A behavioral cobweb model with heterogeneous speculators

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Abstract

According to empirical studies, speculators place significant orders in commodity markets and may cause bubbles and crashes. This paper develops a cobweb model that takes into account the behavior of technical and fundamental speculators. We find that interactions between consumers, producers and heterogeneous speculators may indeed produce price dynamics which mimics the cyclical price motion of actual commodity markets, i.e., irregular switches between bullish and bearish price developments. We analytically show that as the number of speculators increases we first observe a pitchfork bifurcation followed by a period doubling bifurcation. After infinitely many period doubling bifurcations the dynamics becomes chaotic.

Keywords

commodity markets, cobweb models, heterogeneous speculators, bifurcation analysis

JEL classification

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1 Introduction

A key characteristic of commodity price dynamics is their strong cyclical behavior. Cashin et al. (2002), who examine the price action of 36 commodities in the period from 1957 to 1999, report that the average price fall across all commodities was 46 percent during slumps, while the average price rise across all commodities was 42 percent during booms. Individual price series are, of course, even more volatile: The price for coconut oil dropped by around 88 percent between June 1984 and August 1986 and the price of coffee arabica increased by around 84 percent from April 1975 to April 1977. Further empirical evidence of commodity price fluctuations is provided by Borenzstein et al. (1994) and Deaton (1999). Alterations between bull and bear markets have important implications for many developing countries dependent on commodity exports. Dramatic price changes may cause severe fluctuations in earnings from commodity exports. A thorough understanding of commodity price dynamics is thus of great significance, especially for policy makers who plan to conduct counter-cyclical stabilization policies (Newberry and Stiglitz 1981).

Several theories have been proposed which give us valuable insight into the dynamics of commodity prices. The focus of this paper is on cobweb models (e.g. Coase and Fowler 1937, Ezekiel 1938 or Nerlove 1958) which describe the price dynamics in a market of a non-storable good that takes one time unit to produce. As a result, suppliers must form price expectations one period ahead. Such a view is not unrealistic. Consider, for instance, the cultivation of crops. The growing season guarantees a finite lag between the time the production decision is made and the time the crop is ready for sale. The decision about how much should be produced is based on current and past experience. Remember that classical linear cobweb models with naive expectations are able to reproduce oscillatory price movements with decreasing amplitude.

The cobweb approach has been extended in several directions. Exploiting nonlinearities in demand and supply, Day (1994) and Hommes (1994, 1998) analytically show the possibility of chaotic price dynamics for different adaptive expectation schemes of the producers. Our framework is related to the model of Brock and Hommes (1997). In their contribution, the demand and supply curves are linear, but producers switch between different forecasting strategies. Depending on publicly available fitness measures, producers opt either for naive or (costly) rational expectations. The choice is rational in the sense that predictors with a high level of fitness are preferred. The model not only yields complex price dynamics but suggests that irregular dynamics may be part of a fully rational notion of equilibrium.

This paper seeks to offer a new perspective of commodity price fluctuations by adding heterogeneous speculators, i.e. interacting chartists and fundamentalists, to the traditional cobweb framework. In fact, there exists widespread evidence that private and professional speculators apply both technical and fundamental analysis to predict commodity price movements. For instance, Smidt (1965) reports that the majority of the speculators relies at least partially on price charts to render trading decisions in commodity markets. Similar results are reported in questionnaire studies of Draper (1985) and Canoles et al. (1998). In addition, Sanders et al. (2000) find strong evidence of positive feedback trading in several commodity markets and Weiner (2002) detects herding behavior in the petroleum market. Overall, these studies indicate that chart and fundamental speculation is a major factor for price variation in commodity markets.

In line with the early cobweb literature, we construct a behavioral cobweb model with a supply response lag. The demand and supply schedules of the consumers and producers are linear and the producers have naive expectations. The market is cleared by the price sensitive demand of the consumers. But the supply available to consumers also depends on the trading decisions of the speculators, i.e. their excess selling (buying) increases (decreases) the supply.

The speculators apply both technical and fundamental methods to predict prices. While technical analysis extrapolates past price trends into the future, fundamental analysis assumes that prices converge towards their fundamental values. The speculators are boundedly rational in the sense that they tend to use forecast rules with a high level of fitness. Note that the speculators´ switching between technical and fundamental rules introduces a non-linearity into the model.

We are interested in how speculators may influence the evolution of commodity prices. Overall, our model is able to generate price dynamics which mimic the cyclical swings of commodity prices quite well. We analytically derive the following results. Suppose that the cobweb market is stable without speculators. Then a pitchfork bifurcation, followed by a period doubling bifurcation, emerge as the total number of speculators increases. Further simulation analysis reveals that after infinitely many period doubling bifurcations the dynamics becomes chaotic. For certain parameter values, we observe the emergence of bull and bear markets, as well as irregular price fluctuations between bull and bear markets.

However, if the demand and supply schedules of the consumers and producers violate the stability condition, we are able to show analytically that the presence of a critical mass of speculators may stabilize the market. Instead of a price explosion, the price may settle down on a complicated attractor, a limit cycle or even a fixed point. This finding is quite remarkable: The common suggestion to crowd out speculators may not always be beneficial to market stability. In fact, complex interactions between technical and fundamental speculators may prevent unstable price trajectories.

The remainder of this paper is organized as follows. Section 2 develops a behavioral cobweb model with heterogeneous boundedly rational speculators. In section 3, we present our analytical results and in section 4, we numerically illustrate the dynamics. The last section offers some conclusions and points out some extensions.

2 A cobweb model with consumers, producers and speculators

2.1 The behavior of consumers and producers

Remember that traditional versions of the cobweb model describe a dynamic price adjustment process on a competitive market for a single non-storable good with a supply response lag. Market clearing occurs in every period

$$D_t = S_t, \tag{1}$$

where D and S denote demand and supply, respectively. To keep the model as simple as possible, we focus on linear demand and supply curves. Consumer demand depends negatively upon the current market price P

$$D_t = \frac{a - P_t}{b}. \tag{2}$$

The output decision of the producers depends on their price expectations. We assume that producers have naive expectations (i.e. $E[P_t] = P_{t-1}$), which entails a so-called supply response lag. Hence, the supply of the producers in period t is

$$S_t^P = \frac{E[P_t] - c}{d} = \frac{P_{t-1} - c}{d}. \tag{3}$$

The coefficients a, b, c and d are non-negative.

In the absence of speculators ($S_t = S_t^P$), the law of motion of the price, obtained by combining (1)-(3), is a one-dimensional linear map

$$P_t = \frac{ad + bc}{d} - \frac{b}{d} P_{t-1}, \tag{4}$$

which has a unique fixed point at

$$F = \frac{ad + bc}{b + d}. \tag{5}$$

We regard the fixed point F as the fundamental value of the market. The law of motion may be simplified by rewriting (4) in terms of deviations from the fundamental value. Defining $X_t = P_t - F$, (4) becomes

$$X_t = -\frac{b}{d} X_{t-1}. \qquad (6)$$

As is well known, market stability requires

$$b/d < 1. \qquad (7)$$

If (7) holds, *P* is attracted by *F*, and *X* converges towards 0.[1] Furthermore, since the parameters *b* and *d* are positive, the price adjustment is oscillatory.

2.2 The behavior of speculators

Our perspective is that producers such as farmers are mainly concerned with the production process. At the stock exchange, where commodities are usually traded, many additional speculators are active. As revealed by empirical studies, private and professional speculators use technical and fundamental trading strategies to determine their investment decisions (Smidt 1965, Draper 1985, Canoles et al. 1998, Sanders et al. 2000). Speculators apparently have a marked influence on the evolution of commodity prices – an aspect which surprisingly has not received much attention so far.

Interactions between chartists and fundamentalists have already been explored in detail in several stock market models. So-called fundamentalists are agents who believe in mean reversion, i.e. they expect prices to return towards fundamentals. Agents using technical analysis, so-called chartists, bet on the persistence of past price trends. Models by Day and Huang (1990), Huang and Day (1993), de Grauwe et al. (1993), Brock and Hommes (1998) or

[1] Note that the parameters *a* and *c* just shift the demand and supply curves vertically upwards or downwards. Hence, the price and its fundamental value both increase in *a* and *c*, yet *X* – the law of motion – is independent of *a* and *c*. Without loss of generality one may assume that *a* and *c* take values such that prices and production quantities are always positive.

Lux and Marchesi (2000) demonstrate that the behavior of heterogeneous speculators may endogenously create complex financial market dynamics.

Following this branch of research, we assume that speculators are selling (i.e. increasing the supply) if they expect a decrease in the price and vice versa. The speculators are heterogeneous with respect to their expectation formation. We enrich the simple cobweb model (6) by two types of speculators: chartists and fundamentalists. Note that we do not keep track of the behavior of individual traders; here we are interested in their aggregated impact on the price dynamics. The total supply may be expressed as

$$S_t = S_t^P + N\,(W_t\ S_t^C + (1-W_t)\ S_t^F\,)\,, \tag{8}$$

where S^C and S^F are the supply generated by the application of the technical and the fundamental trading rule, respectively. W stands for the fraction of agents who follow the technical rule. The market share of fundamentalists is (1-W). N denotes the number of speculators, which we normalize to $0 \le N \le 1$.

To characterize the behavior of chartists, we adapt a predictor first suggested by Day and Huang (1990). Chartists optimistically believe in the persistence of a bull market as long as the price is above its fundamental value. Conversely, in a bear market $(P < F)$, chartists pessimistically think that the price will decline further (i.e. $E^C[P_t] = P_{t-1} + e''(P_{t-1} - F)$ with $0 < e'' < 1$). The orders generated by technical analysis are thus given as

$$S_t^C = -e'(E^C[P_t] - P_{t-1}) = -e(P_{t-1} - F)\,, \tag{9}$$

where $e = e'e'' > 0$. Chartists are buying into a rising (bull) market and selling into a falling (bear) market. Note that (9) implies that changes in the demand of chartists are positively correlated with changes in the price.

Fundamentalists expect the price to converge towards its fundamental value. Such regressive expectations may be formalized as $E^F[P_t] = P_{t-1} + f''(F - P_{t-1})$ with $0 < f'' < 1$. The supply due to the fundamental trading rule is

$$S_t^F = -f'(E^F[P_t] - P_{t-1}) = -f(F - P_{t-1}), \quad (10)$$

where $f = f'f'' > 0$. Fundamental analysis suggests selling (buying) if the good is overvalued (undervalued).

The selection of a trading rule depends on market circumstances. The more the price deviates from its fundamental value, the greater the speculators perceive the risk that the price path will collapse (i.e. return to *F*). Hence, the attractiveness of the technical trading rule may be written as

$$A_t^C = g\ Log\,|\,1/(F - P_{t-1})\,|, \quad (11)$$

where $g > 0$. Clearly, the fitness of technical analysis declines with increasing mispricing. Conversely, the attractiveness of fundamental analysis may be formalized as

$$A_t^F = h\ Log\,|\,F - P_{t-1}\,|. \quad (12)$$

Since *h* is a positive coefficient, speculators regard fundamental analysis as more suitable if the distance between the price and its fundamental value increases.

The fraction of chartists is given as

$$W_t = \frac{Exp[i\ A_t^C]}{Exp[i\ A_t^C] + Exp[i\ A_t^F]}. \quad (13)$$

According to (13), the fraction of speculators who apply the technical trading rule increases if the attractiveness of that rule increases. The fraction of speculators who follow the fundamental trading rules is defined as $(1-W)$. The coefficient $i \geq 0$ measures how sensitive the mass of traders is to selecting the most attractive rule. For instance, if $i = 0$, then the traders do not discriminate between the options ($W = 0.5$). The higher *i*, the more speculators select the rule with the highest level of attractiveness. Inserting (11) and (12) into (13) yields

$$W_t = \frac{1}{1+|F-P_{t-1}|^j}, \tag{14}$$

where $j = i(g+h)$.

The modified law of motion of the price, in terms of deviations from the fundamental value, becomes

$$X_t = -\frac{b}{d}X_{t-1} + \frac{b\ N\ X_{t-1}(e-f\,|X_{t-1}|^j)}{1+|X_{t-1}|^j}. \tag{15}$$

Due to its second term, (15) is a one-dimensional non-linear difference equation.

3 Theoretical Analysis

In agreement with the literature (e.g. de Grauwe et al. 1993, Hommes 2001, Westerhoff 2003), we approximate the decision behavior of the speculators by a bell-shaped switching function. Thus, (14) entails a quadratic term in the denominator. We are now able to derive important theoretical insights into the dynamics of our cobweb model which hopefully will improve our understanding of the workings of commodity markets. In section 4, we also explore the dynamics of the model for $j \neq 2$.

For convenience, we express (15) as

$$X_t = H(X_{t-1}, N), \tag{16}$$

where $H(X,N) = X\left(-\frac{b}{d} + \frac{bN(e-fX^2)}{1+X^2}\right)$.

The parameter N plays a key role in our stability and bifurcation analysis. Denote $N_1 = \frac{b-d}{bde}$, $N_2 = \frac{b+d}{bde}$ and $N_3 = \frac{-be+f(b+2d)+\sqrt{(e+f)(eb^2+f(b+2d)^2)}}{2efbd}$, where $N_1 < N_2 < N_3$. We are now able to derive the following results (proven in the appendix).

Theorem 1: The difference equation (16) possesses three fixed points at $X_1 = 0$ and

$X_{2,3} = \pm \frac{\sqrt{b + d - bdeN}}{\sqrt{-b - d - bdfN}}$, where $X_{2,3}$ only exist if $N > N_2$.

Theorem 2: If *b*<*d* then the difference equation (16) possesses

(a) a locally stable fixed point at $X_1 = 0$ for $0 < N < N_2$,

(b) a pitchfork bifurcation at $X_1 = 0$ for $N = N_2$,

(c) two locally stable fixed points at $X_{2,3} = \pm \frac{\sqrt{b + d - bdeN}}{\sqrt{-b - d - bdfN}}$ for $N_2 < N < N_3$,

(d) a period doubling bifurcation at $X_{2,3} = \pm \frac{\sqrt{b + d - bdeN}}{\sqrt{-b - d - bdfN}}$ for $N = N_3$.

Theorem 3: If *b*>*d* then the difference equation (16) possesses

(a) a locally unstable fixed point at $X_1 = 0$ for $0 < N < N_1$,

(b) a locally stable fixed point at $X_1 = 0$ for $N_1 < N < N_2$,

(c) for $N \geq N_2$ see Theorem 2 (b), (c) and (d).

Observe the fundamental difference between Theorems 2 and 3. If the basic cobweb market is locally stable (*b*/*d*<1), an increase in the number of speculators tends to decrease the efficiency of the market. A locally stable fixed point equal to the fundamental value is transformed into one of the two locally stable fixed points unequal to the fundamental value and then into one of the two period-two cycles. From that point of view it might appear desirable to ban speculators from the market, for instance by imposing a transaction tax. But such popular policy advice may easily turn out to be a mixed blessing. If the market becomes unstable (*b*/*d*>1), either due to a technology shock or a shift in the preferences of the consumers, the existence of a critical mass of speculators ($N>N_1$) guarantees at least some kind of stability. Instead of an exploding price trajectory, the price either converges towards its fundamental value, to one of the two non-fundamental fixed points, to a limit cycle in the bull or the bear market, or displays complex motion. Thus it might be better to tolerate the activity of speculators and to accept some mispricing to prevent unstable price paths.

4 Numerical Analysis

Let us carry on the analysis by numerical investigation. Figure 1 presents the bifurcation diagrams for the model´s six parameters. The bifurcation parameter is increased in 400 steps as indicated on the axis while the remaining parameters are given as

$N = 0.57$, $b = 1$, $d = 2$, $e = 10$, $f = 1$ and $j = 2$.

For each parameter combination, 100 observations are plotted. A transient period of 500 periods has been erased to allow the system to settle down on its attractor.

Figure 1 goes about here

The first panel of figure 1 continues the bifurcation analysis of the previous section. As can be seen, the more speculators enter the market, the more complex the dynamics. After infinitely many period doubling bifurcations the dynamics becomes chaotic. Comparable routes into chaos are observed for parameters *b* and *e*. However, there also exist different routes into chaos, indicating that the model has the potential to produce quite complex dynamics for a large variety of parameter combinations. Put differently, the emergence of endogenous price movements is not a special case. From a qualitative point of view one may say that an increase in both *b* and *e* seems to increase the range in which the fluctuations takes place, whereas an increase in *f* tends to shrink the upper and lower price boundaries. Parameters *d* and *j* hardly allow such conclusions, which again reveals the model´s strength to generate complex dynamics.

Figure 2 compares actual prices of agricultural products with simulated prices. The first, second and third panels display monthly hog prices, corn prices and hog-corn price ratios between 1970 and 2002, respectively. All three time series reveal cyclical motion, as is often reported for commodity markets. Especially the hog-corn price ratio data switches between high and low price periods.[2] A simulated time series with 250 observations is depicted in the bottom panel (we use the same parameter setting as in the bifurcation analysis). Our simple model obviously has the power to mimic complex cyclical patterns. Although the model is completely deterministic, prices move up and down erratically.

Figure 2 goes about here

Let us try to understand what is going on in the (artificial) market. Since we have set $b/d<1$, the price would – in the absence of speculators – settle down at its fundamental value. However, the presence of speculators destabilizes the market. If the price is close to its fundamental value, chartism is popular and thus the price is driven away from its fundamental value. But as the mispricing increases, fundamental trading becomes more fashionable and a convergence sets in. Since this again favors chartism, the pattern continually repeats itself, but in a very intricate manner. If the price crosses the fundamental value, a temporary bull market turns into a temporary bear market or vice versa. To sum up, the price seems to move up and down erratically in a bull or bear market, and also seems to swing erratically between bull and bear markets. Such a trajectory qualitatively resembles those displayed in the top three panels.

Let us finally turn to the case in which the basic cobweb market is unstable ($b/d>1$). If there are no speculators ($N=0$), the price diverges from its fundamental value. To be precise, the dynamics evolves as follows. Suppose the price is above its fundamental value in period $t-1$. In the next period, the consumers face an increased supply of the commodity and

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[2] Since corn is the primary ingredient of the hog´s diet (it constitutes around 60-65 percent of the total costs of pig production), the hog-corn price ratio is a general indicator of profitability and, therefore, future changes in pork production. Therefore, it is not surprising that Ezekiel´s (1938) hog-corn price ratio figure, ranging from 1900 to 1935, looks, despite technological progress, quite similar to the one displayed here.

consequently the price drops below its fundamental value. As a result, the producers reduce their output in period t+1 and the price increases again. Due to b/d>1, the price is now higher than in period t–1. This pattern repeats itself and the price path explodes.

Contrary to the intuition Theorem 3 shows that speculators may stabilize an otherwise unstable market although their behavior is in general destabilizing. Figure 3 illustrates this puzzling feature. It presents a simulation run for the following parameter setting

$N = 0.242$, $a = 20$, $b = 2.2$, $c = 10$, $d = 2$, $e = 10$, $f = 1$ and $j = 2$.

The first, second, third and fourth panel of figure 3 displays the price of the commodity, the demand of the consumers, the output of the producers and the net supply of the speculators, respectively. Note that the price of the commodity switches erratically between bull and bear markets (as in figure 2), yet does not explode. The reason for this becomes obvious in the bottom three panels. Note that when the price is high (i.e. above its fundamental value), the output of the producers is also high. However, this does not automatically lead to a crash in the next period since the net supply available to the consumers now also depends on the activity of the speculators. If the price is high, the speculators buy the commodity (see bottom panel). This reduces the net supply and thus hinders the system from crashing. The same is true in the opposite case. If the price is low (i.e. below its fundamental value), the output of the producers is also low. But now the speculators are selling the commodity so that the price remains in the bull market. Overall, we observe intricate dynamics, mainly due to the behavior of the speculators, but the system does not run away from its fundamental value. In this sense, speculators stabilize the dynamics. The popular recommendation to crowd out speculators may thus not be beneficial to market stability.

Figure 3 goes about here

5 Conclusions

For more than 100 years, the regularly recurring cycles in the production and prices of particular commodities have been studied with great interest (for an early review see Ezekiel 1938). The cobweb framework has become the basic workhorse to explore this phenomenon. While the early theoretical contributions have been linear in design, a number of promising non-linear models have also been formulated (Day 1994, Hommes 1994, Brock and Hommes 1997). Without question, all these papers help explain the empirical evidence.

But to our understanding one important aspect has been overlooked. As indicated in many empirical studies, speculators have a marked impact on the price formation process (Smidt 1965, Draper 1985, Canoles et al. 1998, Sanders et al. 2000, Weiner 2002). Our paper extends the basic linear cobweb model with naive expectations and a supply response lag by the incorporation of heterogeneous interacting speculators. Even in its simple form the model has the potential to generate cyclical, yet complex, price dynamics. The dynamics live from the fact that a sufficient fraction of the speculators applies destabilizing technical trading rules. However, as we have shown analytically, banning speculators from the market may only be a mixed blessing. Although the speculators might, on average, be regarded as destabilizing there also exist situations in which they stabilize the dynamics.

The simplicity of our model has been achieved by ignoring some of the details of commodity markets. We would thus like to point out a number of extensions. First, in our model, producers and speculators operate on the same time scale. A more reasonable perspective would be that the producers update their production decision on, say, a weekly or monthly basis while speculators trade on a daily basis (or even more frequently). An interesting modification of this model would be to combine different time scales for speculators and producers. Second, in many agriculture markets we often see interventions by governments that try to influence prices. Cobweb models are suitable to pre-study the consequences of such actions. However, if one omits the impact of speculators, the strategies may be ill designed. Third, our model is deterministic. By adding dynamic noise one may try to calibrate the model even more closely to the stylized facts. For instance, one may investigate the consequences of demand and supply shocks. Finally, since the structure of the proposed model is quite simple it may be possible to test the model statistically using real data.

Appendix

Proof for Theorem 1:

Solving

$$-\frac{b}{d}X+\frac{bNX(e-fX^2)}{1+X^2}-X=0 \tag{A1}$$

with respect to *X* one obtains $X_1=0$ and

$$X_{2,3}=\pm\frac{\sqrt{b+d-bdeN}}{\sqrt{-b-d-bdfN}}. \tag{A2}$$

The latter two fixed points only exist if $N>N_2$ (see below).

Proof for Theorem 2:

(a) Recall that $\partial H/\partial X$ denotes the eigenvalue of our one-dimensional map. Since $b<d$,

$$\left.\frac{\partial H}{\partial X}\right|_{X=0,N=0}=-\frac{b}{d}>-1 \tag{A3}$$

and

$$\left.\frac{\partial H}{\partial X}\right|_{X=0,N=N_2}=1. \tag{A4}$$

Thus, the fixed point at $X=0$ is locally stable for $0<N<N_2$.

(b) From (A4) we know that $X=0$ for $N=N_2$ is a nonhyperbolic fixed point with eigenvalue 1. A few transformations reveal that

$$\left.\frac{\partial H}{\partial N}\right|_{X=0,N=N_2}=0,$$

$$\left.\frac{\partial^2 H}{\partial X^2}\right|_{X=0,N=N_2}=0,$$

$$\left.\frac{\partial^2 H}{\partial X\partial N}\right|_{X=0,N=N_2}=eb\neq 0,$$

$$\left.\frac{\partial^3 H}{\partial X^3}\right|_{X=0,N=N_2}=\frac{6(e+f)(-b-d)}{ed}\neq 0.$$

Thus, $X=0$ undergoes a pitchfork bifurcation at $N=N_2$. See, for instance, Wiggins (1990) for the requirements of a pitchfork bifurcation.

(c) Note that

$$\left.\frac{\partial H}{\partial X}\right|_{X=X_{2,3},N=N_3} = -1 \tag{A5}$$

Thus, the fixed points at $X_{2,3} = \pm\frac{\sqrt{b+d-bdeN}}{\sqrt{-b-d-bdfN}}$ are locally stable for $N_2 < N < N_3$.

(d) From (A5) we know that $X_{2,3} = \pm\frac{\sqrt{b+d-bdeN}}{\sqrt{-b-d-bdfN}}$ for $N = N_3$ are nonhyperbolic fixed points with eigenvalue -1. A few transformations show that

$$\left.\frac{\partial H^2}{\partial N}\right|_{X=X_{2,3},N=N_3} = 0,$$

$$\left.\frac{\partial^2 H^2}{\partial X^2}\right|_{X=X_{2,3},N=N_3} = 0,$$

$$\left.\frac{\partial^2 H^2}{\partial X \partial N}\right|_{X=X_{2,3},N=N_3} =$$

$$\frac{2(e+f)bd\left(eb^2+f(b+2d)^2\right)+2(be-f(b+2d))\sqrt{(e+f)b^2d^2\left(eb^2+f(b+2d)^2\right)}}{(e+f)d(b+d)^2} \neq 0,$$

$$\left.\frac{\partial^3 H^2}{\partial X^3}\right|_{X=X_{2,3},N=N_3} =$$

$$\frac{1}{e(e+f)bd^2(b+d)^2}\Big(-48\left(eb^2+f(b+d)^2\right)\sqrt{(e+f)b^2d^2\left(eb^2+f(b+2d)^2\right)}$$
$$+48bd\left(-b^3e^2-f^2(b+d)^2\right)(b+2d)-efb\left(2b^2+4bd+3d^2\right)\Big) \neq 0.$$

Thus, the fixed points at $X_{2,3} = \pm\frac{\sqrt{b+d-bdeN}}{\sqrt{-b-d-bdfN}}$ undergo a period-doubling bifurcation for $N = N_3$. See again Wiggins (1990) for the requirements of a period doubling bifurcation.

Proof for Theorem 3:

(a) Since $b>d$,

$$\left.\frac{\partial H}{\partial X}\right|_{X=0,N=0} = -\frac{b}{d} < -1 \tag{A6}$$

and

$$\left.\frac{\partial H}{\partial X}\right|_{X=0,N=N_1} = -1\,. \tag{A7}$$

Thus, $X_1 = 0$ is a unstable fixed point for $0 < N < N_1$.

(b) $X_1 = 0$ is a stable fixed point for $N_1 < N < N_2$ because of (A4) and (A5).

(c) See proofs of Theorem 2 (b), (c) and (d).

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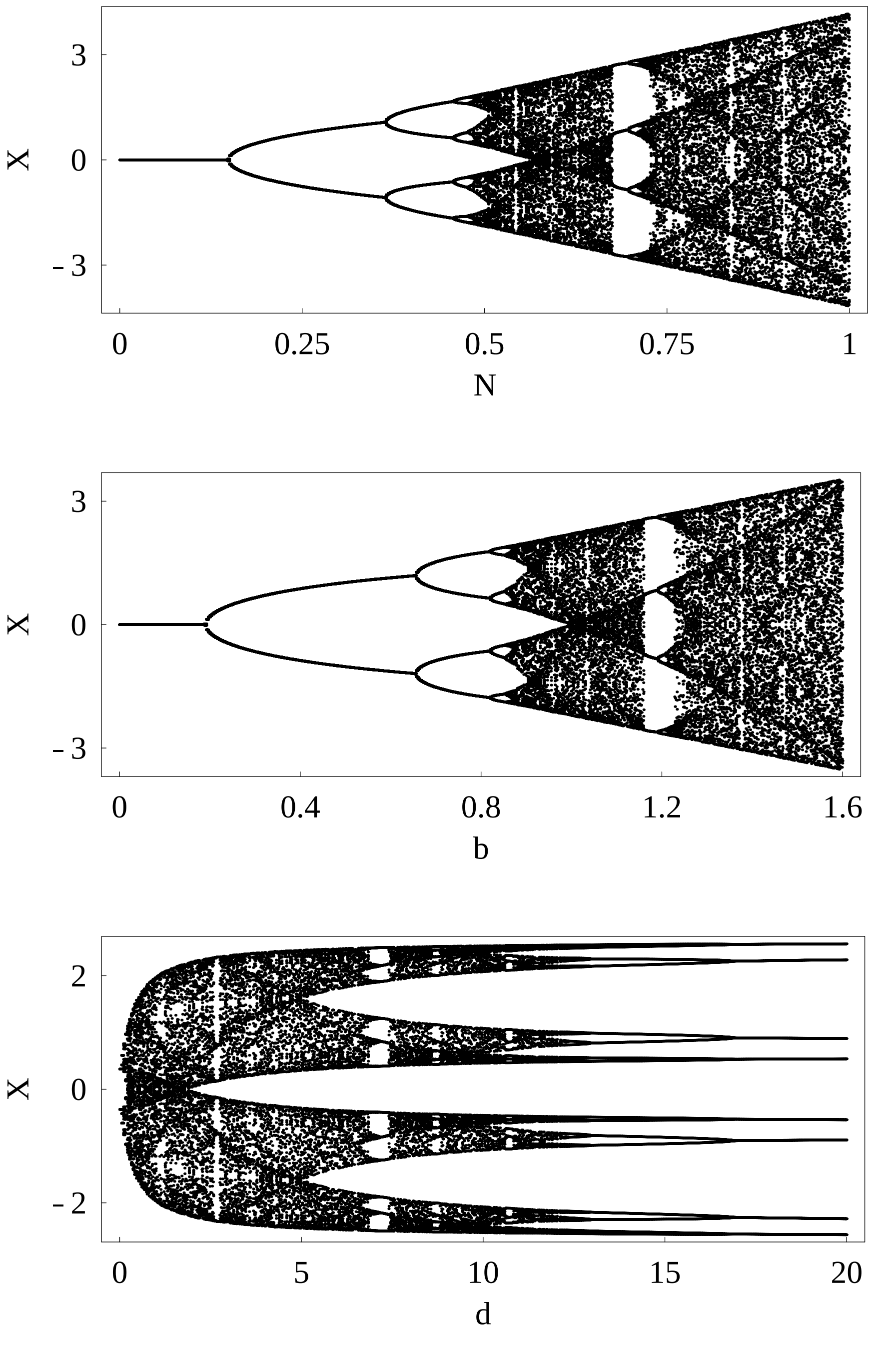

Figure 1: Bifurcation diagrams. The bifurcation parameter is increased in 400 steps. For each value, 100 observations are plotted (a transient period of 500 periods has been erased). Parameter setting as in section 4 and as indicated on the axis.

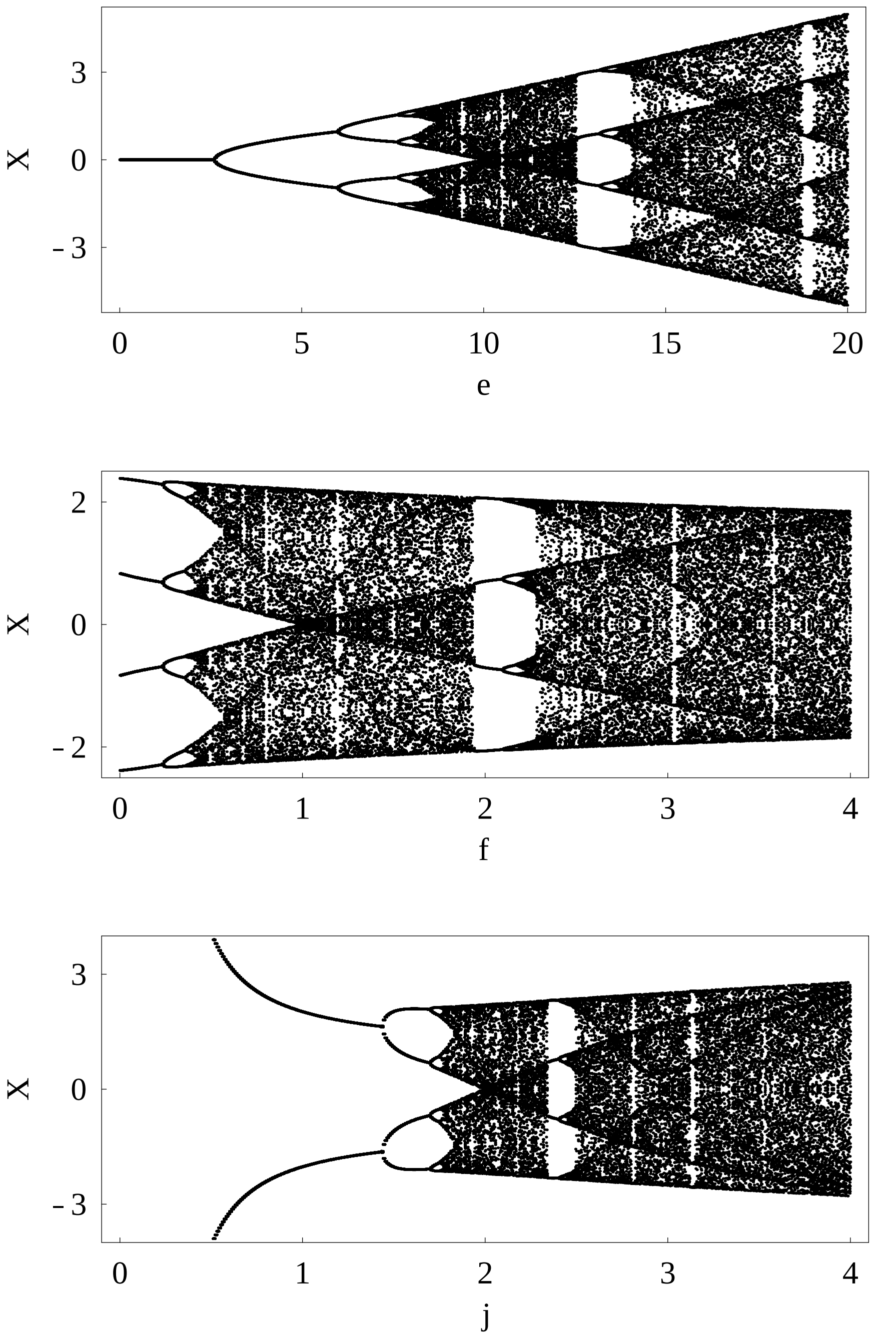

Figure 1: continued.

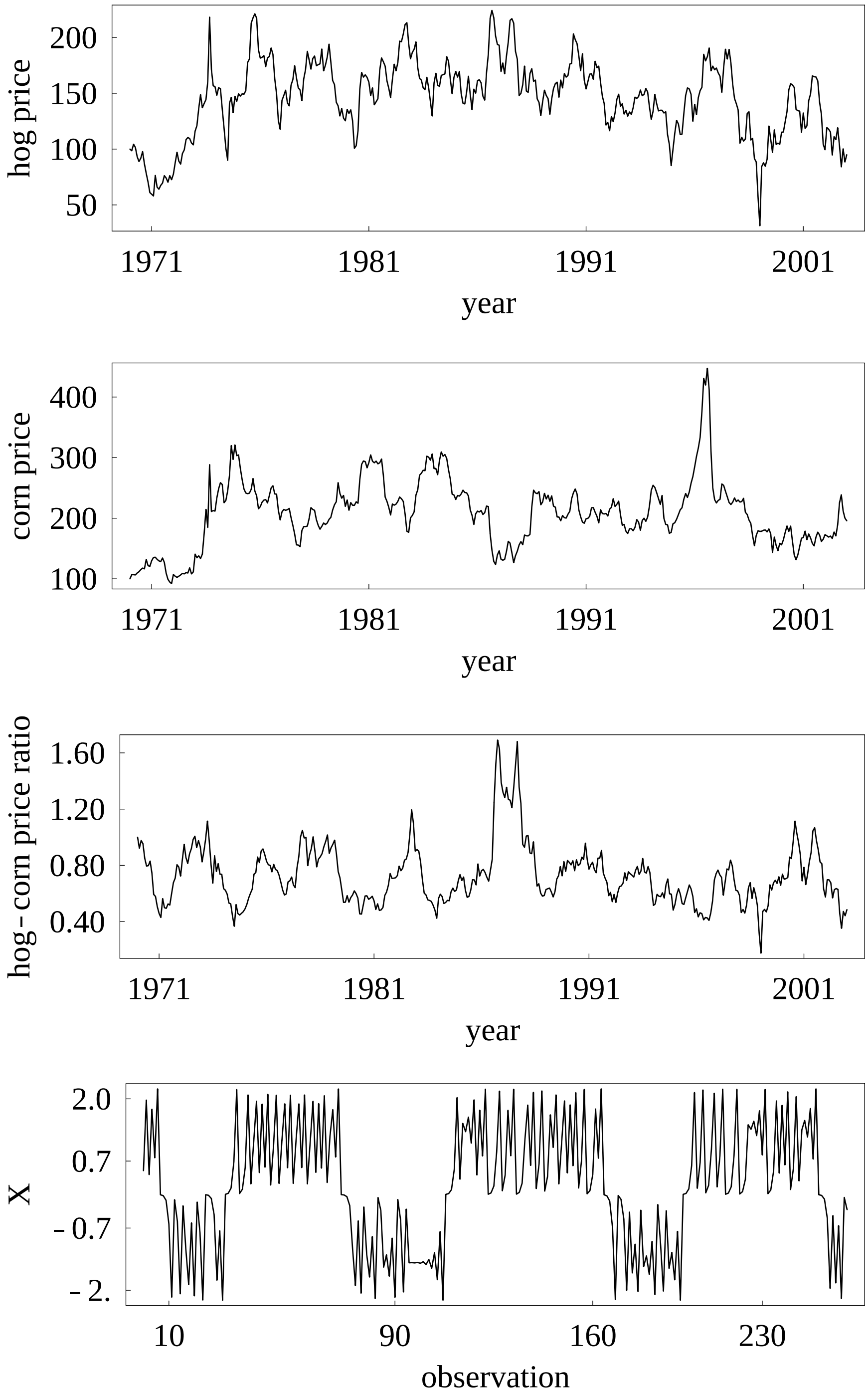

Figure 2: Cycles in cobweb markets. The first three panels show the hog price, the corn price and the hog-corn price ratio between 1970 and 2002 (monthly data, 1970=100). The fourth panel displays simulated prices for 250 observations.

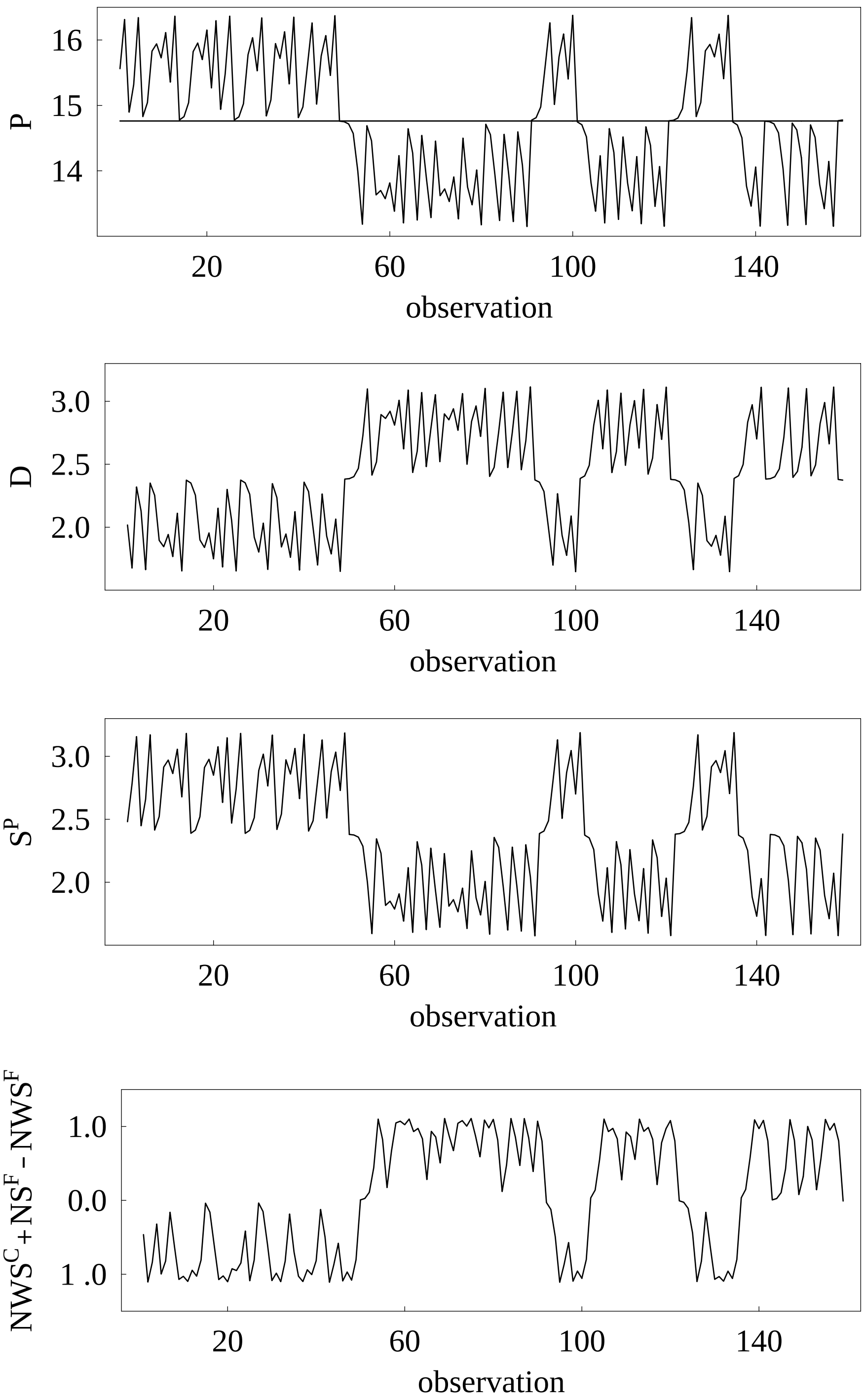

Figure 3: The impact of speculators when *b>d*. The first, second, third and fourth panel shows the price of the commodity, the demand of the consumers, the output of the producers and the supply of the speculators, respectively.