Filtering Long-Run Inflation Expectations with a Structural Macro Model of the Yield Curve

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Abstract

This paper proposes a methodolgy to estimate structural macroeconomic models including non-stationary steady state dynamics. Using a transitory-permanent decomposition of the Euler equations, the method first solves for the transitory dynamics and subsequently provides the solution for the full model by substituting back in the steady state dynamics. The method is applied to models linking the macroeconomic dynamics to the term structure of interest rates. We find that non-stationary variables play a crucial role in this respect. More specifically, long-run inflation expectations, estimated on the macroeconomic variables, turn out to be extremely important in the determination of the term structure.

Keywords: Structural model, New-Keynesian model, filtering procedure, essentially affine term structure model, time-varying inflation expectation.

J.E.L.: E43, E44, E52.

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1 Introduction

Since the seminal papers by Vasicek (1977) and Cox, Ingersoll, and Ross (1985), the finance literature seems to agree that term structure models must exclude arbitrage opportunities and be both econometrically and numerically tractable. Only in this way such models can be useful, for instance, in the pricing of fixed income derivatives and in the assessment of the risks implied by fixed income portfolios. More recently, however, a number of requirements have been added to the modeling of the term structure dynamics. Satisfactory models should (*i*) be able to identify the economic forces behind movements in the yield curve, (*ii*) take into account the way central banks implement their monetary policies, and (*iii*) have a macroeconomic framework consistent with the stochastic discount factor implied by the model. In this paper, we present a methodology that allows one to fulfill all the above requirements. We consider, moreover, the effect of a possible asymmetry between the targets announced by the central bank and those perceived by the agents.

The methodology presented in this paper is the result of three distinct phases in the line of research making use of *affine term structure models*. The first phase is characterized by the use of latent or unobservable factors, as defined in Duffie and Kan (1996) and summarized in Dai and Singleton (2000).[1] Although this framework excludes arbitrage opportunities and is reasonably tractable, the factors derived from such models do not have a direct economic meaning and are simply labeled according to their effect on the yield curve (i.e. as a "level", a "slope", and a "curvature" factor).

The second phase involves the inclusion of macroeconomic factors in the standard affine term structure model. Ang and Piazzesi (2003) show that such inclusion improves the forecasting performance of Vector Autoregression (VAR) models in which no-arbitrage restrictions are imposed.[2] Their model, nevertheless, still includes unobservable factors without a clear economic interpretation. The model is also estimated in two stages based on the assumption that the short-term interest rate does not affect the macroeconomic dynamics. Kozicki and Tinsley (2001, 2002) indicate the importance of long-run inflation expectations in modeling the yield curve. This fact is confirmed by Dewachter and Lyrio (2003), who estimate a affine term structure model in one stage and based solely on factors with a well-specified macroeconomic interpretation.[3] Most of these make use of a Taylor (1993) type of rule to represent the monetary policy interest rate. The mentioned papers do not attempt, however, to propose a macroeconomic framework consistent with the pricing kernel implied by their models.

The third and current phase in this line of research is marked by the use of structural macro relations together with the standard affine term structure model. The structural macro model replaces the unrestricted VAR set-up adopted in previous research[4], and has commonly been based

[1] Duffee (2002) and Duarte (2004) propose more flexible specifications for the market prices of risk.

[2] Other papers following this approach include Diebold, Rudebusch and Aruoba (2004).

[3] A related approach can be found in Berardi (2004).

[4] For instance, the models presented in Ang and Piazzesi (2003) and Dewachter and Lyrio (2003).

on a New-Keynesian framework. Hördahl, Tristani and Vestin (2003) find that the forecasting performance of such model is comparable to that of standard latent factor models. They are also able to explain part of the empirical failure of the expectations hypothesis. A similar approach is adopted by Rudebusch and Wu (2003). Bekaert, Cho and Moreno (2003) go one step further and estimate a similar model based on deep parameters. They also make sure that the pricing kernel they formulate is consistent with their proposed macro model. These models have, however, one common drawback. Although they allow for temporary changes in the inflation target of the central bank, its long-run inflation target is assumed to be constant over time. Recent empirical evidence for the US suggests, however, the presence of permanent changes in the inflation target of the central bank. Kozicki and Tinsley (2001, 2002), for example, provide evidence that long-run inflation targets (endpoints in their terminology) are time varying.

As pointed out in Dewachter and Lyrio (2003), a necessary condition to model in a consistent way the central bank's long-run inflation expectation (inflation target) is to assume that it follows a martingale process. The presence of such stochastic trend is in line with the models proposed by Kozicki and Tinsley (2001, 2002). It introduces, however, a nonstationary characteristic in the dynamics of the model and, more specifically, in the interest rate rule adopted by the central bank. In this paper, we propose a methodology to estimate a structural macro model jointly with a affine term structure model containing unobserved stochastic trends. It involves, therefore, the solution of a rational expectations model in a nonstationary environment. Standard solution methods become infeasible since they typically assume stationarity of the dynamic system. The proposed solution is based on a transitory-permanent decomposition of the equilibrium conditions and is performed in two steps. We first solve the transitory component of the model using standard solution methods (e.g. Sims 2001). We then substitute this solution back in the nonstationary system. This two-step approach allows us to obtain the reduced form dynamics consistent with the rational expectations macroeconomic model including nonstationary variables.

We use standard no-arbitrage conditions to link the macroeconomic dynamics to the term structure of interest rates. Under suitable conditions on the prices of risk, it is well known that the term structure is affine in the (macroeconomic) state vector. Given the nonstationarity in the macroeconomic dynamics and the affine property of the term structure, the nonstationarity carries over to the term structure as well. While nonstationarity of interest rates is still not rejected by standard unit root tests, there is strong evidence against nonstationarity in yield spreads. In order to impose the stationarity of yield spreads, we need to impose some cointegrating restrictions on the term structure model. We provide sufficient conditions on the prices of risk that generate term structure models satisfying both the no-arbitrage principles and the stationarity of all yield spreads. These conditions are, however, not sufficient to guarantee consistency between the implied pricing kernel and the proposed macro model (see Bekaert, Cho and Moreno 2003). As is shown in their paper, the necessary additional conditions, i.e. constant market prices of risk, are in accordance with the mentioned cointegrating restrictions.

The introduction of time-variation in the central bank's long-run inflation target makes the assumptions regarding the information set of agents crucial to the solution of the model. This is the case since it is the private agents' expectations that define the term structure of interest rates. We analyze two cases. In a first case, we assume that the inflation target announced by the central bank is fully credible and enforceable. In this situation, agents have *full information* with respect to the actual target being implemented. Since agents observe the time-varying inflation target, their inferred target coincides with the actual target of the central bank. The econometrician, on the other hand, does not observe this target and is, therefore, obliged to filter it from the data. In a second case, we assume the presence of *asymmetric information* between the inflation target announced by the central bank and the one perceived by the agents. Since agents do not observe the actual time-varying target being implemented, they are forced to infer (filter) it from observable macro variables. In this scenario, the estimated inflation target corresponds to the beliefs of private agents. Since agents filter from observable macroeconomic factors, the econometrician knows the target inferred by the agents. In each case, we analyze the estimated long-run inflation expectation of the agents, their effect on the yield curve, and the final fit of the model.

We estimate a structural macro model with five factors: three observable ones (inflation, output gap, and the short-term interest rate), and two unobservable ones (the natural real interest rate and the long-run inflation target of the central bank). The unobservable variables are filtered with the use of a Kalman filter. We avoid, therefore, the inversion of the yield curve, which involves the arbitrary choice of yields observed without measurement errors. We use two filtering approaches corresponding to the two cases described above. The first one uses the term structure as an information variable for filtering the central bank's inflation target. The second one only makes use of macroeconomic information for the identification of long-run inflation expectations.

We apply the mentioned model to the US and German data. Our results show that nonstationary variables play a crucial role in linking the macroeconomic dynamics to the term structure of interest rates. The time-varying inflation expectation implied by our model seems to be in line with survey data available for the US market. The fit of the term structure, specially for the full information case, is comparable to the one obtained making use of standard latent factor models.

The remainder of the paper is divided in three sections. In Section 2, we present the method used to solve the macroeconomic dynamics when nonstationary components are included. Next we relate the term structure to the macroeconomic dynamics and derive conditions for stationarity of the yield spreads. In Section 3, we estimate the model for the two cases considered: the full information and the asymmetric information. We conclude in Section 4 by summarizing the main findings of the paper.

2 Macro-finance models with stochastic endpoints

This section explores the implications of a nonstationary steady state system for both the macroeconomic and the term structure dynamics. We first propose a decomposition of the Euler equations separating the transitory from the permanent dynamics. Under suitable assumptions, the macroeconomic dynamics can be solved explicitly. Subsequently, we present a simple framework in which the possibility of a change in the central bank's long-run inflation target naturally gives rise to a nonstationary macro dynamics. Finally, we analyze the implications for the term structure deriving from the inclusion of nonstationarity in the macroeconomic dynamics. Conditions are provided such that (i) the yield curve satisfies some cointegrating restrictions, implying stationary yield spreads, and (ii) the term structure model is consistent with the macroeconomic set-up.

2.1 Solving for absolute macroeconomic dynamics

In this paper we consider a rational expectation (RE) model for a set of macroeconomic variables collected in the $n \times 1$ vector X_t. We furthermore introduce an $n \times 1$ vector F_t with elements being the stochastic endpoints for the respective elements of X_t:

$$\lim_{s \to \infty} E_t \left[X_{t+s} \right] \to F_t. \tag{1}$$

The vector F_t is determined by a set of stochastic trends Z_t and an $n \times k$ matrix T, which maps the stochastic trends into the respective stochastic endpoints for X_t:

$$F_t = T Z_t \tag{2}$$

with the dynamics of Z_t a system of independent, possibly degenerate, random walk processes:

$$Z_t = Z_{t-1} + H \eta_t. \tag{3}$$

Equations (1) to (3) define the dynamics of the stochastic endpoints and thus model the properties of the steady state of the economy. Finally, we assume a set of n restrictions on the equilibrium dynamics relative to the steady state.[5] More specifically, defining the macroeconomic state relative to steady state by $\tilde{X}_t = X_t - F_t$, we assume the existence of $n \times n$ matrices A, B, C, and W such that the n restrictions can be written in matrix form as:

$$A \tilde{X}_t = B E_t \left[\tilde{X}_{t+1} \right] + C \tilde{X}_{t-1} + W w_t, \tag{4}$$

where w_t denotes a set of temporary shocks, i.e. shocks that determine the temporary deviations from the steady state dynamics. If the relative dynamics of $\tilde{X}_t$ has a unique stationary solution,

[5] These restrictions can be obtained from the log-linearization of the equilibrium first order conditions of the underlying structural models. Log-linearization is typically done relative to the steady state dynamics and, hence, deliver automatically the required type of restrictions.

a standard modeling assumption, then the following (uniform) growth conditions are satisfied (see Sims 2001):

$$\lim_{s\to\infty} E_t\left[\tilde{X}_{t+s}\xi^{-s}\right] \to 0, \text{ for } \xi \geq 1. \tag{5}$$

Consider now a set of n equilibrium conditions, possibly including some nonstationary terms. These equilibrium conditions could be derived from structural models if the necessary log-linearization is done relative to a zero steady state. The set of equilibrium conditions differs from the standard Euler equations in that (stochastic) steady states modeled as nonstationary variables enter in the equilibrium conditions. That is, instead of solving macroeconomic dynamics relative to the steady state, we are now interested in solving for the absolute dynamics, i.e. the dynamics including the steady state dynamics. The set of equilibrium conditions is, instead of (4), equal to:

$$AX_t = BE_t\left[X_{t+1}\right] + CX_{t-1} + DZ_t + Vv_t \tag{6}$$

with v_t a set of shocks affecting the macroeconomic variables (not only temporary deviations) and with the matrix D an $n \times k$ matrix containing the relations between the macroeconomic variables and the stochastic trends Z_t. This general representation of the absolute dynamics can be decomposed into a transitory-permanent decomposition, identifying the stochastic endpoints and the relative dynamics.

Result 1 [Permanent-temporary decomposition]: Suppose that the absolute dynamics of a model are given by the system (6). Suppose also that $A - B - C$ is invertible, there exists then a unique steady state vector $F_t = TZ_t$ consistent with (6), and a unique identification of temporary shocks Vv_t:

$$T = (A - B - C)^{-1} D, \quad W = [V, \ -CTH] \quad \text{and} \quad w_t = [v_t', \eta_t']'.$$

Moreover, if A, B and C are such that a unique, stationary, equilibrium exists, then the dynamics of $\tilde{X}_t$ are given by

$$A\tilde{X}_t = BE_t\left[\tilde{X}_{t+1}\right] + C\tilde{X}_{t-1} + Ww_t$$

and the steady state F_t is attracting:

$$\lim_{s\to\infty} E_t\left[\tilde{X}_{t+s}\xi^{-s}\right] \to 0 \Longrightarrow \lim_{s\to\infty} E_t[X_{t+s}] \to E_t[F_{t+s}] = F_t.$$

This result is instrumental in the solution of the absolute dynamics. Solution methods (based on standard QZ decompositions) for standard rational expectations models are now well established (e.g. Sims 2001). However, these methods do not readily allow for the presence of nonstationary driving processes (Klein 2000). Given that standard solutions do not work for this type of problem, they are only applied to solve for the relative dynamics, which can safely be assumed to be stationary. The above result allows one to extend the standard RE solution methods to solve for absolute dynamics by using the transitory-permanent decomposition of the system. More specifically, one should first solve for the solution of the transitory part, and then, given the solution for the transitory part, solve for the absolute dynamics. This procedure is summarized in Result 2.

Result 2 [Solution for absolute dynamics]: Consider the transitory dynamics obtained from system (6). Using Result 1, the dynamics has the form $A\tilde{X}_t = BE_t\left[\tilde{X}_{t+1}\right] + C\tilde{X}_{t-1} + Ww_t$. Based on the assumption of a unique equilibrium, there exist matrices Θ_0 and Θ_1, characterizing the solution to the transitory dynamics $\tilde{X}_t$ such that $\tilde{X}_t = \Theta_1\tilde{X}_{t-1} + \Theta_0 w_t$ (see Sims 2001). The solution for the absolute dynamics is given by

$$\begin{bmatrix} X_t \\ Z_t \end{bmatrix} = \begin{bmatrix} \Theta_1 & (I-\Theta_1)T \\ 0 & I \end{bmatrix} \begin{bmatrix} X_t \\ Z_t \end{bmatrix} + \begin{bmatrix} \Theta_{0,v} & \Theta_{0,\eta} + TH \\ 0 & H \end{bmatrix} \begin{bmatrix} v_t \\ \eta_t \end{bmatrix}$$

with $\Theta_{0,v}$ the part of Θ_0 corresponding to the shocks v_t and, $\Theta_{0,\eta}$ the part of Θ_0 corresponding to the shocks η_t.

Denoting the full macroeconomic state vector by $\breve{X}_t = [X_t', Z_t']'$, and defining the $(n+k)\times(n+k)$ matrices Φ_0 and Φ_1, the solution for the absolute dynamics can now be concisely written as

$$\breve{X}_t = \Phi_1\breve{X}_t + \Phi_0 w_t. \tag{7}$$

In the next section, we apply the above framework to solve for the absolute dynamics of a standard new-Keynesian model augmented with a interest policy rule that incorporates time-varying inflation targets.

2.2 A simple macro-model with time-varying inflation targets

We apply the above methodology to solve a monetary macroeconomic model. The model incorporates standard AS and AD equations. It is,however, different from standard specifications since we allow for a time-varying, long-run inflation target of the central bank.

The model starts with a standard inflation equation (Phillips curve) relating current inflation, π_t, to expected future and past inflation and current values of the output gap. This type of equation has a theoretical underpinning in the Calvo-type of price-setting behavior by firms (Calvo 1983). The appearance of past inflation expresses the idea that non-optimizing firms index according to past, observed, inflation (Galí and Gertler 1999). The output gap dependence of inflation models a cost-push inflation effect, assuming a linear relation between real marginal costs and the output gap.[6] The inflation equation can be summarized as

$$\pi_t = \alpha_\pi E_t\left[\pi_{t+1}\right] + (1-\alpha_\pi)\,\pi_{t-1} + \alpha_y y_t + \sigma_\pi v_{\pi,t}. \tag{8}$$

The output equation is also standard. The IS curve models the output gap, y_t, as a function of future expected and past output gap values, and of the ex ante real interest rate differential relative

[6] See Galí, Gertler and López-Salido (2001) for a discussion of the approximation of real marginal costs by means of output gaps

to the natural real rate of interest (ρ_t):

$$y_t = \beta_y E_t [y_{t+1}] + \left(1 - \beta_y\right) y_{t-1} + \beta_{yi} (i_t - E_t [\pi_{t+1}] - \rho_t) + \sigma_y v_{y,t}. \tag{9}$$

The first term on the RHS represents the standard consumption smoothing component, and the presence of lagged output gaps can be explained by the presence of a significant number of non-optimizing agents or in terms of a significant degree of habit persistence (Galí, Gertler and López-Salido 2001). Furthermore, we only allow for a real interest rate effect if the expected real interest rate differs from ρ_t. The latter variable can be interpreted as the inflation or equivalently output neutral real interest rate. The dynamics of the inflation neutral real interest rate is assumed to be exogenous and is further simplified to follow an AR(1) process:

$$\rho_t = c_\rho + \delta_\rho(\rho_{t-1} - c_\rho) + \sigma_\rho \varepsilon_{\rho,t}. \tag{10}$$

Finally, we close the model by assuming an interest rate policy rule determining the short-run interest rate. Although the equation is based on the standard Taylor rule, we introduce time-varying inflation targets for the central banker. This feature is atypical. Most papers implementing Taylor rules assume constant inflation targets. This assumption is, however, often rejected in empirical tests. Kozicki and Tinsley (2001, 2002), for example, show that introducing time variation in the targets is empirically important. Also, in the same line of research, Dewachter and Lyrio (2003) show that the time-varying inflation target correlates well with the level factor in the term structure of interest rates. Here, we incorporate these empirical results by postulating a specific martingale model for the inflation target at time t, π_t^*. More specifically, we assume that the dynamics of π_t^* follows a standard random walk process:

$$\pi_t^* = \pi_{t-1}^* + \sigma_{\pi^*} \eta_t. \tag{11}$$

The policy rule, conditional on the inflation target, is a hybrid of the standard backward and forward looking versions of the Taylor rule:

$$\begin{aligned} i_t = \rho_t^* + \pi_t^* + \gamma_{E\pi}(E_t [\pi_{t+1}] - \pi_t^*) + \gamma_{Ey} E_t [y_{t+1}] + \gamma_\pi(\pi_t - \pi_t^*) + \gamma_y y_t \\ + \gamma_{\pi-1}(\pi_{t-1} - \pi_t^*) + \gamma_{y-1} y_{t-1} + \gamma_i(i_{t-1} - \pi_t^* - \rho_t) + \sigma_i v_{i,t} \end{aligned} \tag{12}$$

This model is an example where the absolute dynamics are identified. More specifically, denoting the vector of macroeconomic variables by $X_t = [\pi_t, y_t, i_t, \rho_t]'$ and denoting the set of stochastic trends, possibly degenerated by $Z_t = [\pi_t^*, 1]$, the above equilibrium equations can be restated in terms of the following system:

$$AX_t = BE_t [X_{t+1}] + CX_{t-1} + DZ_t + Vv_t \tag{13}$$

with

$$
\begin{aligned}
A &= \begin{bmatrix} 1 & -\alpha_y & 0 & 0 \\ 0 & 1 & -\beta_i & \beta_i \\ -\gamma_\pi & -\gamma_y & 1 & -(1-\gamma_i) \\ 0 & 0 & 0 & 1 \end{bmatrix}, \; B = \begin{bmatrix} \alpha_\pi & 0 & 0 & 0 \\ -\beta_i & \beta_y & 0 & 0 \\ \gamma_{E\pi} & \gamma_{Ey} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \\
C &= \begin{bmatrix} 1-\alpha_\pi & 0 & 0 & 0 \\ 0 & 1-\beta_y & 0 & 0 \\ \gamma_{\pi-1} & \gamma_{y-1} & \gamma_i & 0 \\ 0 & 0 & 0 & \delta_\rho \end{bmatrix}, \; D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ -\gamma_{E\pi} - \gamma_\pi - \gamma_{\pi-1} + (1-\gamma_i) & 0 \\ 0 & c_\rho(1-\delta_\rho) \end{bmatrix}
\end{aligned} \tag{14}
$$

and

$$
V = \begin{bmatrix} \sigma_\pi & 0 & 0 & 0 \\ 0 & \sigma_y & 0 & 0 \\ 0 & 0 & \sigma_i & 0 \\ 0 & 0 & 0 & \sigma_\rho \end{bmatrix}.
$$

Using the above results we have a transitory-permanent decomposition if $(A-B-C)$ is invertible[7]. Assuming invertibility we find the permanent part, $F_t = TZ_t$, to be:

$$
\begin{aligned}
F_t &= TZ_t, \\
T &= (A-B-C)^{-1} D
\end{aligned} \tag{15}
$$

with

$$
Z_t = Z_{t-1} + \begin{bmatrix} \sigma_{\pi^*} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \eta_{1,t} \\ \eta_{2,t} \end{bmatrix}. \tag{16}
$$

yielding

$$
T = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 1 & c_\rho \\ 0 & c_\rho \end{bmatrix} \text{ and } F_t = \begin{bmatrix} \pi_t^* \\ 0 \\ \pi_t^* + c_\rho \\ c_\rho \end{bmatrix}. \tag{17}
$$

Also, the transitory dynamics $\tilde{X}_t = [\pi_t - \pi_t^*, y_t, i_t - \pi_t^* - c_\rho, \rho_t - c_\rho]'$ can be expressed relative

[7] Note that the conditions for invertibility can easily be obtained computing the determinant of $(A-B-C)$. It can be shown that the determinant equals $\alpha_y\beta_i(1-\delta_\rho)(1-\gamma_\pi-\gamma_{E\pi}-\gamma_{\pi-1}-\gamma_i)$. From this equation invertibility conditions can easily be derived as

$$
\begin{aligned}
&\beta_i\alpha_y \neq 0 \\
&\delta_\rho \neq 1 \\
&\gamma_\pi + \gamma_{E\pi} + \gamma_{\pi-1} + \gamma_i \neq 1
\end{aligned}
$$

Note that obviously existence of a steady state does not imply that it will be stable. Additional conditions will have to be imposed to guarrantee the stability of the solution.

to this stochastic endpoints as:

$$\begin{bmatrix} 1 & -\alpha_y & 0 & 0 \\ 0 & 1 & -\beta_i & \beta_i \\ -\gamma_\pi & -\gamma_y & 1 & -(1-\gamma_i) \\ 0 & 0 & 0 & 1 \end{bmatrix} \tilde{X}_t = \begin{bmatrix} \alpha_\pi & 0 & 0 & 0 \\ -\beta_i & \beta_y & 0 & 0 \\ \gamma_{E\pi} & \gamma_{Ey} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} E_t\left[\tilde{X}_{t+1}\right] +$$

$$\begin{bmatrix} 1-\alpha_\pi & 0 & 0 & 0 \\ 0 & 1-\beta_y & 0 & 0 \\ \gamma_{\pi-1} & \gamma_{y-1} & \gamma_i & 0 \\ 0 & 0 & 0 & \delta_\rho \end{bmatrix} \tilde{X}_{t-1} + \begin{bmatrix} \sigma_\pi & 0 & 0 & 0 & -(1-\alpha_\pi)\sigma_{\pi^*} \\ 0 & \sigma_y & 0 & 0 & 0 \\ 0 & 0 & \sigma_i & 0 & \left(-\gamma_{\pi-1}-\gamma_i\right)\sigma_{\pi^*} \\ 0 & 0 & 0 & \sigma_\rho & 0 \end{bmatrix} \begin{bmatrix} \varepsilon_{\pi,t} \\ \varepsilon_{y,t} \\ \varepsilon_{i,t} \\ \varepsilon_{\rho,t} \\ \eta_{1,t} \end{bmatrix} \tag{18}$$

Equation (18) specifies the temporary macroeconomic dynamics. A well-defined model will yield a solution where temporary dynamics eventually converge to 0. As such, it is now reasonable to assume that the structure of the model, i.e. the matrices A, B and C are such that they allow for a unique solution such that the growth condition (5) applies. As is well known, under the assumption of a unique solution, and satisfying the growth condition, there exists a solution for the temporary dynamics $\tilde{X}_t = \Theta_1\tilde{X}_{t-1} + \Theta_0 v_t$ which in its turn can then be transformed back into a solution for the full dynamics specification of the form:

$$\check{X}_t = \Phi_c + \Phi_1\check{X}_{t-1} + \Phi_0 v_t. \tag{19}$$

Definitions of the above matrices were made explicit in the previous section and can be obtained from the structural matrices A, B, C and F after applying the standard QZ decomposition described in Sims (2001).

2.3 Macro-economic models for the term structure

A number of papers have recently linked the macroeconomic dynamics with the term structure.[8] Given the linear structure of the reduced form macroeconomic dynamics, the standard model used in this type of analysis belongs to the class of affine term structure models. The advantage of this class of models is that it translates the linear state-space dynamics into linear relations between the yield curve and the state vector. Given the linear structure of the macroeconomic dynamics, it becomes relatively easy to relate the yield curve to the macroeconomic state.

The class of affine term structure models posits a linear (affine) functional form for the prices of risk, Λ_t:

$$\Lambda_t = \Lambda_0 + \Lambda_1\check{X}_t \tag{20}$$

where Λ_0 is an $n+k$ vector and Λ_1 an $(n+k)\times(n+k)$ matrix. Given this specification for the price of risk, the term structure becomes linear in the state variable $\check{X}_t$. Let $\bar{y}_t(\tau_i)$ denote the time

[8] For example, Ang and Piazzesi (2003), Bekaert, Cho and Moreno (2003), Berardi (2004), Dewachter and Lyrio (2003), Diebold, Rudebusch and Aruoba (2004), Hördahl, Tristani and Vestin (2003), Kozicki and Tinsley (2001, 2002), and Rudebusch and Wu (2003).

t yield on a bond with maturity τ_i, $i = 1, ..., n_y$ and denote by Y_t the n_y vector collecting these yields. The affine class of term structure models proves the existence of an n_y vector $\bar{A}_Y$ and an $n_y \times (n+k)$ matrix $\bar{B}_Y$ such that no-arbitrage conditions allow the following representation:

$$Y_t = \bar{A}_Y + \bar{B}_Y \check{X}_t \tag{21}$$

No-arbitrage conditions (i.e. the expectations hypothesis holds under the risk neutral measure) further identifies $\bar{A}_Y$ and $\bar{B}_Y$ by means of a set of ordinary differential equations (ODEs). Specifying $\bar{A}_Y = [\bar{a}_y(\tau_1), ..., \bar{a}_y(\tau_{n_y})]$ and $\bar{B}_Y = [\bar{b}_y'(\tau_1), ..., \bar{b}_y'(\tau_{n_y})]$ with $\bar{a}_y$ scalar functions and $\bar{b}_y$ a $1 \times (n+k)$ vector, being $\bar{a}_y(\tau) = -a_y(\tau)/\tau$ and $\bar{b}_y(\tau_1) = -b_y(\tau)/\tau$ we have that:

$$\begin{aligned} a_y(\tau+1) &= a_y(\tau) + b_y(\tau)(\Phi_c - \Phi_0\Lambda_0) + \tfrac{1}{2} b_y(\tau)\Phi_0\Phi_0' b_y(\tau)' - \delta_0 \\ b_y(\tau+1) &= b_y(\tau)\left[\Phi_1 - \Phi_0\Lambda_1\right] - \delta_1 \end{aligned} \tag{22}$$

where δ_0 and δ_1 are defined by $i_t = \delta_0 + \delta_1 \check{X}_t$. A proof of this solution is provided by Ang and Piazzesi (2003). Equation (21) thus defines the term structure relation between macroeconomic state and the yields, consistent with the absence of arbitrage opportunities. Note that while in principle the prices of risk are not restricted in the standard affine term structure model, i.e. Λ_0 and Λ_1 can be full and do not have to satisfy any restriction, some problems still remain. More specifically, there are potential problems related to the introduction of nonstationary stochastic endpoints. A second type of problem refers to the joint consistency of the macroeconomic dynamics and the term structure model.

Introducing a set of nonstationary factors, i.e. stochastic endpoints, entering the interest rate policy equation, renders the interest rates also nonstationary. Although nonstationarity for interest rates is not the standard finance approach, one cannot reject the null hypothesis of unit roots in interest rates. This result tends to hold across the maturity spectrum of interest rates and across countries. Without further restrictions, however, the model would go against a second robust empirical finding, i.e. the stationarity of the yield spreads. The general affine model of the term structure, in conjunction with the presence of nonstationary factors, allows for nonstationarity of the yield spreads. Condition $C1$ defines the necessary and sufficient conditions on the prices of risk to guarantee the stationarity of the yield spreads.

Condition C1 [Stationary yield spreads]: Assume that all of the above macroeconomic restrictions are satisfied and decompose Λ_1

$$\Lambda_1 = \begin{bmatrix} \Lambda_{1,n\times n}^{NN} & \Lambda_{1,n\times k}^{NK} \\ \Lambda_{1,k\times n}^{KN} & \Lambda_{1,k\times k}^{KK} \end{bmatrix}$$

then yield spreads will be stationary under the following restrictions:

$$\Lambda_{1,k\times n}^{KN} T = -\Lambda_{1,k\times k}^{KK}$$

$$\Lambda_{1,n\times n}^{NN} T = -\Lambda_{1,n\times k}^{NK}$$

A heuristic proof of these conditions, guaranteeing stationarity of the yield spreads is available in Appendix A.

A second type of problem is related to the joint modeling of the term structure and the macroeconomic dynamics. Although both the sub-model for the macroeconomic dynamics is consistent with the Euler equations and although the term structure is derived from no-arbitrage conditions, the two models taken together are not necessarily consistent. More specifically, a pricing kernel implicit in the term structure model is not necessary the pricing kernel implicit in the macroeconomic model and more in particular in the IS equation. Beckaert et al. (2003) show that consistency across models can only be obtained in a linear, homoskedastic macroeconomic model if risk premia are constant. This condition is restated here in terms of condition $C2$.

Condition C2 [Consistency between macro and term structure model]: Assuming normally distributed shocks in the macroeconomic dynamics, the term structure representation is consistent with the macroeconomic dynamics only if $\Lambda_1 = 0$.

Obviously, condition C2 implies condition C1 so that consistent models will also imply stationary yield spreads. In this paper, we take the most stringent approach by imposing full consistency across models. That is, we restrict the risk premia to constants an therefore obtain a reduced set of ODEs of the form:

$$\begin{aligned} a_y(\tau+1) &= a_y(\tau) + b_y(\tau)(\Phi_c - \Phi_0\Lambda_0) + \tfrac{1}{2}b_y(\tau)\Phi_0\Phi_0' b_y(\tau)' - \delta_0 \\ b_y(\tau+1) &= b_y(\tau)\left[\Phi_1\right] - \delta_1 \end{aligned} \tag{23}$$

3 Empirical results

In this section, we estimate the structural macroeconomic and term structure model for the US and the German economies. The underlying assumption throughout this section is that inflation, output gap and interest rates are observed. The other two macroeconomic variables, ρ and π^* are, however, not in the information set of the econometrician. The latent character of the natural real interest rate, ρ, and the long-run inflation target, π^*, necessitates the use of a filtering procedure to recover their respective time series. To this end, we apply the Kalman procedure. We do differentiate, however, between two cases. The first case is the *full information setting*. In this case, both macroeconomic and term structure information are included in the filtering procedure, i.e. both types of variables (observable and unobservable) enter in the measurement equation. The advantage of this approach is that, from an econometric point of view, all relevant information is taken into consideration. Nevertheless, from an economic point of view, it assumes the agents consider the exogenous inflation target as fully credible. The second case, the *restricted information setting*, limits the information set only to observable macroeconomic variables. The main advantage of this approach is that only macroeconomic information is used in filtering the two unobservable macroeconomic variables. The absence of term structure feedback, therefore, prevents potential distortions in the filtering procedure. From an economic point of view, this approach corresponds to some asymmetric information models with learning rules based on macroeconomic surprises (see, for instance, Kozicki and Tinsley 2003). In what follows, we first discuss the data used in the empirical analysis and then analyze the results for each of the mentioned cases.

3.1 Data

We estimate the proposed model using monthly data for the USA and Germany. The US data set covers the period 1970:01 until 2000:12 (372 observations). The German data set covers a shorter period, ranging from 1987:03 until 1998:12 (142 observations). This is due to the lack of swap rates data for the German mark before 1987 and in order to exclude the European Economic and Monetary Union (EMU) period, avoiding then the use of a sample period with different monetary regimes.

Each data set contains three series of macroeconomic observations obtained from Datastream: the year-on-year inflation based on the consumer price index (CPI), the output gap (constructed based on industrial production) and a short-term interest rate (maturity 1 month), representing the policy rate. The output gaps are constructed using the standard HP filter with lambda equal to 14400. For Germany, we use industrial production excluding construction in order to avoid possible effects from the German unification. Next to the macroeconomic variables, the data sets include ten yields with maturities 3, 6, and 9 months and 1, 2, 3, 4, 5, 7 and 10 years. For the US, we use data provided by Waggoner (1997), using the unsmoothed Fama-Bliss data sets. For Germany, the yields are constructed based on swap rate data also retrieved from Datastream.

Table 1 presents some descriptive statistics on the data sets described above. These statistics point to the usual observations: average term structures are increasing both in the USA and in Germany; the volatility of yields is decreasing in the maturity, an observation found both in the USA and in Germany; normality is rejected for both data sets (based on JB statistics); and,finally, all variables present strong inertia, with a first order autocorrelation coefficient typically higher than 0.95. The exception is the German output gap with an autocorrelation of approximately 0.73.

Insert Table 1

Table 2 presents the correlation structure for the data. We make two relevant observations. First, the yields are extremely correlated across the maturity spectrum. This points to the well-known fact that a few factors are able to explain a large part of the comovement of the yields. This conclusion holds for all maturities and both data sets. Second, there is a strong correlation between the term structure and the macroeconomic variables. The correlation is strongest between inflation and the term structure and remains significant even for long-term yields. The correlation between the yield curve and the output gap is smaller and becomes rather weak at the long end of the maturity spectrum. These correlation patterns suggest that macroeconomic variables might play an important role in dynamics of the term structure of interest rates.

Insert Table 2

Finally, since this paper introduces nonstationarity both in the interest rates and in the inflation series, we also present a third type of descriptive statistic of the data. In Table 3, we report the results of two standard unit root tests, the ADF and KPSS tests. The ADF test has the unit root as the null hypothesis while the KPSS test adopts the null hypothesis of mean (KPSS_μ) and trend (KPSS_τ) stationarity. Results are presented for the macroeconomic variables, and for a selection of yields and yield spreads. For the US data set, the test statistics unambiguously point to the rejection of stationarity in inflation and interest rates, while yield spreads are found to be stationary. This evidence on the yield spreads suggests that yields have identical loadings on the stochastic trend. Results are more ambiguous for the German data set. Although stationarity of inflation and interest rates are in general rejected, pointing again to nonstationarity, the evidence on the stationarity of yield spreads is not strong. The ADF test cannot reject the nonstationarity of the spreads, while the KPSS_μ test could not reject the stationarity hypothesis. The trend stationarity hypothesis, however, tends to be rejected. For the German yield spreads, no clear conclusions can be drawn. Given the limited data span and hence the relatively low power of the tests, some caution is appropriate in interpreting the test results. We do not draw any definitive conclusions concerning the nonstationarity of inflation and interest rates. The statistics are shown not to contradict the main modeling assumption, i.e. the nonstationarity in interest and inflation rates.

Insert Table 3

3.2 Full-information models

The framework presented in Section 2 allows the estimation of a variety of models. We restrict the estimation to the most restrictive version. In other words, we impose the consistency requirement across the macroeconomic and the term structure model.[9] The estimated models incorporate three features: (*i*) the macroeconomic dynamics are consistent with the structural macroeconomic model set out in Section 2.2; (*ii*) the term structure satisfies the no-arbitrage conditions and, more specifically, the expectation hypothesis; and (*iii*) the model is based on a unique pricing kernel consistent both with the term structure and the macroeconomic dynamics. Note that consistency also imposes stationary restrictions on the yield spreads.

3.2.1 Econometric issues

In the adopted setting, Kalman filter estimates are efficient. Conditional on a set of parameters collected in the vector ϑ, including the structural parameters in the matrices A, B, C, T and V, the

[9] See Bekaert et al. (2003) for a detailed discussion of this issue.

transition equation is defined by equation (19), which we repeat here for convenience:

$$\breve{X}_t = \Phi_c + \Phi_1 \breve{X}_{t-1} + \Phi_0 v_t. \tag{24}$$

The measurement equation includes both the (to the econometrician) observable macroeconomic variables and the yield vector. Let S be a selection matrix that identifies the observable macroeconomic variables in $\breve{X}_t$, i.e. $S\breve{X}_t$ is a vector of observable variables, a measurement equation can then be constructed on $S\breve{X}_t$ and Y_t:

$$\begin{bmatrix} S\breve{X}_t \\ Y_t \end{bmatrix} = \begin{bmatrix} 0 \\ \bar{A}_y \end{bmatrix} + \begin{bmatrix} S \\ \bar{B}_y \end{bmatrix} \breve{X}_t + \begin{bmatrix} 0 & 0 \\ 0 & Q \end{bmatrix} \chi_t \tag{25}$$

which we restate in matrix notation as

$$X_{M,t} = M_c + M_1 \breve{X}_t + M_0 \chi_t \tag{26}$$

where χ denotes the measurement errors in the yield data.[10] Note that this procedure implies that information in the term structure is used when filtering the unobservable variables ρ and π^*. This has the advantage that the information in the term structure, i.e. market expectations about future macroeconomic developments, is used efficiently. Conditional on the parameter vector ϑ, unobserved factors can be filtered consistently using the standard Kalman filter updating equations. Denoting by $\breve{X}_{t|t}$ the filtered vector of macroeconomic dynamics, $\breve{X}_{t|t}$ can be obtained recursively:

$$\breve{X}_{t|t} = \breve{X}_{t|t-1} + P_{t|t-1} M_1' (M_1 P_{t|t-1} M_1' + M_0' M_0)^{-1} (X_{M,t} - M_c + M_1 \breve{X}_{t|t-1}) \tag{27}$$

with

$$\begin{aligned} \breve{X}_{t|t-1} &= \Phi_c + \Phi_1 \breve{X}_{t-1} \\ P_{t|t-1} &= \Phi_1 P_{t-1|t-1} \Phi_1 + \Phi_0 \Phi_0' \\ P_{t|t} &= P_{t|t-1} - P_{t|t-1} M_1' (M_1 P_{t|t-1} M_1' + M_0 M_0')^{-1} M_1 P_{t|t-1}. \end{aligned} \tag{28}$$

The above filtering procedure is conditional on a set of structural parameters contained in ϑ. In a second step, these parameters can be estimated consistently by standard QML methods. Applying the Kalman filter thus allows for the identification of the model if the above stated assumptions are made explicit in the optimization procedure. From an operational perspective, a consistent set of estimates is obtained by solving a constrained maximization process on the loglikelihood $l(v;\vartheta)$:

$$\begin{aligned} l(\vartheta) \;\; = \;\; & \sum_t -\frac{1}{2} \left| M_1 P_{t|t-1} M_1' + M_0 M_0' \right| \\ & -\frac{1}{2} (X_{M,t} - M_c + M_1 \breve{X}_{t|t-1})' \left(M_1 P_{t|t-1} M_1' + M_0 M_0' \right)^{-1} (X_{M,t} - M_c + M_1 \breve{X}_{t|t-1}) \end{aligned} \tag{29}$$

[10] Note that we impose the perfect updating condition for the observable macroeconomic variables. This updating condition is equivalent to a zero measurement error restriction on the maroeconomic observable variables.

under the condition that:

$$\lim_{s\to\infty} E_t\left[\tilde{X}_{t+s}\xi^{-s}\right] \to 0, \quad \xi \geq 1, \quad \text{and the solution is unique.} \tag{30}$$

These conditions can be easily imposed using the algorithms provided by Sims (2001).

3.2.2 Results

Table 4 presents the estimation results of the structural model under the full information assumption. First we discuss parameters related to the structural macroeconomic model. For the US, parameter estimates are generally in line with estimates presented in the literature. We find strong evidence in favor of the Calvo-type of price-setting theory. Inflation tends to be affected both by the forward and backward looking components. Our results suggest that both components are almost equally important with a slight dominance of the forward looking part. The parameter estimates for the inflation equation conform well with other studies. For instance Cho and Moreno (2002), using quarterly data, report an estimate for α_π in between 0.52 and 0.6. The Phillips curve parameter α_y is estimated significantly at 0.0042. This is larger than the value obtained by Cho and Moreno (0.0011) and unlike many studies is statistically significant. Similar parameter estimates have been reported in various papers, e.g. Galí and Gertler (1999) or Fuhrer and Moore (1995).

Insert Table 4

The output dynamics also follows closely estimates reported in the literature. We find that both the backward and forward components are of importance, suggesting the presence of significant habit formation. This finding is in line with the results of Fuhrer and Rudebusch (2003). Our parameter estimate for β_y of 0.52 is again in line with, for instance, Cho and Moreno (2002), who obtain a value of 0.49. Also for the interest rate elasticity of output (β_i), we find a significant estimate of about -0.01. Other papers typically find values around -0.005. This is the case in Cho and Moreno, while Fuhrer and Rudebush report estimates between -0.008 and -0.02. In contrast to various results reported in the literature, this interest rate parameter is statistically significant. The introduction of a natual real interest rate, ρ_t, seems to improve the support for significant real interest rate effects.

Finally, we find a significant effect of inflation and output gap in the policy rule. Moreover, by construction, the policy rule is stable. Somewhat unlike standard models, we estimate a relatively low interest rate smoothing coefficient. The estimate of 0.7 on a monthly frequency corresponds to a quarterly value of 0.34 on a quarterly basis. This estimate is considerably smaller than most findings in the literature but has some support in Rudebush (2002) who finds that current interest smoothing parameters suffer from an upward omitted variable bias. Given that we introduce two additional factors, i.e. the inflation neutral real interest rate and the inflation target, it should come as no surprise the finding that the interest inertia parameter drops significantly. Overall, the main conclusion is that the estimated parameters are reasonable and in line with most existing studies. The introduction of the term structure in the measurement equation does not distort in any significant way the estimation of the structural parameters.

The parameter estimates for the German data are in line with the US data. Both the inflation equation and the output equations have significant backward and forward components. Note, moreover, that also the output elasticity of inflation and the interest rate effect on output are estimated to be of the same order as in the US case. For the German data, however, the latter two coefficients are not significant. Empirical studies for Germany are not abundant. Our estimates conform,however, with some of the existing studies. The (almost) equal weighting of forward and backward components is also found in Smets (2000) for a yearly frequency. Smets (2000) also reports similar backward and forward weights for the output equation. Additional evidence supporting our coefficients can be found in Chadha et al. (1992). The estimates of Hördahl et al. (2003) differ significantly from ours. In this study, backward components are larger than our estimates. As far as the policy rule is concerned, we find the US and the German rule to differ significantly. Similar to the US case, we find a moderate interest rate inertia (0.88 on a monthly basis, or 0.68 on a quarterly basis), which could be attributed to the inclusion of ρ and π^* in the policy rule. However, while the US policy rule loads primarily on current values of inflation and output, we find more moderate responses in the German case (0.074 and about 0.01 for inflation and output, respectively).

Next to estimating the macroeconomic part of the model, the model also provides an explanation for the term structure dynamics. Concerns of consistency between the macro and the term structure model restrict significantly the flexibility of affine term structure models. Despite the imposed tight parameterization, we find, surprisingly, that the macro model performs well in fitting the term structure. Table 5 presents the measurement errors for the US and German yields. In both cases, we find relatively small measurement errors. For the US, the standard deviation of the measurement errors is smaller than 25 basis points, being in most cases close to 10 basis points. For comparison, full latent models (standard three factor models) typically find values of about 5 to 10 basis points.[11]

Insert Table 5

Figure 1 presents the observable macroeconomic variables and the filtered series for the two unobservable factors, ρ_t and π_t^*. The model fit of the term structure is depicted in Figure 2 and shows a very accurate fit for the whole range of maturities. Figure 3 depicts for each maturity the loadings on the different macroeconomic variables. The three traditional macroeconomic variables are particularly important for the shorter maturities, while the ρ and π^* become dominant for the longer maturities. Note, however, that this interpretation should be taken with some caution as interest rates and inflation also depend on the variables ρ and π^*. In order to disentangle the effects of ρ and especially π^* from the other observable factors, we plot the yield loadings that correspond

[11] See, for example, de Jong (2000) and Dewachter and Lyrio (2003)

to the transitory-permanent decomposition of the state vector.[12] Figure 4 presents these loadings. This figure shows three types of effects. First, the inflation target π^* now exerts an identical effect across the entire yield curve. The fact that loadings on this factor are identical stems from the restrictions on the prices of risk and guarantees the stationarity of yield spreads (see Section 2.3). This factor thus models shifts of the entire yield curve and can be related to the level effect, alluded to in the finance literature. Next, the loadings on (transitory) interest deviations decrease monotonically in the maturity. This factor models thus primarily the slope factor. Temporary interest rate disequilibria are therefore associated with the yield spreads. More specifically, all else equal, restrictive monetary policy is associated with low spreads, while expansionary policy tends to increase the yield spread. Finally, disequilibrium in either output, inflation or the real interest rate bring about a curvature effect. This type of disequilibrium will primarily affect the intermediate maturities.

Insert Figures 1 to 4

The observable variables and the filtered series for the German case are shown in Figure 5. The fit of the German yield curve can be seen in Figure 6 and is as precise as the one for the US term structure. In the German case, we find the measurement errors to be of the order of 10 basis points. Figures 7 and 8 depict the loadings for each of the factors. As in the US case, we find a clear level effect represented by π^*, a slope factor in terms of temporary interest rate disequilibrium, and curvature factors in terms of temporary inflation, output and real interest rate deviations. The conclusion that emerges over the two data sets is that in general the two models give a rather homogeneous explanation of both the macroeconomic and the term structure dynamics.

Insert Figures 5 to 8

3.3 Restricted information models

In the previous section, the term structure was used as an additional information variable. While this is an efficient econometric approach, it also has some drawbacks. The main econometric disadvantage is that term structure information feeds back into the filtered macroeconomic variables ρ and π^*. This feedback generates uncertainty about the determinants of the filtered variables. More specifically, the filtered values could be distorted so as to fit better the term structure dynamics. In order to avoid this feedback from the term structure into the filtered macroeconomic variables, we exclude the term structure from the measurement equation. We use this set-up primarily as a means to investigate how much of the inflation target could be rationalized in terms of observable macroeconomic variables.

[12] The transitory-permanent decomposition transforms the state vector $\check{X}_t$ into the state vector $\mathring{X}_t$,which contains the temporary deviations, $X_t - F_t$, on the first n rows and the stochastic trends on the last k rows. Defining the matrix performing this operation by $L : \mathring{X}_t = L\check{X}_t$, the transformed loadings are defined by: $\bar{B}_y L^{-1}$. The matrix L is specfied in the appendix.

3.3.1 Econometric issues

The econometric procedure can easily be adapted to incorporate only observable macroeconomic variables in the filtering of long-run inflation targets. Using the definition introduced in the previous section, observable macroeconomic variables are given by $S\breve{X}_t$. By changing the measurement equation (26) to the identity $S\breve{X}_t = S\breve{X}_t$, we effectively eliminate term structure information entering into the filtering equation. More specifically, conditional on the transition equation (24) and the redefined measurement equation, the updating equations reduce to:

$$\breve{X}_{t|t} = \breve{X}_{t|t-1} + P_{t|t-1}S'(SP_{t|t-1}S')^{-1}(S\breve{X}_t - S\breve{X}_{t|t-1}) \tag{31}$$

with

$$\begin{aligned} P_{t|t-1} &= \Phi_1 P_{t-1|t-1}\Phi_1 + \Phi_0\Phi_0' \\ P_{t|t} &= P_{t|t-1} - P_{t|t-1}S'(SP_{t|t-1}S')^{-1}SP_{t|t-1} \end{aligned} \tag{32}$$

The updating equations are only a function of the prediction errors of observable macroeconomic variables and thus no longer depend on the prediction errors with respect to term structure variables. Therefore, term structure information is no longer used in the filtering procedure. We do use, however, the term structure information in the estimation of the parameter of the model. The likelihood function is modeled in terms of the full set of prediction errors as

$$\begin{aligned} l(\vartheta) \;=\; & \sum_t -\frac{1}{2}\left|M_1 P^A_{t|t-1}M_1' + M_0M_0'\right| \\ & -\frac{1}{2}(X_{M,t} - M_c + M_1\breve{X}_{t|t-1})'\left(M_1P_{t|t-1}M_1' + M_0M_0'\right)^{-1}(X_{M,t} - M_c + M_1\breve{X}_{t|t-1}), \end{aligned} \tag{33}$$

which can be maximized with respect to ϑ under the constraints imposed by equation (30).

Equation (31) has an interesting interpretation in terms of asymmetric information models. Recently, various papers model learning effects in reduced-form VAR systems. One example of this type of models is Kozicki and Tinsley (2003). The main idea in this line or research is to estimate transition equations that are adapted to the information set of private agents. Typical in this literature is the assumption that agents do not observe or believe in the inflation target announced by the central bank but, instead, filter it from prediction errors. Kozicki and Tinsley (2003), for instance, assume that agents change the perceived target by a fraction of the interest rate surprise. The current version of the updating equation can be interpreted in this context. First, the implied VAR dynamics now correspond to a underlying structural model. Second, it generates a transition equation (eq. (31)) adapted to the information generated from observable macroeconomic variables only. The implicit learning rule U in the macroeconomic dynamics maps the prediction errors $S(\breve{X}_t - \breve{X}_{t|t-1})$ into updated beliefs, $\breve{X}_{t|t} = \breve{X}_{t|t-1} + US(\breve{X}_t - \breve{X}_{t|t-1})$. Moreover, the learning rule U is not defined in an ad hoc way. Given that L corresponds to the Kalman filter updating matrix, the learning rule is a mean-squared optimal rule. The unconditional version of the learning rule is given by:

$$U_{\infty} = P_{\infty|\infty} S'(SP_{\infty|\infty} S')^{-1} \tag{34}$$

where $P_{\infty|\infty}$ denotes the unconditional version of the prediction error variance-covariance matrix. This version of the model thus provides as an interesting side-effect a learning rule for agents conditioning only on macroeconomic information.

3.3.2 Results

The estimated parameters for the restricted information case can be seen in Table 4. Interestingly, for both the US and German samples, we find that the structural macroeconomic parameters are not significantly affected by the reduction in the information set. The inclusion of the term structure does not seem, therefore, to have great influence over the parameter estimates of the structural model. Nevertheless, the inferences regarding the inflation target do seem to be affected in a important way. Comparing Figures 1 and 9 (or the top panels in Figure 17) for the USA and Figures 5 and 13 for Germany, one observes a clear drop in the variability of the long-run inflation targets. In this aspect, the information set used, including term structure information (full information) or only macroeconomic information (restricted information), seems to affect the results in a strong way. This fact is not really surprising given that the yield curve is treated and used differently in these two versions of the model. In the full information version, the term structure is used to filter both ρ and π^*. This filtering procedure implies that the filter for ρ and π^* take into consideration the prediction errors of both the macroeconomics and the term structure. The filtered variables are, therefore, partially determined by the term structure characteristics. In the alternative model, filters for ρ and π^* are only based on macroeconomic information. The filtering procedure thus completely ignores the term structure variables. Note, however, that as far as ρ is concerned, filtered values do not differ significantly between the two versions of the model. Both for the US and Germany, we find the filtered values to be qualitatively similar.

Insert Figures 9 to 16

Inferences for π^* only based on macroeconomic information turn out to be much better at replicating agent's expectations. Figure 17 presents the implied one-year and ten-year average inflation forecasts based on the respective models and compares it to survey expectations. These survey expectations were only available for the US and are provided by the Federal Reserve Bank of Philadelphia (Survey of Professional Forecasters). The left panels present model-based forecasts and survey forecasts for the full information case. The right panels present the equivalent forecasts for the model using macroeconomic information only. Both models track with some success the survey inflation forecasts, although both models tend to be excessively volatile compared to the survey expectations. Comparing the one-year inflation forecasts, we observe hardly any difference between the two models. Differences between the full and the restricted information models become more evident for longer forecasting horizons. However, the restricted model clearly tracks better the survey data than the full information model. The latter still displays some excess volatility while the former tracks survey expectations closely.

Insert Figure 17

Restricting information variables to the set of observable macroeconomic variables has, however, one major drawback. Where the full information version fits the term structure extremely well, the restricted model fails significantly in modeling the yield curve adequately. Comparing Figure 2 to 10 for the US and Figure 6 to 14 for Germany, illustrates clearly the misfit of restricted models. The fitting errors, moreover, tend to increase with the maturity of the yields. Some descriptive statistics for these fitting errors is presented in Table 6. One observes that for both countries and all yields, the fitting errors for the restricted case are more volatile and present a higher first order autocorrelation.

Insert Table 6

As mentioned above, a by-product of the restricted information model is the learning matrix U. This matrix gives the optimal learning rules for private agents in the versions of asymmetric information models introduced by Kozicki and Tinsley (2003). Below, the respective learning matrices are presented for the USA and Germany:

$$U^{USA} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ -0.45 & -0.18 & 0.92 \\ 0.31 & 0.02 & 0.02 \end{bmatrix}, \quad U^{GER} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ -0.39 & -0.01 & 1.89 \\ 0.19 & -0.01 & 0.04 \end{bmatrix}.$$

The optimal learning matrices, consistent with the estimated macroeconomic models, are qualitatively similar. This is due to the similarity of the parameter estimates for both countries. More important is the observation that mainly interest rate and inflation surprises are used as information variables. More specifically, both for the US and German data, interest rate surprises, corrected for inflation and output surprises, are primarily seen as accommodating changes in ρ. Changes in the long-run inflation target are filtered mainly through inflation surprises, both in the US and in the German case. These results contrast with the findings of Kozicki and Tinsley (2003), who find that interest rate surprises significantly affect the inferences concerning the inflation target. This apparent contradiction in results could, however, be due to the fact that in our setting an additional interest rate variable (ρ) is included.

4 Conclusions

This paper proposes an econometric methodology that allows the solution and estimation of the macroeconomic dynamics in nonstationary environments. The method uses a two-step procedure.

First, a transitory-permanent decomposition on the Euler equations is performed. In a second step, the transitory dynamics are solved using standard QZ-based solution techniques and the permanent dynamics are substituted back into the solution of the transitory dynamics. This procedure extends the standard macroeconomic models by solving the macro model with the inclusion of the steady state dynamics.

Solving for the full dynamics is important in many types of applications. This paper focuses on models linking the macroeconomic dynamics to the term structure of interest rates. Our model differs from the standard approach in that we do not assume stationary long-run inflation expectations. Instead, we assume that the central bank's inflation target is time-varying and that it follows a martingale process, which renders the macroeconomic dynamics nonstationary. The above mentioned technique is the applied to solve for the macroeconomic dynamics. The inclusion of the nonstationary components yields reasonable structural parameters for the macroeconomic dynamics. The implied time variation in the long-run inflation expectations (target) turns out to be significant both for the US and German models. Finally, we find that the introduction of time-varying long-run expectations is crucial for the fitting of the term structure. When both the macro and the yield curve information is used to filter the unobserved factors in the model, we find that macroeconomic factors, including long-run inflation expectations, are able to explain the term structure in a very accurate way. The proposed methodology, therefore, contributes to the further development of fully consistent, rational, and arbitrage-free models of the term structure of interest rates.

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5 Appendices

5.1 Appendix A

In this appendix, we give a heuristic proof of the conditions set out in condition C1. Consider the following state space dynamics:

$$\breve{X}_t = \Phi_c + \Phi_1 \breve{X}_{t-1} + \Phi_0 w_t \tag{35}$$

and define a selection matrix L performing a transitory-permanent decomposition on $\breve{X}_t$. That is, defining $\mathring{X}_t = L\breve{X}_t$ we construct L in such a way that the first n entries in $\mathring{X}_t$ are stationary by construction and the last k elements of $\mathring{X}_t$ denote the stochastic trends. Within the setting of our model, we have that the selection matrix L takes the following form:

$$L = \begin{bmatrix} I_{n\times n} & -T_{n\times k} \\ 0_{k\times n} & I_{k\times k} \end{bmatrix} \tag{36}$$

Given the dynamics of $\breve{X}_t$, the dynamics of $\mathring{X}_t$ are defined by:

$$\begin{aligned} \breve{X}_t &= \Phi_c + \Phi_1 \breve{X}_{t-1} + \Phi_0 w_t \\ L\breve{X}_t &= L\Phi_c + L\Phi_1 L^{-1} L\breve{X}_{t-1} + L\Phi_0 L^{-1} L w_t \\ \mathring{X}_t &= \mathring{\Phi}_c + \mathring{\Phi}_1 \mathring{X}_{t-1} + \mathring{\Phi}_0 \mathring{w}_t. \end{aligned} \tag{37}$$

The matrices $\mathring{\Phi}_1$ and $\mathring{\Phi}_0$ are defined in terms of the matrices generated by the Sims (2001) procedure, i.e. Θ_0, Θ_1 and Θ_c :

$$\mathring{\Phi}_1 = \begin{bmatrix} \Theta_1 & 0_{n\times k} \\ 0_{k\times n} & I_{k\times k} \end{bmatrix}, \mathring{\Phi}_0 = \begin{bmatrix} \Phi_{0,n\times n} & \Phi_{0,n\times n}T - T\Phi_{0,k\times k} \\ 0_{k\times n} & \Phi_{0,k\times k} \end{bmatrix}.$$

Given the above transitory-permanent decomposition, we now adapt the bond loadings to the redefined state space. This new loadings correspond to the loadings on the transitory and permanent (nonstationary) components. In order to have stationary yield spreads, the loadings on the non-stationary components need to be identical across yields. Imposing this condition yields, as shown below, a set of necessary conditions in terms of the prices of risk.

Using the affine term structure model, yields satisfy the following relation:

$$Y_t = \bar{A}_Y + \bar{B}_Y \breve{X}_t \tag{38}$$

which can be rewritten in terms of the new state vector $\mathring{X}_t$ as:

$$Y_t = \bar{A}_Y + \bar{B}_Y L^{-1} L\breve{X}_t = \bar{A}_Y + \mathring{B}_Y \mathring{X}_t \tag{39}$$

with $\mathring{B}_Y = \bar{B}_Y L^{-1}$. Consider a yield with maturity τ. Based on the above relation between the yields and the state vector, the yield will load on the transitory and permanent variables with loadings $[-\tau^{-1}\mathring{b}_T(\tau)_{1\times n}$, -$\tau^{-1}\mathring{b}_P(\tau)_{1\times k}]$, where $\mathring{b}_T(\tau)_{1\times n}$ denotes non-scaled loadings on the transitory variables and $\mathring{b}_P(\tau)_{1\times k}$ a $1\times k$ vector of (non-scaled) loadings on the non-stationary variables. The loading of yield spreads can hence be defined as the difference of the loadings of the respective yields. Letting $y(\tau_1)$ and $y(\tau_2)$ denote yields with maturities τ_1 and τ_2, the loadings on the yield spreads are given by

$$\begin{aligned} y(\tau_1)-y(\tau_2) &= c(\tau_1,\tau_2) \\ &+[-\tau_1^{-1}\mathring{b}_T(\tau_1)_{1\times n}-(-\tau_2^{-1}\mathring{b}_T(\tau_2)_{1\times n}), -\tau_1^{-1}\mathring{b}_P(\tau_1)_{1\times k}-(-\tau_2^{-1}\mathring{b}_P(\tau_2)_{1\times k})]\mathring{X}_t \end{aligned}$$

Stationarity of yield spreads now implies that the loadings of yield spreads on the nonstationary factors equal zero. This implies that conditions need to be imposed only on the loadings of the nonstationary factors:

$$\mathring{b}_P(\tau_1)_{1\times k}=\frac{\tau_1}{\tau_2}\mathring{b}_P(\tau_2)_{1\times k} \text{ for all } \tau_1,\tau_2>0.$$

Consider the system of ODEs generating the loadings:

$$\begin{aligned} a_y(\tau+1) &= a_y(\tau)+b_y(\tau)(\Phi_c-\Phi_0\Lambda_0)+\tfrac{1}{2}b_y(\tau)\Phi_0\Phi_0'b_y(\tau)'-\delta_0 \\ b_y(\tau+1) &= b_y(\tau)\left[\Phi_1-\Phi_0\Lambda_1\right]-\delta_1 \end{aligned}$$

The loadings $\mathring{b}_P(\tau)$ and $\mathring{b}_T(\tau)$ can be obtained by transforming the loadings obtained from the above system of ODEs. More specifically, the transformed loadings are generated by the system of ODEs:

$$\begin{aligned} b_y(\tau+1)L^{-1} &= b_y(\tau)\left[\Phi_1-\Phi_0\Lambda_1\right]L^{-1}-\delta_1L^{-1} \\ b_y(\tau+1)L^{-1} &= b_y(\tau)L^{-1}L\left[\Phi_1-\Phi_0L^{-1}L\Lambda_1\right]L^{-1}-\delta_1L^{-1} \\ \mathring{b}_y(\tau+1) &= \mathring{b}_y(\tau)\mathring{\Phi}_1-\mathring{b}_y(\tau)\mathring{\Phi}_0\mathring{\Lambda}_1-\mathring{\delta}_1 \end{aligned} \tag{40}$$

with $\mathring{\Phi}_0\mathring{\Lambda}_1$:

$$\begin{bmatrix} \Phi_{0,n\times n}\Lambda_1+(\Phi_{0,n\times n}-T\Phi_{0,n\times k})\Lambda_{1,k\times n}^{KN} & \Phi_{0,n\times n}(\Lambda_{1,n\times n}^{NN}T+\Lambda_{1,n\times k}^{NK})+(\Phi_{0,n\times k}-T\Phi_{0,k\times k})(\Lambda_{1,k\times n}^{KN}T+\Lambda_{1,k\times k}^{KK}) \\ \Phi_{0,k\times k}\Lambda_{1,k\times n}^{KN} & \Phi_{0,k\times k}(\Lambda_{1,k\times n}^{KN}T+\Lambda_{1,k\times k}^{KK}) \end{bmatrix}$$

The dynamics of the loadings on the nonstationary components are now separated from the loadings on the transitory components and the dynamics of the loadings van be written as:

$$\begin{aligned} &\mathring{b}_P(\tau+1)=\mathring{b}_P(\tau)-\mathring{b}_P(\tau)(\Phi_{0,k\times k}(\Lambda_{1,k\times n}^{KN}T+\Lambda_{1,k\times k}^{KK})) \\ &-\mathring{b}_T(\tau)(\Phi_{0,n\times n}(\Lambda_{1,n\times n}^{NN}T+\Lambda_{1,n\times k}^{NK})+(\Phi_{0,n\times k}-T\Phi_{0,k\times k})(\Lambda_{1,k\times n}^{KN}T+\Lambda_{1,k\times k}^{KK})) \\ &-\mathring{\delta}_1 \end{aligned} \tag{41}$$

Sufficient conditions for stationarity of yield spread are given by the following conditions:

$$\begin{aligned}
&(\Phi_{0,n\times n}(\Lambda^{NN}_{1,n\times n}T+\Lambda^{NK}_{1,n\times k})+(\Phi_{0,n\times k}-T\Phi_{0,k\times k})(\Lambda^{KN}_{1,k\times n}T+\Lambda^{KK}_{1,k\times k}))=0\\
&(\Phi_{0,k\times k}(\Lambda^{KN}_{1,k\times n}T+\Lambda^{KK}_{1,k\times k}))=0\\
&\mathring{b}_P(\tau)=0
\end{aligned}\tag{42}$$

such that $\mathring{b}_P(\tau)=-\tau\mathring{\delta}_1$ which automatically satisfies the conditions for stationarity of the yield spreads. Without further restrictions on the matrix Φ_0, the above restrictions can be satisfied by restricting the matrix Λ_1. More specifically, the condition is satisfied for any matrix Φ_0 if:

$$\begin{aligned}
\Lambda^{KN}_{1,k\times n}T&=-\Lambda^{KK}_{1,k\times k}\\
\Lambda^{NN}_{1,n\times n}\,T&=-\Lambda^{NK}_{1,n\times k}.
\end{aligned}\tag{43}$$

6 Tables and Figures

Table 1: Summary of data statistics

	USA				Germany			
	Mean	Std.	Auto	JB	Mean	Std.	Auto	JB
π	4.990	2.897	0.997	86.834^{***}	2.484	1.476	0.994	10.982^{***}
y	-0.021	2.199	0.948	52.392^{***}	-0.010	1.914	0.731	2.142
i	6.445	2.582	0.995	148.919^{***}	5.795	2.311	0.998	15.317^{***}
$\overline{y}_{3m}$	6.757	2.655	0.996	141.364^{***}	5.816	2.303	0.998	15.226^{***}
$\overline{y}_{6m}$	6.984	2.662	0.996	120.786^{***}	5.804	2.259	0.998	15.011^{***}
$\overline{y}_{9m}$	7.106	2.639	0.996	109.349^{***}	5.766	2.188	0.998	14.723^{***}
$\overline{y}_{1y}$	7.202	2.569	0.996	95.342^{***}	5.750	2.130	0.998	14.536^{***}
$\overline{y}_{2y}$	7.458	2.443	0.997	88.951^{***}	6.013	1.983	0.998	13.546^{***}
$\overline{y}_{3y}$	7.631	2.341	0.997	89.662^{***}	6.177	1.771	0.998	12.107^{***}
$\overline{y}_{4y}$	7.769	2.284	0.998	86.687^{***}	6.341	1.585	0.998	9.942^{***}
$\overline{y}_{5y}$	7.841	2.248	0.998	78.792^{***}	6.495	1.453	0.998	7.438^{**}
$\overline{y}_{7y}$	7.987	2.182	0.998	79.840^{***}	6.748	1.234	0.998	2.223
$\overline{y}_{10y}$	8.047	2.135	0.999	76.080^{***}	7.007	1.063	0.998	0.917

Notes: *Mean* denotes the sample average, expressed as percentage per year, *Std* standard deviation, *Auto* the first order monthly autocorrelation, *JB* the Jarque-Bera normality test statistic, where *** indicates that the null of normality can be rejected at the 1% significance level, and ** at the 5% confidence level.

Table 2: Summary of data statistics - Correlation matrix

USA

	π	y	i	$\overline{y}_{3m}$	$\overline{y}_{6m}$	$\overline{y}_{9m}$	$\overline{y}_{1y}$	$\overline{y}_{2y}$	$\overline{y}_{3y}$	$\overline{y}_{4y}$	$\overline{y}_{5y}$	$\overline{y}_{7y}$	$\overline{y}_{10y}$
π	1.000												
y	0.101	1.000											
i	0.639	0.284	1.000										
$\overline{y}_{3m}$	0.649	0.274	0.995	1.000									
$\overline{y}_{6m}$	0.654	0.272	0.987	0.996	1.000								
$\overline{y}_{9m}$	0.647	0.262	0.980	0.990	0.998	1.000							
$\overline{y}_{1y}$	0.622	0.253	0.973	0.984	0.994	0.998	1.000						
$\overline{y}_{2y}$	0.575	0.203	0.944	0.958	0.973	0.982	0.989	1.000					
$\overline{y}_{3y}$	0.538	0.162	0.916	0.932	0.949	0.961	0.971	0.994	1.000				
$\overline{y}_{4y}$	0.512	0.148	0.894	0.911	0.928	0.942	0.954	0.985	0.997	1.000			
$\overline{y}_{5y}$	0.498	0.126	0.877	0.894	0.912	0.927	0.940	0.977	0.993	0.998	1.000		
$\overline{y}_{7y}$	0.476	0.120	0.851	0.868	0.888	0.904	0.918	0.962	0.982	0.991	0.996	1.000	
$\overline{y}_{10y}$	0.465	0.111	0.827	0.845	0.865	0.881	0.896	0.944	0.968	0.981	0.988	0.995	1.000

Germany

	π	y	i	$\overline{y}_{3m}$	$\overline{y}_{6m}$	$\overline{y}_{9m}$	$\overline{y}_{1y}$	$\overline{y}_{2y}$	$\overline{y}_{3y}$	$\overline{y}_{4y}$	$\overline{y}_{5y}$	$\overline{y}_{7y}$	$\overline{y}_{10y}$
π	1.000												
y	0.080	1.000											
i	0.843	0.159	1.000										
$\overline{y}_{3m}$	0.823	0.182	0.997	1.000									
$\overline{y}_{6m}$	0.799	0.209	0.990	0.997	1.000								
$\overline{y}_{9m}$	0.781	0.235	0.982	0.992	0.998	1.000							
$\overline{y}_{1y}$	0.766	0.256	0.974	0.986	0.995	0.999	1.000						
$\overline{y}_{2y}$	0.711	0.311	0.936	0.955	0.972	0.983	0.989	1.000					
$\overline{y}_{3y}$	0.679	0.330	0.909	0.929	0.949	0.963	0.973	0.995	1.000				
$\overline{y}_{4y}$	0.653	0.332	0.884	0.905	0.926	0.942	0.953	0.984	0.996	1.000			
$\overline{y}_{5y}$	0.623	0.333	0.856	0.878	0.900	0.917	0.930	0.968	0.987	0.997	1.000		
$\overline{y}_{7y}$	0.566	0.313	0.797	0.819	0.842	0.861	0.875	0.924	0.954	0.975	0.989	1.000	
$\overline{y}_{10y}$	0.508	0.287	0.735	0.756	0.779	0.797	0.812	0.868	0.906	0.936	0.959	0.990	1.000

Table 3: Summary of data statistics - Unit root tests

	USA			Germany		
	ADF	KPSS$_\mu$	KPSS$_\tau$	ADF	KPSS$_\mu$	KPSS$_\tau$
π	-1.761	1.270*	0.165*	-1.330	0.278	0.259*
y	-5.654*	0.020	0.020	-2.952*	0.051	0.051
i	-2.303	0.603*	0.338*	-2.198	0.455*	0.245*
$\overline{y}_{3m}$	-2.134	0.621*	0.346*	-1.945	0.475*	0.241*
$\overline{y}_{1y}$	-2.103	0.686*	0.386*	-1.490	0.510*	0.227*
$\overline{y}_{2y}$	-2.044	0.708*	0.432*	-1.227	0.523*	0.218*
$\overline{y}_{5y}$	-1.895	0.736*	0.496*	-0.918	0.534*	0.212*
$\overline{y}_{7y}$	-1.829	0.731*	0.515*	-0.796	0.541*	0.210*
$\overline{y}_{10y}$	-1.822	0.734*	0.532*	-0.610	0.545*	0.204*
$\overline{y}_{1y} - i$	-5.194*	0.182	0.118	-2.585	0.193	0.178*
$\overline{y}_{2y} - i$	-4.356*	0.123	0.124	-2.412	0.166	0.167*
$\overline{y}_{5y} - i$	-3.339*	0.161	0.138	-2.228	0.244	0.191*
$\overline{y}_{7y} - i$	-3.303*	0.184	0.138	-2.193	0.271	0.197*
$\overline{y}_{10y} - i$	-3.128*	0.203	0.146	-2.156	0.290	0.201*

Notes: The critical value for the ADF (uniformly estimated with 12 lags is -2.88 at the 5% significance level. Critical values for the KPSS$_\mu$ (null hypothesis of stationarity) and KPSS$_\tau$ (null hypothesis of trend stationarity) at the 5% confidence level are 0.463 and 0.146, respectively.

Table 4: Parameter estimates

		USA				Germany			
		Full		Asymmetric		Full		Asymmetric	
π-eq.	α_π	0.5061	(0.0055)	0.5085	(0.0025)	0.5007	(0.0085)	0.5072	(0.0049)
	α_y	0.0043	(0.0011)	0.0035	(0.0008)	0.0056	(0.0019)	0.0051	(0.0019)
y-eq.	β_y	0.5252	(0.0085)	0.5301	(0.0061)	0.5669	(0.0136)	0.5555	(0.0114)
	β_{yi}	-0.0116	(0.0055)	-0.0136	(0.0031)	-0.0016	(0.0015)	-0.0005	(0.0009)
i-eq.	γ_π	0.3092	(0.0204)	0.3310	(0.0074)	0.0743	(0.0630)	0.0825	(0.0248)
	γ_y	0.2703	(0.0346)	0.2858	(0.0276)	0.0089	(0.0148)	0.0090	(0.0132)
	$\gamma_{\pi-1}$	-0.0006	(0.0145)	-0.0008	(0.0012)	0.0429	(0.0670)	0.0313	(0.0219)
	γ_{y-1}	-0.1301	(0.0306)	-0.1672	(0.0235)	-0.0091	(0.0112)	-0.0084	(0.0082)
	γ_i	0.6914	(0.0066)	0.6698	(0.0072)	0.8828	(0.0098)	0.8862	(0.0095)
ρ-eq.	c_ρ	7.4e-9	(4.5e-7)	7.4e-9	(6.3e-8)	3.9e-5	(1.2e-4)	8.7e-5	(9.5e-5)
	δ_ρ	0.9872	(0.0014)	0.9937	(0.0007)	0.9969	(0.0034)	0.9958	(0.0023)
	σ_π	0.00165	(0.00005)	0.00169	(0.00005)	0.00173	(0.00010)	0.00172	(0.00008)
	σ_y	0.00408	(0.00011)	0.00408	(0.00011)	0.00785	(0.00048)	0.00728	(0.00038)
	σ_i	0.00563	(0.00012)	0.00554	(0.00009)	0.00280	(0.00012)	0.00269	(0.00011)
	σ_ρ	0.00765	(0.00021)	0.00627	(0.00030)	0.00583	(0.00045)	0.00595	(0.00083)
	σ_{π^*}	0.00335	(0.00015)	0.00108	(0.00004)	0.00200	(0.00018)	0.00066	(0.00007)
	λ_π	2.5688	(0.8706)	1.8144	(0.3217)	-0.0104	(0.0486)	0.0001	(0.0009)
	λ_y	-2.4383	(0.9940)	-2.4194	(0.4080)	-0.2469	(0.1073)	-0.0304	(0.0550)
	λ_i	-0.0507	(0.2354)	-0.0125	(0.0918)	0.0132	(0.0611)	0.0184	(0.0473)
	λ_ρ	-0.0685	(0.0624)	-0.0368	(0.0127)	-0.1461	(0.0663)	-0.0983	(0.0312)
	λ_{π^*}	-0.0062	(0.0259)	-0.0038	(0.0245)	-0.0050	(0.0204)	-0.0050	(0.0307)
average lnlik		73.4928		72.3263		78.6562		76.9474	

Notes: Maximum likelihood estimates with robust standard errors between brackets. The total average likelihood excludes a constant.

Table 5: Parameter estimates - Standard deviation of measurement error

	USA		Germany	
	Full	Asymmetric	Full	Asymmetric
R_{3m}	0.00176	0.00171	0.00106	0.00101
R_{6m}	0.00111	0.00101	0.00108	0.00106
R_{9m}	0.00077	0.00064	0.00128	0.00111
R_{1y}	0.00111	0.00116	0.00163	0.00135
R_{2y}	0.00110	0.00116	0.00056	0.00049
R_{3y}	0.00093	0.00091	0.00020	0.00025
R_{4y}	0.00085	0.00080	0.00032	0.00032
R_{5y}	0.00070	0.00069	0.00031	0.00026
R_{7y}	0.00148	0.00163	0.00052	0.00066
R_{10y}	0.00227	0.00244	0.00117	0.00136

Table 6: Summary statistics of fitting errors

	USA					
	Full			Asymmetric		
	Mean	Std.	Auto	Mean	Std.	Auto
R_{3m}	0.030	0.172	0.286	0.014	0.303	0.432
R_{6m}	0.012	0.101	0.343	-0.002	0.469	0.679
R_{9m}	-0.007	0.056	0.546	-0.016	0.569	0.746
R_{1y}	-0.006	0.099	0.438	-0.012	0.609	0.778
R_{2y}	0.004	0.104	0.606	-0.006	0.719	0.827
R_{3y}	0.003	0.083	0.454	-0.010	0.750	0.854
R_{4y}	0.012	0.073	0.230	0.001	0.763	0.847
R_{5y}	-0.012	0.052	0.584	-0.018	0.767	0.877
R_{7y}	0.013	0.142	0.523	0.014	0.778	0.876
R_{10y}	-0.001	0.222	0.720	-0.012	0.795	0.903
	Germany					
	Full			Asymmetric		
	Mean	Std.	Auto	Mean	Std.	Auto
R_{3m}	0.022	0.099	0.249	0.033	0.330	0.025
R_{6m}	0.003	0.111	0.548	0.016	0.400	0.374
R_{9m}	-0.050	0.112	0.721	-0.040	0.470	0.541
R_{1y}	-0.091	0.116	0.823	-0.086	0.526	0.629
R_{2y}	0.019	0.048	0.639	0.009	0.688	0.750
R_{3y}	0.001	0.012	0.318	-0.012	0.741	0.807
R_{4y}	-0.005	0.029	0.577	-0.018	0.760	0.841
R_{5y}	-0.002	0.026	0.655	-0.013	0.775	0.870
R_{7y}	0.005	0.040	0.607	-0.006	0.796	0.910
R_{10y}	-0.006	0.107	0.811	-0.028	0.782	0.932

Notes: *Mean* denotes the sample average, expressed as percentage per year, *Std* standard deviation, *Auto* the first order monthly autocorrelation.

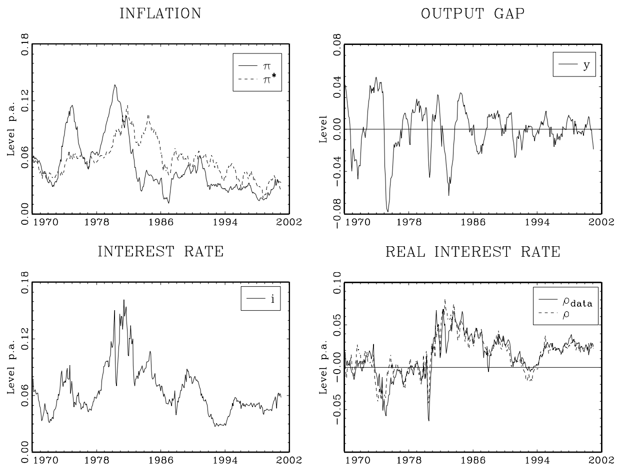

Figure 1: Macroeconomic factors (USA, 1970:01-2000:12) - Full Information.

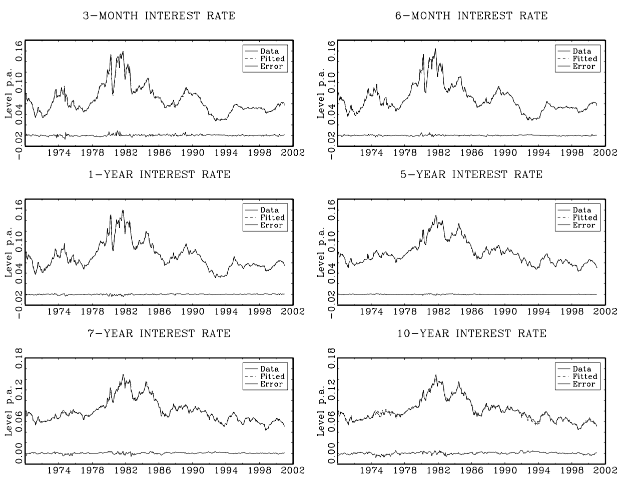

Figure 2: Model fit of the term structure of interest rates (USA, 1970:01-2000:12) - Full Information.

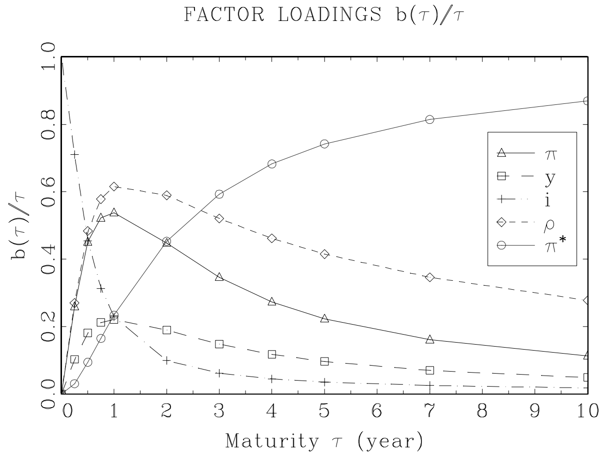

Figure 3: Factor loadings (USA, 1970:01-2000:12) - Full Information.

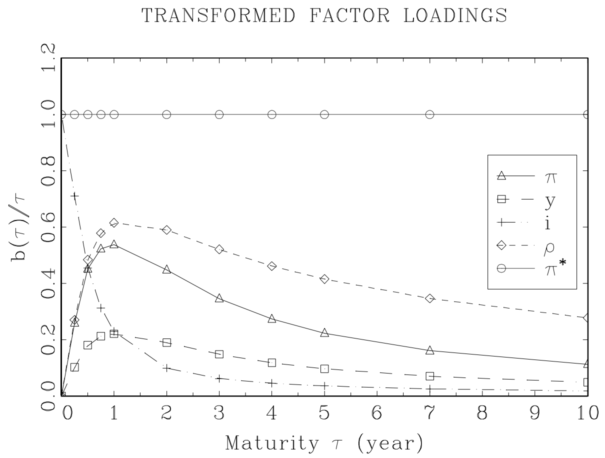

Figure 4: Transformed factor loadings (USA, 1970:01-2000:12) - Full Information.

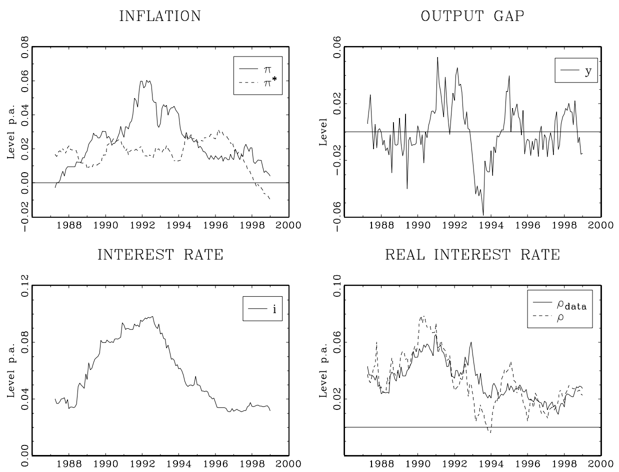

Figure 5: Macroeconomic factors (Germany, 1987:03-1998:12) - Full Information.

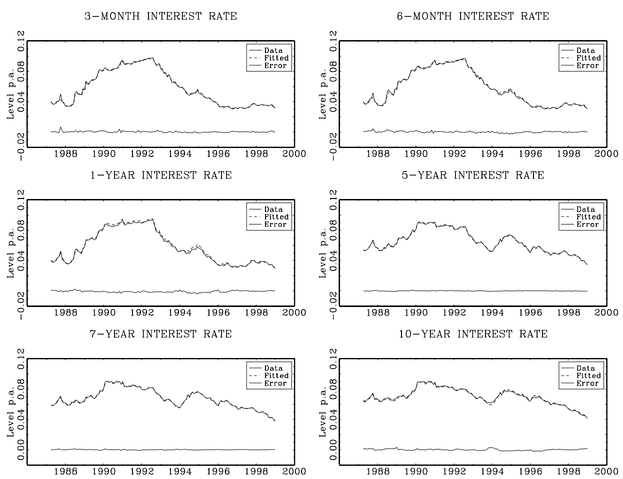

Figure 6: Model fit of the term structure of interest rates (Germany, 1987:03-1998:12) - Full Information.

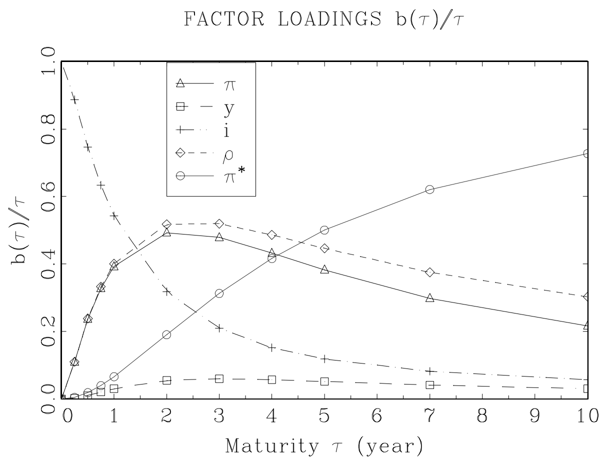

Figure 7: Factor loadings (Germany, 1987:03-1998:12) - Full Information.

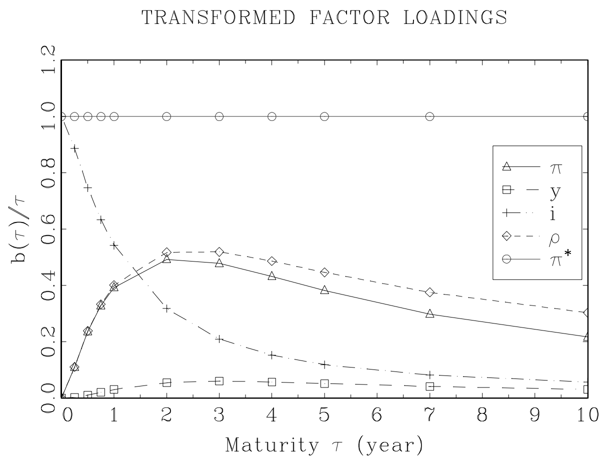

Figure 8: Transformed factor loadings (Germany, 1987:03-1998:12) - Full Information.

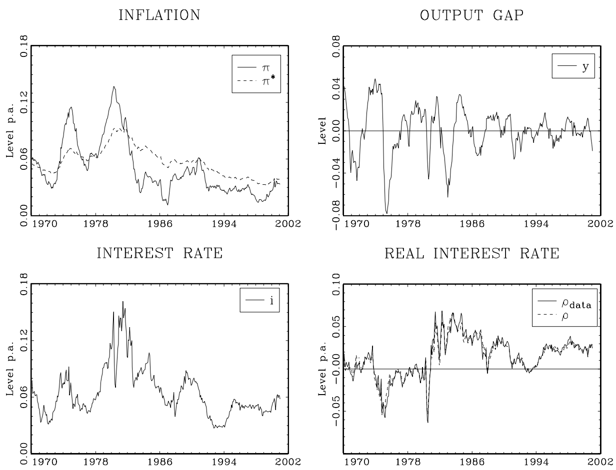

Figure 9: Macroeconomic factors (USA, 1970:01-2000:12) - Asymmetric Information.

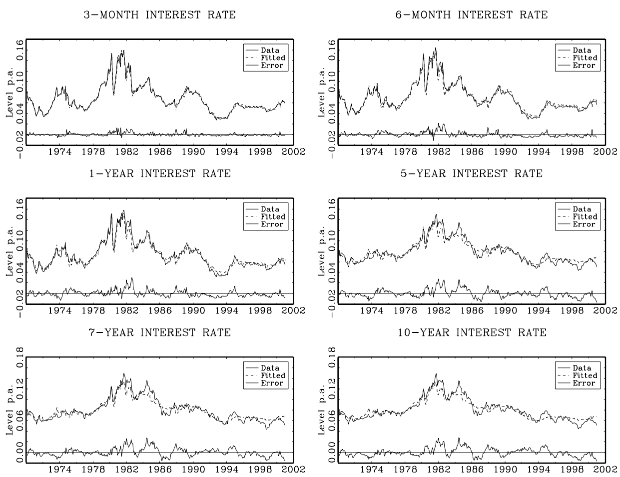

Figure 10: Model fit of the term structure of interest rates (USA, 1970:01-2000:12) - Asymmetric Information.

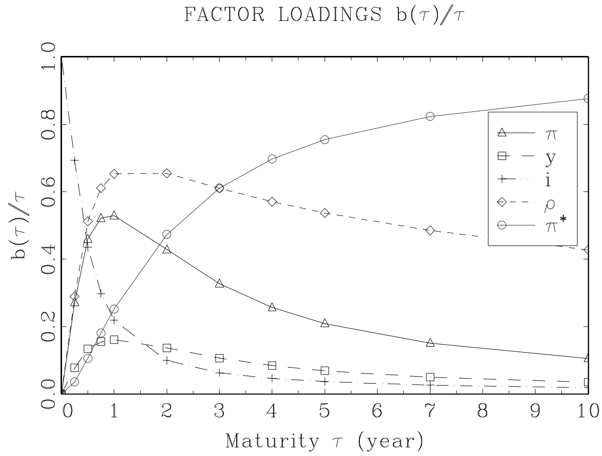

Figure 11: Factor loadings (USA, 1970:01-2000:12) - Asymmetric Information.

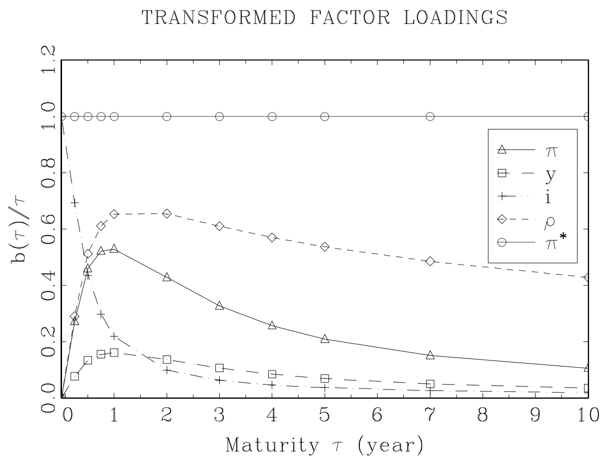

Figure 12: Transformed factor loadings (USA, 1970:01-2000:12) - Asymmetric Information.

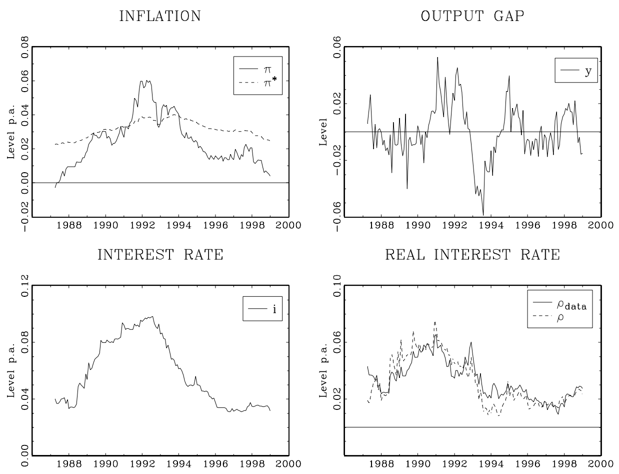

Figure 13: Macroeconomic factors (Germany, 1987:03-1998:12) - Asymmetric Information.

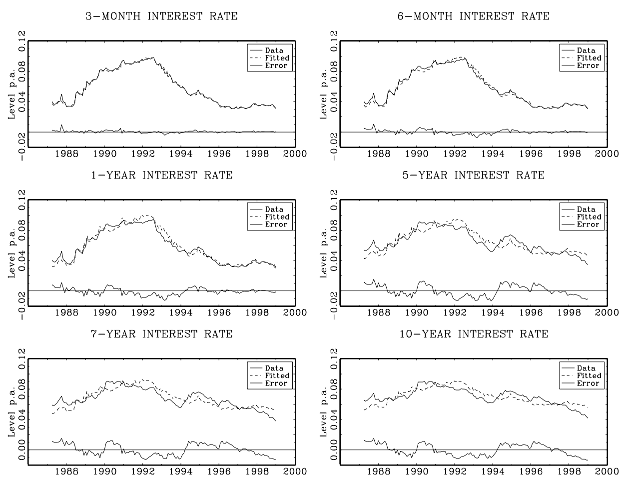

Figure 14: Model fit of the term structure of interest rates (Germany, 1987:03-1998:12) - Asymmetric Information.

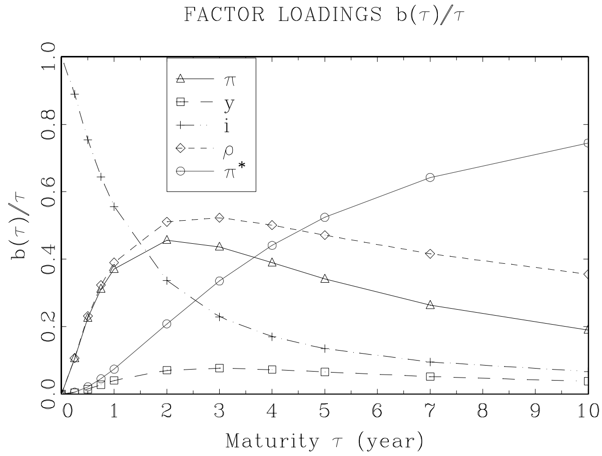

Figure 15: Factor loadings (Germany, 1987:03-1998:12) - Asymmetric Information.

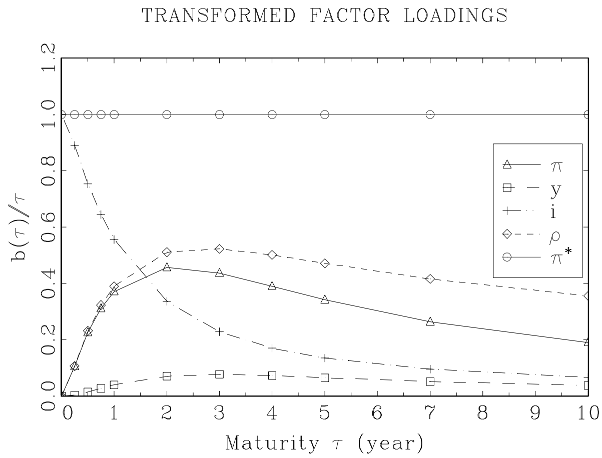

Figure 16: Transformed factor loadings (Germany, 1987:03-1998:12) - Asymmetric Information.

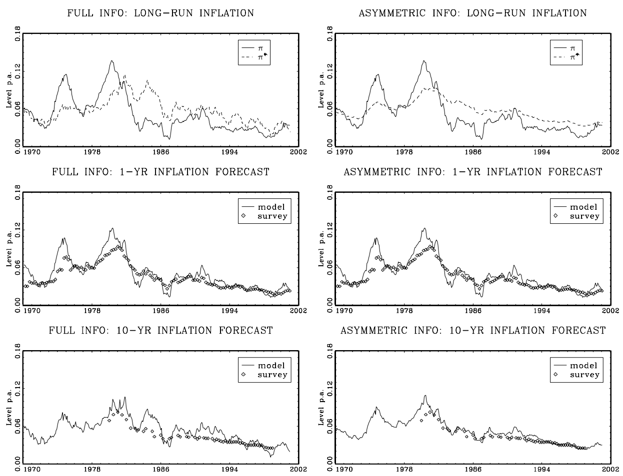

Figure 17: Long-run inflation forecast, 1-year and 10-year inflation forecast (USA, 1970:01-2000:12) - Full and Asymmetric Information.