The Manufacturing Flexibility to Switch Products: Valuation and Optimal Strategy

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Abstract

This paper applies a dynamic programming methodology to the valuation problem for the flexibility to switch. In our model, flexibility provides an investor with the right, or option, to perform a switch between a less profitable and a more profitable project at no cost. In contrast to previous analyses, the option to switch can be exercised in the future at any time during the decision horizon. We present the solution methodology that allows to determine the value of the flexibility and to identify the optimal timing of the switching decision. Comparative statics demonstrate how changes in the input parameters affect the values of the problem's solution. The results partially explain why investing in flexible manufacturing systems is reported to have both low profitability and rate of diffusion.

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Introduction

       Many manufacturing firms are investing today in flexible manufacturing systems (FMS) in an attempt to improve their responsiveness to unforeseen changes in product markets and manufacturing technology[1]. FMS are designed to provide their adopters with the capability to meet the ever-increasing market demand for product variety, improved product quality, shorter delivery times, faster product innovation and higher delivery reliability. Moreover, this improved market performance can be achieved at reduced costs of operations, with shorter processing, set-up, and manufacturing lead times as well as increased machine utilization. Despite numerous potential advantages of FMS, their achievement in practice have been impeded by a number of factors, including difficulties with the financial appraisal of a new technology in the capital budgeting process, the technical complexity of FMS and changes in the organization's structure that are required for their successful implementation (Boer et al. [1989]). These factors have contributed to the relatively low profitability of FMS - as expressed by the traditional financial methods - and consequently, have led to a lower rate of diffusion of these systems as compared to other industrial innovations (Mansfield[1993]).

       One of the major requirements for the increased adoption of FMS is the development of financial methods that would allow one to adequately evaluate the benefits of these systems during the capital budgeting analysis, and thus demonstrate their superiority over investments in less costly, dedicated technologies. There exists a growing consensus that traditional capital budgeting methods, such as net present value techniques, are not appropriate for analyzing investments in FMS (Kaplan [1986], Kulatilaka [1988]). In particular, these traditional approaches are not suited for capturing the value of manufacturing flexibility, which is considered to be a major strategic benefit of FMS. Although the importance of flexibility has been found to gain widespread acceptance (DeMeyer et al. [1987]), attempts to develop valid and reliable measures of flexibility have incurred a number of obstacles. Flexibility measures are needed to advance both theoretical and applied research on manufacturing flexibility. In particular, flexibility measures will help managers justify investments in flexible technology and determine the

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[1] See for example the Wall Street Journal from September 15, 1999 article about FMS at Honda.

performance levels of their firms. Consequently, operationalizing flexibility is considered today the most important priority in research on manufacturing flexibility (Gerwin [1993]).

Flexibility is widely recognized as one of the key components of a successful manufacturing strategy and defined as a capability of a firm to quickly and economically respond to various types of environmental uncertainty (Chung and Chen[1990]). In many situations, the investment in flexibility is equivalent to "banking" flexibility that is storing it for its future use in changing environments (Gerwin[1993]). In this sense, flexibility is a source of real options that a firm may choose to exercise in the future. These real options are typically provided by the means of various flexibility facets or dimensions such as product flexibility (the capability of a production system to quickly introduce new products), mix flexibility (the ease with which the firm offers different combinations of multiple products), volume flexibility (the capability of a system to operate economically at different aggregate production volumes), and process flexibility (the ability to produce the same set of products using different processes and materials). The literature on manufacturing flexibility provides numerous examples illustrating how firms can use these flexibility dimensions, either defensively to adapt to sudden changes in market conditions, or proactively to redefine competitive conditions (see Chen et al. [1992] and Gerwin [1993]).

The aforementioned flexibility dimensions provide specific examples of the flexibility or option to switch, which enables production systems to switch between alternative modes of operation in response to changing market conditions. The valuation of benefits resulting from investing in the flexibility to switch has been recently addressed, in a real option framework, with mathematical tools such as dynamic programming (Kulatilaka [1988], Kulatilaka and Trigeorgis [1994]) and contingent claims analysis (Triantis and Hodder [1990], Tannous [1990]). Most of these analyses compare investments in flexible technologies, allowing for switching between alternative modes of operation and inflexible or rigid technologies representing irreversible commitment to only one of the operating modes. The decision horizon has typically a finite length of N periods and switching decisions can be made only at preset and fixed points in time (typically, at the beginning of each period). Operating in a given mode results in a stream of cash flows to the firm, contingent upon the realization of uncertainty modeled as a stochastic process. A typical framework models one source of uncertainty (such a price, demand, exchange rate) which affects all operating modes.

Decision to switch to one of the alternative modes involves trading off the costs of switching and the expected profits incurred as a result of switching decisions. Under these assumptions, the value of flexibility to switch is defined as a difference between the expected profits from investing in flexible rather then inflexible technology.

The above framework appears first in Kulatilaka[1988], who develops a stochastic dynamic program to compute the value added to the firm as a result of the investment in flexibility to switch. The set of operating modes includes alternative modes of production, waiting to invest, temporarily shutting down a plant, and abandoning the production. Kulatilaka and Trigeorgis[1994] consider the value of flexibility by comparing the values of flexible and rigid technologies, where flexible technology allows for switching between two mutually exclusive projects. The valuation of flexibility is conducted in the absence as well as in the presence of (asymmetric) switching costs. Triantis and Hodder[1990] apply the option pricing methodology to value the capability of a production facility to offer different combinations of multiple products. The switching decisions involve adjustments in product production rates subject to the capacity of the facility constraint. Tannous[1996] applies contingent claims analysis to quantify the benefits of volume flexibility and develops a model which can be used in determining the optimal level of investment in volume flexibility. Similar analyses are recently applied to value the flexibility of multinational production networks in Kogut and Kulatilaka[1994] and Huchzermeier and Cohen[1996].

The critical mass achieved by these theoretical investigations made possible the transmission of knowledge to practitioners. However, for a successful implementation of these calculations in practice simpler methods should be investigated. Recently Copeland and Antikarov [2001] discuss a few simple solutions to the valuation of the flexibility of switching technologies in their real option book written for practitioners.

In line with the objective of making simpler solutions to complicated problems available to practitioners this paper develops a simple real option framework for the measurement of the value of flexibility to switch between alternative projects. The value of flexibility is derived in the context of a firm facing the choice between the two investment opportunities referred to as rigid and flexible scenarios. In the former, the firm invests in one of the two available projects at the beginning of the planning horizon without the right to reverse its investment decision later, whereas in the latter the firm additionally acquires the option to switch between the alternative

projects in the future. Assuming the knowledge of the stochastic evolutions of the projects, we provide a mathematical formulation to the real option valuation problem using a dynamic programming approach. The solution methodology allows one to value the flexibility to switch and provides a decision rule indicating the critical ratio of the projects' values at which the timing of the switch becomes optimal.

This study differs from the existing research on the measurement of flexibility to switch in that it allows the switching between the projects to take place at any point in time during the planning horizon. In effect, the real option to switch corresponds to an American-style call option rather then a European-style call, as in the previous analyses. This assumption makes the analysis more complex and implies the need for an application of an analytic approximation to the option-valuation problem. The solution methodology applied in this paper is based on the approximation proposed in Barone-Adesi and Whaley[1987] for American options written on commodities. We extend their methodology to solve the differential equation for the value of the option to switch, contingent upon the evolution of two state variables representing the stochastic evolutions of the projects' values. Our model is similar to previous analyses in that it assumes two operating modes (Kulatilaka and Trigeorgis [1994]), irreversibility of the switching decision and the absence of switching costs (Triantis and Hodder [1991]). Despite these restrictions, our model can be considered general enough to assess the value of various types of flexibility options, including various dimensions of manufacturing flexibility.

The rest of the paper is organized as follows: Section 2 develops the theoretical framework for analyzing the option value of flexibility to switch and outlines the solution methodology. Section 3 presents the sensitivity analysis. Conclusions are given in the last section.

The flexibility to switch: the theoretical framework

Consider a firm facing a decision to invest in one of the two mutually exclusive projects, say A and B, at the beginning of the time horizon [0,T]. Each project results in a different stream of cash flows to the firm. We assume, as customary, that the present value of the future cash flows for each project follows a geometric Brownian motion stochastic process. Under these assumptions, we consider two investment scenarios that can be adopted by the firm. One situation, called a "rigid scenario," is characterized by the absence of flexibility to switch

between alternative projects during the time horizon [0, T]. Under this scenario, the firm invests at time zero in the more profitable project. This means that based on the information about the project values available at that time the project with the highest net present value (NPV) is undertaken. In other words, the firm invests in project B rather then project A if NPV(B) > NPV(A). Moreover, once the investment decision is made, it cannot be reversed later. Under the second scenario, called further a "flexible scenario," the firm additionally acquires at time zero the flexibility or an option to switch between alternative projects, which may be exercised any time during the time interval [0, T]. This flexibility enables the firm to invest initially in a more profitable project (say project B) and to receive additionally the right to the difference of present values PV(A) – PV(B) at the time it chooses to exercise the option to switch from project B to project A. The foregoing analysis assumes that the option to switch can be exercised only once (in particular, the reverse switch is not permitted) and involves no switching costs.

The firm facing the choice between the rigid and flexible investment scenarios selects the one with the higher profitability at time zero (i.e. higher NPV). If project B is selected at time zero, the investment in the rigid scenario brings to the firm a value equal to PV(B) which is acquired at the initial investment cost I. On the other hand, the flexible scenario brings the value of PV(B) + V(A, B, 0), where V(A, B, t) denotes the value of the flexibility to switch operation from B to A at time t. The initial investment required under this scenario is I + C(A, B), where C(A, B) denotes the incremental cost required to invest in flexibility to switch from B to A. Obviously, the firm chooses the flexible scenario only if the value of the flexibility V(A, B, 0) exceeds the extra cost C(A, B) needed to acquire it. The symmetric argument holds for the valuation of the option to switch from project A to project B.

The formulation of the dynamic program for V(A, B, t) provided below assumes knowledge of the stochastic processes followed by the two project values, PV(A) and PV(B). In general, one could identify a number of sources of uncertainty affecting the PV of each project (stochastic cashflows, discount rates, etc.), and possibly incorporate the impact of the stochastic evolutions of these variables on the two project values. In our analysis, we choose to consider that the impact of these uncertainties can be collapsed into processes representing geometric Brownian motion specifications(we simplify the notation by using symbols A and B to represent PV(A) and PV(B), respectively). Thus:

$$dA = A\alpha dt + A\sigma_A dz_A \qquad dB = B\beta dt + B\sigma_B dz_B \qquad [1]$$

where α, β represent the growth rates, σ_A, σ_B are the instantaneous variances, and dz_A, dz_B are Wiener processes for projects A and B, respectively. The uncertainties in the stochastic processes A and B are correlated, with the coefficient of correlation ρ_{AB}.

Now, the valuation problem can be formulated as follows. At time t, 0<t<T, the firm that adopts the flexible scenario attempts to maximize its profits by choosing between exercising the right to switch between projects B and A or postponing the decision until later to obtain more information about the evolutions of the projects. Assuming that the next decision instant is at time t + dt, the optimal switching strategy becomes:

$$V(A,B,t) = MAX \left\{ A_t - B_t, E_t(V(A + dA, B + dB, t + dt)\exp(-\gamma\, dt)) \right\} \qquad [2]$$

Equ.[2] expresses the value of the option to switch between projects B and A at time t. It indicates that, if the switch is exercised immediately, the option is worth the difference between the PVs of projects A and B. If the decision is postponed until time t + dt, the option is worth the expectation of its future value discounted to time t at a discount rate γ. The expectation is computed based on the information at time t. The maximization reflects the fact the firm makes its choice optimally bearing in mind not only the immediate payout (such as a positive A_t-B_t) but also the consequences of the future evolutions of projects A and B. Equ.[2] also applies to the value of the option to switch at the beginning of the decision interval, V(A,B,0). This value represents the maximum price that the firm is willing to pay for the flexibility given by the right to switch between alternative projects.

Equ. [2] is known as the Bellman equation (see, for example, Oksendal [1991]) and represents the dynamic programming problem in continuous time. Assuming that it is not optimal to exercise the option at time t but rather postpone the decision until time t + dt, the following successive steps modify the right-hand side of equ.[2]. First, apply Taylor's theorem to expand the term $\exp(-\gamma dt)$. Second, replace the term V(At+dA,Bt+dB,t+dt) by its equivalent $V(A_t,B_t,t) + dV(A_t,B_t,t)$ and apply Itô's lemma for two variables (see,for example, Ingersoll [1987]) to expand differential dV. Next, apply the expectancy operator to the expanded expression, keeping in mind that $E(dz_A) = E(dz_b) = 0$ and $E(dz_A dz_B)=\rho_{AB}\, dt$. These steps yield the following expression for the

value of the option (time subscripts are dropped for convenience of notation):

$$V = \{\frac{1}{2}\sigma_A^2 A^2 V_{AA} + \frac{1}{2}\sigma_B^2 B^2 V_{BB} + \rho_{AB}\sigma_A\sigma_B ABV_{AB} + \alpha_{PV}V_{PV} + \beta BV_B + V_t\}dt + (1-\gamma dt)V + o(dt)$$

[3]

with o(dt) representing the terms that go to zero faster then dt as dt -> 0. Dividing by dt and proceeding to limit as dt->0, we get the second-order partial differential equation for the value of the option to switch:

$$\frac{1}{2}\sigma_A^2 A^2 V_{AA} + \frac{1}{2}\sigma_B^2 B^2 V_{BB} + \rho_{AB}\sigma_A\sigma_B ABV_{AB} + \alpha AV_A + \beta BV_B + V_t - \gamma V = 0$$

[4]

 To specify the value of the option in the situation when it becomes optimal to exercise the switch at time t, we add the following boundary conditions:

$$V(0,B,t)=0$$
$$V(A, B, t^*) = A_{t*} - B_{t*}$$ [5]
$$V_A(t*)=1, \quad V_B(t*)=1$$

The boundary conditions,[5] are known as the value matching and the high contact conditions (Dixit and Pindyck [1994]) and correspond to the American-style call option to switch, which may be exercised any time before or at the expiration date T. On the other hand, if the expiration is limited to time T only, equations [4] and [5] represent the valuation problem of the European-style call option, for which the solution methodology is known (see McDonald and Siegel [1985]).

To solve differential equ. [4] s.t. [5], one utilizes the fact that function V(A, B, t) is homogeneous of degree 1 in (A, B) (Ingersoll [1987], p.210). This allows the reduction of equ. [4] to a one dimensional problem expressed in terms of the ratio of projects A and B. The assumption of homogeneity allows the following substitution:

$$V(A, B, t) = B W(A/B, t) = B W(S, t)$$ [6]

where W(S, t) is the value of the option to switch contingent on the ratio S of projects A and B with the exercise price equal to unity, at time t. Successive differentiation of V(A, B, t) yields:

$$V_A = W_S \qquad\qquad V_B = W - S W_S$$
$$V_{AA} = W_{SS}/B \qquad\qquad V_{BB} = S^2 W_{SS}/B$$

$$V_{AB} = -S \, W_{SS}/B \qquad\qquad V_t = BW_t \qquad\qquad [6']$$

Substituting these derivative expressions into [4], one gets the following second-order differential equation:

$$1/2[\sigma_A^2 + \sigma_B^2 - \rho_{AB}\sigma_A\sigma_B]S^2 W_{SS} + (\alpha - \beta)SW_S + W_t + (\beta - \gamma)W = 0$$

[7]

Similarly, the boundary conditions [5], become:

$$W(0,t) = 0$$
$$W(S_{t*}, \; t*) = S_{t*} - 1 \qquad\qquad [8]$$
$$W_S = 1$$

Equ.[7] with the boundary conditions [8] may be solved through approximation using an approach proposed by Barone-Adesi and Whaley [1987], (BAW) who showed how to solve a similar differential equation for American options written on commodities. The mathematical details of the solution's derivation are similar to BAW and we do not present the resulting (somewhat complex) formulas[2]. A by product of the derivation is the ratio S* for which switching is optimal. In what follows we present numerical examples and provide insights on valuing the flexibility to switch and the ratio S* for which switching is the optimal strategy.

Numerical Examples and Sensitivity Analysis

This section explains how the values of parameters entering the valuation model affect the value of the option to switch W(S, t) and the optimal ratio to switch S*, at any point in time t, in the interval [0, T]. As explained previously, the set of input parameters in the model includes: the growth rates of projects A and B, the variances of the two projects, the discount rate, the correlation coefficient between projects A and B, the time to expiration of the option T - t, and the ratio S of projects A and B at time t. To simplify the analysis, we choose the time at which the solution is calculated to be the beginning of the time horizon [0, T] and consider the time to expiration T as the only time-related variable in the model. In the same spirit we take the value of project B as numeraire so the solution for V is identical with the solution for the companion transformation W.

There are eight input parameters and two outputs in the model. Studying the impact of joint variation in values of eight input

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[2] The derivation is available from the authors upon request.

parameters on the solution (outputs) is obviously limited. We can, however, form pairs of input parameters, and observe the impact of their variation on the optimal solution, for fixed values of the remaining six parameters. To present the results of our analysis, we define the base case as: $S = 1$, $\alpha = 0.05$, $\beta = 0.03$, $\sigma_A = 0.3$, $\sigma_B = 0.2$, $\gamma = 0.15$, $T = 1$, $\rho_{AB} = 0$. This means that whenever one of the input parameters is kept fixed, its value is given by the base case. One easily verifies that it is possible to form 28 different pairs of input parameters for V and 21 pairs of input parameters for S* (the difference between the two is because the optimal ratio S* is independent of S). While it is impossible to present in a short study the analyses of all possible pairs, we present the results of the most important, in our opinion, numerical scenarios. The analysis focuses on the impact S the initial ratio, T time to expiration, project volatilities and correlation have on the value V of the flexibility to switch. At the same time the impact of T time to expiration, project volatilities and correlation on S* is analyzed.

First, it is interesting to see how the value of the option V depends upon the project values ratio S and the time to expiration T (recall that S is the ratio of the project values, and T is the length of time remaining in the decision horizon). The values of the option for different pairs (S,T) are reported in Table 1. One observes that flexibility has very little value for small values of S and increases along with the increase in S. Moreover, one can see how V increases with the increase in time, and that this increase is more significant for larger values of S. As explained previously, one may use the values of V reported in Table 1 to assess the investment in the option to switch: if the price one has to pay to acquire the flexibility does not exceed V at time 0, the investment is justified. Finally, the last row in Table 1 shows the optimal ratio to switch S* corresponding to various values of T. These numbers indicate when the switch between the alternative projects should be undertaken: if the "current" ratio S exceeds S*, the switch should be exercised. Figure 1 gives the graphical account of the sensitivities that the value of the option has with respect to the ratio of the projects and time. It appears that S has a more significant impact on the value of the flexibility than time. It is clear from the analysis that the value of the option erodes with the passage of time if the ratio S does not change.

From a managerial point of view, if the ratio of the two projects is not expected to change considerably, investing in the real option to

switch does not make too much sense. If the price one pays at time 0 is equal to the value of the option at time 0 and the ratio S does not change, investing in flexibility is value destroying. To make this point clear, let's consider that the initial ratio is S=1 and the option is open for 3 years and 3 months (T=3.25). The value of the option declines to .07 (for .25 years remaining) from .23 (for 3.25 years remaining) if three years pass by and nothing else changes. However if the ratio suddenly moves to 1.25 when three months remain in the life of the real option, the value of the real option is restored to its initial value. If on the other hand at any time t, S* is reached then switching becomes optimal and the initial decision to invest in flexibility makes sense. Table 1 shows that we are dealing here with a term structure of optimal ratios S*: the ratio changes its value if times goes by. This shows that ignoring time in such an analysis can lead to erroneous interpretations.

Whenever the initial ratio is greater than S* we have the trivial case when the value of the option is equal to the difference between the values of the two projects. In Table 1 if S=1.5 and the option to switch is open for only 3 months (T=.25) the value is .5, reflecting the fact that S* for t=.25 is 1.48.

Second, one examines the impact of the correlation between the projects and time. We present the results of the analysis for the pairs (T, ρ_{AB}), 0 < T < 2, 1 ≥ ρ_{AB} ≥ −1. Figure 2A (see also Table 2A) demonstrates how V decreases in T for different values of ρ_{AB}, given that S = 1. One notes that, in general, the value of V is always higher for negatively than positively correlated projects. Moreover, it decreases more sharply for negatively correlated projects as T approaches zero. Figure 2B (see also Table 2B) shows a similar pattern for the ratio S*. Whereas, the decrease in the value of the option with the decrease in time to expiration (with all other variables, including S, kept constant) is consistent with the option theory, the impact of correlation on V and S* is less obvious. It justifies a higher price for the option to switch between the negatively rather than positively correlated projects. At the same time, it shows that the flexibility of switching between positively correlated projects has, in general, less value and therefore one should not overstate the benefits of such flexibility.

Running the analysis of (T, ρ_{AB}) for values of S significantly different from one (not shown), we observed that the ratio S not only significantly affects V but also influences the pattern observed in Figure 2A and 2B and Table 2A and 2B. As S significantly departs from one, the surface representing V tends to flatten and thus the impact of

time and correlation is weaker than that observed for values of S close to 1.

Third, we examine the effect of the uncertainties in the project values on the flexibility value and S*. Table 3A demonstrates how variances of the projects' values affect the value of the option, given that $0.5 \geq \sigma_i \geq 0.1$, i = A, B, and the project values are negatively correlated (ρ_{AB}=-.5). Table 3B shows how variances affect the optimal ratio S*. One observes that in this scenario, the higher the uncertainties, the higher is the value of the flexibility and the higher ratio to switch. Figures 3A and 3B summarize these findings graphically. It appears that uncertainty is an important factor that influences both the investment and the switching decisions.

Interestingly, the pattern observed in Figures 3A and 3B, while true for negative ranges of correlation, is not exactly replicated in case of positively correlated projects. In the latter case, the surface representing V (S*) achieves its minimum at some positive value with respect to σ_A for a given value of σ_B (and vice versa). Table 4A and 4B as well as Figure 4A and 4B tell this very interesting story. Therefore, we conclude that one should study the impact of uncertainty in the model in conjunction with the correlation between the projects. Mixtures of variances (volatilities) and correlation will decrease both the flexibility value and the optimal ratio to switch. The general idea that uncertainty increases the flexibility to switch value[3] is incorrect. When projects are positively correlated, managers should pay attention both to the projects' level of uncertainty (volatility) and to the ratio project A volatility to project B volatility.

The numerical examples we have studied demonstrate the impact of selected input parameters on the solution to the valuation model. It appears that the ratio of the projects, at the time the decisions to invest or to switch are considered, is the primary factor influencing these decisions. The research has also studied the impact of time, correlation, and projects' variances on the value of option to switch and the optimal exercise of the flexibility option. These observations imply that inappropriate assessment of the projects' characteristics may lead to erroneous investment decisions or inappropriate timing of switching decisions.

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[3] See for instance Grinblatt and Titman (1998)

The Initial Ratio and Time

Given the importance this research finds for the initial project value A
to project value B ratio, some further analysis[4] is in order.

Let's recall that S is the ratio of two geometric Brownian motions
$S_t=A_t/B_t$. Applying Itô's lemma, one gets the following expression for dS:

$$dS = S(\alpha - \beta + \sigma_A^2 - \sigma_A\sigma_B\rho_{AB})dt + S(\sigma_A dz_A - \sigma_B dz_B) \qquad [9]$$

The parenthesis in the last term of [9] can be rewritten as:

$$\sigma_S dz_S = \sigma_A dz_A - \sigma_B dz_B \qquad [10]$$

with

$$\sigma_S = \sigma_A^2 + \sigma_B^2 - 2\sigma_A\sigma_B\rho_{AB} \qquad [10']$$

Now one can solve [9] as for a simple Brownian motion and obtains:

$$S(t) = S(0)\exp[\sigma_S dz_S]\exp\left\{[(\alpha-.5\sigma_A^2) - (\beta-.5\sigma_B^2)]t\right\} \qquad [11]$$

The solution given by [11]consists of a deterministic component, usually
called signal, represented by the second exponent and a stochastic
component, usually called noise, represented by the first exponent. In
order to asses the true ratio S(t), one would like to have a signal which
dominates the noise. From [11] it is obvious that one can find a time t
for which the signal dominates the noise with a required degree of
confidence. The condition is for the exponent representing the signal to
be greater than the exponent representing the noise. Solving for the
inequality, one obtains:

$$t > \frac{C^2\sigma_S^2}{[(\alpha-.5\sigma_A^2) - (\beta-.5\sigma_B^2)]^2} \qquad [12]$$

where C is the number of standard deviations for the required confidence
level. Assuming that, in our base case ,the degree of confidence,
required is 55 percent (C=.13), it should take almost 88 years to obtain
a ratio where the signal dominates the noise. For higher levels of
confidence the number of years is very large. It is obvious that, under
the circumstances, the observed ratio at one point in time is just
incomplete information. To obtain more information, it will mean to wait
much longer then the opportunity window exists. Therefore, one can
conclude that the switch is always based on a very noisy estimate. It is

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[4] Ambarish and Seigel (1995) discuss te same idea in another context.

very much possible for this to be the reason of the low profitability of investing in flexibility (Mansfield, [1993]).

Conclusions

In this paper, we have developed a model for evaluating the value of the flexibility to switch. We defined flexibility as the capability to switch between the alternative projects at no cost, and contrasted investment in flexibility with the situation, in which one makes the irreversible investment in only one of the projects. We developed our real option model under the assumption that the flexibility to switch has a value only during the limited time horizon. We formulated the valuation problem as a dynamic programming problem in a continuous time. The resulting partial differential equation for the value of the option to switch, contingent upon the ratio of the projects, can be solved through approximation. We examined how the values of the input parameters entering the valuation model affect the value of its outputs: the value of the flexibility to switch and the ratio of the projects, at which the switching decision becomes optimal.

We believe that our formulation offers a valid alternative to the recent attempts to quantify the flexibility to switch and incorporate this value into the capital budgeting process. Our valuation model expands the existing literature on the measurements of flexibility and provides a mathematical tool that can support managers facing choices associated with investments in manufacturing flexibility. At the same time, we recognize the limitations of our formulation. In the current model, switching costs are negligible and the switching decision cannot be reversed. We are planning to address these issues in future studies.

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TABLE 1

The value V of the flexibility to switch as a function of the initial ratio S and time T

S	T						
	.25	.75	1.25	1.75	2.25	2.75	3.25
.50	0.00000	0.00113	0.00532	0.01142	0.01828	0.02531	0.03220
.75	0.00381	0.02659	0.04847	0.06732	0.08360	0.09778	0.11023
1.00	0.07256	0.12349	0.15619	0.18101	0.20109	0.21787	0.2322
1.25	0.25975	0.29340	0.32038	0.34211	0.36015	0.37544	0.38859
1.50	0.5	0.50863	0.52315	0.53737	0.55031	0.56186	0.57214
1.75	0.75	0.75	0.75232	0.75809	0.76488	0.77177	0.77839
2.00	1.	1.	1.	1.	1.00049	1.00236	1.00495
S^*	1.48804	1.72664	1.86923	1.97356	2.05591	2.12356	2.18047

The last row indicates the critical value S^* for which the projects should be switched.

FIGURE 1

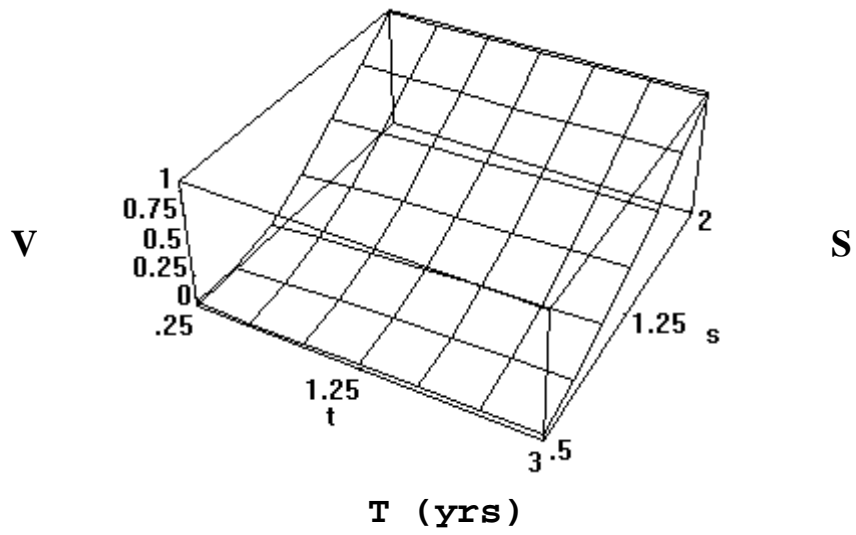## The value V of the flexibility as a function of the initial ratio S and time T

TABLE 2

Sensitivity analysis for changes in correlation and time

PANEL A
The value V of flexibility as a function
of correlation and time

ρ	T				
	.25	.50	.75	.1.00	1.25
-1	0.099641	0.1392	0.168289	0.19181	0.2117
-.8	0.094871	0.132621	0.160419	0.182923	0.20197
-.6	0.089837	0.125673	0.152101	0.173525	0.19168
-.4	0.084489	0.118288	0.143254	0.163522	0.18072
-.2	0.078764	0.110376	0.133769	0.15279	0.16895
0	0.072569	0.101808	0.123492	0.141154	0.15619
.2	0.065767	0.092394	0.112192	0.128352	0.14213
.4	0.058136	0.081829	0.099502	0.113966	0.12633
.6	0.049277	0.069558	0.084756	0.097239	0.10794
.8	0.038299	0.054349	0.066475	0.076497	0.08513
1.	0.021936	0.031735	0.039336	0.045741	0.05134

FIGURE 2 A

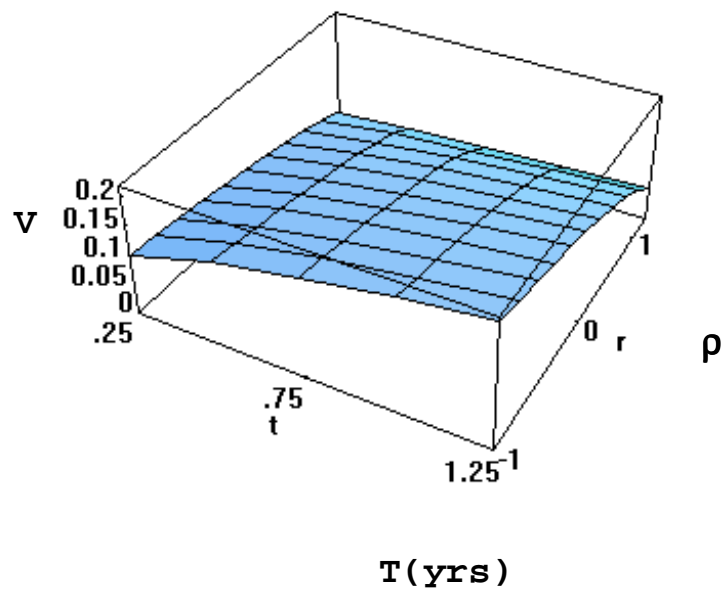## The value V of flexibility as a function of correlation and time

T(yrs)

TABLE 2

Sensitivity analysis for changes in correlation and time

PANEL B
The value of S* as a function of correlation and time

ρ	T				
	.25	.50	.75	1.00	1.25
-1.	1.6883	1.90914	2.06757	2.19395	2.29995
-.8	1.65072	1.85596	2.0027	2.1195	2.21732
-.6	1.61213	1.80155	1.93652	2.04372	2.13335
-.4	1.57232	1.74566	1.86875	1.96627	2.04769
-.2	1.53106	1.68794	1.79897	1.88673	1.95987
0.0	1.48804	1.62793	1.72664	1.80448	1.86923
.2	1.44284	1.56496	1.65095	1.71862	1.7748
.4	1.39495	1.49804	1.57068	1.62775	1.67508
.6	1.34392	1.42564	1.48378	1.52951	1.56742
.8	1.2909	1.34564	1.38668	1.4194	1.44669
1.	1.24309	1.26171	1.27705	1.29055	1.30258

FIGURE 2 B

The value of S* as a function of correlation and time
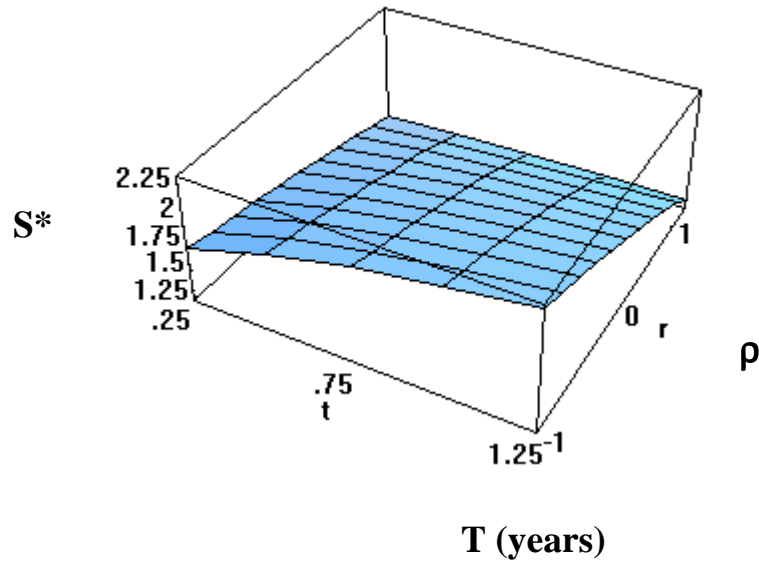
TABLE 3

Sensitivity analysis for changes in volatilities
with a correlation of -.5

	PANEL A				
	The value V of flexibility as a function of volatilities with a correlation of -.5				
s_A	s_B				
	.1	.2	.3	.4	.5
.1	0.07239	0.10596	0.14115	0.17671	0.21221
.2	0.10596	0.13598	0.16860	0.20230	0.23635
.3	0.14115	0.16860	0.19887	0.23059	0.26298
.4	0.17671	0.20230	0.23059	0.26046	0.29118
.5	0.21221	0.23635	0.26298	0.29118	0.32032

	PANEL B				
	S^* as a function of volatilities with a correlation of -.5				
s_A	s_B				
	.1	.2	.3	.4	.5
.1	1.39937	1.57977	1.80448	2.06915	2.3757
.2	1.57977	1.7692	2.00523	2.28551	2.6118
.3	1.80448	2.00523	2.25516	2.55333	2.9017
.4	2.06915	2.28551	2.55333	2.87292	3.2467
.5	2.37575	2.61188	2.90176	3.24676	3.6499

FIGURE 3 A

The value V of flexibility as a function of volatilities with a correlation of -.5
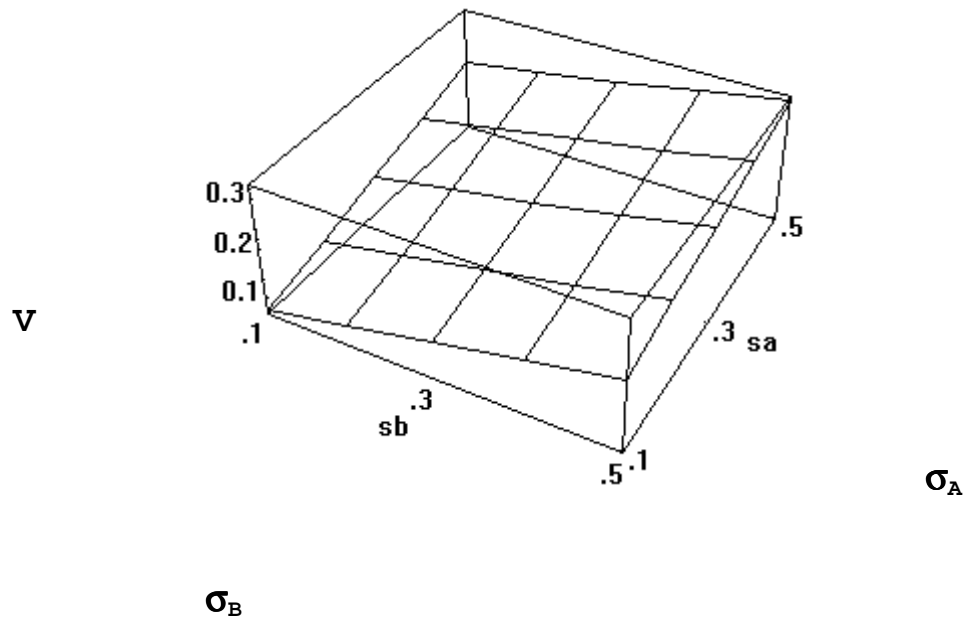

V

σ_A

σ_B

FIGURE 3 B

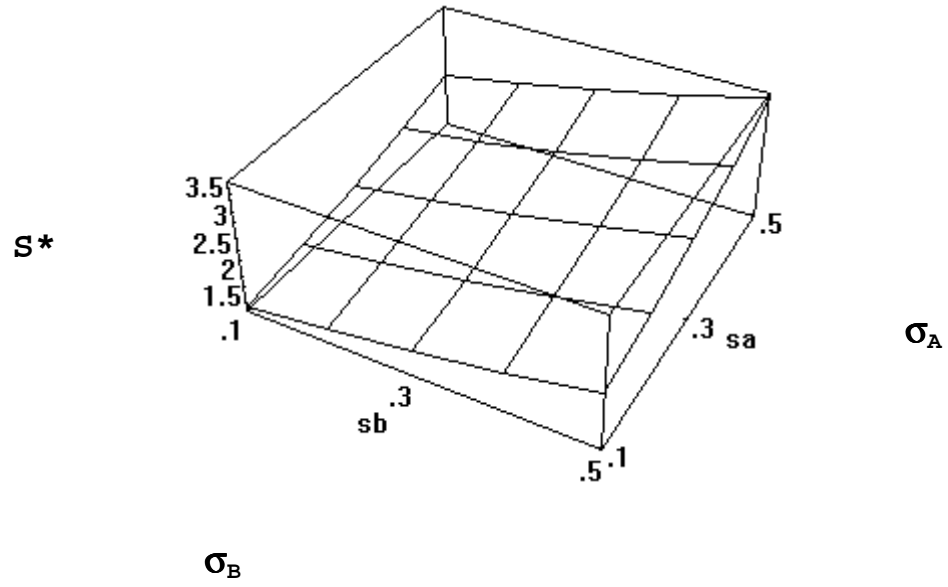S* as a function of volatilities with a correlation of -.5

TABLE 4

Sensitivity analysis for changes in volatilities
with a correlation of .5

PANEL A
The value V of flexibility as a funciton
of volatilities with a correlation of .5

s_A	s_B				
	.1	.2	.3	.4	.5
.1	0.0457413	0.0723926	0.105966	0.141154	0.176719
.2	0.0723926	0.0822305	0.105966	0.13598	0.168606
.3	0.105966	0.105966	0.118974	0.141154	0.168606
.4	0.141154	0.13598	0.141154	0.15555	0.176719
.5	0.176719	0.168606	0.168606	0.176719	0.19181

PANEL B
S* as a function of volatilities with a correlation of .5

s_A	s_B				
	.1	.2	.3	.4	.5
.1	1.29055	1.39937	1.57977	1.80448	2.06915
.2	1.39937	1.44843	1.57977	1.7692	2.00523
.3	1.57977	1.57977	1.65871	1.80448	2.00523
.4	1.80448	1.7692	1.80448	1.90684	2.06915
.5	2.06915	2.00523	2.00523	2.06915	2.19395

FIGURE 4 A

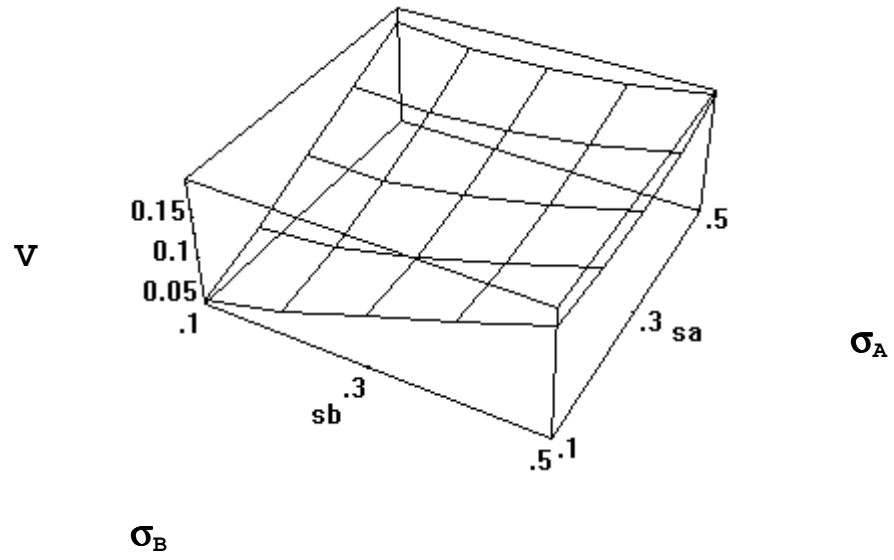The value V of flexibility as a function
of volatilities with a correlation of .5

FIGURE 4 B

S* as a function of volatilities with a correlation of .5