Surrogate Data Analysis and Stochastic Chaotic Modelling: Application to Stock Exchange Returns Series

by

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Extended Abstract

Financial markets are highly complex feedback systems in which investors collectively or independently overreact to information or withhold action in the face of information. Such feedback processes characterise the dynamics of nonlinear systems because they are based on a non-proportional relationship between a cause and its effect. The presence of non-linear dependence in financial markets undercuts the EMH, which is founded on assertions about the independence of successive stock price changes. Non-linear dependence suggests that there are deeper structural forces, possibly including noisy chaotic dynamics, that affect financial markets outcomes. The advantage of a noisy chaotic perspective is that it considers structural factors in addition to external noise for explaining markets fluctuations, although for traditional stochastic theory only exogenous forces exist.

The EMH in finance is closely related to the rational expectations hypothesis in economics, according to which agents try to maximising their expected returns and given any observed information, all agents agree on the mean interpretation of such information (homogenous agents). Agents are supposed to be rational and this is common knowledge. In such context, all financial risks and observed volatility arise from causes which are external to the information system. In a rational expectation model, market equilibrium equations are usually assumed to be in the information set, and agents use these underlying market equilibrium equations in forming their rational expectations forecast.

The conclusions of the theory of rational expectations are contradicted by many empirical observations and common experience of markets agents. Indeed, the implications of the theory have been rejected in broad areas of economics. In reality, traders face different transaction costs, have different information sets, work with different time scales and time horizons, and have different expectations about future dividends and stock prices. A

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considerable number of recently introduced structural financial models suggests that when heterogeneity is taken into account, we can obtain results certainly different from those of the homogenous agents rational expectations models, and thus we can be closer to the empirical properties of stock series, i.e. fat tails, volatility clustering etc. (Lux (1998), Iori (1999), Malliaris and Stein (1999), Hommes (2000), Brock and Hommes (2001), Gaunersdorfer and Hommes (2001)).

Brock and Hommes (1998), Lux (1995, 1998), Chen, Lux and Marchesi (2000), Malliaris and Stein (1999), Gaunersdorfer (2000, 2001), and Chiarella *et al.* (2000) show that due to heterogeneity in expectations, structural non-linear financial models produce chaotic dynamics. Non-linear dynamic models can generate a wide variety of irregular patterns. A non-linear chaotic model, buffeted with dynamic noise, with almost no autocorrelations in returns but at the same time persistence in squared returns, with slowly decaying autocorrelations, may thus provide a structural explanation of the unpredictability of stock returns and volatility clustering. Nevertheless, even if the theoretical power of the structural modelling is well proved, the impossibility to provide empirical estimations constitutes an important disadvantage.

The aim of this article is double. We first provide evidence that the underlying structures of three stock exchange returns series (UK, USA, Italy) present strong high-dimensional, non-linear and complex behaviour. Secondly, as a hypothesis we propose a specific structure for the observed behaviour and we test this by statistical inference.

The recent research results by Antoniou and Vorlow (2003a, 2003b) suggest that the underlying dynamics of stock return sequences are characterised by strong aperiodic cyclical behaviour. The dynamical structures observed in their data generating processes are qualitatively similar to those of chaotic systems that exhibit unstable periodic orbits and nonlinear recurrent behaviour. Based on this evidence, in order to avoid the limitations of structural modelling and capture the complex underlying structures, we use the Mackey-Glass-GARCH(p,q) model (MG-GARCH) initially proposed by Kyrtsou and Terraza (2003), that has for different values of the constants either zero or significant autocorrelations in the conditional mean, and a rich structure in the conditional variance. We compare the results of estimating the MG-GARCH on original and simulated return sequences. To provide further evidence for the existence of noisy chaos and complex behaviour, we simulate return processes according to the Surrogate Data Analysis (SDA) framework proposed by Theiler *et* *al.* (1992). SDA enables us essentially to test whether the dynamics are consistent with linearly filtered noise or a nonlinear dynamical process.[4]

The results indicate that there is a strong complex-deterministic component in the dynamical underlying process. Qualitative and quantitative results suggest that the assumption of stochastic randomness (according to the EMH) is not supported by the evidence and markets can be assumed to be highly complex, high-dimensional, open and dissipative dynamical systems that need feedback as well as other kinds of inputs in order to operate (Kyrtsou and Terraza, 2002, Kyrtsou *et al.,* 2003). These inputs may come in the guise of noise or news. The inputs may also control the evolution of the system dynamics and the knowledge of their nature may allow us to forecast the future states of the market with greater accuracy. To this extent the MG-GARCH model provides a valuable insight on how a feedback mechanism can operate within the structure of stock returns processes and explain stylized facts about these.

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[4]According to SDA, assuming to know very little about the origin of fluctuations of the returns sequences, we attempt to refute the hypothesis that these were generated by a Gaussian random process with linear correlations, "possibly distorted by a nonlinear measurement function" (Kantz and Schreiber, 1997). To demonstrate this we need to generate artificial data sets (called "surrogates") which exhibit the same linear properties as the original data but do not contain any further deterministic components. The surrogates consist of random numbers, rescaled to the distribution of the original data (i.e., same mean and variance) and can be further filtered exhibit the same spectral properties (i.e., same power spectrum and autocorrelation function).

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