Robust Control: A Note on the Timing of Model Uncertainty*

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Abstract. In this note a one-state, one-control variable quadratic linear problem with robust control and discount factor is developed to examine the optimal response of the first-period control to changes in future model uncertainty. A change in future model uncertainty has an effect on the optimal first-period control response going in the same direction as the one caused by an equal size change in current model uncertainty. However, both analytical and numerical results show that such effect is much lower than the one derived from a change in current model uncertainty. Moreover, such effect is even much lower as the change in model uncertainty moves farther away into the future. Finally, the infinite horizon result confirms the reinforcing nature of the effects on the optimal first-period control response of current and future changes in model uncertainty.

Key words: optimal control; model uncertainty; robustness; macroeconomic policy

1. Introduction

The recent interest in the application of robust control methods to monetary policy has prompted some work on comparing them to the more familiar expected value controllers. Zakovic, Rustem and Wieland (2003) simultaneously apply both methods to monetary policy and find rules that limit worst-case outcomes while providing a reasonably good performance on average. In particular, one would like to characterize the response of the control to increases in model and parameter uncertainty with robust control and expected value control, respectively.

In this note I distinguish between "model uncertainty" which can be stepped up by decreasing the "free" parameter in the criterion function and "parameter uncertainty" which can be stepped up by increasing a parameter's variance. Gonzalez and Rodriguez (2003) characterize the response of the control to changes in current model uncertainty.

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Mercado and Kendrick (2000) examine the effect of an increase in future parameter uncertainty on the optimal use of the first-period control variables. Mercado (2001) finds that caution will always prevail over intensity given an equal increase in current and future *parameter uncertainty*. In this note, the effect of an increase in *future model uncertainty* on the optimal response of the first-period control variable is both analyzed with the Riccati equations derived from a QLP with discounting and compared to the effect corresponding to an equal size increase in *current model uncertainty*. Unlike the results in Mercado (2001) when comparing current to future parameter uncertainty, the analytical results show that the change in *future* model uncertainty has an effect on the first-period control variable response going in the same direction as the one caused by an equal size change in *current* model uncertainty –i.e. both effects are more aggressive. However, both analytical and numerical results show that such effect is much lower than the one derived from a change in current model uncertainty. This is the same as Mercado's (2001) result - i.e. the effect of changes in current uncertainty is larger than the effect of changes in future uncertainty. Moreover, the analytical results show that the effect on the first-period control variable response becomes smaller as the change in model uncertainty moves farther into the future. Finally, the infinite horizon result confirms the reinforcing nature of the effects on the optimal first-period control response of current and future changes in model uncertainty.

In the next section a Quadratic Linear Problem (QLP) with one-state, one-control variable, discounting and robust control is set up. Section 3 shows the response of the first-period control variable to changes in future model uncertainty and compares that response to one caused by changes in current model uncertainty. Moreover, the decaying effect of future model uncertainty on the first-period control variable response as the uncertainty moves farther into the future is proved. Section 4 provides a numerical example. Section 5 studies the infinite horizon case. Section 6 contains the concluding remarks.

2. Problem statement

A QLP with one-state, one control variable, discounting and robust control is adapted from the one in Gonzalez and Rodriguez (2003). It is used here to examine the response of the first-period control variable to changes in future model uncertainty. Formally, the robust control problem consists of choosing u_k and ω_k to minimize and maximize the quadratic criterion function, respectively. Since both the Riccati equation for the QLP emerges from first-order conditions alone and the first-order conditions for extremizing a quadratic criterion function match those for an ordinary (non-robust) QLP with two controls (see Hansen and Sargent, 2003, pp 29-30), the robust control problem can be written as

$$\text{finding} \quad \left(\mathbf{u_k}\right)_{k=0}^{N-1}$$

in order to

$$\text{ext}\{\frac{1}{2}\beta^{N}[x_N - \tilde{x}_N]'W[x_N - \tilde{x}_N]$$
$$+\frac{1}{2}\sum_{k=0}^{N-1}\beta^{k}([x_k - \tilde{x}_k]'W[x_k - \tilde{x}_k]+[\mathbf{u_k} - \tilde{\mathbf{u}}_\mathbf{k}]'\mathbf{\Lambda_k}[\mathbf{u_k} - \tilde{\mathbf{u}}_\mathbf{k}]\}, \qquad (1)$$

where

x_k = state variable for period k,

$\mathbf{u_k} = \begin{pmatrix} u_k \\ \omega_k \end{pmatrix}$,

u_k = control variable for period k,

ω_k = bounded uncertainty variable that is fed back on the history of the state variable in period k,

$\tilde{x}_k$ = target scalar for state variable in period k,

$\tilde{\mathbf{u}}_\mathbf{k}$ = target vector for the control variables in period k,

β = discount factor[1] ($0 < \beta \leq 1$),

W = penalty scalar on deviations of state variable from target path,

[1] The case of no discounting of the future ($\beta = 1$) is also included for generalization purposes.

$$\mathbf{\Lambda_k} = \begin{pmatrix} \lambda_{11} & 0 \\ 0 & -\theta_k \end{pmatrix},$$

λ_{11} = penalty scalar on deviations of control variable from target path,

θ_k = "free" parameter in robust control models.

subject to the system equations

$$x_{k+1} = ax_k + \mathbf{bu_k} + \varepsilon_k \tag{2}$$

$k = 0,1,\ldots,N\text{-}1$

and

$$\varepsilon_k \sim N(0,\sigma) \tag{3}$$

$$x_0 \quad \text{given,} \tag{4}$$

where

a = state variable coefficient,

$\mathbf{b}' = \begin{pmatrix} b_1 \\ 1 \end{pmatrix}$ = 2-element coefficient vector.

The desired path for both the control and state variable is zero[2]- i.e. $\tilde{\mathbf{u}}_{\mathbf{k}} = \mathbf{0}$, $\tilde{x}_k = 0$ for $k = 0,1,\ldots,N$. The absolute value of the state parameter a is assumed to be smaller or equal to one - i.e. the state equation is not unstable. The penalty weight on the state variable W is set equal to one. The first-order conditions (FOCs) with $\beta = 1$ are the following (see Kendrick, 1981, Ch. 2)

$$\begin{bmatrix} k_{k+1}b_1^2 + \lambda_{11} & k_{k+1}b_1 \\ k_{k+1}b_1 & k_{k+1} - \theta_k \end{bmatrix} \begin{bmatrix} u_k \\ \omega_k \end{bmatrix} + \begin{bmatrix} ab_1k_{k+1} \\ ak_{k+1} \end{bmatrix} x_k = \mathbf{0} \tag{5}$$

[2] As mentioned by Mercado and Kendrick (2000) and Mercado (2001), a zero path is common with variables expressed in logs or percentage deviations from a base case.

The solution to this extremization problem is the feedback rule (see Kendrick, 1981, p. 17).

$$\mathbf{u_k} = \mathbf{G_k} x_k \tag{6}$$

By adapting the relevant equations from Gonzalez and Rodriguez (2003) to our case where the timing of model uncertainty and discounting matter, the following feedback coefficients for all periods are obtained:[3]

$$\mathbf{G_k} = \begin{pmatrix} G_{u_k} \\ G_{\omega_k} \end{pmatrix} = \begin{pmatrix} \dfrac{ab_1\theta_k \beta k_{k+1}}{\det_k} \\ -\dfrac{a\lambda_{11}\beta k_{k+1}}{\det_k} \end{pmatrix} \tag{7}$$

where k_k is the Riccati equation and given by

$$k_k = W + a^2 \beta k_{k+1} \left(1 - \frac{\lambda_{11}\beta k_{k+1} - \theta_k \beta k_{k+1} b_1^2}{\det_k} \right) \tag{8}$$

and

$$k_N = W \tag{9}$$

$$\det_k = -\lambda_{11}\theta_k + \lambda_{11}\beta k_{k+1} - \theta_k \beta k_{k+1} b_1^2 \tag{10}$$

Eq. (7) can be rewritten as

$$G_{u_k} = \frac{ab_1\theta_k \beta k_{k+1}}{\det_k} \tag{11}$$

$$G_{\omega_k} = -\frac{a\lambda_{11}\beta k_{k+1}}{\det_k} \tag{12}$$

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[3] See Appendix A in Amman et al. (1995) for derivation of the Riccati equations in a model with discounting.

In the next section equations (8) - (11) will be used to examine the response of the first-period control variable to changes in future model uncertainty and compare it to that one caused by changes in current model uncertainty.

3. Future and Current Model Uncertainty: The Optimal First-Period Response

An increase in future model uncertainty is represented here by a reduction of the robustness "free" parameter θ_T where T can take any value between 1 and N-1. An increase in current model uncertainty corresponds to lower values of θ_0. The link between the change in future model uncertainty and the value of the first-period feedback gain coefficient G_{u_0} is obtained by applying the chain rule from calculus.[4]

The first-period feedback gain coefficient G_{u_0} is obtained from Eq. (11) and is given by[5]

$$G_{u_0} = \frac{ab_1\theta_0\beta k_1}{\det_0} \tag{13}$$

A change in future model uncertainty θ_T and the first-period feedback gain coefficient G_{u_0} are linked, after applying the chain rule, according to the following:

$$\frac{\partial G_{u_0}}{\partial \theta_T} = \frac{\partial G_{u_0}}{\partial k_1} \cdots \frac{\partial k_k}{\partial k_{k+1}} \cdots \frac{\partial k_T}{\partial \theta_T} \tag{14}$$

[4] Mercado (2001) applies the chain rule from calculus to examine the link between changes in future parameter uncertainty and the absolute value of the first-period feedback gain coefficient G_{u_0} .

[5] Unlike Mercado and Kendrick (2000) and Mercado (2001), the absolute value of the gain coefficient G_{u_0} is not used here because its denominator is either positive or negative and contains the variable whose changes are analyzed.

From Eq. (13) the partial derivative of the first-period feedback gain coefficient G_{u_0} with respect to k_1 is obtained:

$$\frac{\partial G_{u_0}}{\partial k_1} = -\frac{ab_1\lambda_{11}\theta_0^2\beta}{\det_0^2} \tag{15}$$

From Eq. (8) the partial derivative of the Riccati equation k_k with respect to next period's Riccati equation k_{k+1} is obtained:

$$\frac{\partial k_k}{\partial k_{k+1}} = \frac{a^2\lambda_{11}^2\theta_k^2\beta}{\det_k^2} > 0 \tag{16}$$

From Eq. (8) the partial derivative of the Riccati equation k_T with respect to θ_T is also obtained:

$$\frac{\partial k_T}{\partial \theta_T} = -\frac{a^2\lambda_{11}^2 k_{T+1}^2\beta^2}{\det_T^2} \tag{17}$$

By substituting the right side of equations (15) – (17) into the right side of Eq. (14) and setting the "free" parameter $\theta_k = \theta$ for all $k < T$ the following is obtained:

$$\frac{\partial G_{u_0}}{\partial \theta_T} = \frac{ab_1\lambda_{11}\beta}{\det_0^2} \cdots \frac{a^2\lambda_{11}^2\theta^2\beta}{\det_k^2} \cdots \frac{a^2\lambda_{11}^2\theta^2 k_{T+1}^2\beta^2}{\det_T^2} \tag{18}$$

Comparing the effect of a change in future model uncertainty on the first-period response of the control variable to that one caused by an equal size change in current model uncertainty will give us a better idea of its relative strength.

The effect of a change in current model uncertainty on the first-period response of the control variable is obtained from Gonzalez and Rodriguez (2003) and adapted to our case with discounting. It is given by

$$\frac{\partial G_{u_0}}{\partial \theta_0} = \frac{ab_1 k_1^2 \lambda_{11} \beta^2}{\det_0^2} \tag{19}$$

By comparing the qualitative effect of a change of model uncertainty on the first-period response of the control variable across both cases, it can be seen that they share an effect going in the same direction. This finding contrasts with the one obtained by Mercado (2001) in relation to the opposite direction of the effects on the first-period control response of changes in current and future parameter uncertainty

Next it will be assumed that the effect of a change in current model uncertainty is greater than the effect of a change in future model uncertainty on the first-period response of the control variable. That is,

$$\frac{ab_1 k_1^2 \lambda_{11} \beta^2}{\det_0^2} > \frac{ab_1 \lambda_{11} \beta}{\det_0^2} \ldots \frac{a^2 \lambda_{11}^2 \theta^2 \beta}{\det_k^2} \ldots \frac{a^2 \lambda_{11}^2 \theta^2 k_{T+1}^2 \beta^2}{\det_T^2} \tag{20}$$

In order to determine whether inequality (20) is true or not, it is necessary to simplify it to get the following:[6]

$$k_1^2 > \beta \frac{a^2 \lambda_{11}^2 \theta^2 \beta}{\det_1^2} \ldots \frac{a^2 \lambda_{11}^2 \theta^2 \beta}{\det_k^2} \ldots \frac{a^2 \lambda_{11}^2 \theta^2}{\det_T^2} k_{T+1}^2 = a^{2T} \beta^T \frac{\lambda_{11}^2 \theta^2}{\det_1^2} \ldots \frac{\lambda_{11}^2 \theta^2}{\det_k^2} \ldots \frac{\lambda_{11}^2 \theta^2}{\det_T^2} k_{T+1}^2 \tag{21}$$

[6] Note that $\frac{ab_1 \lambda_{11}}{\det_0^2}$ is canceled out as it appears on both sides of the inequality. Same case with β^2.

Since $|a| < 1$ and

$$\frac{\lambda_{11}^2\theta^2}{\det_k^2} = 1 - \frac{\lambda_{11}^2\beta^2 k_{k+1}^2 - 2\lambda_{11}^2\theta\beta k_{k+1} - 2\lambda_{11}\theta b_1^2\beta k_{k+1}(-\theta + \beta k_{k+1}) + \theta^2\beta^2 k_{k+1}^2 b_1^4}{\det_k^2} < 1 \quad (22)$$

for all k

then

$$k_1^2 > (fraction) k_{T+1}^2 \quad (23)$$

The numerator of the right side of Eq. (22) will always be smaller than the denominator for any value of θ different from zero.[7] Moreover, the value of the right side of inequality (21) decreases with T since $|a| < 1$ and $\frac{\lambda_{11}^2\theta^2}{\det_k^2} < 1$.

In order to determine if inequality (23) is true or not, the analysis will only be done for the case when $\det_k < 0$. Such case is the only one that represents the domain of θ for which the robust control solution arises – i.e. as indicated by an anonymous referee, a saddle point arises when the overall impact of the omegas on the objective function is negative and it is being maximized with respect to omega.

Now, it will be proved why the robust control solution arises only when $\det_k < 0$. From Eq. (5), two functions will be defined. These two functions are

$$f_u = k_{k+1} b_1^2 u_k + \lambda_{11} u_k + k_{k+1} b_1 \omega_k + a b_1 k_{k+1} x_k = 0 \quad (24)$$

$$f_\omega = k_{k+1} b_1 u_k + k_{k+1}\omega_k - \theta_k \omega_k + a k_{k+1} x_k = 0 \quad (25)$$

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[7] The denominator contains all the terms of the numerator plus one more positive term: $\lambda_{11}^2\theta^2$. When the Riccati equation is positive, the case for a positive numerator is very strong given the poor contribution of the term $\lambda_{11}^2\theta^2$ due to the relative low values of λ_{11} and θ. Indeed, the extreme case of infinite uncertainty when $\theta = 0$ would make $\frac{\lambda_{11}^2\theta^2}{\det_k^2}$ attain its lowest value which is zero.

from which the second order conditions and the Hessian matrix are derived

$$f_{uu} = k_{k+1} b_1^2 + \lambda_{11} \tag{26}$$

$$f_{u\omega} = k_{k+1} b_1 \tag{27}$$

$$f_{\omega u} = k_{k+1} b_1 \tag{28}$$

$$f_{\omega\omega} = k_{k+1} - \theta_k \tag{29}$$

$$H = \begin{bmatrix} f_{uu} & f_{u\omega} \\ f_{\omega u} & f_{\omega\omega} \end{bmatrix} \tag{30}$$

A saddle point arises when $f_{uu} f_{\omega\omega} < f_{u\omega}^2$ (see Chiang, 1967, p. 317)

or

$$-k_{k+1} b_1^2 \theta_k - \lambda_{11}\theta_k + k_{k+1}^2 b_1^2 + \lambda_{11} k_{k+1} < k_{k+1}^2 b_1^2 \tag{31}$$

After solving for θ_k in inequality (31), the following is obtained

$$\theta > \underline{\theta} = \frac{\lambda_{11} k_{k+1}}{\lambda_{11} + k_{k+1} b_1^2} \tag{32}$$

This is the case that corresponds to the right side of the discontinuity ($\det_k < 0$) in the response of the control to changes in the "free" parameter θ.[8] By simplifying the term inside the parenthesis of the Riccati equation, Eq. (8) can be rewritten as

$$k_k = W + a^2 \beta k_{k+1} \left(-\frac{\lambda_{11}\theta_k}{\det_k} \right) \tag{33}$$

Since $\det_k < 0$, $k_N = 1$ and $W = 1$, then $k_k > k_{k+1}$. This in turn implies that $k_1 > k_{T+1}$ or $k_1^2 > k_{T+1}^2$. Therefore $k_1^2 > (fraction) k_{T+1}^2$. Consequently, it validates the assumption that the effect of a change in current model uncertainty is greater than the effect of a change in future model uncertainty on the first-period response of the control variable.

[8] See Gonzalez and Rodriguez (2003) for a further characterization of the control response to changes in model uncertainty.

4. Numerical Example

A numerical example will help to illustrate the previous findings. The parameters obtained with the relevant data from Gonzalez (2003) are substituted into equations (1) and (2).

The parameter values are:

$$a = 0.7737$$
$$b_1 = -3.389\text{E} - 04$$
$$\beta = 1$$
$$x_0 = 0.020$$
$$\lambda_{11} = 1.0\text{E} - 15$$
$$N = 6$$

The desired path for the state variable (pollution stock) and control variables is zero for every time period. Moreover, the parameter λ_{11} must be zero since there is no penalty on deviations of the control (taxes) from its desired path. However, setting $\lambda_{11} = 0$ would make the feedback matrix singular (see Kendrick, 1981, p. 17). Consequently, a very small value for λ_{11} was chosen. Finally, the estimated value for b_1 is very low since taxes were measured in \$/tons and the pollution stock in parts per million (ppm). *Table 1* shows the effects on the first period control variable of equal increases in current and future model uncertainty.[9]

[9] Both current and future model uncertainty increases are obtained by subtracting one tenth of $E - \#$ from the base case theta denoted by $cE - \#$. For example, the model uncertainty increase for the first row would be represented by a theta equal to 5.00E-08 – 0.1xE-08 = 4.9E-08.

Table 1
First-period control response to changes in current and future model uncertainty

Base case theta	Control response in base case	Control response with a current model uncertainty increase (Base case theta - 0.1E#)	Control response with a future model uncertainty increase (Base case theta - 0.1E#)
5.00E-08	55.2869	55.5258	55.2869
2.00E-08	80.8617	84.2816	80.8617
1.00E-08	353.0643	1401.4	353.0643
9.2E-09	851.6582	1056.6	851.6582

It can be seen from the table above that changes in future model uncertainty do not have an effect on the first-period control response as opposed to changes in current model uncertainty. It is important to mention that the relevant range for θ depends on the values of the model parameters.

5. Infinite Horizon

Consider now the infinite horizon case. Here the feedback gain coefficient of the control given by Eq. (11) and the Riccati equation given by Eq. (8) become stationary. These new equations are:

$$G_u = \frac{ab_1\theta\beta k}{\det} \tag{34}$$

$$k = W + a^2\beta k\left(1 - \frac{\lambda_{11}\beta k - \theta\beta k b_1^2}{\det}\right) \tag{35}$$

where $G_u = G_{uss}$ and $k = k_{ss}$, where 'ss' stands for steady-state. Thus, for an infinite horizon problem, the time-varying optimal control policy given by Eq. (11) is replaced by the constant optimal control policy implied by equations (34) and (35).

By taking the derivative of Eq. (34) with respect to θ and after simplifying, the following expression is obtained:[10]

[10] In this footnote Eq. (36) is obtained.

$$\frac{\partial G_u}{\partial \theta} = \frac{ab_1\beta\lambda_{11}\left(k^2\beta - \theta^2 \frac{\partial k}{\partial \theta}\right)}{\det^2} \tag{36}$$

By comparing Eq. (36) with equations (18) and (19), they will share the same sign if $\left(k^2\beta - \theta^2 \frac{\partial k}{\partial \theta}\right)$ is positive.

In order to obtain $\frac{\partial k}{\partial \theta}$, the implicit function theorem is applied to Eq. (35). The resulting expression is:[11]

$$\frac{\partial G_u}{\partial \theta} = \frac{ab_1\beta k + ab_1\theta\beta \frac{\partial k}{\partial \theta}}{\det} - \frac{ab_1\theta\beta k\left(-\lambda_{11} + \lambda_{11}\beta \frac{\partial k}{\partial \theta} - \beta k b_1^2 - \theta\beta \frac{\partial k}{\partial \theta} b_1^2\right)}{\det^2}$$

$$\frac{\partial G_u}{\partial \theta} = \frac{ab_1\beta\left[\left(k + \theta \frac{\partial k}{\partial \theta}\right)\det - \theta k\left(-\lambda_{11} + \lambda_{11}\beta \frac{\partial k}{\partial \theta} - \beta k b_1^2 - \theta\beta \frac{\partial k}{\partial \theta} b_1^2\right)\right]}{\det^2}$$

$$\frac{\partial G_u}{\partial \theta} = \frac{ab_1\beta\left[-k\lambda_{11}\theta + k^2\lambda_{11}\beta - \theta\beta k^2 b_1^2 - \theta^2 \frac{\partial k}{\partial \theta}\lambda_{11} + \theta \frac{\partial k}{\partial \theta}\lambda_{11}\beta k - \theta^2 \frac{\partial k}{\partial \theta}\beta k b_1^2\right]}{\det^2} + \frac{ab_1\beta\left[\theta k\lambda_{11} - \theta k\lambda_{11}\beta \frac{\partial k}{\partial \theta} + \theta k^2\beta b_1^2 + \theta^2\beta k \frac{\partial k}{\partial \theta} b_1^2\right]}{\det^2}$$

$$\frac{\partial G_u}{\partial \theta} = \frac{ab_1\beta\lambda_{11}\left(k^2\beta - \theta^2 \frac{\partial k}{\partial \theta}\right)}{\det^2}$$

11 In this footnote Eq. (37) is obtained. Let us define the function F from the Riccati expression given by Eq. (35) as

$$\frac{\partial k}{\partial \theta} = -\frac{\frac{\lambda_{11}^2 \beta k}{\det^2}}{1 - a^2 \beta \frac{\lambda_{11}^2 \theta^2}{\det^2}} \tag{37}$$

The denominator of Eq. (37) will always be positive according the explanation given before in footnote 9 of Section 3. Thus, $\frac{\partial k}{\partial \theta} < 0$, which in turn implies that $\frac{\partial G_u}{\partial \theta} = \frac{a b_1 \beta \lambda_{11} \left(k^2 \beta - \theta^2 \frac{\partial k}{\partial \theta} \right)}{\det^2}$ shares the same sign of the expressions given by equations (18) and (19).

Consequently, for an infinite horizon dynamic problem, an increase in model uncertainty induces either a more aggressive or a more cautious control response

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$F = k - W + a^2 \beta k \left(\frac{\lambda_{11} \theta}{\det} \right)$

Now the implicit function theorem states that $\frac{\partial k}{\partial \theta} = -\frac{\frac{\partial F}{\partial \theta}}{\frac{\partial F}{\partial k}}$

$$\frac{\partial F}{\partial \theta} = \frac{\lambda_{11}}{\det} - \frac{\lambda_{11}\theta \left(-\lambda_{11} - \beta k b_1^2 \right)}{\det^2}$$

$$\frac{\partial F}{\partial \theta} = \frac{-\lambda_{11}^2 \theta + \lambda_{11}^2 \beta k - \theta \lambda_{11} \beta k b_1^2 + \lambda_{11}^2 \theta + \lambda_{11} \theta \beta k b_1^2}{\det^2}$$

$$\frac{\partial F}{\partial \theta} = \frac{\lambda_{11}^2 \beta k}{\det^2}$$

and

$$\frac{\partial F}{\partial k} = 1 + a^2 \beta \left(\frac{\lambda_{11} \theta}{\det} \right) + a^2 \beta k \left[\frac{-\lambda_{11} \theta \left(\lambda_{11} \beta - \theta \beta b_1^2 \right)}{\det^2} \right]$$

$$\frac{\partial F}{\partial k} = 1 + a^2 \beta \lambda_{11} \theta \left(\frac{-\lambda_{11} \theta + \lambda_{11} \beta k - \theta \beta k b_1^2}{\det^2} \right) + a^2 \beta k \left(\frac{-\lambda_{11}^2 \beta \theta + \lambda_{11} \theta^2 \beta b_1^2}{\det^2} \right)$$

$$\frac{\partial F}{\partial k} = 1 - a^2 \beta \frac{\lambda_{11}^2 \theta^2}{\det^2}$$

then

$$\frac{\partial k}{\partial \theta} = -\frac{\frac{\lambda_{11}^2 \beta k}{\det^2}}{1 - a^2 \beta \frac{\lambda_{11}^2 \theta^2}{\det^2}}$$

depending on the sign of the ab_1 product.[12] Notice that in this case the timing of model uncertainty, the main focus of this note, is removed from consideration.

6. Conclusions

In this note, the effect of an increase in future model uncertainty on the optimal response of the first-period control variable is analyzed with a one-state, one-control variable model in a standard QLP with robust control and discounting incorporated. The Riccati equations are used to compare that effect to the one corresponding to an equal size increase in current model uncertainty. Unlike the results in Mercado (2001) when comparing current to future changes in parameter uncertainty, the analytical results show that the change in future model uncertainty has an effect on the first-period control variable response going in the same direction as the one caused by an equal size change in current model uncertainty. However, both analytical and numerical results show that such effect is much lower than the one derived from a change in current model uncertainty. That result is the same as the one in Mercado (2001) – i.e. the prevalence of the effect of changes in current uncertainty over changes in future uncertainty is maintained. Moreover, the analytical results show that the effect on the first-period control variable response becomes smaller as the change in model uncertainty moves farther into the future. Finally, the infinite horizon result confirms the reinforcing nature of the effects on the optimal first-period control response caused by current and future changes in model uncertainty.

[12] Just like in Mercado (2001), notice that the infinite horizon problem could also be assimilated to the case of a change in model uncertainty expanding into the future. Thus, this result confirms the reinforcing nature of the effects on the optimal first-period control response of current and future changes in model uncertainty.

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