# Exchange Rate Pass-through and Monetary Policy: How Strong is the Link?

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#### Abstract

Several authors have presented reduced-form evidence suggesting that the degree of exchange rate pass-through to the consumer price index has declined in Canada since the early 1980s and is currently close to zero. Authors such as Taylor (2000) suggest that this may be due to a change in the conduct of monetary policy. Specifically, if monetary policy is seen to be responding more aggressively to shocks that affect inflation then the expected persistence of these shocks will decline and by consequence, so will the degree of pass-through to consumer prices. This paper investigates the extent to which monetary policy, under commitment to an inflation target, can influence exchange rate pass-through in a dynamic stochastic general equilibrium model of a small open economy. We first define pass-through in the context of a reduced-form Phillips curve and then compute numerically the relationship between pass-through and the parameters of a simple monetary policy rule, holding constant the model's remaining structural parameters. Our results suggest that for reasonable increases in the aggressiveness of policy, such as those observed since the 1980s in Canada, measured pass-through can effectively be eliminated. However, this result depends critically on the inclusion of price-mark-up shocks in the model. When these are excluded, a more modest, albeit still negative, relationship prevails. This is also true when pass-through is defined as the response of prices to an exchange rate shock in the structural model.

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## 1 Introduction

The goal of this paper is to quantify the link between changes to the conduct of monetary policy and exchange rate pass-through in Canada. Specifically, we investigate to what extent the observed decline in pass-through experienced in Canada since the early 1980s is attributable to a monetary policy that more aggressively targets inflation.

Several authors have presented reduced-form evidence for Canada indicating that the degree of exchange-rate pass-through to the consumer price index has declined since the early 1980s and is currently close to zero. Furthermore, this phenomenon of reduced pass-through has been shown to exist for many countries and appears to coincide with decreases in the average rate of price inflation. As argued in Devereux and Yetman (2002), decreases in average inflation should result in longer price contracts. Thus, lower average inflation itself may explain reduced pass-through. Alternatively, lower inflation may simply coincide with more aggressive inflation targeting, whether the target is explicitly announced or not. For example, those countries who have successfully reduced their inflation rates may have, at the same time, committed to responding more aggressively to shocks that threaten to undermine hard-fought gains to credibility. Taylor (2000) argues that if monetary policy is perceived to be responding more aggressively to shocks that affect inflation then the expected persistence of these shocks will decline and by consequence, so will the degree of pass-through to consumer prices. Taylor goes on to show, using a simple model, how the perceived persistence of an expansionary money shock can influence firms' desire to 'pass on' cost increases in the form of higher prices.

While compelling as a theoretical argument, it is less clear how quantitatively important this effect is for reasonable changes in the conduct of policy, particularly when the central bank targets inflation and not the price level. Rudebusch (2003), for example, demonstrates that the parameters of reduced-form Phillips curves are largely invariant to historical shifts in the aggressiveness of monetary policy under inflation targeting in the United States over the last several decades.

In order to quantify the influence of monetary policy on exchange rate pass-through, we develop a small open-economy dynamic stochastic general equilibrium model (DSGE) for Canada. The model is consistent with the New Open-Economy Model (NOEM) paradigm, in that it extends the basic closed-economy, optimizing-agent/sticky-price, or New Neoclassical Syn-

<sup>&</sup>lt;sup>1</sup>Assuming fixed menu costs represent the main rationale for nominal contracting.

thesis (NNS, see Goodfriend and King 1997), framework to allow for international trade in goods and credit. The model includes sticky nominal wages and sticky prices for domestically produced and imported goods (as in Smets and Wouters 2002), the latter implying incomplete exchange rate pass-through to import prices in the short run. The model is closed using a Taylor rule that includes a role for interest-rate smoothing. Using the model and the estimated variances of the historical structural shocks, we then generate artificial data for a range of parameter values for the Taylor rule. Finally, we estimate, for each parameterization of the rule, a reduced-form equation that models inflation as function of lags of inflation, the output gap and changes in the real exchange rate. Pass-through is then computed by extrapolating the price-level response to a one percent change in the exchange rate.

Overall, we find that for reasonable changes to the policy rule, large changes in estimated pass-through can be generated. Specifically, parameter values of between 1.6 and 2.1 on the deviation of inflation from target (in a Taylor rule) are sufficient to drive estimated pass-through to zero. Furthermore, this range of values is not inconsistent with estimated policy rules for Canada since the 1980s. However, our results also indicate that the reduction stems more from the effect of changes to policy on the correlation between prices and the exchange rate in mark-up shocks than in exchange rate shocks. When mark-up shocks are excluded from the model, pass-through declines by at most 50 per cent. This is also true when we define pass-through in terms of the response of consumer prices to a deterministic exchange rate shock in the structural model. Thus our results indicate that the strength of the negative relationship between monetary policy and pass-through depends strongly on how one chooses to measure the latter.

# 2 A Small Open Economy Model

Our objective here is to elaborate a model sufficiently rich in detail and structure so as to produce realistic dynamics for output and inflation while at the same time remaining reasonably tractable. Toward this end, we begin with a core structure of optimizing consumers and producers with rational expectations and add to it sticky final-goods prices, imported intermediategoods prices and nominal wages. In addition to these nominal rigidities, we allow for costly adjustment of the capital stock, time to build with ex-post inflexibilities, variable utilization of capital and habit formation in consump-

tion. In terms of external linkages, we assume that while producer currency pricing prevails in the long run, import prices are temporarily rigid in the currency of the importing country. Furthermore, imports are treated as an input to the production of final goods. The model contains 6 structural shocks (preference, technology, monetary policy, risk premium and 2 markup shocks) as well as a non-structural shock to foreign demand.

The basic structure of the model can be summarized as follows; A perfectly competitive firm purchases differentiated labour services and investment goods to produce a domestic good using a CES technology. There also exists an imperfectly-competitive imported-good sector. Domestic goods and imports are sold to monopolistically-competitive producers of final consumption, investment and non-commodity export goods. These 3 types of finished goods differ only in their import concentrations. <sup>2</sup> Commodities are produced using the domestic good and a fixed factor, which we refer to as land, and are sold for export only. Moreover, commodity producers are assumed to be price takers on world markets. We view it as important to specify an explicit role for commodities for a resource-rich, open economy such as Canada when analyzing the exchange rate since a significant proportion of Canada's exchange-rate volatility can be linked to terms-of-trade fluctuations (see Chen and Rogoff (2003) and Amano and van Norden (1995)).

Consumers supply heterogenous labour and purchase the final consumption good with labour income, firm dividends and interest from foreign bond holdings so as to maximize lifetime utility. The model assumes no role for fiscal policy.

#### 2.1 Domestic Production

We begin by assuming the existence of a representative, perfectly competitive, firm that produces a domestic good using a constant elasticity of substitution production (CES) technology that combines effective labour,  $A_tL_t$ , with capital services,  $u_tK_t$ :

$$\mathcal{F}(A_t L_t, u_t K_t) = \left[ \delta^{\frac{1}{\sigma}} (A_t \cdot L_t)^{\frac{\sigma - 1}{\sigma}} + (1 - \delta)^{\frac{1}{\sigma}} (u_t \cdot K_t)^{\frac{\sigma - 1}{\sigma}} \right]^{\frac{\sigma}{\sigma - 1}} \quad \sigma \neq 1, (1)$$

<sup>&</sup>lt;sup>2</sup>Thus, final-good producers do not explicitly choose their capital to labour mix. This allows us to differntiate the import intensities across sectors without having to model separately the investment and labour decision for each of the 3 sectors. In Canada, the import share's of consumption, investment and exports differ substantially.

where  $A_t$  is labour-augmenting technology,  $L_t$  and  $K_t$  are aggregate labour and capital and  $u_t$  is capacity utilization.<sup>3</sup>  $A_t$  evolves according the first-order autoregressive process:

$$\log(A_t) = (1 - \rho_A)\log(A) + \rho_A\log(A_{t-1}) + \varepsilon_t^A \qquad \varepsilon_t^A \sim (0, \sigma_A^2) \tag{2}$$

Capital accumulation is constrained by time-to-build with ex-post inflexibilities (Edge 2000a, b). We assume complementarity between investment expenditures in a given project across time, which discourages firms from diverging ex post from their original investment plan. We formally incorporate this interdependence by specifying what we call the firm's "effective investment,"  $I_t^E$ , as a CES aggregator of past and current investment expenditures:

$$I_t^E = \left(\sum_{j=0}^{\tau} (\phi_j I_{t-j,t})^{\theta}\right)^{1/\theta}.$$
(3)

The first subscript on the investment terms denotes the time of the investment expenditure; the second denotes the period in which the project is to be completed.  $\theta$  controls the degree of intertemporal complementarity between investment expenditures: as  $\theta \to -\infty$ , investment expenditures become perfect complements, and the investment plan is completely inflexible ex post. The  $\phi$ 's can account for a planning phase at the start of a project in which expenditures are typically relatively small as, for example, building plans are drawn up (see Christiano and Todd 1996). We allow a 1-quarter planning period by allowing  $\phi_{\tau}$  to vary relative to  $\phi_{0}...\phi_{\tau-1}$ . For the project length,  $\tau+1$ , we assume 5 quarters.

The firm's capital stock at the start of a period is the sum of the last quarter's depreciated capital stock plus the amount of effective investment, or new capital, installed at the end of the previous period:

$$K_{t+1} = (1 - \omega)K_t + I_t^E. (4)$$

ment is given as 
$$I_t = \left[\int_0^1 I_{it}^{\frac{\epsilon_I - 1}{\epsilon_I}} di\right]^{\frac{\epsilon_I}{\epsilon_I - 1}}$$
 and labour as  $L_t = \left[\int_0^1 L_{ht}^{\frac{\epsilon_w - 1}{\epsilon_w}} dh\right]^{\frac{\epsilon_w}{\epsilon_w - 1}}$  where

 $I_{it}$  is the output of the i<sup>th</sup> investment good producer and  $L_{ht}$  is the labour supplied by the h<sup>th</sup> household.

<sup>&</sup>lt;sup>3</sup>In addition to producing the domestic good, we assume that the representative firm purchases and bundles differentiated investment goods and labour services. Total invest-

Aggregate investment,  $I_t$ , is the sum of the firm's investment expenditures on projects currently underway:

$$I_t = \sum_{j=0}^{\tau} I_{t,t+j}.$$
 (5)

The firm incurs a quadratic cost when it adjusts the level of the capital stock, which takes the form of a deadweight loss of the produced good. We also assume that the firm can vary its rate of capital utilization at the cost of foregone output. When we incorporate quadratic capital adjustment costs in addition to convex costs of capital utilization, output evolves according to

$$Y_t^d = \mathcal{F}(A_t L_t, u_t K_t) - \frac{\chi}{2K_t} (I_t^E)^2 - \psi \left(1 - e^{\rho(u_t - 1)}\right) K_t,$$
 (6)

where  $\chi$  determines the size of capital adjustment costs and  $\psi$  and  $\rho$  determine the costs of variable capital utilization.<sup>4</sup>

The competitive firm's objective is to choose  $P_{d,t}$ ,  $Y_t^d$ ,  $L_t$ ,  $K_{t+1}$ ,  $I_t^E$ ,  $u_t$ , and  $I_{t,t+k}$  ( $k = 0, 2..., \tau$ ), subject to equations (3),(4), (5) and (6) to maximize the value of the firm:

$$\mathcal{V}_t = \mathcal{E}_t \sum_{s=t}^{\infty} \mathcal{R}_{t,s} \left[ P_{d,s} Y_s^d - W_s L_s - P_{I,s} \sum_{k=0}^{\tau} I_{s,s+k} \right], \tag{7}$$

where  $P_{I,t}$  is the price of investment,  $W_t$  is the aggregate nominal wage and the stochastic discount factor,  $\mathcal{R}_{t,s}$ , is defined as

$$\mathcal{R}_{t,s} \equiv \prod_{v=t}^{s} \left( \frac{1}{1 + R_v} \right). \tag{8}$$

The solution to (7) gives rise to the following optimality conditions (ignoring first-order conditions with respect to the Lagrangians):

 $<sup>^4</sup>$ We impose a restriction on the parameter  $\psi$  such that, in steady state, the utilization rate is one and the cost of utilization is zero.

$$W_{t} = \lambda_{t} \mathcal{F}_{l}(\cdot), \qquad (9)$$

$$q_{t} = \mathcal{E}_{t} \mathcal{R}_{t,t+1} \left[ \lambda_{t+1} \left( \mathcal{F}_{k}(\cdot) + \frac{\chi}{2} \left( \frac{I_{t+1}^{E}}{K_{t+1}} \right)^{2} - \psi \left( 1 - e^{\rho(u_{t}-1)} \right) \right) + (1 - \omega)q_{t+1} \right], \qquad (10)$$

$$I_{t,t+k} = \mathcal{E}_t \left[ \frac{\left(I_{t+k}^E\right)^{1-\theta} \phi^{\theta} \mathcal{R}_{t,t+k}}{P_{I,t}} \left( q_{t+k} - \chi \lambda_{t+k} \frac{I_{t+k}^E}{K_{t+k}} \right) \right]^{\frac{1}{1-\theta}}, \tag{11}$$

$$\mathcal{F}_u(\cdot) = -\psi \rho e^{\rho(u_t - 1)} K_t, \tag{12}$$

$$P_{d,t} = \lambda_t, \tag{13}$$

where  $\lambda_t$  is the constraint that equates demand and supply, which may be interpreted as the marginal cost of production. The variable  $q_t$  is the shadow value of capital, or the discounted contribution of capital to future dividends. Finally,  $\mathcal{F}_j(\cdot)$  is the partial derivative of  $\mathcal{F}(\cdot)$  with respect to j.

#### 2.2 Imported Goods Sector

In addition to domestic goods, we assume the existence of a continuum of intermediate imported goods,  $M_{jt}$ ,  $j \in [0, 1]$ , that are bundled into an aggregate import,  $M_t$ , by the aggregator and sold to final-goods producers,

$$M_t = \left[ \int_0^1 M_{jt}^{\frac{\epsilon - 1}{\epsilon}} dj \right]^{\frac{\epsilon}{\epsilon - 1}}.$$
 (14)

Demand by the aggregator for the differentiated goods is given by the familiar cost-minimizing demand functions,

$$M_{jt} = \left(\frac{P_{m,jt}}{P_{m,t}}\right)^{-\epsilon} M_t,\tag{15}$$

where  $P_{m,t}$  is the aggregate import-price deflator, given as

$$P_{m,t} = \left[ \int_0^1 P_{m,jt}^{1-\epsilon} dj \right]^{\frac{1}{1-\epsilon}}.$$
 (16)

We follow Smets and Wouters (2002) in assuming that the price of the imported good is temporarily rigid in the currency of the importing country. Consequently, exchange rate pass-through to import prices is partial in the short run and complete in the long run. Exchange rate fluctuations are absorbed by the importers' profit margins in the short run, since they purchase goods according to the law of one price. Importers therefore take into consideration the future path of foreign prices and the nominal exchange rate when deciding on their time t price. As for the source of rigidity, we follow the bulk of the literature in assuming the existence of multi-period price contracts. We follow Dotsey, King, and Wolman (1999)<sup>5</sup> and Wolman (1999) and allow for the possibility that firms fix their prices for up to j (j > 1) periods.<sup>6</sup> We first introduce the following notation. Let  $\alpha$  be a jdimensional vector in which the ith row,  $\alpha_i$ , represents the probability that a firm adjusts its price, conditional on having last adjusted i periods ago. By assumption,  $\alpha_j = 1$ . The fraction of firms,  $\varpi_i$ , in a given period that charge prices that were set i periods ago is therefore given by

$$\overline{\omega}_i = (1 - \alpha_i) \cdot \overline{\omega}_{i-1} \quad i = 1, 2, ..., j - 1,$$
(17)

and the probability,  $\Lambda_i$ , of a contract price remaining in effect *i* periods in the future is equal to the product of the probabilities of not changing prices in each of the preceding periods up to the *ith* 

$$\Lambda_i \equiv \frac{\overline{\omega}_i}{\overline{\omega}_0} = \prod_{q=0}^i (1 - \alpha_q) \qquad \alpha_0 = 0, \tag{18}$$

 $\varpi_0$  represents the (constant) proportion of firms that adjust their price in any given period. Given this setup, the importers' optimal decision rule is given as:

$$P_{m,it} = \left(\frac{\epsilon}{\epsilon - 1}\right) \mathcal{E}_t \left(\frac{\sum_{s=t}^{t+j-1} \mathcal{R}_{t,s} \Lambda_{s-t} P_{m,s}^{\epsilon}(e_s P_s^*) M_s}{\sum_{s=t}^{t+j-1} \mathcal{R}_{t,s} \Lambda_{s-t} P_{m,s}^{\epsilon} M_s}\right), \tag{19}$$

<sup>&</sup>lt;sup>5</sup>For this version of the model, we exclude the state-dependent component discussed in Dotsey, King, and Wolman (1999). Thus, our price-change probabilities are invariant to the state of the economy.

<sup>&</sup>lt;sup>6</sup>For a more a discussion of the merits of this pricing model, see Murchison, Rennison and Zhu (2004).

where we can again replace individual price-resetters with a cohort of firms,  $p_{m,t}$ , each of which resets at time t. The aggregate import price level is then determined as a CES aggregate of past contract prices:

$$P_{m,t} = \left(\sum_{k=0}^{j-1} \varpi_{m,k} (p_{m,t-k})^{1-\epsilon}\right)^{\frac{1}{1-\epsilon}}.$$
 (20)

#### 2.3 Final Goods Sector

The domestic and imported goods,  $Y_t^d$  and  $M_t$ , are then used in the production of the consumption, investment, commodity or non-commodity export subject to the constraints that:

$$Y_t^d = C_t^d + I_t^d + X_{NC,t}^d + X_{C,t}^d (21)$$

$$M_t = C_t^m + I_t^m + X_{NC,t}^m (22)$$

where  $C_t^d$ , for instance, refers to the quantity of the domestic good used in the production of the final consumption. Hence, resource constraints (21) and (22) simply states that the sum of the domestic and imported goods used in the production of final goods cannot exceed total domestic or import production.

#### 2.3.1 Consumption Good

We assume a continuum of monopolistically competitive firms that each produce a differentiated consumption good and charge a price for their good that maximizes expected profits. Thus, the representative firm,  $i, i \in [0, 1]$ , will produce  $C_i$  and receive price  $P_{c,i}$  in return. Aggregate consumption,  $C_t$ , and its corresponding deflator,  $P_{c,t}$ , are defined by

$$C_t = \left[ \int_0^1 C_{it}^{\frac{\epsilon_t - 1}{\epsilon_t}} di \right]^{\frac{\epsilon_t}{\epsilon_t - 1}} \tag{23}$$

$$P_{c,t} = \left[ \int_0^1 P_{c,it}^{1-\epsilon_t} di \right]^{\frac{1}{1-\epsilon_t}}.$$
 (24)

Note that we treat the elasticity of substitution between finished consumption goods as stochastic as in Smets and Wouters (2003), Steinsson (2003) and Ireland (2004). In addition, we assume the following process

$$\log(\epsilon_t) = (1 - \rho_{\epsilon})\log(\epsilon) + \rho_{\epsilon}\log(\epsilon_{t-1}) + \varepsilon_t^{\epsilon} \qquad \varepsilon_t^{\epsilon_c} \sim (0, \sigma_{\epsilon}^2)$$
 (25)

Cost minimization in the production of a unit of C by the aggregator implies that firm i faces the demand schedule

$$C_{it} = \left(\frac{P_{c,it}}{P_{c,t}}\right)^{-\epsilon_t} \cdot C_t \tag{26}$$

for its product. In addition, firms produce goods using a CES production technology that combines the domestic good,  $C_t^d$ , with the imported good,  $C_t^m$ , to produce final consumption

$$C_{it} = \left[ (1 - \gamma_c)^{\frac{1}{\varphi}} (C_{it}^d)^{\frac{\varphi - 1}{\varphi}} + \gamma_c^{\frac{1}{\varphi}} (C_{it}^m)^{\frac{\varphi - 1}{\varphi}} \right]^{\frac{\varphi}{\varphi - 1}}, \tag{27}$$

The  $i^{th}$  firm's problem is to choose  $P_{c,it}$ ,  $C_{it}$ ,  $C_{it}^d$  and  $C_{it}^m$  subject to (26) and (27) so as to maximize the value of the firm. This leads to following set of first order conditions

$$C_{it}^{d} = (1 - \gamma_c) \left(\frac{P_{d,t}}{\lambda_{it}^c}\right)^{-\varphi} \cdot C_{it}$$
 (28)

$$C_{it}^{m} = \gamma_c \left(\frac{P_{m,t}}{\lambda_{it}^c}\right)^{-\varphi} \cdot C_{it} \tag{29}$$

$$P_{c,it} = \mathcal{E}_t \left( \frac{\sum_{s=t}^{t+j-1} \mathcal{R}_{t,s} \zeta_{s-t} P_{c,s}^{\epsilon_s} \lambda_{is}^c C_s \epsilon_s}{\sum_{s=t}^{t+j-1} \mathcal{R}_{t,s} \zeta_{s-t} P_{c,s}^{\epsilon_s} C_s \left( \epsilon_s - 1 \right)} \right), \tag{30}$$

where we can again replace individual price-resetters with a cohort of firms,  $p_{c,t}$ , each of which resets at time t.<sup>7</sup> The aggregate import price level is then determined as a CES aggregate of past contract prices:

 $<sup>{}^{7}\</sup>zeta$  is defined in a manner analogous to  $\Lambda$  for the import sector.

$$P_{c,t} = \left(\sum_{k=0}^{j-1} \overline{\omega}_{c,k} (p_{c,t-k})^{1-\epsilon_t}\right)^{\frac{1}{1-\epsilon_t}}.$$
 (31)

### 2.3.2 Investment and Non-Commodity Export Goods

The structure of the investment and non-commodity export goods sector is identical to the consumption sector except that we allow for a different import intensities,  $\gamma_{inv}$  and  $\gamma_{x,nc}$ , in the production process. This reflect the fact that historically, the import shares of these components of GDP have differed substantially. Thus, relative prices across the components of GDP will differ from one to the extent that import intensities differ.

#### 2.3.3 Commodity Exports

We assume a representative, perfectly competitive domestic firm produces commodities and exports them to the rest of world. For its product, the firm receives the rest-of-world price of commodities adjusted for by the nominal Canada/rest-of-world exchange rate:

$$P_{xc,t} = e_t \cdot P_{xc,t}^* \tag{32}$$

The commodity export,  $X_{C,t}$ , is produced by combining the value added good, as defined above, with a fixed factor, which we refer to as land,  $LD_t$ :

$$X_{C,t} = \left[ (\gamma_{xc})^{\frac{1}{\varphi_{xc}}} \left( A_t \cdot LD_t \right)^{\frac{\varphi_{xc}-1}{\varphi_{xc}}} + (1 - \gamma_{xc})^{\frac{1}{\varphi_{xc}}} (X_{C,t}^d)^{\frac{\varphi_{xc}-1}{\varphi_{xc}}} \right]^{\frac{\varphi_{xc}-1}{\varphi_{xc}}} - \chi_2 \Omega_t^2 \cdot A_t,$$

$$(33)$$

where  $X_{C,t}^d$  is the amount of value added good used in commodity production and  $\Omega_t = \left(\frac{X_{C,t}^d/A_t}{X_{C,t-1}^d/A_{t-1}} - 1\right)$ . The second term implies that it is costly for the firm to adjust the share of the value-added good in the production of commodities; one can think of this cost as slowing the reallocation of factors of production across sectors.

The commodity producing firm chooses the quantity to produce,  $X_{C,t}$ , in order to maximize the value of the firm, or the discounted flow of profits:

$$\mathcal{V}_t = \mathcal{E}_t \sum_{s=t}^{\infty} \mathcal{R}_{t,s} \left[ X_{C,s} P_{xc,s} - P_{d,s} X_{C,s}^d \right], \tag{34}$$

The first-order condition which, in conjunction with (33) and (32), determines commodities production and the use of the value added good in the commodities sector is:

$$P_{d,s} = P_{xc,t} \left( \frac{\gamma_{xc} X_{C,t}}{X_{C,t}^d} \right)^{\varphi_{xc}} - \chi_1 \left( \frac{\Omega_t A_{t-1}}{X_{C,t-1}^d} - \mathcal{R}_{t,t+1} \frac{P_{xc,t+1} \Omega_{t+1} (1 + \Omega_{t+1}) A_t}{X_{C,t}^d} \right), \tag{35}$$

Finally, nominal gross domestic product in this economy is given by

$$P_t Y_t = P_{c,t} C_t + P_{I,t} I_t + P_{xc,t} X_{C,t} + P_{xnc,t} X_{NC,t} - P_{m,t} M_t$$
(36)

#### 2.4 Consumers

A continuum of households indexed by h,  $h \in [0, 1]$ , purchase domestically produced and imported goods and consume leisure to maximize their lifetime utility. Each household is assumed to supply differentiated labour services to the intermediate-goods sector. Furthermore, the labour market is assumed to be monopolistically competitive, which motivates the existence of wage contracts. Household labour services are purchased by an aggregator and bundled into composite labour according to the Dixit-Stiglitz aggregation function:

$$L_t = \left[ \int_0^1 L_{ht}^{\frac{\epsilon_{w,t}-1}{\epsilon_{w,t}}} dh \right]^{\frac{\epsilon_{w,t}}{\epsilon_{w,t}-1}}.$$
 (37)

Similarly, the aggregate nominal wage index is given as

$$W_t = \left[ \int_0^1 W_{ht}^{1 - \epsilon_{w,t}} dh \right]^{\frac{1}{1 - \epsilon_{w,t}}}.$$
 (38)

where we assume a time-varying mark-up for wages where  $\epsilon_{w,t} \sim NIID(\epsilon, \sigma_{\epsilon_w}^2)$ . The aggregator purchases differentiated labour services to minimize costs. Thus, the demand for labour services from individual h is given as

$$L_{ht} = \left(\frac{W_{ht}}{W_t}\right)^{-\epsilon_{w,t}} \cdot L_t. \tag{39}$$

Finally, we assume that wages are reset according to the same model presented for import and final-goods prices in the previous section. Specifically, we allow for the possibility that households fix their wages for up to q, (q > 1) periods. As with prices, the aggregate nominal wage,  $W_t$ , can be expressed as a CES aggregate of the individual "cohort" wage contracts signed up to q - 1 periods in the past:

$$W_t = \left(\sum_{k=0}^{q-1} \varpi_{w,k}(w_{t-k})^{1-\epsilon_{w,t}}\right)^{\frac{1}{1-\epsilon_{w,t}}}.$$
(40)

The instantaneous utility function for the  $h^{th}$  household is given as<sup>8</sup>

$$\mathcal{U}_{ht} = \frac{\mu}{\mu - 1} (C_{ht} - H_t)^{\frac{\mu - 1}{\mu}} \exp\left(\frac{\eta(1 - \mu)}{\mu(1 + \eta)} \cdot L_{ht}^{1 + 1/\eta}\right),\tag{41}$$

where  $H_t$  is the external habit, which is assumed to be proportional to lagged aggregate consumption:

$$H_t = \xi C_{t-1}. \tag{42}$$

Thus, household consumption will depend positively on lagged aggregate consumption according to the parameter  $\xi$ . Thus, we assume that individuals enjoy high consumption in and of itself (provided  $\xi < 1$ ), but that they also derive utility from high consumption relative to that of the general population.

Households maximize lifetime utility according to

<sup>&</sup>lt;sup>8</sup>Equation (41) is non-standard primarily in the sense that consumption and leisure are not additively separable (see King, Plosser, and Rebelo 1988 and Basu and Kimball 2000; for a model application, see Smets and Wouters 2003). Consequently, the marginal utility of consumption (leisure) will depend on labour (consumption).

$$\mathcal{E}_0 \sum_{t=0}^{\infty} \beta^t \varepsilon_t \mathcal{U}_{ht}, \tag{43}$$

where  $\varepsilon_t$  is a temporary shock to the rate of time preference that is assumed to follow the process,

$$\log(\varepsilon_t) = \rho_\beta \log(\varepsilon_{t-1}) + \nu_{\beta,t}, \quad \nu_{\beta,t} \sim (0, \sigma_\varepsilon^2)$$
(44)

subject to the dynamic budget constraint,<sup>9</sup>

$$P_{c,t}C_{ht} + \frac{B_{ht}}{1 + R_t} + \frac{e_t B_{ht}^*}{(1 + R_t^*)(1 + \kappa_t)} = B_{h,t-1} + e_t B_{h,t-1}^* + W_{ht}L_{ht} + \Pi_t,$$
(45)

where  $B_{ht}^*$  and  $B_{ht}$  are, respectively, the value of foreign (domestic) currency-denominated bonds held at time t and  $e_t$  is the Canadian dollar price of unit of foreign exchange.  $\Pi_t$  represents dividends paid by the firm.  $\kappa_t$  is interpreted as the country-specific risk premium and is assumed follow the process

$$\kappa_t = \rho_{\kappa} \kappa_{t-1} + \nu_{\kappa,t} \quad \nu_{\kappa,t} \sim (0, \sigma_{\kappa}^2)$$
(46)

Maximizing (43) with respect to  $C_{ht}$ ,  $W_{ht}$ ,  $L_{ht}$ ,  $B_{ht}^*$ , and  $B_{ht}$  subject to (39) and (45) yields the following first-order conditions:

$$P_{c,t}\Phi_t = (C_t - H_t)^{-1/\mu} \exp\left(\frac{\eta(1-\mu)}{\mu(1+\eta)} \cdot L_{ht}^{1+1/\eta}\right) \cdot \varepsilon_t,$$
 (47)

$$\Phi_t = \beta \mathcal{E}_t \Phi_{t+1}(1+R_t), \tag{48}$$

$$e_t \Phi_t = \beta \mathcal{E}_t e_{t+1} \Phi_{t+1} (1 + R_t^*) (1 + \kappa_t).$$
 (49)

Consumers, when given the chance to reset their wages, will do so according to the following dynamic rule:

<sup>&</sup>lt;sup>9</sup>In addition to the budget constraint, the no-Ponzi game condition is enforced for domestic and foreign bonds. Also, we assume that consumption is identical across households despite differences in wage income out of steady state.

$$W_{ht} = \mathcal{E}_t \left( \frac{\sum_{s=t}^{t+q-1} \mathcal{R}_{t,s} \Xi_{s-t} P_s^c \Phi_s W_s^{\epsilon_{w,s}(1+1/\eta)} L_s^{1+1/\eta} (C_s - H_s) \epsilon_{w,s}}{\sum_{s=t}^{t+q-1} \mathcal{R}_{t,s} \Xi_{s-t} \Phi_s W_s^{\epsilon_{w,s}} L_s (\epsilon_{w,s} - 1)} \right)^{\frac{\eta}{\epsilon_{w,t} + \eta}}.$$
(50)

Moreover, since all consumers who choose to reset their wage at the same time will choose the same wage, we can replace  $W_{ht}$  with the cohort wage,  $w_t$ . Equation (50) can then be combined with (40) to solve for the behaviour of the aggregate nominal wage,  $W_t$ .

#### 2.5 Foreign Economy

In order to close our small open economy, it is necessary to specify processes that describe foreign demand for Canadian exports, foreign import prices, the economy-wide price level, interest rates and the foreign-dollar-denominated price of commodities. Foreign demand for Canadian non-commodity exports is given by the following derived demand function

$$X_{NC,t} = \gamma^* \cdot \left(\frac{P_t^{*x}}{P_t^*}\right)^{-\vartheta} \cdot Y_t^*,\tag{51}$$

where  $P_t^{*x}$  and  $P_t^*$  are, respectively, the foreign price of domestic output and the foreign general price level, and  $Y_t^*$  is foreign output. Here, we assume that the foreign import price  $(P_t^{*x})$  is determined in the same manner as the home import price;  $-\vartheta$  is the elasticity of substitution between domestic exports and foreign-produced goods. Foreign output (GDP),  $Y_t^*$ , and its corresponding deflator,  $P_t^*$ , foreign-dollar denominated commodity prices,  $P_{xc,t}^*$ , and foreign nominal interest rates,  $R_t^*$ , are modelled using a reduced-form restricted VAR(1). The foreign output shock is simply a shock to the foreign output-gap equation. A positive shock has the effect of raising foreign output, commodity prices, inflation and interest rates.

# 3 Solution and Calibration

The model presented here is non-linear and contains unobserved expectations of future state variables. Before solving the model we first log-linearize it numerically about its stationary steady state using a first-order Taylor-series expansion (implemented numerically in Troll). Second, we solve the

log-linear version of the model using Sparse AIM (see Anderson and Moore 1985 and Anderson 1997).

As is now the custom with DSGE models, we divide the unknown structural parameters into two sets. The first set is calibrated so that the model will generate steady-state ratios that conform to historical averages found in the data. The results are summarized in Table 1. The quarterly subjective discount rate is set to 0.99, which corresponds to a quarterly steady-state real interest rate of just over one per cent, given the stability condition  $\beta(1+r)=1$  is enforced. The parameters  $\epsilon_p$  is set to 11.0, which yields a markup of price over marginal cost of 10 per cent. In addition, we assume that  $\epsilon_w = \epsilon_p$  in steady state. The parameter  $\delta$  was set to 0.33 to replicate the historical steady-state labour share of income. The parameter  $\omega$ , which is the quarterly depreciation rate of installed capital, is set to 0.05. In the absence of trend growth, it is necessary to calibrate this parameter at a level above the typical value of 0.025 to ensure a plausible steady-state investment-to-output ratio.  $\psi$  is simply a calibration parameter that is set so that the steady-state behaviour of the model is unaffected by the introduction of variable capacity utilization.  $\theta$  is set to -20, implying an elasticity of substitution across investment expenditures of -0.05. This calibration ensures that investment plans are costly to revise ex post.  $\phi_4$ , which governs investment in the planning period, is estimated (see Table 2), while the remaining  $\phi$ 's ( $\phi_0$  through  $\phi_3$ ) are calibrated so that, in steady state, the capital stock generated by the model is equal to the traditional capital stock measure (i.e., that generated by replacing  $I_t^E$  in (4) with  $I_t$ ).  $\gamma_c, \gamma_I$ , and  $\gamma_{x,nc}$  were, respectively, chosen to replicate in steady state the historical average import shares for consumption, investment, and exports.  $\gamma_{xc}$  is then chosen to replicate the average share of commodities in total exports.

In order to avoid having to calibrate j-1 price-change probabilities for each pricing model, we have imposed a non-linear functional form on the  $\alpha$  vector to reduce the free parameter set to 1:

$$\alpha_k = \left(\frac{1}{1+S}\right)^{j-k} \qquad S > 0; \quad k = 1, 2, ..., j-1,$$
 (52)

where S is a freely estimated parameter (subject to being positive). Furthermore,  $\partial \alpha_k/\partial k > 0$  and  $\partial^2 \alpha_k/\partial k^2 > 0$ , ensuring that the conditional probability of a price change is increasing (at an increasing rate) in the time since the last price change. We select the wage duration parameter,  $S_w$ , so as to produce an average wage duration of about 6 quarters. This value

is chosen to be somewhat shorter than the average of private sector wage settlements between 1978 and 1984, since this survey includes explicit contracts only.  $S_c$  and  $S_m$  are chosen to yield price-contract durations of about 3 quarters, as in Ambler, Dib and Rebei (2003). In addition, we assume for simplicity that price contract durations are equal across consumption, investment and non-commodity exports.

The remainder of the parameters are taken from Murchison, Rennison and Zhu (2004), who estimate a very similar model by matching the theoretical impulse responses from their model to those of a VAR using a demand, exchange rate and monetary policy shock. These parameter values are listed in Table 2. Finally, conditional on these parameter values, the AR(1) coefficients ( $\rho$ 's) are estimated using least squares thereby rendering the structural shocks to be (approximately) white noise.

The model presented here remains too stylized to adequately capture the low frequency movements in Canadian data witnessed over the last 30 years. For instance, factors such as trade liberalization have allowed both exports and imports to grow faster than GDP over the last 25 years. In addition, there has been more than one discrete change in inflation regime during this period. Factors such as this render our model, its current form, unable to reproduce all of the trends in the historical data.<sup>10</sup> Thus, we elect to de-trend the raw data both for the purpose of estimation and for calculating the structural shocks. De-trended data have been used with DSGE models by Ireland (2001), Smets and Wouters (2002) and Bouakez, Cardia, and Ruge-Murcia (2002). However, we view this an interim solution only since de-trending remains controversial and substantial differences can arise depending on the detrending technique used. For our purpose here, we compute each series as the difference between the log of the raw series and the Hodrick-Prescott-filtered (HP) series with lambda set to 1600. We have also experimented with lambda settings of 3200 and 6400 with no appreciable changes to the results.

# 4 Monetary Policy and Exchange Rate Pass-through

# 4.1 Characterizing Monetary Policy

In terms of specify a policy rule, we must address two related issues. First, how to best characterize the behaviour of policy over the last 35 years in

<sup>&</sup>lt;sup>10</sup>We are currently elaborating a second version of this model that will have several addon feature that will allow it to better track low-frequency historical trends. This version will replace QPM as the Bank of Canada's main projection model.

Canada - a period characterized by several different regimes - in terms of a simple rule. Here we follow much of the literature and specify a Taylor-style rule that includes a role for interest rate smoothing

$$R_t = \Gamma_r R_{t-1} + (1 - \Gamma_r)(\overline{r} + \Theta \cdot (\Gamma_\pi \pi_t + \Gamma_u \widetilde{y}_t)) + u_t$$
(53)

where  $\tilde{y}_t$  is detrended output and  $u_t$  is an i.i.d. error term (hereafter referred to as the monetary policy shock). Thus the time-t-response of nominal interest rates to an increase current inflation (output relative to trend) is  $(1 - \Gamma_r)\Theta\Gamma_{\pi}$   $((1 - \Gamma_r)\Theta\Gamma_y)$  whereas the long-run response is simply  $\Theta\Gamma_{\pi}$   $(\Theta\Gamma_y)$ . The inclusion of  $\Theta$  will allow us to vary proportionately the response of policy to both inflation and the output gap, thus reducing our parameters of interest to just one in the next section. For now we set it equal to one and ignore it.

The second question we face is what values should be chosen  $\{\Gamma_r, \Gamma_{\pi}, \Gamma_{\eta}\}$ for our baseline historical rule. Unfortunately, we know of only one paper that estimates a Taylor rule of the form given by (53) over a high passthrough period only. Gagnon and Ihrig (2002), following the approach taken by Clarida, Gali and Gertler (1998), estimate a rule from 1971 to 1984 using GMM. While they do not report  $\Gamma_r$ ,  $\Gamma_\pi$  and  $\Gamma_y$  are respectively estimated to be 0.5 and 0.7. Any value for  $\Gamma_{\pi}$  less than one creates a problem in that the 'Taylor principal' for determinacy is not satisfied and a stable, unique rational expectations solution for the model does not exist. As noted by Rudebusch (2003) for the United States, this may reflect the tendency for central banks during this period to have followed unstable rules (or no rule at all) until the economy began to get out of control, at which point they would implement a stable rule. In any event, we are required to work only with stable rules. As a check on the Gagnon and Ihrig result, we estimate (53), again on HP-filtered data, from 1970Q1 to 1983Q4 using both OLS and GMM.<sup>11</sup> Both OLS and GMM yield estimates of about 0.7 for  $\Gamma_r$ . By contrast, the GMM estimate yields a stable rule ( $\Gamma_{\pi} = 1.06, \Gamma_{\eta} =$ 0.62) whereas the OLS result is unstable ( $\Gamma_{\pi} = 0.54, \Gamma_{y} = 0.97$ ). Moreover, even the GMM result suggests that policy only just satisfies the stability condition. This result is quite similar to that of Clarida, Gali and Gertler (2000) who estimate a rule for the United States from 1960Q1 through 1979Q2 and obtain ( $\Gamma_r = 0.73, \Gamma_{\pi} = 0.86, \Gamma_y = 0.34$ ), which is almost stable. Estrella and Fuhrer (2000), also using data for the United States, obtain a

<sup>&</sup>lt;sup>11</sup>We use 2 lags each of the interest rate, output gap and inflation as instruments. Results are available on request from the author.

value of 1.46 for  $\Gamma_{\pi}$ . To the extent that Canadian monetary policy may have tracked policy in the United States over this period, these estimates provide us with an idea of the aggressiveness with which the Bank of Canada has responded to economic developments that affect inflation relative to its mean. We elect to use the GMM result for the baseline rule.

#### 4.2 Characterizing Pass-through

Exchange rate pass-through is typically measured indirectly by first estimating a Phillips curve of the form

$$\pi_t = A(L)\pi_{t-1} + B(L)(\pi_{t-1}^* + \Delta z_{t-1}) + C(L)\widetilde{y}_{t-1} + u_t$$
(54)

where  $\pi_t$  and  $\pi_t^*$  are respectively measures the domestic and foreign inflation such as the consumer price index (CPI) and  $\Delta z_t$  is the change in the nominal exchange rate. Pass-through from the exchange rate to the price level at a particular horizon can then be calculated based on the estimated lag polynomials A(L) and B(L).

Estimates of the average pass-through to the CPI (or core CPI) in Canada are typically between 0.15 and 0.4 for Canada for samples that span the last 30 years or so. However, the parameters of the Phillips curve also change through time and the degree of pass-through begins to fall around the mid-1980s. For instance, Kichian (2001) estimates average pass-through to be 0.42 based on a Phillips curve estimated from 1972Q3 to 1999Q4, whereas over the sub-samples 1972Q3 to 1989Q4 and 1990Q1 to 1994Q4 it is, respectively, 0.53 and 0.04. Similarly, Gagnon and Ihrig (2002) estimate pass-through of 0.41 from 1971 to 2000, 0.3 from 1971 to 1984 and 0.01 from 1985 to 2000. Campa and Goldberg (2002) show that pass-through to import prices falls from 0.91 to 0.68 when the years 1990-99 are added to a sample beginning in 1977. Since the high pass-through years remain in the sample, however, 0.68 likely over-estimates the degree of pass-through in the most recent regime. <sup>12</sup>Thus, while there is some debate about just how large pass-through was pre-1980s, the consensus appears to be that it has fallen dramatically since this time.

As discussed in calibration and solution section, the historical shocks for our model are calculated using HP filtered date with lambda set to 1600.

<sup>&</sup>lt;sup>12</sup>Choudri and Hakura (2001) estimate pass through in Canada to be 0.19 after 20 quarters based on a Phillips curve estimated from 1979 to 2002 but do not test for break for Canada.

The previously mentioned studies, however, used raw rather than filtered data. Thus, before proceeding further it is useful to inquire as to whether the same decline in pass-through is evident using our transformed series. Thus, we specify and estimate a simple Phillips curve relation of the form

$$\pi_t = a\pi_{t-1} + \sum_{i=1}^4 b_i (\pi_{t-i}^* + \Delta z_{t-i}) + c\widetilde{y}_{t-1} + u_t$$
 (55)

where  $\pi_t$  corresponds to the quarterly growth rate of  $P_{c,t}$  and define pass-through as

$$\Pi(\Theta) \equiv \frac{\sum_{i=1}^{4} b_i}{1-a}.\tag{56}$$

Over the sample 1970 to 2003, we estimate pass-through to be 0.11, smaller than the average value obtained in previous studies. However, the evidence of a decline remains evident; pass-through from 1970 to 1983 is 0.16 whereas from 1984 to 2003 is just 0.02.

While there are several potential causes for the decline in pass-through (see Campa and Goldberg (2001)), it is difficult to ignore how well it seems to coincide with declines in average inflation. For instance, Choudri and Hakura (2001) state

A positive and significant association between the pass-through and average inflation rate across these [71] countries. Further evidence in support of a robust link between inflation and the pass-through is provided by a small number of countries that experienced a dramatic shift in the inflation environment.

A similar conclusion is reached by Gagnon and Ihrig (2002), who test the relationship between inflation regime and pass-through and find a positive link for 18 of the 20 countries. Campa and Goldberg (2001) find weaker evidence of a positive relation between inflation volatility and pass-through to import prices for several OECD countries.

Taylor (2000) argues that this decline in observed pass-through may be due to a change in the conduct of monetary policy. Specifically, if monetary policy is seen to be responding more aggressively to shocks that affect inflation then the expected persistence of these shocks will decline and by consequence, so will the degree of pass-through to consumer prices. Taylor shows, using a simple model, how the perceived persistence of an expansionary money shock can influence firms' desire to 'pass on' cost increases in the

form of higher prices. Gagnon and Ihrig (2002) and Choudri and Hakura (2001) show, using small calibrated models, that an explicit link does exist between the aggressiveness of policy in achieving its inflation target and the pass-through of exchange rate movements into import prices. What these studies fail to adequately address is the quantitative significance of this relationship in a realistic business cycle model. We take up this issue for Canada in the next section.

#### 4.3 How strong is the link

Having defined monetary policy aggressiveness  $(\Theta)$  and exchange rate passthrough  $(\Pi(\Theta))$ , we are now in a position to examine the link. The basic experiment is as follows; we first compute historical time series for the structural shocks using detrended data and compute their variances. We then generate 10000 stochastic synthetic times series of length 75 periods using the model structure. The choice of 75 observations corresponds roughly to the time period over which pass-through has been found to be low and high, i.e. 1970 to 1983 and 1984 to 2002. For each 75-period sample we estimate equation (55) and compute  $\Pi(\Theta)$ . Our measure of pass-through, for a given  $\Theta$ , then corresponds to the median value of the 10000 observation distribution of  $\Pi(\Theta)$  (see Table 3). We then incrementally increase  $\Theta$  and repeat the process. Our reference result is generated assuming  $\Theta = 1$  and we normalize the corresponding median value of  $\Pi(\Theta)$  to equal one. Thus, for instance, the value taken for  $\Pi(\Theta)$  evaluated at  $\Theta = 1.5$  measures exchange rate pass-through as a percentage of pass-through for  $\Theta = 1$ .

In setting up our experiment in this fashion, a number of assumptions have implicitly been made. First, we assume the process generating the structural shocks is invariant to the value taken by  $\Theta$ . Second, we assume no transitional dynamics associated with changes to  $\Theta$ , agents are assumed to know  $\Theta$  at all times. In addition, we assume that monetary policy behaves under commitment to the aforementioned Taylor rule and seeks to achieve an inflation target that is known to agents.<sup>13</sup> These assumptions allow us to compute the unique rational expectations solution to the model.

Table 3 provides results for the relationship between  $\Theta$  and  $\Pi(\Theta)$ .  $\Theta = 1$  corresponds to the baseline policy rule, which was estimated from 1970 to 1983 and just satisfies the Taylor principal for determinacy. Thus, in the context of inflation targeting rules, this is just about the weakest response possible.

 $<sup>^{13}{\</sup>rm The~target}$  inflation rate is set to zero for simplicity.

From Table 3 (see column 1, labelled *All shocks*) it is striking to observe just how strong the relationship is between the aggressiveness of policy on the one hand, and pass-through on the other. For instance, as  $\Theta$  moves from 1 to 1.5 we see pass-through fall very quickly and essentially go to zero. In other words, the empirical finding that pass-through has fallen to close to zero can be explained wholly by a 50 per cent increase to the aggressiveness of monetary policy, which raises a question; is a value of 1.5 for  $\Theta$  a fair characterization of average response of policy since 1984. Gagnon and Ihrig (2002) estimate values of ( $\Gamma_{\pi} = 1.43, \Gamma_{y} = 0.87$ ) from 1985 to 2000, which corresponds to  $\Theta = 1.4$ . More recently, Lam and Tkacz (2004) estimate  $\Theta = 1.9$  (from 1990 to 2000 they estimate ( $\Gamma_{r} = 0.82, \Gamma_{\pi} = 2.1, \Gamma_{y} = 1.1$ )).

If we take values between 1.5 and 2.0 as reasonable, we see that indeed policy can exert a profound effect on measured pass-through. It is worth noting that the decline comes through a combination of a fall in a and a lower sum  $\sum_{i=1}^{4} b_i$  (see equation (56)). The precise value of  $\Theta$  at which passthrough is zero does depend on the calibration of the model. For instance, making capacity utilization more expensive to adjust (increasing  $\rho$  in equation (12)) will tend to flatten the  $\Pi(\Theta)$  function. Moreover, even with 10000 samples, there remains some sampling uncertainty. With 75 observations, the distribution of  $\Pi(\Theta)$  is very wide. Nevertheless, based on several robustness checks, we conclude that  $\Pi(\Theta) \approx 0$  for  $(1.5 \lesssim \Theta \lesssim 2.0)$ , when all 7 shocks are used, which corresponds to  $(1.6 \lesssim \Gamma_{\pi} \lesssim 2.1)$  and  $(0.9 \lesssim \Gamma_{y} \lesssim 1.2)$ in a standard Taylor rule. Thus, based on these results alone, one is tempted to accept the Taylor argument as quantitatively important. However, when we check the robustness of this result by varying the types of shocks, in the model we arrive at a somewhat different conclusion. For instance, with just exchange rate shocks  $(\kappa_t)$  in the model (column 2, Table 3) we observe a much flatter  $\Pi(\Theta)$  function. For reasonable increases to  $\Theta$ , pass-through asymptotes at about 50 percent of its baseline level. <sup>14</sup> In other words, relative to a rule that just satisfies the Taylor principal, policy is capable of cutting pass-through to consumer prices in half.

A similar result obtains when we measure pass-through directly in deterministic exchange rate shocks to the structural model.<sup>15</sup> For instance, one

 $<sup>^{14}</sup>$ For this particular calibration, pass-through begins increasing again for  $\Theta > 2.0$ . This rather strange result is not robust to the calibration of the model. Several other calibrations we tried resulted in pass-through stabilizing at about 50 per cent. Also, pass-through falls monotonically for the deterministic exchange rate shock (see column 4 of Table 3).

<sup>&</sup>lt;sup>15</sup>The persistence of the shock to  $\kappa$  is the same as in the stochastic environment (i.e.  $\rho_{\kappa} = 0.94$ )

may define model-based pass-through simply as  $\Pi_{\varrho}(\Theta) \equiv \widehat{P}_{c,t+5}/\widehat{e}_t$  where  $\widehat{P}_{c,t+5}$  and  $\widehat{e}_t$  denote log deviations of the consumer price level and nominal exchange rate from arbitrary control solutions. Thus, our measure captures the percent change in the price level at time t+5 quarters relative to the exchange rate at time t when the source of the exchange-rate movement is a shock to the risk premium only,  $\varrho$ .<sup>16</sup> As indicated in the last column of Table 3, this definition yields a qualitatively-similar result to our Phillipscurve based measure,  $\Pi(\Theta)$ , when there are just exchange-rate shocks. For instance, for  $\Theta = 2$ ,  $\Pi_{\varrho}(\Theta) = 0.52$  versus  $\Pi(\Theta) = 0.5$ .

A similar conclusion is reached if all shocks are used except the price and wage mark-up shocks ( $\epsilon$  and  $\epsilon_w$ ) (see column 3, Table 3). Thus, the difference appears to stem from changes in the aggressiveness of policy in the presence of what we can loosely refer to as price and wage shocks. To see why, consider again equations (48) and (49), which can be combined to obtain the standard nominal uncovered interest parity (UIP) condition

$$e_t = \mathcal{E}_t \frac{e_{t+1}(1 + R_t^*)(1 + \kappa_t)}{(1 + R_t)}$$
(57)

Fixing  $R_t^* = R^*$ ,  $\kappa_t = 0$  and log-linearizing around steady-state (57) yields

$$\widehat{z}_t = \widehat{z}_{t+1} - \widehat{r}_t = \widehat{z} - \sum_{i=0}^{\infty} \widehat{r}_{t+i}$$

where  $\hat{z}_t$  is the deviation of the real exchange rate from control and the real interest rate is given by  $1 + r_t = (1 + R_t)P_{c,t}/P_{c,t+1}$ . If the long-run real exchange rate change  $(\hat{z})$  from a shock to  $\epsilon$  is small (relative purchasing power parity (PPP) holds) then we can say that the difference in behaviour between the nominal exchange rate and the consumer price level can be explained by the future stream of real interest rate deviations from steady state (real UIP holds). If we take the limiting case where  $\Theta = 1$  then the Taylor principal (which states that real interest rates must rise above control at some point following an inflationary shock) is just satisfied and therefore  $\sum_{i=0}^{\infty} \hat{r}_{t+i}$  will be small. In this instance, the nominal exchange rate and price level will essentially move in tandem (or the real exchange rate will

<sup>&</sup>lt;sup>16</sup>This measure of pass-through differs somewhat from that measured through the Phillips curve since the former will include the impact of movements in output relative to steady state on inflation. Also, the choice of 5 quarters is arbitrary.

move very little). Thus there should be a high correlation between prices and the exchange rate in mark-up shocks, which will be picked up in the reduced-form Phillips curve. However, as the policy response becomes more vigorous,  $\hat{z}_t$  will fall (increase) by more in a positive (negative) price shock and at some point the nominal exchange rate and price level will move in opposite directions in the short run, thereby inducing a negative correlation. This will occur when the influence of UIP on the nominal exchange rate dominates the PPP effect in the short-run.

In summary, our results are twofold. First, the negative relationship between monetary policy and pass-through proposed by Taylor (2000) is quantitatively important for Canada, regardless of whether pass-through is measured using an estimated Phillips curve equation or as the price response to a structural exchange rate shock in the model. Reasonable increases to the responsiveness of policy to inflation can generate an appreciable reduction to the degree of exchange rate pass-through to consumer prices. Second, whether pass-through can be altogether eliminated by a policy of inflation targeting does depend strongly on the method used to measure pass-through. When measured through a Phillips curve, pass-through is eliminated for very reasonable levels of aggressiveness, which serves to validate much of the reduced-form evidence for Canada indicating that passthrough has been close to zero since 1984. When measured by the response of prices to an exchange-rate shock in the structural model, however, a 50 per-cent reduction in pass-through is about the best the central bank can do. Finally, the distinction across the two measures appears to be due to the presence of mark-up shocks in the model. Future work should be aimed at applying this same methodology to a fully-estimated model that uses raw, rather than HP-filtered, data to check the robustness of these results.

# 5 Conclusion

The purpose of this paper is to quantify the link between changes to the aggressiveness of monetary policy and exchange rate pass-through in Canada. We define pass-through in the context of a reduced-form Phillips curve equation and then explore, using an open-economy DSGE model closed with a Taylor-style monetary-policy rule, the magnitude of the relationship between the response of interest rates to inflation and the measured degree of exchange rate pass-through. We find that, when measured in this manner, a strong negative relationship does indeed exist between monetary policy and pass-through. Specifically, reasonable increases in the aggressiveness

of policy will lead to small, statistically-insignificant exchange-rate terms in the Phillips curve. We go on to show, however, that this should not be taken to mean that the exchange rate no longer feeds through to consumer prices. Rather, it reflects more the reduced-form nature of Phillips curve. In particular, small changes in policy can have a profound effect on the correlation between prices and the exchange rate in the presence of mark-up shocks and this is largely responsible for the result. When mark-up shocks are excluded from the model or when pass-through is defined in terms of the response of prices to a deterministic exchange-rate shock, we conclude that more aggressive monetary policy in Canada has likely reduced pass-through by about 50 per cent relative to its level prior to 1984.

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Table 1: Calibrated Parameters

Parameter	Value
Preferences	
eta	0.99
Competition	
$\epsilon=\epsilon_w$	11
Production	
$\delta$	0.33
$\omega$	0.05
heta	-20
$\phi_0,,\phi_3$	4.85
Import shares	
${\gamma}_c$	0.17
$\gamma_I$	0.36
$\gamma_{x,nc}$	0.53
Land share	
$\gamma_{xc}$	0.5

Table 2: Estimated Parameters

D .	3.7.1			
Parameter	Value			
Contract duration				
$\mathcal{S}_c$	0.15			
$\mathcal{S}_m$	0.15			
$\mathcal{S}_w$	0.32			
Production				
$\sigma$	0.53			
$\varphi$	0.8			
$\varphi_{xc}$	1.5			
$\chi$	20			
$\rho$	0.1			
$\phi_4$	10			
Preferences				
$\mu$	0.92			
$\eta$	0.90			
ξ	0.85			
Trade				
$\vartheta$	0.5			
Monetary policy				
$\Gamma_r$	0.7			
$\Gamma_{\pi}$	1.06			
$\Gamma_y$	0.62			
Risk				
ς	0.021			
Shocks				
$ ho_A$	0.9			
$\rho_{eta}$	0.4			
$ ho_{\kappa}$	0.93			
$ ho_{\epsilon_c}$	0.8			

Table 3: Exchange Rate Pass-through

Pass-through ( $\Pi(\Theta)$ ) (%. rel. to $\Theta = 1.0$ )				
Policy $(\Theta)$	All shocks	Just $\kappa_t$ shocks	All but $\epsilon$ shocks	$\Pi_{\varrho}(\Theta)$
1.00	1.00	1.00	1.00	1.00
1.10	0.56	0.81	0.83	0.86
1.25	0.22	0.65	0.70	0.75
1.50	~0.0	0.53	0.60	0.64
1.75	-0.19	0.50	0.55	0.57
2.00	-0.25	0.50	0.52	0.52
2.50	-0.28	0.53	0.49	0.47
3.00	-0.26	0.57	0.48	0.44