Durable Consumption as a Status Good: A Study of Neoclassical Cases

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Abstract

In this paper we extend the representative agent model of the consumer to incorporate durable consumption goods that generate status, where status depends on relative consumption. The analysis is done in the neoclassical context. In the closed economy framework both endogenous and fixed employment cases are considered. A small open economy version of the model is also developed. We derive the intertemporal equilibria and establish that in all instances they are saddlepoint stable. Among our principle results, we show in the closed economy context with endogenous work effort that an increase in the degree of status preference raises durable consumption, its stock, employment, and physical capital. These results extend, in general, to the small open economy.

JEL Codes: D9, E21

Key Words: Status Seeking, Durable Consumption, Relative Consumption

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1. Introduction

Interactions between individuals and economy-wide aggregates or social groups are a pervasive phenomenon. Recent researchers who have explored this aspect of economic life with respect to an individual's social status and welfare include Easterlin (1974, 1995), Clark and Oswald (1996), Oswald (1997), and Frank (1997), to name just a few. One aspect of this question that has drawn increasing attention in the last decade or so is the issue of how social status affects overall economic performance, including its implications for the long-run rate of economic growth and potential public policy interventions. Examining the recent literature in this area, we observe that there are two primary ways in which status is modelled in macroeconomic settings. The first approach, represented by authors such as Galí (1994), Persson (1995), Harbaugh (1996), Rauscher (1997b), Grossmann (1998), Ljungqvist and Uhlig (2000), and Fisher and Hof (2000a, b) specifies that status arises from an individual's comparison—in terms of his instantaneous preferences—of his own consumption to some economy-wide measure of aggregate or average consumption, which can be modelled as an agent's consumption *relative* to this macroeconomic variable. The second approach, adopted by Corneo and Jeanne (1997, 2001a, 2001b), Rauscher (1997a), Futagami and Shibata (1998), Fisher (2004a, 2004b), and Hof and Wirl (2002) specifies that status arises from an agent's stock of relative wealth, which can consist of durable physical capital, financial assets, or both.

In this paper, we assume that social status is generated by *relative consumption*, as in the first approach, but specify, as in the second approach, that it depends on a *stock* variable, here the stock of durable consumption. To our knowledge, this is an extension that has not been undertaken in the existing literature. Given the importance of durable consumption for countries such as the United States, this seems to us a worthwhile exercise.[1] The specific form of durable consumption we adopt has been developed by Mansoorian (1998) in his study of the implications of durability for the dynamics of the current account.[2] The present work is also related to the approach used by Carrol, *et. al.* (1997, 2000) in

[1]According to Obstfeld and Rogoff (1996), spending on durables accounted for 18.1% of overall consumption spending in 1994.

[2]Mansoorian (2000) extends this work by considering the implications of commercial policy. In Matsuyama (1990) the stock housing is the durable consumption good.

their endogenous growth models, which assume that agents compare current consumption to some measure of past consumption history, or "habits."[3]

The modelling framework in which we conduct this exercise is otherwise standard. It is in the general category of the neoclassical Ramsey framework that assumes infinitely-lived consumer-producers. We consider both closed and small open economy models in this study. Moreover, in our closed economy setting, similar to that employed by Fisher and Hof (2000b), we analyze the implications of a status preference for durable consumption in both endogenous and fixed employment specifications. In considering implications of changes in agents attitudes toward relative social position, we also employ a specific parameterization of instantaneous preferences used by Rauscher (1997b), among others.

The small open economy framework we develop is based on the specifications of Bhandari, *et. al.* (1990), Fisher (1995), and Fisher and Terrell (2000). These specifications assume an otherwise "small" open economies that cannot freely borrow or lend at the prevailing world interest rate. Rather, they are subject to an upward-sloping interest rate relationship that implies that the interest rate the country can borrow from the international capital market rises with the level of national indebtedness. Among the advantages of this framework, it generates, as in the closed economy cases, a fourth-order differential equation system that possesses a saddlepoint, which facilitates comparisons between the closed and open economies.

Among our principle results, we derive the intertemporal equilibria of the closed and open economies, including their dynamics, show that they are all of a fourth order, and establish that the corresponding steady states are saddlepoints. In terms of the closed economy model we consider particular two issues: i) the effect of an increase in the degree of status preference on the steady-state equilibrium with endogenous employment and ii) the implications in the special case of fixed employment of increase in the importance of relative social position on speeds of intertemporal adjustment.[4] Regarding the first question, we find that an increase in the degree of status preference raises the long-run levels of durable consumption, its corresponding stock, employment, and the physical

[3]Mansoorian (1998, 2000) also incorporates habits (of durable consumption) in his work.

[4]Since the Jacobian matrices of these models have two negative eigenvalues, they possess two distinct "speeds" of stable transitional adjustment.

capital stock (and, thus, steady-state output). These results represent an extension of the Fisher and Hof (2000b) findings to the case of durable relative consumption.[5] With regard to the second issue, we find that whether the speeds of adjustment rise or fall depends on whether the rise in the degree of status preference raises or lowers the intertemporal elasticity of substitution. Finally, we show that the long-run implications of an increase in status considerations are generally extend to the small open economy model.

The rest of the paper is structured as follows. Section 2 describes the closed economy model with durable consumption and derives its intertemporal equilibrium. The first part of section 2 outlines the general framework with endogenous employment, while the second part of section 2 discusses the implication for the economy's speed of adjustment in the special case of fixed employment. In section 3 we develop the small open economy framework and analyze its dynamic and long-run properties. The paper closes with a brief conclusion and a mathematical appendix that contains some results that are important for our subsequent analysis.

2. The Model and Intertemporal Equilibrium

2.1. Variable Employment

We assume that the decentralized economy is populated by a large number of identical, infinitely-lived consumer-producers.[6] Without loss of generality, we specify that the population is constant. In this framework, agents derive positive utility from aggregate consumer durables, $c + a$, and status, s. In contrast, work effort, l, yields disutility. We assume that durable consumption and status are additively separable from work effort in the instantaneous utility function. Aggregate consumer durables consist of current goods purchased at time t, c, and the inherited stock of consumer durables, a. For expositional convenience, we refer to c as durable consumption and to a as its stock. In this continuous-

[5] Among other results, Fisher and Hof (2000b) show that an increase in the degree of status preference increases the steady-state values of non-durable consumption, employment, and physical capital.

[6] We abstract from a public sector in this model, which means that we do address the question of optimal policy in the context durable consumption externalities. We leave this issue for future work.

time framework, the stock of consumer durables corresponds to:

$$a = \int_{-\infty}^{t} e^{\delta(\tau - t)} c(\tau) d\tau \tag{2.1a}$$

and accumulates according to:

$$\dot{a} = c - \delta a \tag{2.1b}$$

where δ is the rate of depreciation of consumer durables.[7] We specify that each agent possesses the following general instantaneous utility function over own durable consumption, $c + a$, and status, s, and work effort l:

$$U(c + a, s) + V(l), \tag{2.2}$$

where U and V have the following properties:

$$U_c > 0, \quad U_s > 0, \quad U_{cc} < 0, \quad U_{ss} \leq 0, \quad U_{cc}U_{ss} - U_{cs}^2 \geq 0, \quad V' < 0, \quad V'' < 0, \tag{2.3a}$$

$$U_{sc}U_c - U_sU_{cc} > 0, \tag{2.3b}$$

$$\lim_{c \longrightarrow 0} U_c(c,\, s) = \infty, \qquad \lim_{c \longrightarrow \infty} U_c(c,\, s) = 0. \tag{2.3c}$$

According to (2.3a), the representative agent derives positive, though diminishing, marginal utility from own consumption and positive and non-increasing marginal utility from status, with the instantaneous utility function U jointly concave in c and s.[8] In addition, work effort generates disutility and V is concave. The condition (2.3b) imposes normality on preferences, i.e., the marginal rate of substitution of status for consumption, U_s/U_c, de-

[7]This specification of durable consumption is found in Mansoorian (1998, 2000).

[8]We use the following notational conventions. In general, we suppress a variable's time dependence, i.e., $x \equiv x(t)$. The time derivative of x will be denoted by $\dot{x}$; a steady-state value by $\tilde{x}$. Unless otherwise indicated, the partial derivative of a function F with respect to x will be denoted by F_x, while "primes" indicate that the derivative of a function of a single variable is being taken.

pends positively on c, while (2.3c) describes the limiting behavior of the marginal utility of consumption. Regarding status, we assume that it depends positively on relative durable consumption, denoted by $z \equiv c+a/C+A$, and takes the following "ratio" functional form:

$$s \equiv s(z) = s\left(\frac{c+a}{C+A}\right), \qquad s' > 0, \quad s'' \leq 0, \tag{2.4}$$

where $C + A$ is the average (or aggregate) level of durable consumption in the economy. Given the specification of status in (2.4), we impose an additional condition of the function $U(c, s)$:

$$U_{cc} + U_{cs}\frac{s'(1)}{c+a} - U_s\frac{s'(1)}{(c+s)^2} < 0. \tag{2.5}$$

This condition guarantees that there exists a negative relationship between the (flow) of durable consumption goods and its shadow value. Finally, the agent possesses a constant returns to scale production function that satisfies the following standard neoclassical properties of positive and diminishing marginal physical productivity in capital, k, and labor, l:[9]

$$y = F(k, l); \qquad F_k > 0, \quad F_{kk} < 0, \quad F_l > 0, \quad F_{ll} < 0, \tag{2.6}$$

where y represents output. In addition, the consumer-producer accumulates physical capital according to:[10]

$$\dot{k} = F(k, l) - c. \tag{2.7}$$

[9]The constant returns of scale property implies that the production function obeys the following relationships:

$$F_{kl} > 0, \quad F_{kk}F_{ll} = F_{kl}^2, \quad F_{kk}F_l - F_{kl}F_k = F_{kk}(y/l), \quad F_{kl}F_l - F_{ll}F_k = F_{kl}(y/l).$$

[10]To keep the exposition of the model as simple as possible, we do not specify that physical capital is subject to depreciation. It would, of course, be straightforward to do so, although none of our qualitative results would be affected.

The consumer-producer's problem is, thus, formalized as follows:

$$\max \int_0^\infty \left\{ U \left[c + a, s\left(\frac{c+a}{C+A}\right)\right] + V(l) \right\} e^{-\beta t}\, dt, \tag{2.8a}$$

subject to:

$$\dot{a} = c - \delta a, \tag{2.8b}$$

$$\dot{k} = F(k, l) - c, \tag{2.8c}$$

and the initial stocks of durable consumption and physical capital, $a(0) = a_0 > 0$ and $k\left(0\right) = k_0 > 0$, where β is the exogenous rate of time preference. To solve the consumer-producer's problem, we form the current value Hamiltonian, which is given by:

$$U \left[c + a, s\left(\frac{c+a}{C+A}\right)\right] + V(l) + \phi\left(c - \delta a\right) + \mu\left[F(k, l) - c\right], \tag{2.9}$$

where ϕ and μ are the costate variables corresponding, respectively, to the constraints (2.8b) and (2.8c). Maximizing equation (2.9), we calculate the following first order optimality conditions:

$$U_c\left[c + a, s\left(z\right)\right] + \frac{U_s\left[c + a, s\left(z\right)\right] s'\left(z\right)}{C + A} = \mu - \phi, \tag{2.10a}$$

$$V'(l) = -\mu F_l\left(k, l\right), \tag{2.10b}$$

$$\dot{\phi} = \left(\beta + \delta\right)\phi - U_c\left[c + a, s\left(z\right)\right] - \frac{U_s\left[c + a, s\left(z\right)\right] s'\left(z\right)}{C + A}, \tag{2.10c}$$

$$\dot{\mu} = \mu\left[\beta - F_k(k, l)\right]. \tag{2.10d}$$

The optimality conditions (2.10a)–(2.10d) have a straightforward interpretation. Equation

(2.10a) is the first order condition for own consumption in which the agent takes the average level of durable consumption in the economy, $C + A$, as given in performing his optimization. This is also the case in equation (2.10c), which describes the dynamics shadow value ϕ when the stock of own durable consumption goods a is chosen optimally. Equation (2.10b) is a standard first order condition for work effort in the neoclassical context, while equation (2.10d) defines the dynamics of the shadow value μ when physical capital k is chosen optimally. Our specification of preferences in equations (2.3a)–(2.3b) guarantees that the Hamiltonian (2.9) is jointly concave in the control variables c and l and the state variables a and k. This implies that if the limiting transversality conditions $\lim_{t\to\infty} a\phi e^{-\beta t} = \lim_{t\to\infty} k\mu e^{-\beta t} = 0$ hold, then necessary conditions (2.10a)–(2.10d) are sufficient for optimality.

As is the usual practise in models of the type, we restrict our subsequent analysis to symmetric equilibria in which identical agents make identical choices. This is the procedure followed by Galí (1994), Persson (1995), Harbaugh (1996), Rauscher (1997b), Grossmann (1998), Ljungqvist and Uhlig (2000), and Fisher and Hof (2000a, b), among others. In the context of our model, we specify that the individual quantities of durable consumption (current flows and aggregate stocks) equal their average levels, i.e., $c + a \equiv C + A$. Substituting this relationship into (2.10a) and combining with (2.10c), we obtain:

$$U_c\left[c+a, s\left(1\right)\right] + \frac{U_s\left[c+a, s\left(1\right)\right] s'\left(1\right)}{c+a} = \mu - \phi = (\beta+\delta)\phi - \dot{\phi}, \tag{2.11}$$

where the optimality conditions for work effort and capital accumulation remain unchanged.

Using (2.11) and (2.10b), it is straightforward to calculate the following instantaneous solutions for consumption and work effort in terms of the state and costate variables:

$$c = c\left(a, \mu, \phi\right); \qquad c_a < 0, \qquad c_\mu = -c_\phi < 0, \tag{2.12a}$$

$$l = l\left(k, \mu\right); \qquad l_k > 0, \qquad l_\mu > 0, \tag{2.12b}$$

where the expressions for the partial derivatives are given in the appendix. The partial derivatives in (2.12a, b) are interpreted as follows: an increase in *stock* of durable consumption a lowers its *current* level c. Indeed, as we show in the appendix, an increase in a lowers c by one-for-one. In addition, current durable consumption depends negatively the marginal utility of wealth μ and positively on the shadow value of the stock of durable consumption ϕ. Regarding the short-run response of work effort, (2.12b) indicates an increase in the shadow value μ raises work effort. Moreover, a given increase in the capital stock k also encourages labor supply, since $F_{kl} > 0$, which is implied by constant returns to scale production technology.

From the equations (2.11), (2.8b), (2.10d) and (2.8c), we state the independent dynamics of the economy:

$$\dot{\phi} = (1 + \beta + \delta)\,\phi - \mu, \tag{2.13a}$$

$$\dot{a} = c\,(a, \mu, \phi) - \delta a, \tag{2.13b}$$

$$\dot{\mu} = \mu\left[\beta - F_k\left(k, l\left(\mu, k\right)\right)\right], \tag{2.13c}$$

$$\dot{k} = F\left[\left(k, l\left(\mu, k\right)\right) - c\left(a, \mu, \phi\right)\right], \tag{2.13d}$$

where we have substituted for $l = l\,(k, \mu)$ in (2.13c, d) and $c = c\,(a, \mu, \phi)$ in (2.13b, d), respectively. Letting $\dot{\phi} = \dot{a} = \dot{\mu} = \dot{k} = 0$, the long-run equilibrium equals:

$$U_c\left[(1+\delta)\,\tilde{a}, s\,(1)\right] + \frac{U_s\left[(1+\delta)\,\tilde{a}, s\,(1)\right] s'\,(1)}{(1+\delta)\,\tilde{a}} = (\beta+\delta)\tilde{\phi}, \tag{2.14a}$$

$$V'\,(l) = -\,(1 + \beta + \delta)\,\tilde{\phi} F_l\left(\tilde{k}, \tilde{l}\right), \tag{2.14b}$$

$$F_k\left(\tilde{k}, \tilde{l}\right) = \beta, \tag{2.14c}$$

$$F\left(\tilde{k},\tilde{l}\right)=\delta\tilde{a}, \tag{2.14d}$$

where $\tilde{c}=\delta\tilde{a}$ and $\tilde{\mu}=(1+\beta+\delta)\,\tilde{\phi}$. Equations (2.14a, b) are the steady-state versions of the first order conditions (2.10a, b), while equation (2.14c) is the standard long-run representation of the Euler equation, which implies, given our technological assumptions, that the marginal physical product of capital, and, thus, the capital-labor ratio $\left(\tilde{k}/\tilde{l}\right)=\tilde{\kappa}$, is pinned-down by the exogenous rate of time preference β. Finally, due to the fact that there is no depreciation of physical capital in this model, equation (2.14d) states that steady-state output equals steady-state durable consumption, which corresponds to the long-run level of durable goods deprecation.

Linearizing the differential equation system (2.13a)–(2.13d) about the steady state (2.14a)–(2.14d), we calculate the fourth-order matrix equation:

$$\dot{\mathbf{z}}=\mathbf{J}\mathbf{z}=$$

$$\begin{pmatrix}\dot{\phi}\\ \dot{a}\\ \dot{\mu}\\ \dot{k}\end{pmatrix}=\begin{pmatrix}(1+\beta+\delta) & 0 & -1 & 0\\ -c_{\mu} & -(1+\delta) & c_{\mu} & 0\\ 0 & 0 & -l_{\mu}\tilde{\mu}F_{kl} & -\tilde{\mu}\left(F_{kk}+F_{kl}l_{k}\right)\\ c_{\mu} & 1 & F_{l}l_{\mu}-c_{\mu} & \beta+F_{l}l_{k}\end{pmatrix}\begin{pmatrix}\phi-\tilde{\phi}\\ a-\tilde{a}\\ \mu-\tilde{\mu}\\ k-\tilde{k}\end{pmatrix} \tag{2.15}$$

where $\mathbf{z}=(\phi,a,\mu,k,)'$and $\mathbf{J}$ denotes the Jacobian matrix of (2.15) in the case of endogenous employment. Observe that functions of variables are evaluated at the steady-state equilibrium (2.14a)–(2.14d). To determine the stability properties of the equilibrium, we first consider the trace and determinant of the Jacobian matrix

$$\mathbf{tr}\left(\mathbf{J}\right)=\omega_1+\omega_2+\omega_3+\omega_4=2\beta-l_{\mu}\tilde{\mu}F_{kl}+F_{l}l_{k}=2\beta>0,$$

$$\det\left(\mathbf{J}\right)=\omega_1\omega_2\omega_3\omega_4$$

$$=(\beta+\delta)\delta c_{\mu}\tilde{\mu}\left(F_{kk}+F_{kl}l_{k}\right)-(1+\delta)(1+\beta+\delta)l_{\mu}\tilde{\mu}F_{kk}\left(\tilde{y}/\tilde{l}\right)>0$$

where $\omega_i = 1-4$ are the eigenvalues of $\mathbf{J}$.[11] The condition $\mathbf{tr}\,(\mathbf{J}) > 0$ rules out the case of four negative eigenvalues, while $\det(\mathbf{J}) > 0$ implies that the following cases do not obtain: i) one negative and three positive eigenvalues; and ii) three negative and one positive eigenvalue. This leaves the cases that are not ruled-out by $\mathbf{tr}\,(\mathbf{J}) > 0$ and $\det(\mathbf{J}) > 0$: i) two negative and two positive eigenvalues; and ii). four positive eigenvalues. To determine which of the two cases holds, we use the fact that the characteristic equation, denoted by $\det(\mathbf{J}-\omega\mathbf{I}) = 0$, can be factored as:

$$\det(\mathbf{J}-\omega\mathbf{I}) = (\omega-\omega_1)(\omega-\omega_2)(\omega-\omega_3)(\omega-\omega_4)$$

$$= \omega^4 - \mathbf{tr}\,(\mathbf{J})\,\omega^3 + (\omega_1\omega_2 + \omega_1\omega_3 + \omega_1\omega_4 + \omega_2\omega_3 + \omega_2\omega_4 + \omega_3\omega_4)\,\omega^2$$

$$-(\omega_1\omega_2\omega_3 + \omega_1\omega_2\omega_4 + \omega_1\omega_3\omega_4 + \omega_2\omega_3\omega_4)\,\omega + \det(\mathbf{J}) = 0. \tag{2.16a}$$

Calculating $\det(\mathbf{J}-\omega\mathbf{I})$ from (2.15), we find:

$$\det(\mathbf{J}-\omega\mathbf{I}) =$$

$$\omega^4 - \mathbf{tr}\,(\mathbf{J})\,\omega^3 + \left\{\beta^2 - (1+\delta)(1+\beta+\delta) - \left[c_\mu\tilde{\mu}\,(F_{kk} + F_{kk}l_k) - l_\mu\tilde{\mu}F_{kk}\left(\tilde{y}/\tilde{l}\right)\right]\right\}\omega^2$$

$$+\beta\left\{(1+\delta)(1+\beta+\delta) + \left[c_\mu\tilde{\mu}\,(F_{kk} + F_{kk}l_k) - l_\mu\tilde{\mu}F_{kk}\left(\tilde{y}/\tilde{l}\right)\right]\right\}\omega + \det(\mathbf{J}) = 0. \tag{2.16b}$$

Matching the coefficients of (2.16a, b), we observe that:

$$\omega_1\omega_2\omega_3 + \omega_1\omega_2\omega_4 + \omega_1\omega_3\omega_4 + \omega_2\omega_3\omega_4$$

$$= -\beta\left\{(1+\delta)(1+\beta+\delta) + \left[c_\mu\tilde{\mu}\,(F_{kk} + F_{kk}l_k) - l_\mu\tilde{\mu}F_{kk}\left(\tilde{y}/\tilde{l}\right)\right]\right\} < 0,$$

[11] In calculating $\mathsf{tr}(\mathsf{J}) = 2\beta$, we substitute for l_μ and l_k in the expression for $\mathsf{tr}(\mathsf{J})$. In deriving $\det \mathsf{J} > 0$, we use the fact that

$$F_{kk} + F_{kl}l_k = F_{kk} - \frac{\tilde{\mu}F_{kl}^2}{V'' + \tilde{\mu}F_{ll}} = \frac{F_{kk}V''}{V'' + \tilde{\mu}F_{ll}} < 0,$$

and substitute for $F_{kk}F_{ll} = F_{kl}^2$ and $(F_{kl}\beta - F_{kk}F_l) = -F_{kk}(\tilde{y}/\tilde{l})$.

which implies that we can rule out the case in which all the eigenvalues are positive. Thus, the equilibrium of (2.15) is a saddlepoint with two negative and two positive eigenvalues ordered according to:

$$\omega_1 < \omega_2 < 0 < \omega_3 < \omega_4.$$

Using standard methods, we can solve (2.15) for the paths of (ϕ, a, μ, k). This procedure is outlined in the appendix. While a detailed analysis of the solution paths of (ϕ, a, μ, k) is beyond the scope of this paper and is left for future work, the methods used by Eicher and Turnovsky (2001) can be employed to do so.

We next investigate the implications of status preference on the long-run equilibrium of the economy. To do so, we choose a convenient specification of $U(c + a, s)$, similar to that employed by Rauscher (1997), in which own consumption (inclusive of its durable stock) is additively separable from status:[12]

$$U(c+a,s) = (1-\gamma)^{-1}(c+a)^{1-\gamma} + \eta s\left(\frac{c+a}{C+A}\right), \qquad \gamma > 0, \quad \eta > 0. \tag{2.17}$$

We interpret the parameter η as a measure of the "importance" of status for consumer-producers, or the as the "degree" of status preference. Under the specification (2.17), the steady-state condition (2.14a) becomes:

$$[(1+\delta)\tilde{a}]^{-\gamma} + \frac{\eta s'(1)}{(1+\delta)\tilde{a}} = (\beta+\delta)\tilde{\phi}. \tag{2.14a$'$}$$

Differentiating (2.14a$'$) and (2.14b)–(2.14d) with respect to η, we calculate the following long-run comparative statics expressions for $\left(\tilde{\phi}, \tilde{\mu}, \tilde{a}, \tilde{c}, \tilde{k}, \tilde{l}, \tilde{y}\right)$:

$$\frac{\partial\tilde{\phi}}{\partial\eta} = (1+\beta+\delta)^{-1}\frac{\partial\tilde{\mu}}{\partial\eta} = \frac{s'(1)\delta V'' F_{kk}}{(1+\delta)\tilde{a}\Delta} > 0, \tag{2.18a}$$

[12] Rauscher (1997) restricts his attention to non-durable consumption. In addition, we retain in this part of the paper the general specification of the disutility of work effort $V(l)$ stated in (2.2) and (2.3a). The conditions $\gamma > 0$, $\eta > 0$, in (2.17) guarantee $c_\mu < 0$, which is a sufficient condition, given the other model assumptions, that the equilibrium of (2.15) is a saddlepoint.

$$\frac{\partial \tilde{a}}{\partial \eta} == \frac{1}{\delta}\frac{\partial \tilde{c}}{\partial \eta} = \frac{1}{\delta}\frac{\partial \tilde{y}}{\partial \eta} = -\frac{s'(1)\,(1+\beta+\delta)\,F_{kk}F_l(\tilde{y}/\tilde{l})}{(1+\delta)\tilde{a}\Delta} > 0, \tag{2.18b}$$

$$\frac{\partial \tilde{k}}{\partial \eta} = -\frac{F_{kl}}{F_{kk}}\frac{\partial \tilde{l}}{\partial \eta} = \frac{s'(1)\,\delta\,(1+\beta+\delta)\,F_{kl}F_l}{(1+\delta)\tilde{a}\Delta} > 0 \quad \Rightarrow \quad \frac{\partial\left(\tilde{k}/\tilde{l}\right)}{\partial \eta} = \frac{\partial \tilde{\kappa}}{\partial \eta} = 0, \tag{2.18c}$$

where[13]

$$\Delta = (\beta+\delta)\,\delta V'' F_{kk} - (1+\beta+\delta)\,F_l\left[\gamma\left[(1+\delta)\tilde{a}\right]^{-(1+\gamma)} + \frac{\eta s'(1)}{(1+\delta)\tilde{a}^2}\right]F_{kk}(\tilde{y}/\tilde{l}) > 0.$$

The signs of the expressions in (2.18a)–(2.18c) can be directly explained: a higher weight on status considerations causes consumer-producers to place a greater value on consumption at the expense of leisure, which is reflected in (2.18a) in the higher long-run levels of $\tilde{\mu}$ and $\tilde{\phi}$. This leads, in turn, to an increase in work effort $\tilde{l}$, which, since $F_{kl} > 0$, causes physical capital $\tilde{k}$ to accumulate. Nevertheless, the steady-state capital-labor ratio $\tilde{\kappa}$, determined by the rate of time preference β and the curvature of the production function, is independent, according to (2.18c), of changes in the status preference parameter η. The long-run rise in output $\tilde{y}$ leads, as indicated in (2.18b), to an increase in durable consumption $\tilde{c}$ and its steady-state stock $\tilde{a}$. Our results represent, then, an extension of those of Fisher and Hof (2000b), who in their model non-durable consumption show that "status consciousness" leads to "too much" consumption, work effort, and physical capital accumulation relative to a socially planned economy in which status considerations do not play a role.[14]

2.2. Fixed Employment Case

In order to explore in greater detail the dynamic properties of the durable consumption model, we simplify the framework by assuming that employment fixed. The description of the fixed employment version of the model, along with its optimality conditions and

[13] In calculating (2.18b) and Δ, we have used the fact that $(F_{kk}F_l - F_{kl}\beta) = F_{kk}\left(\tilde{y}/\tilde{l}\right)$.

[14] See Fisher and Hof (2000b) proposition 1 and section 4, pp. 10-12. In the context of our model, the social planner would set $c + a = C + A$, implying that (2.17) becomes:

$$U(c+a,s) = (1-\gamma)^{-1}(c+a)^{1-\gamma} + \eta s\,(1)\,, \quad \gamma > 0, \quad \eta > 0.$$

In this case there is no relative consumption externality arising from status preference.

intertemporal equilibrium, is given below in the appendix. Examining the steady-state system in (A7a)–(A7c), it is clear that the per-capita values of $\left(\tilde{c}, \tilde{a}, \tilde{k}\right)$ are independent of the parameters of the instantaneous utility function $U(c + a, s)$. Indeed, they are solely a function, as in the standard neoclassical framework without status preference, of the time rate of preference β and the properties of the per-capita production function $y = f(k)$. With respect to the steady-state equilibrium, shifts in the status parameter η only influence the values of the costate variables $\tilde{\mu}$ and $\tilde{\phi}$.[15] Nevertheless, changes in the importance that status consciousness individuals place on the relative consumption of durable goods do affect, through their influence on the values of c_μ and $\tilde{\mu}$, the stable speeds of adjustment of the economy toward steady-state equilibrium. Investigating the relationship between η and the stable eigenvalues, denoted ψ_1 and ψ_2 in the special case of fixed employment, is the focus in this part of the paper.

To do so, we choose the following very simple numerical parameterization of the neoclassical economy with durable goods:

$$\beta = 0.04, \qquad \delta = 0.10, \qquad y = k^{0.36}. \tag{2.19a}$$

Substituting these values in the steady-state equilibrium conditions (A7b, c), we obtain the solutions for the long-run stocks of physical capital and durable goods and the (flow) of durable consumption (=output):

$$\tilde{k} = 31.0, \qquad \tilde{a} = 34.4, \qquad \tilde{c} = \tilde{y} = 3.44. \tag{2.19b}$$

In this exercise we retain the parameterized functional form for $U(c + a, s)$ given in (2.17) and specify two alternative values of the preference parameter γ: $\gamma = 2.5$ and $\gamma = 0.4$.[16]

[15] Using the parameterized instantaneous utility function (2.17) and differentiating (A7a) with respect to η, we find:

$$\frac{\partial \tilde{\phi}}{\partial \eta} = (1 + \beta + \delta)^{-1} \frac{\partial \tilde{\mu}}{\partial \eta} = \frac{s'(1)}{(\beta + \delta)(1 + \delta)\,\tilde{a}} > 0.$$

[16] The available empirical evidence supports an estimate of γ that is closer to 2.5 to 0.4.

According to (2.17), the expressions for c_μ and $\tilde{\mu}$ correspond to:

$$c_\mu = -\left\{\gamma\left[(1+\delta)\tilde{a}\right]^{-(1+\gamma)} + \eta s'(1)/\left[(1+\delta)\,\tilde{a}\right]^2\right\}^{-1}$$

$$\tilde{\mu} = \frac{(1+\beta+\delta)}{(\beta+\delta)}\left\{\left[(1+\delta)\tilde{a}\right]^{-\gamma} + \frac{\eta s'(1)}{(1+\delta)\,\tilde{a}}\right\} \tag{2.20}$$

To calculate the stable eigenvalues ψ_1 and ψ_2, we substitute (2.19a, b)–(2.20), along with the other relevant parameter values, into the appropriate elements of the Jacobian matrix for the fixed employment economy, denoted by $\mathbf{J}^{\tilde{l}}$. We then calculate the eigenvalues of $\mathbf{J}^{\tilde{l}}$, permitting the status parameter take on the following values: $\eta = (0.0, 0.2, 0.4, 0.6, 0.8, 1.0)$. The results are given in Tables 1a and 1b, where we report the absolute values, and, hence, the speeds of adjustment, of the stable eigenvalues, $\mid \psi_1 \mid$ and $\mid \psi_2 \mid$.[17] We find in Table 1a for the case $\gamma = 2.5$ that greater values of η lead to faster stable speeds of adjustment, although after $\eta = 0.4$, these increases are negligible. In contrast, in Table 1b for the case $\gamma = 0.4$ higher values of the status parameter η result in slower speeds of stable adjustment, although, as in Table 1a, the changes in $\mid \psi_1 \mid$ and $\mid \psi_2 \mid$ fall after $\eta = 0.4$. The reason for the distinct responses in Tables 1a and 1b is that increases in η have opposite effects on the intertemporal elasticity of substitution—and, thus, on the stable speeds of adjustment—depending on the value of the preference parameter γ. If consumer-producers have instantaneous preferences described by (2.17), then the intertemporal elasticity of substitution, denoted by $\sigma = \sigma(c+a)$, is equal to:[18]

$$\sigma(c+a) = -\frac{(\beta+\delta)c_\mu\tilde{\mu}}{(1+\beta+\delta)\,(1+\delta)\,\tilde{a}} = \frac{(c+a)^{-\gamma} + (c+a)^{-1}\,\eta s'(1)}{\left[\gamma\,(c+a)^{-\gamma} + (c+a)^{-1}\,\eta s'(1)\right]}. \tag{2.21}$$

[17] All numerical simulations are performed using Mathematica 4.1.

[18] If status depends on relative consumption, Fisher and Hof (2000a) show that the formula for the decentralized intertemporal elasticity of substitution in the symmetric state is given by

$$\sigma(c) = -\frac{V_c(c,1) + c^{-1}V_z\,(c,1)}{c\left[V_{cc}(c,1) + c^{-1}V_{cz}(c,1) - c^{-2}V_z(c,1)\right]}$$

where $U(c,C) = V(c,z)$, $z \equiv c/C$ and C is the aggregate level of (non-durable) consumption. In (2.21) we apply this expression to our specification in which consumption is a durable good.

It is then straightforward to show:

$$\operatorname{sgn}\frac{\partial\sigma(c+a)}{\partial\eta}=\operatorname{sgn}(\gamma-1),$$

which implies that if $\gamma>1$, (resp. $\gamma<1$), then an increase in η raises (resp. lowers) $\sigma(c+a)$ and, thus, the stable speeds of adjustment, consistent with the numerical results in Tables 1a and 1b.

3. Small Open Economy Equilibrium

In this section of the paper we extend the model and its equilibrium properties to the small open economy context.[19] While we assume that the preference structures in (2.2)–(2.5) are the same as in the closed economy specification, we alter the model in two important ways. The first modification of the model is to assume that the small open economy has an upward-sloping supply function of debt. This specification, which is used by authors such as Bhandari, Haque, and Turnovsky (1990), Fisher (1995), and Fisher and Terrell (2000), states that open economies, otherwise satisfying the "small country" assumption, cannot freely borrow or lend at the prevailing world interest rate r^*. Instead, reflecting the imperfect substitutability of domestic and foreign assets, the interest rate on domestic assets, denoted by $r^n(n)$, rises as the national indebtedness of the country increases. Letting the variable n denote the stock of international debt, the domestic interest rate relationship is given as:

$$r^n(n)=r^*+\alpha(n), \qquad \alpha'>0, \qquad \alpha''>0, \tag{3.1}$$

where the convex function $\alpha(n)$ can be considered a country-specific "cost" or "risk premium."[20] The reason why we incorporate (3.1) into our durable consumption model is

[19] In the open economy durable consumption goods are traded at a unitary price.

[20] For convenience, we assume throughout this section of the paper that the small open economy is always a net debtor, i.e., $n>0$, $\forall t\geq 0$. An alternative specification of (3.1) would scale the level of indebtedness by the ability to pay, measured, for example, by the level of output.

because it generates interior intertemporal equilibria with saddlepath dynamics.[21] Moreover, incorporating (3.1), together with durable consumption goods, into the neoclassical small open economy model yields a fourth-order differential equation system, as in the case of our closed economy model. This characteristic makes our closed and open economy specifications more directly comparable.

The other modification of the basic model is the assumption that employment l is the sole factor of production so that: $y = F(l)$, $F' > 0$, $F'' < 0$. The reason why we abstract from physical capital in the small open economy extension is for the sake of analytical tractability and because its dynamic behavior, in the absence of specifying an installation-cost, Tobin's q framework for physical capital, is not particularly revealing.[22] Taken together, these two modifications imply that the accumulation of net debt is given by

$$\dot{n} = r^n(n)n + c - F(l) \tag{3.2}$$

where, as before c is the level of current durable goods. The consumer-producer's maximization problem in this small open economy contest becomes

$$\max \int_0^\infty \left\{ U\left[c + a, s\left(\frac{c+a}{C+A}\right) + V(l)\right] e^{-\beta t} dt \right\} \tag{3.3a}$$

subject to

$$\dot{a} = c - \delta a \tag{3.3b}$$

[21]The recent papers of Fisher and Hof (2003) and Fisher (2004a) have an extensive discussion of the conditions required for the small open economy to possess interior equilibria, particularly the circumstances in which an interior equilibrium is attained without imposing equality between the world interest rate and the domestic rate of time preference, $r^* = \beta$. This issue is also addressed in the standard references of Barro and Sala-i-Martin (1995), ch. 2, and Turnovsky (1997), chs. 2 and 3.

[22]It is straightforward to show that the condition $\beta = r^*$ must be imposed for the small economy Ramsey model, in the absence of other modifications, to achieve an interior steady state and possess saddlepath dynamics. If the domestic economy also possesses physical capital k, then this condition implies that k is always at its steady-state value. In terms of our model with durable consumption goods, it can be demonstrated that the only variables that exhibit non-degenerate dynamics are the stock of consumer durables a and the stock of net debt n. All other variables always equal their steady-state values.

$$\dot{n} = r^n(n)n + c - F(l) \tag{3.3c}$$

and the No–Ponzi-Game condition $\lim_{t\to\infty} ne^{-r^n t} \geq 0$, together with the initial stocks of durable consumption and net debt: $a(0) = a_0 > 0$, $n(0) = n_0 > 0$. The current-value Hamiltonian for this problem corresponds to

$$H = U\left[c + a, s\left(\frac{c+a}{C+A}\right)\right] + V(l) + \phi\left(c - \delta a\right) + \mu\left[r^n(n)n + c - F(l)\right] \tag{3.4}$$

where μ now corresponds to the shadow value of international assets. As in the closed-economy framework, consumers make their choices taking the aggregate levels of durable goods and their stocks as given. Furthermore, agents make their consumption/savings decision holding the interest rate on bonds $r^n(n)$ constant. This implies that the first order conditions are equal to the following expressions

$$U_c\left[c + a, s\left(z\right)\right] + \frac{U_s\left[c + a, s\left(z\right)\right] s'\left(1\right)}{C + A} = \mu - \phi, \tag{3.5a}$$

$$V'(l) = -\mu F'(l) \tag{3.5b}$$

$$\dot{\phi} = (\beta + \delta)\phi - U_c\left[c + a, s\left(z\right)\right] - \frac{U_s\left[c + a, s\left(z\right)\right] s'\left(1\right)}{C + A}, \tag{3.5c}$$

$$\dot{\mu} = \mu\left[\beta - r^n(n)\right] \tag{3.5d}$$

The optimality conditions (3.5a)–(3.5c) for the open economy have an interpretation similar to their counterparts (2.10a)–(2.10c) for the closed economy. The exception is equation (3.5d), which describes the optimal path of the shadow value μ if the stock of net debt is chosen optimally. Our specification of preferences in equations (2.3a)–(2.3b) guarantees that the Hamiltonian (3.4) is jointly concave in the control variables c and l and the state variables a and n. This implies that if the limiting transversality conditions $\lim_{t\to\infty} a\phi e^{-\beta t} = \lim_{t\to\infty} n\mu e^{-\beta t} = 0$ hold, then necessary conditions (3.5a)–(3.5d) are sufficient for optimality.

As in the closed economy model, we restrict the analysis to symmetric equilibria in which identical agents make identical choices, implying, $c + a \equiv C + A$. Substituting this relationship into (3.5a) and combining with (3.5c), we obtain:

$$U_c\left[c+a, s\left(1\right)\right] + \frac{U_s\left[c+a, s\left(1\right)\right] s'\left(1\right)}{c+a} = \mu - \phi = (\beta+\delta)\phi - \dot{\phi} \tag{3.6}$$

which repeats equation (2.11). The optimality conditions for work effort and international borrowing in the symmetric state remain unchanged.

Consequently, the independent dynamics of the small open economy model corresponds to the following system of equations

$$\dot{\phi} = \left(1+\beta+\delta\right)\phi - \mu \tag{3.7a}$$

$$\dot{a} = c\left(a, \mu, \phi\right) - \delta a \tag{3.7b}$$

$$\dot{\mu} = \mu\left[\beta - r^n(n)\right] \tag{3.7c}$$

$$\dot{n} = r^n(n)n + c\left(a, \mu, \phi\right) - F\left[l(\mu)\right] \tag{3.7d}$$

where we have substituted for $c = c\left(a, \mu, \phi\right)$ in equations (3.7b, 3.7d) and $l = l\left(\mu\right)$ in (3.7d).[23] A key distinction between the closed and small open economy models is the ability of small open economy to borrow (and lend) from abroad. This is reflected in equation (3.7d), which describes the (negative of) the current account balance. It is the difference between domestic durable consumption, inclusive of interest service, and domestic output.

Letting $\dot{\phi} = \dot{a} = \dot{\mu} = \dot{n} = 0$, the long-run equilibrium equals

$$U_c\left[(1+\delta)\tilde{a}, s\left(1\right)\right] + \frac{U_s\left[\left(1+\delta\right)\tilde{a}, s\left(1\right)\right] s'\left(1\right)}{(1+\delta)\tilde{a}} = (\beta+\delta)\tilde{\phi}, \tag{3.8a}$$

[23] This instantaneous consumption function, together with its partial derivatives, is the same as in the closed economy framework, while for $l = l(\mu)$, $\partial l/\partial\mu = -\left(V'' + \mu F''\right)^{-1} > 0$.

$$V'(\tilde{l}) = -(1+\beta+\delta)\,\tilde{\phi}F'(\tilde{l}), \tag{3.8b}$$

$$r^n(\tilde{n}) = r^* + \alpha(\tilde{n}) = \beta \tag{3.8c}$$

$$\delta\tilde{a} - F\left(\tilde{l}\right) = r^n(\tilde{n})\tilde{n} = [r^* + \alpha(\tilde{n})]\,\tilde{n}, \tag{3.8d}$$

where $\tilde{c} = \delta\tilde{a}$ and $\tilde{\mu} = (1+\beta+\delta)\,\tilde{\phi}$. The first two steady-state conditions are quite straightforward: equation (3.8a) describes the long-run first order condition for own durable consumption, while equation (3.8b) is the long-run optimality condition for employment if it is the sole factor of production. In turn, equation (3.8c) describes the steady-state maximum condition for foreign debt: the real return on debt in steady-state equilibrium equals the given consumer-producer rate of time preference. Correspondingly, this condition determines the steady-state stock of debt $\tilde{n}$, which is a function of the world interest rate r^*, the domestic rate of time preference β, and the curvature of the "risk premium" function $\alpha(\cdot)$. Finally, equation (3.8d) is the steady-state version of the current account balance in which the difference between long-run durable consumption spending and output equals steady-state interest payments on the outstanding stock of international debt.

Linearizing (3.7a)–(3.7d) about the steady-state equilibrium described by (3.8a)–(3.8d), we obtain the following matrix differential equation

$$\dot{\mathbf{z}} = \mathbf{J}^{\text{soe}}\mathbf{z} =$$

$$\begin{pmatrix} \dot{\phi} \\ \dot{a} \\ \dot{\mu} \\ \dot{n} \end{pmatrix} = \begin{pmatrix} (1+\beta+\delta) & 0 & -1 & 0 \\ -c_\mu & -(1+\delta) & c_\mu & 0 \\ 0 & 0 & 0 & -\tilde{\mu}\alpha' \\ -c_\mu & -1 & -(F'l_\mu - c_\mu) & \beta + \alpha'\tilde{n} \end{pmatrix} \begin{pmatrix} \phi - \tilde{\phi} \\ a - \tilde{a} \\ \mu - \tilde{\mu} \\ n - \tilde{n} \end{pmatrix} \tag{3.9}$$

where $\mathbf{z} = (\phi, a, \mu, n,)'$and $\mathbf{J}^{\text{soe}}$ denotes the Jacobian matrix in the small open economy (soe) case. To determine the stability properties of the equilibrium, we first consider the trace and determinant of the Jacobian matrix

$$\mathbf{tr}\left(\mathbf{J}^{\text{soe}}\right) = \theta_1 + \theta_2 + \theta_3 + \theta_4 = 2\beta + \alpha'\tilde{n} > 0,$$

$$\det\left(\mathbf{J}^{\text{soe}}\right) = \theta_1\theta_2\theta_3\theta_4 = \mu\alpha'\left[(1+\beta+\delta)\left(1+\delta\right)F'l_\mu - (\beta+\delta)\delta c_\mu\right] > 0,$$

where $\theta_i = 1-4$ are the eigenvalues of $\mathbf{J}^{\text{soe}}$. The condition $\mathbf{tr}\left(\mathbf{J}^{\text{soe}}\right) > 0$ rules out the case of four negative eigenvalues, while $\det\left(\mathbf{J}^{\text{soe}}\right) > 0$ eliminates, respectively, the cases of: i) one negative and three positive eigenvalues; and ii) three negative and one positive eigenvalue. As in the closed economy framework, we must directly evaluate the characteristic equation to determine whether $\mathbf{J}^{\text{soe}}$ has: i) two negative and positive eigenvalues; or ii) four positive eigenvalues. In the small open economy case the characteristic equation, denoted by $\det\left(\mathbf{J}^{\text{soe}} - \theta\mathbf{I}\right) = 0$ equals:

$$\det\left(\mathbf{J}^{\text{soe}} - \theta\mathbf{I}\right) =$$

$$\theta^4 - \mathbf{tr}\left(\mathbf{J}^{\text{soe}}\right)\theta^3 + \left[\left(\beta + \alpha'\tilde{n}\right)\beta - (1+\delta)(1+\beta+\delta) - \tilde{\mu}\alpha'\left(F'l_\mu - c_\mu\right)\right]\theta^2$$

$$+\left[(1+\beta+\delta)(1+\delta)\left(\beta+\alpha'\tilde{n}\right) + \tilde{\mu}\alpha'\left(F'l_\mu - c_\mu\right)\beta\right]\theta + \det\left(\mathbf{J}^{\text{soe}}\right) = 0 \tag{3.10}$$

Matching the coefficients of (2.16a) and (3.10), we observe that $\theta_1\theta_2\theta_3 + \theta_1\theta_2\theta_4 + \theta_1\theta_3\theta_4 + \theta_2\theta_3\theta_4 = -\left[(1+\beta+\delta)(1+\delta)\left(\beta+\alpha'\tilde{n}\right) + \tilde{\mu}\alpha'\left(F'l_\mu - c_\mu\right)\beta\right] < 0$, which implies that we can rule out the case in which all the eigenvalues are positive. Thus, the fixed-employment equilibrium of (3.9) is a saddlepoint with two negative and two positive eigenvalues ordered according to:

$$\theta_1 < \theta_2 < 0 < \theta_3 < \theta_4.$$

We have established that both closed and open economies display saddlepath dynamics. As in the case of the closed economy, (3.9), using standard procedures, can be solved for the solutions paths $(\phi, a, \mu, n,)$. This is done in the last part of the appendix.

We close this part of the paper with an analysis of the impact of an increase in the degree of status consciousness in the small open economy context. Differentiating (2.14a$'$)

and (3.8b)–(3.8d) with respect to η, we calculate the following long-run comparative statics expressions for $\left(\tilde{\phi}, \tilde{\mu}, \tilde{a}, \tilde{c}, \tilde{l}\right)$:

$$\frac{\partial \tilde{\phi}}{\partial \eta} = (1+\beta+\delta)^{-1} \frac{\partial \tilde{\mu}}{\partial \eta} = -\frac{s'(1)\,\alpha'(1+\beta+\delta)\left[V'' + (1+\beta+\delta)\,\tilde{\phi}F''\right]}{(1+\delta)\,\tilde{a}\Gamma} > 0, \tag{3.11a}$$

$$\frac{\partial \tilde{a}}{\partial \eta} = \frac{1}{\delta}\frac{\partial \tilde{c}}{\partial \eta} = \frac{s'(1)\,\alpha'(1+\beta+\delta)(F')^2}{(1+\delta)\,\tilde{a}\Gamma} > 0, \tag{3.11b}$$

$$\frac{\partial \tilde{l}}{\partial \eta} = \frac{s'(1)\,\alpha'(1+\beta+\delta)\,(F')^2}{(1+\delta)\,\tilde{a}\Gamma} > 0, \qquad \Rightarrow \qquad \frac{\partial \tilde{y}}{\partial \eta} = F'\frac{\partial \tilde{l}}{\partial \eta} > 0, \tag{3.11c}$$

$$\frac{\partial \tilde{n}}{\partial \eta} = 0, \tag{3.11d}$$

where

$$\Gamma = -\alpha'(\beta+\delta)\,\delta\left[V'' + (1+\beta+\delta)\,\tilde{\phi}F''\right]$$
$$-\alpha'\left[\gamma\left[(1+\delta)\tilde{a}\right]^{-(1+\gamma)} + \frac{\eta s'(1)}{(1+\delta)\tilde{a}^2}\right](1+\beta+\delta)\,F''F' < 0.$$

How do the small open economy results in (3.11a)–(3.11d) compare to those calculated in (2.18a)–(2.18c) for the closed economy case? As in the closed economy framework, a greater degree of status preference raises the shadow values $\tilde{\phi}$ and $\tilde{\mu}$ and causes a long-run increase in work effort $\tilde{l}$, along with durable consumption $\tilde{c}$ and its aggregate stock $\tilde{a}$. These results, then, represent an extension of the Fisher and Hof (2000b) findings that an increase in the degree of status preference leads to a long-run increase in consumption, work effort, and output to the small open economy. One key distinction between the small open economy and closed economies is, of course, the ability of the small open economy to finance its durable consumption by borrowing from abroad. In this model, however, the steady-state increase in durable consumption is exactly offset by the long-run rise in domestic output, which implies that there is no change in the steady-state stock of debt $\tilde{n}$. Of course, the fact that the long-run stock of debt $\tilde{n}$ is independent of the status parameter η does not imply a lack of current account dynamics subsequent to an increase in η. For example, it is straightforward to show, using the solution for n derived in the appendix, that the current account, depending on the relative intertemporal dynamics of durable consumption and output, initially improves (resp. deteriorates) before reaching a time $t = t^*$, such that $\dot{n}(t^*) = 0$. Afterwards, i.e., for $t > t^*$, the current deteriorates (resp. improves) as the stock of debt returns to its initial and steady-state level, $n_0 = \tilde{n}$, as $t \rightarrow \infty$.[24]

4. Conclusions

In this paper we study the dynamic properties of neoclassical, representative agent models of the consumer-producer in which status depends on relative consumption. Our extension is to model in closed and open economy contexts relative consumption as a durable good. Among our major results, we derive the optimizing equilibria of the closed and open economies, including their dynamic properties, show that they are all of a fourth order, and establish that the corresponding steady states in all cases possess the saddlepoint property. Using a particular specification of relative consumption preferences, we investigate the implications of changing the importance of status considerations. Among our results, we find: i) an increase in the degree of status preference in the closed economy with endogenous work effort raises the long-run levels of durable consumption, its corresponding stock, employment, and physical capital; and ii) an increase in the status preference parameter affects the stable speeds of adjustment in the special case of fixed employment, depending on whether it raises or lowers the intertemporal elasticity of substitution. Finally, the long-run implications of an increase in status considerations in the small open economy model are generally similar to those in the closed economy.

[24]The non-monotonic behavior of the current account reflects the fact that it depends on two stable eigenvalues. Because $\tilde{n} = n_0$ in response to a (permanent) increase in the status parameter η, the solution of n simplifies because in (A15b), $Q_2 = -Q_1$. From the corresponding expression for n, we can show

$$\dot{n}(t^*) = 0 \qquad \Leftrightarrow \qquad t = t^* = \frac{\ln(\theta_2/\theta_1)}{\theta_1 - \theta_2}.$$

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5. Appendix

5.1. Partial Derivatives of (2.12a, b)

Taking the total derivatives of (2.11) and (2.10b), we obtain the expressions for the partial derivatives of durable consumption and work effort with respect to the model's state and costate variables:

$$c_a = -1, \qquad c_\mu = -c_\phi = \left[U_{cc} + U_{sc}s'(1)/(c+a) - U_s s'(1)/(c+a)^2\right]^{-1} < 0, \tag{A1a}$$

$$\frac{\partial l}{\partial \mu} = \frac{-F_l}{V'' + \mu F_{ll}} > 0, \qquad \frac{\partial l}{\partial k} = -\frac{\mu F_{kl}}{V'' + \mu F_{ll}} > 0. \tag{A1b}$$

5.2. Solution for (ϕ, a, μ, k) in the Variable-Employment Economy

The general stable solution to the differential equation system (2.15) is represented by the following expressions:

$$\phi = \tilde{\phi} + A_1 e^{\omega_1 t} + A_2 e^{\omega_2 t} \tag{A2a}$$

$$a = \tilde{a} + B_1 e^{\omega_1 t} + B_2 e^{\omega_2 t} \tag{A2b}$$

$$\mu = \tilde{\mu} + C_1 e^{\omega_1 t} + C_2 e^{\omega_2 t} \tag{A2c}$$

$$k = \tilde{k} + D_1 e^{\omega_1 t} + D_2 e^{\omega_2 t} \tag{A2d}$$

where A_i, B_i, C_i and $D_i, i = 1, 2$ are constants (eigenvectors) corresponding to the stable eigenvalues ω_1 and ω_2 and $\left(\tilde{\phi}, \tilde{a}, \tilde{\mu}, \tilde{k}\right)$ are the long-run solutions calculated from (2.14a)–(2.14d). Since only two of these constant are independent, the first step in obtaining the complete solution is to solve A_i, B_i, C_i in terms of D_i, $i = 1, 2$. Using (2.15), (A2a)–(A2d) and letting $\mathbf{x} = (A_i, B_i, C_i, D_i)'$, these relationships are calculated from following the homogeneous system:

$$(\mathbf{J}-\omega\mathbf{I})\mathbf{x} =$$

$$\begin{pmatrix} (1+\beta+\delta)-\omega_i & 0 & -1 & 0 \\ -c_\mu & -[(1+\delta)+\omega_i] & c_\mu & 0 \\ 0 & 0 & d_{33}-\omega_i & d_{34} \\ c_\mu & 1 & d_{43} & d_{44}-\omega_i \end{pmatrix} \begin{pmatrix} A_i \\ B_i \\ C_i \\ D_i \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{A3a}$$

where

$$d_{33} = -l_\mu \tilde{\mu} F_{kl} > 0, \quad d_{34} = -\tilde{\mu}\left(F_{kk} + F_{kl} l_k\right) > 0, \quad d_{43} = F_l l_\mu - c_\mu > 0, \quad d_{44} = \beta + F_l l_k > 0.$$

From (A3a), the constants A_i, B_i, C_i and D_i, $i = 1, 2$ are written as:

$$A_i = \frac{C_i}{(1+\beta+\delta)-\omega_i} = \frac{C_i}{\Phi(\omega_i)} = \frac{-d_{34}D_i}{(\theta_{33}-\omega_i)\Phi(\omega_i)},$$

$$B_i = \frac{c_\mu d_{34}}{(\theta_{33}-\omega_i)\left[(1+\delta)+\omega_i\right]}\left(\frac{1-\Phi(\omega_i)}{\Phi(\omega_i)}\right) D_i = \Omega_i\left(\frac{1-\Phi(\omega_i)}{\Phi(\omega_i)}\right) D_i, \tag{A3b}$$

where

$$\Phi(\omega_i) = (1+\beta+\delta)-\omega_i, \qquad \Omega_i\left(\omega_i\right) = \frac{c_\mu d_{34}}{(\theta_{33}-\omega_i)\left[(1+\delta)+\omega_i\right]}.$$

To solve for the constants D_i and complete the solution of (A3a), we use the fact that the stock of durable goods and physical capital evolve continuously from their initial conditions, $a(0) = a_0$ and $k(0) = k_0$. From (A2b, d) this gives us the two equation system

$$\tilde{a} + \Omega_1\left(\omega_1\right)\left(\frac{1-\Phi(\omega_1)}{\Phi(\omega_1)}\right) D_1 + \Omega_2\left(\omega_2\right)\left(\frac{1-\Phi(\omega_i)}{\Phi(\omega_i)}\right) D_2 = a_0, \qquad \tilde{k} + D_1 + D_2 = k_0 \tag{A4a}$$

from which we solve for D_1, D_2:

$$D_1 = \frac{\Phi(\omega_1)\Phi(\omega_2)\left[-(\tilde{a}-a_0) + \Omega_2\left(\omega_2\right)\frac{(1-\Phi(\omega_2))}{\Phi(\omega_2)}(\tilde{k}-k_0)\right]}{\Omega_1\left(\omega_1\right)\Phi(\omega_2)\left[1-\Phi(\omega_1)\right] - \Omega_2\left(\omega_2\right)\Phi(\omega_1)\left[1-\Phi(\omega_2)\right]},$$

$$D_2 = \frac{\left[\Phi(\omega_1)\Phi(\omega_2)(\tilde{a} - a_0) - \Omega_1(\omega_1)\Phi(\omega_2)(1 - \Phi(\omega_1))(\tilde{k} - k_0)\right]}{\Omega_1(\omega_1)\Phi(\omega_1)[1 - \Phi(\omega_1)] - \Omega_2(\omega_2)\Phi(\omega_1)[1 - \Phi(\omega_2)]}. \tag{A4b}$$

Substitution of equations (A4b) into (A3b) and the resulting expressions into (A2a)–(A2d) yield the stable solutions for (ϕ, a, μ, k) in the endogenous employment case.

5.3. Fixed Employment Equilibrium

If consumer-producers supply a fixed (unitary) level of work effort $l = \bar{l}$, instantaneous preferences reduce to $U = U(c + a, s) + V(\bar{l})$, where the conditions (2.2)–(2.5) obtain. Furthermore, we can simplify production technology to $y = f(k)$, where y and k now represent per-capita output and physical capital, respectively, and the following standard restrictions on $f(k)$ hold: $f'(k) > 0$, $f''(k) < 0$, $f(0) = 0$, $f(0) = 0$, $f(k) \to \infty$ as $k \to \infty$. Furthermore, we assume the usual Inada conditions are satisfied. It is then straightforward to demonstrate that the symmetric equilibrium in the fixed employment case corresponds to:

$$U_c[c + a, s(1)] + \frac{U_s[c + a, s(1)]\, s'(1)}{c + a} = \mu - \phi, \tag{A5a}$$

$$\dot{\phi} = (\beta + \delta)\phi - U_c[c + a, s(1)] - \frac{U_s[c + a, s(1)]\, s'(1)}{c + a}, \tag{A5b}$$

$$\dot{a} = c - \delta a, \tag{A5c}$$

$$\dot{\mu} = \mu[\beta - f'(k)], \tag{A5d}$$

$$\dot{k} = f(k) - c. \tag{A5e}$$

Equations (A5a)–(A5c) repeat (2.11) and (2.1b) from the model with endogenous work effort, while (A5d, e) are the corresponding differential equations for the capital stock and its costate variable in the fixed employment case. As in the general model with endogenous employment, we calculate an instantaneous consumption function $c = c(a, \mu, \phi)$ from (A5a), which possesses the same partial derivatives stated above in (A1a).

The equilibrium dynamics of the fixed employment model then equals:

$$\dot{\phi} = (1 + \beta + \delta)\,\phi - \mu, \tag{A6a}$$

$$\dot{a} = c(a, \mu, \phi) - \delta a, \tag{A6b}$$

$$\dot{\mu} = \mu[\beta - f'(k)], \tag{A6c}$$

$$\dot{k} = f(k) - c(a, \mu, \phi). \tag{A6d}$$

Letting $\dot{\phi} = \dot{a} = \dot{\mu} = \dot{k} = 0$, the long-run equilibrium equals

$$U_c\left[(1+\delta)\tilde{a}, s(1)\right] + \frac{U_s\left[(1+\delta)\,\tilde{a}, s(1)\right] s'(1)}{(1+\delta)\tilde{a}} = (\beta + \delta)\tilde{\phi}, \tag{A7a}$$

$$f'\left(\tilde{k}\right) = \beta, \tag{A7b}$$

$$f\left(\tilde{k}\right) = \delta\tilde{a}, \tag{A7c}$$

where $\tilde{c} = \delta\tilde{a}$ and $\tilde{\mu} = (1 + \beta + \delta)\,\tilde{\phi}$. Linearizing (A6a)–(A6d) about the steady-state equilibrium described by (A7a)–(A7c), we obtain the following matrix differential equation:

$$\dot{\mathbf{z}} = \mathbf{J}^{\bar{l}}\mathbf{z} =$$

$$\begin{pmatrix} \dot{\phi} \\ \dot{a} \\ \dot{\mu} \\ \dot{k} \end{pmatrix} = \begin{pmatrix} (1+\beta+\delta) & 0 & -1 & 0 \\ -c_\mu & -(1+\delta) & c_\mu & 0 \\ 0 & 0 & 0 & -\tilde{\mu}f'' \\ c_\mu & 1 & -c_\mu & \beta \end{pmatrix} \begin{pmatrix} \phi - \tilde{\phi} \\ a - \tilde{a} \\ \mu - \tilde{\mu} \\ k - \tilde{k} \end{pmatrix} \tag{A8}$$

where $\mathbf{z} = (\phi, a, \mu, k,)'$and $\mathbf{J}^{\bar{l}}$ denotes the Jacobian matrix of (A8) in the fixed employment case. To determine the stability properties of the equilibrium, we first consider the trace and determinant of the Jacobian matrix

$$\mathbf{tr}\left(\mathbf{J}^{\bar{l}}\right) = \psi_1 + \psi_2 + \psi_3 + \psi_4 = 2\beta > 0,$$

$$\det\left(\mathbf{J}^{\bar{l}}\right) = \psi_1\psi_2\psi_3\psi_4 = (\beta+\delta)\delta c_\mu \tilde{\mu} f'' > 0,$$

where $\psi_i = 1 - 4$ are the eigenvalues of $\mathbf{J}^{\bar{l}}$. The condition $\mathbf{tr}\left(\mathbf{J}^{\bar{l}}\right) > 0$ rules out the case of four negative eigenvalues, while $\det\left(\mathbf{J}^{\bar{l}}\right) > 0$ eliminates, respectively, the cases of one negative and three positive eigenvalues, and three negative and one positive eigenvalue. As in the more general framework with endogenous employment, we must directly evaluate the characteristic equation to determine whether $\mathbf{J}^{\bar{l}}$ has two negative and positive eigenvalues or four positive eigenvalues. In this case the characteristic equation, denoted by $\det\left(\mathbf{J}^{\bar{l}} - \psi\mathbf{I}\right) = 0$ equals:

$$\det\left(\mathbf{J}^{\bar{l}} - \psi\mathbf{I}\right) =$$

$$\psi^4 - \mathbf{tr}\left(\mathbf{J}^{\bar{l}}\right)\psi^3 + [\beta^2 - (1+\delta)(1+\beta+\delta) - c_\mu\tilde{\mu}f'']\psi^2 + \beta[(1+\delta)(1+\beta+\delta) + c_\mu\tilde{\mu}f'']\psi.$$

$$+\det\left(\mathbf{J}^{\bar{l}}\right) = 0 \tag{A9}$$

Matching the coefficients of (2.16a) and (A9), we observe that

$$\psi_1\psi_2\psi_3 + \psi_1\psi_2\psi_4 + \psi_1\psi_3\psi_4 + \psi_2\psi_3\psi_4 = \beta[(1+\delta)(1+\beta+\delta) + c_\mu\tilde{\mu}f''] < 0,$$

which implies that we can rule out the case in which all the eigenvalues are positive. Thus, the fixed-employment equilibrium of (A9) is a saddlepoint with two negative and two positive eigenvalues ordered according to:

$$\psi_1 < \psi_2 < 0 < \psi_3 < \psi_4.$$

5.4. Solution for (ϕ, a, μ, k) in the Fixed-Employment Economy

The general stable solution to the differential equation system (A9) is represented by the following expressions:

$$\phi = \tilde{\phi} + E_1 e^{\psi_1 t} + E_2 e^{\psi_2 t} \tag{A10a}$$

$$a = \tilde{a} + F_1 e^{\psi_1 t} + F_2 e^{\psi_2 t} \tag{A10b}$$

$$\mu = \tilde{\mu} + G_1 e^{\psi_1 t} + G_2 e^{\psi_2 t} \tag{A10c}$$

$$k = \tilde{k} + L_1 e^{\psi_1 t} + L_2 e^{\psi_2 t} \tag{A10d}$$

where E_i, F_i, G_i and $L_i, i = 1, 2$ are constants (eigenvectors) corresponding to the stable eigenvalues ψ_1 and ψ_2 of the fixed employment economy $(l = \bar{l})$ and where $\left(\tilde{\phi}, \tilde{a}, \tilde{\mu}, \tilde{k}\right)$ are the steady-state solutions derived from the system (A7a)–(A7c). Since only two of these constants are independent, the first step in obtaining the complete solution is to solve E_i, F_i, G_i in terms of L_i, $i = 1, 2$. These relationships, using (A8) and (A10a)–(A10d), are calculated from the homogeneous system:

$$(\mathbf{J}^{\bar{l}} - \psi \mathbf{I})\mathbf{x}$$

$$= \begin{pmatrix} (1+\beta+\delta) - \psi_i & 0 & -1 & 0 \\ -c_\mu & -[(1+\delta) + \psi_i] & c_\mu & 0 \\ 0 & 0 & -\psi_i & -\tilde{\mu} f'' \\ c_\mu & 1 & -c_\mu & \beta - \psi_i \end{pmatrix} \begin{pmatrix} E_i \\ F_i \\ G_i \\ L_i \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{A11a}$$

where $\mathbf{x} = (E_i, F_i, G_i, L_i)'$and where the constants for $i = 1, 2$ are written as

$$E_i = \frac{G_i}{(1+\beta+\delta) - \psi_i} = \frac{G_i}{\Phi^{\bar{l}}(\psi_i)} = \frac{-\tilde{\mu} f'' L_i}{\psi_i \Phi^{\bar{l}}(\psi_i)},$$

$$F_i = \frac{c_\mu \tilde{\mu} f''}{\psi_i \left[(1+\delta) + \psi_i\right]} \left(\frac{1 - \Phi^{\bar{l}}(\psi_i)}{\Phi^{\bar{l}}(\psi_i)}\right) L_i = \Omega_i^{\bar{l}} \left(\frac{1 - \Phi^{\bar{l}}(\psi_i)}{\Phi^{\bar{l}}(\psi_i)}\right) L_i, \tag{A11b}$$

where:

$$\Phi^{\bar{l}}(\omega_i) = (1+\beta+\delta) - \psi_i, \qquad \Omega_i^{\bar{l}}(\psi_i) = \frac{c_\mu \tilde{\mu} f''}{\psi_i \left[(1+\delta) + \psi_i\right]},$$

are the solution coefficients for the fixed employment case. To solve for the constants L_i and complete the solution of (A10a)–(A10d), we use the fact that the stock of durable goods and physical capital evolve continuously from their initial conditions, $a(0) = a_0$ and $k(0) = k_0$. From (A10b, d) this gives us the two equation system

$$\tilde{a} + \Omega_1^{\bar{l}}(\psi_1) \left(\frac{1 - \Phi^{\bar{l}}(\psi_1)}{\Phi^{\bar{l}}(\psi_1)}\right) L_1 + \Omega_2^{\bar{l}}(\psi_2) \left(\frac{1 - \Phi^{\bar{l}}(\psi_2)}{\Phi^{\bar{l}}(\psi_2)}\right) L_2 = a_0, \qquad \tilde{k} + L_1 + L_2 = k_0, \tag{A12a}$$

so that L_1, L_2 equals:

$$L_1 = \frac{\Phi^{\bar{l}}(\psi_1)\Phi^{\bar{l}}(\psi_2)\left[-(\tilde{a} - a_0) + \Omega_2^{\bar{l}}(\psi_2)\frac{[1-\Phi^{\bar{l}}(\psi_2)]}{\Phi^{l}(\psi_2)}(\tilde{k} - k_0)\right]}{\Omega_1^{\bar{l}}(\psi_1)\Phi^{\bar{l}}(\psi_2)\left[1 - \Phi^{\bar{l}}(\psi_1)\right] - \Omega_2^{\bar{l}}(\psi_2)\Phi^{\bar{l}}(\psi_1)\left[1 - \Phi^{\bar{l}}(\psi_2)\right]},$$

$$L_2 = \frac{\left[\Phi^{\bar{l}}(\psi_1)\Phi^{\bar{l}}(\psi_2)(\tilde{a} - a_0) - \Omega_1^{\bar{l}}(\psi_1)\Phi^{\bar{l}}(\psi_2)[1 - \Phi^{\bar{l}}(\psi_1)](\tilde{k} - k_0)\right]}{\Omega_1^{\bar{l}}(\psi_1)\Phi^{\bar{l}}(\psi_1)\left[1 - \Phi^{l}(\psi_1)\right] - \Omega_2^{\bar{l}}(\psi_2)\Phi^{\bar{l}}(\psi_1)\left[1 - \Phi^{\bar{l}}(\psi_2)\right]}. \tag{A12b}$$

Substitution of equations (A12b) into (A11b) and the resulting expressions into (A10a)–(A10d) yield the stable solutions for (ϕ, a, μ, k) in the fixed employment case.

5.5. Solution for (ϕ, a, μ, n) in the Small Open Economy

The general stable solution to the differential equation system (3.9a) is represented by the following expressions:

$$\phi = \tilde{\phi} + M_1 e^{\theta_1 t} + M_2 e^{\theta_2 t} \tag{A13a}$$

$$a = \tilde{a} + N_1 e^{\theta_1 t} + N_2 e^{\theta_2 t} \tag{A13b}$$

$$\mu = \tilde{\mu} + P_1 e^{\theta_1 t} + P_2 e^{\theta_2 t} \tag{A13c}$$

$$n = \tilde{n} + Q_1 e^{\theta_1 t} + Q_2 e^{\theta_2 t} \tag{A13d}$$

where M_i, N_i, P_i and $Q_i, i = 1, 2$ are constants (eigenvectors) corresponding to the stable eigenvalues θ_1 and θ_2 of the small open economy and where $\left(\tilde{\phi}, \tilde{a}, \tilde{\mu}, \tilde{n}\right)$ are the steady-state solutions derived from the system (3.8a)–(3.8d). Since only two of these constants are independent, the first step in obtaining the complete solution is to solve M_i, N_i, P_i in terms of Q_i, $i = 1, 2$. These relationships, using (3.9a) and (A13a)–(A13d), are calculated from the homogeneous system:

$$(\mathbf{J}^{\text{soe}} - \theta \mathbf{I})\mathbf{x} = \tag{14a}$$

$$\begin{pmatrix} (1+\beta+\delta) - \theta_i & 0 & -1 & 0 \\ -c_\mu & -[(1+\delta) + \theta_i] & c_\mu & 0 \\ 0 & 0 & -\theta_i & -\tilde{\mu}\alpha' \\ -c_\mu & -1 & -\left(F' l_\mu - c_\mu\right) & (\beta + \alpha' \tilde{n}) - \theta_i \end{pmatrix} \begin{pmatrix} M_i \\ N_i \\ P_i \\ Q_i \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

where $\mathbf{x} = (M_i, N_i, P_i, Q_i)'$and where the constants in the small open economy (soe) for $i = 1, 2$ are written as:

$$M_i = \frac{P_i}{(1+\beta+\delta) - \theta_i} = \frac{P_i}{\Phi^{\text{soe}}(\theta_i)} = \frac{-\tilde{\mu}\alpha' Q_i}{\theta_i \Phi^{\text{soe}}(\theta_i)},$$

$$N_i = \frac{c_\mu \tilde{\mu} \alpha'}{\theta_i \left[(1+\delta) + \theta_i \right]} \left(\frac{1 - \Phi^{\mathrm{soe}}(\theta_i)}{\Phi^{\mathrm{soe}}(\theta_i)} \right) Q_i = \Omega_i^{\mathrm{soe}} \left(\frac{1 - \Phi^{\mathrm{soe}}(\theta_i)}{\Phi^{\mathrm{soe}}(\theta_i)} \right) Q_i, \tag{A14b}$$

where:

$$\Phi^{\mathrm{soe}}(\theta_i) = (1 + \beta + \delta) - \theta_i, \qquad \Omega_i^{\mathrm{soe}}(\theta_i) = \frac{c_\mu \tilde{\mu} \alpha'}{\theta_i \left[(1+\delta) + \theta_i \right]},$$

are the solution coefficients for the small open economy case. To solve for the constants Q_i and complete the solution of (A13a)–(A13d), we use the fact that the stock of durable goods and net debt evolve continuously from their initial conditions, $a(0) = a_0$ and $n(0) = n_0$. From (A13b, d) this gives us the two equation system

$$\tilde{a} + \Omega_1^{\mathrm{soe}}(\theta_1) \left(\frac{1 - \Phi^{\mathrm{soe}}(\theta_1)}{\Phi^{\mathrm{soe}}(\theta_1)} \right) Q_1 + \Omega_2^{\mathrm{soe}}(\theta_2) \left(\frac{1 - \Phi^{\mathrm{soe}}(\theta_2)}{\Phi^{\mathrm{soe}}(\theta_2)} \right) Q_2 = a_0, \qquad \tilde{n} + Q_1 + Q_2 = n_0, \tag{A15a}$$

from which we solve Q_1, Q_2 as:

$$Q_1 = \frac{\Phi^{\mathrm{soe}}(\theta_1) \Phi^{\mathrm{soe}}(\theta_2) \left[-(\tilde{a} - a_0) + \Omega_2^{\mathrm{soe}}(\theta_2) \frac{[1 - \Phi^{\mathrm{soe}}(\theta_2)]}{\Phi^{\mathrm{soe}}(\theta_2)} (\tilde{n} - n_0) \right]}{\Omega_1^{\mathrm{soe}}(\theta_1) \Phi^{\mathrm{soe}}(\theta_2) \left[1 - \Phi^{\mathrm{soe}}(\theta_1) \right] - \Omega_2^{\mathrm{soe}}(\theta_2) \Phi^{\mathrm{soe}}(\theta_1) \left[1 - \Phi^{\mathrm{soe}}(\theta_2) \right]},$$

$$Q_2 = \frac{\left[\Phi^{\mathrm{soe}}(\theta_1) \Phi^{\mathrm{soe}}(\theta_2) (\tilde{a} - a_0) - \Omega_1^{\mathrm{soe}}(\theta_1) \Phi^{\mathrm{soe}}(\theta_2) [1 - \Phi^{\mathrm{soe}}(\theta_1)] (\tilde{n} - n_0) \right]}{\Omega_1^{\mathrm{soe}}(\theta_1) \Phi^{\mathrm{soe}}(\theta_1) \left[1 - \Phi^{\mathrm{soe}}(\theta_1) \right] - \Omega_2^{\mathrm{soe}}(\theta_2) \Phi^{\mathrm{soe}}(\theta_1) \left[1 - \Phi^{\mathrm{soe}}(\theta_2) \right]}. \tag{A15b}$$

Substitution of equations (A15b) into (A14b) and the resulting expressions into (A13a)–(A13d) yield the stable solutions for (ϕ, a, μ, n) in the small open economy with an upward-sloping supply function of debt.

Table 1a: Effects of the Degree of Status Preference on the Stable Speeds on Adjustment for $\gamma = 2.5$

η	$\mid \psi_1 \mid$	$\mid \psi_2 \mid$
0.0	1.14	0.0181
0.2	1.20	0.0320
0.4	1.21	0.0322
0.6	1.21	0.0324
0.8	1.21	0.0325
1.0	1.21	0.0325

Table 1b: Effects of the Degree of Status Preference on the Stable Speeds on Adjustment for $\gamma = 0.4$

η	$\mid \psi_1 \mid$	$\mid \psi_2 \mid$
0.0	1.35	0.0516
0.2	1.35	0.0509
0.4	1.34	0.0502
0.6	1.33	0.0496
0.8	1.33	0.0491
1.0	1.32	0.0485