Nonlinear Mean Reversion in Stock Prices

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Abstract

We investigate evidence for nonlinear mean reversion in yearly S&P500 data from 1871 until 2001. We find that up to 1990 there is significant evidence of nonlinear mean reversion. In particular, stock prices are characterized by a persistent process close to the fundamental value. However, when prices deviate significantly a mean reverting regime is activated and prices adjust to fundamental values. Instead, the stock price run-up of the late 90s exacerbated the persistence of the deviations and there is no evidence for a mean reverting regime that drives prices back to fundamentals.

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1 Introduction

Does the stock market rationally reflect fundamental values? The stock price run-up of the late 90s revived the debate about the rationality of stock prices. In 2000 the Price-to-Dividends (PD) ratio for the S&P500 index reached a level of 85 against an historical average of approximately 25. The extreme behaviour compared to historical standards has been explained in different ways.

According to rational explanations, the rapid increase in stock prices reflects changes occurred in fundamental factors. They argue that the required rate of return has lowered significantly because of higher participation of investors to the stock market and changes occurred in consumers preferences. If investors discount future pay-offs at a lower rate, prices will increase. A similar result is obtained when the expected growth rate of dividends or earnings increases. These arguments were proposed by Heaton and Lucas (1999). However, they found that these explanations are not able to account for the large increase of the late 90s.

On the other hand, Campbell and Shiller (2001) argue that changes in fundamental factors are not large enough to explain changes in stock prices. In addition, historical evidence suggests that in periods followed by large collapses of stock prices the valuation ratios never reached such extraordinary levels. An alternative explanation is that prices experience large swings from fundamental valuations due to fads in investors expectations. Summers (1986) suggested that irrational fads create persistent deviations of prices from intrinsic valuations that are difficult to arbitrage away by rational investors. According to this approach, a combination of irrational expectations of some investors and limits to the arbitraging activities of rational investors explains the deviations of stock prices from rational valuations. This view is also consistent with the empirical evidence of mean reversion and long-run predictability of stock prices. If the stock price reverts (in the long-run) back to its intrinsic value, a positive (negative) deviation predicts that prices will decrease (increase). Hence, the adjustment process creates a negative relation between the changes in prices and the deviation from the fundamentals that emerges at long horizons. Some theoretical models that try to capture this idea are de Long *et al.* (1990) and Brock and Hommes (1998).

In this paper we investigate the role that fundamental factors played in the recent increase of stock prices. In particular, we use a dynamic version of the Present Value Model (PVM) that allows for time variation in the discount rate and the growth rate of dividends. The analysis of more than a century of the S&P500 index shows that the fundamental factors fail to explain the persistence of the deviations from intrinsic valuations, in particular in the late 90s. Shocks to the growth rate of dividends or to proxies for the discount rate, such as interest rates and returns volatility, die out very quickly compared to shocks to the stock prices. This indicates that the excessive persistence of the deviations from fundamentals could be caused by the overreaction of investors to fundamental news: they expect the effects of positive (negative) news about the fundamentals to be more persistent than it is rational. This evidence is consistent with the explanation of Summers (1986) that assumes that deviations follow a persistent AR(1) process. Recently, there has been a growing interest in modelling deviations of asset prices from instrisic valuations using nonlinear models. A common result is that asset prices can be characterized as switching between two regimes: when deviations are small they follow a random walk process but when they are large they follow a stable AR process that contributes to the reversion of the price toward the fundamentals. Some studies along these lines are Gallagher and Taylor (2001) for stock prices and Kilian and Taylor (2002) and Taylor and Peel (2000) for exchange rates.

We investigate the issue of nonlinear mean reversion for yearly observations of the S&P500 index from 1871 until 2001. Estimation results for stock price data up to 1990 show that there is evidence for nonlinearity in the mean reversion process. In particular, when the price is close to the intrinsic value the deviations are very persistent and mean reversion is weak; however, when deviations are large the speed of adjustment increases and the price reverts back toward the fundamental value. The results suggest that in the mean reverting regime the half-life of a shock is approximately 3 years. When the 90s are included in the sample, there is strong evidence of nonlinearity in the transitory component. The estimation results indicate that the pattern of mean reversion has changed compared to the previous findings. Both close and far from the long-run equilibrium deviations are very persistent. So, there is no evidence that the speed of mean reversion becomes stronger for large deviations. We interpret these results as evidence that the extreme behaviour of prices in the 90s exacerbated the persistence of the mean reversion process. Before the 90s, when a fad was driving the stock price away from the fundamentals, stabilizing forces were activated to weaken the persistence of the process. However, in the 90s the persistence became stronger and drove the PD ratio to unprecedented levels.

The chapter is organized as follows: section (2) introduces different notions of fundamental values used for empirical investigation. Section (3) describes the nonlinear model used for the deviations of stock prices from fundamentals. Section (4) discusses the estimation results and the evidence in support of the hypothesis of nonlinear mean reversion. Finally, section (5) concludes.

2 Fundamentals

A standard approach in asset valuation is to assume that the price satisfies

$$P_t = E_t \left[\frac{1}{1 + r_{t+1}} (P_{t+1} + D_{t+1}) \right], \tag{1}$$

where P_t is the price of the asset at the end of period t, D_{t+1} is the cash flow paid during period $(t+1)$ and r_{t+1} is the required rate of return at time $(t+1)$. $E_t(\cdot)$ indicates the expectation conditional upon information available at time t. Solving Equation (1) forward for T periods and applying the law of iterated expectations, we obtain

$$P_t = E_t\left[\sum_{j=1}^{T}\left(\prod_{i=1}^{j}\frac{1}{1+r_{t+i}}\right)D_{t+i}\right] + E_t\left[\prod_{i=1}^{T}\frac{1}{1+r_{t+i}}P_{t+T}\right]. \tag{2}$$

The present value of holding the asset for T periods is equal to the expected discounted value of its cash flows and the expected discounted value of the resale price. A typical assumption introduced to rule out the occurrence of bubbles is

$$\lim_{T\to\infty} E_t\left[\prod_{i=1}^{T}\frac{1}{1+r_{t+i}}P_{t+T}\right] = 0, \tag{3}$$

called the transversality condition. If we assume that $T\to\infty$ and the transversality condition holds, the asset price is equal to the expected discounted value of its future cash flows

$$P_t^* = E_t\left[\sum_{j=1}^{\infty}\left(\prod_{i=1}^{j}\frac{1}{1+r_{t+i}}\right)D_{t+i}\right], \tag{4}$$

where we indicate P_t^* as the fundamental value. We define the growth rate of the dividend process g_t as $D_{t+1} = (1+g_{t+1})D_t$, so that the fundamental value is given by

$$P_t^* = E_t\left[\sum_{j=1}^{\infty}\left(\prod_{i=1}^{j}\frac{1+g_{t+i}}{1+r_{t+i}}\right)D_t\right]. \tag{5}$$

The time variation of g_t and r_t and the nonlinearity in the pricing equation complicate the derivation of analytically tractable formulas. One approach to simplify the problem consists of assuming that the dividends growth rate and the required return are constant and equal to g and r, respectively. Under these assumptions, Equation (5) implies that

$$P_t^* = mD_t, \tag{6}$$

where $m = (1+g)/(r-g)$. The stock price at time t is given by the cash flow times a multiple that depends on the ex-ante rate of return and the growth rate of dividends. This model is also known as the Gordon valuation formula and has recently been used by Heaton and Lucas (1999) to determine the rational valuation of stock prices and by Fama and French (2002) to evaluate the size of the risk premium. The model is very simple and makes some clear predictions about the behaviour of prices: prices will increase if r is lowered, that is, if investors discount at a lower rate future cash flows, or if dividends are expected to grow at a faster rate. Another implication of the model is that the PD ratio should be constant over time.

However, the assumption that the dividend growth rate and the expected returns are constant seems unrealistic. It is possible to allow for time variation by following the approach of Poterba and Summers (1986). They approximate the pricing formula given in (5) by a first-order Taylor expansion around the mean of the required return, r, and the mean of the growth rate, g,

$$P_t^* \approx E_t \left[\sum_{j=1}^{\infty} \left(\frac{1+g}{1+r} \right)^j + \frac{\partial P_t^*}{\partial r_{t+j}} |_r (r_{t+j} - r) + \frac{\partial P_t^*}{\partial g_{t+j}} |_g (g_{t+j} - g) \right] D_t \tag{7}$$

where the partial derivatives are given by

$$\frac{\partial P_t^*}{\partial r_{t+j}} |_r = -\frac{D_t}{r-g} \beta^j, \tag{8}$$

$$\frac{\partial P_t^*}{\partial g_{t+j}} |_g = \frac{(1+r)D_t}{(1+g)(r-g)} \beta^j, \tag{9}$$

and $\beta = (1+g)/(1+r)$. Substituting the derivatives into Equation (7), we get

$$P_t^* = \left\{ \frac{1+g}{r-g} - \frac{1}{(r-g)} E_t \left[\sum_{j=1}^{\infty} \beta^j (r_{t+j} - r) \right] + \frac{1+r}{(1+g)(r-g)} E_t \left[\sum_{j=1}^{\infty} \beta^j (g_{t+j} - g) \right] \right\} D_t. \tag{10}$$

The pricing formula depends on the expectations of investors about future ex-ante returns and dividends growth rates. A typical assumption made in the literature is that the expectations follow an AR(1) process, that is

$$E_t(r_{t+j} - r) = \rho^j (r_t - r) \tag{11}$$

$$E_t(g_{t+j} - g) = \phi^j (g_t - g), \tag{12}$$

and the approximated pricing formula in Equation (10) becomes

$$P_t^* = m_t D_t, \tag{13}$$

where m_t is the time-varying multiplier given by

$$m_t = \left\{ \frac{1+g}{r-g} - \frac{\rho(1+g)}{(r-g)(1+r-\rho(1+g))} (r_t - r) + \frac{\phi(1+r)}{(r-g)(1+r-\phi(1+g))} (g_t - g) \right\}. \tag{14}$$

This version of the fundamental value is known in the literature as the dynamic Gordon model because it defines asset prices as a time-varying multiplier of the dividends. The multiplier in Equation (14) has a straightforward interpretation: if the required rate of return and the growth rate of dividends are constant and equal to their mean then it collapses to the static multiplier of Equation (6); however, time variation in the required rate of return and/or in the dividend growth rate changes the level of the multiplier. The response of prices to changes in r_t and g_t is similar to the case of the static Gordon: if investors require at time t a return higher (lower) than the average r, this will decrease (increase) the multiplier and consequently prices. On the other hand, if dividends grow at a higher (lower) rate at time t, this will increase (decrease) the multiplier and will affect positively (negatively) stock prices. Equation (14) shows that the multiplier depends also on the AR coefficients in the expectations of the required return and the dividend growth rate. High ρ and ϕ imply that shocks to g_t and r_t will have a persistent effect on the multiplier and on prices. Analogously to the static case, the multiplier can be interpreted as the PD ratio: in this case the forcing variables, ex-ante returns and dividend growth rate, determine the dynamics of the ratio. The required rate of return is unobserved and many variables have been used as proxies. Campbell and Shiller (1989) used different notions of required returns: the risk-free interest rate plus a constant risk premium, the expected growth of real consumption times the coefficient of relative risk aversion plus a constant risk premium and another version in which the risk-free rate is constant and the risk premium is given by the conditional volatility of stock returns times the coefficient of relative risk aversion.

The extension to the dynamic Gordon model takes into account the possibility that time variation in interest rates, risk premia or growth rates could explain the large deviations of the PD ratio from its mean. The top plot in Figure (1) shows the PD ratio for yearly data from 1871 to 2001 of the S&P500 index[1].

Figure (1)

It is clear that the static Gordon model is rejected by the large and persistent deviations of the ratio from its mean. It is also striking how the PD ratio increased during the 90s: while it has historically oscillated between approximately 10 to 35, after 1995 it exceeded this range to reach levels as high as 85. This is also apparent in the bottom plot of Figure (1) that shows the log of the real stock price and the log of the fundamental value.

It makes then sense to use the dynamic version of the Gordon formula in order to explain the large deviations by changes occurred in fundamentals. Figure (2) shows the time series properties of the dividend growth rate, the real riskless interest rate and the yearly volatility measured by the average squared monthly returns. The autocorrelation plots show that at yearly frequency only the interest rate has some significant linear dependence whereas both the growth rate of dividends and the volatility of the stock returns have no significant dependence. In addition, the autocorrelation in the riskless rate is quite small to be able to explain the large deviations of the stock price from the fundamental price. The last column of Figure (2) depicts the multiplier (equivalent to the PD ratio) in Equation (14) when the dividend growth rate or the required

[1] The dataset used is described in Shiller (1989). It consists of yearly observations of the price and dividends for the S&P500 Composite Stock Price Index from 1871 until 2001. We deflated the series by CPI index. The interest rate used is the return on four to six months commercial paper.

rate are allowed to vary. We follow the approach of Campbell and Shiller (1989) and use the risk-free rate (plus a constant risk premium) and the stock return volatility as proxies for ex-ante returns. In all cases, the multipliers do not have the persistence and variability displayed by the PD ratio in Figure (1).

Figure (2)

The evidence discussed here and the results of Campbell and Shiller (1989) and, more recently by Zhong *et al.* (2002), suggest that the fundamental factors should have high persistence to explain stock prices. Barsky and de Long (1993) assume that prices are formed according to Equation (6) with the dividend growth rate following an ARIMA(0,1,1). This process contains a unit root and gives more persistence to the warranted fundamental value. However, there is no empirical evidence to support the assumption of a unit root in the dividend growth rate. Bansal and Lundblad (2002) provide evidence that at monthly frequency an ARMA(1,1) process has quite large AR and MA coefficients. However, Figure (2) suggests that for the yearly data analyzed here there is no evidence of statistical significance of an ARMA specification.

These results point to the fact that fundamental factors are not able to give a full account of the dynamics of stock prices. On the other hand, the failure of rational valuation could be caused by misspecification of the fundamental process. This issue has been investigated by Donaldson and Kamstra (1996) in order to give a rational explanation for the bubble occurred in 1929. They used monthly data and simulated paths from Equation (5) assuming that the discount factor included a nonlinear component and ARCH innovations. In this way, they allow for time variation in the fundamental factors without relying on the approximated pricing formula. They found that the stock price run-up and crash of 1929 was not caused by a bubble but it could be rationalized by considering nonlinear effects and heteroscedasticity in the discount factor and the growth rate of dividends. However, the results are based on the findings of significant linear structure in the monthly growth rate of dividends. As it is clear from Figure (2), the yearly data do not show significant evidence of ARMA dependence. Hence, it is unlikely that their method would perform successfully on the data analyzed here.

In the literature, there are two alternative interpretations to explain this failure: rational bubbles and irrational fads. The deviations are associated with rational bubbles when they satisfy Equation (1), while irrational fads do not. In both cases, prices are decomposed into a permanent (or fundamental) and a transitory (or non-fundamental) component

$$P_t = P_t^* + X_t, \tag{15}$$

where P_t^* is as in Equation (5) or (10) and X_t represents the deviations from the fundamentals.

To be consistent with a rational bubble model X_t has to satisfy the condition

$$X_t = (1 + r)^{-1} E_t(X_{t+1}), \qquad (16)$$

where, to simplify notation, we focus on the simple static Gordon case. This is a more general solution to Equation (2) because it does not require the transversality condition. The characteristic of a bubble is that it grows indefinitely at rate $(1 + r) > 1$. Blanchard and Watson (1982) proposed a model in which the bubble switches between two states, one in which the bubble survives ($X_t > 0$) with probability q and one in which it collapses ($X_t = 0$) with probability $1 - q$. A further refinement is the periodically bursting bubble model proposed by Evans (1991). In this model, the bubble grows faster than the rate $(1 + r)$ if X_t is below a positive threshold while beyond it has a positive probability to burst. Many tests for the existence of bubbles in asset prices or exchange rates were proposed. For a survey see Flood and Hodrick (1990). The results based on data until the beginning of the 90s did not show evidence to support the hypothesis of rational bubbles in asset prices.

An alternative explanation for the transitory component X_t is that it represents an irrational fad in investors sentiment that causes temporary deviations from fundamental valuations. This approach has been proposed by Summers (1986) and Poterba and Summers (1988). The assumption that X_t is a persistent stationary process is consistent with the evidence of mean reversion and long-term predictability in stock prices. Mean reversion was investigated mainly by using variance ratio tests that showed that stock prices do not follow a random walk because the variance of returns over k periods is significantly lower than k times the variance of one period return. In addition, long-horizon returns are negatively related to measures of deviations from the fundamentals, such as the PD ratio.

However, few attempts have been made to explicitly model the transitory component and investigate the possibility that it evolves in a nonlinear fashion. In the next section we introduce a nonlinear model to explain the time variation in the mean reversion of stock prices to the fundamental value.

3 Nonlinear Dynamics

A simple way to consider nonlinear effects consists of a smooth (nonlinear) transition between 2 linear regimes. The model is called STAR (Smooth Transition AR)[2] and assumes that the process X_t for the transitory component in stock prices evolves as

[2]We largely simplify the discussion of STAR models according to the application at hand. For a more detailed discussion of this family of models see Teräsvirta (1994) and van Dijk *et al.* (2002).

$$X_t = \{\phi_1' G_t(S_t, \gamma, c) + \phi_2'[1 - G_t(S_t, \gamma, c)]\} \mathbf{X}_{t-1} + \epsilon_t \tag{17}$$

where $\mathbf{X}_{t-1} = (1, X_{t-1}, ..., X_{t-p})'$ and the disturbance term ϵ_t is *i.i.d.* with constant variance σ^2. $G_t(S_t, \gamma, c)$ is the function that regulates the transition from the first regime, with coefficient vector ϕ_1, to the second regime, where the dynamics evolves according to ϕ_2. S_t is the variable that determines the switch between regimes. In the application in the next section we use $S_t = X_{t-d}$ for $d \geq 1$. Two common choices of $G_t(S_t, \gamma, c)$ are the logistic and the exponential function. The logistic version of the STAR model (called in the literature LSTAR) has transition function

$$G_t(S_t, \gamma, c) = \{1 + \exp[-\gamma(S_t - c)]\}^{-1}, \tag{18}$$

where $\gamma > 0$ determines the speed of transition and the threshold c determines the regime that is active. The logistic function varies smoothly from 0 to 1 as the transition variable, S_t, becomes increasingly larger than the threshold c. Another common choice for the transition function is the exponential (and the model is termed ESTAR), given by

$$G_t(S_t, \gamma, c) = 1 - \exp[-\gamma(S_t - c)^2] \tag{19}$$

In this case the transition function smoothly approaches 1, the further S_t deviates (in either directions) from the threshold value c.

These transition functions imply different dynamics for the process of mean reversion: the logistic is characterized by an asymmetric adjustment of X_t to its past values depending on the transition variable, S_t, being above or below the threshold c. In contrast, the exponential implies a symmetric adjustment in both directions of $(S_t - c)$. In other words, when using the logistic function we assume that negative and positive deviations revert back to the fundamentals at different speeds, whereas using the exponential the speed of mean reversion is equal for negative and positive deviations. The choice of the transition function is a crucial issue for the interpretation of the results. We will test which type of transition seems to accommodate better the dynamics in the deviations of stock prices from the fundamental value.

The null hypothesis of linearity against STAR holds if either $H_0 : \phi_1 = \phi_2$ or $H_0' : \gamma = 0$. As discussed more extensively in Teräsvirta (1994), under both null hypotheses the test statistics are affected by the presence of nuisance parameters that complicate the derivation of the asymptotic distribution. In order to overcome this identification problem, Luukkonen *et al.* (1988) proposed to approximate the transition function $G_t(S_t, \gamma, c)$ with a Taylor-expansion around $\gamma = 0$. This allows to derive an LM type statistic with a standard χ^2 distribution. A 2^{nd} order Taylor-series expansion of the exponential transition function around $\gamma = 0$, leads to the auxiliary regression

$$x_t = \beta_0' \mathbf{X}_{t-1} + \beta_1' \tilde{\mathbf{X}}_{t-1} S_t + \beta_2' \tilde{\mathbf{X}}_{t-1} S_t^2 + \beta_3' \tilde{\mathbf{X}}_{t-1} S_t^3 + \beta_4' \tilde{\mathbf{X}}_{t-1} S_t^4 + e_t \tag{20}$$

where $\tilde{\mathbf{X}}_{t-1} = (X_{t-1}, ..., X_{t-p})'$ and the β_j are reparametrizations of the vector of parameters $(\phi'_1, \phi'_2, \gamma, c)'$. The null hypothesis that $\gamma = 0$ against ESTAR corresponds to test that $H_0 : \beta_1 = \beta_2 = \beta_3 = \beta_4 = 0$. Similarly, a 3^{rd} order expansion of the logistic function involves only the first four elements of the RHS of Equation (20) and the null of linearity can be tested as $H_0 : \beta_1 = \beta_2 = \beta_3 = 0$. The artificial regression in Equation (20) could also be used to guide the specification of the transition function. The reparametrizations of the expansion of the logistic function imply that the null holds if $\beta_1 = 0$ and $\beta_3 = 0$, whereas the expansion of the exponential function under the null involves only the second order term, that is, $\beta_2 = \beta_4 = 0$. We can design the following null hypotheses in order to test for evidence of STAR dynamics and the type of transition function that is more appropriate. The null hypotheses are

$(LM_4) \qquad H_0 : \beta_1 = \beta_2 = \beta_3 = \beta_4 = 0$

$(LM_3) \qquad H_0 : \beta_1 = \beta_2 = \beta_3 = 0 | \beta_4 = 0$

$(H_{0,L}) \qquad H_0 : \beta_1 = \beta_3 = 0$

$(H_{0,E}) \qquad H_0 : \beta_2 = \beta_4 = 0$

LM_4 and LM_3 are used as general tests of linearity against STAR dynamics. Instead, rejection of $H_{0,L}$ suggests that a logistic transition should be preferred while rejection of $H_{0,E}$ points to an exponential specification. The testing procedure is conditional on the lag d used for the transition variable. By testing for different values of d, the tests are also useful in the selection of the optimal lag for the transition variable. In order to robustify inference in small samples we will use the F-version of the tests. A relevant issue in the implementation of these models is the choice of p, the order of the AR regimes. We follow the approach of Teräsvirta (1994) by looking at the PACF and the order selected by AIC.

4 Estimation Results

We use the static Gordon valuation as our notion of fundamental value for the yearly S&P500 Index data described earlier. From Figures (1) and (2) it is clear that considering the dynamic PVM would not change significantly the dynamics in the transitory component of stock prices. We estimate the STAR model to the deviations of the price from the fundamental value scaled by the dividends, that is, we define

$$X_t = \frac{P_t - P_t^*}{D_t}. \tag{21}$$

Using the static Gordon model in Equation (6), X_t is equivalent to the deviation of the PD ratio from the multiplier m. In what follows we analyze the time series both in the full sample and in the sub-sample from 1871 to 1990. This seems a natural choice because the late 90s might have changed dramatically the time series properties and the mean reversion dynamics of the deviations from the fundamentals.

Figure (3) shows the time series of the PD ratio, X_t, and the PACF in the restricted and the full sample. In the subperiod, the PACF suggests that there is dependence up to lag 3. This is also confirmed by the AIC selection criterion. However, in the full sample the dependence in the third lag is not significant and $p = 2$ seems appropriate.

Figure (3)

First, we tested for linearity of the time series of the transitory component against a STAR alternative. The p-values of the tests described in the previous section are given in Table (1).

Table (1)

In the sample up to 1990 the LM_3 and LM_4 tests reject at 5% significance level the null hypothesis of linearity. For both tests the rejection occurs in the 4^{th} lag of the transition variable. The rejection for LM_4 is stronger than for the other test and it is evidence in favour of the choice of the exponential as transition function. More insights about the specification of the transition function come from the $H_{0,L}$ and $H_{0,E}$ tests. For both tests we reject the null of linearity: for $H_{0,L}$ the third and seventh lag have p-value 0.02 and for $H_{0,E}$ the lowest p-value is 0.04 in the fourth lag. Thus, contrary to the previous more general tests, the rejections favour the logistic specification.

The pattern of rejections changes dramatically when the sample is extended to include the last 10 years. Both LM_3 and LM_4 reject for d up to lag 8. The test for logistic transition, $H_{0,L}$, rejects strongly from the second lag up to the fifth and also in the eight and tenth lag. Instead, $H_{0,E}$ rejects on the forth lag and in the seventh lag. These results might be explained by the run-up of the late nineties that attributes a higher weight to one tail of the distribution and gives more support to a logistic specification. As discussed in detail in Teräsvirta (1994), LSTAR and ESTAR are to some extent substitutes. This might happen when an ESTAR model has most of the observations lying in one side of the threshold such that it can be reasonably approximated by an LSTAR specification.

The tests for linearity suggest that there is evidence to reject the null hypothesis both in the period 1871-1990 and in the full sample until 2001. The evidence about the specification of the transition function is mixed: there seems to be support for both specifications up to 1990 while the full sample favours more clearly the logistic function. Given the mixed evidence of the tests, we chose the best specification by estimating and selecting the models based on the

AIC selection criteria for the period 1871-1990. The best model is an ESTAR specification with $d = 4$ as was also found by the tests. This result suggests that there is no evidence of asymmetry in the adjustment process of the stock price toward its long-run equilibrium. This confirms the evidence of Kilian and Taylor (2002) and Taylor and Peel (2000) that estimated ESTAR models to exchange rates. It is also consistent with the results of Gallagher and Taylor (2001) that used an ESTAR model to quarterly stock prices data from 1926 to 1997. However, the late 90s can be interpreted as evidence that positive deviations might have become more persistent than negative. A definitive answer to this issue requires to observe the evolution of stock prices in the following years to conclude that there has been a break in the symmetry of the adjustment process.

We use the same specification both in the estimation for the subsample and in the full sample. In this way we can interpret the changes that might occur in the estimation results. The estimation results are shown in Table (2). We performed a grid search for γ and c to initialize the NLLS estimation procedure. When a coefficient was not significant we dropped it from the regression and fitted the reduced model.

Table (2)

In the sample 1871-1990 the estimated coefficients are statistically significant and the residuals diagnostics proposed in Eitrheim and Teräsvirta (1996) do not show significant model misspecification. The mid-regime is characterized by an AR(3) with modulus 0.962, that is, very persistent and close to a unit root. The outer regime is a stationary process with dependence only in the first lag. The estimated model suggests that the dynamics of the deviations from the fundamentals is characterized by a very persistent process in the inner regime that drives the price away from the fundamental value; when the deviations get large the outer regime is activated and the process mean revert to the fundamental value. The outer regime has an AR(1) coefficient of 0.794. When this regime is completely active the half-life of a shock is approximately 3 years. It is interesting to analyze the behaviour of the transition function $G_t(X_{t-4}, \gamma, c)$ in Figure (4). The top plot shows the evolution over time of $G_t(\cdot)$. It is clear that there are wide variations in the transition function but the external regime is never completely active.

Figure (4)

This is due to the small estimated value of γ that implies a very smooth transition clear in the bottom plot of Figure (4). These results suggest that the speed of mean reversion of the transitory component (toward its mean) varies over time and depends in a nonlinear fashion on the magnitude of the deviation. They can interpreted as evidence that investors are uncertain about the direction of the stock market when the price is close to the fundamentals while they become increasingly concerned of the irrational mispricing when the transitory component becomes large. Similar results where also found in the exchange rate literature by Kilian and Taylor (2002) that show that a linear mean reverting process is not consistent with the findings of long-horizon predictability.

The estimation results of the previous model specification for the sample 1871-2001 are shown in Table (2). The LM_{NL} adequacy test rejects the null hypothesis of no further nonlinearity in the data. This is to expect given the pattern of strong rejections in the linearity tests and the fact that the model was selected on the shorter sample. The tests for residuals autocorrelation do not reject at 10% significance level but they are much lower than in the shorter sample. Also the test for parameter constancy, LM_{PC}, has a much lower p-value. The results for the mid-regime confirm the previous findings. The coefficients of the AR(3) process have very similar magnitude and the modulus is 0.971. However, the estimated coefficients of the outer regime is 0.989 while before it was 0.794. The interpretation of this result is clear: to accommodate for the price behaviour after 1995 the model has to allow for more persistence in the outer regime. Figure (5) shows the transition function for the full sample estimation. It is quite similar to the plot in the shorter sample. However, the upper limit of 1 is reached in the last years of the sample, suggesting that for those years the transitory component had a random walk type of dynamics.

Figure (5)

Contrary to the results in the sub-sample, the outer regime is also close to have a unit root. This highlights the fact that the late 90s represented a discontinuity in the time series properties of the data. While until the 90s large deviations were adjusted by the activation of a stabilizing regime, the sample until 2001 is heavily characterized by the unprecedented run-up in stock prices. The persistence found in the outer regime is probably related to the fact that the sample stops in 2001. If stock prices are mean reverting, the addition of more observations in the coming years would allow us to test if a structural break has occured in the adjustment pattern of stock prices.

5 Conclusion

It is a well documented fact that rational valuation models are not able to account for the dynamics of stock prices that are too volatile and take long swings away from intrinsic valuations. As we showed in this paper, allowing for time variation in the discount rate and in the dividends growth rate does not improve significantly the explanatory power of the PVM presented in

Section (2). The deviations of stock prices from the fundamental value are much more persistent than warranted by the factors that are assumed to determine the asset price dynamics.

An explanation proposed by Summers (1986) is that stock prices contain a temporary component associated with the sentiment of investors. When investors observe positive (negative) news about the fundamentals of an asset they expect the effect on the stock price to be more persistent than it is rational. This implies that shocks to stock prices are more persistent than warranted by shocks to fundamental factors as it is clear from Figure (2). In Summers (1986) and Poterba and Summers (1988) it is assumed that the transitory component follow a persistent AR process while the fundamental value evolves according to a random walk. This model implies that stock returns have small negative autocorrelations at short-horizons while they become large and negative at long-horizons. In other words, they display the same type of mean reversion that was found for various assets, such as stocks and exchange rates.

In this paper we show that the assumption of a linear process for the deviations from the fundamentals is inappropriate. The transitory component is better explained by a nonlinear model that behaves like a unit root process when prices are close to the intrinsic value and follow a stable AR process when the the deviations are large. This model implies that the speed at which stock prices revert towards the fundamentals is higher when deviations are large. This could be the result of the arbitraging activities of smart investors that act to correct the mispricing of a stock. When the stock price is farther away from the fundamentals, they will act more aggressively to correct the deviation that will cause the adjustment toward the mean.

However, this explanation seems not appropriate to explain the rapid stock price run-up of the late 90s. The stabilizing role of the outer regime has significantly lowered and there is weak evidence of mean reversion. After 1996, instead of experiencing fast adjustment toward the mean the stock price continued to deviate from the intrinsic value until 2000 when it started to correct downward. This fact is at odd with the previous interpretation of the role of rational arbitrageurs that cause the stock price to revert back. Probably, in the late 90s the irrational expectations of a majority of investors about the persistence of stock prices prevailed on the stabilizing role of rational agents and drove the transitory component to unprecedented levels. A situation that could be probably associated with a bubble in stock prices.

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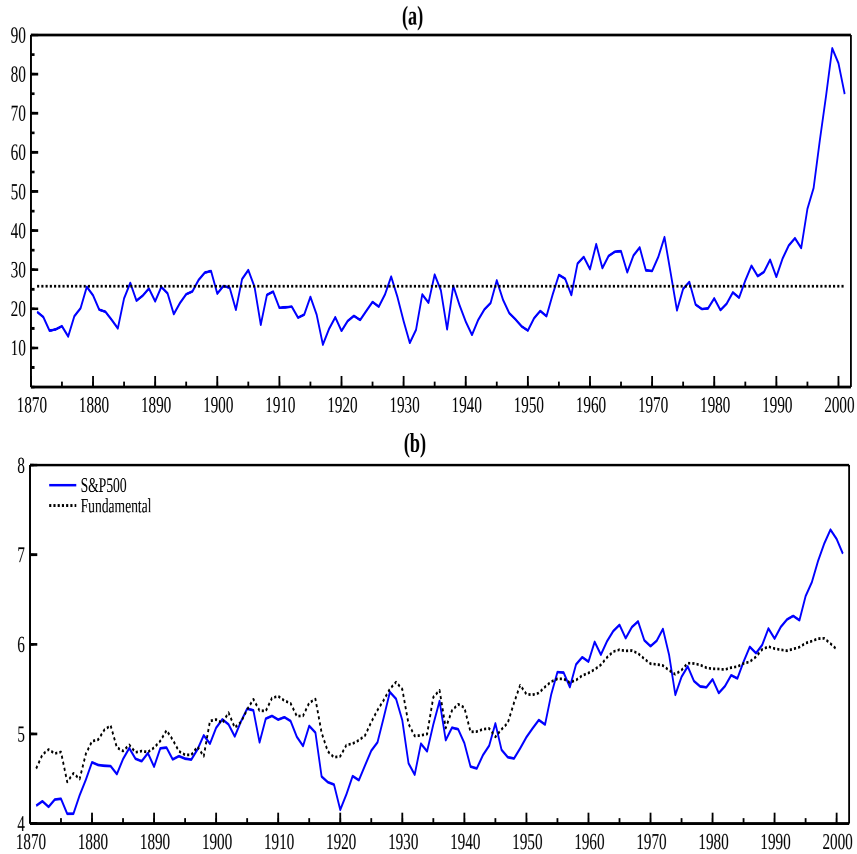
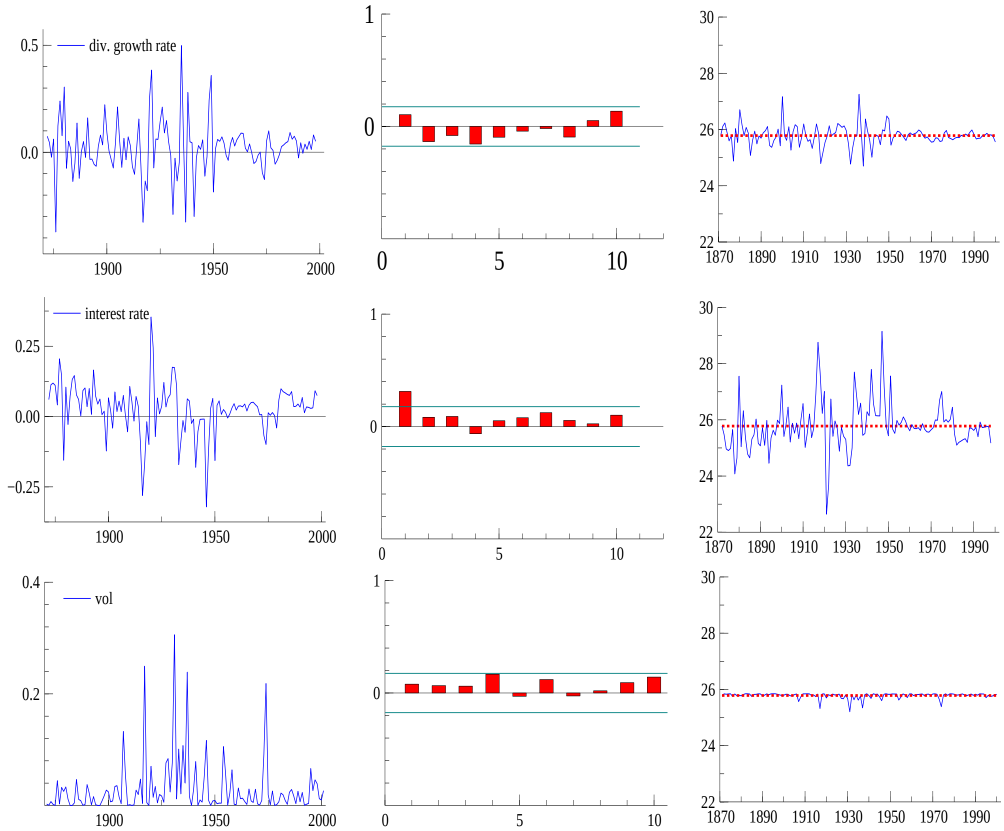
Figure 1: **PD ratio**



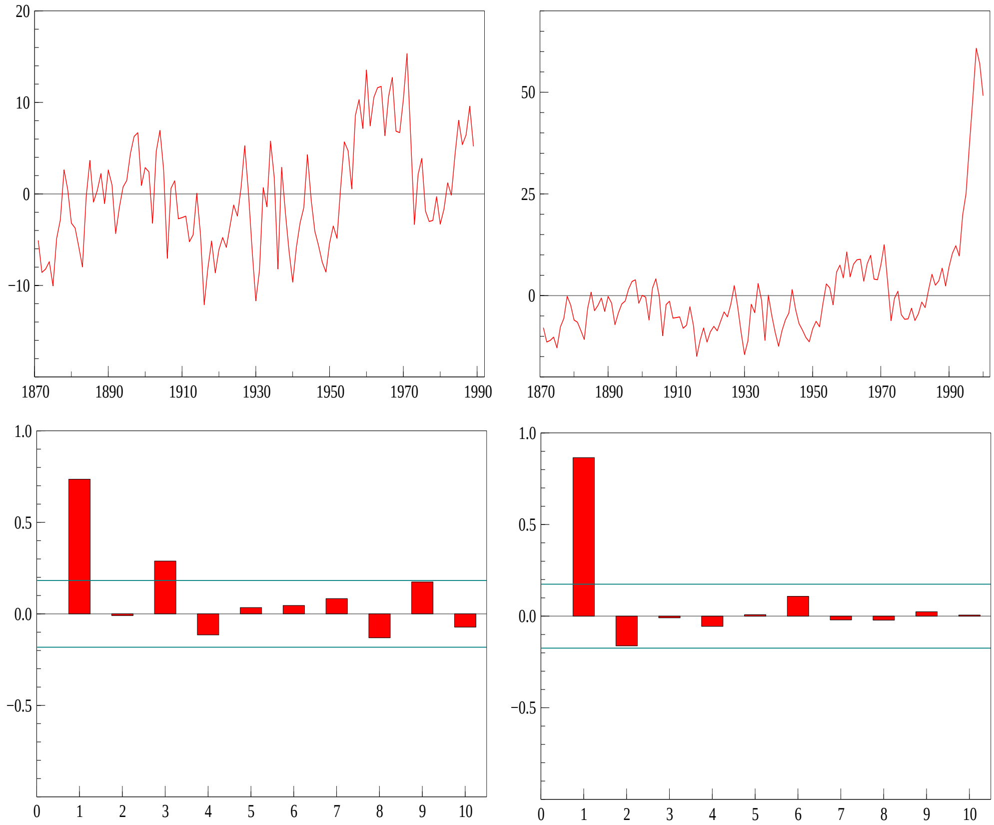
(a) Price-Dividend ratio for the S&P500 Composite Index from 1871 to 2001. The line indicates the average PD ratio of 25.78. *(b)* Log of the stock price and the static PVM fundamental value. The multiplier is obtained by assuming $g = 0.018$ and $r = 0.057$.

Figure 2: **Fundamental Factors**



Plot of the time series (left), autocorrelation function (center) and multiplier as given in Equation (14) when the dynamic PVM includes time varying dividends growth rates, real interest rate and stock return volatility.

Figure 3: **Deviations from the Fundamental**



Time series of X_t, the deviation from the fundamentals scaled by the dividends. It can also be interpreted as the deviation of the PD ratio from its multiplier. The bottom plots are the PACF up to lag 10.

Table 1: **Linearity Tests**

d	LM_3	LM_4	$H_{0,L}$	$H_{0,E}$	LM_3	LM_4	$H_{0,L}$	$H_{0,E}$
		1871 - 1990				**1871 - 2001**		
1	0.53	0.33	0.39	0.39	**0.00**	**0.00**	0.08	0.42
2	0.11	0.11	0.06	0.28	**0.00**	**0.00**	**0.02**	0.17
3	0.12	0.09	**0.02**	0.39	**0.00**	**0.00**	**0.01**	0.55
4	**0.03**	**0.01**	0.08	**0.04**	**0.00**	**0.00**	**0.00**	**0.04**
5	0.48	0.57	0.41	0.78	**0.00**	**0.00**	**0.01**	0.62
6	0.21	0.36	0.92	0.08	**0.01**	**0.02**	0.18	0.52
7	0.12	0.08	**0.02**	0.27	**0.01**	**0.00**	0.11	**0.02**
8	0.92	0.92	0.90	0.79	**0.01**	**0.00**	**0.00**	0.20
9	0.50	0.68	0.60	0.65	**0.03**	0.10	0.07	0.42
10	0.07	0.09	0.27	0.06	**0.01**	**0.01**	**0.01**	0.04

The tests are described in Section (3) and are applied to X_t, the deviations of stock prices from the static PVM. The autoregressive order, p, was set to 3. The tests are implemented as F-tests. In bold the p-values that are smaller than 5% significance level.

Table 2: **ESTAR Estimation**

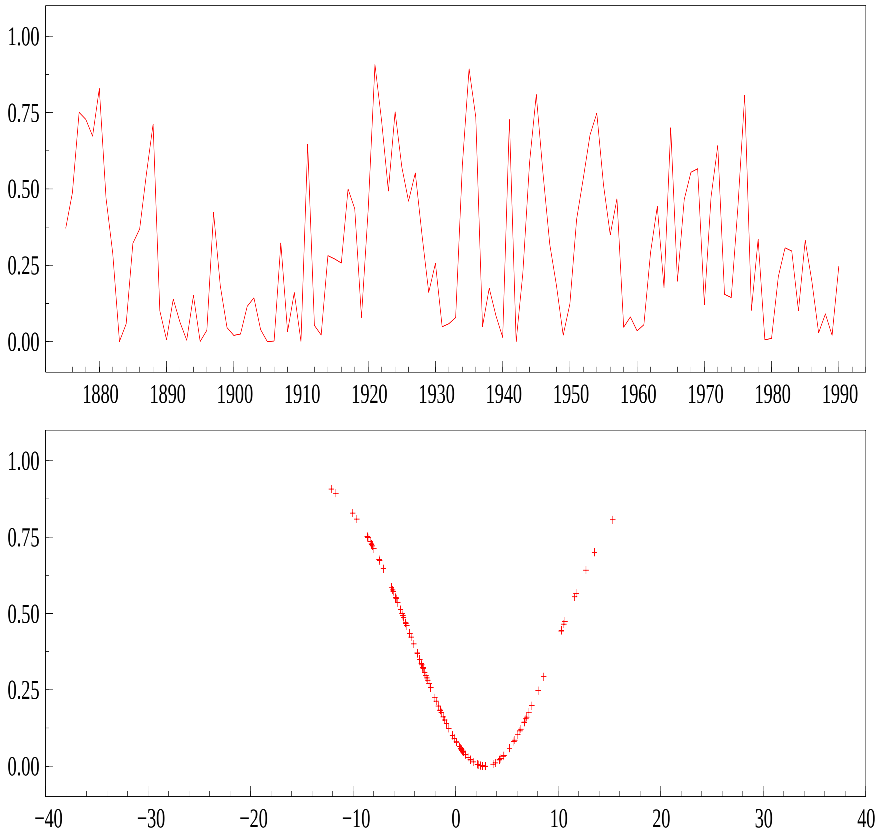
	1871-1990		1871-2001	
	$G(\cdot)=0$	$G(\cdot)=1$	$G(\cdot)=0$	$G(\cdot)=1$
$\phi_{0,i}$	-1.053	2.265	-1.152	3.033
	[-2.175]	[1.368]	[-1.538]	[2.094]
$\phi_{1,i}$	0.712	0.794	0.738	0.989
	[5.193]	[4.869]	[4.951]	[13.51]
$\phi_{2,i}$	-0.311		-0.308	
	[-2.135]		[-1.97]	
$\phi_{3,i}$	0.531		0.52	
	[4.758]		[4.137]	
γ	0.35		2.08	
	[3.364]		[12.8]	
c	2.87		-1.421	
	[2.03]		[2.21]	

1871-1990: R^2 =0.619 , AIC = 2.637, $\hat{\sigma}$ = 3.645,AR(1) = 0.34,
AR(4) = 0.565, LM$_{NL}$ =0.276, LM$_{PC}$=0.892

1871-2001: R^2 = 0.881, AIC =2.929, $\hat{\sigma}$ = 4.228, AR(1) = 0.11,
AR(4) = 0.21, LM$_{NL}$ = 0.01, LM$_{PC}$ = 0.16

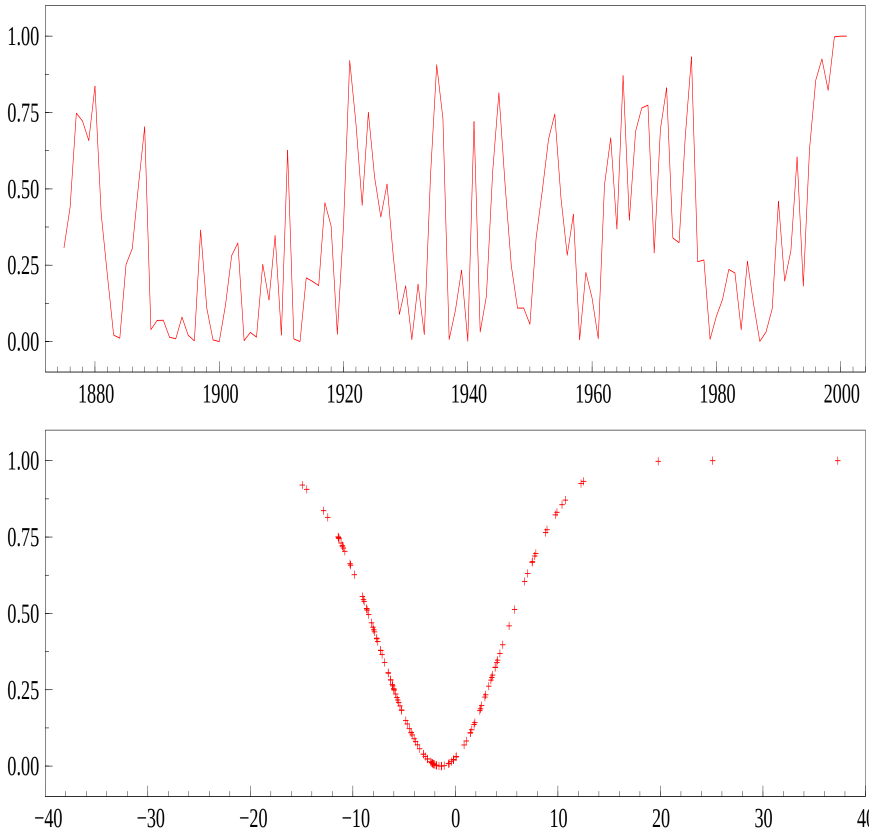
Estimation results for the sample period 1871-1990 and the full sample; the t-values in parenthesis are obtained by Newey-West variance-covariance estimator. The adequacy tests are as in Eitrheim and Terasvirta (1996): $AR(q)$ is a test for residual serial independence of order q, LM_{NL} tests for no remaining nonlinearity and LM_{PC} is a test for parameter constancy.

Figure 4: **Transition Function: 1871-1990**



Transition function $G_t(X_{t-4}, \hat{\gamma}, \hat{c})$ plotted in time and against X_{t-4} for the period 1871-1990.

Figure 5: **Transition Function: 1871-2001**



Transition function $G_t(X_{t-4}, \gamma, c)$ plotted in time and against X_{t-4} for the period 1871-2001.