

Uninsurable investment risks*

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Abstract

This paper studies a general equilibrium economy in which agents have the ability to invest in a risky technology. The investment risk cannot be fully insured with optimal contracts because shocks are private information. We show that the presence of these risks may lead to under-accumulation of capital relative to an economy where idiosyncratic shocks can be fully insured. We also show that, although the availability of state-contingent (optimal) contracts cannot provide full insurance, it brings the aggregate stock of capital close to the complete market level. Institutional reforms that make possible the use of these contracts have important welfare consequences.

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1 Introduction

A large body of literature that studies the saving behavior in the presence of uninsurable idiosyncratic risks assumes that these risks are not associated with investment. As in Bewley (1986), the most common assumption is that earnings or endowments are subject to shocks that cannot be insured away. See for example Aiyagari (1994, 1995), Hansen & İmrohorođlu (1992), Huggett (1993, 1996), İmrohorođlu (1989), Ríos-Rull (1994). In this class of models the inability to fully insure the idiosyncratic risk implies that the equilibrium interest rate is smaller than in the complete market economy, whether market incompleteness is taken as given or modelled endogenously. Because the interest rate is equal to the marginal productivity of capital, the presence of uninsurable risks implies that the stock of capital is larger than in the complete market economy (over-accumulation). Aiyagari (1995) shows that in this case a positive capital income tax is desirable in the long-run. Golosov, Kocherlakota, & Tsyvinski (2003) also show that a positive capital income tax may improve the allocation when market incompleteness is endogenous, but the mechanism that justifies the positive tax is different.

Although earnings or labor income uncertainty is an important source of idiosyncratic risk, investment activities are also subject to uninsurable risks. For instance, entrepreneurs invest heavily in their own business¹ and managers of corporations hold a large number of the firm's shares.² Even the return from investing in education is highly uncertain and cannot be insured away. What differentiates investment risks from earnings or endowment risks is that the agent can avoid them by choosing safer allocations of savings. On the contrary, earnings or endowment risks in the class of Bewley's economies are beyond the control of the agent. The agent can only use the available markets to (incompletely) insure them.

The goal of this paper is to model explicitly investment risks. We consider three environments. In the first environment agents can sign optimal state-contingent contracts. These contracts, however, cannot provide full insurance because there are agency problems in the form of asymmetric information. We will refer to this environment as the "Optimal Contract Economy". In the second environment agents cannot sign state-contingent contracts. Only non-contingent contracts (borrowing and lending) are available. We will refer

¹See Cagetti & DeNardi (2002), Carroll (2002), Gentry & Hubbard (2000), Hurst & Lusardi (2002) and Quadrini (1999).

²See Mikkelson, Partch, & Shah (1997) and Himmelberg, Hubbard, & Love (2000).

to this environment as the “Bond Economy”. Finally, the third environment is the “Complete Markets Economy” in which there are no agency problems, and therefore, full insurance against investment risks is possible.

By comparing these three economies we show that:

1. In the two economies with incomplete markets (the Bond Economy and the Optimal Contract Economy) the equilibrium risk-free interest rate is smaller than in the Complete Markets Economy. However, for certain specifications of the model, the aggregate stock of capital is smaller than in the Complete Markets Economy, (*i.e.*, there is under-accumulation).
2. Even with very large agency problems, the availability of optimal contracts brings the aggregate stock of capital and the equilibrium interest rate very close to the corresponding levels in the complete market economy. Also, the feasibility of optimal contracts increases welfare significantly.

The first result, that is, the under-accumulation of capital, may change our conclusion about the desirability of long-term capital taxes. Because in Aiyagari (1995) the optimality of capital taxes derives from the over-accumulation of capital, if the model does not generate over-accumulation, the rationale for the taxation of capital may also vanish. The full investigation of this conjecture, however, is beyond the scope of this paper.

The second result points out the importance of factors that make state-contingent contracts feasible. Among these factors, formal and informal institutions play a central role. The reason state-contingent contracts are not extensively used in practice is because the enforcement system may be highly inefficient and costly. For instance, the resolution of contractual disputes may be extremely long and uncertain. Cross-country studies show that the degree of contract enforcement is correlated with the degree of financial development. See Levine (1997) and Dolar & Meh (2002) for reviews of the empirical literature. In this study we interpret the economy with state-contingent contracts as an economy in which financial markets are more developed in part as a result of higher efficiency of the institutional enforcement. Our study then provides a welfare assessment of institutional reforms leading to greater contract enforceability.

The model studied in this paper has some similarities with the model studied in Khan & Revikumar (2001). There are two important differences.

The first difference is that their model allows for endogenous growth. Consequently, agency problems affect the long-term growth of the economy. In our model, instead, agency problems have only level effects. The second difference relative to Khan and Revikumar’s paper is that they only compare the Optimal Contracts Economy to the Complete Markets Economy. In our paper, instead, we are primarily interested in comparing the Optimal Contract Economy with the economy in which state-contingent contracts are not available (the Bond Economy). This comparison is more relevant for the question we are interested in, that is, the welfare implications of institutional reforms that make possible the availability of state-contingent contracts.

Our paper is also related to Angeletos (2003) who also shows that uninsurable investment risks may induce under-accumulation of capital. In his paper, however, market incompleteness is not endogenous, and therefore, it does not answer the question of whether the availability of state-contingent contracts has large welfare implications in the presence of agency problems. Our analysis is also more general because we shows that the under-accumulation result requires very specific assumptions about the specification of the model.

The plan of the paper is as follows. In the next section we describe the basic theoretical framework and characterize the problems solved by agents when state-contingent contracts are available (Optimal Contract Economy) and when they are not available (Bond Economy). Section 3 conducts the quantitative analysis using parameterized versions of the model. Section 4 considers several extensions and Section 5 concludes.

2 The basic model

There is a continuum of households that maximize the expected lifetime utility:

$$E \sum_{t=0}^{\infty} \beta^t U(c_t), \quad U(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma} \quad (1)$$

where c_t is consumption at time t and β is the intertemporal discount factor. Households are endowed with one unit of time supplied inelastically at the market wage rate w_t .

Each household can run a risky technology that returns $F(k_t, l_{t+1}, z_{t+1})$ in the next period with the inputs of capital k_t and labor l_{t+1} . The variable z_{t+1} is an idiosyncratic *iid* shock that is unknown when k_t is chosen but it is known when l_{t+1} is chosen. For simplicity we assume that the shock can

take only two values denoted by z_L and z_H , with $z_L < z_H$. The probability, denoted by $p(z)$, is strictly positive for both realizations of the shock. The function F is strictly concave in the production inputs and satisfies $\lim_{k_t \rightarrow 0} EF_k(k_t, l_{t+1}, z_{t+1}) = \lim_{l_t \rightarrow 0} EF_l(k_t, l_{t+1}, z_{t+1}) = \infty$.

The agent has the ability to divert the retained capital to get a private benefit. Diversion of capital is not observable and generates efficiency losses in the form of a lower probability of the good shock z_H . More specifically, we assume that the probability of the good shock becomes zero in case of diversion. The private and unobservable return from diversion is additive to consumption. Given c_t the agent's consumption, the current utility is $U(c_t + \alpha k_t)$, where α is a utility parameter which is constant in the model. When we later specify the functional form for $F(k_t, l_{t+1}, z_{t+1})$, we will also impose some restrictions on the parameter α that guarantee the inefficiency of diversion.

For the analysis that follows it would be convenient to define the following function:

$$R(w_{t+1}; k_t, l_{t+1}, z_{t+1}) = F(k_t, l_{t+1}, z_{t+1}) - w_{t+1}l_{t+1} \quad (2)$$

This is the gross revenue net of the labor cost. Given the specification of the return from diversion, the optimal input of labor is fully determined by the input of capital, the shock and the wage rate, that is, $l_{t+1} = l(k_t, w_{t+1}, z_{t+1})$. We can then eliminate l_{t+1} as an explicit argument of the gross revenue and write it simply as $R(w_{t+1}; k_t, z_{t+1})$.

In addition to the risky investment, there are state-contingent assets that pay $b(z_{t+1})$ units of output in the next period conditional on the realization of z_{t+1} . The current value of these assets is $\delta_t \sum_{z_{t+1}} p(z_{t+1})b(z_{t+1})$ where $\delta_t = 1/(1 + r_t)$ is the market discount rate and r_t is the equilibrium riskless interest rate.

2.1 The agent's problem

Denote by a the agent's wealth or net worth before consumption. Given the sequence of prices $P^t \equiv \{r_j, w_{j+1}\}_{j=t}^{\infty}$, the optimization problem can be written as follows:

$$V_t(a) = \max_{c,k,b(z_i)} \left\{ U(c) + \beta \sum_i V_{t+1}(a(z_i))p(z_i) \right\} \quad (3)$$

subject to

$$a = c + k + \delta_t \sum_i p(z_i)b(z_i) \quad (4)$$

$$a(z_i) = w_{t+1} + b(z_i) + R(w_{t+1}; k, z_i), \quad \text{for } i = L, H \quad (5)$$

$$U(c) + \beta \sum_i V_{t+1}(a(z_i))p(z_i) \geq U(c + \alpha k) + \beta V_{t+1}(a(z_L)) \quad (6)$$

$$a(z_i) \geq \underline{a}_{t+1} \quad (7)$$

This is the optimization problem for any deterministic sequence of prices, not only steady states. The time subscript t in the value function is motivated by the non-stationarity of the problem. Notice that z_i , with $i \in \{L, H\}$, denotes the next period realization of the shock which is unknown when the agent chooses the consumption and investment plan. Equation (4) is the budget constraint. Equation (5) is the law of motion for next period net worth before consumption, the variable a . Equation (6) is the incentive-compatibility constraint and equation (7) imposes limited liability. Limited liability is justified by the assumption that the agent can renegotiate any liability for which its net worth is smaller than a minimum value \underline{a}_{t+1} . The size of this lower bound depends on the particular assumptions about the penalty that can be imposed on a defaulting agent. Following is the description of two possibilities:

- **No market exclusion:** One possibility is to assume that there is no market exclusion if the contract is renegotiated and the investor can only confiscate the current net worth of the agent. This can be justified using an argument similar to Kiyotaki & Moore (1997). In this case the lower bound is $\underline{a}_{t+1} = 0$. A variation would assume that labor income cannot be confiscated. In this case the lower bound is $\underline{a}_{t+1} = w_{t+1}$.
- **Exclusion from the investment:** An extreme form of punishment assumes that in case of repudiation the agent is precluded from running the risky technology and a fraction ϕ of his current and future (labor) income is confiscated in every period. The lifetime utility after

repudiation is $\underline{V}_{t+1} = \sum_{j=0}^{\infty} \beta^j U((1-\phi)w_{t+1})$. The lower bound is then determined by the condition $V_{t+1}(\underline{a}_{t+1}) = \underline{V}_{t+1}$.

These are only two possibilities. Throughout the paper we will adopt the first assumption and we impose $\underline{a}_{t+1} = 0$.

The structure of problem (3) is not standard because the unknown value functions V_j , for $j = t, t+1, \dots$, enter the constraints of the problem and there are no guarantees that the problem is concave. We will describe in the next section how we deal with these analytical problems. For the moment we assume that a solution exists. This solution consists of the sequence of policy functions $\{c_j(a), k_j(a), b_j(a)(z_i)\}_{j=t}^{\infty}$. Given the solution to the agent's problem and the initial distribution of households over asset a —which we denote by $M_t(a)$ —the general equilibrium can be defined as follows:

Definition 1 *Given the initial distribution $M_t(a)$, a general equilibrium is defined by (i) a sequence of prices $P^t \equiv \{r_j, w_{j+1}\}_{j=t}^{\infty}$; (ii) a sequence of aggregate demands for labor $L(P^t) \equiv \{L_{j+1}(P^t)\}_{j=t}^{\infty}$; (iii) a sequence of aggregate capital $K(P^t) \equiv \{K_j(P^t)\}_{j=t}^{\infty}$; and (iv) a sequence of aggregate consumption $C(P^t) \equiv \{C_j(P^t)\}_{j=t}^{\infty}$. These sequences must satisfy: (i) the aggregate demands of labor, capital and consumption are the aggregation of individual demands and they satisfy $L_{j+1}(P^t) = 1$ and $C_j(P^t) + K_j(P^t) = \int a M_j(da)$; (ii) the distribution $M_t(a)$ evolves according to individual decisions and the stochastic properties of the shock.*

2.2 Complete Markets and Bond Economies

One of the goal of this paper is to compare the allocation obtained when state-contingent contracts are feasible with the allocations in two alternative environments: when state-contingent contracts are not available (Bond Economy) and when shocks are public information (Complete Markets Economy).

The optimization problems solved in the economy with complete markets and in the bond economy are special cases of problem (3). More specifically, in the Complete Markets Economy the agent's problem is not subject to the incentive-compatibility constraint (6). This allows the agent to self-insure against the investment risk and the first order conditions imply that

$ER_k(w_{t+1}; k_t, z_{t+1}) = 1 + r_t$, where R_k is the derivative of the gross revenue with respect to k . Of course, in the steady state it must be that $1 + r_t = 1/\beta$ for all t .

The optimization problem solved in the bond economy is also a special case of problem (3). This is obtained by restricting $b(z_L) = b(z_H) = b$. In this case the incentive-compatibility constraint never binds and the optimization problem simplifies to:

$$V_t(a) = \max_{c,k,b} \left\{ U(c) + \beta \sum_i V_{t+1}(a(z_i))p(z_i) \right\} \quad (8)$$

subject to

$$a = c + k + \delta_t b \quad (9)$$

$$a(z_i) = w_{t+1} + b + R(w_{t+1}; k, z_i) \quad (10)$$

$$a(z_i) \geq 0 \quad (11)$$

This is a standard concave problem as formally stated in the following proposition:

Proposition 1 *For any sequence of prices, there is a unique solution to problem (8) and the function $V_t(a)$ is strictly increasing, concave and differentiable at all t .*

Proof 1 *It can be verified that the feasible set in problem (8) is convex and the objective function is strictly concave. Therefore, if V_{t+1} is concave, V_t is strictly concave. Moving backward we can establish that $\lim_{t \rightarrow -\infty} V_t$ is concave. Because the objective of problem (8) is strictly concave, the solution is unique. Standard arguments can be used to prove that the value function is differentiable. Q.E.D.*

Given proposition (1), the solution to the agent's problem can be characterized by the following first order conditions:

$$U'(c_t) = \beta (1 + r_t) E\{U'(c_{t+1})\} + \lambda_t \quad (12)$$

$$U'(c_t) = \beta E\{U'(c_{t+1}) \cdot R_k(w_{t+1}; k, z)\} + \lambda_t \cdot R_k(w_{t+1}; k, z_L) \quad (13)$$

where λ_t is the Lagrange multiplier associated with the limited liability constraint (11). This is positive if the solution is binding.

The first order conditions make clear that the expected return from the risky investment is always greater than the return from the risk-free asset, that is, $1 + r_t < ER_k(w_{t+1}; k, z)$. To see this, consider the case in which the solution is not binding. Then (12) and (13) imply,

$$(1 + r_t) \cdot EU'(c_{t+1}) = ER_k(w_{t+1}; k, z) \cdot EU'(c_{t+1}) + \text{Cov}\left(R_k(w_{t+1}; k, z), U'(c_{t+1})\right) \quad (14)$$

Because $U'(c_{t+1})$ is negatively correlated with $R_k(w_{t+1}; k, z)$, the last term on the right-hand-side is negative, and therefore, $1 + r_t < ER_k(w_{t+1}; k, z)$.

Let's compare this to the case in which $z_L = z_H = z$ (no shocks). In this case the covariance term in equation (14) is zero and the marginal returns from the two investments are equal, that is, $1 + r_t = ER_k(w_{t+1}; k, z)$. In this case the environment is similar to Aiyagari (1995). The only difference is that w_{t+1} is deterministic in our framework. However, even if w_{t+1} is stochastic at the individual level, the condition $1 + r_t = ER_k(w_{t+1}; k, z)$ still holds. Because the equilibrium interest rate r_t is smaller than the intertemporal discount rate, the model with only earnings risks generates an over-accumulation of capital.

With investment risks, the result that the interest rate is smaller than the intertemporal discount rate still holds. However, the marginal return on capital is not necessarily smaller than the intertemporal discount rate and there could be an under-accumulation of capital. This result will be shown numerically in Section 3.

2.3 Optimal contract economy

One of the complication in solving problem (3) is that the unknown function V_t enters the constraints of the problem. It is then convenient to study the dual problem which minimizes the cost of providing a certain level of utility to the agent.

Denote by v_t the lifetime utility of the agent and by $A_t(v_t)$ the cost for the intermediary. This is defined as:

$$A_t(v) = \min_{c,k,v(z_i)} \left\{ c + k + \delta_t \sum_i \left[-w_{t+1} - R(w_{t+1}; k, z_i) + A_{t+1}(v(z_i)) \right] p(z_i) \right\} \quad (15)$$

subject to

$$v = U(c) + \beta \sum_i v(z_i) p(z_i) \quad (16)$$

$$U(c) + \beta \sum_i v(z_i) p(z_i) \geq U(c + \alpha k) + \beta v(z_L) \quad (17)$$

$$v(z_i) \geq \underline{v}_{t+1}, \quad \text{for } i = L, H \quad (18)$$

Equation (16) is the promise-keeping constraint, equation (17) is the incentive-compatibility constraint and equation (18) imposes limited liability. The lower bound \underline{v}_{t+1} is the equivalent of \underline{a}_{t+1} imposed in the original problem.

This can be interpreted as the problem solved by a financial intermediary that enters into a long-term contractual relation with the agent. If we can show that the long-term contract is equivalent to a sequence of short-term contracts, we can claim that the solution of the dual problem is equivalent to the solution of the original problem.

There are two main difficulties with the dual problem. The first difficulty derives from the fact that the constraint set is not convex. Consequently, we cannot prove that the problem is concave and use first order conditions to characterize the solution. Therefore, in solving the problem we use a direct optimization technique described in the Appendix.

The second difficulty is to show that the optimal long-term contract is free from renegotiation and can be implemented with a sequence of short-term contracts. As shown in Fudenberg, Holmstrom, & Milgrom (1990), if the utility frontier is downward sloping, the long-term contract is free from renegotiation and can be implemented with a sequence of short-term contracts. In our model, the utility frontier is represented by the negative of the function $A_t(v)$. Therefore, it is enough to show that $-A_t(v)$ is not increasing for all $v < \underline{v}_t$. In Section 3 we will show this result numerically for the particular parameterizations of the model considered in this paper.

Once we have (numerically) established that the solution of the dual problem (15) is equivalent to the solution of the original problem (3), we can

easily see the correspondence between the two problems. More specifically, the cost value $A_t(v)$ is equal to the net worth a in the original problem. Likewise, the agent's value $V_t(a)$ in the original problem corresponds to the agent's promised utility v in the dual problem. Therefore, $a = A_t(v)$ and $v = V_t(a)$. Finally, the lower bound \underline{v}_{t+1} is such that $A(\underline{v}_{t+1}) = 0$. This guarantees that the limited liability constraint $a(z_i) \geq 0$ is satisfied in the original problem.

3 Numerical analysis

The goal of this section is to show numerically the macroeconomic and welfare implications of market incompleteness. Although the analysis is not aimed at matching specific observations, nevertheless it provides important information about the potential magnitude of these implications.

Parameterization: We assign the following parameter values. The period in the model is one year and the intertemporal discount rate is $\beta = 0.95$. The risk aversion parameter is $\sigma = 1.5$.

We assume that the shock affects the efficiency units of capital. More specifically, if the investment at time t is k_t , the efficiency units of capital at the beginning of the next period (before choosing labor) is $\tilde{k}_{t+1} = z_{t+1}k_t$. The total resources returned by the risky technology is:

$$F(k_t, l_{t+1}, z_{t+1}) = \tilde{k}_{t+1} + (\tilde{k}_{t+1}^\epsilon l_{t+1}^{1-\epsilon})^\theta$$

The first component is non-depreciated capital and the second component is the output produced. After setting $z_L = 0.5$ and $z_H = 1.0$, the probability of the low shock is chosen to have an expected depreciation rate of 8 percent, that is, $p(z_L) \cdot z_L + (1 - p(z_L)) \cdot z_H = 0.92$. This implies that, with 16 percent probability, capital depreciates by 50 percent and with 84 percent probability there is no depreciation. A sensitivity analysis will be conducted by changing the value of z_L (keeping the average depreciation rate constant). The return-to-scale parameter is set to $\theta = 0.95$ and the parameter $\epsilon = 0.35$. This implies a labor income share of 60 percent. Finally we set $\alpha = 0.2$. This value guarantees that diversion is always inefficient. We will also conduct a sensitivity analysis with respect to this parameter. Table 1 reports the full set of parameter values for the baseline economy.

Table 1: Parameter values for the baseline economy.

Discount rate	β	0.95
Risk-aversion	σ	1.50
	θ	0.95
	ϵ	0.35
Technology $zk + [(zk)^\epsilon l^{1-\epsilon}]^\theta$	z_L	0.50
	z_H	1.00
	p_L	0.16
	α	0.20

Steady state properties: Figure 1 plots several variables for an individual household in the steady state equilibrium, for the Bond Economy (left panels) and for the Optimal Contract Economy (right panels). The top panels plot the household's value as a function of assets, that is, the function $V(a)$. In the case of optimal contracts, this is the inverse of the function $A(v)$ derived from solving the dual problem. Because $V(a)$ is monotonically increasing, the function $A(v)$ is also increasing. This guarantees that the long-term contract is free from renegotiation and can be implemented as a sequence of short-term contracts. Therefore, the solution of problem (15) is equivalent to the solution of the original problem (3).

The other panels plot the investment in the risky technology, k , the investment in the state-contingent asset, $b(z)$, and the next period wealth $a(z)$. In the Bond Economy there are no state contingent assets and b represents the investment in the riskless asset or bond. In both economies the next period wealth depends on the realization of the shock. It is interesting to observe that state-contingent contracts reduce significantly the volatility of assets, and therefore, the risk of investing in the risky technology (see the last two panels of Figure 1). This explains why the availability of these contracts can have substantial macroeconomic and welfare consequences.

Table 2 reports the steady state interest rate, aggregate capital and the concentration of wealth as measured by the Gini index. In the Complete Markets Economy the interest rate is equal to the intertemporal discount rate and the stock of capital (normalized to 1) satisfies $ER_k(w; k, z) = 1/\beta$. In the two versions of market incompleteness, instead, the interest rate is smaller than the intertemporal discount rate. This is not surprising given the results of Huggett (1993) and Aiyagari (1994). What differs here is that the

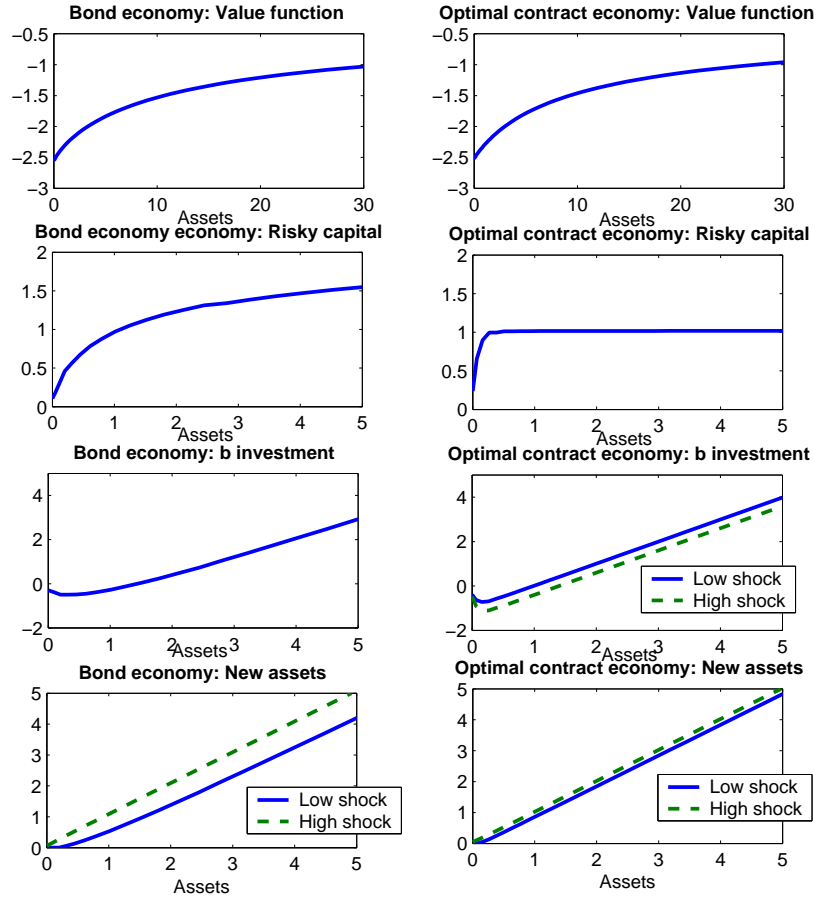


Figure 1: Value function and policy rules in the Bond Economy and in the Optimal Contract Economy.

aggregate stock of capital is smaller than in the Complete Markets Economy. In other words, market incompleteness may lead to under-accumulation of capital. This is the direct consequence of the fact that the accumulation of real capital is risky and agents require a premium.

Table 2 also shows that the availability of state-contingent contracts brings the steady state level of capital very close to the complete markets level. It is also interesting to observe that the availability of state-contingent contracts reduces the inequality in the distribution of wealth but only slightly. The Gini index for wealth is relatively small relative to the data. This is because shocks are i.i.d. and there is no other sources of heterogeneity. If we

Table 2: Steady state interest rate, capital stock, and wealth inequality for different degrees of market completeness.

	Interest rate	Aggregate capital	Gini index
Bond Economy	4.22	0.911	43.8
Optimal Contract Economy	5.21	0.995	42.4
Complete Markets Economy	5.26	1.000	

assume that only a sub-group of agents have access to the risky technology—as we will do in the next section—the model will generate a much higher concentration of wealth. Notice that in the Complete Markets Economy the distribution of wealth is not determined. In other words, any distribution of wealth is a steady state equilibrium as long as in aggregate there is the same (steady state) level of capital. See Chatterjee (1994) for a proof of this result.

Institutional reforms and welfare: The steady state comparisons conducted above show that market incompleteness may have substantial macroeconomic consequences in the absence of state-contingent contracts. We now study the welfare implications. We will ask the following question: Assuming the existence of institutions that make the use of state-contingent contracts feasible, what are the welfare consequences of introducing such institutions?

Figure 2 plots the transition dynamics for the interest rate, the wage rate, the aggregate stock of capital and the Gini index. After the introduction of state-contingent contracts, the interest rate increases sharply and then it converges gradually to the new steady state level. This is because the introduction of state-contingent contracts increases the demand of capital immediately, while the supply responds only gradually through capital accumulation. As shown in panel (c), the aggregate stock of capital converges to a higher level but only gradually. As capital increases, the demand of labor also increases and to clear the labor market the wage rate must rise (see panel (b)). The increase in the wage rate reduces profits, and therefore, the propensity to invest in the risky technology. This effect, however, does not totally compensate the higher incentive induced by better insurance possibilities provided by state-contingent contracts. Finally, panel (d) shows that

the introduction of state-contingent contracts reduces the concentration of wealth as measured by the Gini index.

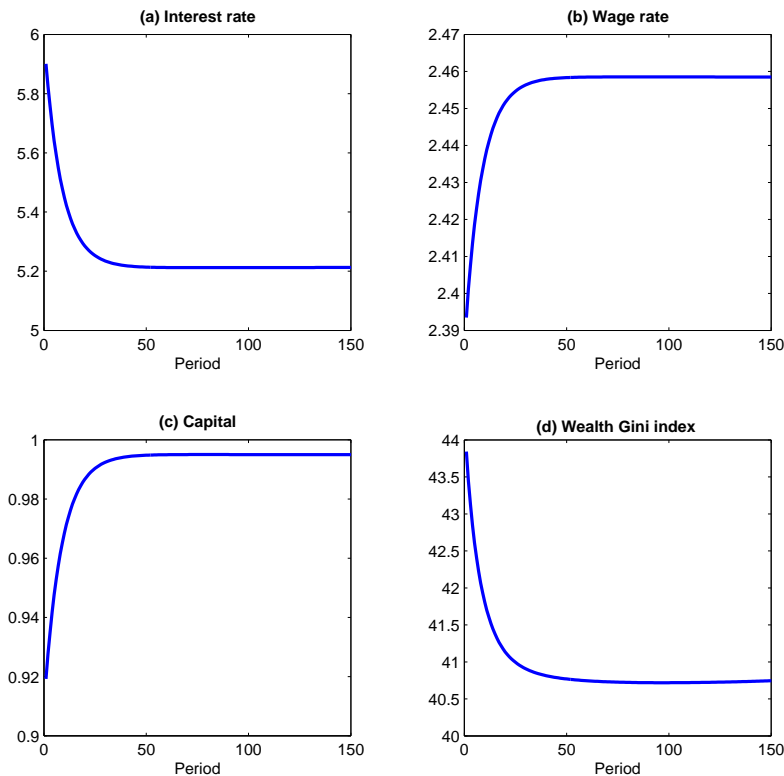


Figure 2: Transition to the steady state with state-contingent contracts.

The welfare consequences are calculated as the aggregate additional consumption (appropriately distributed among agents) required to make all agents indifferent between remaining with the existing institutions (and being unable to use state-contingent contracts) and undertaking a transition to the new steady state equilibrium after the introduction of the new institutions (and having access to state contingent contracts).

Let $V^{Bond}(a) = E \sum_{t=0}^{\infty} \beta^t U(c_t^{Bond})$ be the expected lifetime utility of an agent with net worth a that lives in the steady state of the Bond Economy. The distribution of agents over a is denoted by $M(a)$. Moreover, define by $V^{OptCon}(a) = E \sum_{t=0}^{\infty} \beta^t U(c_t^{OptCon})$ the expected lifetime utility of an agent with net worth a after the introduction of state-contingent contracts (and therefore, after undertaking the transition to the new steady state). The

consumption gain from transition for an agent with net worth a is denoted by $g(a)$. This is determined by the following condition:

$$V^{OptCon}(a) = E \sum_{t=0}^{\infty} \beta^t U(c_t^{Bond} \cdot (1 + g(a))) = (1 + g(a))^{1-\sigma} \cdot V^{Bond}(a)$$

In other words, the consumption gain is determined by equalizing the lifetime utility reached in the transition with the lifetime utility obtained by increasing the consumption in the Bond Economy by $c_t^{Bond} g(a)$ for all t .

The aggregate consumption gains are given by:

$$\text{Gains} = \frac{\int_a c^{Bond}(1 + g(a))M(da)}{\int_a c^{Bond}M(da)} - 1$$

For the baseline parameterization the average gains are 2.32 percent of aggregate consumption.

Although the average gains are positive, these gains are not uniformly distributed across agents. The top panel of Figure 3 plots the welfare gains as a function of the initial wealth. It is interesting to note that the gains are larger for (initially) wealthier agents. For example, an agent with the average wealth would gain less than 2 percent. For an agent with 10 times the average wealth, the welfare gains are 12 percent. The bottom panel plots the initial and final distribution of agents over assets. This informs us about the relative importance of poorer agents (who do not gain much from the transition) and wealthier agents (who are the largest beneficiaries).

The distribution of the welfare gains can be explained as follows. After the introduction of state-contingent contracts, the aggregate demand of capital increases. Because the supply responds slowly, the interest rate increases (see the first panel of Figure 2). The increase in the interest rate is beneficial for the holders of wealth, that is the richest agents. For the poorer agents, instead, the increase in the interest rate represents an increase in the cost of financing because they are net borrowers. We may have expected that the relaxation of financial constraints are more beneficial for agents with tighter constraints, that is, the poor. This would have been the case if the interest rate had remained constant. However, due to general equilibrium effects, the interest rate does increase, and this benefits those who receive interest payments, that is, the rich.

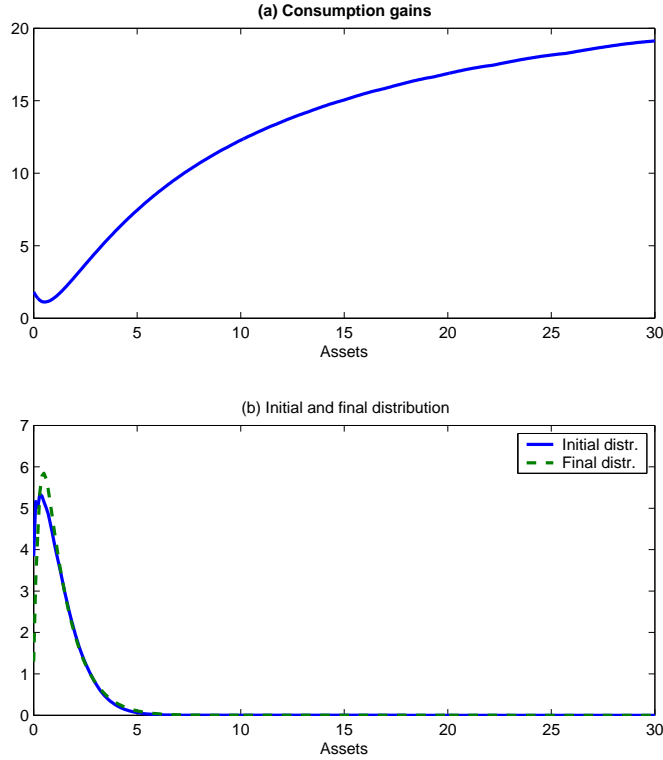


Figure 3: Distribution of the welfare gains following the introduction of state-contingent contracts.

Sensitivity analysis: We close this section by conducting a sensitivity analysis with respect to some key parameters. In particular, the utility parameter for diversion, α , the concavity of the production function, θ , and the volatility of the shock, $z_H - z_L$. Key statistics for the steady state equilibrium and the welfare gains from the transition are reported in Table 3.

First, we observe that the higher utility from diversion does not affect significantly our results. A similar conclusion seems to hold per the curvature of the production function. Now the Gini index for the Bond economy is smaller but the difference is not large. The volatility of the shock, instead, seems to play an important role. The increase in the volatility has significant macroeconomic consequences when state-contingent contracts are not available. For example, the aggregate stock of capital drops by 8 percent when the low realization of the shock changes from 0.5 to 0.25. The drop

Table 3: Sensitivity analysis: Steady state values and welfare gains from transition.

	Interest rate	Aggregate capital	Gini index	Welfare gains
Baseline, $\alpha = 0.2$, $\theta = 0.95$, $z_L = 0.5$				
Bond Economy	4.22	0.911	43.8	2.32
Optimal Contract Economy	5.21	0.995	42.4	
Complete Markets Economy	5.26	1.000		
Higher utility from diversion, $\alpha = 0.3$				
Bond Economy	4.22	0.911	43.8	2.19
Optimal Contract Economy	5.18	0.992	41.1	
Complete Markets Economy	5.26	1.000		
Higher curvature of production, $\theta = 0.915$				
Bond Economy	4.19	0.911	38.8	2.25
Optimal Contract Economy	5.21	0.994	41.7	
Complete Markets Economy	5.26	1.000		
Higher volatility of shocks, $z_L = 0.25$				
Bond Economy	2.96	0.832	48.3	4.73
Optimal Contract Economy	5.23	0.997	42.1	
Complete Markets Economy	5.26	1.000		

in the risk-free interest rate is also large. However, the availability of state-contingent contracts still brings the aggregate stock of capital very close to the complete markets level. As a result, the introduction of state-contingent contracts leads to much larger welfare gains, almost 5 percent.

4 Extensions of the model

The model studied in the previous sections is very stylized. For example, we have assumed that all agents in the economy have access to the risky investment. It seems more reasonable to think that only a sub-group of households have access to this investment. Also, we have assumed that agents do not face any earnings risks. Another assumption is that labor supply is fixed while in an actual economy it may respond to wages. Finally, we have

assumed that the input of labor is chosen after the observation of the shock. The goal of this section is to extend the previous model by considering these alternative assumptions.

4.1 Only a fraction of the population has access to the risky technology

One possible interpretation of the risky investment is that it captures the risk associated with entrepreneurial activities. Therefore, we can assume that the households that have access to this type of investment are the ones engaged in entrepreneurial activities and/or high managerial positions. If we adopt this interpretation, then about 10 percent of households are in the position of investing in the risky technology (see Quadrini (1999)). We will refer to these households as “entrepreneurs” and to the others as “workers”.

In this economy, entrepreneurs solve the same problem we have studied earlier. Workers, instead, solve a simpler problem. Because workers face no risk, the consumption path can be easily determined using the Euler equation, $U'(c_t) \leq \beta(1 + r_t)U'(c_{t+1})$, the budget constraint, $a_t = c_t + \delta_t b_t$, and the law of motion for wealth, $a_{t+1} = w_{t+1} + b_t$. The Euler equation is satisfied with the inequality sign if $a_{t+1} = 0$, that is, if the borrowing limit is binding. In the steady state the interest rate is smaller than the intertemporal discount rate and the liability constraint will be binding, that is, $a_t = 0$ for all t . The level of consumption is then equal to $c_t = \delta w$, where δ and w are constant in a steady state.

Table 4: Steady state values and welfare gains from transition when 10 percent of the population has access to the risky investment.

	Interest rate	Aggregate capital	Gini index	Welfare gains
Bond Economy	1.84	0.873	95.1	5.81
Optimal Contract Economy	5.24	0.993	94.9	
Complete Markets Economy	5.26	1.000		

The basic results do not change by assuming that only a fraction of the population has access to the risky technology. In particular, the aggregate

stock of capital is still smaller than in the Complete Markets Economy. Furthermore, the availability of optimal contracts brings the aggregate stock of capital close to the complete markets level. The most notable change is the increase in the Gini index. This is because only a small fraction of agents (the entrepreneurs) save. Although the model is stylized, this shows how entrepreneurial activities can generate a much larger concentration of wealth. Also significant is the increase in the welfare gains from the introduction of state-contingent contracts. These larger gains come from the increase in the wage rate. Because 90 percent of the population are workers with low level of consumption, the increase in the wage rate, and therefore consumption, has an important impact in their utilities.

4.2 Agents also face earnings risks

Would the result change if agents also face idiosyncratic risks to earnings as in the Bewley economy? To investigate this question we now assume that agents have different earnings abilities which we denote by ε . Individual labor income is then the product of the earnings ability with the wage rate, that is, εw . Earnings abilities follow a two-state Markov process with symmetric transition probability $\Gamma(\varepsilon'/\varepsilon)$.

To keep the problem simple, we assume that earnings abilities are observable. This implies that with optimal contracts the earnings risk is insurable. Therefore, the problem solved in the Optimal Contract Economy is the same problem solved before. In the Bond Economy the optimization problem is also similar. The only difference is that now we take expectations also with respect to the earnings ability ε .

In Table 5 we report the results for the economy with earnings risks where the process for earnings abilities has been calibrated by assuming an autocorrelation of 0.5 and a standard deviation of 0.33. These are the baseline numbers used in Aiyagari (1994).

Even with earnings risks the aggregate stock of capital is smaller than in the Complete Markets Economy. However, we observe that the difference between the two levels of capital is somewhat reduced. This is because now there is an extra incentive to save which reduces the equilibrium interest rate. The lower interest rate then facilitates more investment in the risky technology.

Table 5: Steady state values and welfare gains from transition when agents face earnings risks.

	Interest rate	Aggregate capital	Gini index	Welfare gains
All agents have access to risky investment				
Bond Economy	3.09	0.972	44.5	5.97
Optimal Contract Economy	5.21	0.995	42.4	
Complete Markets Economy	5.26	1.000		
Only 10% have access to risky investment				
Bond Economy	0.01	0.931	88.6	9.33
Optimal Contract Economy	5.24	0.993	94.9	
Complete Markets Economy	5.26	1.000		

4.3 Elastic labor supply

In this section we show how the results would change if labor is elastic. We consider the extreme case in which labor is perfectly elastic. There are two ways to incorporate this in the model. One possibility is to assume that the utility function is of the form $U(c - \varphi \cdot l)$. Alternatively, we could assume that wages are not set competitively and the wage rate is above the market clearing rate with involuntary unemployment. For the calculation of the welfare gains we use the first assumption.

Table 6 reports steady state values for the economy with elastic labor. As can be seen from the table, market incompleteness has a much larger impact on the macroeconomy when labor is elastic. In particular, the aggregate stock of capital is substantially smaller (with and without state-contingent contracts) than in the Complete Markets Economy. This is because, with inelastic supply, the fall in the demand of labor induces a fall in the equilibrium wage rate which in turn increases the return from the risky investment (that is, the expected profit rate increases). This reduces the fall in the demand of risky capital and in equilibrium the capital stock is higher. When the supply is perfectly elastic, instead, the lower demand of labor does not lead to lower wages. Consequently, the fall in investment is bigger.

Figure 4 plots the transition path for several variables when labor is elastic and when it is not elastic. The plots are constructed using the baseline economy in which all agents have access to the risky investment. The case

Table 6: Steady state values and welfare gains from transition when labor is elastic and all agents have access to the risky technology.

	Interest rate	Aggregate capital	Gini index	Welfare gains
Bond Economy	4.57	0.650	42.5	1.37
Optimal Contract Economy	5.21	0.970	42.4	
Complete Markets Economy	5.26	1.000		

in which only a fraction of agents invest in this technology are qualitatively similar (which we omitted for economy of space).

Of course, the assumption that labor is perfectly elastic is an abstraction. With a more reasonable assumption in which the elasticity of labor is positive but not infinity, the effects of market incompleteness on the accumulation of capital are smaller. However, the point we would like to make here is that the elasticity of labor tends to increase the under-accumulation of capital when markets are incomplete.

4.4 The input of labor is chosen in advance

Until this point we have assumed that the input of labor is decided after the observation of the shock. Suppose that both capital and labor have to be decided one period in advance. To simplify the problem, we make some small changes in the specification of the technology. The total resources returned by the risky technology is as follows:

$$F(k_t, l_t, z_{t+1}) = (1 - d)k_t + z_{t+1}(k_t^\epsilon l_t^{1-\epsilon})^\theta$$

The only relevant change to the technology is that the shock only affects output, and therefore, the depreciation of capital is not stochastic. We also modify the benefit from diversion as follows:

$$U(c_t + \alpha y_{t+1})$$

where $y_{t+1} = E z_{t+1} (k_t^\epsilon l_t^{1-\epsilon})^\theta$. With this changes the capital-output ratio chosen by the firm depends only on the wage and interest rates and not on the asset position of the agent. This facilitates the computation of the agent's problem (15).

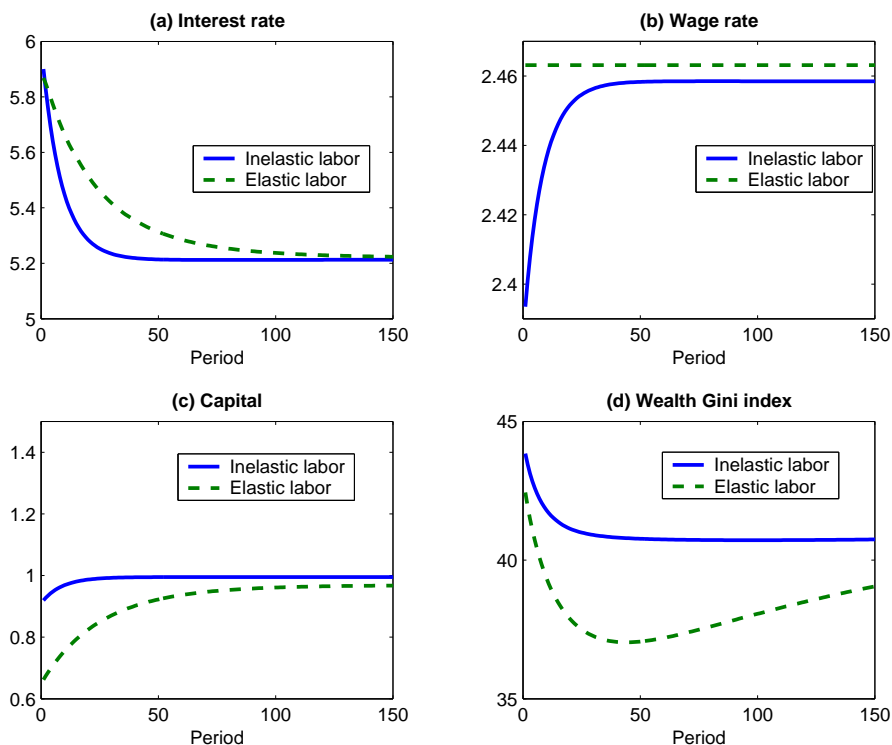


Figure 4: Transition to the Optimal Contract Economy for different degrees of labor elasticity.

Table 7 reports the steady state results for the following parameter values: $\alpha = 1.0$, $z_L = 0$, and $p(z_L) = 0.5$. The most important result is that the aggregate stock of capital is higher than the complete markets level when markets are incomplete (both in the Bond Economy and in the Optimal Contracts Economy). Also notice that the over-accumulation of capital is quite large in the bond economy. The introduction of state-contingent contracts brings it very close to the complete markets level.

The over-accumulation of capital can be explained as follows: Because labor is chosen before the realization of the shock, the employment choice is also risky. In other words, if the agent employs more labor, the return from the risky investment is more volatile. This reduces the demand of labor which, in turn, reduces the wage rate. Because of the lower wage rate, the expected profit per unit of capital is higher. This provides an incentive to invest more in the risky technology. As a result, market incompleteness

Table 7: Steady state values when labor is chosen in advance and all agents have access to the risky technology.

	Interest rate	Aggregate capital	Gini index
Bond Economy	1.70	1.201	63.5
Optimal Contract Economy	4.81	1.010	52.3
Complete Markets Economy	5.26	1.000	

generates over-accumulation of capital as in the model with only earnings risks. Notice, however, that with only earnings risks the wage rate is higher than in the Complete Markets Economy. Here, instead, the wage rate is still lower than in the complete market economy.

5 Conclusion

This paper has studied an economy in which agents have investment opportunities in a risky technology. The consideration of uninsurable investment risks may overturn the previous conclusion that uninsurable risks induce agents to over-accumulate capital. We have shown that with investment risks, the equilibrium stock of capital may be smaller than in the complete markets economy. This may also change some earlier results emphasizing the benefits of long-run capital taxes. In the final section, however, we have also shown that the under-accumulation of capital depends on the assumption that labor is chosen in advance and there is no risk in the employment choice. When labor is chosen in advance, the model with investment risks may generate an over-accumulation of capital that is bigger than in the simpler model with only earnings risks.

We have also compared economies with different degrees of market incompleteness. We have placed particular attention to economies in which state-contingent contracts are available but they cannot provide full insurance due to information asymmetries. Even if agency problems are quite severe in the sense that agents can obtain large gains from diverting resources, the use of state-contingent contracts can lead to an aggregate stock of capital that is very close to the one with complete markets and substantially higher than the capital that would prevail when state-contingent contracts are not

available. We have also seen that institutional reforms that make the use of optimal contracts feasible can have important welfare consequences. The next step, then, is to understand which types of institutional environments facilitate or make possible the use of these contracts. We leave for future research the study of this and other related issues.

Appendix: Computation of the equilibrium

Steady state for the Bond Economy: We start the procedure by guessing the steady state interest and wage rates. Given the prices, we solve problem (8) on a grid of points for the asset holdings a using value function iteration. After guessing the next period values of $V(a)$ at each grid point, we approximate this function with a quadratic polynomial. Given the next period value function, problem (8) is solved at each grid point using a maximizing routine that do not requires smoothness of the value function. We use the Fortran routine BCPOL.

Once the iteration on the value function has converged, we use the agents' policy rules to find the invariant distribution of agents over a . Starting from an initial distribution we iterate until convergence. After aggregating using the invariant distribution, we verify the clearing conditions in the capital and labor markets. Based on these conditions we update the prices and restart the procedure until all markets (labor and capital) clear.

Steady state for the Optimal Contract Economy: The numerical procedure is similar to the procedure used to solve for the steady state of the Bond Economy based on value function iteration. Because we solve for the dual problem (15), the agent's problem is solved at each grid point of v . In forming the grid for v , however, we do not know the lower bound \underline{v} . Therefore, when we guess the prices r and w we also guess the value of \underline{v} , which is the first point of the grid. After solving for the individual problem on all grid points we verify whether $A(\underline{v}) = 0$. If not we update the guess for \underline{v} until this condition is satisfied.

Transition equilibrium: To compute the transition from the steady state of the Bond Economy to the steady state of the Optimal Contracts Economy, we start the procedure by guessing sequences of prices, r and w , and lower bounds \underline{v} for a certain number of periods. The number of periods is sufficiently long for the economy to get close to the new steady state equilibrium. Given the guessed sequences, we solve the agents' problem backward at each grid point starting from the final transition period. In the final period the economy is supposed to have converged to the new steady state, and therefore, we already know the solution. Once we have solved for all transition periods, we start from the initial period and compute the market clearing

conditions and the condition $A_t(\underline{v}_t) = 0$. We then update the guessed sequences and continue until all the equilibrium conditions are satisfied in all transition periods.

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