Evaluating Policy Feedback Rules using the Joint Density Function of a Stochastic Model

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Abstract
This paper argues that the dominant practise of evaluating the properties of feedback rules in stochastic models using marginal distributions of the variables of interest implies the loss of considerable information. It argues that it is both practical and important to base decisions on the full joint density function. This argument is illustrated by comparing the properties of three rules applied to a large stochastic model under rational expectations.

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1. Introduction

There has been an enormous literature in recent years examining the properties of various policy feedback rules in a stochastic environment. Here we list only a few key references, Taylor 1993, 1999, Svenson 1997, Woodford 1999, Svenson and Woodford,2002, Orphanides and Williams 2002, Fuhrer 1997, Giannoni 2002. One almost universal feature of this literature is that where an author considers a stochastic model decisions are based on the marginal distribution of the endogenous variables and the joint distribution is ignored. That is to say we might look at the variance of output and the variance of inflation but we would not consider the joint distribution of output and inflation.

In this paper we argue a conceptually very simple point. The marginal distribution conceals a great deal of interesting information which is contained in the full joint distribution. Very often in standard analysis we are forced to assign largely arbitrary weights to policy objectives so as to trade them of against each other. However when we understand the full joint distribution a policy maker is presented with a much richer information set about the possible outcomes and often this may make the choice of an appropriate policy rule much easier.

The next section then sets out a framework which contrasts the information in the marginal and joint distribution. Section 3 then shows how this information can be used in a practical investigation of the National Institute's NIGEM model under three alternative policy rules. In the past this model has been used extensively to investigate alternative policy rule structures. Barrell and Dury (2000a and b) discuss alternative frameworks for monetary policy and contrast inflation based rules with nominal aggregate and price level based rules. Barrell and Hurst (2003) use the model to investigate fiscal policy options, as do Barrell and Pina (2004). Our results extend the analysis in these papers. Finally section 4 concludes.

2. A Formal Framework

When we consider the above referenced literature on monetary rules and stabilisation policy the essence of the problem is that we are attempting to evaluate a density subject to certain assumptions about either the structure or the parameterisation of a particular rule. Formally this amounts to analysing how probability forecasts for a set of variables might change as the rule changes. So assume that we are concerned with the m-variable vector, $z_t=(z_{1t},z_{2t},\ldots,z_{mt})'$ and that the density is evaluated subject to a parametric model $M(\phi)$ where ϕ is the parameterisation of the model. We assume that this parameterisation is general enough so that any rules being considered can be viewed as specific parameterisations of the general model M. The possible different density functions produced by different values of ϕ can then be characterised by assuming that ϕ lies in a compact parameter space Φ. Then,

$$M(\phi) = \{ f(z_1, z_{2,} ... z_T; \Phi), \ \ \phi \in \Phi\} \qquad 1.$$

This may be factorised to consider various density function of interest (see Garrat, Lee, Pesaran and Shin(2001)), we might factorise it sequentially to give the conditional distribution of z_s conditional on all earlier z values, this would correspond to the density of a standard VAR analysis. We could decompose z into exogenous and endogenous variables (x and y) and derive the density function of the y variables jointly conditional on the x's. What is conventionally done in the monetary policy literature however is to calculate the marginal distribution of individual z's, that is if we define the marginal distribution of z_{it} to be $F(z_{it})$ then

$$F(z_{nt}) = \int_{-\infty}^{\infty} ... \int_{-\infty}^{\infty} f(z_1, z_2, ... z_T; \phi) dz_{ij} \qquad i = 1...m,\ j = 1...T,\ i, j \neq n, t \qquad 2.$$

If the z's are independent then

$$f(z_1, z_{2,} ... z_T; \phi) = \prod_{i=1}^{m} \prod_{j=1}^{T} F(z_{ij}) \qquad 3.$$

And the set of marginal distributions carries all the information which is of interest in the joint distribution. However if this independence does not hold then there is no simple relationship between the joint probability distribution and the set of marginal distributions. Indeed in the typical applications in the monetary rules literature it seems unlikely that say inflation and the output gap will be independent. Hence the concentration on marginal distributions may be significantly distorting the results.

It is however possible to define the probability of a particular joint set of events occurring. So suppose a joint event was defined as

$$\vartheta(z_1, z_2, ... z_T; \phi) < a \qquad 4.$$

Then we can define the probability of this joint event occurring by

$$\pi(a, \vartheta) = \Pr[\vartheta(z_1, z_2, ... z_T) < a \mid M(\Phi)] \qquad 5.$$

While the marginal distributions are normally what is calculated in the Monte Carlo analysis it is actually very straightforward to also calculate this joint probability from a Monte Carlo experiment.

3. Results

In order to illustrate the relevance of the general points made above we now use the National Institutes world model NIGEM in some stochastic simulations where we test 3 different rules for stabilising inflation and output in the European Monetary Union area. The first rule is discussed in Barrell and Dury (2000a) and reflects the framework adopted by the European Central Bank where there is a nominal target (in their case M3) and an inflation target. This combined policy of nominal aggregate and inflation rate targeting is parametrised with coefficients of .5 on nominal output and 0.75 on inflation[1] (Rule 1).

$$r_t = \gamma_1 (\log(P_t Y_t) - \log(P_t Y_t *)) + \gamma_2 (\Delta \log P_{t+j} - \Delta \log P_{t+j} *) \qquad 6:$$

This rule is contrasted to two different Taylor rules that feedback on the output gap and inflation. We may for our purposes suppress the intercept and write this as

$$r_t = \gamma_1 (\log(Y_t) - \log(Y_t *)) + \gamma_2 (\Delta \log P_{t+j} - \Delta \log P_{t+j} *) \qquad 7$$

[1] As the nominal target includes the price level as a controller the feedback on inflation does not need to exceed 1.0 for the model to be stable. See Barrell, Dury and Hurst (2001) for a discussion.

The Taylor Rule coefficients start with the industry standard 0.5 on output and 1.5 on inflation (Rule 2). Our alternative has an increased weight on GDP at $\gamma_1 = 1.5$ (rule 3) with that on inflation at 1.50.

These rules are then compared using stochastic simulations on NiGEM, which is a large, estimated and calibrated New Keynesian global model with around 300 core relationships describing 20 of the OECD countries (including all members of the Monetary Union) and their links with the rest of the world. NiGEM has model consistent rational expectations in financial markets (exchange rates, long term interest rates and equity prices) as well as in labour markets and consumption, and has an underlying CES production function in each country[2]. The stochastic simulations use bootstrapping to apply equation residuals recovered from 1991q1 to 1999q4 to the forecast baseline. All shocks from one time period are applied and the model is run with rational expectations in place, and this creates a new future history which forms the baseline for the next trial period. The process is repeated in sequence over 20 quarters with the application of a different randomly selected set of shocks from the past. Each sequence constitutes a replication, and each experiment has 150 replications[3]

We set up two targets which we wish to achieve, Inflation should be stabilised to within 0.3% of its target value and GDP should also be within 0.5% of its target value. Table 1 summarises information from the marginal and joint distributions in these trials and Tables 2 to 4 show both the full marginal and joint distributions for the outcomes under the different rules. The second and third columns of Tables 1 to 4 show the probability of meeting each of these targets individually. This is the mean of the two marginal distributions and this is what is usually calculated in the literature. The fourth column shows the probability of jointly meeting both targets at once; this is the mean of the joint probability of the targets being met. Column 1 shows the probability that both targets will fail, this is the mean of the joint probability that both targets are not met. Finally the last two columns define the part of the joint distribution where one target is met but the other is not (clearly when we examine the usual marginal distribution we have no information as to whether the other target is met or not).

Table 1 Summary Density Indicators

	Both fail	GDP pass	INF pass	Both Pass	INF only	GDP only
Combined Rule						
year 1	0.393	0.331	0.480	0.205	0.276	0.126
mean	0.496	0.247	0.356	0.100	0.256	0.147
Standard Taylor Rule						
year 1	0.393	0.333	0.450	0.176	0.274	0.157
mean	0.489	0.248	0.361	0.098	0.262	0.150
Output focussed Taylor Rule						
year 1	0.404	0.393	0.403	0.201	0.202	0.193
mean	0.452	0.311	0.352	0.115	0.237	0.196

Proportions –cols1+2+3-4 = 1, cols 1+4+5+6=1

[2] More details of the model can be found in Barrell et. al (2004)

[3] Barrell, Dury and Hurst (2003) discuss the procedure and demonstrate that only 120 replications are needed for the variances to settle.

We are now able to analyse the decision problem faced by the policy maker much more fully as we can now see the full joint distribution. Table 1 presents summary statistics and tables 2 to 4 present detailed quarterly results for the three experiments. For example we can see that for rule 1, in table 2, in the first period if we meet the GDP target we are almost certain to meet the inflation target as there is only a 6% probability of meeting the GDP target without also meeting the inflation target, as we can see from table 1. For the first year as a whole there is only a 13% chance of hitting the GDP target without also hitting the inflation target. In both cases this is better than the performance of the standard Taylor Rule. In the first period and in the first year the combined rule is more likely to see both targets met, as well as more likely to have inflation within the bounds we set. We can also see that when we increase the weight on GDP (rule 3 relative to rule 2) as you might expect the probability of meeting the GDP target rises and the inflation probability falls. However, less obviously the joint probability of meeting both targets actually rises quite substantially.

The decision maker is not simply being asked to make a trade off between hitting the inflation and GDP target she is also given the information that under this rule it is more likely that both targets will be hit. We can off course use this information to choose between the rules more systematically also. A policy maker might choose the rule that maximises the probability of meeting one target (inflation or GDP) or the joint probability of meeting both targets or even choosing the rule that minimises the probability of missing both targets. Naturally each of these assumptions may cause us to choose a different rule. So for example if we simply look at the marginal mean probability of meeting the GDP target in the first period or the first year this would cause us to choose rule 3, if we consider the marginal distribution for inflation we would choose rule 1. If this is all the information we have then we have to choose between the rules based simply on our relative weighting of inflation against GDP. But here we can also see that rule 1 also maximises the joint probability of hitting both targets and it minimises the joint probability of missing both targets. So the case for rule 1 becomes much stronger without having to apply an arbitrary weighting of inflation and unemployment.

4.Conclusion

In this paper we have argued that the joint probability distribution of a number of policy objectives contains considerable information which is unavailable if we only consider the marginal distributions as is the case with almost all current work. We have shown that it is actually quite easy to calculate the joint distribution and that in a practical example offers some important insights. In particular a rule chosen on the basis of the marginal distribution may not be chosen when the full density is investigated.

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Table 2. Results for the combined rule 1

Date	Both fail	GDP pass	INF pass	Both Pass	INF only	GDP only
200101	0.17045	0.47159	0.76704	0.40909	0.35795	0.0625
200102	0.34659	0.32386	0.55114	0.22159	0.32955	0.10227
200103	0.47727	0.28977	0.33523	0.10227	0.23295	0.1875
200104	0.57955	0.23864	0.26705	0.08523	0.18182	0.15341
200201	0.43182	0.30114	0.35227	0.08523	0.26705	0.21591
200202	0.46023	0.24432	0.36932	0.07386	0.29545	0.17045
200203	0.47159	0.22159	0.38068	0.07386	0.30682	0.14773
200204	0.48295	0.23295	0.38068	0.09659	0.28409	0.13636
200301	0.51705	0.26136	0.32386	0.10227	0.22159	0.15909
200302	0.57386	0.26705	0.20455	0.04545	0.15909	0.22159
200303	0.55114	0.1875	0.32955	0.06818	0.26136	0.11932
200304	0.52273	0.23864	0.30682	0.06818	0.23864	0.17045
200401	0.54545	0.19318	0.32955	0.06818	0.26136	0.125
200402	0.53977	0.20455	0.33523	0.07955	0.25568	0.125
200403	0.54545	0.20455	0.32386	0.07386	0.25	0.13068
200404	0.48295	0.24432	0.34091	0.06818	0.27273	0.17614
200501	0.50568	0.25	0.30682	0.0625	0.24432	0.1875
200502	0.56818	0.21591	0.30682	0.09091	0.21591	0.125
200503	0.52841	0.21591	0.31818	0.0625	0.25568	0.15341
200504	0.625	0.14205	0.29545	0.0625	0.23295	0.07955
mean	0.49631	0.24744	0.35625	0.1	0.25625	0.14744

Table 3: Results or the standard NIGEM Taylor rule (rule 2)

Date	Both fail	GDP pass	INF pass	Both Pass	INF only	GDP only
200101	0.2449	0.41837	0.66837	0.33163	0.33673	0.08673
200102	0.33163	0.33673	0.53571	0.20408	0.33163	0.13265
200103	0.47449	0.31633	0.28061	0.07143	0.20918	0.2449
200104	0.52041	0.2602	0.31633	0.09694	0.21939	0.16327
200201	0.44898	0.29082	0.37755	0.11735	0.2602	0.17347
200202	0.32143	0.27551	0.55612	0.15306	0.40306	0.12245
200203	0.44388	0.2551	0.43367	0.13265	0.30102	0.12245
200204	0.43878	0.25	0.42857	0.11735	0.31122	0.13265
200301	0.5102	0.20408	0.36224	0.07653	0.28571	0.12755
200302	0.56633	0.19898	0.27041	0.03571	0.23469	0.16327
200303	0.48469	0.2602	0.31633	0.06122	0.2551	0.19898
200304	0.51531	0.20918	0.34694	0.07143	0.27551	0.13776
200401	0.56633	0.22449	0.27551	0.06633	0.20918	0.15816
200402	0.60714	0.17347	0.2602	0.04082	0.21939	0.13265
200403	0.54592	0.22449	0.31122	0.08163	0.22959	0.14286
200404	0.58163	0.19898	0.27551	0.05612	0.21939	0.14286
200501	0.55102	0.21429	0.29592	0.06122	0.23469	0.15306
200502	0.55102	0.20918	0.29082	0.05102	0.2398	0.15816
200503	0.46429	0.2398	0.38265	0.08673	0.29592	0.15306
200504	0.61735	0.20918	0.22959	0.05612	0.17347	0.15306
mean	0.48929	0.24847	0.36071	0.09847	0.26224	0.15

Table 4 results for the Taylor rule with increased weight on GDP (Rule 3)

Date	Both fail	GDP pass	INF pass	Both Pass	INF only	GDP only
200101	0.20382	0.57325	0.6242	0.40127	0.22293	0.17197
200102	0.3121	0.43312	0.50955	0.25478	0.25478	0.17834
200103	0.50318	0.32484	0.26752	0.09554	0.17197	0.2293
200104	0.59873	0.24204	0.21019	0.05096	0.15924	0.19108
200201	0.4586	0.28662	0.36943	0.11465	0.25478	0.17197
200202	0.41401	0.28662	0.45223	0.15287	0.29936	0.13376
200203	0.40764	0.25478	0.45223	0.11465	0.33758	0.14013
200204	0.45223	0.24841	0.36306	0.06369	0.29936	0.18471
200301	0.49681	0.3121	0.27389	0.0828	0.19108	0.2293
200302	0.44586	0.29936	0.33758	0.0828	0.25478	0.21656
200303	0.49045	0.24841	0.29936	0.03822	0.26115	0.21019
200304	0.5414	0.27389	0.25478	0.07006	0.18471	0.20382
200401	0.42038	0.3121	0.33758	0.07006	0.26752	0.24204
200402	0.38853	0.33758	0.38853	0.11465	0.27389	0.22293
200403	0.50955	0.3121	0.29299	0.11465	0.17834	0.19745
200404	0.44586	0.33121	0.31847	0.09554	0.22293	0.23567
200501	0.43949	0.30573	0.36943	0.11465	0.25478	0.19108
200502	0.47134	0.28025	0.32484	0.07643	0.24841	0.20382
200503	0.54777	0.25478	0.27389	0.07643	0.19745	0.17834
200504	0.49681	0.30573	0.3121	0.11465	0.19745	0.19108
mean	0.45223	0.31115	0.35159	0.11497	0.23662	0.19618