International Capital Mobility and Aggregate Volatility: the Case of Credit-Rationed Open Economies*

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Abstract

This paper studies how international capital mobility affects aggregate volatility by considering the case of *imperfect* financial markets such that only physical capital serves as collateral for international borrowing, whereas human capital cannot. We find that credit-rationed, small open economies may be destabilized by expectation-driven fluctuations, even in the presence of small externalities, provided that the share of human capital is low enough. It follows that economies that highly borrow on international credit markets are more susceptible, through a financial accelerator effect, to expectation-driven fluctuations. Moreover, opening the capital account may push a saddle-point stable economy on a volatile path. On the contrary, tighter constraints on external borrowing may protect the economy against expectation-driven volatility. In contrast with existing results relying on the assumption of perfect financial markets, both the elasticity of labor supply and the elasticity of intertemporal substitution in consumption, though less traditionally, condition the set of parameter values associated with expectation-driven fluctuations. Finally, we extend the basic model and show, first, that tax progressivity (resp. regressivity) may protect (resp. expose) the economy against (resp. to) expectation-driven volatility and, second, that our main results do not specifically depend on the presence of externalities.

Keywords: international financial markets, endogenous borrowing constraints, progressive taxation, indeterminacy and expectation-driven fluctuations.

Journal of Economic Literature Classification Numbers: E44, E62, F34, F43 .

1 Introduction

For a long time, the potential destabilizing effect of financial market globalization has been a recurrent topic in economic analysis, not to mention the public debate. The concern in this theme has risen parallel to the substantial development of interdependence in modern economies, a phenomenon that one usually, although roughly, dates back to the end of World War II. Recently, a number of studies have reexamined an important aspect of this debated question, by studying how international capital mobility affects volatility in benchmark business-cycle models. In particular, notable contributions by Lahiri [31], Weder [44], Nishimura and Shimomura [34], Meng and Velasco [33], have shown that local indeterminacy and sunspots are more likely in two-sector open economies: in essence, fluctuations driven by self-fulfilling expectations may occur for very small levels of productive externalities, independent of the level of curvature in utility for consumption. However, Lahiri [31], Nishimura and Shimomura [34], Meng and Velasco [33] consider the borderline case of *perfect* capital markets which, as intuition suggests, may be more conducive to instability as it imposes no limit to international capital flows. On the other hand, Weder [44, pp 351-6] incorporates imperfect credit markets by assuming that the international interest rate is an increasing function of the debt/capital ratio. He then shows that indeterminacy may be reconciled with almost constant returns provided that the interest rate is not too responsive to the debt/capital ratio. However interesting this specification is, it leads to a four-dimensional dynamical system that is not easily amenable to analytical study.

Another group of notable and related contributions include Boyd and Smith [13], Aghion, Bacchetta and Banerjee [2], Matsuyama [32]. In a spirit that differs from the papers cited above, Boyd and Smith [13] and Matsuyama [32] show how imperfect financial markets (subject to costly-state verification or potential default, respectively) may result in a polarized world with rich and poor countries and, therefore, explain persistent inequality among nations. On the other hand, Aghion, Bacchetta and Banerjee [2] focus on the impact of credit constraints (due to moral hazard) on endogenous volatility of open economies. However, their framework is designed to describe more adequately financially-based *crises*, rather than business cycles occurring at higher frequency.

Overall, the extent to which existing instability results depend on the assumption of (im)perfect credit markets seems to remain open to discussion and further study.

In the present paper, we focus on a simple framework which models the working of a *credit-rationed*, one-sector, small open economy: we assume that infinitely-lived agents are not allowed to use their *human* capital as collateral for international borrowing, while the stock of *physical* capital determines a limit on what debtors can borrow from international credit markets. In other words, the debt/physical capital ratio is constrained to be less than one, as in Barro, Mankiw and Sala-i-Martin [5], although we reasonably expect similar results to hold in alternative environments, such as the one considered for instance by Cohen and Sachs [19] (which imposes, in a model without human capital, a constant upper bound on the debt/physical capital ratio). This feature is therefore a major difference with the models considered by Lahiri [31], Weder [44], Nishimura and Shimomura [34], Meng and Velasco [33], and it leads arguably to a simpler model than those considered in Boyd and Smith [13], Aghion, Bacchetta and Banerjee [2] or Matsuyama [32].

Our assumption, although simple, seems to accord quite well to the actual working of international financial markets, which implies that many countries belonging to the lower end of the world income distribution do not borrow internationally, presumably because they have low collateral. On the other hand, it is also in agreement with the observation that some countries coping with a high external debt have a large share of physical capital in ouput (as, for instance, Burkina Faso, Cote d'Ivoire, Jordan or Peru, defined as "highly indebted" countries by the statistical appendix of World Bank's *Global Development Finance 2003*, tables A51 and A52, pp. 232-5; see section 3).

The major result of this paper is that local indeterminacy is compatible with almost *constant* social returns to scale - small externalities - provided that the share of human capital is low enough. Therefore, this paper highlights an additional channel through which (short-term) international capital mobility may contribute to increase volatility in modern economies: *indebted economies that face loose borrowing constraints - that is, economies that have large shares of physical capital in total capital - and may, therefore, highly borrow on international credit markets, are more susceptible to expectation-driven fluctuations.* On the contrary, however, tighter borrowing constraints may protect the economy against expectation-driven volatility: when the share of human capital in total capital becomes larger that some threshold values, the economy is then saddle-point stable and, therefore, protected against fluctuations driven by self-fulfilling expectations. This shows that imposing, quite reasonably, some limits on international borrowing does not prevent open economies from being exposed to "animal spirits", which may seem to contrast with one's intuition. In fact, economies that borrow very little internationally, because of low collateral, may still be destabilized by endogenous volatility (in contrast with results by Aghion, Bacchetta and Banerjee [2] or Matsuyama [32]). In this sense, our simple framework sheds some light on the robustness of the results cited earlier: open economies are more susceptible to some form of aggregate instability, driven by expectations, even when international financial markets are *imperfect* and impose some borrowing limits. However, endogenous volatility is more naturally interpreted, in our context, as driving high-frequency business cycles in indebted economies, as opposed to causing unfrequent regime switches (with capital inflows followed by capital outflows, as for instance in Aghion, Bacchetta and Banerjee [2]).

The basic intuition underlying our results may be easily grasped, as shown in section 3: economies that have enough collateral and, therefore, borrow a lot internationally, are more likely to be driven by self-fulfilling expectations because of the following mechanism. Essentially, optimistic expectations drive up current consumption, labor supply, output and investment, which is perfectly consistent with rational expectations, intertemporal equilibria. In our credit-constrained economy, an additional feature of the boom is that the physical stock increases, thereby allowing *foreign debt* to go up. Eventually, increasing interest payments will negatively affect output, so that consumption, investment and debt will go down: pessimistic expectations will then prevail during the temporary recession. In short, a traditional *financial accelerator effect* may exacerbate volatility, through international credit, in credit-constrained, open economies.

Note that, in our framework, instability is consistent with two different aspects that are shown to be more likely when international capital mobility is introduced: the economy may be subject to expectation-driven fluctuations, and may also be perturbed by amplified and persistent effects of temporary shocks. In both views, net foreign debt and interest payments, although cycling, remain positive near the steady state, in the perturbed economy.

Section 3 also discusses, in more detailed way, the effects of financial integration and shows that, for moderate externalities, open economies may be subject to endogenous volatility whereas the corresponding closed economies would experience monotone convergence. This section is also illustrated by the cases of several countries that are classified as "highly indebted" and are also highly volatile. In the real world, there is some evidence that financial integration and aggregate volatility (of output and income) may be positively correlated (this was the case in the 1990s, according to Kose, Prasad and Terrones [29, Table 3]), which is, as discussed in Section 3, broadly consistent with the predictions of our model.

In contrast with existing results relying on the assumption of perfect credit markets, we show that both the elasticity of labor supply and, though less traditionally, the elasticity of intertemporal substitution in consumption condition the set of parameter values associated with endogenous fluctuations. More precisely, we find that indeterminacy requires labor supply to be elastic enough but, quite surprisingly, intertemporal substitution to be *weak enough*. In fact, indeterminacy may be ruled out when intertemporal substitution is almost perfect: this stands also in contrast with results obtained in closed-economy models; see, for instance, Benhabib and Farmer [7, 8], Bennett and Farmer [11], Harrison [25], Hintermaier [26], Pintus [36]. Compare to results by Aghion, Bacchetta and Banerjee [2], who find that financial crises *cannot* occur when the supply of country-specific input is highly elastic, while some of their simulations suggest that regime switches are more frequent when intertemporal substitution is weak enough. On the other hand, Weder [44, pp 351-6] assumes logarithmic utility and, therefore, does not examine this issue, while Boyd and Smith [13] and Matsuyama [32] abstract from such a discussion.

We also combine other mechanisms in our model, which do not appear in the studied cited above. Most importantly, we impose, in Appendix C, distortionary taxes on income such that the tax rate may be either *progressive* - that is, increasing with the tax base - or *regressive*. Although in contrast with the usual assumption of a flat tax rate, this is in agreement with income tax schedules prevailing both in OECD countries, e.g. in the U.S. or European countries (see, for instance, the *Statistical Abstracts of the United States*, or *Tax and the Economy: A Comparative Assessment of OECD Countries, OECD Tax Policy Studies* *n. 6, 2002*) and in developing countries. In the real world, a number of mechanisms introduce regressivity in otherwise progressive income tax schedules, most notably through proportional consumption taxes, upper bounds that are imposed on social security contributions, absent or low capital income taxes (especially in developing countries; see, e.g., Tanzi and Zee [42]). Therefore, one cannot rule out the case of effective taxes being indeed regressive. However, the specific formulation that we study turns out not to be critical for our main conclusion: Appendix B shows that indeterminacy is in fact compatible with *constant* returns to scale, in the absence of externalities, and confirms our main results regarding the impact of credit constraints, in a version of the model with predetermined public expenditures financed by distortionary income taxes. In section 2, we present the simpler model with externalities but without taxes (as we just mentioned, the extension with progressive taxation is studied in Appendix C), as it lightens the exposition by avoiding to deal first with labor taxes and then to add, for robustness purposes, capital taxes, income-elastic government expenditures, and so on.

In addition, we also consider the case of *elastic* labor supply, in contrast with Lahiri [31], Weder [44], Nishimura and Shimomura [34] or Boyd and Smith [13], Matsuyama [32]. Meng and Velasco [33] assume elastic labor but conclude that, in a *two-sector* economy, labor supply elasticity is not relevant, although their analysis relies on a related, but borderline hypothesis, as labor can be *costlessly* reallocated from one sector to the other. In contrast, we show that labor supply elasticity plays a key role. More traditionally, we finally assume that firms benefit from productive externalities that generate increasing social returns to scale (as in Benhabib and Farmer [7, 8]). Therefore, our analysis does not rely on either the questionable assumption of negative externalities or the implicit presence of nonstationary fixed costs implying zero profits (as in Weder [44], Nishimura and Shimomura [34], Meng and Velasco [33]). Again, the presence of externalities is not critical, as we just said.

The present paper follows a large strand of the literature, starting with Scheinkman and Weiss [40], Bernanke and Gertler [12], which has examined the implications of imperfect credit markets on aggregate outcomes (see, e.g., Galor and Zeira [20], Kocherlakota [28], Kiyotaki and Moore [27], Aghion and Bolton [1], among others). Within the enormous literature on international economics, most papers deal with financial or currency *crises* in open-economy models (see Obstfeld [35], Aghion, Bacchetta and Banerjee [2], Tirole [43] for more references), while only few papers provide a theoretical analysis of the possible sources of business cycles, as do Boyd and Smith [13], Matsuyama [32]. In contrast with the latter analyses, we abstract from specific microeconomic aspects of market imperfections and, therefore, study what we think is a simple and more tractable model. In particular, our framework models agents as being infinitely-lived, working and consuming in each period, while Boyd and Smith [13] and Matsuyama [32] consider overlapping generations economies. Moreover, the latter authors introduce indivisible investment, which we do not. Finally, Boyd and Smith [13], Aghion, Bacchetta and Banerjee [2] assume that two distinct classes of agents are present in constant proportion, named borrowers and lenders, and that only borrowers are able to invest in production (in the model studied by Matsuyama [32], the proportion of borrowers is endogenous). In contrast, we make no such distinction.

This paper is also closely related to recent papers by Schmitt-Grohe and Uribe [41], Christiano and Harrison [18], Guo and Lansing [22], Guo and Harrison [21]. These authors have shown that distortionary income taxes may destabilize the economy. In particular, persistent, self-fulfilling fluctuations are more likely when the government follows a balanced-budget rule implying regressive taxation. Here, we consider a quite stylized formulation of an elastic tax rate that was also studied by Guo and Lansing [22]. On the other hand, Altig *et al.* [3], Benabou [6], Cassou and Lansing [15], Caucutt, Imrohoroglu and Kumar [16], among others, have studied the growth effects of non-uniform tax rates. In particular, Cassou and Lansing [15], Caucutt, Imrohoroglu and Kumar [16] consider progressive taxes on personal income when calibrating their models, therefore ignoring regressive aspects of the actual tax systems. However, all the latter authors examine closed-economy versions of neoclassical models.

The rest of the paper is organized as follows. Section 2 presents the open-economy model, derives its dynamics and conditions leading to local indeterminacy. Section 3 discusses, more specifically, how the tightness of international borrowing constraints affects the likelihood of local indeterminacy. Finally, some concluding remarks and directions for future research are gathered in Section 4.

2 Indeterminacy in a Credit-Rationed Open Economy

2.1 Model and Laws of Motion

The economy produces a tradeable good Y by using physical capital K, human capital H, and raw labor L, according to the following technology:

$$Y = AK^{\zeta}H^{\eta}L^{1-\zeta-\eta}, \tag{1}$$

where A is total factor productivity, $\zeta \geq 0$, $\eta \geq 0$ and $\zeta + \eta < 1$.

The Ramsey households have preferences represented by:

$$\int_0^{\infty} e^{-\rho t}\{\frac{C(t)^{1-\theta}}{1-\theta} - \frac{L(t)^{1+\gamma}}{1+\gamma}\}dt, \tag{2}$$

where C is consumption, $\theta \geq 0$ is the inverse of the elasticity of intertemporal substitution in consumption, $\gamma \geq 0$ is the inverse of the elasticity of labor supply with respect to the real wage, and $\rho \geq 0$ is the discount factor. The representive consumer owns the three inputs and rent them to firms through competitive markets. Therefore, we can write down, for sake of brevity, the consolidated budget constraint as:

$$\dot{K} + \dot{H} - \dot{D} = Y - \delta(K + H) - rD - C, \tag{3}$$

where D is the amount of net debt, $0 \leq \delta \leq 1$ is the depreciation rate for both types of capital, and $r \geq 0$ is the world interest rate. The initial stocks $K(0) > 0$, $H(0) > 0$, $D(0)$ and the labor endowment L^* are given to the households. Labor and human capital are not mobile.

We also assume, following Barro *et al.* [5], that international borrowing is limited to the amount of physical capital, that is, $D \leq K$. This hypothesis is, for instance, fulfilled when only physical capital is accepted as collateral for international credit, whereas human capital is not. We focus, as is usual, on the case in which $r = \rho$, the interest rate that would prevail in the corresponding closed economy (that is, when $D = 0$). Moreover, we assume that $K(0) + H(0) - D(0) < H^*$ holds at the outset of the period, where H^* is the steady state value of human capital stock. In other words, the initial net asset position is small enough so that the credit constraint binds, that is, $D = K$ (the economy jumps immediately to the steady state when the latter inequality does not hold; see Barro *et al.* [5, p. 109]).

Equating the return on physical capital to $r + \delta$ yields $K = \zeta Y/(r + \delta)$, and, therefore, by using equation (1),

$$Y = BA^{\frac{1}{1-\zeta}} H^a L^b, \tag{4}$$

where $B = [\zeta/(r + \delta)]^{\zeta/(1-\zeta)}$, $a = \eta/(1 - \zeta)$, $b = 1 - a$, with $0 \leq a < \zeta + \eta$. Therefore, a is simply the share of human capital relative to the share of effective labor, that is, human capital and raw labor.

Our next assumption is, quite traditionally, that firms benefit from productive externalities through human capital and labor. More precisely, we postulate that:

$$A = (H^\eta L^{1-\zeta-\eta})^\chi, \tag{5}$$

where $\chi \geq 0$ is the level of externalities. In fact, direct inspection of equations (1) and (5) shows that social returns to scale are measured by $1 + \chi(1 - \zeta)$ and are increasing whenever χ is positive. Excluding *physical* capital from the external effects somewhat conforms to some evidence offered by Benhabib and Jovanovic [10], who find capital externalities to be nonsignificant. As discussed below and in appendix B, our analysis does not depend in an important way on the presence of externalities. In fact, we will show that local indeterminacy may occur, in the model of this section, for small values of χ.

The budget constraint (3) may be written as the following, by observing that interest payments are proportional to output, that is, $(r + \delta)D = \zeta Y$ when $D = K$:

$$\dot{H} = (1 - \zeta)Y - \delta H - C. \tag{6}$$

Finally, households decisions follow from maximizing (2) subject to the budget constraint (6), using equation (4), given the initial stock $H(0) < H^*$. Straightforward computations yield the following first-order conditions:

$$\theta\frac{\dot{C}}{C} = a(1-\zeta)\frac{Y}{H} - \rho - \delta,\ L^{\gamma}C^{\theta} = b(1-\zeta)\frac{Y}{L}. \tag{7}$$

We may rewrite the budget constraint, from (6), as:

$$\frac{\dot{H}}{H} = (1-\zeta)\frac{Y}{H} - \delta - \frac{C}{H}. \tag{8}$$

Equations (7)-(8), together with equations (4)-(5), characterize the dynamics of intertemporal equilibria. It is not difficult to check that the associated transversality constraint is met in the following analysis, as we consider orbits that converge towards the interior steady state: both the human capital stock and the co-state variable converge. Direct inspection of equations (7)-(8) reveals that the dynamics arising in our credit-rationed economy are somewhat similar to the laws of motion describing *closed* economies, as studied by Benhabib and Farmer [7] and many others since. The major difference is that human capital is here the key state variable.

2.2 Local Indeterminacy when Externalities are Small

Appendix A linearizes equations (7)-(8), (4)-(5), around the interior steady state which is shown to be unique. Straightforward computations yield the following expressions for trace T and determinant D of the Jacobian matrix of the dynamical system derived from equations (7)-(8), (4)-(5) (see equations (14) in appendix A):

$$T = \tfrac{\rho+\delta}{a}[\lambda_1 + \lambda_2\tfrac{a}{\theta}] + \tfrac{\rho+\delta(1-a)}{a} \quad , \quad D = (\tfrac{\rho+\delta}{\theta})(\tfrac{\rho+\delta(1-a)}{a})[\lambda_1+\lambda_2], \tag{9}$$

with $\lambda_1 = \frac{b(1+\chi)+(\gamma+1)[a(1+\chi)-1]}{\gamma+1-b(1+\chi)}$ and $\lambda_2 = \frac{-\theta b(1+\chi)}{\gamma+1-b(1+\chi)}$.

We focus, as most papers in the literature do, on the case for which the steady state is locally a sink - that is, asymptotically stable - and derive conditions on parameter values such that $T < 0$ and $D > 0$. We then show that these conditions may include small values of χ (that is, configurations with small externalities): indeterminacy is compatible with almost *constant returns to scale.* In our discussion, a key parameter is the share of human capital a, as we now show.

Direct inspection of equations (9) shows that the sign of D is given by the sign of $\lambda_1 + \lambda_2$. Moreover, the numerator of $\lambda_1 + \lambda_2$ is negative only if $(\gamma + 1)(1 - a(1 + \chi)) \geq (1 - \theta)(1 - a)(1 + \chi)$. The latter inequality is met if one assumes that $a(1 + \chi) < 1$ - that is, externalities are not large enough to ensure endogenous growth - and $\theta \geq 1$ - that is, the elasticity of intertemporal substitution in consumption is smaller than unity. Moreover, the denominator of D is negative when $\gamma + 1 < (1-a)(1+\chi)$, that is, when $\chi > \chi_c \equiv (\gamma + a)/(1-a)$. On the other hand, T is a increasing function of χ, tends to $+\infty$ when χ tends, from below, to χ_c. When χ increases from χ_c, T increases from $-\infty$ and tends to a negative value. The immediate consequence is that $D > 0$ and $T < 0$ when χ is larger than χ_c.

Proposition 2.1 (Indeterminacy with Small Externalities)

In our credit-constrained open economy, the dynamics of consumption C and human capital H, given by equations (7)-(8), together with equations (4) and (5), have a unique positive steady state (C^*, H^*).

Assume that $a(1 + \chi) < 1$ and $\theta \geq 1$ (that is, the elasticity of intertemporal substitution in consumption $1/\theta$ is smaller than one). Then the steady state (C^*, H^*) is, locally:

(i) a saddle (locally determinate) when $\chi < \chi_c \equiv (\gamma + a)/(1 - a)$ (that is, when externalities are small enough).

(ii) a sink (locally indeterminate) when $\chi > \chi_c$.

In particular, local indeterminacy of the steady state (C^*, H^*) occurs for values of χ that are close to zero (small externalities), provided that γ is close to zero (that is, labor supply is almost perfectly elastic) and a is close to zero (that is, the share of human capital is close to zero).

Appendix B shows that indeterminacy is also likely to occur in a slightly different version of the model *without* externalities (returns to scale are then constant), when government finances a constant level of public expenditures by using distortionary taxes. In that sense, our specific assumption that external effects generate increasing returns is not critical for our main conclusion that credit-rationed open economies may be perturbed by expectation-driven fluctuations. On the other hand, Appendix C extends the basic model and shows that adding tax regressivity implies that indeterminacy may be compatible with (small externalities and) values of a that are larger than in the model without taxes.

In fact, some of our conclusions are somewhat different from existing results. First, in contrast with the results of Meng and Velasco [33], the parameter constellations leading here to indeterminacy depend on labor supply elasticity (that is, $1/\gamma$), as χ_c increases with γ. Therefore, indeterminacy is more likely when labor supply is highly elastic. Moreover, note that the elasticity of intertemporal substitution in consumption - that is, $1/\theta$ - plays also a role, in contrast with the results in Lahiri [31], Weder [44], Nishimura and Shimomura [34], Meng and Velasco [33]. This is not surprising, as consumption is not *perfectly* smoothed out in our small open economy facing borrowing constraints (the return on human capital accumulation differs from the international interest rate, out of steady state). In fact, one corollary of our analysis is that indeterminacy may be ruled out, quite surprisingly, when θ is close enough to zero, i.e. when intertemporal substitution in consumption is almost *perfect*, as we now show.

Proposition 2.2 (Indeterminacy and Strong Intertemporal Substitution in Consumption)

Local indeterminacy of the steady state (C^*, H^*) is ruled out, in the limit, when θ goes to zero, that is, when the elasticity of intertemporal substitution in consumption $1/\theta$ tends to infinity.

Proof: From equations (9), one can show that, here again, $T < 0$ implies that $\chi > \chi_c \equiv (\gamma + a)/(1 - a)$, as T does not depend on θ. However, $\chi > \chi_c$ implies that $D < 0$ when $\gamma = 0$ and θ is close enough to zero. To prove this, remember that D has the same sign as that of $\lambda_1 + \lambda_2$. But when $\chi > \chi_c$, the denominator of

$\lambda_1+\lambda_2 = [(\gamma+1)(a(1+\chi)-1)+(1-a)(1+\chi)]/[\gamma+1-(1-a)(1+\chi)]$ is negative, while its numerator is positive when $\chi \geq \gamma(1-a)/(1+a\gamma)$. It is straightforward to see that $\chi_c \equiv (\gamma+a)/(1-a) > \gamma > \gamma(1-a)/(1+a\gamma)$, so that $D < 0$ when $\chi > \chi_c$ and θ is close to zero. Therefore, the steady state is then a saddle if $\chi > \chi_c$, and a saddle or a source, when $\chi < \chi_c$ and θ is close enough to zero. □

Note that, indeed, most estimates of θ fall within a large interval, including $(0, 2)$ (see, e.g., Hansen and Singleton [24], Campbell [17]). Therefore, the evidence does not seem to provide clear-cut conclusions.

3 Imperfect International Credit Markets and Indeterminacy

3.1 When does financial integration lead to indeterminacy?

As a benchmark, it is useful to consider the case of *closed* economies deciding to open their capital account. We know, by modifying results from Benhabib and Farmer [7, p. 30] to incorporate human capital, that in the absence of international borrowing, a closed economy is saddle-point stable (the steady state is then determinate) if and only if $\gamma+1 > (1-\zeta-\eta)(1+\chi)$, that is, if and only if $\chi < \chi_c^{closed} \equiv (\gamma+\zeta+\eta)/(1-\zeta-\eta)$. On the other hand, Proposition 2.1 has shown that indeterminacy occurs only when χ is close to, but larger than $\chi_c \equiv (\gamma(1-\zeta)+\eta)/(1-\zeta-\eta)$ in the credit-constrained, open economy. As $\chi_c < \chi_c^{closed}$ always holds when $\zeta > 0$, there are parameter values such that the closed economy is saddle-point stable while *indeterminacy* prevails in the open economy: this is the case when χ is such that $\chi_c < \chi < \chi_c^{closed}$.

Corollary 3.1 (Comparing Closed and Open Economies)

Suppose that the level of externalities χ is close enough to, but larger than χ_c. Then indeterminacy prevails in the credit-constrained, open economy, while the corresponding closed economy is saddle-point stable.

Therefore, for moderate externalities (values of χ as low as 0.1, for instance, well within the standard errors of availlable estimates, when a is small enough), our credit-constrained economy would experience sunspots while the corresponding closed economy would be saddle-point stable. On the other hand, remember that our analysis also suggests that "rich" countries - that is, countries such that $K(0) + H(0) - D(0) \geq H^*$ - are not credit-constrained. Therefore, such countries jump immediately to the steady state and, hence, do not experience volatility.

To illustrate Corollary 3.1, let us now examine what would happen to economies deciding to open their capital account. To fix ideas, assume that labor supply is indivisible ($\gamma = 0$) and $\zeta = 0.3$. Then open economies exhibit indeterminacy for small externalities, provided that the share of human capital is low enough: η has to be lower than $0.7\chi/(1+\chi)$. Therefore, indeterminacy in open economies requires $\eta < 0.03$ when $\chi = 0.05$, $\eta < 0.06$ when $\chi = 0.1$, and $\eta < 0.12$ when $\chi = 0.2$. In contrast, closed economies would be saddle-point stable in all three cases, as $\zeta = 0.3 > \chi/(1+\chi) - \eta$.

3.2 Tightness of Credit-Rationing and Indeterminacy

The purpose of this section is to develop a sensitivity analysis with respect to a key parameter of our model: the share of physical capital in GDP - that is, ζ - which also measures the share of interest payments in GDP. This parameter highlights the effect of openness: when $\zeta = 0$, our open economy has no collateral and, therefore, evolves like a *closed* economy in which only human capital is accumulated and generates growth; on the contrary, economies with large ζ's highly borrow on international credit markets.

As a natural indicator of the *looseness* of the credit constraint, we consider the share of collateral in total capital, i.e. $\zeta/(\zeta+\eta)$, given the share of total capital in output $\zeta+\eta$. In other words, we fix the share of labor (given by $1-\zeta-\eta$) and vary the share of physical capital ζ. The lower ζ, the higher the share of human capital η, which is not accepted as collateral, and the less open the economy to international credit markets (and consequently to foreign trade), as $D/Y = \zeta/(r+\delta)$ is then smaller. In contrast, the higher ζ, the easier the access to international borrowing and the higher openness for the corresponding economy. The main specific question that we want to address in this section is the following: how does the severity of the credit constraint affects the likelihood of indeterminacy and endogenous fluctuations in open economies?

When $\zeta+\eta$ is fixed, varying ζ affects $a = \eta/(1-\zeta)$ in the following way. In fact, fixing $\zeta+\eta < 1$ implies that $1-a = (1-\zeta-\eta)/(1-\zeta)$ increases with ζ so that a decreases with ζ. In view of Proposition 2.1, we may infer how conditions leading to indeterminacy are affected by a. First, note that the condition $\chi > \chi_c$ - that is, $\gamma+1 < (1-a)(1+\chi)$ - in Proposition 2.1 is more likely to be met when a is small enough. Therefore, *indeterminacy is more likely when* a *is not too large, in the sense that* χ_c *is then smaller.* This proves the following corollary.

Corollary 3.2 (Indeterminacy and Tightness of Credit Constraint)

Indeterminacy of the steady state (C^*, H^*) *is more likely, as it requires lower externalities, when the share of human capital* $a = \eta/(1-\zeta)$ *is small.*

It follows that, given the share of total capital $\zeta+\eta$*, economies that have a large share of physical capital* ζ*, and are, therefore, less heavily credit-constrained, are more susceptible to indeterminacy and expectation-driven volatility than economies that borrow little on international credit markets.*

To illustrate Corollary 3.2, we now consider benchmark parameter values: indivisible labor supply (that is, $\gamma = 0$), $\rho = 0.065$ and $\delta = 0.1$ (as in, for instance, Benhabib and Farmer [7]). For sake of brevity, we study two configurations depending on the share of total capital: $\zeta+\eta = 0.8$ (the main case considered by

Barro *et al.* [5]) and $\zeta + \eta = 0.4$, that we summarize in the following table. Our numerical experiments

$\zeta + \eta = 0.8$	$\zeta = 0.70$, that is, $a \approx 0.33$	$\chi_c(1-\zeta) \approx 0.15$
	$\zeta = 0.78$, that is, $a \approx 0.09$	$\chi_c(1-\zeta) \approx 0.02$
$\zeta + \eta = 0.4$	$\zeta = 0.20$, that is, $a = 0.25$	$\chi_c(1-\zeta) \approx 0.27$
	$\zeta = 0.39$, that is, $a \approx 0.02$	$\chi_c(1-\zeta) \approx 0.01$

Table i: the indeterminacy range ($\chi > \chi_c$) when the share of collateral ζ varies.

confirm that the critical lower bound χ_c is small (externalities are small) when a is small. Therefore, our numerical cases illustrate that when the credit constraint is getting tighter, that is when ζ decreases for given $\zeta + \eta$, the human capital share a increases and indeterminacy is becoming less likely. On the contrary, open economies that face loose credit constraints - that is, have small a's - are more susceptible to indeterminacy and expectation-driven fluctuations. On the other hand, computing $\chi_c^{closed}(1-\zeta)$ for the values of Table i (from top to bottom) yields 1.20, 0.88, 0.53, 0.41.

Therefore, the predictions of our model are broadly consistent with some evidence that financial integration and aggregate volatility (of output and income) have been positively correlated in the 1990s (see Kose, Prasad and Terrones [29, Table 3]). Also note that indeterminacy may be associated with larger values of a when regressive taxes on labor income are introduced into the model, as shown in Appendix C.

3.3 Interpretation

The intuition for why, in open economies, indeterminacy implies that the human capital share cannot be too large may be easily grasped if one looks at the labor market, following Benhabib and Farmer [7]. More precisely, one may rewrite the static condition in equations (7), in logs, as:

$$\gamma l^s + \theta c = cst + a(1+\chi)h + [(1-a)(1+\chi) - 1]l^d, \tag{10}$$

where lowercase letters are logs of uppercase variable (so that, for instance, $l \equiv \ln L$) and l^s, l^d denote, respectively, labor supply and labor demand (in logs). From equation (10), we immediately see that labor demand has a slope that is smaller than the slope of labor supply when $\gamma + 1 > (1 - a)(1 + \chi)$. The latter condition may be rewritten as $\chi < \chi_c$. But Proposition 2.1 has shown that $\chi < \chi_c$ rules out local indeterminacy, as the steady state is then a saddle (see case (i) in the statement of that proposition). On the contrary, local indeterminacy of the steady state is possible, as is usual in one-sector models, when externalities are large enough ($\chi > \chi_c$). More importantly, our analysis also shows that indeterminacy is possible, with almost constant returns to scale (small values of χ), when γ and a are small: this generates a labor market configuration in which labor demand has a higher slope than that of labor supply. In essence, the basic mechanism is then that optimistic expectations may be fulfilled, in equilibrium, because agents willing to increase current consumption work harder today: this increases, in fact, the real wage, production, investment, and decreases the returns on human capital, so that optimistic expectations are compatible with intertemporal equilibrium. In our credit-constrained economy, an additional feature of the boom is that the physical stock increases, thereby allowing foreign debt to go up. Eventually, increasing interest payments will negatively affect output, so that consumption, investment and debt will go down: pessimistic expectations will then prevail during the temporary recession. In short, a traditional *financial accelerator effect* may exacerbate volatility, through international credit, in open economies.

A main result of this paper is that this mechanism is stronger the easier the access to international borrowing for open economies with enough collateral, in the case of constrained foreign credit. As we just discussed, the key condition for indeterminacy, that is, $\gamma + 1 < (1 - a)(1 + \chi)$ (or $\chi > \chi_c$), is more likely to be met when, other things equal, a is small enough. We have seen that the latter condition may be interpreted as implying that the share of physical capital, which serves as collateral, in total capital should be large enough. However, one can also study the impact of a on the occurrence of indeterminacy when the share of physical capital ζ is fixed. Then one concludes, from Proposition 2.1, that the lower η, the more likely indeterminacy. Other things equal, economies with a low share of human capital may be subject to the basic mechanism described earlier and, for that reason, more susceptible to expectation-driven fluctuations.

Our analysis also suggests that indebted countries having different shares of *total* capital in output $(\zeta + \eta)$ may be more or less likely to be perturbed by expectation-driven fluctuations, depending on their shares of human and physical capital. For example, the first and last lines in Table i suggest that for a given ratio $\zeta/(\zeta+\eta) = 0.975$, indeterminacy requires lower externalities when *both* the human capital share η and the physical capital share ζ are small.

In the real world, different indebted countries have different physical and human capital shares. For instance, Burkina Faso, Cote d'Ivoire, and Zambia are classified, in the statistical appendix of World Bank's *Global Development Finance 2003*, as "severely indebted, low-income" countries (see tables A51 and A52, pp. 232-5), but these nations differ in their physical capital share, the average of which was, respectively, 76%, 66%, and 55% for the period 1960-90[1], and also perhaps differ in their share of human capital. However, the three countries are much more volatile than OECD countries: the standard deviation of output growth was, over the period 1960-90, 1.9 times larger for Burkina Faso than for the US, and 2.7 times larger for Cote d'Ivoire and Zambia than for the US (see Breen and Garcia-Peñalosa [14, Table A1]; see also Ramey and Ramey [38]).

Within the class of "severely indebted, middle-income" countries, as classified by the World Bank, one can find even more volatile economies: ouput volatility in Peru, Jordan and Guyana was, respectively, 2.7, 4.1 and 4.8 times larger than ouput volatility in the US, for the period 1960-90 (the average capital share of Peru, Jordan and Guyana was about 66%, 65% and 30%, respectively, for the period 1960-90, again from UN data).

[1] I thank Emilie Daudey for providing me with the latter figures, computed from UN data.

4 Conclusion

This paper has studied a benchmark open-economy growth model with constraints on international borrowing and it has shown that imposing, quite realistically, some borrowing limits does not necessarily rule out the occurrence of expectation-driven volatility in open economies that have access to international credit markets. We have found that credit-constrained, small open economies may be destabilized by expectation-driven fluctuations, even in the presence of small externalities, provided that the share of human capital is low enough. It follows that economies that highly borrow on international credit markets are more susceptible to expectation-driven fluctuations. Moreover, opening the capital account may push a saddle-point stable economy on a volatile path. The main mechanism through which fluctuations are exacerbated is the presence of a financial accelerator effect that increases the levels of collateral and foreign debt during booms. On the contrary, tighter constraints on external borrowing may protect the economy against expectation-driven volatility.

In contrast with some existing results in the literature, we have shown that the elasticity of labor supply and the elasticity of intertemporal substitution in consumption both condition the set of parameter values associated with expectation-driven fluctuations. Quite surprisingly, indeterminacy requires, under some usual conditions, intertemporal substitution to be quite weak.

A notable lesson that may be drawn from our analysis is that the easier the access to international borrowing (the looser the credit constraint, given the share of total capital), the more likely indeterminacy and endogenous fluctuations. However, we also have shown, in an extension of the model, that international capital mobility may or may not lead to local indeterminacy, depending also on the level of tax *progressivity*. In the real world, countries seem to differ significantly with respect to the progressivity of their tax system, for various reasons including distribution concerns (see, for example, Benabou [6]). Therefore, an interesting application would be to confront the level of tax regressivity compatible with indeterminacy to the corresponding estimates, so as to assess the plausibility of endogenous volatility. Moreover, our analysis has shown that the level of *human* capital plays a key role: indeterminacy is more likely in economies with a low share of human capital. Put together, these results seem in agreement with the observation that a number of indebted, volatile countries are similar in that they have low incomes, but differ in their physical and human capital shares. However, it remains to be seen how the present model is able to match the observed correlations associated with actual business cycles that unfold in those economies. It would also be interesting to test whether indebted, volatile countries tend to have low human capital shares and regressive tax systems, which sounds quite plausible.

On the theoretical side, a relevant extension of our analysis would be to assume, in agreement with existing tax schedules, that labor income taxes and capital income taxes have different progressivity features. Moreover, it would also be relevant to take into account the fact, documented by some studies (e.g. Krusell *et al.* [30]), that skilled labor and physical capital are complements while raw labor and physical capital are substitutes. In view of some results by Barro *et al.* [4, pp. 23-28], one expects that the stronger complementarity between physical and human capital, the higher the concavity in production, in which case the occurrence of indeterminacy and expectation-driven fluctuations should be more likely. I hope this gives directions for future and potentially fruitful research.

A Linearized Dynamics

This appendix derives and linearizes, around the steady state, the dynamical system describing intertemporal equilibria which consists of equations (7)-(8), together with equations (4) and (5). The first step is to rewrite, from the static condition in (7), the following equation:

$$[\gamma + 1 - b(1+\chi)]l = cst + a(1+\chi)h - \theta c, \tag{11}$$

where lowercase variables are logs of uppercase variables (so that, for instance, $l = \ln L$), using the fact that $y = cst + a(1+\chi)h + b(1+\chi)l$ from taking logs in equations (4) and (5). This yields, by using equation (11):

$$y - h = \lambda_0 + \lambda_1 h + \lambda_2 c, \tag{12}$$

where

$$\begin{aligned} \lambda_0 &= \ln(B) + \tfrac{b(1+\chi)[\ln(b(1-\zeta)B)]}{\gamma+1-b(1+\chi)} \quad , \quad \lambda_1 = \tfrac{b(1+\chi)+(\gamma+1)[a(1+\chi)-1]}{\gamma+1-b(1+\chi)}, \\ \lambda_2 &= \tfrac{-\theta b(1+\chi)}{\gamma+1-b(1+\chi)}. \end{aligned} \tag{13}$$

By rewriting equations (7)-(8) in logs and using equations (13), it is easy to get:

$$\begin{aligned} \theta\dot{c} &= a(1-\zeta)e^{\lambda_0+\lambda_1 h+\lambda_2 c} - \rho - \delta, \\ \dot{h} &= (1-\zeta)e^{\lambda_0+\lambda_1 h+\lambda_2 c} - \delta - e^{c-h}. \end{aligned} \tag{14}$$

It is straightforward to show that, under our assumptions, the differential equations (14) admit a unique interior steady state. More precisely, $\dot{c} = 0$ yields, from the first equation of system (14):

$$(1-\zeta)e^{\lambda_0+\lambda_1 h^*+\lambda_2 c^*} = (\rho+\delta)/a, \tag{15}$$

On the other hand, $\dot{h} = 0$ then yields, from the second equation of system (14):

$$e^{c^*} = [(\rho+\delta)/a - \delta]e^{h^*}. \tag{16}$$

One can then easily solve the two latter equations for c^* and h^*, provided that $(\rho+\delta)/a > \delta$.

Straightforward computations lead to the following expressions of T and D, respectively the trace and determinant of the Jacobian matrix associated with equations (14), evaluated at the unique steady state c^*, h^*:

$$\begin{aligned} T &= \tfrac{\rho+\delta}{a}[\lambda_1 + \lambda_2 \tfrac{a}{\theta}] + \tfrac{\rho+\delta(1-a)}{a} = \rho + \tfrac{\chi(\rho+\delta)(\gamma+1)}{\gamma+1-(1-a)(1+\chi)}, \\ D &= (\tfrac{\rho+\delta}{\theta})(\tfrac{\rho+\delta(1-a)}{a})[\lambda_1 + \lambda_2]. \end{aligned} \tag{17}$$

B Predetermined Public Spending and Indeterminacy with Constant Returns

In this appendix, we show how indeterminacy results may also be obtained in our credit-rationed open-economy model *without* productive externalities - that is, with constant returns - when the government finance a constant amount of public expenditures G by using distortionary taxes on labor income. Following Schmitt-Grohe and Uribe [41], we assume that:

$$G = \tau(t)\omega(t)L(t), \tag{18}$$

where ω is the marginal productivity of raw labor and τ is no longer given by equation (26): the tax rate is now determined by the balanced-budget rule in equation (18), given the constant flow of government spending, the real wage and labor supply.

We further assume, for simplicity, that labor is indivisible (as in Hansen [23] and Rogerson [39])) - that is, $\gamma = 0$ - and that consumption utility is logarithmic - that is, $\theta = 1$. The representative agent then maximizes:

$$\int_0^\infty e^{-\rho t}\{\ln C(t) - L(t)\}dt, \tag{19}$$

subject to:

$$\dot{H} = (1-\zeta)Y - \delta H - C - G. \tag{20}$$

Assuming that externalities are absent - that is, $\chi = 0$ - production is now given by:

$$Y = BH^a L^b, \tag{21}$$

where $B = [\zeta/(r+\delta)]^{\zeta/(1-\zeta)}$, $a = \eta/(1-\zeta)$, $b = 1-a$, and it generates a flow of income such that:

$$Y = uH + \omega L, \tag{22}$$

where u is the rental on human capital. If we define Λ as the marginal utility of income, that is, $\Lambda = 1/C$, then straightforward computations lead to the analogs of the first-order conditions in equations (7):

$$\frac{\dot{\Lambda}}{\Lambda} = \rho + \delta - u, \; 1 = \Lambda(1-\tau)\omega. \tag{23}$$

Putting together equations (23), the market clearing condition:

$$\dot{H} + \delta H + \frac{1}{\Lambda} + G = (1 - \zeta)Y, \tag{24}$$

the balanced-budget rule in equation (18), the definition of production in equation (21), and the usual conditions that input rentals equal marginal productivities - that is, $\omega L = bY$ and $uH = aY$ - one gets equilibrium conditions that coincide with the four equations $(5) - (8)$ derived in Schmitt-Grohe and Uribe [41, pp. 979-80].

Consequently, we can borrow from the results established by Schmitt-Grohe and Uribe [41, p. 983] and conclude that *a necessary and sufficient condition for local indeterminacy is that the steady-state tax rate belongs to the interval bounded below by a and above by the tax rate that maximizes tax revenues.* In this version of the model, therefore, local indeterminacy arises under the assumption of constant returns to scale.

Further borrowing from Schmitt-Grohe and Uribe [41], we could also establish that the occurrence of indeterminacy is robust to some extensions of the model. In particular, allowing, more realistically, for the presence of capital income taxes, of income-elastic government expenditures, or of public debt confirms that indeterminacy is compatible with values of income tax rates that are observed for the United States and other OECD countries.

The main lesson from this exercise is that *credit-rationed open economies* are, despite the imposed constaint on international borrowing, susceptible to expectation-driven fluctuations even in the absence of externalities and with an *a priori* flat tax rate, which extends both existing results in the literature and our results of section 2. However, this conclusion relies on the assumption that a is small enough, just as in our main analysis (see sections 2 and 3 above). In fact, actual tax rates on labor income probably belong to $(0.2; 0.5)$ in most OECD countries. Focusing on the lower end of this interval, we conclude that $a = \eta/(1 - \zeta)$ has therefore to be lower than 0.2: the share of human capital must be less than 20% of the share of total labor. This means, if we accept the usual value of $1 - \zeta = 0.7$ for the share of total labor, that indeterminacy occurs only if human capital has a share η below 0.14. On the other hand, fixing the total share of physical and human capital $\zeta + \eta$ means that small a's imply small values for η but high values for ζ: indeterminacy is then more likely to occur in economies that have easily access to international borrowing and, therefore, have a large debt/GDP ratio. Therefore, our main conclusions stated in sections 2 and 3 remain valid in this version of the model.

C Indeterminacy with Progressive Taxes

In this appendix, we extend the basic model presented in the main text and show two results: when the tax rate on labor income is *regressive*, indeterminacy is more likely, while, on the contrary, *progressive* taxes may protect the economy against endogenous volatility.

C.1 Model and Laws of Motion

The consolidated budget constraint is now changed to:

$$\dot{K} + \dot{H} - \dot{D} = (1 - \tau)(Y - R_K K) + (1 - \tau_K) R_K K - \delta(K + H) - rD - C, \tag{25}$$

where $\tau_K \geq 0$ is the constant tax rate on physical capital income $R_K K$, and $\tau \geq 0$ is the labor income tax rate to be specified below.

Government is assumed to finance public spending G that do not affect private decisions by taxing output, i.e. $G = \tau Y$. To keep things simple, we assume that the tax schedule is as follows:

$$\tau = 1 - \nu Y^{-\pi}, \tag{26}$$

where $0 < \nu < 1 - \zeta$ is a parameter that plays only a minor role in the following analysis and $\pi < 1$. For instance, one may interpret ν as depending on the base level of income - the steady state level for instance - that is taken as given. The important feature here is that the tax rate τ increases (resp. decreases) with the tax base when π is positive (resp. negative). Therefore, this tax system exhibits progressivity when π is positive, and regressivity when π is negative. As in Guo and Lansing [22], we assume that households take into account how the tax rate will affect their earnings. Note that we assume only for simplicity that capital income tax rate is flat (Pintus [37] shows, in a related framework with population growth and technical progress, that including progressive taxes on capital income yield a similar model). So as to ensure the existence of a positive steady state, we also assume that taxes are not too regressive, that is, $\pi > \overline{\pi}$, where $\overline{\pi}$ is a negative lower bound to be determined below (see Proposition C.1)

Equating the return on physical capital to $r + \delta$ yields $K = \zeta(1 - \tau_K)Y/(r + \delta)$, and, therefore, using equation (1), this gives equation (4) where, here, $B = [\zeta(1 - \tau_K)/(r + \delta)]^{\zeta/(1-\zeta)}$. Therefore, the budget constraint (25) may be written as the following, by observing that interest payments are proportional to output, that is, $(r + \delta)D = \zeta(1 - \tau_K)Y$ when $D = K$:

$$\dot{H} = (1 - \zeta)(1 - \tau)Y - \delta H - C. \tag{27}$$

Households decisions follow from maximizing (2) subject to the budget constraint (27), using equations (4)-(5) and (26), given the initial stock $H(0) < H^*$. The first-order conditions are:

$$\theta \frac{\dot{C}}{C} = \nu a(1 - \zeta)(1 - \pi)\frac{Y^{1-\pi}}{H} - \rho - \delta, \; L^\gamma C^\theta = \nu b(1 - \zeta)(1 - \pi)\frac{Y^{1-\pi}}{L}. \tag{28}$$

We may rewrite the budget constraint, from (27) and (26), as:

$$\frac{\dot{H}}{H} = \nu(1 - \zeta)\frac{Y^{1-\pi}}{H} - \delta - \frac{C}{H}. \tag{29}$$

Equations (28)-(29), together with equations (4) and (5), characterize the dynamics of intertemporal equilibria. Note that after-tax income is concave in H whenever $a(1 + \chi)(1 - \pi) < 1$, that is, $\pi > \underline{\pi} \equiv 1 - 1/(a(1 + \chi))$.

C.2 Local Indeterminacy when Externalities are Small

Below, we linearize equations (28)-(29) and (4)-(5), around the interior steady state which is shown to be unique. For a steady state to be feasible, we need the additional condition that $\rho + \delta > a\delta(1-\pi)$ to be met (see, below, the proof of Proposition C.1), which imposes a negative lower bound on π, that is, $\pi > \overline{\pi} \equiv 1 - (\rho+\delta)/(a\delta)$. In other words, the tax rate cannot be too regressive.

Straightforward computations yield the following expressions for trace T and determinant D of the Jacobian matrix of the dynamical system derived from equations (7)-(8), (4) and (5) (see equations (34) below):

$$\begin{aligned} T &= \tfrac{\rho+\delta}{a(1-\pi)}[\lambda_1 + \lambda_2 \tfrac{a(1-\pi)}{\theta}] + \tfrac{\rho+\delta(1-a(1-\pi))}{a(1-\pi)}, \\ D &= (\tfrac{\rho+\delta}{\theta})(\tfrac{\rho+\delta(1-a(1-\pi))}{a(1-\pi)})[\lambda_1+\lambda_2], \end{aligned} \tag{30}$$

with $\lambda_1 = \frac{b(1+\chi)(1-\pi)+(\gamma+1)[a(1+\chi)(1-\pi)-1]}{\gamma+1-b(1+\chi)(1-\pi)}$ and $\lambda_2 = \frac{-\theta b(1+\chi)(1-\pi)}{\gamma+1-b(1+\chi)(1-\pi)}$.

We focus, as most papers in the literature do, on the case for which the steady state is locally a sink, that is, asymptotically stable, and we derive conditions on parameter values such that $T < 0$ and $D > 0$. We then show that these conditions include *arbitrarily* small values of χ (that is, configurations with arbitrarily small externalities): indeterminacy is compatible with almost *constant returns to scale*. In our discussion, a key parameter is the level of tax progressivity π, the empirical estimate of which is not easily measured and, therefore, quite uncertain.

Direct inspection of equations (30) shows that the sign of D is in fact the sign of $(1-a)(1+\chi)(1-\pi)-\gamma-1$ (the latter term is the opposite of the denominator of $\lambda_1+\lambda_2$) under the assumptions that $\pi < 1$, $\theta \geq 1$ and $a(1+\chi)(1-\pi) < 1$. The latter assumption is equivalent to $\pi > \underline{\pi} \equiv 1-1/(a(1+\chi))$, where $\underline{\pi} > \overline{\pi}$. On the other hand, T is a decreasing function of π, the denominator of which vanishes when $(1-a)(1+\chi)(1-\pi) = \gamma+1$, i.e. when $\pi = \pi_c \equiv 1-(\gamma+1)/((1-a)(1+\chi))$. Therefore, T tends to $-\infty$ when π tends to π_c from below and decreases from $+\infty$ (to a positive value) when π increases from π_c to one. The immediate consequence is that $D > 0$ and $T < 0$ when π is slightly smaller than π_c. More precisely, we can in fact show that the following statements hold.

Proposition C.1 (Indeterminacy, Arbitrarily Small Externalities, and Progressive Taxes)

In our credit-constrained open economy, the dynamics of consumption C and human capital H, given by equations (28)-(29) and (4)-(5), have a unique positive steady state (C^*, H^*) if $\overline{\pi} \equiv 1-(\rho+\delta)/(a\delta) < \pi < 1$.

Assume that $\theta \geq 1$ (that is, the elasticity of intertemporal substitution in consumption $1/\theta$ is smaller than one) and, moreover, that $a(\gamma+1)[\chi(\rho+\delta)+\rho] < \rho(1-a)$ (that is, the elasticity of labor supply $1/\gamma$ is large enough, or externalities, measured by χ, are small enough).

Then the steady state (C^*, H^*) is, locally:

(i) a saddle (locally determinate) when $\pi_c \equiv 1-(\gamma+1)/((1-a)(1+\chi)) < \pi < 1$ (that is, when the tax rate is progressive enough).

(ii) a sink (locally indeterminate) when $\pi_h < \pi < \pi_c$, undergoes a Hopf bifurcation at $\pi = \pi_h$, and is a source (locally unstable) when $\underline{\pi} < \pi < \pi_h$, with $\pi_h \equiv 1-(\gamma+1)[\chi(\rho+\delta)+\rho]/(\rho(1-a)(1+\chi)) > \underline{\pi} \equiv 1-1/(a(1+\chi)) > \overline{\pi}$.

In particular, local indeterminacy of the steady state (C^*, H^*) occurs for values of χ that are arbitrarily close to zero (arbitrarily small externalities), provided that π is close enough to, but smaller than π_c. Positive but small values of χ imply, in turn, that π_c is then negative.

In summary, local indeterminacy occurs when externalities are small and returns to scale are almost constant, provided that the tax rate is regressive and elastic enough.

Proof: We first derive and linearize, around the steady state, the dynamical system describing intertemporal equilibria which consists of equations (28)-(29), together with equations (4)-(5). The first step is to rewrite, from the static condition in (28), the following equation:

$$[\gamma+1-b(1+\chi)(1-\pi)]l = cst + a(1+\chi)(1-\pi)h - \theta c, \tag{31}$$

where lowercase variables are logs of uppercase variables (so that, for instance, $l = \ln L$), using the fact that

$y = cst + a(1+\chi)h + b(1+\chi)l$ from taking logs in equations (4) and (5). This yields, by using equation (31):

$$(1-\pi)y - h = \lambda_0 + \lambda_1 h + \lambda_2 c, \tag{32}$$

where

$$\begin{aligned} \lambda_0 &= \ln(B^{1-\pi}) + \tfrac{b(1+\chi)(1-\pi)[\ln(\nu b(1-\zeta)(1-\pi)B^{1-\pi})]}{\gamma+1-b(1+\chi)(1-\pi)} \quad , \quad \lambda_1 = \tfrac{b(1+\chi)(1-\pi)+(\gamma+1)[a(1+\chi)(1-\pi)-1]}{\gamma+1-b(1+\chi)(1-\pi)}, \\ \lambda_2 &= \tfrac{-\theta b(1+\chi)(1-\pi)}{\gamma+1-b(1+\chi)(1-\pi)}. \end{aligned} \tag{33}$$

By rewriting equations (28)-(29) in logs and using equations (33), it is easy to get:

$$\begin{aligned} \theta\dot{c} &= \nu a(1-\zeta)(1-\pi)e^{\lambda_0+\lambda_1 h+\lambda_2 c} - \rho - \delta, \\ \dot{h} &= \nu(1-\zeta)e^{\lambda_0+\lambda_1 h+\lambda_2 c} - \delta - e^{c-h}. \end{aligned} \tag{34}$$

It is straightforward to show that, under our assumptions, the differential equations (34) admit a unique interior steady state. More precisely, $\dot{c} = 0$ yields, from the first equation of system (34):

$$\nu(1-\zeta)e^{\lambda_0+\lambda_1 h^*+\lambda_2 c^*} = (\rho+\delta)/(a(1-\pi)), \tag{35}$$

On the other hand, $\dot{h} = 0$ then yields, from the second equation of system (34):

$$e^{c^*} = [(\rho+\delta)/(a(1-\pi)) - \delta]e^{h^*}. \tag{36}$$

One can then easily solve the two latter equations for c^* and h^*, provided that $(\rho+\delta)/(a(1-\pi)) > \delta$, i.e. that $\pi > \overline{\pi} \equiv 1 - (\rho+\delta)/(a\delta)$.

Straightforward computations lead to the following expressions of T and D, respectively the trace and determinant of the Jacobian matrix associated with equations (34), evaluated at the unique steady state c^*, h^*:

$$\begin{aligned} T &= \tfrac{\rho+\delta}{a(1-\pi)}[\lambda_1 + \lambda_2\tfrac{a(1-\pi)}{\theta}] + \tfrac{\rho+\delta(1-a(1-\pi))}{a(1-\pi)}, \\ D &= (\tfrac{\rho+\delta}{\theta})(\tfrac{\rho+\delta(1-a(1-\pi))}{a(1-\pi)})[\lambda_1 + \lambda_2]. \end{aligned} \tag{37}$$

Note that expressions (37) slightly differ from the corresponding formula appearing in Guo and Lansing [22, p. 488] (once we take into account that the latter authors impose $\theta = 1$).

Direct inspection of equations (33) and (37) shows that D has the same sign as that of $\lambda_1 + \lambda_2$, when $\overline{\pi} < \pi < 1$, with $\overline{\pi} \equiv 1 - (\rho+\delta)/(a\delta)$. But $\lambda_1 + \lambda_2 = [(1-\theta)(1-a)(1+\chi)(1-\pi) + (\gamma+1)(a(1+\chi)(1-\pi) - 1)]/[\gamma+1-(1-a)(1+\chi)(1-\pi)]$ has a negative numerator when $\theta \geq 1$ and $\pi > \underline{\pi} \equiv 1 - 1/(a(1+\chi))$, so that its sign is the opposite of the sign of its denominator. Therefore, D is positive, under the assumptions just stated, if and only if $\gamma+1 < (1-a)(1+\chi)(1-\pi)$, or equivalently if and only if $\pi < \pi_c \equiv 1-(\gamma+1)/((1-a)(1+\chi))$.

The steady state is then a saddle if and only if $\pi > \pi_c$, which proves (i).

When $\pi < \pi_c$, the steady state may be a sink if $T < 0$ or a source if $T > 0$, and it may undergo a Hopf bifurcation at $T = 0$. Straightforward computations from equations (33) and (37) yield:

$$T = \rho + \frac{\chi(\rho+\delta)(\gamma+1)}{\gamma+1-(1-a)(1+\chi)(1-\pi)}. \tag{38}$$

Therefore, equation (38) shows that T is a decreasing function of π. In particular, T increases from a positive value to $+\infty$ when π decreases from 1 to π_c, as the denominator of the fraction in equation (38) tends to zero from above while its numerator is positive. Moreover, T increases from $-\infty$ to zero when π decreases from π_c toward $\pi_h \equiv 1 - (\gamma+1)[\chi(\rho+\delta)+\rho]/(\rho(1-a)(1+\chi))$, where π_h is the value such that $T = 0$. Therefore, $T > 0$ if $\pi < \pi_h$, $T = 0$ when $\pi = \pi_h$, and $T < 0$ when $\pi_h < \pi < \pi_c$ (cases covered by (ii)).

It is straightforward to check that $1 > \pi_c > \pi_h$ and that $\underline{\pi} > \overline{\pi}$ under the assumptions on our primitive parameters. Moreover, $\pi_h > \underline{\pi}$ when $a(\gamma+1)[\chi(\rho+\delta)+\rho] < \rho(1-a)$. Note that the latter assumption implies that externalities cannot be too large (χ must be small enough), that labor supply has to be elastic enough (γ must be small enough), and that a has to be small enough.

Finally, our last result shows that local indeterminacy of the steady state (when $D > 0$ and $T < 0$) may be associated with arbitrarily small externalities (values of χ close to zero), when $a(\gamma+1)[\chi(\rho+\delta)+\rho] < \rho(1-a)$ is met. In fact, π_c and $\pi_h (< \pi_c)$ respectively increases and decreases with χ, and π_h tends to π_c when χ tends to zero. Therefore, indeterminacy occurs, for small χ's, when π is close enough to, but smaller than π_c. Finally, one observes that $\pi_c = 1 - (\gamma+1)/((1-a)(1+\chi))$ is negative when χ is close enough to zero, as $(\gamma+1)/(1-a) > 1$ when $a < 1$. $\square$

The main result of Proposition C.1 is that local indeterminacy is associated with values of π that belong to (π_h, π_c). Therefore, indeterminacy is *a priori* compatible with progressive or regressive taxes, that is, with positive or negative values of π. In fact, π_c is negative (and π_h is also negative and close to π_c) when χ is close to zero, but this value increases with χ and is positive if $\chi > (\gamma + a)/(1 - a)$. Therefore, indeterminacy implies *regressive* taxation when externalities are arbitrarily small.

Appendix B shows that indeterminacy is also likely to occur in a version of the model *without* externalities, when government finances a constant level of public expenditures by using distortionary taxes. In that framework, imposing to the public budget to be balanced implies that the tax rate is regressive, which is reminiscent of the results established in Proposition C.1. Therefore, our specific assumptions that the tax rate is elastic to the tax base and that external effects generate increasing returns are not critical for our main conclusion that credit-rationed open economies may be perturbed by expectation-driven fluctuations.

Note that indeterminacy is ruled out when the tax rate is *progressive* enough (when π is close enough to one), independent of whether externalities are small or large (as we just noted, large values of χ are associated with a large threshold value π_c, above which the steady state is saddle-point stable), in contrast with results by Guo and Harrison [21]. Finally, note that the elasticity of intertemporal substitution in consumption - that is, $1/\theta$ - plays also a role here, in contrast with the results in Lahiri [31], Weder [44], Nishimura and Shimomura [34], Meng and Velasco [33].

Proposition C.2 (Indeterminacy and Strong Intertemporal Substitution in Consumption)

Assume that $\gamma = 0$ (that is, labor is indivisible). Then indeterminacy of the steady state (C^*, H^*) is ruled out, in the limit, when θ goes to zero (that is, when the elasticity of intertemporal substitution in consumption $1/\theta$ tends to infinity).

Proof: From equations (30), one can show that, here again, $T < 0$ implies that $\pi < \pi_c \equiv 1 - (\gamma + 1)/((1-a)(1+\chi))$, as T does not depend on θ. However, $\pi < \pi_c$ implies that $D < 0$ when $\gamma = 0$ and θ is close enough to zero. To prove this, remember that D has the same sign as that of $\lambda_1 + \lambda_2$, when $\rho > 0$, $\delta > 0$, $a < 1$, $\overline{\pi} < \pi < 1$, with $\overline{\pi} \equiv 1 - (\rho+\delta)/(a\delta)$. But when $\pi < \pi_c$, the denominator of $\lambda_1 + \lambda_2 = [(\gamma+1)(a(1+\chi)(1-\pi)-1) + (1-a)(1+\chi)(1-\pi)]/[\gamma+1-(1-a)(1+\chi)(1-\pi)]$ is negative, while its numerator is positive when $\gamma = 0$ and $\theta = 0$ (implying $\lambda_2 = 0$). Therefore, the steady state is then a saddle if $\pi < \pi_c$, and a saddle or a source when $\pi > \pi_c$. □

Note that the condition $a(\gamma+1)[\chi(\rho+\delta)+\rho] < \rho(1-a)$ in Proposition C.1 is more likely to be met when a is small enough. Moreover, both $\underline{\pi}$ and $\overline{\pi}$ tend to $-\infty$ when a goes to zero: the lower bounds on π are small when a is small. In fact, one has $\pi_h < \pi_c < \underline{\pi}$ when a is greater than 0.5.

Second, manipulating the corresponding expressions appearing in Proposition C.1 lead one to conclude that both π_h and π_c decrease with a, although the size of the range $\pi_c - \pi_h$ increases with a (however, the size of $\pi_c - \pi_h$ is proportional to $\chi/(1+\chi)$ and is, therefore, small when χ is close to zero). Therefore, *indeterminacy is more likely when a is not too large, in the sense that π_h and π_c are then not too negative* (when χ is close to zero). This shows that Corollary 3.2 still holds in this economy.

To illustrate, in the present context, Corollary 3.2, we now consider benchmark parameter values: indivisible labor supply (that is, $\gamma = 0$), small externalities ($\chi = 0.001$), $\rho = 0.065$ and $\delta = 0.1$ (as in, for instance, Benhabib and Farmer [7]). These values imply that condition $a(\gamma+1)[\chi(\rho+\delta)+\rho] < \rho(1-a)$ in Proposition C.1 is now simply $a < 0.3935$. Moreover, note that all cases considered below imply that $\chi(1-\zeta) < 0.001$.

For sake of brevity, we study two configurations depending on the share of total capital: $\zeta + \eta = 0.8$ (the main case considered by Barro *et al.* [5]) or 0.4, that are summarized in the following table.

Our numerical experiments confirm that although the range (π_h, π_c) is quite small when χ is small (externalities are then negligible), indeterminacy is compatible with slightly negative values of π when a is

$\zeta + \eta = 0.8$	$\zeta = 0.7$, that is, $a \approx 0.33$	$(\pi_h, \pi_c) \approx (-0.502, -0.499)$
	$\zeta = 0.75$, that is, $a = 0.2$	$(\pi_h, \pi_c) \approx (-0.251, -0.249)$
$\zeta + \eta = 0.4$	$\zeta = 0.2$, that is, $a = 0.25$	$(\pi_h, \pi_c) \approx (-0.335, -0.332)$
	$\zeta = 0.35$, that is, $a \approx 0.08$	$(\pi_h, \pi_c) \approx (-0.084, -0.083)$

Table ii: indeterminacy range (π_h, π_c) when the share of collateral ζ varies.

small enough, that is, when ζ is large: the tax rate need not necessarily be highly regressive. Therefore, our numerical cases illustrate that when the credit constraint is getting tighter, that is when ζ decreases for given $\zeta + \eta$, the human capital share a increases and indeterminacy is becoming less likely. On the contrary, open economies that face loose credit constraints - that is, have small a's - are more susceptible to indeterminacy and expectation-driven fluctuations.

Let us, again, consider the case of closed economies. We know, by modifying results from Guo and Lansing [22, p. 488] to incorporate human capital, that in the absence of international borrowing, a closed economy is saddle-point stable (the steady state is then determinate) if and only if $\gamma + 1 > (1 - \zeta - \eta)(1 + \chi)(1 - \pi)$, that is, if and only if $\pi > \pi_c^{closed} \equiv 1 - (\gamma + 1)/((1 - \zeta - \eta)(1 + \chi))$. On the other hand, Proposition C.1 has shown that indeterminacy occurs only when π is close to, but smaller than $\pi_c \equiv 1 - (\gamma + 1)/((1 - \eta/(1 - \zeta))(1 + \chi))$ in the credit-constrained, open economy. Then there are parameter values such that the closed economy is saddle-point stable while *indeterminacy* prevails in the open economy: this is the case when $\pi_c > \pi_c^{closed}$, that is, if $1 - \zeta - \eta < 1 - \eta/(1 - \zeta)$. It is not difficult to check that the latter condition is always met when $\zeta + \eta < 1$, which we have assumed. Therefore, we can state the following result.

Corollary C.1 (Comparing Closed and Open Economies)

Suppose that the level of tax progressivity π is close enough to, but smaller than π_c. Then indeterminacy prevails in the credit-constrained, open economy, while the corresponding closed economy is saddle-point *stable.*

References

[1] P. Aghion, P. Bolton, A theory of trickle-down growth and development, Rev. Econ. Stud. **64** (1997), 151-72.

[2] P. Aghion, P. Bacchetta, A. Banerjee, Capital markets and the instability of open economies, CEPR Discussion Paper Series n. 2083, November 2000.

[3] D. Altig, A.J. Auerbach, L.J. Kotlikoff, K.A. Smetters, J. Walliser, Simulating fundamental tax reform in the United States, Am. Econ. Rev. **91** (2001), 574-95.

[4] R. Barro, N.G. Mankiw, X. Sala-i-Martin, Capital mobility in neoclassical models of growth, NBER working paper series n. 4206, 1992.

[5] R. Barro, N.G. Mankiw, X. Sala-i-Martin, Capital mobility in neoclassical models of growth, Am. Econ. Rev. **85** (1995), 103-15.

[6] R. Benabou, Tax and education policy in a heterogeneous-agent economy: what levels of redistribution maximize growth and efficiency?, Econometrica **70** (2002), 481-518.

[7] J. Benhabib, R.E.A. Farmer, Indeterminacy and increasing returns, J. Econ. Theory **63** (1994), 19-41.

[8] J. Benhabib, R.E.A. Farmer, Indeterminacy and sector specific externalities, J. Monet. Econ. **421-43** (1996), 421-43.

[9] J. Benhabib, R.E.A. Farmer, Indeterminacy and sunspots in Macroeconomics, in J. Taylor & M. Woodford (eds.), "Handbook of Macroeconomics", Volume 1A, Chapter 6, Amsterdam: Elsevier Science, 1999.

[10] J. Benhabib, B. Jovanovic, Externalities and growth accounting, Am. Econ. Rev. **81** (1991), 82-113.

[11] R. Bennett, R.E.A. Farmer, Indeterminacy with nonseparable utility, J. Econ. Theory **118-43** (2000), 118-43.

[12] B. Bernanke, M. Gertler, Agency costs, net worth, and business fluctuations, Am. Econ. Rev. **79** (1989), 14-31.

[13] J.H. Boyd, B.D. Smith, Capital market imperfections, international credit markets, and nonconvergence, J. Econ. Theory **73** (1997), 335-64.

[14] R. Breen, C. Garcia-Peñalosa, Income inequality and macroeconomic volatility: an empirical investigation, GREQAM working paper series n. 99B11, 1999.

[15] S. Cassou, K. Lansing, Growth effects of shifting from a progressive tax system to a flat tax, Federal Reserve Bank of San Francisco Working Paper 00-15, april 2002.

[16] E.M. Caucutt, S. Imrohoroglu, K.B. Kumar, Growth and welfare analysis of tax progressivity in a heterogeneous-agent model, forthcoming in Review of Economic Dynamics, may 2002.

[17] S. Campbell, Asset prices, consumption and the business cycle, in J. Taylor & M. Woodford (eds.), "Handbook of Macroeconomics", Volume 1C, Chapter 19, Amsterdam: Elsevier Science, 1999.

[18] L. Christiano, S. Harrison, Chaos, sunspots and automoatic stabilizers, J. Monet. Econ. **44** (1999), 3-31.

[19] D. Cohen, J. Sachs, Growth and external debt under risk of debt repudiation, Eur. Econ. Rev. **30** (1986), 526-60.

[20] O. Galor, J. Zeira, Income distribution and macroeconomics, Rev. Econ. Stud. **60** (1993), 35-52.

[21] J.-T. Guo, S. Harrison, Tax policy and stability in a model with sector-specific externalities, Rev. Econ. Dyn. **4** (2001), 75-89.

[22] J.-T. Guo, S. Lansing, Indeterminacy and stabilization policy, J. Econ. Theory **82** (1998), 481-90.

[23] G.D. Hansen, Indivisible labor and the business cycle, J. Monet. Econ. **16** (1985), 309-27.

[24] L.P. Hansen, K. Singleton, Stochastic consumption, risk aversion, and the temporal behavior of asset returns, J. Polit. Econ. **91** (1983), 249-65.

[25] S. Harrison, Indeterminacy in a model with sector-specific externalities, J. Econ. Dyn. Cont. **25** (2001), 747-64.

[26] T. Hintermaier, On the minimum degree of returns to scale in sunspot models of the business-cycle, forthcoming in J. Econ. Theory, 2003.

[27] N. Kiyotaki, J. Moore, Credit cycles, J. Pol. Econ. **105** (1997), 211-48.

[28] N.R. Kocherlakota, Consumption, commitment, and cycles, J. Monet. Econ. **37** (1996), 461-74.

[29] A. Kose, E. Prasad, M. Terrones, Financial integration and macroeconomic volatility, mimeo IMF, november 2002.

[30] P. Krusell, L. Ohanian, G. Violante, J. Rios-Rull, Capital-skill complementarity and inequality: a macroeconomic analysis, Econometrica **68** (2000), 1029-1054.

[31] A. Lahiri, Growth and equilibrium indeterminacy: the role of capital mobility, Econ. Theory **17** (2001), 197-208.

[32] K. Matsuyama, Financial market globalization, symmetry-breaking, and endogenous inequality of nations, Northwestern University, December 2002. Forthcoming in Econometrica.

[33] Q. Meng, A. Velasco, Indeterminacy in a small open economy with endogenous labor supply, Econ. Theory **22** (2003), 661-9.

[34] K. Nishimura, K. Shimomura, Indeterminacy in a dynamic small open economy, J. Econ. Dyn. Cont. **27** (2002), 271-81.

[35] M. Obstfeld, The global capital market: benefactor or menace, J. Econ. Persp. **12** (1998), 9-30.

[36] P.A. Pintus, Indeterminacy with almost constant returns: capital-labor substitution matters, GREQAM working paper series, June 2003.

[37] P.A. Pintus, On convergence in credit-rationed open economies when taxes are progressive, GREQAM working paper series n. 03A11, May 2003.

[38] G. Ramey, V. Ramey, Cross-country evidence on the link between volatility and growth, Am. Econ. Rev. **85** (1995), 1138-1151.

[39] R. Rogerson, Indivisible labor, lotteries and equilibrium, J. Monet. Econ. **21** (1988), 3-16.

[40] J.A. Scheinkman, L. Weiss, Borrowing constraints and aggregate economic activity, Econometrica **54** (1986), 23-45.

[41] S. Schmitt-Grohe, M. Uribe, Balanced-budget rules, distortionary taxes, and aggregate instability, J. Pol. Econ. **105** (1997), 976-1000.

[42] V. Tanzi, H. Zee, Tax Policy for Developing Countries, IMF Economic Issues no. 27, 2001.

[43] J. Tirole, Financial crises, liquidity, and the international monetary system, Princeton, N.J.: Princeton University Press, 2002.

[44] M. Weder, Indeterminacy in a small open economy Ramsey growth model, J. Econ. Theory **98** (2001), 339-56.