A Fully-Rational Liquidity-Based Theory of IPO Underpricing and Underperformance

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Abstract

I present a fully-rational symmetric-information model of an IPO, as well as a dynamic imperfectly competitive model of the aftermarket trading that follows. The model helps explain why IPO share allocations favor large institutional investors. It also helps to explain IPO underpricing, and underperformance, and the large fees charged by underwriters. The critical assumption in the model is that underwriters need to sell a fixed number of shares at the IPO or soon thereafter in the aftermarket, but they want to avoid selling in the aftermarket because there are some aftermarket investors who have market power and can affect the prices received by the underwriter. To maximize revenue and avoid unnecessary aftermarket sales, the underwriter distorts share allocations toward those those investors who have market power, and he sets the offer price at the IPO below the aftermarket price that will prevail shortly after the IPO. In the aftermarket model, I show that there are share allocations that can generate arbitrarily high levels of return underperformance for very long periods of time. In some simulations, the distorted share allocations at the IPO generate return underperformance that persists for more than one year. The underwriter can dilute investor's market power by participating for longer periods of time in aftermarket trading. By doing so, he sometimes substantially increase the revenue that is raised by the IPO issuer.

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1 Introduction

Two of the principle functions of a well performing financial system are to facilitate risk sharing among investors, and capital formation by firms. The initial public offering (IPO) process serves both of these functions by allowing the initial owners of a firm to raise capital while simultaneously transferring and sharing some of the firm's risk with the wider investing public.

IPOs are special events in capital markets because the amounts of risk that are transferred during the share allocation process of an IPO dwarfs the amount of risk that is transferred during the regular trading process for individual stocks. If the IPO risk transfer process was fully efficient, then the investors who place the most value on the shares should receive them, and they should pay a high price. Additionally, in the absence of firm specific news or private information, there should be little trading volume after the shares are initially allocated. Relative to this efficient benchmark, IPOs appear to be highly inefficient: share trading is very heavy on the first day after a share has been allocated.[1] Additionally, shares are apparently allocated at too low a price: the closing share price on the first trading day of U.S. IPO's is on average about 17 percent higher than the price at which the shares were allocated earlier in the day. This phenomenon, known as IPO underpricing, represents a loss of revenue to the issuer who could presumably do better by selling directly at the high prices that occur in the aftermarket following the IPO.

In addition to underpricing and frequent trading, the returns on newly issued shares underperform; that is, following the first day of trading, the returns on new issues underperform the return on the market and underperform the returns of shares of firms that have the same risk characteristics, but are not new issues. Moreover, this underperformance appears to persist for periods of time as long as five years [Loughran and Ritter (1991), Ritter and Welch (2002)]. An additional source of inefficiency is that underwriters charge and receive very high fees for their services; these fees are equal to about 7% of the revenues raised in the new issue.

In this paper I present a fully-rational, symmetric information, theoretical model of the IPO share allocation and price-setting process, and of the aftermarket trading that follows the IPO. The theoretical model helps to explain IPO underpricing and underperformance, and very preliminary results suggest it may help to rationalize the high fees charged by underwriters.[2] The theory is also consistent with the stylized facts that investors are often rationed at the IPO offer price, and that IPO share allocations are biased towards institutional investors.

[1] In Ellis, Michaely, and O'Hara's (2000) study of NASDAQ IPO's, they report that a stock's daily turnover (measured as a percentage of shares traded) on its first trading day following its IPO is equal to about 1/3rd of the turnover that a typical NASDAQ stock experiences over an entire year.

[2] The results on underwriters are still highly preliminary, but encouraging. I find that in some circumstances the underwriters trading activities in the aftermarket were found to add 25% to the total proceeds raised by the issue.

The IPO process is modeled as a simple bargaining game between the underwriter and investors: The underwriter has a fixed number of shares that must be sold at the IPO or shortly afterwards in aftermarket trading. At the IPO, the underwriter sets a uniform IPO offer price and makes take it or leave it share allocations to the investors. Any shares that are not sold in the IPO, are sold by the underwriter in the aftermarket.

The IPO aftermarket is modeled using a dynamic imperfect competition framework in which investors trade multiple risky assets over a total of T time periods. The imperfect competition takes the form that there are large investors who have market power in the sense that their trades move prices; and take their price impact into account when trading. Large investors market power provides them with bargaining power at the IPO because each large investor knows that if he turns down his share allocation, the underwriter will be forced to sell those shares in the aftermarket, where large investors can influence (and lower) the price received by the underwriter. To avoid this outcome, the underwriter optimally distorts the IPO asset allocations towards investors with market power, and he sets the IPO offer price below the aftermarket price that will prevail shortly after the IPO.

An important feature of the aftermarket is that the more a large investor buys or sells, the more he moves prices. Because large investors cannot buy or sell all of the shares that they want at current prices, the market is not perfectly liquid; and this illiquidity causes them to break up their desired trades through time to reduce its price impact. This illiquidity also affects the pattern of equilibrium returns in the aftermarket. More specifically, in aftermarket trading investors asset holdings and returns adjust towards those associated with efficient risk sharing, but to minimize the price impact of large investors trades, the adjustment occurs over time. That is, from any inefficient asset allocation, the model generates a unique equilibrium adjustment path of trades and expected asset returns. Because of the slow adjustment in positions, one can always find asset allocations that generate adjustment paths along which returns underperform the market by arbitrary amounts for all T trading periods. Whether underperformance actually results and persists depends on how assets are allocated at the IPO. The preliminary results on underpricing are encouraging. In some circumstances, the equilibrium allocations at the IPO generates post-IPO underpricing relative to the market portfolio that persists for longer than one year. This suggests illiquidity and allocation distortions at the IPO could help to explain post-IPO return underperformance.

If the underwriter did not have to sell shortly after the IPO, but could instead sell shares not allocated at the IPO over a longer period following the IPO, then doing so, as well as the threat of doing so, dilute the market power of large investors. In some circumstances these aftermarket "stabilization" activities were found to substantially increase the revenues received by the issuer.

There is a voluminous literature on IPO underpricing and underperformance.[3] One strand of the underpricing literature is based on information-asymmetries. In Rock (1986), uninformed investors face an adverse selection problem at the IPO: they are allocated too many shares of bad firms and too few shares of good firms. Equilibrium underpricing results

[3] Recent reviews of this literature are provided by Ritter and Welch (2002) and Ljungvist (2004).

in order to compensate uninformed investors for their adverse selection costs. In the IPO bookbuilding literature, that begins with Benveniste and Spindt (1989) and has since been refined by many others, some investors have private information about the value of the IPO firm. The IPO share allocation and price setting process is a mechanism that is designed to raise money for the issuer while simultaneously eliciting information from the informed investors. To compensate investors for revealing their information, share allocations are tilted towards informed investors, and the offer price is set below the price in aftermarket trading.[4]

This paper is the most closely related to a small theoretical literature on IPO underpricing and liquidity.[5] In Booth and Chua (1996), IPO underpricing is used to encourage investors to gather costly information and to participate in the IPO; it is assumed that such participation increases liquidity in aftermarket trading and increases the value of the firm.[6] A key prediction of the Booth and Chua model is that more underpricing is associated with more liquidity. In Ellul and Pagano (2003), some investors that participate in the IPO may need to sell their share holdings soon thereafter into an illiquid IPO aftermarket. These investors require a liquidity premium to participate in the IPO. The liquidity premium takes the form of IPO underpricing.[7] In contrast with Booth and Chua, the Ellul and Pagano model predicts that more underpricing is associated with less liquidity in the aftermarket.

This paper makes two contributions to the theoretical literature on underpricing. First, this paper is one of a handful of papers that studies underpricing and underperformance within the same framework. In much of the underpricing literature modeling both is not possible because many of the theoretical models are essentially two or three-period models that contain too few periods to study underperformance in aftermarket trading.[8] By contrast, my model of aftermarket trading is fully dynamic, and allows me to study aftermarket trade over thousands of time periods. A second contribution of this paper is that it highlights how the strategic environment in aftermarket trading can generate IPO underpricing. This represents a departure from most theoretical models of underpricing, because many of them do not model the aftermarket trading environment all, and those that do so, usually model it competitively [Booth and Chua (1996) and Ellul and Pagano (2003)].

[4] Other rational theories of underpricing are based on the underwriter deliberately underpricing in order to generate trading revenue for himself in the IPO aftermarket (Boehmer and Fishe, 2000), or the underwriter colluding with other investors against the issuer (Bias et. al. 2002).

[5] To date, I am aware of only two theoretical papers on this subject.

[6] Westerfield (2003) is similar to Booth and Chua in that underpricing is used to change the base of investors in the IPO aftermarket. In Westerfield, there are irrational noise traders, and their presence in the investor base reduces the value of the new issue because a risk premium is required for noise-trader risk. Underpricing is assumed to reduce the relative share of noise traders in the investor-base, and hence enhances the value of the firm.

[7] The Ellul and Pagano model is essentially a three period model, but it is very rich in some dimensions. For example, it incorporates asymmetric information, illiquidity, and risk averse investors within the same framework. Additionally, their paper contains a substantial empirical section where they show that more illiquidity after the IPO is associated with more IPO underpricing.

[8] For example, see Rock(1986), Benveniste and Spindt(1989), Booth and Chua (1996), and Ellul and Pagano(2003)

There is a small theoretical literature on underperformance. Ritter and Welch (2002) claim that there are no rational theoretical models of IPO underperformance. If so, then a rational theory of underperformance is a unique contribution of this paper. In a very interesting paper, Ljungqvist, Nanda, and Singh (2003) present a behavioral model of IPO underpricing and underperformance. Their key behavioral assumption is that there are irrationally exuberant sentiment investors in the aftermarket whose presence causes long-run return underperformance. They also assume the demand of sentiment investors will grow through time, or end abruptly, causing a price collapse. Their paper shows if legal constraints prevent the underwriter from selling above the offer price in the aftermarket to sentiment investors, then the next best strategy for the underwriter is to sell to rational investors at the IPO, who in turn sell to sentiment investors in the aftermarket. The rational investors require a premium for the risk that sentiment will end before they can sell. This risk premium takes the form of underpricing at the IPO.

My model shares some elements in common with Ljungqvist, Nanda, and Singh. In both models, the underwriter optimally distorts share allocations towards one class of investors at the IPO, and those investors slowly sell their assets to other investors through time in the aftermarket. Additionally, the strength of the results in both models depends on strategic considerations as measured by the competitiveness of aftermarket trading.[9] Despite the similarities, the two models are very different. The main contribution of my approach is that I show that a fully rational model can generate underpricing and underperformance.

The rest of the paper proceeds in six parts. Sections 2, 3, and 4, provide a model overview followed by details on the IPO aftermarket, and on the process for share allocation and price setting at the IPO. Section 5 uses simulations to study whether the model generates underpricing and underperformance; section 6 discusses the empirical implications of the model and provides a brief review of the most closely related empirical literature; a final section concludes.

2 Model Overview

The basic model is a stylized IPO in which a firm that wishes to raise capital by selling X^{IPO} shares of stock enlists a single underwriting firm to market the issue. To abstract from agency issues, the underwriter is assumed to act on behalf of the issuer.[10] The underwriter sells the issue to an investor base that consists of M risk-averse investors who participate in the IPO and trade in the aftermarket. Investor 1 represents a continuum of small investors who each take prices as given. Investors 2 through M are large investors whose desired aftermarket trades are large enough to move asset prices. Because of differences in their size, the small investors can be viewed as representing the demands of retail investors, while the

[9] Ljungqvist, Nanda, and Singh assume the aftermarket is not perfectly competitive because they assume that their rational investors coordinate their trades in the aftermarket.

[10] Some of the research in the IPO literature attributes underpricing to agency problems between the underwriter and the issuer [Biais, Bossaerts, and Rochet (2002); Boehmer and Fishe (2000)].

large investors represent the demands of institutional investors. The process for setting the IPO offer price and share allocations is modeled as a two-stage game. In the first stage, the underwriter assesses the demand for the new issue by learning about the characteristics of the investor base, and about aftermarket trading conditions. Based on his information, the underwriter sets a uniform IPO offer price and offers take-it or leave-it share allocations to the investors.[11] In the second stage, investors decide whether to accept their allocations. Any shares that are turned down at the IPO are sold by the underwriter in the aftermarket.

To rule out the possibility that the results are driven by informational differences between the underwriter and investors, or between the investors themselves, I assume that information on investors risk preferences, asset holdings, and the entire model of aftermarket trading is publicly available at all points of time and is common knowledge. The next section formally models the IPO aftermarket; and the following section models the share allocation and price-setting process at the IPO.

3 The IPO aftermarket

The model of trading in the IPO aftermarket is a partial equilibrium extension of Pritsker's (2004) multiple-asset heterogeneous agent model of imperfect competition in asset markets.[12] Investors trade a riskfree asset and two distinct sets of risky assets. The first set are shares of firms that belong to a particular market segment or group (for example manufacturers of semi-conductor parts); the new issue is one of the assets within the segment. I will refer to these assets as the segment-assets, and I will refer to the vector of their returns as the segment returns. The second set of assets are proxies for systematic risk factors. For simplicity, the only systematic risk factor is the market portfolio. Additionally, in order to focus on the liquidity of the segment-assets, I abstract from other sources of illiquidity by assuming that the traded proxy for the market portfolio is perfectly liquid.

I assume there is market segmentation which takes the form that, for informational or other reasons, the continuum of small investors and investors 2 through M are the only investors that trade the segment-assets and participate in the IPO. These investors are so small relative to the economy that their collective trades have no effect on interest rates or on the market return; but their actions do affect the returns of the segment-assets.

The investors are modeled as being infinitely lived, but the segment-assets are only traded for a large but finite number of periods T. After period T investors continue to hold their segment shares; they continue to trade all other assets; they continue to receive dividends; and they continue to consume. The requirement that there is a final period of trade facilitates

[11] The first stage share allocation process resembles IPO bookbuilding: in both processes the underwriter collects information about market demand, and then allocates shares and sets an offer price based on the information that he collects.

[12] Closely related models of imperfect competition in asset markets include Urosevic (2002a & b), DeMarzo and Urosevic (2000), and Vayanos (2001).

solution of the model through backwards induction. The requirement can also be understood as an an assumption that market liquidity for the segment-assets eventually dries up. The time until the liquidity dries up influences the dynamic behavior of the model.

The Assets

Investors trade a risk-free asset, the segment assets, and the market proxy. The gross per-period risk free rate is fixed at $r > 1$. There are N^1 segment assets whose total supply is denoted the by $N^1 \times 1$ vector X^1.[13] The time t prices of the segment-assets and the market are denoted $P^1(t)$ and $P^2(t)$ respectively. $P(t)$ denotes the stacked vector of risky asset prices at time t. Similar naming conventions will be followed throughout the rest of the paper. The risky assets pay i.i.d. dividends $D(t)$ in each period:

$$D(t) \sim \text{i.i.d.} \quad \mathcal{N}(\bar{D}, \Omega) \tag{1}$$

where

$$\Omega = \begin{pmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{pmatrix}$$

is the partitioned variance-covariance matrix of the assets' returns.

Because dividends are normally distributed, the risky assets are not limited liability instruments; and hence their share price can drop below zero. Because of this possibility, the returns in excess of the risk free rate are best expressed in units of return per share instead of units of return per dollar invested. This means that assets excess return over the riskless rate per share are given by the vector:

$$Z(t) = P(t) + D(t) - rP(t-1) \tag{2}$$

Because the model is partial equilibrium, I assume that $P^2(t)$ is exogenous, and for simplicity, fixed for all $t = 1, \ldots \infty$. This implies that excess returns on the market portfolio are i.i.d. through time with mean $\bar{Z}^2$ and variance Ω_{22}.

It is useful to decompose the vector of segment-assets returns into a vector of components that are perfectly correlated with the market and into a residual return vector $e(t)$:

$$Z^1(t) = \beta_{12} Z^2(t) + e(t) \tag{3}$$

where $\beta_{12} = \frac{\text{Cov}[Z^1(t), Z^2(t)]}{\text{Var}[Z^2(t)]} = \Omega_{21}\Omega_{22}^{-1}$, has the same interpretation as β in the CAPM.

The market component of the segment-assets returns can be hedged by trading the market portfolio. The residual component is not hedgeable, but it can usually be diversified under the assumption that a very large number of investors can each take a very small piece of the

[13]The supply of assets for the market proxy does not play a role in the analysis because the segment-investors are such a small part of the economy that collectively their asset demands do not affect the return on the market portfolio.

residual risk. In the current setting, the residual risk is not diversifiable because it is only shared by M investors. Therefore, the expected return for holding the residual risk will not necessarily be equal to 0. The variance of $e(t)$ is denoted Ω_e; in equilibrium it turns out to be constant through time and is given by:

$$\Omega_e = \Omega_{11} - \Omega_{21}\Omega_{22}^{-1}\Omega_{12}.$$

Note: Ω_e will not be diagonal if the dividends of the segment-firms have a common component that is uncorrelated with the market.

Investors

There are M investors in the model. With great loss of generality, each investor m has a time-separable per period utility of consumption that takes the CARA form with absolute risk aversion parameter A_m. Investors choose their consumption and asset holdings to maximize their discounted expected CARA utility of consumption:

$$U_m(C_m(1), ...C_m(\infty)) = \sum_{t=1}^{\infty} -\delta^t e^{-A_m C_m(t)}. \tag{4}$$

Investor $m's$ holdings of risky assets at the beginning of time t is denoted by $Q_m(t)$ which is the stacked vector of his holdings of the segment-assets and the market. The change in his risky asset holdings during period t is denoted by $\Delta Q_m(t)$ $(= Q_m(t+1) - Q_m(t))$. Investors choose their consumption and asset holdings subject to the standard set of intertemporal budget constraints:

$$W_m(t) = Q_m(t-1)'Z(t) + r[W_m(t-1) - C_m(t-1)] \quad t = 1, \ldots T, \tag{5}$$

where $W_m(t)$ denotes total wealth at the beginning of time t.

Although the budget constraint will be formally satisfied for all investors, the interpretation of $W_m(t)$ is different for large and small investors. For small investors, $W_m(t)$ is the liquidation value of their wealth because each small investor is infinitesimal and hence his sales have no effect on prices. By contrast, $W_m(t)$ is not the liquidation value of a large investors wealth because her attempts to sell the segment-assets would depress their prices. Large investors liquid wealth (which can be sold without loss of value) appears as a separate argument in their value functions; therefore, it is useful to express large investors intertemporal budget constraints in terms of the evolution of their liquid wealth. Investor $m's$ liquid wealth at the beginning of time t, denoted by $W_{ml}(t)$, consists of dividends on their beginning of time t share holdings plus the value of their bond portfolio plus the value of their holdings of the market portfolio. Their intertemporal budget constraints expressed in terms of liquid wealth have form:

$$\begin{aligned} W_{ml}(t) = {} & Q_m^1(t)'D^1(t) + Q_m^2(t)'Z^2(t) \\ & + r\left[W_{ml}(t-1) - \Delta Q_m^1(t-1)'P^1(t-1) - C_m(t-1)\right] \quad t = 1, \ldots T. \end{aligned} \tag{6}$$

It will turn out that when there is imperfect competition, the stacked vector of investors segment-asset holdings is a crucial state variable. This state variable is denoted by denoted by $Q^1(t) = \text{vech}(Q_1^1(t)', \ldots, Q_M^1(t)')'$.

Trading Dynamics

In each time period $t \leq T$, investors enter the period with with their holdings $Q_m(t), m = 1, \ldots M$. They receive dividends on their risky asset holdings; they choose their risky asset trades $\Delta Q_m(t)$, and these trades determine risky asset prices $P(t)$; investors then make their consumption choices, and then the period ends.

The process of trade for the segment-assets is modeled as a dynamic Cournot-Stackelberg game of full information. In each period $t \leq T$, each small investor computes his demand for the segment assets is conditional on $(Q^1(t), t)$. The aggregated demands of the small investors form a schedule of prices at which they are willing to absorb all possible quantities of the large investors demand for the segment assets. Given this price schedule, during period t large investors play a Cournot game in which they choose their trades while taking the price schedule and other large investors trades as given. The equilibrium trades are those for which each large investors trade is a best response to the trades of all of the other large investors. Within the period, the price schedule and the set of equilibrium trades is a Stackelberg Cournot Nash equilibrium. The entire model of trading is solved by backwards induction from period T; therefore investors optimal trading strategies are fully rational and subgame perfect.

For ease of exposition, the above discussion only focuses on the investors demand for the segment-assets. It should be understood that as investors alter their holdings of the segment assets they also alter their holdings of the market portfolio. The appendix solves for investors demand for the market portfolio and for the segment assets together.

To illustrate the derivation of the price schedule for the segment-assets at period t, without loss of generality assume that an equilibrium price function has been derived for time $t+1$ that maps investors holdings of the segment-assets at the beginning of period $t+1$ into equilibrium prices during time $t+1$.[14] The presence of such a price function is necessary so that small investors can compute their expected future wealth at time $t+1$. Given the price function, and state variable $Q^1(t)$, large investors submit risky-asset orderflow $\Delta Q_m^1(t)$, $m = 2, \ldots M$. Based on this orderflow, there exists a market clearing price $P^1(., t)$, for which the risky asset trade vector $\Delta Q_s(t)$, of each infinitesimal investor s, $s \in [0, 1]$, solves the maximization problem:

$$\max_{C_s(t), \Delta Q_s(t)} -e^{-A_s C_s(t)} + \delta E_t\{V_s(W_s(t+1); Q^1(t) + \Delta Q^1(t), t+1)\}, \tag{7}$$

[14] This is without loss of generality because I derive equilibrium price functions for all trading periods using dynamic programming from time infinity until the last period of trade, and then use backwards induction from the last period of trade.

subject to the budget constraint,

$$W_s(t+1) = Q_s(t+1)'Z(t+1) + r[W_s(t) - C_s(t)]$$

where, $Q_s(t+1) = Q_s(t) + \Delta Q_s(t)$.

Equation (7) represents the portfolio choice and consumption problem of each small investor in its dynamic programming form. The arguments of small investors value function are time, their future wealth, and the state variable $Q^1(t+1) = Q^1(t) + \Delta Q^1(t)$. The state variable $Q^1(t+1)$ affects the demand of each small investor, but because each small investor is infinitesimal, his asset demands do not affect the state variable.

The equilibrium price schedule $P^1(.,t)$ enters equation (7) because it affects $Z^1(t+1)$ in the budget constraint. For the price schedule $P^1(.,t)$ to be market clearing, each small investors net purchases of the segment-assets, denoted by $\Delta Q^1_s(t)$ must satisfy equation (7) and prices must be set so that the net orderflow of the small and large investors sums to 0:

$$\int_0^1 \Delta Q^1_s(t)\ ds + \sum_{m=2}^{M} \Delta Q^1_m(t) = 0. \tag{8}$$

The price schedule must also be consistent with an additional internal consistency condition for small investors orderflow. Recall that small investors are infinitesimal. This means that they take the orderflow of the other small investors as given and treat it as a state-variable. For small investors beliefs about the state variable to be internally consistent, $\Delta Q^1_1(t)$, their beliefs about the net trades of all small investors in equation (7), must be consistent with the optimal behavior of small investors conditional on their beliefs; i.e. internal consistency requires that[15]:

$$\Delta Q^1_1(t) = \int_0^1 \Delta Q^1_s(t) ds \tag{9}$$

For any given set of trades by the large investors, I solve for equilibrium prices which satisfy the market clearing and internal consistency conditions. Each such price $P^1(.,t) = P^1(\Delta Q^1(t), Q^1(t), t)$ is one point on the price schedule which is faced by the large investors. The full price schedule is found by solving the above problem for all possible $Q^1(t)$ and all possible $\Delta Q^1(t)$. The resulting price schedule turns out to a linear function of the state variable $Q^1(t)$ and large investors trades for the segment assets $\Delta Q^1_m(t)$, $m = 2, \ldots M$:

$$P^1(.,t) = \frac{1}{r}\left(\beta_0(t) - \beta_{Q^1}(t)Q^1(t) - \sum_{m=2}^{M} \beta_m(t)\Delta Q^1_m(t)\right). \tag{10}$$

[15] $\Delta Q^1_1(t)$ corresponds to the first row of the $Q^1(t)+\Delta Q^1(t)$ argument of the small investors value function in equation (7).

Given the demand curve in equation (10), large investors choose trades and consumption to solve the maximization problem:

$$\max_{\substack{C_m(t),\\ \Delta Q_m(t)}} \quad -e^{-A_m C_m(t)} - \delta\, \mathrm{E}_t\, V_m(W_{ml}(t+1), Q^1(t) + \Delta Q^1(t), t+1) \tag{11}$$

subject to the budget constraint:

$$W_{ml}(t+1) = Q^1_m(t+1)'D^1(t+1) + Q^2_m(t+1)'Z^2(t+1) + r\left[W_{ml}(t) - \Delta Q^1_m(t)'P^1(.,t) - C_m(t)\right]. \tag{12}$$

In equation (11), the arguments of large investors value function include time, liquid wealth $W_{ml}(t+1)$, and the state variable $Q^1(t+1)$. Each large investors trades affect the state variable and prices. Large investors account for both of these effects when trading.

The trade and consumption choices of large and small investors are an equilibrium, if small investors demand and consumption choices satisfy equation (7), large investors trades are a Nash Equilibrium of the Cournot game, and their trade and consumption choices satisfy equation (11), investors choices satisfy the market clearing and internal consistency conditions given in equations (8) and(9). Finally, large and small investors value functions in every time must be subgame perfect. Because they are solved by backwards induction, they are subgame perfect by construction.

The form of investors value functions, and the form of the equilibrium price function in each period is given in the following proposition:

Proposition 1 *At time $t < T$ when the state vector of segment-asset holdings is Q^1, small investors value function of entering period t with wealth W_s is given by:*

$$\begin{aligned} V_s(W_s, Q^1, t) &= -K_1(t)\ F(Q^1, t)\ e^{-A_s(t)W_s}, \\ \textit{where } F(Q^1, t) &= e^{-Q^1(t)'\bar{v}_s(t) - Q^1(t)'\theta_s(t)Q^1(t)}. \end{aligned} \tag{13}$$

Additionally, large investor m's value function for entering period t with liquid wealth is W_{ml} is given by:

$$V_m(W_{ml}, Q^1, t) = -K_m(t)e^{-A_m(t)W_{ml} - A_m(t)Q^{1\prime}\Lambda_m(t) + .5A_m(t)^2 Q^{1\prime}\Xi_m(t)Q^1} \qquad m = 2, \ldots M, \tag{14}$$

and the price function for segment-assets has the functional form:

$$P^1(t) = \frac{1}{r}(\alpha(t) - \Gamma(t)Q^1) \tag{15}$$

Proof: See section B of the appendix.

In the small investors value function, the parameters $A_s(t)$, $\bar{v}_s(t)$ and $\theta_s(t)$ are a scalar, an $N^1M \times 1$ vector, and an $N^1M \times N^1M$ matrix respectively.[16] The parameters $A_m(t)$, $\Lambda_m(t)$, and $\Xi_m(t)$ from large investors value functions are similarly dimensioned. The parameters of the value functions in each time period are the solution of a system of nonlinear Riccati difference equations that are solved backwards from date T. The details are in the appendix.

An important property of the model is that investors value functions depend on how the segment assets are distributed among the investors. This suggests that how the segment assets are distributed among investors just after the IPO will influence asset demands and equilibrium asset returns. This topic is addressed in the next section when I discuss how the assets are priced.

3.1 Asset Pricing

In order to interpret the main results on asset pricing when there is imperfect competition, it is useful to first consider a competitive benchmark model that is the same as the model in all respects except that all of the investors are price-takers. The competitive benchmark is examined below.

Asset Pricing with Perfect Competition

The results on asset pricing in a competitive framework are provided in the next proposition:

Proposition 2 *If the segment-assets are traded in a perfectly competitive environment in which all investors take asset prices as given, then the equilibrium expected excess return for the segment-assets has a 2-factor structure:*

$$\bar{Z}^1(t) = \beta_{12}\bar{Z}^2(t) + \lambda_{[X^1]}\Omega_e X^1, \tag{16}$$

with market price of risk for the second factor given by

$$\lambda_{[X^1]} = \frac{1-(1/r)}{\sum_{m=1}^{M} 1/A_m}. \tag{17}$$

Investors equilibrium holdings of the segment assets are constant in all periods. The equilibrium segment-asset holdings of investor m are denoted Q_m^{1W} and given by:

$$Q_m^{1W} = \frac{(1/A_m)X^1}{\sum_{m=1}^{M}(1/A_m)}. \tag{18}$$

[16] Recall that there are N^1 segment assets.

Proof: See section D.1 of the appendix.

Equation (16) shows that the expected excess return of the segment-assets consists of a reward for its systematic risk plus an additional reward for its residual risk. The reward for the market risk is standard. To interpret the reward for residual risk, note that the investors can hedge the systematic component of the segment-assets returns but have to share their residual risk. Therefore, one can view the investors as trading in a submarket for the residual risk. Recall X^1 is the $N_1 \times 1$ supply vector of the segment-assets. The quantity $X^{1\prime}e(t)$ can be interpreted as the "market portfolio" of the segment-assets' residual-returns; and $\Omega_e X^1$ denotes the vector of covariances of the segment-assets' residual returns with this "market portfolio". Because the residual returns are normally distributed and shared by investors who have CARA utility, intuition suggests there should be a CAPM-like pricing relationship in which the reward for bearing each segment-assets residual-risk should be based on its covariances with the "market portfolio" of the segment-assets residual-risk.[17] This intuition is confirmed by the second term on the right hand side of equation (16). In the equation, the price for bearing a segment-assets residual-risk, $\Lambda_{[X^1]}$, depends on the sum-total of investors risk tolerances $(1/A_m)$. I refer to this sum as the risk bearing capacity of the investors in the segment.

Because the segment is perfectly competitive, risk sharing among market participants is efficient; and investors efficient risky asset holdings are intuitive: the proportion of the asset supply that each investor holds is equal to his risk bearing capacity $(1/A_m)$ as a proportion of the segments total risk bearing capacity.

Because of market segmentation, it might be more appropriate to label the risk sharing among investor as *constrained* efficient. If instead of market segmentation, all investors in the economy could freely trade the segment-assets, they would drive the reward for residual risk to 0.

Asset Pricing with Imperfect Competition

When there is imperfect competition in asset markets, examination of equation (10) shows that the more segment assets that a large investor attempts to buy or sell within a period, the more he moves their price. This suggests if large investors holdings of the segment-assets are not efficient, then they will tend to trade slowly towards efficient asset holdings in order to minimize the price impact of their trades. This slow trading affects how risks are shared along an equilibrium adjustment path, and may affect how the segment assets are priced. This intuition is confirmed in the next proposition:

Proposition 3 *When investors holdings of the segment assets are not efficient, then the segment-assets equilibrium excess expected returns satisfy a linear factor model in which the*

[17]Stapleton and Subrahmanyam (1978) derive circumstances in which the CAPM holds dynamically through time when investors have CARA utility and trade risky assets whose dividend payments are normally distributed.

first factor is the market portfolio, the second factor is the "market portfolio" of segment-asset residual risk, and the remaining factors correspond to the deviations of large investors asset holdings from those associated with efficient sharing of the residual risk:

$$\bar{Z}^1(t) = \beta_{12}\bar{Z}^2(t) + \lambda_{[X^1]}\Omega_e X^1 + \sum_{m=2}^{M} \lambda(m,t)\Omega_e(Q_m^1(t) - Q_m^{1W}) \tag{19}$$

Proof: See section D of the appendix.

The proposition shows that if investors asset holdings are the same as in the competitive benchmark of the model, then the segment-assets' returns will also be the same. However, if a large investors segment asset holdings deviate from those associated with efficient risksharing, then the deviation, measured as $[Q_m^1(t) - Q_m^{1W}(t)]'e(t)$ for large investor m, behaves like a priced factor.[18] In equation (19), the scalars $\lambda(m,t)$ represents the prices of risk for these additional factors at time t. These prices of risk are negative because if a large investor holds more than his efficient amount of risky assets, then because he will only sell it slowly through time, the marginal investor, in this case the small investors, expect to hold less and hence require a smaller premium for holding the residual risk.

The theoretical results on asset pricing generate potential explanations for post-IPO return underperformance.

Potential Explanations for Underperformance

A segment-assets return underperforms the market when its expected excess return is less than its market beta times the expected return on the market. Examination of equation (19) shows that an assets excess return can underperform the market when the sum of the second and third terms on the right hand side of the equation is less than zero. The imperfect competition model provides two potential channels for underperformance. The first channel is that a segment-asset's residual risk could be negatively correlated with the "market portfolio" of the segment-assets' residual-risk. Under this condition, if sharing of the segment residual-risk is efficient (which makes the third term 0), then the asset will underperform the market. This channel for underperformance is not as far-fetched as it might seem; and it might explain underperformance for some firms.[19] This channel also admits the more plausible possibility of an IPO outperforming the market if its residual returns are positively correlated with the "market-portfolio" of the segment-assets residual returns.

The second channel for underperformance comes from the third term in equation (19), which represents inefficient risk sharing among the investors. I want to study whether there

[18] That is, each segment-assets' expected excess return depends on its covariances with these factors.

[19] For example, if the firm that does the IPO competes with other firms in its segment, then good news for it might mean bad news for its competitors. More specifically, good news about the residual component of the IPO firm's business might be associated with bad news for the residual component of its competitors businesses.

are initial inefficient segment-asset holdings that can cause the returns of a newly issued asset to underperform the market for long periods of time. To analyze this question, pretend for a moment that all investors holdings of all assets in the segment are efficient. Without loss of generality, assume the first asset in the segment is the new issue. If I perturb asset holdings away from efficiency by increasing investor 2's holdings of the new issue, while holding the supply of the risky assets constant, I need to change the asset holdings of another investor; because the holdings of the other large investors are constant, it is the holdings of the small investors that are being implicitly changed when I do such a perturbation in equation (19). Because $\lambda_{m,t}$ is nonzero, it is clear from the equation that for any target amount of return underperformance at time t, there is a perturbation of investor 2's holdings of the IPO firm away from efficient asset holdings such that the model generates that amount of underperformance. In other words, the imperfect competition model makes arbitrarily large amounts of underperformance theoretically possible over a single period. A corollary of proposition 3 shows that inefficient risk sharing at period t affects equilibrium excess returns at future time periods as well:

Corollary 1 *When segment-asset holdings are not efficient at time t, then the expected value of τ period ahead 1-period excess returns follow a factor model in which the market portfolio, the "market portfolio" of segment-asset residual-risk, and the deviation of large investors time t segment asset holdings from efficient segment-asset holdings are factors:*

$$\mathrm{E}_t[Z^1(t+\tau+1)] = \beta_{12}\bar{Z}^2 + \lambda_{[X^1]}\Omega_e X^1 + \sum_{m=2}^{M} \lambda_m(t,\tau)\Omega_e(Q_m^1(t) - Q_m^{1W})$$

Proof: See section D of the appendix.

Provided that the risk prices $\lambda_m(t, \tau)$ are nonzero for all τ, then using the same reasoning as for 1-period returns, the corollary shows that there are initial asset allocations in the imperfect competition model that can generate arbitrary amounts of return underperformance over the time horizon from periods 1 to T.

The corollary shows that underperformance relative to the market for long periods of time is a theoretical possibility in the model, but the result is not intuitive. To provide intuition, pretend for a moment that there is only 1 large investor and a continuum of small investors and that the large investor has a very large long position and the small investors have a very large short position. In the appendix I show that because all risky assets are liquid from the perspective of each small investor, small investors demand for risky assets only depend on the assets 1 period return and variance-covariance matrix. As a result, when small investors take a short position on assets in a segment, they require the expected return on the assets to underperform the market. Standard intuition suggests that return underperformance cannot represent an equilibrium because the large investor who has a long position should sell and this will cause returns to equilibrate. This intuition is largely correct; with one addendum: because of imperfect competition the large investor sells slowly to reduce his price impact, and thus the equilibration takes time. As a result, the segment asset holdings and trades follow an equilibrium adjustment path; along this path small investors initially have a very short position and require a very negative rate of return to justify holding it; over time large investors sell to the small investors, this reduces their short position, and it reduces the amount of return underperformance that is required over the next period. Eventually, the large investors sell enough to eliminate much if not all of the return underperformance, but along large portions of the adjustment path, the returns of the segment assets can underperform the market.

Although I have shown that in theory the model can generate segment-asset returns that underperform by an arbitrary magnitude for long periods of time, whether there is underperformance following an IPO depends on the competitiveness of the aftermarket.[20] If the aftermarket is sufficiently competitive, then large investors trades will not have much price impact, and they will be able to trade more quickly towards efficient risk sharing. Therefore, when the aftermarket is very competitive, the asset allocations that are needed to generate large amounts of underperformance will be very extreme—and the IPO might not generate such asset allocations. If the aftermarket is not very competitive, whether there is underpricing due to imperfect risk-sharing will also depend on which investors receive the assets at the IPO, and it will depend on the quantities they receive. To see why it matters which investors receive the assets, note that the $\lambda_m(t, \tau)$ coefficients that determine how imperfect risk sharing today affects future excess returns, and underpricing, varies by large investor, and is greater in magnitude for those large investors who have more market power, where market power measures an investors ability to influence asset prices. In the model, large investors have more market power the greater is their risk tolerance as a share of all investors risk tolerances.[21] Therefore, the question that needs to be answered is whether the asset allocations at the IPO are sufficiently distorted towards investors with market power,

[20] There are many possible methods to measure the competitiveness of the aftermarket. In the empirical analysis I use the Herfindahl index, which is a measure of the concentration of risk bearing capacity among the investors.

[21] As intuition for why large investors who are more risk tolerant have more market power suppose that a syndicate of M investors with CARA utility who differ in their absolute risk aversion bid the syndicate's reservation price for a pool of segment assets that have a 1-period residual risky expected payoff $\bar{D}$ with variance σ^2. If all syndicate members participate the reservation price is $\frac{\bar{D}/r}{\sigma^2 \sum_{m=1}^{M}(1/A_m)}$ and if investor j does not participate the reservation price is $\frac{\bar{D}/r}{\sigma^2[\sum_{m=1}^{M}(1/A_m)-(1/A_j)]}$. It is straightforward to show that syndicate members with greater risk tolerance have more ability to influence the syndicate's reservation price by not participating.

that return underperformance results.[22] This question is examined as part of the simulations. Before turning to the simulations, the next section describes how the IPO offer price is set, and how the shares are allocated.

4 IPO Share Allocation and Price Setting

The motivation for the analysis in this section is based on Pritsker (2004). Pritsker studies a situation in which a distressed seller has a given number of shares to sell into an imperfectly competitive market. Because the seller is essentially selling to the equivalent of an oligopoly in financial markets, it is not surprising that the seller receives a price that is worse than the competitive price. The size of the price discount depends on the intensity of competition for the distressed sellers orderflow; and it depends on the amount of impatience that the distressed seller has when selling his shares. Regarding the intensity of competition, it turns out that it depends on cross-sectional dispersion of large investors risk tolerances. If one large investor is far more risk tolerant than the others, then that large investor has significant market power because if he purchases a smaller amount in the distressed sale, then the asset sales will have to be absorbed by investors with greater risk aversion who will require a large price discount in order to hold the assets. By contrast, if the risk tolerances are spread more evenly among large investors, then the competition for the distressed sales is more intense and the drop in price due to the distressed sales is consequently smaller.[23]

The distressed seller may be able to sell at better prices if he is more patient and breaks up his trades through time instead of selling all at once. This forces the large investors to compete for the distressed sales through time and dilutes their market power.

Pritsker's distressed seller analysis is applicable to the IPO setting. In the IPO, the distressed seller is the issuing firm. For simplicity, in this version of the paper I abstract

[22] Which investors receive the assets at the IPO also influences their initial aftermarket price. To illustrate this point, recall from from proposition 1 that the price for the segment assets in period t will be equal to

$$\begin{aligned} P^1(t) &= \frac{1}{r}(\alpha(t) - \Gamma(t)Q^1) \\ &= \frac{1}{r}(\alpha(t) - \sum_{m=1}^{M} \gamma_m(t)\Omega_e Q^1_m), \end{aligned}$$

where the scalars $\gamma_m(t)$ differ by investor. Because the $\gamma_m(t)$ coefficients differ by investor, how shares are allocated at the IPO affects the equilibrium price in aftermarket trading. It turns out that when the asset holdings are distorted towards investors with more market power this raises the price at time 1 above the long-run equilibrium price of the issue.

[23] In Pritsker (2004) large investors can be interpreted as trading on behalf of identical small investors. Under this interpretation, large investors are agents who purchase risk and then spread it to their base of small investors. The large investors absolute risk tolerance is equal to the small investors risk tolerance multiplied by the mass of small investors that the large investor represents. This result is intuitive because the large investor should be more risk tolerant if he can spread a given amount of risk that he purchases among a larger base of investors.

away from how the size of the issue is chosen, and simply assume the issuer needs to sell X^{IPO} shares. The underwriter acts on his behalf by lining up investors to buy the issue, and by supporting the issue in the aftermarket. For his efforts, the underwriter receives a fee. I assume that the investors that seek shares in the IPO are the same investors that are modeled as trading in the IPO aftermarket. The IPO process resembles bookbuilding as practiced in the United States. The underwriter gathers demand information on the issue. In the model this information consists of knowledge about the other risky assets in the segment; the investors holdings of the segment-assets; the investors risk preferences; and whether there are investors who have market power in aftermarket trading. Based on this information, the underwriter sets an IPO offer price P^{IPO} and makes take it or leave offers of share allocations to the large and small investors. The large investors allocations' are denoted by $X_m^{IPO}, m = 2, \ldots M$. I assume that the small investors that are offered share allocations are offered identical amounts of shares. The fraction of small investors that are offered share allocations is denoted by ϕ.

The relevance of the distressed seller analysis is that if a large or small investor turns down the share allocation that he is offered, then I assume that the unallocated shares are sold immediately by the underwriter in the IPO aftermarket. The possibility that an investor can force distressed sales in the aftermarket serves as a threat that constrains how the issuer allocates shares and chooses the IPO offer price.[24] In particular, if an investor receive shares, the allocations and offer price must be set so that it cannot be in the interest of the investor to refuse their allocation and instead force the shares to be sold by the underwriter in the aftermarket. Of course, it is possible in theory that the underwriter might find it optimal to sell some shares in the aftermarket; denote these shares as X_U^{IPO} and the aftermarket price on the first day of trading as P_A^{IPO}.[25] This suggests that the underwriter chooses share allocations and the IPO offer price to maximize:

$$P^{IPO} \times (\phi X_1^{IPO} + \sum_{m=2}^{M} X_m^{IPO}) + P_A^{IPO} X_U^{IPO}, \tag{20}$$

where the first term measures revenues raised at the IPO, and the second represents revenues raised by distressed sales in the IPO aftermarket.

This maximization takes place subject to the constraints that the total issue is allocated:

$$\phi X_1^{IPO} + \sum_{m=2}^{M} X_m^{IPO} + X_U^{IPO} = X^{IPO}, \tag{21}$$

that there are no short-sales[26]:

$$X_U^{IPO} \geq 0, \text{ and } X_m^{IPO} \geq 0, m = 1, \ldots M, \tag{22}$$

[24]There are many other possible ways to model the threats that available to the large investors and the threats that are available to the underwriter.

[25]The aftermarket price on the first day of trading is equal to the equilibrium price function for time period 1 in the aftermarket.

[26]This constraint will be eventually relaxed to examine how underwriter short-selling affects the results.

and subject to incentive compatibility constraints that those who receive allocations in the IPO will accept the allocations. For small investors who receive allocations this condition takes the form that the value associated with participating in the IPO is greater than the value from not participating:

$$V_s[Q_s^{IPO}; Q^{IPO}, t^{IPO}+1] \geq V_s[Q_s; Q^{IPO}, t^{IPO}+1], \tag{23}$$

where the IPO occurs at time t^{IPO} and investors decide whether or not to participate based on the effect that the IPO has on their time $t^{IPO}+1$ value functions. The value functions that large and small investors use to evaluate whether to participate in the IPO are value functions that were derived for time 1 post-IPO trading in the aftermarket. For convenience, I have suppressed most, but not all, of the notation in the value functions. Specifically, small investors that participate have post IPO risky asset holdings Q_s^{IPO}. The post-IPO risky asset holdings of all investors is denoted Q^{IPO}. If a small investor chooses not to participate in the IPO, his post IPO risky asset holdings are Q_s. Note: that the above expression is for a multiple-asset context in which I assume that the shares of one of the assets is an IPO and the others are not. Note also that whether or not a small investor participates in the IPO has no effect on the state vector Q^{IPO} because each small investor is infinitesimal.

For large investors who receive share allocations, the incentive compatibility constraints take the form:

For every $m > 1$ such that $Q_m^{IPO} > 0$

$$V_m[Q^{IPO}, t^{IPO}+1] \geq V_m[Q_{-m}^{IPO}, t^{IPO}+1] \tag{24}$$

where large investor m's share allocation in the IPO is Q_m^{IPO} and Q_{-m}^{IPO} is the post-IPO share allocation if large investor m chooses not to accept his allocation.[27]

The assumption that the distressed sales occur immediately following the IPO is very strong. A more reasonable assumption is that any shares that the underwriter fails to sell at the IPO will instead be sold over τ_S periods following the IPO. This modeling assumption is consistent with empirical evidence, reported in Ellis et. al. (2000), that IPO underwriters engage in price support activities in the IPO aftermarket, and with evidence reported by Ellis et. al. (2002) which shows that underwriters are active participants in the IPO aftermarket for long periods of time.[28].

I assume that when the underwriter sells shares over τ_s time periods he will sell them optimally. By optimality I mean that the underwriter buys shares at the IPO offer price,

[27]When there is the possibility of distressed sales, as there is here, the equilibrium value functions and equilibrium price that that is associated with entering period $t+1$ have a similar form to those given in equations (13), (14), and (15), but with the state vector supplemented by an additional argument, which is the amount of distressed sales.

[28]In Ellis et. al.ś (2002) sample of 313 NASDAQ IPOs, the lead underwriter participated in an average of more than 90 percent of post IPO NASDAQ trades during the first day of the IPO; this amount tapers down over the next 140 days, but remained above 40 percent on average on the 140th day

and then trades his shares over the following τ_S time periods in order to maximize his own utility subject to the constraint that by time τ_S the underwriter holds no shares of the issue. It is assumed that the certainty equivalent value of the underwriters utility from buying and trading the shares is turned over to the issuing firm at the time of the IPO. For tractability I assume that the underwriter has CARA utility like the other large investors. Let $CE_U(Q^{IPO}, \tau_s)$ denote the underwriters certainty equivalent. Then, under the less restrictive assumption, the underwriter maximizes:

$$P^{IPO} \times (\phi X_1^{IPO} + \sum_{m=2}^{M} X_m^{IPO}) + CE_U(Q^{IPO}, \tau_s), \tag{25}$$

subject to the constraints that the total issue is allocated [equation (21)], that there are no short sales [equation (22)], and subject to a new set of participation constraints that account for the new behavior of the underwriter:

$$V_s[Q_s^{IPO}; Q^{IPO}, U(\tau_s), t^{IPO} + 1] \geq V_s[Q_s; Q^{IPO}, U(\tau_s), t^{IPO} + 1], \tag{26}$$

and

$$V_m[Q^{IPO}, U(\tau_s), t^{IPO} + 1] \geq V_m[Q_{-m}^{IPO}, , U(\tau_s), t^{IPO} + 1]. \tag{27}$$

The addition of the argument $U(\tau_s)$, which denotes the possibility that an underwriter optimally liquidates over τ_s periods differentiates the incentive compatibility constraints in equations (26) and (27) from those when the underwriter must sell his holdings immediately after the IPO (equations (23) and (24)). Because the underwriter is modeled as selling any unallocated shares over a longer amount of time, it alters large investors market power after the IPO. I expect that this will raise the IPO offer price and revenues raised through the IPO. Below I investigate whether it actually does so in the simulations that follow.[29]

5 Simulation Analysis

To study whether imperfect competition in the aftermarket can help explain underpricing and underperformance, I studied the behavior of the model when only a single risky asset, the new issue, is traded in the aftermarket. Liquidity in the aftermarket depends on two state-variables. The first is the distribution of risk tolerances across investors, which was alluded to above, and the second is the number of post-IPO trading periods. When the number of post IPO trading periods is small, there is little opportunity to spread risks across investors

[29]The solution for the model with distressed sales is closely based on Pritsker (2004). To save space, it is not presented in the appendix.

through time. Consequently, investors who take on positions require more compensation for doing so and the market becomes more illiquid. The market is most illiquid when no trading periods remain. Conversely, as the number of remaining time periods gets large, the market becomes increasingly liquid and in the limit becomes perfectly competitive.[30] I believe that in reality financial markets are not perfectly competitive; the only way to accommodate this within the present model is through a finite number of trading periods. I study the behavior of the model when the number of trading periods after the IPO ranges from a high of 2000 trading periods, to a low of 200. Each trading period is interpreted as 1-business day. To date, I have solved the model for 4 configurations of investors. In all configurations, large investors are labeled as "Institutional Investors" and the small investors as "Retail Investors". Results are presented when there is a continuum of small investors and 5 large investors who differ in their risk aversion. Recall that when risk-sharing is efficient, the proportion of each risky asset's supply that should be held by each large investor is equal to his risk tolerance as a fraction of the sum total of all investor's risk tolerances. I refer to this quantity as the investor's share of risk bearing capacity. Intuitively, an investor with a higher share of risk bearing capacity has more market power. One gauge of the competitiveness of trading in this segment is the concentration of risk bearing capacity among the investors. The concentration of risk bearing is measured by using the Herfindahl index from Industrial Organization. The Herfindahl index is equal to 10,000 times the sum of the squares of each investors share of risk bearing capacity.[31] The maximum size of the index is 10,000 which corresponds to the extreme case in which all of the risk bearing capacity is held by one investor; the minimum size of the index is 0 which corresponds to perfect competition, which formally requires that all investors are infinitesimal.[32]

Before presenting the simulation results, it is important to emphasize that solving the model is numerically challenging. The parameters of investors value functions are solved backwards for thousands of periods using a system of nonlinear Riccati difference equations; and each step backward in the solution involves a matrix inversion. The parameters of investors value functions are then used as inputs to solve the pricing and allocation problem in the IPO. The constraints in the IPO allocation and pricing problem are themselves nonlinear; and it is not certain that my optimization routines are finding global maxima. Given the numerical difficulties, the simulation results should be treated as preliminary.

The results from the simulations are provided in Tables 1 through 4; and are sorted by Herfindahl indices with the results for the least competitive cases presented first. The

[30] Recall that in my model, the concepts of illiquidity is that trades move prices, which is the same as the concept of market power. Additional intuition for the relationship between illiquidity and the number of post-IPO trading periods is based on Coasian analysis of the market power of a durable goods monopolist. Coase argues that the monopolist can get a higher price if he can commit to selling over a single time period; the possibility that he will sell over several periods erodes his market power. Kihlstrom argues that the Coasian analysis applies to stocks because stocks are durable goods; and he too shows that additional periods of retrade erode the monopolists market power. In the model of aftermarket trading, I suspect that the Coasian argument also holds; and that a larger number of periods of retrade erodes the oligopolists' (large investors) market power.

[31] Each small investors ideal percentage share of the market is 0.

[32] When each investor is infinitesimal, and indexed on $s \in [0, 1]$ then his risk bearing capacity is $1/A_s$ ds; and the Herfindahl index is 0 because the integral of the investors squared risk bearing capacities is 0.

simulations shed light on five questions. First, how does imperfect competition in the IPO aftermarket affect asset allocations at the IPO.

Asset Allocations

The distorting influence of imperfect competition on allocations at the IPO is measured in terms of percentage deviations from each investors efficient asset holdings. For example, Table 1, Panel B, shows that retail investors should receive 10 percent of shares in the IPO if the assets at the IPO are allocated to ensure efficient risk-sharing. Panel C, shows that retail investors were distorted by -100 percent from their optimal holdings; which means that in the optimal IPO allocations in the simulations, retail investors receive nothing and institutional investors receive everything. This pattern of allocation distortion away from retail investors is repeated in all of the results (Tables 1 - 4, panel C) and is consistent with stylized empirical evidence that retail investors perceive that they are cut out of IPO allocations, and that institutional investors benefit at their expense.

The fact that retail investors are cut out and institutional investors receive more allocations raises the question of which institutional investors receive the allocations, and how does this depend on the institutions' risk bearing capacities. The simulations suggest that the relationship is complicated. Intuition suggests that asset holdings should be distorted towards those investors with the greatest risk bearing capacity because risk bearing capacity is a proxy for market power. The results are partially consistent with this intuition: for a given Herfindahl index, when the number of Post-IPO trading periods is small enough, then asset holdings are distorted towards those institutional investors with the greatest amounts of risk bearing capacity (Tables 1-3, panel C). However, the simulations show that the intuition is incomplete because when there are a large enough number of post-IPO trading periods, asset holdings can be distorted away from large investors with the most risk bearing capacity and towards large investors that have less risk bearing capacity (Table 2 and 3, panel C).

Aftermarket trading

The second question is can the model rationalize the large amounts of trading volume after the IPO? The results on asset allocation distortions provide one potential explanation. Recall, that if the assets were allocated to those investors who valued them most, and if there was no private information, then there should be no trade. However, if asset allocations at the IPO are distorted away from efficient allocations and towards investors with market power, then trading volume will be generated in the aftermarket as investors adjust their asset holdings towards those associated with optimal risk sharing. I plan to present more detailed results on whether the model matches the time series pattern of post-IPO trading volume in future revisions.

Underperformance

The third question is can imperfect competition lead to return underperformance after the IPO. At the outset, it is important to note that because the new issue is the sole risky asset in the segment, in the long-run, when its share holdings become efficient, its excess return will outperform the market (equation (19)). The relevant questions are whether there is short-term underperformance, and whether the underperformance persists for a fairly long period of time.

The intuition that was provided earlier suggested that when asset allocations after the IPO are sufficiently distorted toward large investors, the returns would underperform and persist for a time. There are two notions of return underperformance: the first is relative to the market portfolio, and the second is short-term underperformance, which occurs if the returns on the new issue following the IPO are lower than the returns will be in the long run. The tables report results on the expected component of the new issue's residual returns. Return underperformance relative to the market occurs if the expected residual return is negative. Examination of Tables 1 through 3 shows that return underperformance relative to the market occurs when the Herfindahl index is high, or the number of trading periods following the IPO is sufficiently small. This result confirms that the model can generate underperformance relative to the market. Additionally, the second type of underperformance is present in all market configurations (Tables 1 - 4, panel C). An important question that is not answered by these simulation results is how long does the underperformance persist. Unfortunately, for these configurations, I did not compute the answer to this question. However, although I have not yet computed a full set of results, experiments with other market configurations have generated underperformance relative to the market that persists for periods of more than one year. I view these results on underperformance as encouraging.

Underpricing

The fourth question is whether imperfect competition generates underpricing in the IPO. The answer is a qualified yes: when the Herfindahl index is high enough, and the number of remaining trading periods is small enough, then underpricing does result (Panel A of Tables 1 - 3); and the amount of underpricing increases when the number of post-IPO trading periods is small. As the market becomes more competitive, the underpricing vanishes, but overpricing does not result. Hence, when averaging across different market configurations, it is clear that the model produces underpricing on average.

Can the underwriter's fees be rationalized?

Finally, the fifth question is whether the fees that are received by underwriters can be rationalized. I have attempted to provide an answer by simulating equilibrium offer prices when the underwriter can trade over many periods in the aftermarket, but I have encountered some numerical difficulties. Nevertheless, I but do have some very preliminary results. The first set of results were computed for the configurations in tables 1 through 4. In Table 1, results were computed for the case of 1000 Post-IPO trading periods. For this case, the presence of an underwriter who sells over 200 trading periods has essentially no effect on the revenues of the issuer; and little effect on the allocations or underpricing in the IPO. For the results in Table 2, the underwriter generates more revenue, but the allocations are little changed. By contrast, for the results in Tables 3 and 4, the optimum involves the underwriter allocating none of the issue in the IPO; instead he sells it over 200 periods in the aftermarket — and this increases the revenues received by the issuer. Although these preliminary findings are discouraging, in a set of additional recent simulations that are not fully reported here, I sometimes found circumstances when the underwriter keeps a large (33 percent) but not 100 percent stake in the issue and then sells it through time. By doing so, he increases the proceeds that the issuer receives by 25%. This suggests that the underwriter can sometimes provide very significant value to the seller by trading in the aftermarket. It is important to reiterate that this finding is a result of a purely strategic setting that does not contain any informational asymmetries. Although some of the most recent results on the underwriter are very encouraging, it is important to stress again that the results on the underwriter are preliminary and the numerical optimizations need to be carefully checked.

Interpretations

The simulation results provide qualitative evidence that imperfect competition in the aftermarket might help to explain observed patterns of IPO underpricing and underperformance. Closer examination of the tables shows that the reported percentage underpricing and underperformance are not quantitatively close to the amounts reported in the empirical literature. This is true, but caution is needed in interpreting the reported magnitudes, because the model's parameters can be altered to much more closely match the empirically observed patterns of percentage underpricing and underperformance, but doing so would probably give an unrealistic view of the model's true explanatory power. Alternatively, the model's parameters could be tied down through calibration, but that too would would be misleading because the model is highly stylized. A much better method for assessing the quantitative importance of the theory is through testing the empirical predictions of the model. That topic is addressed in the next section.

6 Empirical Implications

6.1 Testable predictions

The key features of the theoretical model are the assumptions that participation in the IPO and the aftermarket is limited, and the assumption that there are large investors that have market power in the IPO and in the aftermarket. These features of the model generate three testable predictions.

1. **Participation limits**: The theory predicts that underpricing and underperformance should be more prevalent among new issues for which there are greater barriers to participating in the IPO and trading in the aftermarket. Therefore, cross-sectional differences in participation costs should be positively correlated with cross-sectional differences in underpricing and underperformance.

2. **Market power**: Simulations based on the theoretical model show that cross-sectional differences in summary measures of investors market power across market-segments are positively related to cross-sectional differences in observed levels of IPO underpricing and underperformance.[33]

3. **Correlations**: The theory predicts that underpricing, and underperformance are positively related cross-sectionally, and both are positively related to the magnitude of allocation distortions at the IPO.

The most novel of these predictions is that the paper predicts that there is a theoretical relationship between investors market power and the magnitudes of IPO underpricing, and underperformance. Hopefully, this paper will stimulate additional research that studies the role of market power. To close this section, it is useful to briefly review the most closely related empirical literature on underpricing and underperformance.

6.2 Related literature

The empirical literature that is most closely related to this paper studies the relationship between after-market liquidity and underpricing or underperformance. The relationship between IPO underpricing and illiquidity has been empirically studied by Booth and Chua (1996), Hahn and Ligon (2004), and Ellul and Pagano (2003).[34] Although the Booth and Chua model makes predictions about the relationship between underpricing and aftermarket liquidity, they don't test this implication of their model; instead their tests focus on underpricing as compensation for costs of information gathering. Because such costs could generate underpricing irrespective of illiquidity, the implications of their tests for the relationship between underpricing and aftermarket liquidity are unclear. Hahn and Ligon attempt to

[33] Measures of market power could include measures of concentration in risk-bearing capacity (for example size of mutual funds), as used in this paper, but could also include informational notions of market power. For example, a firm might have significant market power in an IPO if other firms decision about whether to participate in the IPO and trade in the aftermarket is predicated on that firms decision to trade and participate.

[34] In related research that does not address asset pricing per se, Corwin, etâlŝtudy the evolution of market microstructure measures of liquidity through time following an IPO. A special aspect of their research is that they observe the limit order book, and hence can study the evolution of liquidity measures such as the depth of the limit order book, and the depth of the book relative to trading volume.

directly test the Booth and Chua hypothesis that underpricing is used to increase liquidity by running OLS regressions of market microstructure measures of aftermarket liquidity on IPO underpricing. In regressions that account for other determinants of illiquidity, their results are mixed; with coefficients on underpricing sometimes statistically significant and positive, sometimes statistically significant and negative, and sometimes not statistically significant at all. A potential difficulty with the Hahn and Ligon methodology is that causality may run from underpricing to illiquidity (as in Booth and Chua) as well as from illiquidity to underpricing (as in Ellul and Pagano). The possibility that causality runs in both directions suggests that an instrumental variable approach is needed. In Ellul and Pagano, they regress underpricing on a set of determinants for underpricing, including measures of aftermarket liquidity. Additionally, they recognize the potential for simultaneity bias and instrument for it in some of their regressions.[35] In all of Ellul and Pagano's regressions they find that more aftermarket illiquidity increases the amount of IPO underpricing. This finding is consistent with both their theory and my theory of IPO underpricing.

Although Ellul and Pagano's findings are favorable for liquidity-based theories of IPO underpricing, there is reason for caution in interpreting their results. One reason for caution is if underpricing is a risk premium for aftermarket illiquidity, then the logical extension of Ellul and Pagano's theory would suggest that in the aftermarket, IPO's should earn a positive and significant risk premium for aftermarket illiquidity. The fact that IPO returns underperform in the aftermarket, suggests that the mechanism driving aftermarket returns is more complicated than the theory of illiquidity considered by Ellul and Pagano.[36] Eckbo and Norli (2002) take this argument one step further; they claim that newly issued stocks are more liquid than other stocks with similar risk characteristics; and thus their returns should underperform. To establish this point empirically, Eckbo and Norli compare the returns of a rolling portfolio of newly issued stocks that are held for up to five years against the returns a portfolio of more seasoned issues that are matched on size and book to market. They find that after adjusting for these factors, and controlling for differences in liquidity, new issues do not underperform.

The Eckbo and Norli analysis highlights an important empirical question: what is the appropriate method to risk-adjust the returns on new issues. The theory in this paper suggests that adjusting returns for book-to-market is problematic. The reason is that the theory shows that the allocation distortions at the IPO biases the price of the new issue upward in aftermarket trading; that is, the prices will be higher just after the IPO than they will be in the long-run.[37] The temporarily high stock price will cause new issues to initially have a low book to market. At the same time the theory also predicts there will be return underperformance following the IPO. Because the theory's predictions of low returns and low book-to-market are consistent with the empirical evidence on how the Fama-French"book-to-market" factor affects returns, tests that adjust for book-to-market

[35] They do not report any results on tests for the strength of the instruments, nor do they report any results of tests for instrument validity.

[36] This critique does not rule out my theory that underperformance is caused by how imperfect competition in the aftermarket distorts share allocations at the IPO.

[37] See footnote 22 for details.

will remove the predictions of my liquidity/imperfect competition theory from the data being analyzed. Such tests will then have low power to detect underperformance due to illiquidity even when such underperformance is present.

6.3 Summary

In summary, it remains an open empirical question whether imperfect competition and illiquidity play a significant role in explaining IPO underpricing and underperformance. The theory in this paper suggests a new direction for empirical research on IPOs that uses differences in investors market power across market segments to help explain cross-sectional differences in IPO underpricing and underperformance.

7 Conclusions

In this paper I have presented a fully-rational symmetric information model of IPO bookbuilding that is followed by imperfect competition and illiquidity in a dynamic post-IPO trading environment. For some parameter values the model generates IPO allocations and offer prices that are consistent with underpricing at the IPO, return underperformance following the IPO, and a tilt in share allocations toward institutional investors and away from retail investors. I have also begun a highly preliminary analysis of the behavior of the underwriter in the IPO aftermarket; and have found that for some model parameterizations the underwriter, by trading in the IPO aftermarket, can dilute other investors market power and substantially increase the revenues raised by the issuer.

An important question going forward is determining the percentages of underpricing, underperformance, and underwriter fees, that can plausibly be attributed to imperfect competition and illiquidity in aftermarket trading. The best way to answer the question is through empirical research that studies the relationship between aftermarket competitiveness and the inefficiencies that are associated with the IPO process. Hopefully the results in this paper will stimulate further research along these lines.

Appendix

A Notation

There are M investors and $N = N_1 + N_2$ risky assets. The first N_1 assets are illiquid. The next N_2 assets are perfectly liquid. The risky asset holdings of investor m at time t are denoted by

$$Q_m(t) = \begin{pmatrix} Q_m^1(t) \\ Q_m^2(t) \end{pmatrix}$$

where $Q_m^1(t)$ and $Q_m^2(t)$ are investor $m's$ holdings of illiquid and liquid risky assets respectively. $Q^1(t)$ denote the $N_1M \times 1$ vector of all investors illiquid asset holdings at time t where

$$Q^1(t) = \begin{pmatrix} Q_1^1(t) \\ \vdots \\ Q_M^1(t) \end{pmatrix}.$$

$Q_1^1(t)$ represents the net asset holdings of a continuum of infinitesimal small investors indexed by s:

$$Q_1^1(t) = \int_0^1 Q_s^1(t)\mu(s)ds.$$

The small investors are often collectively referred to as the competitive fringe. $Q_2^1(t)$ through $Q_M^1(t)$ denotes the net illiquid risky asset holdings of large investors, and is denoted by the $N_1 \times (M-1)$ vector $Q_B^1(t)$. The change in investors illiquid risky asset holdings from the beginning of time period t to the beginning of time period $t+1$ is denoted by the $N_1M \times 1$ vector $\Delta Q^1(t)$. Similarly, $\Delta Q_1^1(t)$ and $\Delta Q_B^1(t)$ denote the change in the competitive fringe's illiquid asset holdings, and the change in the illiquid asset holdings of the large investors.

The algebra which follows requires many matrix summations and the use of selection matrices. Rather than write summations explicitly, I use the matrix $S = \iota_M' \otimes I_N$ to perform summations where ι_M is an M by 1 vector of ones, and I_N is the $N \times N$ identity matrix.[38] In some cases, the matrix S may have different dimensions to conform to the vector whose elements are being added. In all such cases, S will always have N, or N_1 rows. The matrix S_i is used for selecting submatrices of a larger matrix. S_i has form

$$S_i = \iota_{i,M}' \otimes I_N,$$

where $\iota_{i,M}$ is an M vector has a 1 in its i'th element, and has zeros elsewhere.[39] As above S_i will sometimes have different dimensions to conform with the matrices being summed, but it will always have N or N_1 rows.

In the rest of the exposition, I will occasionally suppress time subscripts to save space.

[38] For example, $SQ(t) = \sum_{m=1}^{M} Q_m(t)$

[39] To illustrate the use of the selection matrix, $Q_m(t) = S_m Q(t)$.

B Proof of Proposition 1

Proposition 1: *Small investors value functions for entering period t with liquid wealth W_s, when investors' state vector of illiquid asset holdings is given by Q^1 is given by:*

$$\begin{aligned} V_s(W_s, Q^1, t) &= -K_1(t)\ F(Q^1, t)\ e^{-A_s(t)W_s}, \\ \textit{where } F(Q^1, t) &= e^{-Q^1(t)'\bar{v}_s(t) - Q^1(t)'\theta_s(t)Q^1(t)}. \end{aligned} \tag{13}$$

Large investor m's value function for entering period t when the state vector of illiquid asset holdings is Q and his holdings of liquid wealth is W_m is given by:

$$V_m(W_m, Q^1, t) = -K_m(t)e^{-A_m(t)W_m - A_m(t)Q^{1\prime}\Lambda_m(t) + .5A_m(t)^2 Q^{1\prime}\Xi_m(t)Q^1} \quad m = 2, \ldots M, \tag{14}$$

and the price function for illiquid assets has the functional form:

$$P^1(t) = \frac{1}{r}(\alpha(t) - \Gamma(t)Q^1) \tag{A1}$$

Proof: The proof is by induction. Part I of the proof establishes that if the value function has this form at time t, then it has the same form at time $t-1$. Part II of the proof establishes the result for time T, the first period in which trade cannot occur.

B.1 Part I:

Suppose the form of the value function is correct for time t. Then, to establish the form of the value function at time $t-1$, I first solve for the competitive fringe's demand curve for absorbing the net order flow of the large investors. I then solve the large investors and competitive fringe's equilibrium portfolio and consumption choices, and then solve for the value function at time $t-1$.

The competitive fringe's demand curve

The competitive fringe represents a continuum of infinitesimal investors that are distributed uniformly on the unit interval with total measure 1, i.e. $\mu(s) = 1$ for $s \in [0, 1]$. At time $t-1$, each participant s of the competitive fringe solves:

$$\max_{\substack{C_s(t-1),\\ Q_s,\\ q_s}} \quad -e^{-A_s C_s(t-1)} - \delta\, \mathrm{E}[K_s(t) F(Q^1, t) e^{-A_s(t) W_s(t)}] \tag{A2}$$

where, Q_s is the stacked vector of small investor s's holdings of illiquid (Q_s^1) and perfectly liquid (Q_s^2) risky assets:

$$Q_s = \begin{pmatrix} Q_s^1 \\ Q_s^2 \end{pmatrix};$$

$Z(t)$ is the stacked vector of excess returns for the illiquid and liquid assets:

$$Z(t) = \begin{pmatrix} Z^1(t) \\ Z^2(t) \end{pmatrix} = \begin{pmatrix} P^1(t) + D^1(t) - rP^1(t) \\ P^2(t) + D^2(t) - rP^2(t) \end{pmatrix}; \tag{A3}$$

and small investors liquid wealth is given by

$$W_s(t) = Q_s' Z(t) + r[W_s(t-1) - C_s(t-1)].$$

Note: Although I refer to the first set of assets as illiquid, they are only illiquid for large investors whose trades have price impact. Because each small investor is infinitesimal, their trades do not have price impact and hence both assets are perfectly liquid from their perspective.

In equation (A3),

$$\mathrm{E}\, Z(t) \equiv \bar{Z}(t) \equiv \begin{pmatrix} \bar{Z}_1(t) \\ \bar{Z}_2(t) \end{pmatrix},$$

and

$$\mathrm{Var}\, Z(t) \equiv \Omega \equiv \begin{pmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{pmatrix}.$$

Substituting the expression for W_s in (A2) and taking expectations shows that small investors maximization becomes:

$$\max_{\substack{C_s(t-1),\\ Q_s}} \quad -e^{-A_s C_s(t-1)} - \delta F(Q^1, t) e^{-A_s(t) r[W_s(t-1) - C_s(t-1)] - A_s(t) Q_s' \bar{Z}(t) + .5 A_s(t)^2 Q_s' \Omega Q_s} \tag{A4}$$

In solving the model, it is useful to break small investors maximization into pieces by first solving for optimal Q_s^2 as a function of Q_s^1, and then solving for optimal Q_s^1. For given

Q_s^1, the first order condition for optimal Q_s^2 shows that optimal Q_s^2 is given by

$$Q_s^2 = \frac{1}{A_s(t)} \Omega_{22}^{-1} \bar{Z}_2(t) - \beta_{12}' Q_s^1, \tag{A5}$$

where $\beta_{12} = \Omega_{12}\Omega_{22}^{-1}$.

Plugging the solution for Q_s^2 into the small investors value function and simplifying then shows that the small investors maximization problem reduces to:

$$\max_{\substack{C_s(t-1),\\ Q_s^1}} \quad -e^{-A_s C_s(t-1)} - \delta F(Q^1, t) K_s(t) \operatorname{Exp}\left\{-.5 \bar{Z}_2' \Omega_{22}^{-1} \bar{Z}_2 - A_s(t) r [W_s(t-1) - C_s(t-1)]\right\}$$
$$\times \operatorname{Exp}\left\{-A_s(t) Q_s^{1\prime} [\bar{Z}_1(t) - \beta_{12}\bar{Z}_2(t)] + .5 A_s(t)^2 Q_s^{1\prime} \Omega_e Q_s^1\right\} \tag{A6}$$

where Ω_e is given by

$$\Omega_e = \Omega_{11} - \Omega_{12}\Omega_{22}^{-1}\Omega_{21}.$$

To gain intuition for the above expression, note that the excess return on each illiquid asset can be decomposed into a component that is correlated with the liquid assets and into a second idiosyncratic component.

$$Z_1(t) = \beta_{12} Z_2(t) + \epsilon_1(t)$$

$\bar{Z}_1 - \beta_{12}\bar{Z}_2(t)$ is the vector of expected returns on the idiosyncratic components at time t and Ω_e is the variance covariance matrix of the idiosyncratic returns. The expression shows that small investors portfolio maximization problem can equivalently be written in terms of choosing an exposure to the returns of the liquid assets, and to the idiosyncratic component of returns of the illiquid assets.

Solving for optimal $Q_s^1(t)$ then shows

$$Q_s^1(t) = \frac{1}{A_s(t)} \Omega_e^{-1} [\bar{Z}_1(t) - \beta_{12}\bar{Z}_2(t)] \tag{A7}$$

The aggregate demand for Q^1 at time t by all small investors can be found by integrating both sides of equation (A7) with respect to μ_s, the density of small investors, yielding:

$$\begin{aligned} Q_1^1(t) &= \int_0^1 Q_s^1(t)\mu_s ds \\ &= \left[\int_0^1 \frac{1}{A_s(t)}\mu_s ds\right] \Omega_e^{-1}[\bar{Z}_1(t) - \beta_{12}\bar{Z}_2(t)] \\ &= \frac{1}{A_1(t)}\Omega_e^{-1}[\bar{Z}_1(t) - \beta_{12}\bar{Z}_2(t)] \end{aligned} \tag{A8}$$

The Price Schedule Faced by Large Investors

The price schedule faced by large investors at time $t-1$ maps large investors desired orderflow of the illiquid assets into the time $t-1$ prices at which the competitive fringe is willing to absorb the net orderflow. To solve for the price schedule, I solve for prices $P(.,t-1)$ in equation (A8) such that when the large investors choose trade $\Delta Q^1_B(t-1)$ at time t-1, then the competitive fringe chooses trade $-S\Delta Q^1_B(t-1)$.

Rearranging, equation (A8) while making the substitutions

$$\begin{aligned} Q^1(t) &= Q^1(t-1) + \Delta Q^1(t-1), \\ Q^1_1(t) &= S_1[Q^1(t-1) + \Delta Q^1(t-1)], \\ \Delta Q^1(t-1) &= \begin{pmatrix} -S\Delta Q^1_B(t-1) \\ I\Delta Q^1_B(t-1) \end{pmatrix} \end{aligned}$$

and

$$\begin{aligned} \bar{Z}^1(t) &= P^1(t) + \bar{D}^1 - rP^1(t-1,.) \\ P^1(t) &= \frac{1}{r}\left(\alpha(t) - \Gamma(t)[Q^1(t-1) + \Delta Q^1(t-1)]\right) \end{aligned}$$

then produces the price schedule faced by large investors at time $t-1$:

$$P^1(.,t-1) = \frac{1}{r}\left(\beta_0(t-1) - \beta_{Q^1}(t-1)Q^1(t-1) - \beta_{Q^1_B}(t-1)\Delta Q^1_B(t-1)\right), \tag{A9}$$

where,

$$\begin{aligned} \beta_0(t-1) &= \bar{D}^1 + (1/r)\alpha(t) - \beta_{12}\bar{Z}^2 && \text{(A10)} \\ \beta_{Q^1}(t-1) &= (1/r)(\Gamma(t) + rA_1(t)\Omega_e S_1) && \text{(A11)} \\ \beta_{Q^1_B}(t-1) &= (1/r)\Gamma(t)\begin{pmatrix} -S \\ I \end{pmatrix} - A_1(t)\Omega_e S && \text{(A12)} \end{aligned}$$

Given the price schedule in equation (A9), large investors at time $t-1$ solve the maximization problem:

Large Investors Maximization Problem

$$\max_{\substack{C_m(t-1),\\ Q_m}} \quad -e^{-A_m C_m(t-1)} - \mathrm{E}\left\{\delta K_m(t)\,\mathrm{Exp}\left(-A_m(t)W_m - A_m(t)Q^{1\prime}\Lambda_m(t) + .5A_m(t)^2 Q^{1\prime}\Xi_m(t)Q^1\right)\right\} \tag{A13}$$

where, substituting in the budget constraint, liquid wealth at the beginning of time t is given by

$$\begin{aligned} W_m(t) =& Q^1_m(t)'D^1(t) + Q^2_m(t)'Z^2(t) \\ &+ r(W_m(t-1) - \Delta Q^1_m(t-1)'P^1(t-1,.) - C_m(t-1)) \end{aligned} \tag{A14}$$

Note: Because dividends are paid in cash, the dividend payments received for holdings of illiquid asset are counted as part of liquid wealth even though the illiquid assets themselves are not counted.

Note that in equation (A13), $\Lambda_m(t)$ and $\Xi_m(t)$ are deterministic functions of time that are parameters of the value function. Keeping this in mind, large investors holdings of the liquid assets are solved in the same way as for small investors. Taking expectations in equation (A13), solving for optimal Q^2_m given Q^1, and substituting the optimal choice back into the large investor's value function, transforms the large investors maximization problem so that it has the following form:

$$\begin{aligned} \max_{\substack{C_m(t-1),\\ Q^1_m}} \quad & - e^{-A_m C_m(t-1)} \\ & - \delta K_m(t)\left\{ \text{Exp}(-.5\bar{Z}^{2\prime}\Omega_{22}^{-1}\bar{Z}_2 - A_m(t)r[W_m(t-1) - \Delta Q_m(t-1)'P^1(t-1,.) - C_m(t-1)] \right. \\ & \left. \times \text{Exp}(-A_m(t)Q^{1\prime}\bar{v}_m(t) + .5A_m(t)^2 Q^{1\prime}\theta_m(t)Q^1) \right\} \end{aligned} \tag{A15}$$

where,

$$\bar{v}_m(t) = S_m'(\bar{D}_1 - \beta_{12}\bar{Z}_2) + \Lambda_m(t) \tag{A16}$$
$$\theta_m(t) = S_m'\Omega_e S_m + \Xi_m(t) \tag{A17}$$

The large investors play a Cournot game in which each choose his time $t-1$ trade $\Delta Q_m(t-1)$ in the illiquid assets to solve the maximization problem in (A15) while taking the trades of the other large investors as given, but while taking into account the effect that his own trades have on the prices of the illiquid assets. Recall the price impact function for the illiquid assets at time $t-1$ is given by equation (A9).

The first order condition for large investors illiquid asset choices is given by:

$$\begin{aligned} 0 = -A_m(t)[(-S_1 + S_m)\bar{v}_m(t)] &+ A_m(t)^2(-S_1 + S_m)[(\theta_m(t) + \theta_m(t)')/2](Q^1 + \Delta Q^1) \\ &+ A_m(t)\left[rP^1(.,t-1) - S_m\beta_{Q^1_B}(t-1)'S_m\Delta Q^1_B\right], \end{aligned} \tag{A18}$$

After substituting for $P^1(.,t-1)$ from equation (A9), writing $Q^1 + \Delta Q^1$ as $Q^1 + \begin{pmatrix} -S\Delta Q^1_B \\ \Delta Q^1_B \end{pmatrix}$ and simplifying, this produces the following reaction function for large investor m:

$$\pi_m(t-1)\Delta Q_B^1 = \chi_m(t-1) + \xi_m(t-1)Q^1, \tag{A19}$$

where,

$$\begin{aligned}\pi_m(t-1) =& A_m(t)(-S_1+S_m)[(\theta_m(t)+\theta_m(t)')/2]\begin{pmatrix}-S\\ I\end{pmatrix}\\ &-\beta_{Q_B^1}(t-1) - S_m\beta_{Q_B^1}(t-1)'S_m\end{aligned} \tag{A20}$$

$$\chi_m(t-1) = (-S_1+S_m)\bar{v}_m(t) - \beta_0(t-1) \tag{A21}$$

$$\xi_m(t-1) = \beta_{Q^1}(t-1) - A_m(t)(-S_1+S_m)[(\theta_m(t)+\theta_m(t)')/2] \tag{A22}$$

Stacking the (M-1) reaction functions produces a system of $(M-1)N$ linear equations in $(M-1)N$ unknowns:

$$\Pi(t-1)\Delta Q_B^1(t-1) = \chi(t-1) + \xi(t-1)Q^1(t-1) \tag{A23}$$

Assume that $\Pi(t-1)$ is invertible. Then the solution for $\Delta Q_B^1(t-1)$ is unique, and given by

$$\Delta Q_B^1(t-1) = \Pi(t-1)^{-1}\chi(t-1) + \Pi(t-1)^{-1}\xi(t-1)Q^1(t-1) \tag{A24}$$

Equilibrium Asset Holdings

The solution for $\Delta Q_1^1(t-1)$ is $-S\Delta Q_B^1(t-1)$. Therefore, the solution for $\Delta Q^1(t-1) = (\Delta Q_1^1(t-1)', \Delta Q_B^1(t-1)')'$ can be written as:

$$\Delta Q^1(t-1) = H_0(t-1) + H_1(t-1)Q^1(t-1). \tag{A25}$$

where,

$$H_0(t-1) = \begin{pmatrix}-S\Pi(t-1)^{-1}\chi(t-1)\\ \Pi(t-1)^{-1}\chi(t-1)\end{pmatrix}, \quad \text{and} \quad H_1(t-1) = \begin{pmatrix}-S\Pi(t-1)^{-1}\xi(t-1)\\ \Pi(t-1)^{-1}\xi(t-1)\end{pmatrix}. \tag{A26}$$

With the above notation, the equilibrium purchases by large participant m in period $t-1$ are given by

$$\Delta Q_m^1(t-1) = S_m[H_0(t-1) + H_1(t-1)Q^1(t-1)] \tag{A27}$$

Additionally, the equilibrium transition dynamics for beginning of period illiquid risky asset holdings are given by:

$$Q^1(t) = G_0(t-1) + G_1(t-1)Q^1(t-1) \tag{A28}$$

where $G_0(t-1) = H_0(t-1)$ and $G_1(t-1) = H_1(t-1) + I$.

Equilibrium Price Function

Recall that the equilibrium price function in each time period maps investors beginning of period holdings of risky assets to an equilibrium price after trade. The equilibrium price function for period $t-1$ is found by plugging the solution for large investors equilibrium trades from equation (A24) into the price schedule faced by large investors (equation (A9)). The resulting price function for illiquid asset in period $t-1$ has form:

$$P^1(t-1, Q^1) = \frac{1}{r}\left(\alpha(t-1) - \Gamma(t-1)Q^1\right) \tag{A29}$$

where,

$$\begin{aligned} \alpha(t-1) &= \beta_0(t-1) - \beta_{Q_B^1}(t-1)\pi(t-1)^{-1}\chi(t-1) && \text{(A30)} \\ \Gamma(t-1) &= \beta_Q(t-1) + \beta_{Q_B^1}(t-1)\pi(t-1)^{-1}\xi(t-1) && \text{(A31)} \end{aligned}$$

Large Investors Consumption

Large investors optimal time $t-1$ consumption depends on optimal time $t-1$ trades. After plugging the expressions for equilibrium prices, and equilibrium trades [equations (A28), (A29), and (A25)] into equation (A15), large investors consumption choice problem has form:

$$\max_{C_m(t-1)} -e^{-A_m C_m(t-1)} - \delta k_m(t) e^{rA_m(t)C_m(t-1)} \times \psi_m(Q^1(t-1), W_m(t-1), D(t-1), t-1), \tag{A32}$$

where

$$\begin{aligned} \psi_m(Q^1, W_m(t-1), t-1) = & e^{-.5\bar{Z}^{2\prime}\Omega_{22}^{-1}\bar{Z}^2 - A_m(t)rW_m(t-1)} \\ & \times e^{+A_m(t)r[S_m(H_0(t-1)+H_1(t-1)Q^1(t-1)]'(\alpha(t-1)-\Gamma(t-1)Q^1(t-1))/r} \\ & \times e^{-A_m(t)(G_0(t-1)+G_1(t-1)Q^1(t-1))'\bar{v}_m(t)} \\ & \times e^{.5A_m(t)^2[G_0(t-1)+G_1(t-1)Q^1(t-1)]'\theta_m(t)[G_0(t-1)+G_1(t-1)Q^1(t-1)]} \end{aligned} \tag{A33}$$

The first order condition for choice of consumption implies that optimal consumption is given by:

$$C_m(t-1) = \frac{-1}{A_m(t)r + A_m} \ln\left(\frac{\delta k_m(t)A_m(t)r\psi_m(Q^1(t-1), W_m(t-1), t-1)}{A_m}\right) \tag{A34}$$

Large investors value function at time $t-1$

Define $V_m(t-1, Q^1, W_m(t-1))$ as the value function to large investor m from entering period $t-1$ when the vector of illiquid risky asset holdings is Q^1, and his liquid asset holdings are $W_m(t-1)$. After substituting the optimal consumption choice in (A34) into equation (A32), this value function is given by:

$$V_m(W_m(t-1), Q^1, t-1) = -\left[\frac{1 + r_m^*(t)}{r_m^*(t)}\right] \left[\delta k_m(t) r_m^*(t) \psi_m(Q^1, W_m(t-1), t-1)\right]^{\frac{1}{1+r_m^*(t)}} \tag{A35}$$

where,

$$r_m^*(t) = A_m(t) r / A_m \tag{A36}$$

Tedious algebra then shows that large investor m's value function at time $t-1$ has form:

$$V_m(t-1, Q^1, W_m(t-1)) = -k_m(t-1) \times e^{-A_m(t-1)W_m(t-1) - A_m(t-1)Q^{1\prime}\Lambda_m(t-1) + .5A_m(t-1)^2 Q^{1\prime}\Xi_m(t-1)Q^1} \tag{A37}$$

where the parameters of the value function at time $t-1$ are given by the following Riccati difference equations.

$$A_m(t-1) = A_m(t) r / (1 + r_m^*(t)) \tag{A38}$$

$$\begin{aligned} k_m(t-1) = &\left[\frac{r_m^*(t) + 1}{r_m^*(t)}\right] \left[\delta k_m(t) r_m^*(t)\right]^{\frac{1}{1+r_m^*(t)}} \\ &\times e^{\frac{-.5\bar{Z}^{2\prime}\Omega_{22}^{-1}\bar{Z}^2}{1+r_m^*(t)}} \\ &\times e^{A_m(t-1)H_0(t-1)' S_m' \alpha(t-1)/r - A_m(t-1)G_0(t-1)'\bar{v}_m(t)/r + .5A_m(t-1)^2((1+r_m^*(t))/r^2)(G_0(t-1)'\theta_m(t)G_0(t-1))} \end{aligned} \tag{A39}$$

$$\begin{aligned} \Lambda_m(t-1) = &- H_1(t-1)' S_m' \alpha(t-1)/r + \Gamma(t-1)' S_m H_0(t-1)/r + G_1(t-1)'\bar{v}_m(t)/r \\ &- A_m(t-1)(1 + r_m^*(t)) G_1(t-1)' \left(\frac{\theta_m(t) + \theta_m(t)'}{2}\right) G_0(t-1)/r^2 \end{aligned} \tag{A40}$$

$$\Xi_m(t-1) = \frac{-2H_1(t-1)' S_m' \Gamma(t-1)}{r A_m(t-1)} + (1 + r_m^*(t)) G_1(t-1)' \theta_m(t) G_1(t-1)/r^2 \tag{A41}$$

Small investors optimal consumption

The solution for each small investors consumption depends on small investors optimal trades. To solve for optimal consumptions, I first use equation (A7) to substitute out for Q_s^1 in equation (A6). I then substitute out for $\bar{Z}^1(t) - \beta_{12}\bar{Z}^2(t)$ with the expression:

$$\bar{Z}^1(t) - \beta_{12}\bar{Z}^2(t) = a_0(t-1) + a_1(t-1)Q^1(t-1), \tag{A42}$$

where,

$$\begin{aligned} a_0(t-1) &= \frac{\alpha(t)}{r} - \alpha(t-1) + \bar{D}^1 - \beta_{12}\bar{Z}^2(t) - \frac{\Gamma(t)G_0(t-1)}{r} && \text{(A43)} \\ a_1(t-1) &= \Gamma(t-1) - \frac{\Gamma(t)G_1(t-1)}{r}. && \text{(A44)} \end{aligned}$$

Finally I substitute out $Q^1(t)$ with $[G_0(t-1) + G_1(t-1)Q(t-1)]$. With these substitutions, small investors choice of optimal consumptions simplifies to:

$$\max_{C_s(t-1)} -e^{-A_s C_s(t-1)} - \delta k_s(t) e^{rA_s(t)C_s(t-1)} \times \psi_s(Q^1(t-1), W_s(t-1), t-1), \tag{A45}$$

where,

$$\begin{aligned} \psi_s(Q^1(t-1), W_s(t-1), t-1) = {} & e^{-A_s(t)rW_s(t-1) - .5\bar{Z}_2'\Omega_{22}^{-1}\bar{Z}_2} \\ & \times e^{-.5[a_0(t-1)+a_1(t-1)Q^1(t-1)]'\Omega_e^{-1}[a_0(t-1)+a_1(t-1)Q^1(t-1)]} \\ & \times e^{-[G_0(t-1)+G_1(t-1)Q^1(t-1)]'\bar{v}_s(t)} \\ & \times e^{-[G_0(t-1)+G_1(t-1)Q^1(t-1)]'\theta_s(t)[G_0(t-1)+G_1(t-1)Q^1(t-1)]} \end{aligned} \tag{A46}$$

The first order condition for choice of optimal consumption implies that optimal consumption is given by:

$$C_s(t-1) = \frac{-1}{A_s(t)r + A_s} \ln\left(\frac{\delta k_s(t)A_s(t)r\psi_s(Q^1(t-1), W_s(t-1), t-1)}{A_s}\right) \tag{A47}$$

Small investors value function at time $t-1$

Define $V_s(W_s(t-1), Q^1(t-1), t-1)$ as the value function to small investor s from entering period $t-1$ when the vector of illiquid risky asset holdings is $Q^1(t-1)$, and his liquid wealth is $W_s(t-1)$. After substituting the optimal consumption choice in (A47) into equation (A45), this value function is given by:

$$V_s(W_s(t-1), Q^1(t-1), t-1) = \\ -\left[\frac{1 + r_s^*(t)}{r_s^*(t)}\right] \left[\delta k_s(t) r_s^*(t) \psi_s(Q^1(t-1), W_s(t-1), t-1)\right]^{\frac{1}{1+r_s^*(t)}} \tag{A48}$$

where,

$$r_s^*(t) = A_s(t)r/A_s \tag{A49}$$

Simplification then shows that the value function has form:

$$\begin{aligned} V_s(W_s(t-1), Q^1(t-1), t-1) &= -K_s(t-1)\ F(Q^1, t-1)\ e^{-A_s(t-1)W_s(t-1)}, \\ \text{where } F(Q^1(t-1), t-1) &= e^{-Q^1(t-1)'\bar{v}_s(t-1) - Q^1(t-1)'\theta_s(t-1)Q^1(t-1)} \end{aligned} \tag{A50}$$

The parameters in the small investors value functions at time $t-1$ are a function of time t parameters as expressed in the following Riccati equations:

$$A_s(t-1) = \frac{rA_s(t)}{1 + r_s^*(t)} \tag{A51}$$

$$\begin{aligned} k_s(t-1) = \left[\frac{r_s^*(t)+1}{r_s^*(t)}\right] \left[\delta k_s(t-1) r_s^*(t) e^{-.5\bar{Z}^{2\prime}\Omega_{22}^{-1}\bar{Z}^2}\right]^{\frac{1}{1+r_s^*(t)}} \\ \times \text{Exp}\left\{\frac{-a_0(t-1)'\Omega_e^{-1}a_0(t-1) - G_0(t-1)'\bar{v}_s(t) - G_0(t-1)'\theta_s(t)G_0(t-1)}{1+r_s^*(t)}\right\}, \end{aligned} \tag{A52}$$

$$\bar{v}_s(t-1) = \frac{a_1(t-1)'\Omega_e^{-1}a_0(t-1) + G_1(t-1)'\bar{v}_s(t) + G_1(t-1)'(\theta_s(t) + \theta_s(t)')G_0(t-1)}{1+r_s^*(t)}, \tag{A53}$$

$$\theta_s(t-1) = \frac{.5a_1(t-1)'\Omega_e^{-1}a_1(t-1) + G_1(t-1)'\theta_s(t)G_1(t-1)}{1+r_s^*(t)} \tag{A54}$$

This completes part I of the proof because equations (A37) and (A50) verify that the value functions at time $t-1$ have the same form as at time t.

B.2 Part II

To establish part II of the proof, I need to show that investors value functions for entering entering period T, the last period of trade, has the same functional form as given in the proposition. To establish this result, I first need to solve for investors value function at time $T+1$, the first period when investors cannot trade the illiquid assets (recall they can continue to trade the riskless asset and the liquid assets indefinitely). Then, given this value function, I use backwards induction to solve for investors value function at time T.

Investors Value Functions at Time T+1

Recall that investors are infinitely lived but that from time T onwards they cannot alter their holdings of illiquid assets, but they can continue to alter their consumption, and their holdings of liquid and riskless assets. Because investors cannot trade in period $T+1$ and after, the distinction between small and large investors after this period is irrelevant. Hence, the index m used below could be for either a large or small investor. Using the Bellman principle, the value function $V_m(.)$ of entering period $t+1$ ($t \geq T$) with illiquid asset holdings Q_m^1 and liquid wealth W_m satisfies the functional equation:

$$V_m(W_m(t+1), Q_m^1) = \max_{\substack{C_m(t+1) \\ Q_m^2(t+2)}} -\exp^{-A_m C_m(t+1)} + \delta\, \mathrm{E}\{V_m(W_m(t+2), Q_m^1)\}, \quad t \geq T, \tag{A55}$$

where,

$$W_m(t+2) = Q_m^1{}' D^1(t+2) + Q_m^2{}' Z^2(t+2) + r[W_m(t+1) - C_m(t+1)], \tag{A56}$$

and,

$$Z^2(t+2) = P^2(t+2) + D^2(t+2) - rP^2(t+1).$$

Inspection shows that the function

$$V_m(W_m, Q_m^1) = -K_m \exp^{-A_m[1-(1/r)]W_m - A_m[1-(1/r)]Q_m^1{}' \frac{(1/r)[\bar{D}^1 - \beta_{12}\bar{Z}^2]}{1-(1/r)} + \frac{1}{2}A_m^2[1-(1/r)]^2 Q_m^1{}' \frac{(1/r)\Omega_e}{1-(1/r)} Q_m^1} \tag{A57}$$

with

$$K_m = \frac{r}{r-1} \times (\delta r)^{\frac{1}{r-1}} \times \exp^{-.5 \frac{\bar{Z}^2{}' \Omega_{22}^{-1} \bar{Z}^2}{r-1}},$$

satisfies the Bellman equation (A55) for all time periods $\geq T+1$.

Given the value function at time $T+1$, to solve for investors value functions at time T, I follow the same steps as in equations (A2) through equation (A54). Therefore, substituting in from equation (A57), small investors maximization problem at time T has form:

$$\max_{\substack{C_s(T), \\ Q_s}} -e^{-A_s C_s(T)} - \delta\, \mathrm{E}\left\{K_s(T+1) e^{-A_s(T+1)W_s(T+1) - A_s(T+1)Q_s^{1\prime}\Lambda_s(T+1) + \frac{1}{2}A_s(T+1)^2 Q_s^{1\prime}\Xi_{se}(T+1)Q_s^1}\right\} \tag{A58}$$

such that,

$$W_s(T+1) = Q_s^{1\prime} Z^1(T+1) + Q_s^{2\prime} Z^2(T+1) + r[W_s(T) - C_s(T)], \tag{A59}$$

where,

$$
\begin{aligned}
K_s(T+1) &= \frac{r}{r-1} \times (\delta r)^{\frac{1}{r-1}} \times \exp^{-.5\frac{\bar{Z}^{2\prime}\Omega_{22}^{-1}\bar{Z}^2}{r-1}}, && \text{(A60)}\\
A_s(T+1) &= A_s[1-(1/r)], && \text{(A61)}\\
\Lambda_s(T+1) &= \frac{(1/r)[\bar{D}^1 - \beta_{12}\bar{Z}^2]}{1-(1/r)}, && \text{(A62)}\\
\Xi_{se}(T+1) &= \frac{(1/r)\Omega_e}{1-(1/r)}, && \text{(A63)}\\
Z^1(T+1) &= D^1(T+1) - rP^1(T), && \text{(A64)}\\
Z^2(T+1) &= P^2(T+1) + D^2(T+1) - rP^2(T). && \text{(A65)}
\end{aligned}
$$

Substituting the expression for $W_s(T+1)$ into the value function, taking expectations, and then solving for optimal Q_s^2 given Q_s^1, and substituting that into the value function shows that small investors optimal choice of Q_s^1 and $C_s(T)$ problem has form:

$$
\max_{\substack{C_s(T),\\ Q_s^1}} \quad -e^{-A_s C_s(T)} - \delta K_s(T+1)\,\mathrm{Exp}\left\{-.5\bar{Z}_2'\Omega_{22}^{-1}\bar{Z}_2 - A_s(T)r[W_s(T) - C_s(T)]\right\}
$$
$$
\times\,\mathrm{Exp}\left\{-A_s(T+1)Q_s^{1\prime}[\bar{v}_s(T+1) - rP^1(T)] + .5A_s(T+1)^2 Q_s^{1\prime}\Omega_e(T+1)Q_s^1\right\} \tag{A66}
$$

where

$$
\begin{aligned}
\bar{v}_s(T+1) &= \left[\frac{\bar{D}^1(T+1) - \beta_{12}\bar{Z}_2(T+1)}{1-(1/r)}\right] && \text{(A67)}\\
\Omega_e(T+1) &= \left[\frac{\Omega_e}{1-(1/r)}\right] && \text{(A68)}
\end{aligned}
$$

Integrating the solution for optimal Q_s^1 over the set of small investors then reveals that the net demand for the illiquid assets by the competitive fringe is:

$$
Q_1^1(T+1) = \frac{1}{A_1(T+1)}[\Omega_e(t+1)]^{-1}[\bar{v}_s(T+1) - rP(t)] \tag{A69}
$$

Following the approach that was used earlier to solve for the price schedule faced by large investors in equation (A9), inverting the small investors demand schedule for the illiquid assets reveals that the price schedule faced by large investors has the form:

$$
P^1(.,T) = \frac{1}{r}\left(\beta_0(T) - \beta_{Q^1}(T)Q^1(T) - \beta_{Q_B^1}(T)\Delta Q_B^1(T)\right), \tag{A70}
$$

$$\beta_0(T) = \bar{v}_s(T+1) \tag{A71}$$
$$\beta_{Q^1}(T) = A_1(T+1)\Omega_e(T+1)S_1) \tag{A72}$$
$$\beta_{Q_B^1}(T) = -A_1(T+1)\Omega_e(T+1)S \tag{A73}$$

Given the price schedule at time T, and the value function in equation (A57), large investors maximization problem at time T can be written in the form:

$$\max_{\substack{C_m(T),\\ Q_m}} \; - e^{-A_m C_m(T)} - \mathrm{E}\left\{\delta K_m(T+1)e^{-A_m(T+1)W_m(T+1)-A_m(T+1)Q^{1\prime}\Lambda_m(T+1)+.5A_m(t)^2 Q^{1\prime}\Xi_m(T+1)Q^1}\right\} \tag{A74}$$

where,

$$A_m(T+1) = A_m[1-(1/r)] \tag{A75}$$
$$\Lambda_m(T+1) = S_m'\left[\frac{(1/r)[\bar{D}^1 - \beta_{12}\bar{Z}^2]}{1-(1/r)}\right], \tag{A76}$$
$$\Xi_m(T+1) = S_m'\left(\frac{(1/r)\Omega_e}{1-(1/r)}\right)S_m. \tag{A77}$$
$$K_m(T+1) = \frac{r}{r-1} \times (\delta r)^{\frac{1}{r-1}} \times \exp^{-.5\frac{\bar{Z}^{2\prime}\Omega_{22}^{-1}\bar{Z}^2}{r-1}} \tag{A78}$$

Substituting in the budget constraint, liquid wealth at the beginning of time $T+1$ is given by

$$\begin{aligned} W_m(T+1) =& Q_m^1(T+1)'D^1(T+1) + Q_m^2(T+1)'Z^2(T+1) \\ &+ r(W_m(T) - \Delta Q_m^1(T)'P^1(T,.) - C_m(T)) \end{aligned} \tag{A79}$$

Large investors maximization problem at time T has exactly the same form as given in equation (A13). Therefore, the optimal trades and consumption of large investors follow precisely the same equations as given in Part I of the proof. Large investors value function at time T also has the same functional form as in part I. The equilibrium price function at time T also has the same functional form as in part I. Therefore, to complete the proof, it suffices to solve for small investors consumption and then value function and verify that the value function has the appropriate functional form.

To do so, note that from equation (A66), it is straightforward to show that the optimal choice of $Q_s^1(T+1)$ is

$$Q_s^1(T+1) = \frac{1}{A_s(T+1)}[\Omega_e(T+1)]^{-1} \times [\bar{v}_s(T+1) - rP^1(T)],$$

and that after substituting this expression back in the value function, and making the substitution $P^1(T) = \frac{1}{r}(\alpha(t) - \Gamma(t)Q^1(t))$, then the maximization in equation (A66) simplifies to have the form:

$$\max_{C_s(T)} -e^{-A_s C_s(T)} - \delta K_s(T+1)\,\mathrm{Exp}\{A_s(T) r C_s(T)\} \times \Psi_s(T, Q^1) \tag{A80}$$

where,

$$\begin{aligned}\Psi_s(T, Q^1) = &\,\mathrm{Exp}\left\{-.5\bar{Z}_2'\Omega_{22}^{-1}\bar{Z}_2 - A_s(T) r W_s(T)\right\}\\ &\times \mathrm{Exp}\left\{-.5[\bar{v}_s(T+1) - \alpha(T)]'[\Omega_e(T+1)]^{-1}[\bar{v}_s(T+1) - \alpha(T)]\right\}\\ &\times \mathrm{Exp}\left\{-Q^1(T)'\Gamma(T)'[\Omega_e(T+1)]^{-1}[\bar{v}_s(T+1) - \alpha(T)]\right\}\\ &\times \mathrm{Exp}\left\{-.5Q^1(T)'\Gamma(T)'[\Omega_e(T+1)]^{-1}\Gamma(T)Q^1(T)\right\}\end{aligned} \tag{A81}$$

Using the same approach that was used to solve for large investors optimal consumption and then value function in part I of the proof, tedious algebra shows that small investors value function at time T has form

$$-F(Q^1, T) K_s(T)\,\mathrm{Exp}(-A_s(T) W_s(T))$$

where, $F(Q^1, T) = e^{-Q^1(T)'\bar{v}_s(T) - Q^1(T)'\theta_s(T)Q^1(T)}$,

$$r_s^*(T+1) = A_s(T+1) r / A_s, \tag{A82}$$

$$A_s(T) = A_s(T+1) r / (1 + r_s^*(T+1)), \tag{A83}$$

$$\begin{aligned}K_s(T) = &\left[\frac{r_s^*(T+1)+1}{r_s^*(T+1)}\right][\delta K_s(T+1) r_s^*(T+1)]^{\frac{1}{1+r_s^*(T+1)}}\\ &\times \mathrm{Exp}\left(\frac{-.5\bar{Z}^{2\prime}\Omega_{22}^{-1}\bar{Z}^2 - .5[\bar{v}_s(T+1) - \alpha(T)]'[\Omega_e(T+1)]^{-1}[\bar{v}_s(T+1) - \alpha(T)]}{1 + r_s^*(T+1)}\right),\end{aligned} \tag{A84}$$

$$\bar{v}_s(T) = \frac{\Gamma(T)'[\Omega_e(T+1)]^{-1}[\bar{v}_s(T+1) - \alpha(T)]}{1 + r_s^*(T+1)}, \tag{A85}$$

$$\theta_s(T) = \frac{\Gamma(T)'[\Omega_e(T+1)]^{-1}\Gamma(T)}{1 + r_s^*(T+1)}. \tag{A86}$$

This completes the proof by establishing that large and small investors value functions take the hypothesized form in all periods that involve trade. □

C Solutions for Value Function Parameters

Proposition 4 *For all time periods $t = 1, \ldots, T$, and for large investors $m = 2, \ldots M$:*

$$\bar{v}_m(t) = \frac{S_m'(\bar{D}^1 - \beta_{12}\bar{Z}^2)}{1-(1/r)} \tag{A87}$$

$$\alpha(t) = (\bar{D}^1 - \beta_{12}\bar{Z}^2) \tag{A88}$$

$$A_m(t) = A_m[1-(1/r)] \tag{A89}$$

$$r^*(t) = r - 1 \tag{A90}$$

$$k_m(t) = \left(\frac{r}{r-1}\right) \times (\delta r)^{\frac{1}{r-1}} e^{-.5\frac{\bar{Z}^{2\prime}\Omega_{22}^{-1}\bar{Z}^2}{r-1}} \tag{A91}$$

Proof:

For $\bar{v}_m(t)$ and $\alpha(t)$:

The proof is by induction. First, suppose that the results for $\bar{v}_m(t)$ and $\alpha(t)$ are true at time t. Then, from equation (A10), $\beta_0(t-1) = \alpha(t)$. This implies that from equation (A21) that$(-S_1 + S_m)\bar{v}_m(t) - \beta_0(t-1) = 0$. As a result $\chi(t-1) = 0$, which implies from equation (A30) that $\alpha(t-1) = \beta_0(t-1)$ and from equations (A26) and (A28) that $H_0(t-1) = G_0(t-1) = 0$. Substituting for $H_0(t-1)$ and $G_0(t-1)$ in equation (A40) and simplifying then shows:

$$\Lambda_m(t-1) = S_m'\alpha(t)/r. \tag{A92}$$

Finally, substituting this result in equation (A16) proves the result for $\bar{v}_m(t-1)$. To complete the induction, I use equations (A76) and (A16) to solve for $\bar{v}_m(T+1)$; I then substitute the resulting expression as well as the one for $\beta_0(T)$ (equation (A71)) in equation (A21) and use it to show that $\chi(T) = 0$, which implies $G_0(T) = H_0(T) = 0$. Substituting into equation (A30), then shows that $\alpha(T) = \beta_0(T) = S_m'(\bar{D}^1 - \beta_{12}\bar{Z}^2)/[1-(1/r)]$, which confirms the result for $\alpha(T)$. Finally, given the solutions for $\alpha(T)$ and $\bar{v}_m(T+1)$, substitution in equations (A76) and (A16) confirms the result for $\bar{v}_m(T)$ and completes the induction.

For $A_m(t)$ and $r^*(t)$:

The proof is by backwards induction. We know $A_m(T+1) = A_m[1-(1/r)]$ from equation (A75). Using this expression, and iterating on equations (A38) and (A36) proves the result for all times $t = 1, \ldots T$.

For $k_m(t)$:

The proof is by backwards induction. Equation (A78) establishes that it is true at time $T+1$. Plugging the solution for $K_m(T+1)$ into equation (A39) while using the solutions for $r_m^*(t)$ and the result $H_0(t-1) = G_0(t-1) = 0$ confirms the result for periods $1, \ldots T$. $\square$

The next proposition provides information on the value functions of the small investors:

Proposition 5 *For all time periods $t = 1, \ldots, T$, and for each small investor s*

$$\begin{aligned} a_0(t) &= 0 && \text{(A93)} \\ \bar{v}_s(t) &= 0 && \text{(A94)} \\ A_s(t) &= A_s[1 - (1/r)] && \text{(A95)} \\ r_s^*(t) &= r - 1 && \text{(A96)} \\ k_s(t) &= \left(\frac{r}{r-1}\right) \times (\delta r)^{\frac{1}{r-1}} e^{-.5\frac{\bar{Z}^{2\prime}\Omega_{22}^{-1}\bar{Z}^2}{r-1}} && \text{(A97)} \end{aligned}$$

Proof:

For $a_0(t)$ and $\bar{v}_s(t)$: Plugging the solutions for $\alpha(t)$ and $G_0(t-1)$ from proposition 4 into equation (A43) shows that $a_0(t) = 0$ for all times t. Since $G_0(t-1) = 0$ for all times t, it then follows from equation (A53) that if $\bar{v}_s(t) = 0$, then so does $\bar{v}_s(t-1)$. To complete the induction, note that substituting the solutions for $\bar{v}_s(T+1)$ (equation (A67)) and $\alpha(T)$ (proposition 4) into equation (A85) confirms the result.

For $A_s(t)$, $r_s^*(t)$, and $k_s(t)$:

The form of the proof is identical to that given in proposition 4.□

Proposition 6 *Assume that for $t \leq T$, conditional on state variable $Q^1(t)$ the Nash Equilibrium trades of the large investors exists and is unique. Then for all $m = 2, \ldots, M$ and $t = 1, \ldots, T$, $\theta_m(t)$ has form:*

$$\vartheta_m(t) \otimes \Omega_e, \quad \text{(A98)}$$

where, $\vartheta_m(t)$ is $M \times M$; and

$$\Gamma(t) = \gamma(t) \otimes \Omega_e, \quad \text{(A99)}$$

where, $\gamma(t)$ is $1 \times M$.

Proof: The proof is by induction. First, assume that the theorem is true at time t. Then, from equations (A12) and (A11) $\beta_{Q_B}(t-1) = B_{Q_B}(t-1) \otimes \Omega_e$, and $\beta_Q(t-1) = B_Q(t-1) \otimes \Omega_e$, where $B_{Q_B}(t-1)$ is $1 \times M - 1$ and $\beta_Q(t-1)$ is $1 \times M$. Applying these substitutions in large investors reaction functions and then stacking the results reveals that in equation (A23), $\pi(t-1) = \mathcal{P}(t-1) \otimes \Omega_e$ and $\xi(t-1) = Z(t-1) \otimes \Omega_e$. The assumption that the Nash Equilibrium trades in each period are unique implies that $\mathcal{P}(t-1)$ is invertible. Solving for $H_0(t-1)$ and $H_1(t-1)$ then shows that $H_0(t-1) = 0$ and

$$\begin{aligned} H_1(t-1) &= \begin{pmatrix} -S[P(t-1)^{-1}Z(t-1)] \otimes I_{N_1} \\ (P(t-1)^{-1}Z(t-1)) \otimes I_{N_1} \end{pmatrix} && \text{(A100)} \\ &= \begin{pmatrix} [-\iota_M' \mathcal{P}(t-1)^{-1}Z(t-1)] \otimes I_{N_1} \\ (\mathcal{P}(t-1)^{-1}Z(t-1)) \otimes I_{N_1} \end{pmatrix} && \text{(A101)} \\ &= \mathcal{H}_1(t-1) \otimes I_{N_1} && \text{(A102)} \end{aligned}$$

where ι_M is a $1 \times M$ vector of ones, and $\mathcal{H}_1(t-1)$ is $M \times M$. Since $G_1(t-1) = H_1(t-1) + I_{N_1M}$, it follows that $G_1(t-1) = \mathcal{G}_1(t-1) \otimes I_{N_1}$ for $\mathcal{G}_1(t-1) = \mathcal{H}_1(t-1) + I_M$. From here, substitution in equation (A31) shows that $\Gamma(t-1) = \gamma(t-1) \otimes \Omega$ and substitution in equation (A41) and (A17) shows that $\theta_m(t-1) = \vartheta_m(t-1) \otimes \Omega$. To complete the induction, I substitute the expression for $\xi_m(T+1)$ (equation (A77)) into equation (A17) and show that the result is true for $\theta_m(T+1)$. Then, following steps similar to those in the first part of the induction, it is straightforward to show that the result holds for $\Gamma(T)$ and $\theta_m(T)$, which completes the induction. □.

Corollary 2 *For each small investors, and for each time period $t = 1, \ldots T$,*

$$\theta_s(t) = \vartheta_s \otimes \Omega_e,$$

where ϑ_s is $M \times M$.

Proof: Straightforward induction involving application of the results from proposition 6.

D Proofs of Asset Pricing Propositions

Proposition 7 *When asset markets are imperfectly competitive as specified in section 2 of the text, then if market participants initial asset holdings are Q^{1W}, then investors will hold Q^{1W} forever, and asset prices and expected returns will be the same as when there is perfect competition.*

Proof: When investors risky asset holdings are Q^{1W}, then investors asset holdings are identical to those associated with a competitive equilibrium and complete markets in which trading is restricted to the set of market participants that has been modeled. Hence, when trade in the first set of assets is restricted to be among the market participants, asset holdings are pareto optimal in all time periods; and investors asset holdings will remain at Q^{1W} because investors have no basis to trade away from asset holdings that are associated with perfect risk sharing. Because Q^{1W} is the vector of asset holdings from a competitive equilibrium, the resulting prices and expected returns which support Q^W are the same as in the competitive equilibrium. □

Corollary 3 *For all $t \geq T$,*

$$[\Gamma(t) - \frac{1}{r}\Gamma(t+1)G_1(t)]Q^{1W} = \lambda_{[X^1]}\Omega_e X^1.$$

Proof: Algebra shows that when asset holdings of asset 1 at time t are Q^{1W}, then excess returns of asset 1 are equal to:

$$P^1(t+1) + \bar{D}^1 - rP^1(t) = \beta_{12}\bar{Z}^2 + [\Gamma(t) - \frac{1}{r}\Gamma(t+1)G_1(t)]Q^{1W}.$$

Proposition 7 shows that when asset holdings are Q^{1W} then the excess returns of asset 1 are $\beta_{12}\bar{Z}^2 + \lambda_{[X^1]}\Omega_e X^1$. Equating the two expressions confirms the claim in the corollary. $\square$.

Proposition 3: *When investors asset holdings of the first asset are not Q^{1W}, then equilibrium expected asset returns satisfy a linear factor model in which one factor is the returns on asset 2, another factor corresponds to perfect risk-sharing, but imperfect diversification of the idiosyncratic risk of asset 1, and the remaining factors correspond to the deviation of large investors asset holdings from those associated with the large investors perfectly sharing the idiosyncratic risk of asset 1.*

Proof: Let Q^{1W} denote the vector of asset holdings of asset 1 that is associated with perfect risk sharing among the investors that trade in asset 1. Manipulation of the equation for equilibrium prices given in proposition 1, and substitution of $G_0(t) + G(t)Q(t)$ for $Q(t+1)$ shows:

$$P^1(t+1)+\bar{D}^1-rP^1(t) = [\frac{1}{r}\alpha(t+1)+\bar{D}^1-\alpha(t)]-[\frac{1}{r}\Gamma(t+1)G_0(t)]+[\Gamma(t)-\frac{1}{r}\Gamma(t+1)G_1(t)]Q^1(t)$$

Plugging in the solution for $\alpha(t) = \alpha(t-1) = [\bar{D}^1 - \beta_{12}\bar{Z}^2]/[1-(1/r)]$ shows the first term in braces on the right hand side of the equation is equal to $\beta_{12}\bar{Z}^2$. The second term in braces is zero since proposition 4 shows that $G_0(t) = 0$. Adding and subtracting Q^{1W} to $Q^1(t)$, the above equation can be rewritten as:

$$P^1(t+1) + \bar{D}^1 - rP^1(t) = \beta_{12}\bar{Z}^2 + [\Gamma(t) - \frac{1}{r}\Gamma(t+1)G_1(t)](Q(t) - Q^W) + [\Gamma(t) - \frac{1}{r}\Gamma(t+1)G_1(t)]Q^{1W} \tag{A103}$$

Using the fact that $Q_1^1 = X^1 - SQ_B^1$, the vector $Q^1(t) - Q^{1W}$ can be expressed in terms of the deviations of large investors asset holdings from pareto optimal asset holdings:

$$\begin{aligned} Q^1(t) - Q^{1W} &= \begin{bmatrix} (X^1 - SQ_B^1) - (X^1 - SQ_B^{1W}) \\ Q_B^1 - Q_B^{1W} \end{bmatrix} \\ &= \begin{bmatrix} -S \\ I \end{bmatrix} (Q_B^1 - Q_B^{1W}) \end{aligned}$$

Applying the substitution for $Q^1(t) - Q^{1W}$, and the result of corollary 3 in equation (A103) shows

$$P^1(t+1)+\bar{D}^1-rP^1(t) = \beta_{12}\bar{Z}^2+\lambda_{[X^1]}\Omega_e X^1+[\Gamma(t)-\frac{1}{r}\Gamma(t+1)G_1(t)]\begin{pmatrix} -S \\ I \end{pmatrix}(Q_B^1(t)-Q_B^{1W})$$

Finally, applying the algebra used in the derivation of proposition 6 shows

$$[\Gamma(t) - \frac{1}{r}\Gamma(t+1)G_1(t)]\begin{pmatrix} -S \\ I \end{pmatrix} = \lambda(t) \otimes \Omega_e \tag{A104}$$

where $\lambda(t)$ is $1 \times M-1$. Making this substitution then shows:

$$\begin{aligned} P^1(t+1)+\bar{D}^1-rP^1(t) &= \beta_{12}\bar{Z}^2+\lambda_{[X^1]}\Omega_e X^1+[\lambda(t)\otimes\Omega_e](Q^1_B(t)-Q^{1W}_B) \\ &= \beta_{12}\bar{Z}^2+\lambda_{[X^1]}\Omega_e X^1+\textstyle\sum_{m=2}^M \lambda(m,t)\Omega_e(Q^1_m(t)-Q^{1W}_m) \end{aligned} \tag{A105}$$

where $\lambda(m,t)=\lambda(t)s'_{m-1}$. $\square$.

Corollary 1: *When asset holdings at time t are not efficient, then asset returns at time $t+\tau$ follow a factor model in which the market portfolio, the portfolio of segment residual risk, and the deviation of large investors time t asset holdings from efficient asset holdings are factors.*

Proof: Iterating equation (A103), by τ periods shows:

$$\begin{aligned} P^1(t+\tau+1)+\bar{D}^1-rP^1(t+\tau) = \beta_{12}\bar{Z}^2+[\Gamma(t+\tau)-\frac{1}{r}\Gamma(t+1+\tau)G_1(t+\tau)](Q^1(t+\tau)-Q^{1W}) \\ +[\Gamma(t+\tau)-\frac{1}{r}\Gamma(t+\tau+1)G_1(t+\tau)]Q^{1W}. \end{aligned} \tag{A106}$$

Iterating the equation for equilibrium trades in each period shows

$$Q^1(t+\tau)=[\prod_{j=0}^{\tau-1}G_1(t+j)]Q^1(t).$$

Additionally, because the investors will not trade away from efficient asset holdings, it also follows that

$$[\prod_{j=0}^{\tau-1}G_1(t+j)]Q^{1W}=Q^{1W}.$$

Making both of these substitutions in equation (A106) shows that:

$$\begin{aligned} P^1(t+\tau+1)+\bar{D}^1-rP^1(t+\tau) &= \beta_{12}\bar{Z}^2+\lambda_{[X^1]}\Omega_e X^1+[\lambda(t,\tau)\otimes\Omega_e](Q^1(t)-Q^{1W}) \\ &= \beta_{12}\bar{Z}^2+\lambda_{[X^1]}\Omega_e X^1+\sum_{m=2}^M \lambda_m(t,\tau)\Omega_e(Q^1_m(t)-Q^{1W}_m) \end{aligned}$$

where,

$$\lambda(t,\tau)\otimes\Omega_e=[\Gamma(t+\tau)-\frac{1}{r}\Gamma(t+1+\tau)G_1(t+\tau)]\prod_{j=0}^{\tau-1}G_1(t+j),$$

and $\lambda_m(t,\tau)=\lambda(t,\tau)S'_{m-1}$ $\square$.

D.1 Competitive Benchmark Model

It is useful to contrast the behavior in the multi-market model with large investors with the behavior of asset prices and trades in the same model when all investors are price takers and can trade forever.

In this infinite period set-up with competitive markets, the equilibrium risk-premium should be time invariant. Denote this risk premium by ρ, where,

$$\rho = \begin{pmatrix} \rho^1 \\ \rho^2 \end{pmatrix} = \begin{pmatrix} \bar{Z}^1 \\ \bar{Z}^2 \end{pmatrix} = \begin{pmatrix} P^1(t+1) + \bar{D}^1 - rP^1(t) \\ P^2(t+1) + \bar{D}^2 - rP^2(t) \end{pmatrix} \tag{A107}$$

Note that $\bar{Z}^2$ is taken as exogenous. The goal is to solve for $\bar{Z}^1$ and P^1 that makes the prices of the first group of assets (the ones that are illiquid in the imperfect competition model) consistent with equilibrium in all time periods.

Solving the equation for $P^1(t)$ forward while imposing the transversality condition $\lim_{t\to\infty} r^{-t}P^1(t) = 0$, shows that

$$P^1(t) = \frac{\bar{D}^1 - \rho^1}{r-1}$$

for all time periods t.

Given the hypothesized behavior of prices, it remains to solve for ρ^1 and then to show that the hypothesized behavior of prices is consistent with equilibrium.

The function,

$$V_m(W,t) = -\frac{r}{r-1}(r\ \delta)^{\frac{-1}{r-1}} \exp^{-A_m(1-(1/r))W - \frac{.5\bar{Z}^{2\prime}\Omega_{22}^{-1}\bar{Z}^2}{r-1} - \frac{.5\rho^{1\prime}\Omega_e^{-1}\rho^1}{r-1}}$$

and the risk premium solution

$$\rho^1 = \bar{Z}^1 = \beta_{12}\bar{Z}^2 + \lambda_{[X^1]}\Omega_e X^1, \tag{A108}$$

where,

$$\lambda_{[X^1]} = \frac{(1-(1/r))}{\sum_{m=1}^{M}(1/A_m)} \tag{A109}$$

satisfies the Bellman equation,

$$V_m(W,t) = \max_{\substack{C_m(t), \\ Q_m^1(t), \\ Q_m^2(t)}} -e^{-A_m C_m(t)} + E_t\{\delta V_m(W(t+1), t+1)\},$$

such that,

$$W(t+1) = Q_m^1(t)'Z^1(t) + Q_m^2(t)'Z^2(t) + r[W(t) - C_m(t)].$$

In addition, in the competitive equilibrium, investors optimal choices of Q_m^1 are constant through time, and are market clearing for the hypothesized ρ^1. Investor $m's$ competitive equilibrium holdings of Q_m^1 is denoted by Q_m^{1W} and is equal to

$$Q_m^{1W} = \frac{(1/A_m)X^1}{\sum_{m=1}^{M}(1/A_m)}, \quad m = 1, \ldots M. \tag{A110}$$

Substituting the hypothesized ρ^1 into the expression for equilibrium P^1, it follows that in a competitive equilibrium, the equilibrium price is given by

$$P^1(t) = \frac{\bar{D}^1 - \beta_{12}\bar{Z}^2}{r-1} - \frac{\Omega_e X^1}{r\sum_{m=1}^{M}\frac{1}{A_m}}, \quad t = 1, \ldots \infty \tag{A111}$$

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Table 1: IPO Under-Pricing and Under-Performance by Competitiveness: I.

A. IPO Under-Pricing and Under-Performance

Herf.	Periods Liq	P_Offer	P_Open	Und_Price	S-T Return	L-T Return
3543.26	2000	43.94	44.61	-0.67	-0.01	0.13
3543.26	1800	43.93	44.70	-0.78	-0.01	0.13
3543.26	1600	43.92	44.80	-0.88	-0.01	0.13
3543.26	1400	43.91	44.90	-0.99	-0.01	0.13
3543.26	1200	43.90	45.00	-1.10	-0.01	0.13
3543.26	1000	43.89	45.10	-1.21	-0.01	0.13
3543.26	800	43.88	45.20	-1.33	-0.01	0.13
3543.26	600	43.87	45.31	-1.45	-0.01	0.13
3543.26	400	43.85	45.42	-1.56	-0.01	0.13

B. Investors Risk Bearing Capacity

Investor Number	Type	Risk Bearing Capacity
1	Retail	10.00
2	Institutional	54.56
3	Institutional	21.82
4	Institutional	8.73
5	Institutional	3.49
6	Institutional	1.40

C. IPO Allocation Distortions (Percent)

	Investor Number					
Post-IPO Trading Periods	1	2	3	4	5	6
2000	-100.00	83.29	-100.00	-100.00	-100.00	-100.00
1800	-100.00	83.29	-100.00	-100.00	-100.00	-100.00
1600	-100.00	83.29	-100.00	-100.00	-100.00	-100.00
1400	-100.00	83.29	-100.00	-100.00	-100.00	-100.00
1200	-100.00	83.29	-100.00	-100.00	-100.00	-100.00
1000	-100.00	83.29	-100.00	-100.00	-100.00	-100.00
800	-100.00	83.29	-100.00	-100.00	-100.00	-100.00
600	-100.00	83.29	-100.00	-100.00	-100.00	-100.00
400	-100.00	83.29	-100.00	-100.00	-100.00	-100.00

Table 2: IPO Under-Pricing and Under-Performance by Competitiveness II.

A. IPO Under-Pricing and Under-Performance

Herf.	Periods Liq	P_Offer	P_Open	Und_Price	S-T Return	L-T Return
2225.00	2000	44.00	44.00	0.00	0.12	0.13
2225.00	1800	44.00	44.00	0.00	0.12	0.13
2225.00	1600	44.00	44.03	-0.03	0.00	0.13
2225.00	1400	44.03	44.18	-0.16	-0.01	0.13
2225.00	1200	44.05	44.34	-0.29	-0.01	0.13
2225.00	1000	44.08	44.50	-0.42	-0.01	0.13
2225.00	800	44.11	44.67	-0.56	-0.01	0.13
2225.00	600	44.14	44.83	-0.70	-0.01	0.13
2225.00	400	44.16	45.00	-0.84	-0.01	0.13
2225.00	200	44.19	45.17	-0.98	-0.01	0.13

B. Investors Risk Bearing Capacity

Investor Number	Type	Risk Bearing Capacity
1	Retail	10.00
2	Institutional	40.00
3	Institutional	12.50
4	Institutional	12.50
5	Institutional	12.50
6	Institutional	12.50

C. IPO Allocation Distortions (Percent)

	Investor Number					
Post-IPO Trading Periods	1	2	3	4	5	6
2000	-100.00	-19.50	35.60	35.60	35.60	35.60
1800	-100.00	-19.44	35.55	35.55	35.55	35.55
1600	-100.00	150.00	-100.00	-100.00	-100.00	-100.00
1400	-100.00	150.00	-100.00	-100.00	-100.00	-100.00
1200	-100.00	150.00	-100.00	-100.00	-100.00	-100.00
1000	-100.00	150.00	-100.00	-100.00	-100.00	-100.00
800	-100.00	150.00	-100.00	-100.00	-100.00	-100.00
600	-100.00	150.00	-100.00	-100.00	-100.00	-100.00
400	-100.00	150.00	-100.00	-100.00	-100.00	-100.00
200	-100.00	150.00	-100.00	-100.00	-100.00	-100.00

Table 3: IPO Under-Pricing and Under-Performance by Market Competitiveness III.

A. IPO Under-Pricing and Under-Performance

Herf.	Periods Liq	P_Offer	P_Open	Und_Price	S-T Return	L-T Return
2100.00	2000	44.00	44.00	0.00	0.11	0.13
2100.00	1800	44.00	44.00	0.00	0.11	0.13
2100.00	1600	44.00	44.00	0.00	0.11	0.13
2100.00	1400	44.00	44.00	0.00	0.11	0.13
2100.00	1200	44.00	44.00	0.00	0.11	0.13
2100.00	1000	44.00	44.00	0.00	0.11	0.13
2100.00	800	44.00	44.00	0.00	0.11	0.13
2100.00	600	44.00	44.00	0.00	0.11	0.13
2100.00	400	44.00	44.00	0.00	0.04	0.13
2100.00	200	44.13	44.36	-0.23	-0.01	0.13

B. Investors Risk Bearing Capacity

Investor Number	Type	Risk Bearing Capacity
1	Retail	10.00
2	Institutional	30.00
3	Institutional	30.00
4	Institutional	10.00
5	Institutional	10.00
6	Institutional	10.00

C. IPO Allocation Distortions (Percent)

	Investor Number					
Post-IPO Trading Periods	1	2	3	4	5	6
2000.00	-100.00	0.46	0.46	32.42	32.42	32.42
1800.00	-100.00	0.46	0.46	32.42	32.42	32.42
1600.00	-100.00	-0.13	-0.13	33.58	33.58	33.58
1400.00	-100.00	0.46	0.46	32.42	32.42	32.42
1200.00	-100.00	0.46	0.46	32.42	32.42	32.42
1000.00	-99.99	0.46	0.46	32.42	32.42	32.42
800.00	-100.00	0.46	0.46	32.42	32.42	32.42
600.00	-100.00	0.46	0.46	32.42	32.42	32.42
400.00	-100.00	66.67	66.67	-100.00	-100.00	-100.00
200.00	-100.00	66.67	66.67	-100.00	-100.00	-100.00

Table 4: IPO Under-Pricing and Under-Performance by Market Competitiveness IV.

A. IPO Under-Pricing and Under-Performance

Herf.	Periods Liq	P_Offer	P_Open	Und_Price	S-T Return	L-T Return
1620.00	2000	44.00	44.00	0.00	0.11	0.13
1620.00	1800	44.00	44.00	0.00	0.11	0.13
1620.00	1600	44.00	44.00	0.00	0.11	0.13
1620.00	1400	44.00	44.00	0.00	0.11	0.13
1620.00	1200	44.00	44.00	0.00	0.11	0.13
1620.00	1000	44.00	44.00	0.00	0.11	0.13
1620.00	800	44.00	44.00	0.00	0.11	0.13
1620.00	600	44.00	44.00	0.00	0.11	0.13
1620.00	400	44.00	44.00	0.00	0.11	0.13
1620.00	200	44.00	44.00	0.00	0.11	0.13

B. Investors Risk Bearing Capacity

Investor Number	Type	Risk Bearing Capacity
1	Retail	10.00
2	Institutional	18.00
3	Institutional	18.00
4	Institutional	18.00
5	Institutional	18.00
6	Institutional	18.00

C. IPO Allocation Distortions (Percent)

	Investor Number					
Post-IPO Trading Periods	1	2	3	4	5	6
2000	-100.00	11.11	11.11	11.11	11.11	11.11
1800	-100.00	11.11	11.11	11.11	11.11	11.11
1600	-100.00	11.11	11.11	11.11	11.11	11.11
1400	-100.00	11.11	11.11	11.11	11.11	11.11
1200	-100.00	11.11	11.11	11.11	11.11	11.11
1000	-100.00	11.11	11.11	11.11	11.11	11.11
800	-100.00	11.11	11.11	11.11	11.11	11.11
600	-100.00	11.11	11.11	11.11	11.11	11.11
400	-100.00	11.11	11.11	11.11	11.11	11.11
200	-100.00	11.11	11.11	11.11	11.11	11.11