supply-side labor market models today developped show often some mecanic effects, economically little interessant. In this paper, I try to show that can exist asymetric comportments of entry/quit on the labor market.

What is done here continues the works of [Campens, 2003] which are a dynamic consideration of a multiple matching model and [Pietro Garibaldi, 2003] who show an endogenous supply model.

The main model elements are :

-an entry/exit comportment on the job market based on microeconomic fundations

-an asymmetry of these comportments

-the representation of real rigidities on the job market (minimum wage, minimum income, unemployment benefits, wage negociations, mobility costs, etc.)

-heterogeneous workforce

2 The model.

Taking the model of [Campens *et al.*, 2002], it's assumed that the labor market can be segmented as following :

-a unskilled workforce, with no human capital, payed on the minimum wage

-a specific skilled workforce, with specific human capital bringed by the employer, and negociating his wage

-a general skilled workforce, with general human capital, acquired outside the labor market, and negociating his wage.

Both first types of workforce, move one to the other via a specific formation process, which is controled by the employers of the specific-skilled workforce. This process permits to the unskilled workers to acquire a specific human capital and to access a better job. The third part of the workforce concerns the most skilled part of the workforce, it's assumed to be "seal" ; access to this formation doesn't directly exist **in** the job market[1].

Whole labor market is based on matching process *à la* Pissarides [Pissarides, 2000].

2.1 Time repartition, domestic production.

In this model, it's assumed that the agents have one unit of time, that they can share out among three activities :

-productive work, spending time h_e.

-the (eventual) job seek, spending time h_s

-the domestic production, spending time $1 - h_e - h_s$

It's assumed for the moment, that the working time is an exogenous fixed variable, and takes the value e in the case of an employed worker, and 0 else.

$$h_e = \{0, e\}$$

It's also assumed that the job search time is exogenously fixed, and will be noted s if the agent actually searches for a job, and 0 else.

$$h_s = \{0, s\}$$

During the domestic production time, each agent produce a amount x of domestic good. This ability to produce domestic good is assumed to differ among agents, and can vary along time with instantaneous probability λ. With such a shock, this ability can take any value in the existence interval of x. However, this interval is supposed to be finished and borned : $x \in \left[x^{\min}, x^{\max}\right]$

[1] Of course, the access to general human capital exists, but is outside the job market. Thus it's not represented here.

2.2 The flows on the labor market.

The present labor market is based on matching process *à la* Pissarides, and supposes the existence of flows inside the labor market. We will see that matching exists on different levels.

Concerning the unskilled agents, several changes are possible in their life. At each time, such an agent can be an employed worker (then he will earn a legal minimum wage), a job seeker (unemployed, he will then have unemployment benefits). On these jobs, it exists matching between vacant jobs and unemployed, as in the simplest matching case.

However, when these agents are in this type of jobs, they have the opportunity to acquire specific human capital. This opportunity transits by a new matching process between "specific-skilled" vacant jobs and unskilled employed workers. A second floor of matching is then build.

Concerning the general skilled workers, the matching process is the same than in the simplest case.

Note that these flows are not the only ones on our labor market ; the global workforce is not supposed fixed as often done. Actually, via shocks on domestic abilities, the agents will choose to enter or exit on the labor market. In this case, the global workforce have a variable size, and these fluctuations will directly influence the matching process.

Several categories of agents will potentially coexist in the labor market :

-unskilled non-working agents
-unskilled job seeker agents
-unskilled employed workers
-specific skilled employed workers
-general skilled non-working agents
-general skilled job seeker agents
-general skilled employed workers[2]

The matching functions will take the classic form of Coob-Douglas functions. This hypothesis is done in all (or almost) the works using matching process.[3]

They will be written as follows :

-unskilled hirings :

$$M^n = M_0^n U_n^{\gamma_n} V_n^{1-\gamma_n} \tag{1}$$

-specific skilled hirings :

$$M^e = M_0^e N_n^{\gamma_e} V_e^{1-\gamma_e} \tag{2}$$

In this case, the matching has the particularity to be done between vacant jobs and already filled jobs. If the process is not modified in appearance, except the fact that there is a direct interaction between flows of these two categories of employees, we will show later that this particulartiy involves changes from classic matching models[4].

-general skilled hirings :

$$M^q = M_0^q \left(U^q\right)^{\gamma_q} \left(V^q\right)^{1-\gamma_q} \tag{3}$$

where γ_i is the elasticity of the vacant jobs on the matching function concerning the jobs of type i.

If θ_i is the tightness existing on the segment i of the labor market (represented by the ratio $\dfrac{V^i}{S_i}$, where S_i is the number of agents searching a job of type i), it can be deduced that the probability to fill an unskilled vacant job will be :

$$\frac{M^n}{V_n} = M_0^n \left(\frac{V_n}{U_n}\right)^{-\gamma_n} \equiv m_n\left(\theta_n\right)$$

[2] It's supposed, in order to simplify the model that general skilled agents make ever the choice to be active on the labor market.

[3] See the works of Pissarides (1986), Petrongolo (1999) or Blanchard & Diamond (1989).

[4] develop overshooting phenomena

Symetrically, the probability to quit unemployment to employment of an unskilled agent will be :

$$\frac{M^n}{U_n} = M_0^n U_n^{\gamma_n - 1} V_n^{1-\gamma_n} = M_0^n \left(\frac{V_n}{U_n}\right)^{1-\gamma_n} = \theta_n m_n(\theta_n)$$

.

Concerning the case of the specific skilled part of the labor market we will have :

$$\frac{M^e}{V_e} = M_0^n \left(\frac{V_e}{N_n}\right)^{-\gamma_e} \equiv m_e(\theta_e)$$

which is the probability to fill a specific skilled vacant job, and

$$\frac{M^e}{N_n} = M_0^e N_n^{\gamma_e - 1} V_e^{1-\gamma_e} = M_0^e \left(\frac{V_e}{N_n}\right)^{1-\gamma_e} = \theta_e m_e(\theta_e)$$

is the probability for an unskilled worker to access a specific skilled job.

As preceding,

$$m_q(\theta_q)$$

will be the probability to fill a general skilled vacant job and

$$\theta_q m_q(\theta_q)$$

will be the probability to quit unemployment to employement for a general skilled agent.

A the steady state, the flows can be represented by the following equations : :

$$\begin{aligned}
u\left[\theta_n m_n(\theta_n) + n + \lambda\left(1 - F(x^e)\right)\right] &= n + F(x^e)\left[h\lambda + s_e l_e + s_n l_n\right] && \text{(4a)}\\
l_n\left[s_n + \theta_e m_e(\theta_e) + \lambda\left(1 - F(x^{q_n})\right) + n\right] &= u\theta_n m_n(\theta_n) && \text{(4b)}\\
l_e\left[s_e + \lambda\left(1 - F(x^{q_e})\right) + n\right] &= l_n \theta_e m_e(\theta_e) && \text{(4c)}\\
h\left[\lambda F(x^e) + n\right] &= u\lambda\left(1 - F(x^e)\right) + l_n\left[s_n\left(1 - F(x^e)\right) + \lambda\left(1 - F(x^{q_n})\right)\right] \\
&\quad + l_e\left[s_e\left(1 - F(x^e)\right) + \lambda\left(1 - F(x^{q_e})\right)\right] && \text{(4d)}
\end{aligned}$$

where l_i $(i = n, e)$, u, h and n are resepectively the proportion of each type of job, the unskill unemployment rate, the non-working proportion in the total population, and the growth rate of the population.

2.3 The value functions of the agents.

2.3.1 The case of agents without general human capital.

The case of a non-working agent. In the case of a non-working agent, it's assumed that he earns at each instant a legal minimum income noted ϑ, and that the level of his domestic production is x (since he spends his whole time to this "activity"). Concerning the different possibilities of the status evolution of this agent, the only one is the decision to go to unemployment ; this decision depends of the different variables (minimum income[5], minimum wage, etc.) but also on the evolution of his domestic ability (then of the variable x). This event is represented by the variable λ which is the instantaneous probability of shock on the variable x by any event[6].

The value function of a unskilled non-working agent, noted V^{hn}, is then written as :

[5]

[6] To be developped

$$V^{hn}(x) = \frac{1}{1+rdt}\left[(x+\vartheta)\,dt + \lambda dt\int_{x^{\min}}^{x^{e}} V^{un}(\widetilde{x})\,dF(\widetilde{x}) + \lambda dt\int_{x^{e}}^{x^{\max}} V^{hn}(\widetilde{x})\,dF(\widetilde{x}) + (1-\lambda dt)\,V^{hn}(x)\right] \tag{5}$$

where x^{e_n} is the entrance threshold on the labor market concerning an unskilled non-working agent, and V^{un} is the value function of an unemployed unskilled worker.

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Written in utility flow, this value function becomes :

$$(r+\lambda)\,V^{hn}(x) = (x+\vartheta) + \lambda\int_{x^{\min}}^{x^{e}} V^{un}(\widetilde{x})\,dF(\widetilde{x}) + \lambda\int_{x^{e}}^{x^{\max}} V^{hn}(\widetilde{x})\,dF(\widetilde{x}) \tag{6}$$

The case of an unskilled employed worker. The case of an unskilled employed worker is not very different than the case of known models, except the fact that the variable x can be modified by any choc. In such a case, the worker has to make a choice between the decision of staying in his job or leaving the labor market and becoming a non-working unskilled agent. The instantaneous gain of this agent comes from the spent time to domestic production $(1-e)$, and from his net wage, noted $\tau_n w_n$.

An other shock which could occur is an exogenous destruction of the job, which instantaneous probability s_n ; in this case the agent has to make a choice between searching for a new job (becoming unemployed) or becoming a non-working. Finally, the agent, can access to a specific human capital formation with a probability $\theta_e m_e(\theta_e)$ and then to a better job (with a expected value $V^{e1}(x)$)

Consequently, the value function of an agent in this case is :

$$V^{n} = \frac{1}{1+rdt}\left[(1-e)\,xdt + \tau_n w_n dt + \lambda dt\int_{x^{\min}}^{x^{q_n}} V^{n}(\widetilde{x})\,dF(\widetilde{x}) + \lambda dt\int_{x^{q_n}}^{x^{\max}} V^{hn}(\widetilde{x})\,dF(\widetilde{x})\right. \tag{7}$$

$$\left. +\theta_e m_e(\theta_e)\,dtV^{e1}(x) + s_n dt\max\left\{V^{un}(x), V^{hn}(x)\right\} + (1-\lambda dt-\theta_e m_e(\theta_e)\,dt - s_n dt)\,V^{n}(x)\right] \tag{8}$$

When the domestic production ability is modified, the agent can be in three different situations, by a double choice :

-the job is not destroyed and the agent make a direct choice between non-working and his current job

-the job is destroyed by the employer and then the employee makes a choice between unemployement and non-working.

This consideration takes importance when it's considered the employer comportment in front of a domestic ability shock (this concept is developped later)

Written in flows, the value function becomes :

$$(r+\lambda)\,V^{n}(x) = (1-e_n)\,x + \tau_n w_n + \lambda\int_{x^{\min}}^{x^{q_n}} V^{n}(\widetilde{x})\,dF(\widetilde{x}) + \lambda\int_{x^{q_n}}^{x^{\max}} V^{hn}(\widetilde{x})\,dF(\widetilde{x})$$

$$+s_n\left(\max\left\{V^{un}(x), V^{hn}(x)\right\} - V^{n}(x)\right) \tag{9}$$

$$+\theta_e m_e(\theta_e)\left(V^{e1}(x) - V^{n}(x)\right) \tag{10}$$

The case of a specific skilled worker in his new job. A specific skilled worker acessing a new job earns a net wage $\tau_e w_{e1}$, which is negotiated by a Nash bargaining, and receives a direct utility from the spent time to domestic production (or leisure) $1-e_e$. Such an agent has not more possibilities to upgrade his social status (except a extern phenomenon which is not represented here). However, his situation can be modified by two channels :

-exogenous destruction of the job (with instantaneous probability s_e) ; then he may make a choice between inactivity and unemployment

-domestic ability shock (with instataneous probability λ) ; then he may make a choice between his current job, unemployment, or inactivity.

His value function is then :

$$\begin{aligned} V^{e1}(x) &= \frac{1}{1+rdt}\Bigg[(1-e_e)\,xdt+\tau_e w_{e1}(x)\,dt+\lambda dt\int_{x^{\min}}^{x^{q_{e1}}} V^{e2}(\widetilde{x})\,dF(\widetilde{x})+\lambda dt\int_{x^{q_{e1}}}^{x^{\max}} V^{hn}(\widetilde{x})\,dF(\widetilde{x}) \\ &\quad +s_e dt\max\left\{V^{un}(x),V^{hn}(x)\right\} \\ &\quad +(1-\lambda dt-s_e dt)\,V^{e1}(x)\Bigg] \end{aligned} \tag{11}$$

In fact, it appears that the eventuality of volontary unemployment is not represented. Actually, it's easy to show that the threshold of effective labor market exit is upper than the entry threshold ; that is that if the agent decide to going out of the effective labor market, it's because his domestic production ability is sufficiently high to permit himself a higher utility from non-working than unemployment and job search.[7].

The utility flows can be written :

$$\begin{aligned} (r+\lambda)\,V^{e1}(x) &= (1-e_e)\,x+\tau_e w_{e1}(x)+\lambda\int_{x^{\min}}^{x^{q_{e1}}} V^{e2}(\widetilde{x})\,dF(\widetilde{x})+\lambda\int_{x^{q_{e1}}}^{x^{\max}} V^{hn}(\widetilde{x})\,dF(\widetilde{x}) \\ &\quad +s_e\left[\max\left\{V^{un}(x),V^{hn}(x)\right\}-V^{e1}(x)\right] \end{aligned}$$

The case of an already specific-skilled employee. The utility of an already specific skilled employee is little different of the precedent case, the only difference is the level of wage, negotiated on other outside options. The other terms and events are exactly the same as the precedent case.

Then :

$$\begin{aligned} V^{e2}(x) &= \frac{1}{1+rdt}\Bigg[(1-e_e)\,xdt+\tau_e w_{e2}(x)\,dt+\lambda dt\int_{x^{\min}}^{x^{q_{e1}}} V^{e2}(\widetilde{x})\,dF(\widetilde{x})+\lambda dt\int_{x^{q_{e1}}}^{x^{\max}} V^{hn}(\widetilde{x})\,dF(\widetilde{x}) \\ &\quad +s_e dt\max\left\{V^{un}(x),V^{hn}(x)\right\} \\ &\quad +(1-\lambda dt-s_e dt)\,V^{e2}(x)\Bigg] \end{aligned} \tag{12}$$

In flows this equations becomes :

$$\begin{aligned} (r+\lambda)\,V^{e2}(x) &= (1-e_e)\,x+\tau_e w_{e2}(x)+\lambda\int_{x^{\min}}^{x^{q_{e2}}} V^{e2}(\widetilde{x})\,dF(\widetilde{x})+\lambda\int_{x^{q_{e2}}}^{x^{\max}} V^{hn}(\widetilde{x})\,dF(\widetilde{x}) \\ &\quad +s_e\left[\max\left\{V^{un}(x),V^{hn}(x)\right\}-V^{e2}(x)\right] \end{aligned} \tag{13}$$

[7] Annex

The case of an unemployed. An unemployed has made the choice to be in activity but hasn't yet found a job.

He has each instant, an amount b of unemployment benefits (eventually function of his precedent wage and/or his precedent benefits), and an utility flow from his domestic production with time $1-s$. Then this agent spend a time s to search a job and to make the necessary steps to.

At each instant, with a probability $\theta_n m_n(\theta_n)$, he has the opportunity to be proposed to a unskilled job, and then to acceed to the utility V^n. And in the case of change of his domestic production ability (with a probability λ), he'll have to make a choice between activity (unemployment) and inactivity.

The value function of an unemployed unskilled agent is :

$$\begin{aligned} V^{un} &= \frac{1}{1+rdt}\left[(1-s)\,xdt + bdt + \lambda dt \int_{x^{\min}}^{x^e} V^{un}(\widetilde{x})\,dF(\widetilde{x}) + \lambda dt \int_{x^e}^{x^{\max}} V^{hn}(\widetilde{x})\,dF(\widetilde{x}) \right. \\ &\quad \left. +\theta_n m_n(\theta_n)\,dtV^n(x) + (1-\lambda dt - \theta_n m_n(\theta_n)\,dt)\,V^{un}(x)\right] \end{aligned} \tag{14}$$

Or, in flows :

$$(r+\lambda)\,V^{u_n} = (1-s)\,x+b+\lambda \int_{x^{\min}}^{x^e} V^{un}(\widetilde{x})\,dF(\widetilde{x})+\lambda \int_{x^e}^{x^{\max}} V^{hn}(\widetilde{x})\,dF(\widetilde{x})+\theta_n m_n(\theta_n)\,(V^n(x)-V^{u_n}(x)) \tag{15}$$

2.3.2 The agents with general human capital.

In this case, the hypothesis are not applicated for the moment. It's supposed a simple matching process. No more rigidities than frictions and wage negotiations exist on this part of the labor market. The value functions are then very simple.

The case of the employee. At each time, a general skilled worker earn a net negotiated wage $\tau_q w_q$. With an instantaneous probability s_q he can loss his job and become an unemployed.

What is resumed by :

$$V^q = \frac{1}{1+rdt}\left[\tau_q w_q dt + s_q dt V^{uq} + (1-s_q)\,V^q\right]$$

In flows :

$$rV^q = \tau_q w_q + s_q\,(V^{uq} - V^q)$$

The case of the unemployed. The unemployed, as in a classic matching model, has an instantaneous probability $\theta_q m_q(\theta_q)$ to access a job, otherwise he stays unemployed and has an amount b_q of unemployment benefits[8]

$$V^{uq} = \frac{1}{1+rdt}\left[b_q + \theta_q m_q(\theta_q)\,dtV^q + (1-\theta_q m_q(\theta_q)\,dt)\,V^q\right] \tag{16}$$

in flows :

$$rV^{uq} = b_q + \theta_q m_q(\theta_q)\,(V^{uq} - V^q) \tag{17}$$

[8] The terms b_q will be developped later.

2.4 Firms.

The firms are, like the individuals, directly concerned by the possible changes in domestic abilities. Indeed, the negotiated wage is directly influenced by the domestic ability, and will play on the net productivity of jobs, then on the rentability and duration of the job.

2.4.1 Individual posts.

The case of the unskilled post.

The filled post. In this case, the value function will be directly influenced by the immediate net productivity $p_n y_n - w_n$. Then, the job can be destroyed by an exogenous shock with probability s_n, the the job is vacant. The job can be destroyed also the mobility of the employee to a more interessant job, with an instantaneous probability $\theta_e m(\theta_e)$. Finally, the job can be modified by a domestic ability shock, which modify the wage negotiation ; then the job can be maintained or destroyed because unproductive.

$$\begin{aligned}\Pi^n(x) &= \frac{1}{1+rdt}\left[\left(p_n y_n + \eta_n w_n\right)dt + s_n dt \Pi^{vn} + \theta_e m(\theta_e)\, dt \Pi^{vn}\right. \\ &\quad + \lambda dt \int_{\underline{x}}^{x^{fn}} \Pi^n(\widetilde{x})\, dF(\widetilde{x}) + \lambda dt \int_{x^{f,}}^{x^{\max}} \Pi^{vn}(\widetilde{x})\, dF(\widetilde{x}) \\ &\quad \left. + \left(1 - s_n dt - \lambda dt - \theta_e m(\theta_e)\, dt\right)\Pi^n(x)\right] \end{aligned} \tag{18}$$

in utility flows :

$$\begin{aligned}(r+\lambda)\Pi^n(x) &= p_n y_n -_n w_n + \lambda \int_{\underline{x}}^{x^{fn}} \Pi^n(\widetilde{x})\, dF(\widetilde{x}) + \lambda \int_{x^{f,}}^{x^{\max}} \Pi^{vn}(\widetilde{x})\, dF(\widetilde{x}) \\ &\quad + s_n\left(\Pi^{vn} - \Pi^n\right) + \theta_e m(\theta_e)\left(\Pi^{vn} - \Pi^n\right) \end{aligned} \tag{19}$$

The vacant job. This case is trivial, and can be written as in the simplest matching models, except the fact than the value can be **indirectly** influenced by the fluctuations of x.

Then a vacant job, has a instantaneous probability $m_n(\theta_n)$ to be filled.

The value function is then :

$$\Pi^{vn}(x) = \frac{1}{1+rdt}\left[-\mu_n dt + m_n(\theta_n)\, dt \Pi^n(x) + \left(1 - m_n(\theta_n)\, dt\right)\Pi^{vn}(x)\right] \tag{20}$$

in flows :

$$r\Pi^{vn} = -\mu_n + m_n(\theta_n)\left(\Pi^n(x) - \Pi^{vn}\right)$$

The case of the skilled job. As written upper, it will exist a value for a just created job, and a value for an already filled job. The only difference between the two jobs, will be the wage level and the human capital bringing cost.

Such a job cannot know a lot of changes, because it represent the top situation of people without general human capital.

So the value function is simply the immediate gain from the net productivity, $y_e - w_e(x)$, augmented by the discounted value of the job in the future. Then with an instantaneous probability s_e the job is destroyed and becomes vacant ; it can be also destroyed by a shock on the variable x ; in this case there is renegotiation and the employer can decide to not keep the employee-employer couple.

Consequently, the value function is simply :

$$\begin{aligned}\Pi^{e1}(x) &= \frac{1}{1+rdt}\left[-C_f + p_e y_e(x) - \eta_e w_e^1(x) + s_e dt \Pi^{ve}(x)\right. \\ &+\lambda dt \int_{x^{\min}}^{x^{f_{e1}}} \Pi^{e2}(\widetilde{x})\, dF(\widetilde{x}) + \lambda dt \int_{x^{f_{e1}}}^{x^{\max}} \Pi v^e(\widetilde{x})\, dF(\widetilde{x}) \\ &\left.+ (1-\lambda dt - s_e dt)\, \Pi^{e1}(x)\right] \qquad (21)\end{aligned}$$

where C_f is the specific human formation cost, paid by the employer.

In flows, this equation is then :

$$\begin{aligned}(r+\lambda)\,\Pi^{e1}(x) &= -C_f + p_e y_e - \eta_e w_{e1}(x) + \lambda \int_{x^{\min}}^{x^{f_{e1}}} \Pi^{e2}(\widetilde{x})\, dF(\widetilde{x}) + \lambda \int_{x^{f_{e1}}}^{x^{\max}} \Pi^{ve}(\widetilde{x})\, dF(\widetilde{x}) \qquad (22) \\ &+ s_e\left(\Pi^{ve}(x) - \Pi^{e1}(x)\right)\end{aligned}$$

For an already filled job :

$$\begin{aligned}\Pi^{e2}(x) &= \frac{1}{1+rdt}\left[p_e y_e(x) - \eta_e w_e^2(x) + s_e dt \Pi^{ve}(x)\right. \qquad (23) \\ &+\lambda dt \int_{x^{\min}}^{x^{f_{e2}}} \Pi^{e2}(\widetilde{x})\, dF(\widetilde{x}) + \lambda dt \int_{x^{f_{e2}}}^{x^{\max}} \Pi v^e(\widetilde{x})\, dF(\widetilde{x}) \qquad (24) \\ &\left.+ (1-\lambda dt - s_e dt)\, \Pi^{e2}(x)\right] \qquad (25)\end{aligned}$$

in flows :

$$\begin{aligned}(r+\lambda)\,\Pi^{e2}(x) &= p_e y_e - \eta_e w_{e2}(x) \\ &+\lambda \int_{x^{\min}}^{x^{f_{e1}}} \Pi^{e2}(\widetilde{x})\, dF(\widetilde{x}) + \lambda \int_{x^{f_{e1}}}^{x^{\max}} \Pi^{ve}(\widetilde{x})\, dF(\widetilde{x}) + s_e\left(\Pi^{ve}(x) - \Pi^{e2}(x)\right)\end{aligned}$$

The vacant job.

This case is too simple, it's supposed an exogenous cost of vacancy noted μ_e, and the vacant job can be filled with probability $m_e(\theta_e)$.

$$\Pi^{ve}(x) = \frac{1}{1+rdt}\left[-\mu_e dt + \left[m_e(\theta_e)\, dt\left(\Pi^{e1}(x) - C_f\right) + (1 - m_e(\theta_e)\, dt)\, \Pi^{ve}(x)\right]\right] \qquad (26)$$

In flows :

$$r\Pi^{ve} = -\mu_e + \left[m_e(\theta_e)\left(\Pi^{e1}(x) - \Pi^{ve}(x) - C_f\right)\right] \qquad (27)$$

2.4.2 The case of general skilled job.

This case is the same as in the simplest matching models : no shock on domestic production ability and no decision de participate on the labor market.

The filled job. The value function of this job is then :

$$\Pi^{q}=\frac{1}{1+rdt}\left[\left(p_{q}y_{q}-\eta_{q}w_{q}\right)dt+s_{q}dt\Pi^{vq}+\left(1-s_{q}dt\right)\Pi^{q}\right] \tag{28}$$

in flows :

$$r\Pi^{q}=p_{q}y_{q}-\eta_{q}w_{q}+s_{q}\left(\Pi^{vq}-\Pi^{q}\right) \tag{29}$$

The vacant job. Then the following value function :

$$\Pi^{vq}=\frac{1}{1+rdt}\left[-\mu_{q}dt+m_{q}\left(\theta_{q}\right)dt\Pi^{q}+\left(1-m_{q}\left(\theta_{q}\right)\right)\Pi^{vq}\right] \tag{30}$$

where $m_q\left(\theta_q\right)$ is the instataneous probability to fill the job.
What is written, in flows :

$$r\Pi^{vq}=-\mu_{q}+m_{q}\left(\theta_{q}\right)\left(\Pi^{q}-\Pi^{vq}\right) \tag{31}$$

2.4.3 Job creations.

The job creations decisions are simple free entrance conditions. The firms open vacant jobs as long as these ones are not costly.At the equilibrium, the profits on this vacant jobs are zero. These creations are **indirectly** influenced by shocks on domestic productions abilities.

The case of unskilled job. Concerning the unskilled jobs, the job creation equations is simply the free entrance condition $\Pi^{vn}=0$.

Then :

$$\frac{\mu_{n}}{m_{n}\left(\theta_{n}\right)}=\left[\left(\frac{p_{n}y_{n}-\eta_{n}w_{n}+\lambda\int_{\underline{x}}^{x^{f_n}}\Pi^{n}\left(\widetilde{x}\right)dF\left(\widetilde{x}\right)}{r+\lambda+s_{n}+\theta_{e}m\left(\theta_{e}\right)}\right)\right] \tag{32}$$

which determine the number of vacant unskilled jobs to open.

The case of specific skilled job. As previously, the free entrance condition $\Pi^{ve}=0$, gives :

$$\frac{\mu_{e}}{m_{e}\left(\theta_{e}\right)}=\frac{1}{r+\lambda+s_{e}}\left[-C_{ft}+p_{e}y_{e}-\eta_{e}w_{e1}\left(x\right)+\lambda\int_{\underline{x}}^{x^{f_{e1}}}\Pi^{e2}\left(\widetilde{x}\right)dF\left(\widetilde{x}\right)\right] \tag{33}$$

which determine the number of vacant specific skilled jobs to open.

The case of general skilled job. Finally, concerning the general skilled jobs $\Pi^{vq} = 0$, gives :

$$\frac{\mu_q}{m_q(\theta_q)} = \frac{p_q y_q - \eta_q w_q}{r + s_q} \tag{34}$$

which determine the number of vacant general skilled jobs to open.

2.5 The decision strategies.

Agents will chose ot their situation by the way of their value functions.

Indeed, to incite a non-working agent to come (back) on the labor market, his intertemporal utility as a worker must be upper than in non-working ; in other terms, his intertemporal utility as a unemployed must be upper than his current utility.

So the following condition appears :

$$\begin{array}{rcl} V^{un}(x^e) & \geq & V_i^{hn}(x^e) \\ & \Longleftrightarrow & \\ sx^e + \vartheta & \leq & b + \theta_n m_n(\theta_n)\left[\left(V^n - V^{hn}\right)(x^e)\right] \end{array} \tag{35}$$

where x^e is the entrance threshold on domestic ability. Is the ability is higher than this level, the agent doesn't come on the effective labor market.

Symetrically, an active agent, decide to go out of the labor market when his intertemporal utility as non-working agent becomes higher than his current utility.

Exists a double condition :

-concerning unskilled agents

$$\begin{array}{rcl} V^{hn}(x^{q_n}) & \geq & V^n(x^{q_n}) \\ & \Longleftrightarrow & \\ ex^{qn} + \vartheta & \geq & \tau_n w_n + \theta_e m_e(\theta_e)\left[\left(V^{e1} - V^n\right)(x^{qn})\right] + \\ & & \lambda\left[\int_{x_{\min}}^{x^{qn}} V^n(\widetilde{x})\,dF(\widetilde{x}) + \int_{x^{qn}}^{x_{\max}} V^{hn}(\widetilde{x})\,dF(\widetilde{x}) - \int_{x_{\min}}^{x^e} V^{un}(\widetilde{x})\,dF(\widetilde{x}) - \int_{x^e}^{x_{\max}} V^{hn}(\widetilde{x})\,dF(\widetilde{x})\right] \end{array} \tag{36}$$

-concerning general skilled agents

$$\begin{array}{rcl} V^{hn}(x^{q_{e_i}}) & \geq & V^{ei}(x^{q_{e_i}}) \\ & \Longleftrightarrow & \\ ex^{q_{ei}} + \vartheta & \geq & \tau_n w_{ei}(x^{q_{ei}}) + \\ & & \lambda\left[\int_{x_{\min}}^{x^{q_{ei}}} V^{e2}(\widetilde{x})\,dF(\widetilde{x}) + \int_{x^{q_{ei}}}^{x_{\max}} V^{hn}(\widetilde{x})\,dF(\widetilde{x}) - \int_{x_{\min}}^{x^e} V^{un}(\widetilde{x})\,dF(\widetilde{x}) - \int_{x^e}^{x_{\max}} V^{hn}(\widetilde{x})\,dF(\widetilde{x})\right] \end{array} \tag{37}$$

2.6 Wage bargaining.

2.6.1 The unskilled wage.

It's supposed that this wage is exogenous and politically fixed. This can be justified by the fact that unskilled workers have not really bargaining power on the labor market, and by the fact that productive ability is not revealed before the worker has a job.

$$w_n = \underline{w} \tag{38}$$

2.6.2 The specific skilled wage.

Concerning this categories of workers, it's said that two wages exist : an entry wage, and a durable wage.

Concerning the entry wage, the negotiation will be made on outside option which is the unskilled job. So :

$$w_{e1} = \arg\max \left\{ \left[\Pi^{e1}(x) - \Pi^{ve}(x) - C_f \right]^{1-\lambda_{e1}} \left[V^{e1}(x) - V^{n}(x) \right]^{\lambda_{e1}} \right\}$$

Concerning the durable wage, the outside option is the unemployment, or inactivity, then :

$$w_{e2} = \arg\max \left\{ \left[\Pi^{e2}(x) - \Pi^{ve}(x) \right]^{1-\lambda_{e2}} \left[V^{e2}(x) - \max\left(V^{un}(x), V^{hn}(x) \right) \right]^{\lambda_{e2}} \right\} \tag{39}$$

With a few calculus[9], the wage seems to be :

$$\begin{aligned} w_{e2} &= \frac{\lambda_{e_{\acute{e}}}}{\eta_e/\tau_e} \left[p_e y_e + \lambda \int_{x^{\min}}^{x_{f_e}} \Pi^{e2}(\widetilde{x})\, dF(\widetilde{x}) \right] \\ &+ (1-\lambda_{e_2}) \left[(r+\lambda) V^{u}(x) - \lambda \left(\int_{x^{\min}}^{x^{q_{e_2}}} V^{e_2}(\widetilde{x})\, dF(\widetilde{x}) + \int_{x^{q_{e_2}}}^{x^{\max}} V^{hn}(\widetilde{x})\, dF(\widetilde{x}) \right) \right] \end{aligned} \tag{40}$$

2.6.3 The general skilled wage.

The bargaining process is more simple than in previous cases, and is as in simple matching models, the wage is like :

$$w_q = \arg\max \left\{ \left(\Pi^{q} - \Pi^{vq} \right)^{1-\lambda_q} \left(V^{q} - V^{uq} \right)^{\lambda_q} \right\}$$

or :

$$w_{qt} = \frac{\lambda_q}{\eta_q} \left(p_q y_q + \mu_q \frac{V_q}{U_q} \right) + (1-\lambda_q) b_q \tag{41}$$

2.7 The final good market.

It will be assumed that exists a final good market, working with perfect conccurence rules, and where the intermediate goods demands are obtained by a maximisation program of the representative firm of the concerned sector, that is to say :

$$\max_{\left\{N_n^*, N_e^*, N_q^*, k\right\}} \left\{ F\left(N_n^*, N_e^*, N_q^*, k \right) - p_n N_n^* - p_e N_e^* - p_q N_q^* - p_k k \right\} \tag{42}$$

where the price of the final good is normalized to the unity, and where N_i^* is the real quantity of workers of type i used in the economy. Strictly the program may be written as follows :

$$\max_{\{q_n, q_e, q_q, k\}} \left\{ F\left(\sum q_n, \sum q_e, \sum q_q, k \right) - p_n \sum q_n - p_e \sum q_e - p_q \sum q_q - p_k k \right\}$$

Nevertheless, since it's assumed that each worker produces one unity of intermediate good, it's equivalent to consider the quantity of each intermediate good, or the quantity of each part of manpower.

[9] Mathematic annex

Then the solutions of this program are given par the following equatities :
$p_i = F_i (q_n, q_e, q_q, k)$ and then

$$\frac{p_n}{p_e} = \frac{F_{q_n}\left(N_n^*, N_e^*, N_q^*, k\right)}{F_{q_e}\left(N_n^*, N_e^*, N_q^*, k\right)}; \frac{p_n}{p_q} = \frac{F_{q_n}\left(N_n^*, N_e^*, N_q^*, k\right)}{F_{q_q}\left(N_n^*, N_e^*, N_q^*, k\right)}; \frac{p_n}{p_k} = \frac{F_{q_n}\left(N_n^*, N_e^*, N_q^*, k\right)}{F_k\left(N_n^*, N_e^*, N_q^*, k\right)} \tag{43}$$

Consequently, at the equilibrium, the price ratios will depend of the number of jobs in each part of the job market, determined by the matching process and the negotation processes early written.

In the approch here done, and in order to obtain a very general case, the production function used will be a CES one, which permits to obtain all the cases possible in terms of complementarity/substitution between the factor.

Then the production technology of the final good will be :

$$F\left(N_n^*, N_e^*, N_q^*, k\right) = A\left[\alpha\left(\beta_1\left(A_n N_n^*\right)^{\rho_1} + (1-\beta_1)\left(A_e N_e^*\right)^{\rho_1}\right)^{\frac{\rho}{\rho_1}} + (1-\alpha)\left(\beta_2\left(A_q N_q^*\right)^{\rho_2} + (1-\beta_2)\left(A_k k\right)^{\rho_2}\right)^{\frac{\rho}{\rho_2}}\right]^{\frac{1}{\rho}} \tag{44}$$

This representation open large considerations. Then a global productivity factor A could play on shocks on global productivity, and each factors have a productivity factor could simulate biased shocks of productivity. .

The choice of using two agregates in the production technology :

-one grouping the two types of agents without general human capital

-one grouping the workers with general human capital and the physical capital

is justified by a more simple evaluation of the elasticities inside each aggregate, and by the fact that other works consider generally an aggregate skill work-capital.

Then, in this case, the productivity of each agent is done by :

$$\begin{aligned}
p_n &= \alpha.\beta_1.A.A_n^{\rho_1}.B^{\frac{1-\rho}{\rho}}.B_1^{\frac{\rho-\rho_1}{\rho_1}}\left(N_n^*\right)^{\rho_1-1} && (45)\\
p_e &= \alpha.(1-\beta_1).A.A_e^{\rho_1}.B^{\frac{1-\rho}{\rho}}.B_1^{\frac{\rho-\rho_1}{\rho_1}}\left(N_e^*\right)^{\rho_1-1} && (46)\\
p_q &= (1-\alpha).(\beta_2).A.A_q^{\rho_2}.B^{\frac{1-\rho}{\rho}}.B_2^{\frac{\rho-\rho_2}{\rho_2}}\left(N_q^*\right)^{\rho_2-1} && (47)\\
p_k &= (1-\alpha).(1-\beta_2).A.A_k^{\rho_2}.B^{\frac{1-\rho}{\rho}}.B_2^{\frac{\rho-\rho_2}{\rho_2}}(k)^{\rho_2-1} && (48)
\end{aligned}$$

with :

$$\begin{aligned}
B_1 &\equiv \alpha\left(\beta_1\left(A_n N_n^*\right)^{\rho_1} + (1-\beta_1)\left(A_e N_e^*\right)^{\rho_1}\right),\\
B_2 &\equiv \left(\beta_2\left(A_q N_q^*\right)^{\rho_2} + (1-\beta_2)\left(A_k k\right)^{\rho_2}\right)\\
B &\equiv \left[\alpha\left(\beta_1\left(A_n N_n^*\right)^{\rho_1} + (1-\beta_1)\left(A_e N_e^*\right)^{\rho_1}\right)^{\frac{\rho}{\rho_1}} + (1-\alpha)\left(\beta_2\left(A_q N_q^*\right)^{\rho_2} + (1-\beta_2)\left(A_k k\right)^{\rho_2}\right)^{\frac{\rho}{\rho_2}}\right]
\end{aligned}$$

What is giving the price ratios following :

$$\frac{p_e}{p_n} = \frac{\alpha.\left(1-\beta_1\right)..A.A_e^{\rho_1}.B^{\frac{1-\rho}{\rho}}.B_1^{\frac{\rho-\rho_1}{\rho_1}}\ (N_e^*)^{\rho_1-1}}{\alpha.\beta_1.A.A_n^{\rho_1}.B^{\frac{1-\rho}{\rho}}.B_1^{\frac{\rho-\rho_1}{\rho_1}}\ (N_n^*)^{\rho_1-1}} = \frac{1-\beta_1}{\beta_1}\left(\frac{A_e}{A_n}\right)^{\rho_1}\left(\frac{N_e^*}{N_n^*}\right)^{\rho_1-1} \tag{49}$$

$$\frac{p_q}{p_n} = \frac{\left(1-\alpha\right).\beta_2.A.A_q^{\rho_2}.B^{\frac{1-\rho}{\rho}}.B_2^{\frac{\rho-\rho_2}{\rho_2}}\left(N_q^*\right)^{\rho_2-1}}{\alpha.\beta_1.A.A_n^{\rho_1}.B^{\frac{1-\rho}{\rho}}.B_1^{\frac{\rho-\rho_1}{\rho_1}}\ (N_n^*)^{\rho_1-1}} = \frac{1-\alpha}{\alpha}\frac{\beta_2}{\beta_1}\frac{A_q^{\rho_2}}{A_n^{\rho_1}}\frac{B_2^{\frac{\rho-\rho_2}{\rho_2}}}{B_1^{\frac{\rho-\rho_1}{\rho_1}}}\frac{\left(N_q^*\right)^{\rho_2-1}}{(N_n^*)^{\rho_1-1}} \tag{50}$$

$$\frac{p_k}{p_n} = \frac{\left(1-\alpha\right).\left(1-\beta_2\right).A.A_k^{\rho_2}.B^{\frac{1-\rho}{\rho}}.B_2^{\frac{\rho-\rho_2}{\rho_2}}\ (k)^{\rho_2-1}}{\alpha.\beta_1.A.A_n^{\rho_1}.B^{\frac{1-\rho}{\rho}}.B_1^{\frac{\rho-\rho_1}{\rho_1}}\ (N_n^*)^{\rho_1-1}} = \frac{1-\alpha}{\alpha}\frac{1-\beta_2}{\beta_2}\frac{A_k^{\rho_2}}{A_n^{\rho_1}}\frac{B_2^{\frac{\rho-\rho_2}{\rho_2}}}{B_1^{\frac{\rho-\rho_1}{\rho_1}}}\frac{(k)^{\rho_2-1}}{(N_n^*)^{\rho_1-1}} \tag{51}$$

The price of capital (interest rate) is assumed fixed. Then the interest rate will determine implicitly the level of capital necessary to the firm.

3 An application to the french economy.

In order to give a more consistent signification, this model will be applied to the french economy in order to have a real tool to make policy simulations. Then the setting of this model aims at having an equilibrium as near as possible from the situation of the french economy in 2000 (full employment year).

3.1 Calibration.

3.1.1 Workforce repartition.

The considered population is the workers of the private sector, approximatively 14,5milllions working people. According the Enquête Emploi (EE)[10] concerning the year 2000, the general skilled workers represent about one third of the total working population, then about 5millions agents. Concerning the non-general skilled workers, the ACEMO-SMIC survey seems to estimate the number of the minimum wage workers at 2,3millions, since works on the EE 2000 estimate the workers paid under 1,1 time the minimum wage at a little upper level.

3.1.2 Structural parameters.

Minimum wage levels and specific human capital acquisition costs, are set by the final production value. To have acceptable ratios, the cost of specific human capital acquisition is then assumed to be six times the value of the premium minimum wage. Then, it's fixed here to be 6000€.

The bargaining powers are justified by the works of [Abowd and Allain, 1996], who concluded a barganing power fixed at 0,4 on french statistics.

Concerning the general-skilled employees, it's assumed that the bargaining power is perfectly shared. It can indeed assumed that, with their general skill, and the fact that they can be employed in every firm, the bargaining power is the same that the other bargainer, wich is their employer.

The separation rates are set to have some conform annual separation rates. According to the DMMO (Déclaration des Mouvements de Main d'Oeuvre) these annual separation rates are 20% and 15% concerning respectivly the unskilled and the specific-skilled workers. [Cahuc and Zylberberg, 2002] use an annual separation rate of 15%. I use here a monthly exogenous probability of .025 for the unskilled workers, 0.0125 for the specific-skilled workers, and 0.02 for the general-skilled workers.

The potentially active population growth is supposed to be zero, this hypothesis doesn't change fundamentaly the results.

Concerning the matching function elasticities, they're set to 0,5, as used in Cahuc & Zylberberg.

The contributions rates are set on the year 2000, including the cuts on the low wages since 1993. Concerning the lower wages, the employer and employee contribution rates are respectivly 40% and 21% of the premium wage. With the contribution cuts, applied since 1993/1995, the level of employer contribution rate falls to 21%. Concerning the specificly skilled workers, these levels are respectively, 38% and 21%.

The parameters of the production function are set with the works of [Cahuc *et al.*,], [Guy Laroque, 2000].

Concerning the matching functions, a lot of empirical works have been led, with different results. The most simple and neutral case, is to use a parameter 0.5, what is done here for both matching functions. The constant scale parameter of the matching functions are set to obtain some consistant transition rates between unemployement/employment and unskilled/skilled job.

The vacancy costs are set by the works of [John Abowd, 2003].

[10] Official statistic source concerning the french labor market

Parameter	Value
$\lambda_{e1} = \lambda_{e2}$	.4
γ_n	.5
γ_e	.5
λ_q	.5
γ_q	.5
s_n	.025
s_e	.0125
s_q	.02
$\underline{\omega}$	3
ω_e	4
ω_q	10
b_t	$0.25 \cdot w_n$
b_{qt}	$0.25 \cdot w_q$
C_f	$6 \cdot w_n$
$\boldsymbol{\eta}_n$	0.21
$\boldsymbol{\eta}_e$	0.38
$\boldsymbol{\eta}_q$	0.38
τ_n	0.21
τ_e	0.20
τ_q	0.20
α	0.23
β_1	0.15
β_2	0.20
ρ_1	1.1
ρ_2	.65
ρ	.65

3.1.3 Free parameters.

The home productivity is uniformly distributed on $[0; 1]$. This hypothesis could appear extremly simplifying, but this paper interests to average wages, and average decisions. It's not very important to have a real wage distribution here. In future developments, this hypothesis will be changed, in order to have wage distribution conforming the reality.

The monthly arrival rate of the indiosyncratic shock is set to 0.10 as in Garibaldi&Wasmer (2003).

3.2 Model equilibrium.

With these parameters, the stationary equilibrium model gives for the year 2000 :

Variable	Rate %	Value (millions)
Unskilled employment	18.88	2,689
Specific-skill employment	49.42	7.038
Non activity	15,73	2,240
General-skill employment	96,32	4.816
Unskilled unemployment (non-participant excluded)	15,97 (18.95)	2.274
General skill unemployment	3,68	0.184
Average unemployment (public sector included, non-working excluded)	9.16	2.458

This virtual photography of the french economy on the year 2000 seems to be near the real one.

4 What about participation incentives?

In France, it appears that the participation rates are particulary low, comparatively to the other european countries. This low participation, could be a restriction to an efficient production process, and to the wealth of social accounts.

4.1 The french policy to increase participation.

Then, a policy which increases the level of earned incomes on the low wages is led in France since 2002. This principle, is called "Prime pour l'Emploi" and concerns particulary the unskilled workers.

The "Prime pour l'Emploi" is clearly build to incite the participation of the lowest skilled (and the the lowest paid). One condition to be eligible is then that one in the household have an effective job, paid between 30% and 140% of the minimum wage. The tax credit level depends on the level of wage, it increases from 30% to 100% of the minimum wage, then decreases.

The maximum tax credit, is then obtained with a minimum wage income. The surplus of earned income for a full time employee paid on the minimum wage raises 6%, which is a real income increase.

4.1.1 Simulations and results. (First results and interpretations)

To have similar effects than an tax credit, we will make decrease the level of the employee contributions on the minimum wage. Then this contribution rate must modified from 21% to 16,26%. The incentive effects will be exactly the same in this case than with a tax credit, and the cost for the government is too exactly the same.

Concerning the wage the specific-skilled workers, the same hypothesis is done, and the the contribution rate, jumps from 20,6% to 19,6%.

A limited effect on the participation. The results are summarized in the table

Variable	Rate(%)	Value	Evolution
Unskilled employment	18,93	2,695	+6300
Specific-skill employment	49,55	7,056	+18320
Non activity	15,53	2,212	-28220
General-skill employment	96,32	4,816	+300
Unskilled unemployment (non-participant excluded)	18,88	2,278	+3600
General skill unemployment	3,68	0,184	-300
Average unemployment (public sector included, non-working excluded)	9.26	2,462	+3300

Few jobs created and essentially skilled jobs. It can be shown in this paper that the effects of this policy are very light. Only 24620 jobs have been created, in which 6300 unskilled jobs and 18320 specific skilled jobs. The fact that the specific skilled jobs benefit particulary of the policy comes from the final production function. Indeed, the complementarity between unskilled and specific-skilled jobs is given by the production function. In this case, the *ex ante* relative costs don't change, because of the only change of **employee** contribution rates. Then the repartition between the different production factors has not to be changed a priori. It's not exactly the case, because of the changes in the bargained wage of the specific skilled workers, what is explained in the following.

A few evolution of the wages. Of course, the wages of the specific-skilled workers don't change *a priori*, but it's not exactly the case. In fact, with the PPE, the earned income of a unskilled worker increases and directly influences his intertemporal value function. Then it increases too the value function of a unemployed worker, by the way of the expected income as an employed worker. Then the outside option of an employee bargaining is greater, the the negotiated wage is bigger. Then the relative costs between unskilled and specific-skilled workers will be changed ex post, and benefits to the less skilled ones. The global negotiated wage grows approximatively 3%,and then the relative cost grows the same level.

What about the transition probabilities? With the PPE, the firms have not the change *a priori* their vacancy opening policy. The costs are exactly the same with or without the PPE, then their expected value by filled or vacant job are too the same. But the incentive changes the number of candidates to fill the job, then the probability to fill a vacant job increases.

By the arrival of new workers, the probability to find a job decreases, and we could observe some unemployment traps, as we can have some low wages traps with the tax credits concerning employer contributions.

4.2 Why not tax credits on employer contributions, and minimum wage increase?

To be written...

Conclusion.

The effects of the PPE are very light, and don't seem to reach his official and declared objective. The job creations are finally very few, and benefits essentially to skilled workers, who have bargained their wages, clearly upper the minimum wage. Furthermore, by the few job creations, the principle is very costly, because many people are concerned and obtain this incentive, but the effects on employment and production are too small to compensate for this. The optimal mechanism to apply seems to be an increase in the minimum wage (which is then not paid by the government) but compensated by tax credits concerning **employer contributions**, in order to not increase (or to decrease) the labor costs. We knows the real efficiency of the tax credits on employer contributions (cf [Campens *et al.*, 2002] or [Campens, 2003]) and it could be the most efficient way with the minimum wage to increase the partipation on the labor market. Furthermore, these wage growth will be included in the retirement pension calculus, unemployment benefits, etc. ; it's not the case of the PPE, which is a direct income, not included in the wage!

To be concluded...

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