Can Social Security be welfare improving when there is demographic uncertainty?

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Abstract

This paper studies the welfare implications of a PAYG pension system in a neoclassical growth model with overlapping generations, demographic uncertainty and sequentially incomplete markets. In absence of public pensions, small cohorts tend to be favored by the changes in relative prices implied by demographic shocks. As described in Bohn (1999), PAYG Define Benefit systems can help to share the financial risks created by demographic uncertainty across the generations. The overall welfare impact depends on the balance between this insurance effect and the well known crowding-out effect stemming from the unfunded character of the system. Therefore, the question about the total welfare impact of PAYG pensions is intrinsically quantitative. In this paper we use a four-periods OLG model calibrated to the US economy to provide a first quantitative assessment of the relative size of the different effects involved. The findings are unfavorable for PAYG pension systems: the size of the crowding-out effect is large enough to offset the benefits from risk sharing, making the introduction of public pensions a welfare decreasing process (even in ex-ante terms). In particular, with a marginal PAYG pension scheme (providing a 2% replacement rate of the average wage) small cohorts lose the equivalent to a 1.9% of their consumption in the age interval 20/40, while larger cohorts loss is 1.5%. This figures depends on the particular calibration employed, but the quantitative result is very robust.

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1 Introduction

In the last two decades, developed countries have seen growing concerns about the financial sustainability of their Pay As You Go (PAYG) pension systems. The view that most OECD countries' pension schemes are out of budget has become predominant among both practitioners and professional economists.[2] Two different demographic processes are commonly held responsible for the problems ahead: a universal trend towards higher longevity and a mixture of cyclical and long run changes in fertility. The impact of the former will be most apparent in the medium/long run, while the latter will become a reality much earlier: as soon as the first cohorts of baby boomers start leaving the labor force and begin to collect their pension benefits. This gloomy prospect has triggered a very lively public debate, with radical proposals frequently hitting the front pages of newspapers. In these circumstances economist have been keen to explore the *pros* and *cons* of alternative pension arrangements, making social security and demography move to the top of their research agenda.

The search for a welfare improving social security

One very active line of research has focused on assessing the overall welfare impact of PAYG public pensions, in a context of stationary demography.[3] The starting points for this literature are the classic deterministic life-cycle models of Diamond (1965) and Auerbach and Kotlikoff (1987). The only role played by PAYG pension schemes in this type of environment is to crowd out private savings and to distort labor supply. Therefore, there is no scope for a welfare improving PAYG system in such environments, with the possible exception of dynamically inefficient economies.[4]

Things may be different in stochastic economies as social security can substitute for some missing or imperfect private insurance markets. This was first claimed in Diamond (1977), but it has not been until recently (with the development of truly quantitative general equilibrium models) that the trade-off "distortions vs. insurance" has been properly measured. Idiosyncratic risks were explored first, with negative results. In an early contribution, Hubbard and Judd (1987) find that, missing private annuity markets, the insurance of life uncertainty provided by public pensions does not offset the damaging effects of pension contributions with borrowing constraints. Transitory income shocks were added to life uncertainty in later contributions. Imrohoroglu et al (1995) and Storesletten et al (1998) compare the steady states of economies with a without PAYG systems, while Huang et al (1997) study the transition path to a privatized system. In all cases, the cost of the crowding-out of private savings dominate the benefits from insurance in dynamically efficient economies.[5]

In presence of aggregate risk, PAYG transfers may have a positive intergenerational risk sharing effect. Krueger and Kubler (2003b) explore this possibility in a standard neoclassical economy with productivity and depreciation shocks and complete absence of contingent markets.[6] When aggregate shocks are imperfectly correlated, PAYG pensions are a useful diversification device, as they effectively extend labor income into retirement. However, and once again, this positive effect

[2] See Casey at al (2003) for an updated evaluation of future fiscal pressures stemming from population aging in OECD countries. For the US, Kotlikoff (2003) offers the following "menu of pain" to eliminate the expected fiscal gap: either increase income taxes by 60 %, or payroll taxes by 95% or cut federal discretionary spending by 106% or Social Security benefits by 45%.

[3] The literature related to social security is very large by now. In this brief review we only focus on those lines of research more directly related to our target in this paper.

[4] We do not review here dynastic economies or economies populated with altruistic agents. In the former, PAYG transfers are neutral while in the latter the extension of the crowding-out effect is significantly lessened (Fuster et al (2003)). In general, it is easier to find a positive role for Social Security in those worlds, but the empirical evidence supporting a bequest motive for savings is controversial.

[5] In contrast, it is easy to find a positive *ex ante* role for Social Security in economies with permanent differences in individual productivity. See Miles and Sefton (2002) for an example.

[6] Theoretical results rule out any positive role for intergenerational transfers in presence of sufficiently rich financial markets. In particular, when markets are complete, the existence of an asset paying a non-negligible income at each state of the world is enough to guarantee the pareto (*interim*) efficiency of the private solution.

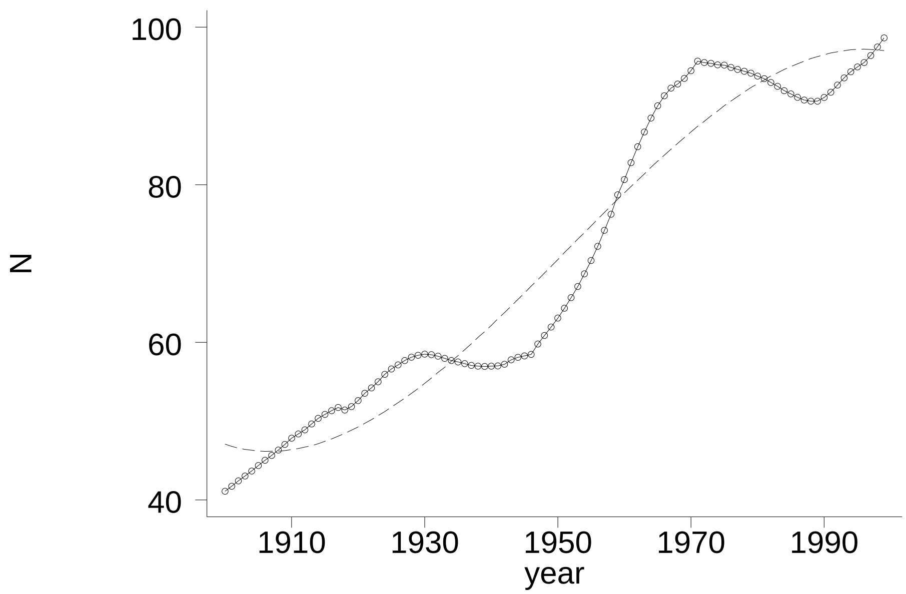

Figure 1: US population aged 25 or below. Source: US census Bureau

do not overcome the cost of crowding-out (for empirically feasible degrees of risk aversion). This quite important result implies that rational individuals would prefer to live in a purely private economy (despise the threat to their retirement savings posed by stock market crashes) rather than facing the low living standards implied by PAYG pensions.[7]

Krueger and Kubler (2003b)'s result leaves little room for any welfare improving role for social security based on aggregate uncertainty. However, one important aspect has been omitted in the previous literature: demographic uncertainty itself has played no part whatsoever in the analysis.[8] All papers mentioned above assume stationary, deterministic demographic patterns. This a bit puzzling, as demographic changes are the very reason for the renewed interest in social security, but quite understandable in the light of the computational difficulties involved in dealing with demographic uncertainty. We examine this in more detail in the next subsection, where we review the empirical evidence supporting the treatment of fertility changes as demographic shocks.

Demographic shocks

Almost 100 million people of less that 25 years of age were living in America in 2000 (see figure 1). At the beginning of the century, the same figure was barely over 40 million, implying that the size of the incoming generations has increased at a considerable 0.9% annual cumulative growth

[7] This result applies to all cohorts after the introduction of the PAYG system. The impact for the cohort going into retirement when the system is first established is different, as their members receive pensions without having made previous contributions, and so normally benefit from the public scheme. The impossibility of capturing this effect is an important drawback of any *steady state* analysis of Social Security.

[8] Auerbach and Kotlikoff (1987) and Huang et al (1997) consider non stationary but deterministic demographic patterns. A pioneer work including stochastic fertility is Rios-Rull (2001) study of savings in Spain, but his analysis does not take into account social security.

rate. However, it would be very misleading to take this average figure as a proxy of the actual evolution of the cohort's growth rate. As the chart 1 makes clear, the actual rate has been anything but constant. On one hand, there is clear evidence of the well know process of "Demographic transition", the long run tendency towards lower fertility and smaller population growth rates. On the other hand, the picture highlights the occurrence of sharp changes in the fertility rates at regular time intervals.

Over the period 1900-1920 the annualized growth rate was 1.2 %, well above the global average. In contrast, between 1920 and 1940 the annualized growth rate was 0.4 %, falling wide short of the long run tendency. After the Second World War the population growth catched up with the long run trend and eventually passed it by, in what has come to be known as the post-war Baby Boom. Overall, the growth rate in the interval 1940-1960 was an spectacular 2.0 %. Not surprisingly, this process reversed again in the following years, displaying this time a mere 0.29% rate in the last two decades of the century. In the light of this behavior, it makes perfect sense to talk about a demographic cycle, made up of a trend plus fertility shocks, that reveals itself in generational frequencies of around 20 years.

Including these shocks in general equilibrium models is very computationally demanding, as they dramatically increase the size of the state vector of the model economy. The whole population distribution by age is needed to compute aggregates and prices at any point in time. This is over an above the usual state variables in these type of models (asset holdings by age). This convoluted situation renders the numerical solution of the problem impossible with the standard computational techniques, in all but the simplest cases.

Bohn (1999) explores one of the few tractable cases: a two periods OLG model. He explores the optimal *dynamic response* to a demographic shock by analytically solving a log-linear approximation of the equilibrium conditions. His findings represent a watershed in this literature: defined-benefits social security systems are more efficient ex-ante than defined-contribution or privatized systems. The argument goes that small cohorts enjoy favorable movements in wages and interest rates. When they enter the labor force, their small size pushes wages up and interest rates down. This is convenient for them, but harming for the cohorts of retirees, whose income depends on the assets return. As individuals do not know *ex ante* if they are going to belong to a large or small cohort, and they can not insurance against this risk in private markets, there is room for public transfer schemes to improve upon the purely private solution. In particular, when a PAYG-DB pension scheme is in operation, the relative-price effects are mitigated: small cohorts have to face larger contributions, while the cohorts of retirees also participate on the wage increases via higher pensions. In this way, this particular type of pension scheme may achieve a more balanced distribution of the burden implied by demographic shocks.

We find this result insightful, but it cannot stand on the same foot as the well established results we reviewed above for two reasons. First, Bohn method do not account for the crowding-out effect. Whether this newly found insurance effect can really lead to welfare improvements is a quantitative question that remains to be explored. Secondly, although Bohn backs the result with some numerical examples, it is not clear yet if the result will extend to models with a larger number of periods[9]. This is so for a number of reasons. In models with larger time-disaggregation, big and small cohorts can overlap in the labor force, with the result of smaller fluctuations in the labor input. Individuals are simultaneously wage earners and capital earners during large parts of their life-cycle, smoothing the effects of changes in relative prices.[10] Shorter periods allow individuals to adapt their behavior according with the state of the economy more frequently along their life cycles. All these elements call into question the quantitative importance of Bohn's result.

[9]A period in Bohn's model stands for roughly 40 years of calendar time. This render any quantitative analysis of fertility shocks meaningless, as this type of shocks only reveal themselves in shorter generational periods. Actually, a 40-years moving average of the US population under 25 is indistinguishable from the long term tendency of the series.

[10]This can be appreciated in Brooks (2002) analysis of the effects of the baby boom on asset prices and the equity premium. In this model workers can short-sell bonds when young. This imply that asset returns do not consistently move against large cohorts, as they can benefit from the risk premium on capital.

Factor prices and demographic changes: empirical evidence.

In all the discussion above, factor prices play a key role in the transmission of demographic shocks. Therefore, it is in order to review the empirical evidence on the association between the population age structure, asset returns and wages. Unfortunately, the studies available so far have come to quite inconclusive results. The basic difficulty is apparent: the low frequency variation in the population age structure implies that each generational observation can demand gathering 20 years of data. As emphasized in Bohn (1999) and Brooks (2002), this situation results in a very small number of generational degrees of freedom for time series analysis. In any case, the general findings are not globally inconsistent with the assumptions in OLG models.

The evidence about the impact of demographic changes on wages is moderately favorable, albeit somewhat indirect. There is a large literature reporting changes in relative wages induced by the arrival of the large cohorts of Baby-Boomers (eg. Welch (1979), Murphy and Welch (1992) or Katz and Murphy (1992)). These analysis confirm the coincidence of drops in new entrants' earnings relative to more experienced workers with the arrival of the peak-sized cohorts spawned by the post-war baby boom.[11] The empirical elasticities seems to be roughly compatible with those normally implemented in the calibration of OLG papers. Less closely related are the cross-country comparisons in per capita income frequently found in the growth literature (eg Barro (1996)). They also point to a negative relation between population growth and wages.

Poterba (2001) is the most relevant reference for the links between asset prices and demographics. He runs a number of regressions using annual and five-year-grouped historical data, to uncover an erratic pattern of correlations.[12] This may reflect (apart from the arguments previously mentioned) that the size of the effects is small in relation to other shocks to asset markets or, perhaps, a small elasticity associated with rather small depreciation rates (Bohn (1999)). He rejects the *asset prices meltdown* conjecture after performing some projection experiments using empirical life-cycle asset profiles. His conclusions, however, are seriously challenged in Abel (2001).

Our target and our contributions

We have seen in the previous sections that PAYG Define Benefit systems can help to share the financial risks created by demographic uncertainty across the generations. Our target in this paper is to assess the quantitative importance of this insurance role of the public pension. In particular, we want to ascertain if that insurance role can compensate for the crowding-out effect of private savings, and so provide an efficient justification for PAYG pension schemes.

To find an answer to this question we proceed as follows. We first illustrate the potentially beneficial role of PAYG pensions when fertility is stochastic in a extremely stylized world: a two-period exchange economy. Section 2 introduces the model and discusses the results. We then move on to our basic model: a OLG economy with a standard neoclassical production function. The model is described in section 3, calibrated to the US economy in section 4, and solved in section 5, where we present the basic findings of the paper. We close the paper with some preliminary conclusions, outlined in section 6. In the remainder of this section we briefly summarize our main findings.

Our quantitative analysis show that:

- It is easy to construct 2-period exchange economies where a PAYG-DB pension system is welfare improving in an *ex ante* way. Although this result can not be extended to all economies in this general class, it nonetheless implies that there is room for a potential welfare improving role for this particular type of pensions, based on its risk-sharing properties.

[11] There is some disagreement about the persistence in the wage differentials. In general, the findings in this labor economics literature points to a more complex model specification: that where workers of different ages are imperfect substitutes in the production function, leading to the existence of different wages depending on the individuals age. This specification will reinforce the positive insurance role of PAYG pensions.

[12] Some strong correlations do show up, specially between Treasury bills and Long-term government bond returns and the share of the population in the "prime saving years" (40 to 64).

- In a 4-period, carefully calibrated production model we find a significant positive effect for the insurance role of PAYG pensions. However, this effect is not strong enough to compensate for the crowding out effect. Overall, the inclusion of a marginal Social Security results in welfare losses for everybody in the economy.

- We test the robustness of this finding to several specifications of the individuals preferences and the persistence of the demographic shocks.

2 The insurance role of PAYG pensions: a minimum model

In this section we present an example of an economy where PAYG pensions have a welfare improving effect: a two-period, exchange economy. This framework is simpler than that used in Bohn (1999), allowing for a remarkably clear presentation of the insurance role of PAYG against demographic shocks.

A two-period, exchange economy

We consider an extremely stylized one-good, close economy. At any point in time, individuals of two possible ages coexists. Young individuals are granted an endowment of the consumption good, w, while nothing becomes available for old individuals. The only way young people can transfer some consumption to the second age is by holding a certain amount of a financial asset, which we assume is in a fixed offer of one unit per period.[13] Young people can then consume part of their endowment (c_1^t) and save the rest by purchasing some of the asset (s^t) at the price p^t from the cohort born in the immediately preceding period. The proceeds from that sale are, in turn, the old-cohort consumption c_2^t. The only source of uncertainty at any point in time is the size of the future generations, $N_1^l \quad l > t$, which we assume follows a 2-state Markov chain ($N_1^t \in \{N_L, N_S\}$), with transition matrix Π:

$$\Pi = \begin{bmatrix} \pi_1 & 1-\pi_1 \\ 1-\pi_2 & \pi_2 \end{bmatrix} \qquad \pi_i = Prob\left[N^{t+1} = i \middle| \, N^t = i\right] \quad i = \{1, 2\}$$

N_L stands for a large cohort, while N_S represents a comparatively smaller cohort.

The structure of the model is completed with a system of pension transfers. Under this program, the old citizens receive a fixed pension b (which for simplicity we assume a fixed proportion θ of the annual endowment w), financed from a proportional payroll contribution ς^t, levied on the young people's endowment. The system is self-balanced on a year-by-year basis by adjusting the contribution rate, ie:

$$N_1^t \varsigma^t \, w = N_2^t \, b$$

Individuals' behavior is represented with a standard life-cycle optimization problem:

$$\max_{s^t} \quad u(c_1^t) + \beta E[\, u(c_2^{t+1})) \, | N_t \,]$$

$$c_1^t + p^t s^t = w\,(1 - \varsigma^t)$$
$$c_2^{t+1} = p^{t+1} s^t + b$$

where u stands for a period utility function with the usual properties.

The equilibrium of this economy is a set of time series for consumption, taxes and prices such that individuals solve the problem above, the pension system is balanced and the asset market clears:

$$N_1^t \, s^t = 1 \tag{1}$$

[13] This is a standard exchange economy. It can be trivially re-interpreted as a production economy with a constant returns to scale technology and where young individuals are endowed with a fixed amount of productive time.

The remarkable aspect of this model is that optimal saving behavior is completely determined by the asset market equilibrium condition (1). This make it possible to use the first order condition (foc) of the agent problem as an asset-pricing device. The downside of the model is the lack of close form solution in all but the simplest preference specification (logarithm utility). This leads to the customary dependence on numerical methods in order to find solutions to particular model economies. Hence, a recursive definition of the equilibrium is more fruitful than the traditional version above. In this really simple model, a recursive equilibrium is just the 3-tuple of asset price and consumption breakdown in every posible combination of current and 1-period lagged shocks (denoted by N_1 and N_2 respectively):

$$\{p, c_1, c_2\}(N_1, N_2) \quad N_1, N_2 \in (N_L, N_H)$$

with the same properties as above.[14]

Equilibrium in the purely private economy

We consider first the case when there are no public pension transfers. We illustrate the properties of the solutions by presenting a numerical example and commenting on the extend to which the results apply more broadly. For concreteness, we explore a world where the incoming cohorts can be either *normal-size* ($N_H = 1$) or *small* ($N_L = 0.8$), cohort sizes are iid distributed, individuals maximize a CRRA utility function, with a parameter value of 2 and the endowment w is normalized to 1.

When a a normal-size cohort is followed by a small one, it is immediately clear that some adjustment must occur. This can be trivially seen by checking the shift in the economy's consumption feasibility frontier, the maximum amount of old-age consumption compatible with any amount of young-age consumption. Its explicit equation at any point in time is simply:

$$N_1 c_1 + N_2 c_2 = N_1 \Leftrightarrow c_2 = (N_1/N_2)(1 - c_1) \tag{2}$$

None of pairs (c_1, c_2) which are feasible when a normal-size cohort follows another normal-size one remains so with a reduced number of youngster.[15] One or both cohorts **must** suffer a reduction in their current consumption following a demographic shock (the arrival of a small-size cohort). This result holds with independence of the way the intergenerational transfers are arranged.

Table 1 shows how the burden is distributed among the generations in a private economy: all the burden is placed on the the larger cohort of elderly people. This is the outcome of the interaction of a smaller number of young buyers with a normal-size number of old sellers in the asset market. According to (1), every young must hold 1.25 units of the asset for the market to clear (1.0 with normal population). Asset prices fall to the point that the young buyers are willing to hold that precise amount of assets. This translates directly into a drop in the elderly's consumption, while young's consumption goes up.[16]

Efficient outcome: the planner solution

After exploring the way private markets deal with a demographic shock, it seems quite natural to inquire about the efficiency of what, at first sight, seems to be a rather extreme distribution of the

[14] The equilibrium is implicitly defined by a system of 4 non-linear equations in the 4 contingent prices $p(N_1, N_2)$:

$$p(N_1,N_2)\, u_c\left(\underbrace{p(N_1,N_2)\frac{1}{N_1} + w(1-\varsigma(N_1,N_2))}_{c1((N_1,N_2))}\right) = \beta\, E_{n'}\left[p(N_1',N_1)\, u_c(\underbrace{p(N_1',N_1)\frac{1}{N_1} + b}_{c2(N_1',N_1)})\ |N_1\right]$$

[15] With the exception of the corner solution where the young consume everything.

[16] Young's consumption is constant only in the isoelastic (log) case.

Private solution				
(N_1, N_2)	c_1	c_2	p	
(1,1)	0.5293	0.4707	0.4707	
(0.8,1)	0.5674	0.3461	0.3461	
Efficient solution				
(N_1, N_2)	c_1	c_2	p	
(1,1)	0.5499	0.4501	-	
(0.8,1)	0.5222	0.3823	-	
Marginal PAYG system				
(N_1, N_2)	c_1	c_2	p	ς
(1,1)	0.5313	0.4687	0.4487	0.0200
(0.8,1)	0.5642	0.3486	0.3286	0.0250

Table 1: Allocations and prices in the exchange model.

cost of the shocks. The efficient benchmark can be obtained by maximizing an aggregate welfare function W, given N_2^1 and specific weights Ω_t for all current and future cohorts:

$$W_1(N_2^1) = E_1\left[\Omega_0 \beta u(c_2^1) + \sum_{t=1}^{\infty} \Omega_t U_t\right] \qquad U_t = u(c_1^t) + \beta u(c_2^{t+1})$$

subject to the resource constraint (2) and the markov process for the demographic shocks. The contingent plans that solve this problem provide the allocations that would be chosen by a planner who is willing to exchange risk across the generation.[17] We are particularly interested in the way this planner allocates consumption among coexisting generations. That is controlled by the following first order condition (assuming for simplicity the existence of a planners' discount rate $(1+R) = \Omega_{t-1}/\Omega_t$, and a CES utility function with IES γ):

$$\frac{u'(c_1^t)}{(1+R)\,\beta u'(c_2^t)} = 1 + N_t \equiv \frac{N_1^t}{N_2^t} \Rightarrow \frac{c_1^t}{c_2^t} = \frac{1}{\tilde{\beta}^{\gamma}} \left(\frac{N_2^t}{N_1^t}\right)^{\gamma}$$

Where $\tilde{\beta} = (1+R)\beta$. This equation implies a proportional relation between the efficient consumption levels of both cohorts, though the proportionality ratio depends on the state of nature. Going one step further, it is easy to prove that although the ratio c_1/c_2 should go up when a small cohort arrives, both cohorts should share the costs of the shocks.[18] This means that the senior citizens may have to pay more than their fair share of the cost, but the young should always pay some part of it, and we can then safely conclude that the extreme private assignment is inefficient. Table 1 illustrates how this result emerges in our example economy.

Equilibrium with PAYG transfers

The inefficiency of the private solution stems form the incompleteness of the contingent markets in our model economy. In those circumstances, a system of intergenerational transfers financed on a PAYG basis may improve upon the purely private solution. In order to explore this possibility we numerically solved our simple economy including the pension system described above. We analyze whether there is any scope for such a program by simulating the introduction of a marginal PAYG system, providing the old with a pension equivalent to a 2% of the young's consumption endowment.

[17] We are using the *ex ante* welfare criterium. Planners who treat differently generations born in different states of the world would rely on the (much weaker) *interim* criterium.

[18] The planner's function in Bohn is a special case of this more general form, where the proportionality is constant along all posible states.

	L (1,1)	L(1,0.8)	S (0.8,1)	S(0.8,0.8)	EX ante
Private	-3.5689	-3.5689	-3.1062	-3.1062	-3.3376
PAYG	-3.5578	-3.5526	-3.1183	-3.1119	-3.3352

Table 2: Welfare in the different allocations in the exchange model

Such a system is kept balanced when a small cohort arrives by charging a higher payroll rate ς to the smaller cohorts, as can be appreciated in the bottom rows of table 1. This, in turn, reduces the demand for private savings and, consequently, the equilibrium price of the asset. The effect on the pensioners is, then, double: on one hand, they can enjoy a new and safe source of income. On the other, the selling price of the private asset goes down, and the total disposable income can move in either direction. If the asset price change is "mild", ie if

$$b > \frac{P^t - P^t_s}{N_1^{t-1}} \Leftrightarrow \varsigma^t > \frac{P^t - P^t_s}{N_1^t}$$

then c_2 is larger and c_1 is smaller with Social Security, which implies a consumption distribution closer to the efficient benchmark. A PAYG system, then, operates as an insurance mechanism that makes large cohorts better off at the expense of the smaller ones. This can be appreciated in the table 2, where we can see how the inclusión of a marginal PAYG scheme is welfare improving in an *ex ante* sense, in the proposed example.

Nevertheless, it is possible to find regular economies where asset prices "overreact" an the final distribution of consumption is more extreme than that in the private solution. Our general conclusion is that a welfare improving role for PAYG pensions is possible although not sure, and that the asset price movement are critical for one outcome or the other.

3 Demographic uncertainty in a production economy

Once we have illustrated the insurance effect of PAYG pensions in an exchange economy, we move on to outline a production model where the crowding out of private savings is also active. The model is a straightforward extension of the standard neoclassical OLG model with stochastic demographic (fertility) shocks.

3.1 The model economy

Time is discrete and the economy is populated by overlapping generations of otherwise identical individuals. There is no lifetime uncertainty and every individual's life spans over I periods: first as children (periods 1 to $I_W - 1$), then as workers (periods I_W to $I_R - 1$) and finally as retirees (from I_R to I). We denote calendar time by t and individual age by i. Individual variables are indexed by the age (as a subscript) and by the calendar time (as a superscript). Aggregate variables are only indexed by calendar time. Each period a new generation of individuals is born and, as in the two-period model in section 2, its size N_1^t is uncertain. We assume the cohort growth rate $n_t = N_1^t / N_1^{t-1}$ follows a two-state Markov chain, featuring "high" and "low" growth shocks. Note that changes in fertility, immigration and mortality patterns underlie the evolution of the aggregate population. As we abstract from these differences, shocks in this model are best interpreted as fertility shocks in a close economy. [19] Individuals can not write contracts contingent

[19] A formal treatment of mortality changes is one of our immediate research targets. Dealing with immigration is also posible, but it may demand (given the widespread extension of the process) a proper consideration of the general aging of the global population. As for the lack of overseas capital flows, we see it as a (admittedly quite extreme) representation of current imperfections in global capital markets. They reveal themselves in eg. high correlations between national saving and investment rates (both across countries and within countries over time),

on the realization of the shock, ie. they can not get insured against the risk of being born in a large generation or being followed by a small cohort.

Some notation is useful to ease the description of the model. L_t stands for the total number of workers in the economy, ψ_t for its one-period-ahead rate of growth and ω_i^t is the size of the cohort of age i in t normalized by the size of the working population in t.

3.1.1 Institutional arrangements

The public sector in the model is made up of two institutions. First, as in section 2, there exists a pay as you go social security system that taxes current workers at rate ς_t to pay a pension b_t to those retired. For simplicity, we assume b_t to be a fixed proportion (θ) of current average per worker earnings, and we adjust ς_t period by period to make sure that the system's budget is continually balanced. Second, the public sector taxes personal income (assets returns and labor earnings net of contributions) to finance some consumption expenditures. These public outlays are simply modelled as a constant fraction ν of total aggregate output. The fiscal system is balanced every period through an endogenous tax rate τ_t.

3.1.2 Technology

The production technology of the economy is given by a constant returns to scale, Cobb-Douglas function that uses capital, K, and efficient units of labor, H, to produce the single good in the model:

$$Y_t = K_t^{\alpha} H_t^{1-\alpha}$$

where $\alpha \in (0, 1)$ is the capital's share in output. We assume that labor productivity increases at a constant rate λ, in accordance with a standard process of exogenous, labor-augmenting, technological growth. Aggregate capital depreciates at a constant rate $\delta \in (0, 1)$.

3.1.3 Individual Decision Problem

Individuals born in calendar year u (ie, belonging to cohort u) start taking decisions after joining the labor force at the age of I_W. From this age onwards, individuals choose their optimal amounts of consumption c_i^{u+i-1} and assets a_i^{u+i-1} at every age $i \in \{I_W, I\}$, in an attempt to smooth consumption over the life-cycle and for precautionary reasons. Saving is made possible by owing some of the economy capital stock. We do not restrict individuals to have a positive amount of assets at every age.[20] In contrast, there is no labor supply decision, as workers provide their age-varying endowment of efficient labor units, ε_i, inelastically. The life-cycle problem solved by cohort u is:

$$\max_{\left\{c_i^{u+i-1}\right\}_{i=I_W}^{I}} \quad \sum_{i=I_W}^{I} \beta^{\,i-I_W} \; E_t \; u(c_i^{u+i-1}) \tag{3}$$

where $u(c)$ is a standard period utility function. This optimization must observe the following sequence of budget constrains (t is the calendar time when cohort-u individual is i years old, ie $t = u + i - 1$):

$$c_i^t + a_{i+1}^{t+1} \leq \chi_i^t + a_i^t \left(1 + r_t(1 - \tau_t)\right) \qquad I_W \leq i \leq I \tag{4}$$

where r_t stands for the gross return to capital and χ_i^t is net labor/pension earnings:

massive home bias in equity ownership and very limited consumption smoothing with respect to country-specific output fluctuations.

[20]Note that there is no idiosyncratic income uncertainty in the model. It is also for computational reasons that we allow individuals to borrow against future earnings.

$$\chi_i^t = \begin{cases} w_t(1-\varsigma_t)(1-\tau_t)\,\varepsilon_i & I_W \le i < I_R \\ b^{t+i-1} = \theta\,\bar{w}_t & i \le I_R \end{cases} \tag{5}$$

w_t is the wage per efficient unit of labor and $\bar{w}_t$ is the current average wage: $\bar{w}_t = w_t \sum_{I_W}^{I_R-1} \omega_i^t\,\varepsilon_i$. Note that we abstract from the costs associated with raising children.[21]

3.1.4 Equilibrium

An equilibrium in our model economy is a set of sequences of cohort sizes, allocations, prices and taxes, $\{N_1^t, (c_i^t, a_i^t)_{I_w}^I, (r^t, w^t), (\tau_t, \varsigma_t)\}_{t=t0}^{\infty}$, given initial conditions for the population and assets holdings in $t0$, with the usual properties: individuals solve their life-cycle problem (3)- (5) given prices and taxes, input markets clear period by period, prices are competitive (match factors' marginal productivity), and taxes keep both public schemes balanced. However, and as is customary in stochastic general equilibrium models, this type of definition is not very fruitful when it comes to solving the model. Furthermore, a recursive definition of the economy's equilibrium provides useful insights into the computacional challenges posed by models with stochastic demography. For these reasons we proceed to restate the definition above in the form of a *recursive competitive equilibrium.* We present this recursive version in a ready-to-compute form, ie, corrected for population and productivity growth (aggregate variables are stated in per worker, deflated terms).

At any point in time, the state of the economy is thoroughly determined by the age structure of the population and the asset holdings by age. We derive the former from the history of *discrete* population shocks $\bar{n} = \{n_{-i}\}_{i=0}^{I-2}$, while the latter is obtained from the aggregate capital stock and the vector of its distribution by age $\bar{s} = \{s_i\}_{i=I_w}^{I-1}$. We group these variables together in the state vector $x = (\bar{n}, k, \bar{s})$.[22]

A competitive equilibrium for our model economy is a set of individuals' policy functions $\{a_{i+1}(x)\}_{i=I_w}^{I}$, public policies $(\tau(x), \varsigma(x))$ and prices for capital and labor $r(x)$ and $w(x)$ such that the following properties apply:

- Individuals' policy rules correspond to the optimal behavior at any age $I_w \le i \le I$. We assume individuals maximize an Epstein and Zin (1989) utility function, which allows to separate risk aversion from preferences for inter-temporal substitution. The expected continuation utility at period t of an individual aged i is given by:

$$U_i(c_i, x) = \left[c_i^{\rho} + \beta(1+\lambda)^{\rho} \left(\sum_{x'} \Gamma(x'/x) U_{i+1}^{\sigma}(c_{i+1}, x') \right)^{\frac{\rho}{\sigma}} \right]^{\frac{1}{\rho}} \tag{6}$$

  where the parameters ρ and σ control the intertemporal elasticity of substitution (IES=$1/1-\rho$) and the degree of relative risk aversion (CRRA= $1-\sigma$). The standard CES utility function is a particular case of the Epstein and Zin's utility function when $\rho = \sigma$.

[21] The analysis in Brooks (2002) makes clear that this element is important to assess the relative welfare of the Baby-Boomers and surrounding generations. As far as demographic shocks are negatively (positively) correlated, the inclusion of this type of cost will dampen (reinforce) the welfare impact of changes in factor prices. Empirical correlation is negative, but is hard to infer a reliable value from just 4-5 observations. This leads Brooks to choice an uncorrelated process for the shocks. We plan to include the cost of raising children in future research, but the qualitative findings in this paper will be most likely unchanged with that inclusion.

[22] Note that, as the computable model is deflated of technological and population growth, we do not need to keep the total population as a state variable. This accounts for the missing variable in $\bar{n}$.

The utility function is maximized subject to the individual budget constraint (similar to equations (4) and (5) above):

$$c_i + (1+\lambda)\, a'_{i+1} \leq \chi_i(x) + s_i(x)\, k\,(\, 1 + r(x)(1-\tau(x))\,)$$

$$\chi_i = \begin{cases} w(x)(1-\varsigma(x))(1-\tau(x))\,\varepsilon_i & i < I_R \\ \theta w(x) h(x) & I_R \leq i \leq I \end{cases}$$

h, the deflated per worker labor supply, is formally defined below. The state vector is updated as follows:

$$x' = (\bar{n}',\ k',\ s'_{I_w}, \ldots, s'_I) \quad \bar{n}' = (n', n, n_{-1}, \ldots, n_{I-3}) \quad n' \sim \Pi(n)$$

$$k' = \sum_{i \in [I_W, I_R]} \omega'_i a'_i \qquad s'_i = \omega'_i\, a'_i / k' \qquad I_W \leq i \leq I_R$$

and where the new population weights are computed according with:

$$\omega'_i = \omega_i(\bar{n}') \qquad \omega_{I_W+j}\,(\bar{n}) = \frac{N_{I_W+j}/N_{I_R-1}}{\sum_{j=0}^{I_R-I_W-1} N_{I_W+j}/N_{I_R-1}} \quad 0 \leq j \leq I_R - 1$$

$$\frac{N_{I_W}}{N_{I_W+k}} = \prod_{i=0}^{k-1} n^{-I_W-i} \qquad \forall\ k > 0$$

- Input markets clear

$$h(x) = \sum_{i=I_W}^{I_R-1} \omega_i(x)\, \varepsilon_i \qquad k(x) = \sum_{i=I_W}^{I} \omega_i(x)(1+\lambda) a_i(x)$$

- Competitive factor prices

$$r(x) = f_k(k(x), h(x)) - \delta \tag{7}$$

$$w(x) = f_h(k(x), h(x)) \tag{8}$$

where $f(k,h) = y = k^{\alpha} h^{1-\alpha}$

- Balance in the public sector accounts:

$$b(x) \sum_{i=I_R}^{I} \omega_i = \varsigma(x)\, w(x) \sum_{i=I_W}^{I_R} h_i(x) \tag{9}$$

$$g(x) = \left[\{h(x) w(x)(1-\varsigma(x)) + k(x) r(x)\} \sum_{i=I_W}^{I_R} \omega_i + b(x) \sum_{i=I_R}^{I} \omega_i \right] \tau(x) \tag{10}$$

- Aggregate feasibility constraint

$$y(x) + (1-\delta) k(x) = \sum_{i=I_W}^{I} \omega_i(x) c_i(x) + g(x) + k'(x)(1+\lambda)\, \psi(x)$$

Solving this recursive equilibrium is a rather daunting task, due to the very large size of the state vector. The literature has dealt with this problem in two ways: approximating the mathematical objects involved with smooth low-dimensional parametric functions (typically orthogonal polinomial, as in Krueger and Kubler (2003)) and approximating the state vector itself (following the general method pioneered by Krusell and Smith (1998)). In this paper we have opted for the first approach, implementing the Smolyak algorithm to get multivariate approximations of the saving functions.[23]

4 Calibration

We align our economic model and our target real-world economy (the US economy) by choosing the parameter values in such a way that our simulations display the following set of properties:

1. Mimic the basic stylized facts of the US demographic process 1900/2000.
2. Reproduce the macroeconomic performance of the US economy in the post-war period (according with the disponibility of aggregate information in the NIPA).
3. Ensure that life-cycle profiles of productivity and income are in agreement with the evidence from micro data.

Demography

As we discussed in section 2, the size of the incoming generations (variable N_1^t in our model) has been anything but stable during the last century, with “Boom” periods being followed by population “Busts”. The average annual cohort growth rate n_t has been 0.9%, but actual figures have been consistently above or below this value for prolonged periods of time. This demographic-cycle is well captured by considering periods of 20 years of length, as shown in figure 1 and discussed in section 2. Accordingly, we set the length of a period in our model to that precise figure. Combining this with a life expectancy of 77 years in 2000, it seems reasonable to assume individuals in the model live for 4 periods: an initial childhood period, two periods of active labor force participation (young-old workers) and a final retirement period.

We stick to the standard procedure of modelling the population process as a discrete binomial tree. This is, of course, for computational convenience. We choose the parameters of the tree so as to mimic the empirical mean and variance of the growth rate of our 20-years-long cohorts. Proceeding in this way, we calibrate our population shocks to alternate between a high growth state featuring a 1.5% annual rate, and a low growth state with a 0.15% annual rate. For the persistence of the process, we coincide with Brooks (2002) in that it is very difficult to pin down this number by resorting to standard econometric techniques (due the the very short number of observations available). The alternative solution is to model uncorrelated shocks as our base case, and explore the robustness of our results to this hypothesis.

Life cycle productivity profiles

We calibrate the life-cycle profile of efficiency labor units $\{\varepsilon_1, \varepsilon_2\}$ to reproduce empirical micro-data on the profile of labor earnings by age. Specifically, we reproduce the relative earnings of male

[23] Smolyak algorithm is a high dimensional interpolation method, characterized by the use of (1) low order (orthogonal) polynomials as interpolating functions and (2) a sparse grid (extrema of Chebyshev polynomials) in which to perform the function evaluations. With this grid choice, it is possible to increase the problem's dimension, d, (for a fixed degree of the approximating polynomials, q) with a small increase in computational cost. Furthermore, the grids obtained with higher q (for a fixed d) are nested. All in all, this method provide almost optimal error bounds using a much lower number of points than with tensor products of univariate polynomials. See Bathelmann et al (2000) or Krueger and Kubler (2003) for detailed descriptions of the method.

aged 45-65 to the earnings of those aged 25-44 using data from the Current Population Survey 1974-2000. The older workers group earn 17% more than their younger workmates.

Macroeconomic performance

We follow the standard procedure in the macroeconomic literature (as described in eg. Cooley and Prescott (1995)) to guarantee a proper correspondence between our model economy and the measurements taken for the US economy. This basically amounts to create, from the original NIPA series, a consistent set of measurements for the aggregate capital stock, income from several capital stocks, and aggregate investment and output in the model. Once this has been accomplished (see the details in appendix A), we proceed in the following way:

Technology The capital's share in output, α, is set to its sample average in the 1947-2000 period: 0.36. The annual productivity growth rate λ is set according with the average empirical growth rate of per capita product (2.1%). Finally, for the depreciation rate δ we obtain a value of 4.1% per annum by applying the procedure described in appendix A). All these estimations are roughly in agreement with the standard values used in the literature.

Preferences The degree of constant relative risk aversion and the intertemporal elasticity of substitution are among the hardest parameters to pin down: there is a richness of empirical studies reporting a wide range of possible values. In these circumstances we have opted for setting both parameters to arbitrary values and checking the robustness of our results by performing specific sensitivity analysis. To characterize the risk attitudes we choice a CRRA of 2 for our benchmark case. This is a standard value in the macro literature, consistent with a moderately low degree of risk aversion. For the IES, in contrast, we select a slightly non-standard value: 0.7. This figure is larger than the most widely used estimations, but perfectly compatible with estimations obtained at the individual level: Attanasio and Weber (1999) report values ranging from 0.2 to 0.8. The other critical preference parameter, the discount factor β, is calibrated to reproduce the average empirical interest rate in our calibration interval: 7.6% (once more, see appendix A for an explanation on how we obtain this number). This value is obtained with very patient individuals, discounting the future at an annual 0.2 % rate (the annual β is 0.998).

Public programs The ratio of public consumption expenditures to GDP is directly set to the average empirical value in our consistent measurements from NIPA (18.5%). For the pension system, we target the current value of the pay-roll tax rate (15 %), by fixing the pension replacement rate in 39%. This is consistent with the empirical estimations (see Brooks (2002) and the references therein).

5 Simulation results

We show in section 2 that PAYG pension can improve the *ex ante* welfare through a better allocation of demographic risks across generations. In this section we explore whether that result extends to the more complex and realistic production economy in section 3. The result in the exchange world is challenged in the new setting by a number of aspects: *(i)* First and foremost, the harmful crowding out of private savings induce by PAYG financing in dynamically efficient economies; *(ii)* enlarged time disaggregation, implying reduced fluctuations in the labor force and a mitigation of the welfare impact of factor price changes (as individuals supply both capital an labor at different moments of their life-cycles); *(iii)* the existence of a one-period advance notice of the arrival of the shock, reinforcing individual's ability to self-insure, and finally *(iv)* a more accurate calibration of the length of the working vs retired phases of individual's life-cycle.

5.1 Demographic shocks in the private economy

We start by exploring the effects of demographic shocks in a version of our baseline economy with no PAYG transfers. We refer to this as the *private* economy. Note that all other parameter values are similar to those presented in section 4.

The impact of an isolated shock

We start by discussing the impact of an isolated negative shock: the arrival of a cohort whose size is only slightly larger than that of its predecesor, implying a growth rate well below the average. All immediately preceding and ensuing cohorts expands at the larger rate. This is just to simplify the exposition, as this particular situation allows for a *clean* presentation of the economic forces at work. [24] Tables 5 and 6 containing the most outstanding simulation results are available in appendix B.

A reduction in the population growth rate in t has an impact over the relative scarcity of capital and labor from date $t+1$ onwards. One year after the shock, both *per worker* labor, h, and capital, k, go up. The former effect reflects that older workers, endowed with a larger amount of efficient labor units, become a bigger share of the labor force. The latter reflects the change in population composition and the first behavioral responses. These induced changes in savings are, however, of second order when compared with the population changes (more on savings below). The change experienced by k/h in $t+1$ is, therefore, a quantitative matter, that in our calibrated economy leads to labor becoming relatively more scarce. This results in an increase in salaries and a reduction in the return to saving. These changes go further away in $t+2$, as k reaches a maximum while h goes back to its initial value. From that point on, k progressively returns to the initial levels, driving wages and interest rates back to their starting positions. Note that all changes after $t+2$ are due to induced effects on savings, as changes in k serve as the propagating mechanism of the demographic shock.

These price changes occur at different points in the life-cycle of the different cohorts involved, causing very uneven welfare effects:

- **Cohorts preceding the shock:** Cohort (born in) $t-2$ suffers from the drop in savings returns at $t+1$. This is in spite of the advanced notice of the arrival of the shock, that allow older workers in t to somewhat protect themselves by increasing savings. Cohort $t-1$ also end up suffering badly, as they bear the brunt of the fall in interest rates (r hits a minimum in $t+2$, precisely when this cohort goes into retirement). Note that this cohort benefits from the large wages prevailing in $t+1$, but this in not enough to compensate for the drops in interest rates.

- **Cohort t**: The relatively small cohort born in t, as with cohort $t-1$, experiences both gains and drawbacks from the change in prices. The overall welfare impact is, however, very different from one another: t-cohort's members enjoy a substantial gain in life-cycle utility. This is because this cohort enjoy the wage rises fully, while interest rate drops touch them only marginally: when they go into retirement the *direct* demographic effect is over, and r is quite close to the initial level.

- **Cohorts born after the shock**: Cohort $t+1$ is the last to experience direct effects from the shock, in form of high wages in $t+2$. All other effects along this cohort life-cycle are induced changes, being particularly important the recovery in the interest rates. The reply to this new scenario takes the form of higher savings, which guarantee a progressive convergence of k to its pre-shock values, spreading in the process the welfare gains from the shock to the ensuing cohorts.

[24] In a world without persistence, isolated shocks are rare. Only by running large simulations is possible to find a relatively large sequence of positive shocks, interrupted half way through by a negative one. We have generated our example in this way.

n_t	w_1^{t+1}	w_2^{t+2}	r_2^{t+2}	r_3^{t+3}	EU
1.03	.20510	.20709	2.11942	2.24510	.01837
1.35	.19623	.19402	2.44909	2.30954	.01783

Table 3: Private economy: expected utility and average prices experienced along the life-cycle by cohort size.

Summing up: relatively small cohorts substantially benefit from the price changes induced by their small size. Their salaries are high and only suffer from low returns to savings when moving from young to senior workers. This allow them to save more and transfer some of the benefits to all ensuing cohorts. In contrast, previous cohorts have to deal with low returns at the end of their life-cycle (the moment when they are more dependent upon the performance of their accumulated assets). Even the immediately precedent cohort, which enjoys some wages improvements, can not escape this fate.

The effect of positive shocks (the occurrence of a large population growth amid slowly growing cohorts) have entirely symmetric effects to those describe above. All in all, the results are qualitatively similar to those encountered in the exchange economy. The main differences are (1) a general weakening of the strength of the effects, as cohorts experience opposite forces at different points of their life-cycles, and (2) the extension of the effects to the following cohorts via changes in the capital stock.

General simulation results

The effects described so far correspond to an economy hit by just one shock over a relatively long time interval. In the absence of persistent demographic shocks, this particular outcome of the stochastic process is rare. It is far more common that cohorts are hit by shocks of opposite sign at different stages of their life-cycle. As a result, we still have to check whether the findings in the previous section extend to the general case. Table 3 shows that this is the case. We report there the average life-cycle prices experienced by cohorts born in both higher and lower-than-the-trend growth rates. The table also provides the average life-cycle utility for both groups. We can see that relatively small cohorts enjoy higher salaries all along their working lives. They also face low returns on savings, but this is mainly concentrated at the beginning of their lives, when their dependency of asset income is low. Overall, their utility is clearly larger than that of relatively large cohorts, which suffers low wages and only slightly higher return on their retirement assets.

The general conclusion is pretty much that of the simple exchange economy: in the absent of PAYG transfer, relatively small cohort are better off, although the quantitative significance of the differences is smaller.

5.2 The impact of a marginal PAYG-DB Social Security

We repeat the experiments above in presence of a marginal PAYG pension scheme, providing a 2% replacement rate of the average wage.

The system is self-balance on a period by period basis, which implies that cohorts made up of relatively less people must pay a larger payroll tax to finance a more stable pension benefit (only depending on shocks indirectly, through the induced effects on wages). Table 7 illustrate how this mechanism work with an isolated shock. The privileged cohort in the private economy (the small cohort born in t) is hit by higher contribution rates all along its working career and relatively lower pensions. Cohorts badly battered by the shocks in the private world (those immediately preceding the shock) can now participate in the wage increases induced through higher PAYG pensions. The cohorts following the shock, which also benefit from it, are beaten by the system of public transfers in two ways: directly via higher interest rates (cohort $t + 1$) and indirectly via larger reductions in

n_t	ς_1^{t+1}	ς_2^{t+2}	b^{t+3}	$\bar{\varsigma}^t$	$a_{2\ SS}^{t+2}/a_2^{t+2}$	EV
1.03	.00849	.00895	.00432	.05306	.97320	-.01881
1.35	.00733	.00682	.00427	.04540	.97576	-.01522

Table 4: Economy with a marginal PAYG-DB pension system: basic statistics by cohort size.

the per worker capital k. This can be appreciated by checking the ratio of retirement savings with and without social security. As PAYG pensions crowd-out private savings, this ratio is always less than one. The scope of this effect changes with the cohort in a systematic way: cohorts t and $t+1$ protect themselves from the discriminatory treatment delivered by the pension system by saving relatively less (and so partly offsetting the larger contributions placed on them). This effectively extends the cost of demographic shocks to the ensuing cohorts through changes in k, a feature that must lead to efficiency gains.

None of these effects are strikingly different from what we found in the exchange economy: social security alters the prices in the production economy in roughly the same direction as before. There are, however, some differences. The most important is a less clean redistribution of gains and losses, as a result of the mixing of the effects along the cohorts' life-cycle. Cohort $t-1$ members, for instance, who are relatively better treated by the PAYG system, also suffer from high contribution rates at the point in their career when the time endowment is most productive. We conclude from all this that self-balanced PAYG-DB pensions have a sizable *ex ante* insurance effect: they redistribute the benefits and costs stemming from demographic shocks in a more even way than the market does. Risk averse individuals who does not know which cohort they are going to be born into will prefer to live in a world with this type of insurance mechanism.

However, this is not the end of the story as the crowding-out effect can overcome the benefits from the insurance and render PAYG schemes undesirable.[25] To achieve a comprehensive evaluation of the welfare impact we compute the Equivalent Variation in first period consumption generated by the introduction of the marginal public pension scheme (last column of table 7).[26] The insurance effect is clearly visible with the help of this statistic: the welfare losses inflicted to cohorts $t-2$ and $t-1$ are lower than a 1.3% reduction in first period consumption, while those suffered by cohorts t and $t+1$ are close to a 1.6% reduction. The figures are, however, uniformly negative, meaning that the crowding-out effect is predominant and the overall impact of social security is a welfare reduction.

The results in our full simulation (table 4 presents simulation averages for the key statistics) confirm all the findings in the paragraphs above. A PAYG-DB system forces relatively small cohorts to pay higher contributions and share in this ways the cost of demographic shocks. The impact on pensions tends to be blurred by the mixing of the shocks, but there is no doubt about the overall impact of the public transfer system. This can be seen in the larger *effective* contribution rates, $\bar{\varsigma}^t$, suffered by the small cohort.[27]. This insurance effect is, however, not enough to counteract the negative consequences of the crowding-out of private savings. Small cohorts lose the equivalent

[25]The crowding-out effect can improve welfare in dynamically inefficient economies (eg Diamond (1977), Imrohoroglu et al (1995), Krueger and Kubler (2003b)) This is not the case in our economy. The implicit annual return from PAYG pension, $n+\lambda$, is 3.55% with a large shock and 2.2% with a small one. The corresponding figures for the net interest rates are 6.3 and 5.94%.

[26]We compute the Equivalent Variation as the change in first period consumption (measure as a percentage) that is needed for the economy with social security to display the same utility level observed in the purely private case. We have also computed an *adjusted* Equivalent Variation, where the percentage change is measured with respect to the steady state first period consumption (an so avoiding the ambiguity derived from the existence of two first period consumptions in the simulations). This refinement does not alter the results, and so is not reported in the text.

[27]The effective contribution rate is the change in life-cycle wealth (express as a proportion of net labor income when young workers) induced by the introduction of the pension system. Its analytical expression is (denoting by

of a 1.5% of their first period consumption with the introduction of PAYG pensions, while larger cohorts loss amounts to a 1.9%.

5.3 Sensitivity analysis

The negative welfare impact of PAYG-DB pensions seems to be very robust to the consideration of individuals with alternative preferences. In principle, individuals with higher degrees of risk aversion or individuals with a larger intertemporal elasticities of substitution could benefit more from Social Security, and so perhaps overturn our previous findings. A few numerical simulations clearly show that that is not the case. When the economy is populated by individuals with a CRRA value of 8 we find that the overall assessment of social security is barely altered. The consideration of more elastic individuals has a larger impact (the equivalent variations are -1.48 and -1.2% for *large* and *small* cohorts respectively), but the qualitative result still holds. We have also checked the robustness of the results to the degree of persistence in the fertility shocks, again without finding any significant variation.

6 Concusions

In a world where the size of the incoming cohorts fluctuates along a long run trend, factor prices can experience significant cyclical variation. In a purely private economy this price movements systematically favor larger-size cohorts at the expense of smaller-size cohorts. Define-Benefit, Pay As You Go pension systems (adjusting the contribution rate on a year by year basis) counteracts these swings in factor prices by charging higher effective contributions on smaller cohorts. From an *ex ante* perspective, then, the existence of a PAYG-DB pension system implies an insurance mechanism against aggregate demographic risk. This opens the door for a welfare improving role for Social Security. In this paper we quantitatively explore the trade off between this insurance effect and the classical crowding out effect of private savings induced by the PAYG financing of the system. Our findings are undoubtedly negative. The welfare benefits created by the insurance role are overwhelmingly dominated by the reductions in per capita income generated by the crowding out effect. For a marginal pension system (with a replacement rate of 2% of the average wage) the welfare losses generated by the public pension scheme are estimated to around 1.9% of young-workers consumption for the small cohorts and 1.5% for large cohorts. This finding seems robust to alternative specifications of individual preferences.

There are a number of issues that we have not taken into account in the present analysis. In particular, we have abstracted from the costs associated with raising children. This could either dampen or reinforce the welfare impact of changes in factor prices depending on whether demographic shocks are negatively or positively correlated. Empirical correlation is negative, but it is hard to infer a reliable value from just a handful of generational observations. Brooks (2002) makes clear that this element is important to assess the relative welfare of the Baby-Boomers and surrounding generations, but we doubt that the qualitative findings in this paper will be seriously altered by that inclusion. It is, in any case, one potential way to improve the current model specification. Another significant extension could be to deal with the uncertainty surrounding the length of the human life. This omission seems important in our quest for an efficient role for PAYG pensions, for a number of reasons. First, public pensions help to cope with lifespan risk at the individual level by substituting for imperfect or missing private annuity markets. Second, there is substantial uncertainty about the speed and scope of longevity increases at the aggregate level. This is a type of risk that cannot be insured in private markets but that can be alleviated

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cot_i^{t+i} the contributions made at age i by a member of cohort t) :

$$\bar{\varsigma}^t = [\, cot_1^{t+1} + cot_2^{t+2}/(1+r^{t+2}) - b^{t+3}/(1+r^{t+2})(1+r^{t+3}) \,]/\chi_1^{t+1}$$

by the social security system. Finally, including this aspect could help to achieve a more realistic representation of the demographic process. All in all, this extension seems to be a quite promising avenue for future work.

References

Abel A. Will bequest attenuate the predicted meltdown in stock prices when baby boomers retire? *The Review of Economic and Statistics*. November 2001, 83(4):589-595.

Attanasio O. and G. Weber. Is Consumption Growth Consistent with Intertemporal Optimization? Evidence for the Consumer Expenditure Survey. *Journal of Political Economy* 1995. p.1121-1157.

Auerbach A. J. and L J. Kotlikoff. Dynamic Fiscal Policy, Cambridge, UK: Cambridge Univ. Press. 1987.

Barthelmann V., E Novak and K Ritter. High Dimensional Polynomial Interpolation on Sparse Grids. *Advances in Computational Mathematics* 12. 273-288.

R. J. Barro Determinants of Economic Growth: A Cross-Country Empirical Study. *NBER* Working Paper No. w5698, 1996.

Bohn H. Social Security and Demographic Uncertainty: the Risk Sharing Properties of Alternative Policies. NBER WP 7030. March 1999.

Bohn H. Should Social Security Trust Fund Hold Equities? An Intergenerational Welfare Analyisis. *Review of Economic Dynamics* 1999. p. 666-697.

Bohn H. Social Security and Demographic Uncertainty: the Risk Sharing Properties of Alternative Policies. Chaper 6 in *Risk Aspects of Investment Based Social Security Reform* . J. Campbell and M Feldstein edts. The University of Chicago Press, 2001

Brooks R. Asset-Market Effects of the Baby Boom and Social Security Reform. *American Economic Review* 2002. p. 403-406.

Caseay B, H Oxley, E Whitehouse, P Antolin, R Duval and W Leibfritz. Policies for an ageing society: recent measures and areas for further reform. *Economics department working papers* No 369.

Cooley T. and E. Prescott. Chapter 1 in *Frontiers of Business Cycle Research* T. Cooley editor, Princeton University Press. p. 1-38.

Diamond P. National Debt in a Neoclassical Growth Model. *American Economic Review* 1965. p.1127-1155.

Diamond P. A Framework for Social Security Analysis. *Journal of Public Economics* 1977. p. 275-298.

Epstein L. and S. Zin. Substitution, Risk Aversion and the Temporal Behavior of Aset Returns: A Theoretical Framework. *Econometric*a 1989. p. 937-969.

Fuster L., A. Imrohoroglu and S. Imrohoroglu. A Welfare Analysis of Social Security in a Dynastic Framework. *International Economic Review* 2003. p.1247-1274.

Huang H., S. Imrohoroglu and T.J. Sargent. Two Computations to Fund Social Security. Macroeconomic Dynamics 1997. p.7-44.

Hubbard G. and K L Judd. Social Security and Individual Welfare: Precautionary Saving, Borrowing Constraints, and the Payroll Tax. *The American Economic Review* vol 77, Sept 1987, pag 630-646

Imrohoroglu A., S. Imrohoroglu and D. H. Joines. A Life Cycle Analysis of Social Security. Economic Theory (1995) p.83-114.

Katz L and K Murphy. Changes in Relative Wages, 1963-1987: Supply and Demand Factors. *Quarterly Journal of Economics* 107 (February 1992): 35- 78.

Kotlikoff L. Fixing Social Security and Medicare for Good. Paper contributed to the 2003 Improving Social Insurance Programs conference held at U. of Maryland.

Krueger D. and F. Kubler. Computing Equilibrium in OLG Models with Stochastic Production. 2003. Forhtcoming in *Journal of Economic Dynamics and Control*.

Krueger D. and F. Kubler. Pareto Improving Social Security Reform when Financial Markets are Incomplete. 2003 Unpublish paper.

Krusell P. and A. Smith. Income and Wealth Heterogeneity in the Macroeconomy. *Journal of Political Economy*, vol 106 (1998) no 5.

Miles D. and J. Sefton. Optimal Social Security Design. CEPR Discussion Paper 2002.

Murphy K. and F. Welch. The Structure of Wages *Quarterly Journal of Economics* n 107 (February 1992) 285-326

Poterba, J. Demographic structure and assets returns. *The Review of Economic and Statistics*. November 2001, 83(4):565-584

Rios-Rull J.V. Population Changes and Capital Accumulation: The Aging of the Baby Boom. 2001.

Storesletten K, C.I. Telmer and A. Yaron. The Risk-sahring Implications of Alternative Social Security Arrangements. Carnegie-Rochester Conference Series on Public Policy 1999.

Welch F. Effects of Cohort Size on Earnings: The Baby Boom Babies's Financial Bus. *Journal of Political Economy*. n 85 (1979) p. 65-97

7 Appendix

A Consistent measurements in the calibration process.

Original NIPA series (we use tables 1.1.5; 1.12; 1.7.5 and 5.7.5B, and Fixed assets table 1.1) have to be modified in order to generate a set of measurements that can meaningfully compare to the variables in the model. We proceed as follows:

1. Construct a proper series of income on fixed private capital (some arbitrary decision is needed to assign the proprietors' income and the difference between national product and national income in the NIPA series).

2. Determine the return on fixed capital (i) from the general equation

$$IC^j = (i + \delta^j)\, K^j \tag{11}$$

   where IC^j stands for income from stock j. Note that NIPA provides an estimation of the fixed capital depreciation.

3. Estimate the depreciation for other stocks not explicitly included in the model (consumer durables). This is achieved by using a steady-state version of the law of motion of any capital stock: $(1 + n)(1 + \lambda)k = x + (1 - \delta)k$. Note that data on the value of the stock and on the value of investments (x) are available in NIPA.

4. Equipped with the depreciation and stock series, use equation in point 2 to compute the income from the other capital stocks.

5. Capital in the model includes household capital (consumer durables and residential structures) and inventories, over and above the stock of fixed private capital. Consequently, income from capital must account for the durable service flows computed above. Output series, in turn, must include an imputation of the flow of services from consumer durables. With this corrections is possible to compute a consistent measure for the capital's share in income, α.

6. An estimation of the return on capital can be obtained by using equation (11) and the Cobb-Douglas assumption:

$$(r + \delta)\, k = \alpha\, y \Rightarrow r = \alpha\, y/k - \delta$$

   Note that the value of δ should be computed as in the point 3 above.

B Simulation results

We reproduce in this section some detailed simulation results.

time	n_t	k	h	k/h	r	w
t-2	1.35	.036583	1.07234	.0341151	2.567659	.189688
t-1	1.35	.035992	1.07234	.033564	2.600392	.18858
t	1.03	.035218	1.07234	.0328422	2.644717	.187108
t+1	1.35	.040058	1.083744	.0369626	2.411197	.195243
t+2	1.35	.041884	1.07234	.0390585	2.308178	.199157
t+3	1.35	.036331	1.07234	.0338801	2.581507	.189217

Table 5: An isolated demographic shock in the private economy: Time series

time	n_t	w_1^{t+1}	w_2^{t+2}	r_2^{t+2}	r_3^{t+3}	a_1^{t+1}	a_2^{t+2}	EU
t-3	1.35	.189688	.18858	2.600392	2.644717	.028844	.072737	.0176452
t-2	1.35	.18858	.187108	2.644717	2.411197	.028882	.073498	.0175806
t-1	1.35	.187108	.195243	2.411197	2.308178	.026876	.072769	.0172029
t	1.03	.195243	.199157	2.308178	2.581507	.027777	.072872	.0179961
t+1	1.35	.199157	.189217	2.581507	2.605963	.031398	.075054	.018152
t+2	1.35	.189217	.188393	2.605963	2.404243	.028754	.073485	.0176192
t+3	1.35	.188393	.195501	2.404243	2.084603	.027183	.073888	.01727

Table 6: An isolated demographic shock in the private economy: Prices, decisions and welfare by cohort. Time= year of birth

time	n_t	ς_1^{t+1}	ς_2^{t+2}	b^{t+3}	$a_{2\ SS}^{t+2}/a_2^{t+2}$	EV
t-3	1.35	.006304	.006304	.003983	.9771553	-.0138422
t-2	1.35	.006304	.006304	.004199	.977948	-.0130724
t-1	1.35	.006304	.007298	.004237	.9774279	-.0122978
t	1.03	.007298	.008263	.004024	.9759787	-.0157345
t+1	1.35	.008263	.006304	.004009	.9763284	-.0158717
t+2	1.35	.006304	.006304	.004204	.9772964	-.013693

Table 7: An isolated demographic shock with marginal PAYG scheme: contribution rates ς, pensions, ratio of retirement savings with and without pensions and equivalent variation in welfare by cohort. Time= year of birth