Endogenous Redistributive Cycles

Christiane Clemens* Maik Heinemann†

February 28, 2004

Abstract

This paper discusses the emergence of endogenous redistributive cycles in a stochastic growth model with incomplete asset markets and heterogeneous agents, where agents vote on the degree of progressivity in the tax–transfer–scheme. We develop two models, the first being highly–stylized, where redistributive cycles occur in a simple majority voting process due to counter–acting effects from inequality aversion and prospects of upward mobility. The second model draws from Bénabou (1996) and ties the bias in the distribution of political power to the degree of inequality in the society, thereby triggering redistributive cycles which then give rise to a nonlinear, cyclical pattern of growth and savings rates over time.

Keywords: Income inequality, Hopf bifurcation, political cycles, redistribution

JEL-Classification: D31, E62, O41, P16

*University of Hannover, Economics Department, D–30 167 Hannover, Germany, e–mail: *clemens@vwl.uni-hannover.de*.

†University of Lüneburg, Institute of Economics, D–21 332 Lüneburg, Germany, e–mail: *maik.heinemann@uni-lueneburg.de*.

1 Introduction

In this paper, we present a model of stochastic growth with incomplete asset markets and heterogeneous agents, where redistributive cycles and cyclical growth emerge as an outcome of voting processes over public tax–transfer schemes. Our analysis is embedded in the framework developed by Bénabou (1996, 2000), which displays the convenient feature that the behavioral relationships between the macroeconomic variables are grounded in the intertemporal optimization decisions of single agents, while preserving analytical tractability and allowing for closed–form solutions of the income dynamics and the endogenously determined wealth distribution.

Individual income mobility is generated by the realization of idiosyncratic shocks in the presence of financial constraints (cf. Loury, 1981; Galor and Zeira, 1993; Piketty, 1997; Matsuyama, 2000). Our argument is based on two competing forces affecting the agents' voting behavior: On the one hand, redistribution provides an insurance against unfavorable outcomes and therefore is preferred by risk averse but, moreover, also by inequality averse agents, who favor equal societies with a comparably low degree of income mobility.[1] On the other hand, to the extent past incomes determine the current level of income and random income components are diversified, individual income mobility is limited. Consequently, an agent facing relatively small prospects of upward income mobility might be inclined to vote against redistribution in this situation.

We follow Bénabou (1996, 2000) in assuming that the political influence is unevenly distributed in the society and pressure groups have the power to enforce redistributive policies which are favorable to them. Yet, contrary to Benabou's approach, we do not fix the rank of the critical pressure group in the wealth distribution at an exogenous *ad hoc* level, but allow for endogenous shifts in the political bias by tying it to income inequality itself. Redistributive politics then affect growth via associated adjustments in the individual savings rates. Combined with a voting system, where the distribution of political power endogenously depends on the current state

[1] See for instance the empirical evidence by Schwarze and Härpfer (2002) on the relation between inequality aversion and the desire for redistribution from the German Socio–economic Panel Study for the years 1985 to 1998.

of wealth inequality, redistributive cycles trigger growth cycles, such that periods of high growth and a low degree of redistribution take turns with intervals of heavy redistribution and correspondingly low growth. In this context, the nonlinear pattern of savings and growth rates observed in our model stand in the tradition of the contributions of Kaldor (1940) Goodwin (1951), although it is important to stress that, here, cycles stem from voting processes over redistributive tax schemes instead of arising from imbalances between saving and investment. The focus on distributional conflicts further relates our work to the literature on political–economic equilibrium, income distribution, and growth; cf. Bertola (1993), Saint-Paul and Verdier (1993, 1997), Persson and Tabellini (1994), Alesina and Rodrik (1994), Perotti (1993, 1996), Piketty (1995), Benhabib and Rustichini (1996), Krusell *et al.* (1997), Krusell and Smith (1998) or more recently Harms and Zink (2002) and Plümper and Martin (2003).

The link between wealth inequality and the bias in political participation is established exogenously, but can be motivated in several ways: First of all, one might argue that the rich have advantages in building up pressure groups, for instance, by employing networks, whereas the poor are less organized. Additionally, it is easier for the rich to raise funds for lobbying activities. Empirical evidence suggests a comparably small degree of political participation in the lower income classes; see Bénabou (2000, and references therein). The low polling rates of the poor can be explained with the presence of opportunity costs, i. e. the poor are primarily concerned with earning their living, as well as with the presence of a certain apathy or frustration regarding the political process. Apart from this, the motivation to engage actively in the political process might be less pronounced in a relatively egalitarian society. Contrary, if the perceived extent of inequality becomes too large, inequality aversion might cause an increase in political participation of the lower income classes.

Altogether, this indicates a dynamic process, where the distribution of political power is shifting over time. Imagine that the bias in public decision–making is moving towards the poor, if the society is highly polarized and wealth inequality is large, while political power shifts towards the upper income classes, if redistribution becomes too equalizing.

The paper is organized as follows: In section 2 we present a short highly–stylized model of income dynamics to give a first intuition of the relevant factors at work in the emergence of endogenous redistributive cycles, without taking into consideration the underlying decisions of maximizing households. Section 3 then develops a framework where wealth dynamics and redistributive cycles are derived within an overlapping generations setting. Section 4 contrasts the outcomes of a simple majority voting mechanism, where preferences of the median voter determine policy outcomes, with biased majority voting in the spirit of Bénabou (1996, 2000) and the shifting bias of pressure groups representing the essential new feature of our model, which lead to redistributive cycles. Section 5 concludes.

2 Redistributive cycles in a stylized model of income dynamics

In this section we develop a simple, highly stylized model of income dynamics, where majority voting over redistributive policies to be implemented or abandoned gives rise to cyclical behavior in the distributional dynamics and the outcomes of the democratic process. The main purpose of this section is to give a first notion of how voting over redistributive schemes and distributional dynamics interact in the presence of (partly) uninsurable income risks and efficiency costs of redistribution. In order to keep the analysis as simple as possible, we focus on exogenous endowments and efficiency costs. Furthermore, we neglect issues of intertemporal optimization. These aspects will be discussed in more detail in the overlapping generations model of section 3.

We assume a continuum of individuals indexed by $i \in [0,1]$. Let $\ln y_t^i$ denote time t (log) income of individual i. We assume an exogenously generated process for individual income dynamics such that, given current income $\ln y_t^i$, the next period's income of individual i is:

$$\ln y_{t+1}^i = \beta \ln y_t^i + \ln u_t^i + \ln z_{t+1}^i \,. \tag{1}$$

Here, $\beta > 0$ is a coefficient measuring income mobility. The higher β, the more does the future income endowment depend on present income and the less it is determined by random effects. This introduces an element of inertia into the model, such that income mobility declines for larger β.

Both i. i. d. income shocks, $\ln u_t^i$ and $\ln z_{t+1}^i$, are assumed to be normally distributed across agents, i. e. $\ln u_t^i \sim \mathcal{N}(-\sigma_u^2/2, \sigma_u^2)$ and $\ln z_{t+1}^i \sim \mathcal{N}(-\sigma_z^2/2, \sigma_z^2)$, such that $\mathrm{E}[u_t] = \mathrm{E}[z_{t+1}] = 1$. The major difference between these two shocks is their date of realization. The underlying timing of shocks allows to explicitly take account of the insurance property of redistribution (cf. Varian, 1980). While $\ln u_t^i$ occurs before redistributive measures are effective, the second shock, $\ln z_{t+1}^i$, takes place after redistribution, by this remaining an uninsurable idiosyncratic risk.

Let us start with restating the well–known benchmark result on the asymptotic properties of the income distribution in the absence of redistributive efforts. If we additionally assume that the initial distribution of income is lognormal, i. e. $\ln y_t^i \sim \mathcal{N}(\mu_t, \sigma_t^2)$, the law of motion of the income distribution is entirely governed by the associated changes in μ and σ^2, given by:

$$\ln y_{t+1}^i \sim \mathcal{N}\left(\mu_{t+1}, \sigma_{t+1}^2\right), \qquad \begin{aligned} \mu_{t+1} &= \beta\,\mu_t - \frac{\sigma_u^2 + \sigma_z^2}{2} \\ \sigma_{t+1}^2 &= \beta^2\,\sigma_t^2 + \sigma_u^2 + \sigma_z^2 \end{aligned}$$

Starting from an arbitrary initial distribution of income endowments, this implies that the long–run the income distribution converges to a stationary lognormal distribution $\mathcal{N}(\mu_*, \sigma_*^2)$, where:

$$\mu_* = -\frac{\sigma_u^2 + \sigma_z^2}{2\,(1-\beta)}, \qquad \sigma_*^2 = \frac{\sigma_u^2 + \sigma_z^2}{1-\beta^2}\,. \tag{2}$$

Let us now briefly describe the income dynamics under redistribution. We assume a highly stylized redistributive scheme, where all agents end up with receiving the mean, $\mathrm{E}[\ln x_t^i] = \ln \bar{x}_t$, of the individual income components $\ln x_t^i = \beta \ln y_t^i + \ln u_t^i$, such that the risk stemming from the random variable u_t^i is perfectly pooled. Additionally, we assume that any redistributive activity induces costs in terms of a uniform income loss $B > 0$, which can be regarded as a wildcard representing efficiency losses and indicating the presence of the well–known tradeoff between equity and efficiency (cf. Mirrlees, 1971).[2]

The time $t+1$ net income of agent i, denoted by $\ln y_{t+1}^{i,R}$, can then be determined as the sum of post–redistribution mean income $\ln \bar{x}_{t+1} = \beta\mu_t +$

[2]In the more elaborated model of the subsequent sections, efficiency losses from redistribution occur in terms of lower growth rates due to distortionary taxation.

$\beta^2\sigma_t^2/2$ (including riskless and pooled income parts) and the uninsurable realization of the shock $\ln z_{t+1}^i$, less the costs B

$$\ln y_{t+1}^{i,R} = \ln \bar{x}_{t+1} - B + \ln z_{t+1}^i = \beta\mu_t + \beta^2\frac{\sigma_t^2}{2} - B + \ln z_{t+1}^i\,.$$

Given the assumptions stated on the distribution of the idiosyncratic random variable z^i the resulting income distribution in the society, too, is lognormal and the distributional dynamics are given as follows:

$$\text{for} \quad \ln y_{t+1}^{i,R} \sim \mathcal{N}(\mu_{t+1}^R, {\sigma^2}_{t+1}^R)\,, \qquad \begin{aligned} \mu_{t+1}^R &= \beta\mu_t + \beta^2\frac{\sigma_t^2}{2} - B - \frac{\sigma_z^2}{2} \\ {\sigma^2}_{t+1}^R &= \sigma_z^2 \end{aligned}$$

As becomes obvious, the extent of income inequality now is solely determined by the variance σ_z^2 of the non–diversifiable risk. The income distribution asymptotically converges towards $\mathcal{N}(\mu_{*,R}, \sigma_{*,R}^2)$, where:

$$\mu_* = -\frac{B + (1-\beta^2)\sigma_z^2}{2(1-\beta)}\,, \quad \text{and} \quad \sigma_{*,R}^2 = \sigma_z^2\,. \tag{3}$$

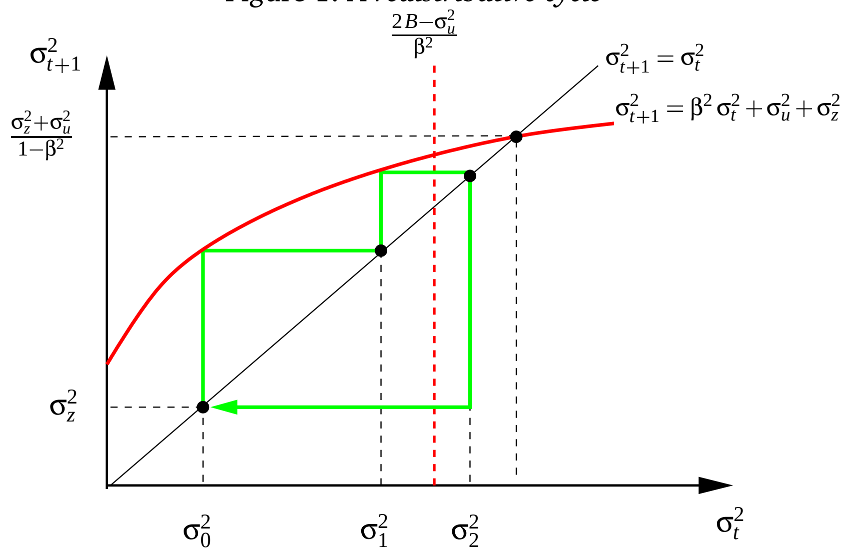
The agents decide in period t over the extent of redistribution in period $t+1$. In order to avoid strategic behavior over time in the political voting process, we assume agents to be myopic. The single agent opts for redistribution, whenever this promises an increase in the next period level of expected utility, which takes on the logarithmic form, $\mathrm{E}[U(y_{t+1}^i)] = \mathrm{E}[\ln y_{t+1}^i]$.

The underlying mechanism of public decision–making is the one of simple majority voting, where the agent m with median income $\ln y_t^m = \mu_t$ acts as a positional dictator. The median voter's preferences decide upon whether or not the redistributive scheme as described above is to be implemented, and she prefers redistribution over laissez–faire if and only if[3]

$$\begin{aligned} \mathrm{E}[U(y_{t+1}^{i,R})] > \mathrm{E}[U(y_{t+1}^m)] \quad &\Longrightarrow \quad \mathrm{E}[\ln y_{t+1}^{i,R}] > \mathrm{E}[\ln y_{t+1}^m] \\ &\Longleftrightarrow \quad \frac{1}{2}\left(\beta^2\sigma_t^2 + \sigma_u^2\right) > B\,. \end{aligned} \tag{4}$$

[3]The underlying redistributive scheme is discontinuous and does not allow for a varying amount of income to be redistributed. In this context, agents face an all–or–nothing decision, either to implement the sketched scheme, thereby perfectly pooling *ex ante* risk u_t^i, or to forgo with redistribution.

Figure 1: *A redistributive cycle*



The left hand side of equation (4) represents the benefits associated with redistribution: Since initial income inequality and at least some part of the income risk are eliminated by the redistributive policy considered here, the benefits of redistribution are larger, the higher σ_t^2 and σ_u. Recall that income mobility is measured by the coefficient β. The larger β, the more is the income of the following period determined by current income, meaning that income mobility is low. This means that agents face only a weak prospect of upward income mobility (cf. Bénabou and Ok, 2001), which also implies that the benefits of redistribution are comparatively large. Since the right hand side of equation (4) represents the costs associated with redistribution, the pivotal individual m prefers redistribution, whenever the gains outweigh the costs.[4]

Combined with the above derived dynamics of the income distribution with and without redistribution, this simple model might give rise to redistributive cycles. Such cycles are characterized by episodes with low income inequality and redistribution followed by episodes with high inequality and no redistribution. Whenever income inequality is low, the pivotal individual votes against redistribution, because the benefits are too small to cover the costs of redistribution. Consequently, the degree of income inequality

[4]From the follows immediately the standard result that the median voter always prefers redistributive policies whenever these do not raise additional costs.

increases over time as σ_t^2 approaches its stationary value determined by (2). This convergence towards σ_*^2 is only disturbed, if the extent of inequality rises above the level $\sigma_t^2 = (2B - \sigma_u^2)/\beta^2$, implicitly given by (4) and describing the threshold value for which the median voter prefers redistribution. Here, the gains from redistribution exceed the costs and the democratic process switches towards the support of redistributive measures. Due to the underlying redistributive scheme the extent of income inequality is solely determined by the non–diversifiable risk, i. e. the time invariant level σ_z^2 from (3). From here, the redistributive cycle is reinitiated.

Figure 1 illustrates this process of slowly increasing inequality in an economy without redistributive activities, taking turns with a sharp drop in the extent of inequality as the majority shifts towards redistribution. Starting from the initial level of a comparably low degree of inequality $\sigma_0^2 = \sigma_z^2$, the median voter decides against redistribution and inequality grows to the level of σ_1^2. The majority of voters still prefers laissez–faire at this degree of inequality, which then rises further towards σ_2^2. But now, we have the case of $\sigma_2^2 > (2B - \sigma_u^2)/\beta^2$, that is, the attained level of inequality exceeds the maximum level the pivotal agent is willing to accept without redistributive measures being effective. Inequality (4) holds and the median voter opts for redistribution, which then causes inequality to drop towards the long–run stationary level σ_z^2. After having reached this point, the cycle starts all over again.

3 Income dynamics and redistribution in an OG growth model

We will now extend our considerations to a model, where income and distributional dynamics are grounded in decisions of intertemporally optimizing agents. Compared to the stylized approach of the preceding section, the model developed here also displays the feature that redistribution is not costlessly available, but now costs in terms of efficiency losses are endogenously determined. We again posit two sources of randomness, once more differentiating between insurable income components and non–diversifiable risk. The two approaches also differ with respect to the underlying redistributive scheme. Now, depending on the outcome of the political process, the amount to be redistributed varies continuously, thereby allowing the agents to choose their preferred extent of redistribution. Growth and redistributive cycles occur, if we introduce deviations from simple majority voting and allow the political process to be biased.

Our analysis contributes to the strand of research explaining distributional dynamics as the outcome of stochastic processes in a dynastic context; see e. g. Becker and Tomes (1979, 1986) as well as Loury (1981). The underlying framework draws from Bénabou (1996, 2000).

The model The economy is again populated by a continuum of overlapping generations families, indexed by $i \in [0,1]$. Agents have preferences defined over their own consumption c_t^i, as well as their child's income y_{t+1}^i

$$u_t^i = \ln c_t^i + \gamma \ln y_{t+1}^i \, . \tag{5}$$

$\gamma > 0$ denotes the utility weight, the parents attach to their children's future income endowment. We disregard population growth, i. e. each agent has exactly one offspring. Additionally, we consider an incomplete market economy, where credit markets are missing. The income of agent i depends on her stock of (human or physical) capital k_t^i, the average stock of capital κ_t, as well as an idiosyncratic random component u_t^i:

$$y_t^i = u_t^i A \, (k_t^i)^{\beta} \kappa_t^{1-\beta} \, , \qquad \ln u_t^i \sim \mathcal{N}(-\sigma_u^2/2, \sigma_u^2) \, . \tag{6}$$

Here, $A > 0$ denotes the usual productivity parameter. The production technology (6) is concave in the individual variables. In the spirit of Romer (1986), the average stock of capital κ_t represents the the level of technical knowledge available in the economy and is enhanced by individual capital investments. Altogether, the technology (6) meets the conditions for ongoing growth of per capita incomes.

Parents are able to invest in the capital stock of their children. However, a redistributive system maps the agent's savings x_t^i into the child's capital endowment according to the following scheme:

$$\hat{x}_t^i = {x_t^i}^{1-\tau_t} \, \tilde{x}_t^{\tau_t} \, , \tag{7}$$

borrowed from Bénabou (1996, 2000), and previously employed for instance by Feldstein (1969) and Kanbur (1979). Here, $\hat{x}_t^i$ denotes post–tax investment. The progressivity of this system is measured by the elasticity of post–tax investment τ_t. For $\tau > 0$, the marginal rate rises with pretax investment, for $\tau < 0$ the scheme is regressive. In what follows, we will restrict our analysis to $\tau \in [0,1)$. The break–even level $\tilde{x}_t$ separates the winners from the tax–transfer–system from the losers, by defining the margin, where pre– and post–tax investment are equal and the associated household receives a zero net gain from redistribution. $\tilde{x}_t$ is determined by the government's budget constraint which requires net transfers summing to zero:

$$\int_0^1 x_t^i \, di = \int_0^1 {x_t^i}^{1-\tau_t} \, \tilde{x}_t^{\tau_t} \, di = \int_0^1 \hat{x}_t^i \, di \, . \tag{8}$$

The offspring's capital stock k_{t+1}^i also is subject to an individual 'ability' shock z_t^i, which is lognormally distributed, i. e. $\ln z_t^i \sim \mathcal{N}(-\sigma_z^2/2, \sigma_z^2)$, and $\mathrm{E}[z_t] = 1$:

$$k_{t+1}^i = z_t^i \, \hat{x}_t^i \, . \tag{9}$$

Maximization of expected utility of agent i with respect to c_t^i and x_t^i, subject to the budget constraint $y_t^i = c_t^i + x_t^i$, the production function (6), and the redistributive scheme (7), then implies that the all agents save the identical proportion $s(\tau_t)$ of their income:

$$x_t^i = s(\tau_t) \, y_t^i = \frac{\beta\gamma(1-\tau_t)}{1+\beta\gamma(1-\tau_t)} \, y_t^i \, . \tag{10}$$

The savings rate, $s(\tau_t)$, depends on the parameter measuring tax progression τ_t, thereby reflecting the well–known result that individual decisions are distorted by the presence of a redistributive tax system, with $\partial s(\tau)/\partial\tau < 0$. Since we argue within an endogenous growth framework, this distortion consequently reduces the long–run growth rate of the economy, thus causing the above mentioned efficiency costs of redistribution (cf. Mirrlees, 1971).

Under the assumption of lognormally distributed exogenous disturbances, the resulting wealth distribution is lognormal, whenever the initial distribution is lognormal, too, as we have already mentioned above. We assume $\ln k_t^i \sim \mathcal{N}(\mu_t, \sigma_t^2)$, where σ_t^2 denotes the variance of $\ln k_t^i$ in period t, measuring wealth inequality. Average (log) wealth is then given by $\ln \kappa_t = \mu_t + \frac{\sigma_t^2}{2}$, where μ_t denotes mean (log) wealth. By using equations (6), (9), and (10), we obtain a stochastic difference equation, describing the evolution of (log) wealth over time for family i:

$$
\begin{aligned}
\ln k_{t+1}^i &= \ln z_t^i + \ln \hat{x}_t^i = \ln z_t^i + (1-\tau_t)\ln x_t^i + \tau_t \ln \tilde{x}_t \\
&= \ln z_t^i + (1-\tau_t)\ln u_t^i + \ln s(\tau_t) + \ln A + (1-\tau_t)\beta \ln k_t^i + [1-\beta(1-\tau_t)]\mu_t \\
&\quad + \left[\beta^2\tau_t(2-\tau_t) + 1 - \beta\right]\frac{\sigma_t^2}{2} + \tau(1-\tau_t)\frac{\sigma_u^2}{2}\,, \qquad (11)
\end{aligned}
$$

where the break–even level of investment, $\ln\tilde{x}_t$, is given by (see the Appendix for derivation):

$$
\ln\tilde{x}_t = \ln s(\tau_t) + \ln A + \mu_t + (1-\tau_t)\frac{\sigma_u^2}{2} + \left[1-\beta+\beta^2(2-\tau_t)\right]\frac{\sigma_t^2}{2}\,. \qquad (12)
$$

Equation (11) completely describes the dynamics of the wealth distribution. In each period, wealth is lognormally distributed, that is $\ln k_t^i \sim \mathcal{N}(\mu_t, \sigma_t^2)$ with mean μ_t and wealth inequality σ_t^2 evolving according to:

$$
\begin{aligned}
\mu_{t+1} &= -(1-\tau_t)^2\frac{\sigma_u^2}{2} - \frac{\sigma_z^2}{2} + \ln s(\tau_t) + \ln A + \mu_t \\
&\quad + \left[\beta^2(1-(1-\tau_t)^2) + 1 - \beta\right]\frac{\sigma_t^2}{2} \qquad (13a) \\
\sigma_{t+1}^2 &= (1-\tau_t)^2\sigma_u^2 + \sigma_z^2 + \beta^2(1-\tau_t)^2\sigma_t^2 \qquad (13b)
\end{aligned}
$$

Mean wealth dynamics in general are negatively related to risk. The impact of the parent's risk (σ_u^2) is mitigated by the redistributive system, thereby reflecting the insurance property of taxation, whereas the offspring's risk (σ_z^2) cannot be diversified. As usual, mean wealth increases with a rise in the propensity to save.

If we look at the evolution of wealth inequality (13b), it becomes obvious how an increase in the progressivity of the tax system reduces wealth inequality. While the effects from the initial wealth inequality and from the individual production risk of the parent generation on the resulting wealth distribution are weakened, the effect of the ability shocks affecting the future generation is left unchanged. This outcome can be ascribed to the fact that the underlying redistributive system does not provide an insurance against these shocks.

The growth rate of income Since we are dealing with a typical model of endogenous growth, the growth rate of average income depends on several factors, the first being the endogenously determined propensity to save, which, indirectly, also establishes a link between the degree of tax progression and growth. Because we assumed imperfect capital markets, differences in the marginal productivity of the individual capital stocks are not leveled out by borrowing and lending. For this reason, the growth rate is also affected by the distribution of wealth. The assumed concavity of the production function, i. e. decreasing returns with respect to individual inputs, implies that a more unequal distribution of wealth goes along with smaller average output; see Aghion *et al.* (1999, p. 1624) and Bénabou (2000). These two effects appear in the following definition of the growth rate of average income $g_{y,t+1} = \Delta \ln \bar{y}_{t+1}$:

$$g_{y,t+1} = g(\tau_t, \sigma_t^2) = \ln A - \beta(1-\beta)\frac{\sigma_z^2}{2} + \underbrace{\ln s(\tau_t)}_{\text{incentive effect}} \underbrace{- \beta(1-\beta)(1-\tau_t)^2\left(\frac{\sigma_u^2}{2} + \beta^2\frac{\sigma_t^2}{2}\right)}_{\text{efficiency effect}} \qquad (14)$$

As can be seen from equation (14), a more unequal distribution of wealth — as measured by σ_t^2 — goes along with a lower growth rate. This is caused by the combination of imperfect capital markets, together with the concavity of the production function. Conversely, growth could be higher in a more equal society, thereby reflecting an *opportunity–enhancing effect* (Aghion *et al.*, 1999) of redistribution. Equation (14) also illustrates that an increase in redistributive taxes results in two competing effects on growth. The first one is an incentive effect which is harmful to growth and arises because the individual savings decisions are distorted, $\partial s(\tau)/\partial\tau < 0$. The second one is related to efficiency and based on the opportunity–enhancing effect. It is promoting growth, because the more equal wealth distribution from an increase in taxes ultimately results in a higher level of output, due to the concavity of the production function. Of course, we restrict parameterization of the model such that positive values of (14) are sustained and the economy evolves along positive endogenous growth path.

4 Redistributive politics under alternative assumptions on the distribution of political power

By now, we have established a link between individual savings, the distribution of wealth and growth for a given tax–transfer scheme. So, the natural next step of the analysis is to discuss the interaction between these variables and the effects of redistributive politics on the economic system, if agents are allowed to vote on the degree of tax progression within a democratic process.

We will start with deriving conditions on the individually preferred degree of redistribution, followed by stating results on the political benchmark case of *one–man–one–vote*, where the preferences of the median voter decide upon policy outcomes. The discussion then turns towards deviations from the median voter assumption. By following Bénabou (1996, 2000), we assume an uneven distribution of political power, where, in a first step, the rank of the pivotal agent in the wealth distribution is varied exogenously. Later on, the bias in the distribution of political power is determined endogenously and assumed to depend on wealth inequality itself, which then results in the emergence of redistributive cycles.

The individually preferred extent of redistribution In what follows, we assume that, in each period, the agents vote on the progressivity of the tax system. The overlapping generations structure of the model, where parents only care about expected income instead of their offspring's expected utility, allows us to disregard strategic interactions in an intertemporal context. Otherwise, voters could have incentives to influence future political outcomes by altering distributional dynamics via present actions.

By (5) and (10), the expected utility of agent i in period t can be determined as:

$$\begin{aligned}\mathrm{E}\,U_t^i &= \mathrm{E}[\ln c_t^i] + \gamma\,\mathrm{E}\,[\ln y_{t+1}^i] \\ &= \overline{U}_t^i + \ln(1 - s(\tau_t)) + \gamma\beta\,\mathrm{E}[\ln k_{t+1}^i] + \gamma(1-\beta)\left[\mu_{t+1} + \frac{\sigma_{t+1}^2}{2}\right]. \qquad (15)\end{aligned}$$

Here, $\overline{U}_t^i$ collects all terms independent from τ_t, therefore being irrelevant for the subsequent analysis.

By utilizing(11), (13a) and (13b), the preferred tax progression of agent i with wealth k_t^i can then be obtained as the the solution to the following necessary condition:

$$0 = -\frac{1}{\gamma}\frac{s'(\tau)}{1-s(\tau)} + \frac{s'(\tau)}{s(\tau)} + \beta(1-\tau)\left[\sigma_u^2 + \beta^2\sigma_t^2\right] - \beta^2[\ln k_t^i - \mu_t]$$

We define the function $G(\tau, \sigma_t^2)$:

$$G(\tau, \sigma_t^2) \equiv \frac{1-\beta(1-\tau)}{(1-\tau)\left[1+\beta\gamma(1-\tau)\right]} - \beta(1-\tau)\left[\sigma_u^2 + \beta^2\sigma_t^2\right] = \beta^2\left[\mu_t - \ln k_t^i\right] , \tag{16}$$

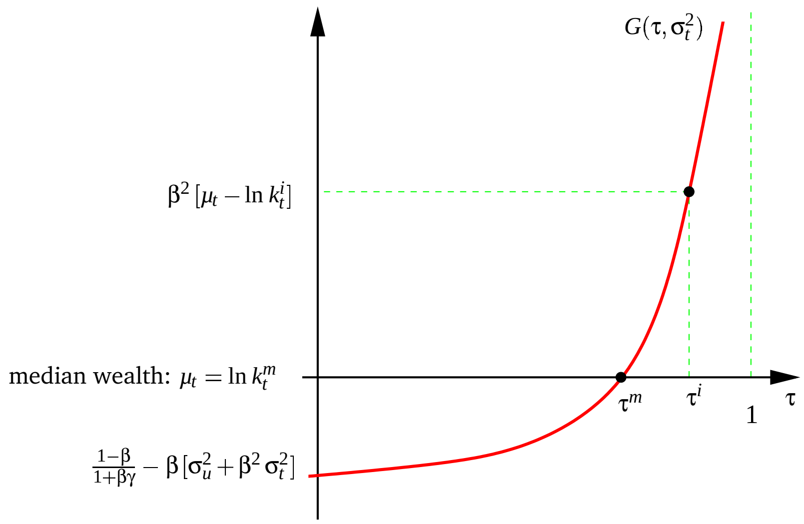
which states, that the individually preferred amount of redistribution is solely determined by the deviation of individual wealth $\ln k_t^i$ from μ_t. The specific structure of the underlying redistributive scheme (7) does not necessarily imply that agents prefer a nonnegative degree of tax progression, reflected by $\tau \in [0,1]$. But, recall that we initially excluded a regressive system from our considerations, that is the case of $\tau < 0$. Given this assumption, the first order condition (16) implies that the agents' preferences over redistributive schemes are single peaked. However, as Bénabou (2000, p. 103) has already pointed out, restricting τ to be nonnegative requires dealing with the possibility of corner solutions as an outcome of majority voting. In order to avoid this, we have to posit further restrictions on the primitives of the model.

The function $G(\tau,\sigma_t^2)$ is monotonically increasing, $G_\tau > 0$, and by (13b), we also have $\sigma_t^2 > \sigma_z^2$ for all $\tau \in [0,1]$. Hence, a sufficient condition for agent m with median wealth (in logs: $\ln k_t^m = \mu_t$), to always prefer a positive degree of progression in the tax–transfer–system, is the function $G(\tau,\sigma_t^2)$ satisfying

$$G(0,\sigma_z^2) = \frac{1-\beta}{1+\gamma\beta} - \beta\left[\sigma_u^2 + \beta^2\sigma_z^2\right] < 0\,, \tag{17}$$

which is obtained by additionally utilizing (13b) for $\tau = 0$. As Figure 2 illustrates, the sufficient condition (16) implies that individuals with wealth below the median always prefer a more progressive system than the pivotal agent, whereas voters with wealth above the median favor a lower degree of redistribution, and, finally, the richest individual preferring a world without taxes.

Figure 2: *Individual wealth and the preferred tax progression*



Dynamics under simple majority voting For now, we assume that the preferences of the median voter, that is the individual m with median wealth $\ln k_t^m$, decide upon the amount of income to be redistributed in each period. Let $\tau_t = \tau(\sigma_t^2)$ denote the associated outcome of the political voting mechanism, stressing the result that, ultimately, the degree of inequality in the society determines the extent of redistribution. Equation (16) then requires that the degree of tax progression $\tau\left(\sigma_t^2\right)$ to be implemented in this period is the solution to:

$$G(\tau, \sigma_t^2) \equiv \frac{1 - \beta\left(1 - \tau(\sigma_t^2)\right)}{\left(1 - \tau(\sigma_t^2)\right)\left[1 + \beta\gamma\left(1 - \tau(\sigma_t^2)\right)\right]} - \beta\left(1 - \tau(\sigma_t^2)\right)\left[\sigma_u^2 + \beta^2 \sigma_t^2\right] = 0\,. \tag{18}$$

This condition implicitly defines the degree of tax progression enforced by the median voter, where, according to condition (17) there always exists a solution satisfying $0 < \tau(\sigma_t^2) < 1$. Condition (18) also implies that greater inequality is always accompanied by a larger amount of redistribution; see also Figure 2.

Given (18), we are now able to rewrite the autoregressive dynamics of wealth inequality from (13b) as

$$\sigma_{t+1}^2 = \sigma_z^2 + \left[1 - \tau\left(\sigma_t^2\right)\right]^2 \left(\sigma_u^2 + \beta^2 \sigma_t^2\right)\,. \tag{19}$$

Let σ_*^2 and $\tau_* = \tau(\sigma_*^2)$ denote the stationary solutions to (19) and (18). The long–run level of inequality in the society, σ_*^2, can then be determined as:

$$\sigma_*^2 = \frac{\sigma_z^2 + (1-\tau_*)^2\,\sigma_u^2}{1-\beta^2\,(1-\tau_*)^2}$$

Long–run wealth inequality unambiguously decreases for a larger long–run extent of redistribution, the upper limiting case given by $\tau_* \to 1$, where the parent's idiosyncratic risk is perfectly pooled. It is straightforward to show that this stationary solution is stable (cf. Bénabou, 2000, Theorem 1). Evaluating the derivative of (19) with respect to σ_t^2 at the steady state level σ_*^2, yields the required stability condition

$$0 < \left|\beta^2\,(1-\tau_*)^2 - 2\,(1-\tau_*)\,(\sigma_u^2 + \beta^2\,\sigma_*^2)\,\tau_{\sigma^2}\right| < 1\,, \tag{20}$$

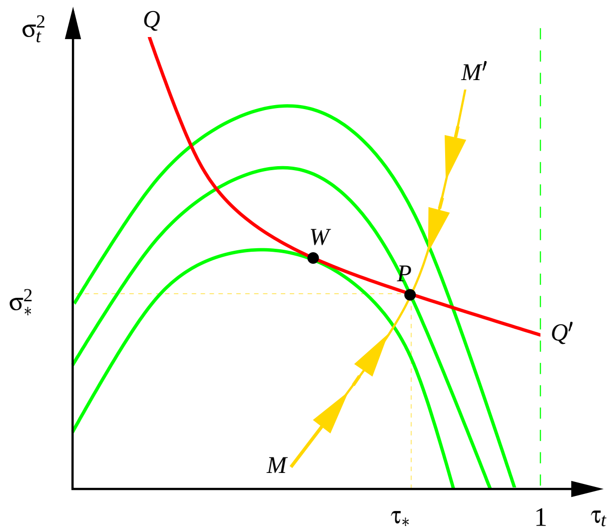
where $\tau_{\sigma^2} = \partial\tau(\sigma_t^2)/\partial\sigma_t^2$ denotes the corresponding partial derivative.

The joint dynamics of the degree of tax progression and wealth inequality are displayed in Figure 3. The curve QQ' represents long–run wealth inequality as a function of the elasticity of post–tax investment τ. The hump–shaped graphs represent curves of iso–growth and can be derived from (14), with growth rates increasing towards the origin. The curve MM' represents the extent of tax progression $\tau(\sigma_t^2)$ that results from the political system for a given level of wealth inequality. Convergence towards the stationary solution occurs along the path MM'. The political equilibrium P of the median voter model is then represented by the stationary values σ_*^2, τ_*.

Figure 3 also displays the well–known result that the growth rate associated with the political equilibrium P falls below the maximum feasible long–run growth rate of the economy which is attained in W, where QQ' is tangent to an iso–growth curve (cf. Bénabou, 1996).

Distribution of political power and inequality Up to now the analysis focused on a simple majority rule in the voting system, where the preferences of the median voter decide upon the outcome of political decision–making. However, empirical evidence suggests a smaller degree of participation of the relatively poor in the democratic process, when compared to the rich. Among others, Bénabou (1996, 2000, and references therein) argues that the economic analysis has to take account of biases in the political system.

Figure 3: *Redistribution, growth, and income inequality*



Deviations from the purely democratic *one–man–one–vote* system can be motivated in several ways: On the one hand, one might argue that it is easier for the rich to raise funds for lobbying activities and that they face less frictions in coordinating themselves in pressure groups by building up networks. The comparably low polling of the lower income classes can then be explained with a less organized structure of interest groups, a general feeling of individual powerlessness or annoyance about political representatives, or, perhaps, with the simple explanation that individuals are more concerned with earning their living and do not actively participate in democratic processes for opportunity costs reasons. On the other hand, one might also take the view that inequality aversion brings masses to raise, whenever from their point of view the perceived extent of inequality becomes too large. Contrary, a large degree of equality, achieved by an extensive amount of redistribution and publicly provided insurance, dampens the chances of upward mobility and provides incentives to vote against redistribution. Altogether, these arguments indicate that inequality itself might be a relevant variable in the explanation of biases in the political system, such that changes in the distribution of wealth also trigger corresponding movements in the degree of political participation.

In what follows, we will assume that the position of the pivotal agent in the wealth distribution changes over time according to the extent wealth inequality evolves. In particular, it is assumed that poor people gain political influence, whenever inequality grows too large and that rich people dominate the voting process, whenever inequality is low.

Let us first consider the consequences of a biased political system, when the political weight ω^i of agent i depends on her absolute level of wealth. We adopt the formulation developed by Bénabou (2000) who has shown that, if $\omega^i = (k_t^i)^\lambda$ for some $\lambda \gtrless 0$, the pivotal voter p has (log) wealth $k_t^p = \mu_t + \lambda\sigma_t^2$.[5] This means that, whenever $\lambda < 0$, the pivotal agent owns less wealth than the median voter, and the political system is biased in favor of the poor. Conversely, the system displays an elitist image, if the pivotal agent is wealthier than the median, that is $\lambda > 0$.

From equation (16), we obtain the associated degree of tax progression, maximizing utility of the pivotal individual in the biased political system:

$$G(\tau_t\,,\sigma_t^2) = -\beta^2\lambda\sigma_t^2\,. \tag{21}$$

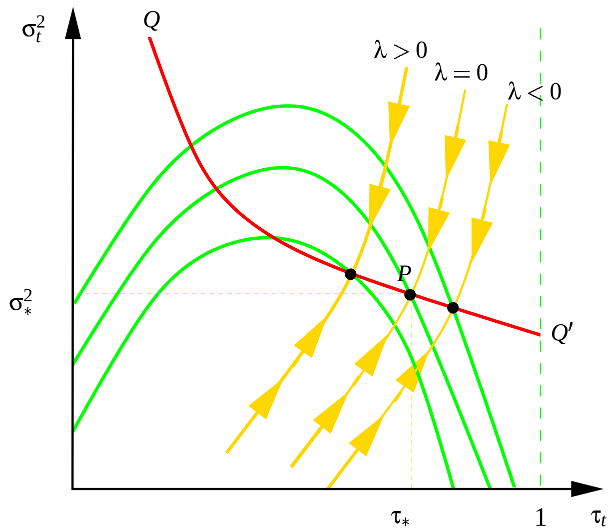
Given an arbitrary value of λ, this equation does not necessarily possess an interior solution. However, under the previously stated assumptions on the parameters of the model, there always exists an open set of biases around $\lambda = 0$, yielding an interior solution.

Figure 4 summarizes the consequences of a biased political system for the dynamics of taxes, the growth rate and wealth inequality. If compared to the outcome of the median voter model, which corresponds to the case of $\lambda = 0$ and is represented by point P, a bias of the political system favoring the rich moves the political equilibrium towards lower taxes accompanied by higher growth and a larger extent of inequality. Contrary, a political bias assigning more political weight to the lower income classes yields an equilibrium characterized by a more progressive tax system which reduces inequality at the cost of lower long–run growth rates.

The figure also gives a first intuition of what might happen, if the bias in the political system varies over time by endogenously adjusting to changes in the wealth distribution.

[5] Cf. Bénabou (2000, Prop. 6) and the related proof.

Figure 4: *Wealth dynamics and redistribution in a biased political system*



Endogenous cycles In order to formalize this idea, we assume that the political bias λ_t depends on the degree of inequality, measured by the variance of wealth. The underlying law of motion is given by $\lambda_{t+1} = H(\sigma_t^2)$. By this we posit that the extent to which the future political system deviates from the 'ideal' of the median voter equilibrium is determined by today's wealth inequality. Regarding the function $H(\sigma_t^2)$, we assume that $H(\sigma_*^2) = 0$, thereby yielding uniform steady state conditions for the biased and the unbiased (median voter) model. The value of H declines with a growing variance of wealth, $H'(\sigma^2) < 0$, thus capturing the idea that an increase in inequality shifts the political power towards the poor.[6]

If we let $\tau_t = \tau(\sigma_t^2, \lambda_t)$ denote the solution to equation (21), we now have to deal with a two–dimensional system, jointly describing the evolution of wealth inequality σ_t^2 and the political bias λ_t:

$$\sigma_{t+1}^2 = \beta^2 \sigma_z^2 + \sigma_u^2 + \beta^2 \left[1 - \tau(\sigma_t^2, \lambda_t)\right]^2 \sigma_t^2 \tag{22a}$$

$$\lambda_{t+1} = H(\sigma_t^2) \tag{22b}$$

Let us now analyze the dynamics of the model in the neighborhood of the stationary point (σ_*^2, λ_*) for the case of $H'(\sigma^2) < 0$. Let $h = H'(\sigma_*^2)$ denote the partial derivative of the political bias with respect to inequality and

[6] Accordingly, $H'(\sigma^2) = 0$ reflects the median voter case.

evaluated at the stationary state. The Jacobian matrix J of the system (22a) and (22b) is given as follows:

$$J = \begin{pmatrix} \beta^2 (1-\tau_*) \left[(1-\tau_*) - 2\sigma_*^2 \tau_{\sigma^2}\right] & -2\beta^2 (1-\tau_*) \sigma_*^2 \tau_\lambda \,, \\ 0 & h \end{pmatrix} \tag{23}$$

where τ_{σ^2}, τ_λ denote the associated partial derivatives of the function $\tau(\sigma^2, \lambda)$ evaluated at the stationary state. The eigenvalues ν_1, ν_2 of the Jacobian matrix are given by the two roots of the characteristic equation:

$$0 = \nu^2 - \nu\beta^2 (1-\tau_*) \left[(1-\tau_*) - 2\sigma_*^2 \tau_{\sigma^2}\right] + \beta^2 2 (1-\tau_*) \sigma_*^2 \tau_\lambda h$$

where we define the function $f(\nu)$:

$$f(\nu) \equiv \nu^2 - \nu\beta^2 (1-\tau_*) \left[(1-\tau_*) - 2\sigma_*^2 \tau_{\sigma^2}\right] = -\beta^2 2 (1-\tau_*) \sigma_*^2 \tau_\lambda h \tag{24}$$

The function $f(\nu)$ on the left hand side of equation (24) is quadratic in ν with roots at $\nu = 0$ and at $\tilde{\nu} = \beta^2 (1-\tau_*) \left[(1-\tau_*) - 2\sigma_*^2 \tau_{\sigma^2}\right]$. Notice that, by utilizing the stability condition (20), we have $0 < \tilde{\nu} < 1$. Moreover, we obtain $\tau_\lambda < 0$, such that the right hand side of equation (24) is always negative as long as $h < 0$ (see also Figure 4). Depending on the value of h we are able to distinguish three cases for the roots associated with equation (24): (a) two real roots with modulus less than one, or (b) conjugate complex roots with modulus less than one, or (c) conjugate complex roots with modulus greater than one.[7]

Since the stability properties of the stationary point (σ_*^2, λ_*) depend on the value of h, this represents a bifurcation parameter. In the present case, the dynamic system undergoes a Hopf–bifurcation, if we reduce the value of h starting from $h = 0$.[8] To see this, notice that the eigenvalues of the Jacobian (23) are complex with modulus one in absolute value, if h equals $h^b \equiv \left[2\beta^2 (1-\tau_*) \sigma_*^2 \tau_\lambda\right]^{-1} < 0$. Since the eigenvalues are of modulus less than one in absolute value for $h < h^b$ and of modulus greater than one for $h > h^b$, the Hopf–bifurcation occurring at $h = h^b$ is supercritical. Thus, for values of h greater than h^b there exists an invariant and stable closed curve in the

[7] Writing $f(\nu)$ as $\nu^2 - a\nu$ we see that $f(\nu)$ attains its minimum $-a^2/4$ at $\nu^* = a/2$. Notice, that $0 > f(\nu^*) - a^2/4 > -1$, since $0 < a < 1$.

[8] A formal proof of this statement is given in the Appendix.

Figure 5: *Dynamics of redistribution and income inequality*

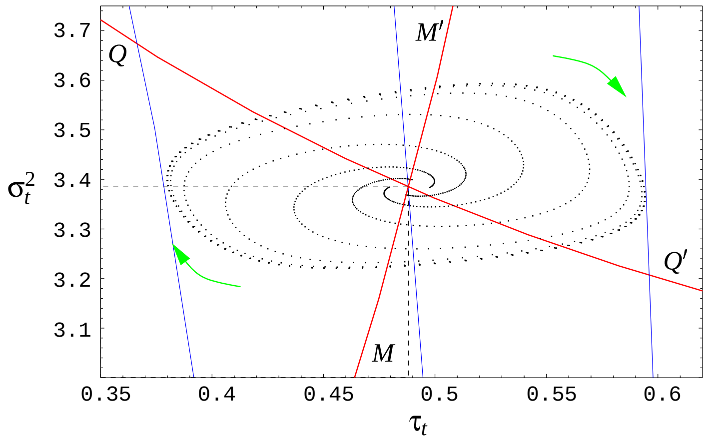
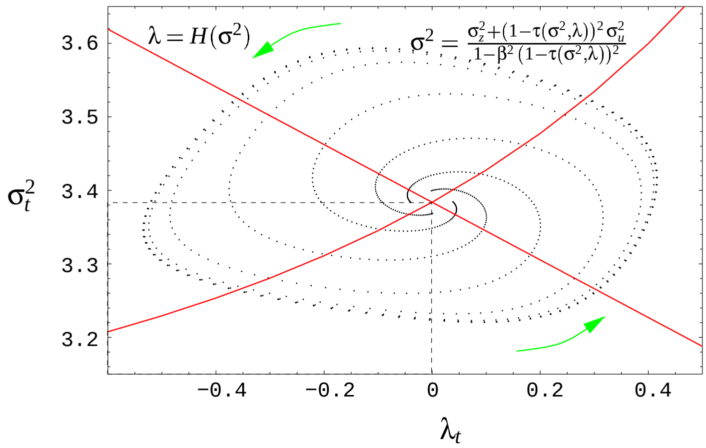


Figure 6: *Dynamics of the political bias and income inequality*



neighborhood of the unstable stationary point (σ_*^2, λ_*). This closed curve then represents an endogenous cycle of political participation, redistribution and growth.

The bifurcation value $h^b = \left[2\beta^2 (1-\tau_*)\,\sigma_*^2\,\tau_\lambda\right]^{-1} < 0$ provides some information on how the emergence of cycles depends on the characteristics of the model. Other things equal, a change in the factors determining h^b affects the bifurcation parameter as follows

$$\frac{\partial h^b}{\partial \beta} < 0, \qquad \frac{\partial h^b}{\partial \sigma_*^2} < 0, \qquad \frac{\partial h^b}{\partial \tau_*} > 0, \qquad \frac{\partial h^b}{\partial \tau_\lambda} < 0\,.$$

The value h^b falls with a rise in the stationary level of wealth inequality σ_*^2 and for a decreasing level of income mobility (i. e. an increase in β). Furthermore it declines whenever a marginal change in the political bias λ induces large changes in the tax rate, whereas it rises with an increase in the stationary tax rate τ_*. Note that c. p. the emergence of cycles is more likely to occur, the smaller h^b is. This also means that, given a small bifurcation value h^b, we only have to establish a weak relationship between λ and σ^2 in order to generate redistributive cycles.

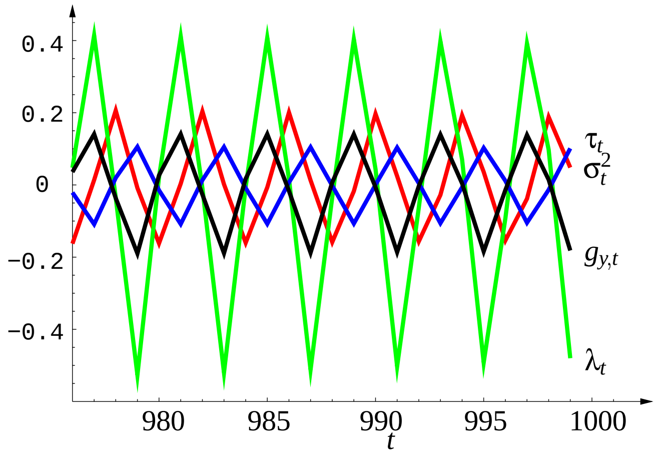
Figures 5 and 6 show the simulations of a numerically specified model. We set the parameters according to $\beta = 0.7$, $\gamma = 0.1$, $\sigma_u^2 = 0.5$ and $\sigma_z^2 = 5$. For the median voter case, represented by $\lambda_* = 0$, the long–run stationary value for wealth inequality results as $\sigma_*^2 = 3.383834$, with an associated elasticity of post–tax investment equal to $\tau_* = 0.488488$. For deviations from the median voter model, such that $\lambda_{t+1} = H(\sigma_t^2)$ and $h \equiv H'(\sigma_*^2)$, the dynamical system undergoes a supercritical Hopf–bifurcation for a critical value of $h^b = -2.51055$. The simulation results presented in Figures 5 and 6 are plotted for an arbitrarily chosen slope of $h = -2.55$, which only has to satisfy the single condition of being smaller than h^b. The dynamics of political power are then given by $\lambda_{t+1} = -2.55\,(\sigma_t^2 - \sigma_*^2)$.

Figure 5 shows the cycle in the σ_t^2/τ_t plane, i. e. the endogenous cycle of redistribution and wealth inequality, whereas Figure 6 depicts the identical cycle in the σ_t^2/λ_t plane, i. e. the endogenous cycle of the political bias and wealth inequality. As becomes obvious, periods of low growth due to a large amount of redistribution go along with a political bias favoring the poor. In the course of decreasing wealth inequality, the political power shifts towards the rich, who enforce tax–transfer–schemes entailing a low degree of redistribution and larger growth rates. This causes inequality to rise again, thereby initiating the shift of power back to the poor.

Figure 7 depicts the time paths of the variables σ_t^2, λ_t, τ_t and $g_{y,t}$ sampled over an arbitrarily fixed time period of the cycles displayed in Figures 5 and 6. For easier comparison, the figure shows the deviations of the respective variables from their stationary values.

The time paths start in a situation where wealth inequality is below its stationary level. This low degree of inequality results in a political process which becomes more and more biased in favor of the rich. The consequence

Figure 7: *Time paths over the cycle*



is a low degree of redistribution, which goes along with a comparably small and further declining tax rate τ_t. From the viewpoint of growth, a decrease in the voted degree of tax progression mitigates the distortionary effect from taxation on saving, subsequently leading to higher growth rates of income.

This phase, where a low degree of redistribution goes along with increasing inequality and rising growth rates might be identified as an episode, where chances of upward income mobility on redistribution dominate the effect of inequality aversion. However, if wealth inequality exceeds a certain threshold, the dynamics switch back. The high level of inequality results in a political process becoming more and more biased towards the poor. Wherease this immediately goes along with a growing degree of redistribution, an increase in tax progressivity and lower growth, at a first instance, wealth inequality is still rising. As can be seen, wealth inequality only begins to decrease, if the degree of tax progressivity becomes sufficiently large. In this case, wealth inequality finally begins to decline, however, by this already triggering the switch back to a more elitist distribution of political power.

5 Summary of results

In this paper, we investigated two models of stochastic growth with incomplete asset markets and heterogeneous agents, where redistributive cycles and cyclical growth emerge as outcomes of voting processes over public tax–transfer schemes. Heterogeneity among agents stems from idiosyncratic risks. The members of the society decide *ex ante* on the implementation of a redistributive scheme. This, consequently, serves the simple purpose of providing an insurance against unfavorable outcomes of current individual income shocks, whereas we assumed future risk to be non–diversifiable.

The redistributive scheme of the stylized model of section 2 took on a very simple all–or–nothing form, either to perfectly insure the current risks or to dispense with redistribution completely. In the second model, we assumed a more sophisticated redistributive scheme, by letting agents vote on the preferred degree of tax progression, thereby introducing the possibility of a continuously varying degree of redistribution over time.

Both approaches display the feature that redistribution is not costlessly available. We started with assuming costs to be exogenously fixed in terms of a certain amount of income to be sacrificed. Later on, in the the overlapping generations model, costs accrued endogenously from the redistributive process in terms of disincentives to save and subsequently forgone growth, since the analysis was embedded in an endogenous growth context.

Redistributive cycles emerge in the stylized model of section 2 as an outcome of a simply majority voting process. In each period, the median voter balances the benefits from the redistributive scheme against the costs and votes in favor of redistribution, if the first outweigh the second, while rejecting redistribution otherwise. Since benefits from redistribution are low in a comparably egalitarian society, while they are large, whenever inequality is high, the state of inequality is determined by the political equilibrium. With costs being exogenously fixed at a certain level, the varying benefits over time give rise to alternating political equilibria representing the aforesaid redistributive cycles.

Redistributive cycles emerge in the overlapping generations model of section 3 for the case of an uneven distribution of political power, in particular, if we tie the degree of political participation to inequality itself. From the economic point of view, agents face two counter–acting forces. On the one hand, whenever inequality grows too large, inequality (and risk) aversion leads to a larger extent of redistribution. On the other hand, the equalizing effects stemming from a comparably high tax progression dampen individual prospects of upward mobility. This induces a shift in the bias of political power towards the relatively rich, thereby causing more and more agents to vote for a lower degree of progressivity in taxation. This is accompanied by less redistribution, and lasts, until inequality again has grown to an extent, where inequality aversion dominates the voting equilibrium and a more progressive tax–transfer–scheme is reestablished. Since redistribution provides negative incentives for individual saving, we also observe a non–linear pattern of saving and growth rates over the political cycle.

From a technical point of view, the dynamic system undergoes a supercritical Hopf–bifurcation, thereby allowing for the emergence of cyclical behavior for an appropriate value of the bifurcation parameter, which here measures the response of the bias in political power to changes in wealth inequality.

References

Aghion, Philippe, Caroli, Eve, and García-Peñalosa, Cecilia (1999), Inequality and Growth: The Perspective of the New Growth Theories, *Journal of Economic Literature*, **37**, 1615–1660.

Alesina, A. and Rodrik, Dani (1994), Distributive Politics and Economic Growth, *Quarterly Journal of Economics*, **109**, 465–490.

Azariadis, Costas (1993), *Intertemporal Macroeconomics*, Blackwell Publishers, Oxford.

Becker, Gary S. and Tomes, Nigel (1979), An Equilibrium Theory of the Distribution of Income and Intergenerational Mobility, *Journal of Political Economy*, **87**, 1153–1189.

— (1986), Human Capital and the Rise and Fall of Families, *Journal of Labor Economics*, **4**, S1–S39.

Bénabou, Roland (1996), Inequality and Growth, in: Ben S. Bernanke and Julio J. Rotemberg (eds.), *NBER Macroeconomics Annual*, MIT Press, Cambridge, MA, vol. 11, pp. 11–74.

— (2000), Unequal Societies: Income Distribution and the Social Contract, *American Economic Review*, **90** (1), 96–129.

Bénabou, Roland and Ok, Efe A. (2001), Social Mobility and the Demand for Redistribution: The POUM Hypothesis, *Quarterly Journal of Economics*, **116**, 447–487.

Benhabib, Jess and Rustichini, Aldo (1996), Social Conflict and Growth, *Journal of Economic Growth*, **1** (1), 129–146.

Bertola, Giuseppe (1993), Factor Shares and Savings in Endogenous Growth, *American Economic Review*, **83**, 1185–1198.

Feldstein, Martin S. (1969), The Effects of Taxation on Risk Taking, *Journal of Political Economy*, **77**, 755–764.

Galor, Oded and Zeira, Joseph (1993), Income Distribution and Macroeconomics, *Review of Economic Studies*, **60**, 35–52.

Goodwin, R. M. (1951), The Nonlinear Accelerator and the Persistence of Business Cycles, *Econometrica*, **19**, 1–17.

Harms, Philipp and Zink, Stefan (2002), Growing in and out of Social Conflict, Working Paper Series 02.06, Studienzentrum Gerzensee, Gerzensee.

Kaldor, Nicholas (1940), A Model of Trade Cycle, *Economic Journal*, **50**, 78–92.

Kanbur, S. M. (1979), Of Risk Taking and the Personal Distribution of Income, *Journal of Political Economy*, **87**, 769–797.

Krusell, Per and Smith, Anthony A. (1998), Income and Wealth Heterogeneity in the Macroeconomy, *Journal of Political Economy*, **106** (5), 867–896.

Krusell, Per, Quadrini, Vincenzo, and Ríos-Rull, José-Víctor (1997), Politico–economic Equilibrium and Economic Growth, *Journal of Economic Dynamics and Control*, **21**, 243–272.

Loury, Glenn C. (1981), Intergenerational Transfers and the Distribution of Earnings, *Econometrica*, **49**, 843–867.

Matsuyama, Kiminori (2000), Endogenous Inequality, *Review of Economic Studies*, **67**, 743–759.

Mirrlees, James A. (1971), An Exploration in the Theory of Optimal Income Taxation, *Review of Economic Studies*, **38**, 175–208.

Perotti, Roberto (1993), Political Equilibrium, Income Distribution, and Growth, *Review of Economic Studies*, **60**, 755–776.

— (1996), Growth, Income Distribution and Democracy: What the Data Say, *Journal of Economic Growth*, **1** (2), 149–187.

Persson, Torsten and Tabellini, Guido (1994), Is Inequality Harmful for Growth?, *American Economic Review*, **84** (3), 600–621.

Piketty, Thomas (1995), Social Mobility and Redistributive Politics, *Quarterly Journal of Economics*, **110**, 551–584.

— (1997), The Dynamics of Wealth Distribution and the Interest Rate with Credit Rationing, *Review of Economic Studies*, **64**, 173–189.

Plümper, Thomas and Martin, Christian W. (2003), Democracy, Government Spending, and Economic Growth: A Political–economic Explanation of the Barro–effect, *Public Choice*, **117**, 27–50.

Romer, Paul M. (1986), Increasing Returns and Long–Run Growth, *Journal of Political Economy*, **94**, 1002–1037.

Saint-Paul, Gilles and Verdier, Thierry (1993), Education, Democracy and Growth, *Journal of Development Economics*, **42** (2), 399–407.

— (1997), Power, Distributive Conflicts, and Multiple Growth Paths, *Journal of Economic Growth*, **2**, 155–168.

Schwarze, Johannes and Härpfer, Marco (2002), Are People Inequality Averse, and Do They Prefer Redistribution by the State?, Discussion Paper 430, IZA, Bonn.

Varian, Hal R. (1980), Redistributive Taxation as Social Insurance, *Journal of Public Economics*, **14**, 49–68.

Appendix

Derivation of the break–even level of investment $\ln \tilde{x}_t$ Let $\mu_{y,t}$ and $\sigma^2_{y,t}$ denote the first and second moments of the distribution of (log) income. By (8), we get

$$\ln \int_0^1 x_t^i \, di = \ln s(\tau) + \mu_{y,t} + \sigma^2_{y,t}/2$$

as well as

$$\ln \int_0^1 \hat{x}_t^i \, di = \ln \left(\int_0^1 {x_t^i}^{1-\tau} \tilde{x}_t^\tau \, di \right) = (1-\tau)[\ln s(\tau) + \mu_{y,t}] + (1-\tau)^2 \sigma^2_{y,t}/2 + \tau \ln \tilde{x}_t$$

Equating the right hand sides of both equations, yields the following expression for the break even income level $\ln \tilde{x}_t$:

$$\ln \tilde{x}_t = \ln s(\tau) + \mu_{y,t} + (2-\tau) \frac{\sigma^2_{y,t}}{2}$$

From $\ln y_t^i = \ln A + \ln u_t^i + \beta \ln k_t^i + (1-\beta) \ln \kappa_t$ follows:

$$\begin{aligned} \mu_{y,t} &= \ln A - \frac{\sigma_u^2}{2} + \mu_t + (1-\beta) \frac{\sigma_t^2}{2} \\ \sigma^2_{y,t} &= \sigma_u^2 + \beta^2 \sigma_t^2 \end{aligned}$$

Substituting these expressions into the definition of $\ln \tilde{x}_t$ leads to equation (12).

Hopf bifurcation: Given a two-dimensional non-linear system of difference equations, let $\mu(h)$ denote the eigenvalues of the Jacobian J dependent on the parameter h. The dynamical system undergoes a Hopf-bifurcation at $h = h^b$, if (cf. Azariadis, 1993, pp. 100), if:

(i) $|\mu(h^b)| = 1$

(ii) $\mu(h^b)^j \neq \pm 1$ for $j = 1, 2, 3, 4$

(iii) $\frac{d|\mu(h)|}{dh}_{h=h^b} \neq 0$

This then implies that there is an invariant closed curve bifurcation from h^b.

In order to simplify the representation, let us write the characteristic polynomial as $f(\mu) = \mu^2 - a\mu = hb$, where $0 < a < 1$ and $b > 0$ Regarding condition (i), the corresponding value of h^b such that $\mu(h^b) = 1$ is $h^b = -1/b$. In this case the roots are complex and can be written as $\mu_{1,2} = Re^{\pm i\theta}$, where $R = \sqrt{-h^b b} = 1$ and $\theta = a/2$. Hence, we have $\mu(h^b)^j \neq \pm 1$ for $j = 1, 2, 3, 4$ and condition (ii) is satisfied too. Since $|\mu(h^b)| = \sqrt{-hb}$, we have $\frac{d|\mu(h)|}{dh} = -b\frac{1}{2}(-hb)^{-1/2}$ and condition (iii) is also satisfied.