

Social Networks in Labor Markets: The Effects of Symmetry, Randomness and Exclusion on Output and Inequality*

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Abstract

In this paper we study how some characteristics of the topology of social networks affect the dynamics of output and inequality. Our main findings are: I) symmetric networks with “strong ties” produce higher output and lower inequality than asymmetric networks; II) the introduction of “weak ties” has a larger positive effect on output and inequality if they are associated with symmetric networks; III) with homogeneous agents, the elimination of social exclusion increases output and reduces inequality; IV) in random networks, an increase in network density increases output and reduces inequality, but there are clear decreasing returns; V) random networks with the same density produce the same level of output and inequality, irrespectively of the relative values of density’s determinants, i. e. the number of agents and the probability of link formation. On the contrary, in fixed networks the same density can be associated to different levels of output and inequality, according to the network geometry.

Keywords: Social Networks Structures, Wage Inequality, Aggregate Output, Weak Ties **JEL Codes:** A14, J31, J38

1 Introduction

Since the seminal work of Granovetter (1974), the sociological literature highlighted the importance of social relationships, like friends, relatives and acquaintances, as sources of information on jobs in labor markets. Such importance is also confirmed by a number of empirical studies reporting that approximately between 40 and 60% of employed workers found

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their jobs through social networks although, in general, these proportions vary with sex, occupations, skills, and workers' socio-economic background.¹ More recently, economists have devoted considerable attention to this topic (see Ioannides and Loury (2004) for a survey), so that the study of individual and aggregate economic outcomes produced by the presence of social relationships in labor markets is becoming a fruitful research area in economics.

An important issue in the studies on social networks refers to how the network structure matters, that is how network characteristics, such as topology, composition and type of connections (e.g. "strong" or "weak" ties, see Granovetter (1973)) play a role in explaining the economic effects of the networks. For instance, the effects of networks density, symmetry and exclusion have been often discussed qualitatively in the sociological literature (e.g. Granovetter (2005)). On the other hand, the quantitative effects that such network's properties may produce on output and wage inequality have still not received the same attention.

In this paper we tackle this issue by extending our previous work (Lavezzi and Meccheri (2005))² in several directions. First, in fixed networks, we study the effects of symmetry and asymmetry in the configuration of agents' connections. Second, we analyze the role of the possible presence of social exclusion, that is of agents who are socially isolated. Finally, we investigate the impact of network density, that is of the average degree of social connections, and for this latter aim we introduce random networks into the analysis.

Our main findings can be summarized as follows: I) symmetric networks with "strong ties" produce higher output and lower inequality than asymmetric networks; II) the introduction of "weak ties", having the function of "structural holes" (see Burt (1992)), has a larger positive effect on output and inequality if they are associated with symmetric networks; III) with homogeneous agents, the elimination of social exclusion increases output and reduces inequality; IV) in random networks, an increase in network density increases output and reduces inequality, but there are clear decreasing returns; V) random networks with the same density produce the same level of output and inequality, irrespectively of the relative values of density's determinants, i. e. the number of agents and the probability of link formation. On the contrary, in fixed networks the same density can be associated to different levels of output and inequality, depending on the network geometry. Moreover, the examples we study allow us to shed some new light on the "strength of weak ties hypothesis" (see Granovetter (1973)).

As already mentioned, the economic literature on social networks in labor

¹See Montgomery (1991) for further discussion and references.

²In Lavezzi and Meccheri (2005) we adopt a particular version of the model by Calvó-Armengol and Jackson (2003) with heterogeneous jobs and agents to assess the effects of social networks on aggregate output and inequality.

market has enormously grown in recent years. A fundamental contribution is the seminal work of Montgomery (1991), who presents an adverse selection model in which job referrals improve the quality of firm-worker matches, when firms cannot perfectly observe workers' ability before hiring. In this model, an increase in the density of social ties increases wage inequality since social ties convey to firms more information on workers' quality, and this increases the gap between the (higher) wage paid to referred workers, and the market wage paid to those who find a job through other channels. Furthermore, Montgomery (1994) analyzes also the role of "weak ties", that is relationships with non frequent social interactions (or transitory relations), and shows that they are positively related to the aggregate employment rate and reduce inequality relatively to the case of absence of a social network, in which individuals are randomly allocated to jobs.

The works of Krauth (2004) and Arrow and Borzekowski (2004) extend Montgomery's contributions in different ways. In particular, Krauth (2004) provides a dynamic model in which employed workers may provide information on the skills of their unemployed friends. Final results show that in the long run the number of connections is positively related to employment (both for individual workers and in the aggregate), and that average employment is positively related to the fraction of weak ties for a given number of connections. Arrow and Borzekowski (2004), instead, propose a model which focuses on wage inequality determined by differences in the number of connections of workers to firms and show, by means of numerical simulations, that workers with different number of connections have on average different incomes. In particular, they find that about 13-15% of the variation in log wages is attributable to the variation in the number of workers' connections.

Differently from all works cited above, in our framework the role of social network is not related to job referrals but to the transmission of information on job opportunities among workers who belong to the same network. Consequently, inequality does not depend on adverse selection issues,³ but on the network structure, that is on how such a structure affects the circulation of information among workers of the network.

Our paper follows Calvó-Armengol and Jackson (2003) and Calvó-Armengol and Jackson (2004), which present a framework in which exogenous social networks facilitate the transmission of information on job vacancies among workers. In particular, Calvó-Armengol and Jackson (2003) show analytically that wages of workers in the same network are positively correlated in the long run, but they do not analyze the role of networks structure. This work, which has to be mainly numerical, is started in Calvó-Armengol and Jackson (2004) that provides some simulations for the case of

³Another paper that studies the effects of social networks on inequality in an adverse selection framework is Finneran and Kelly (2003).

homogeneous workers and jobs.⁴ In this paper we extend the study on the role of network structure to the case of heterogeneous jobs, and consider in detail how some relevant network characteristics affect the dynamics of inequality and aggregate output. In addition, we introduce random networks into the analysis.

The paper is organized as follows: Section 2 presents the theoretical model; Section 3 contains the results of the simulations on the effects of symmetry, asymmetry and relational heterogeneity; Section 4 discusses the case of social exclusion; Section 5 introduces random network to study the effects of network density; Section 6 compares networks with random links to networks with fixed links; Section 7 contains a brief discussion of the results with some policy implications, and concludes.

2 A model of labor market with social networks

2.1 Production, wages and turnover

We present a model of labor market which derives from Calvó-Armengol and Jackson (2003). In particular we study the case with identical workers and two types of jobs. Time is discrete and indexed by $t = 0, 1, 2, \dots$. The economy is populated by a number of risk-neutral, infinitely-lived agents (workers) indexed by $i \in \{1, 2, \dots, N\}$. In each period a worker can be either employed or unemployed. Thus, by indicating with s_{it} the employment status of worker i in period t , we have three possible agents' states:

$$s_{it} = \begin{cases} b, & \text{employed in a bad job} \\ g, & \text{employed in a good job} \\ u, & \text{unemployed} \end{cases}$$

On the production side, we consider one-to-one employment relationships and assume a very simple form of a production function, in which productivity depends on the job offered by a firm to a worker. In particular, we denote by y_{it} the output of a firm employing worker i at time t or, in other words, the surplus generated by the match between a worker and a firm (output price is normalized to one).

We simply assume that output in a good job is higher than in a bad job, for instance because it is a hi-tech job. According to these assumptions, the parameter $y^s, s \in (g, b, u)$, indexing the productivity of a match, follows the rule:

$$y^g > y^b > 0 (= y^u).$$

⁴ Calvó-Armengol and Jackson (2004) also show the key role in labor markets of drop-out rates to explain wage inequality.

Wages are a fraction of the match surplus, and are denoted by $w^s = \beta y^s$ with $\beta \in (0, 1)$.⁵ This produces an ordering of wages obtainable in a given match, which follows the ordering of outputs. Obviously, unemployed workers earn zero wages, and we normalize their reservation utility to zero.

The labor market is subject to the following turnover. Initially, all workers are unemployed. Every period (from $t = 0$ onwards) has two phases: at the beginning of the period each worker receives an offer of a job of type f , with $f \in \{b, g\}$, with arrival probability $a_f \in (0, 1)$.⁶ Parameter a_f captures all the information on vacancies which is not transmitted through the network, that is information from firms, agencies, newspapers, etc. When an agent receives an offer and she is already employed and not interested in the offer, in the sense that the offered job does not increase her wage, she passes the information to a friend/relative/acquaintance who is either unemployed or employed but receiving a lower wage than the one paid for the offered job. At the end of the period every worker loses the job with breakdown probability $d \in (0, 1)$.

2.2 Social links and job information transmission

Social networks may be represented by a graph G summarizing the information on agents' links, where $G_{ij} = 1$ if i and j know each other, and $G_{ij} = 0$ indicates if they do not. It is assumed that $G_{ij} = G_{ji}$, meaning that the acquaintance relationship is reciprocal. Given the assumptions on wages and arrival probabilities, the probability of the joint event that agent i learns about a job and this job ends up in agent's j hands, is described by p_{ij} :

$$p_{ij}(s_{it}^\theta, f) = \begin{cases} a_b & \text{if } f = b \cup j = i \cup s_i = u \\ a_g & \text{if } f = g \cup j = i \cup (s_i = u \cap s_i = b) \\ a_b \frac{G_{ij}}{\sum_{k: s_k = u} G_{ik}} & \text{if } f = b \cup (s_i = b \cap s_i = g) \cup s_j = u \\ a_g \frac{G_{ij}}{\sum_{k: s_k \neq g} G_{ik}} & \text{if } f = g \cup s_i = g \cup (s_j = u \cap s_j = b) \\ 0 & \text{otherwise} \end{cases}$$

In the first two cases, worker i receives an offer with probability a_f , $f \in \{b, g\}$, and takes the offer for herself. This holds if she is either unemployed or employed in a bad job and receives an offer for a good job. In the third case the worker i is employed and receives with probability a_b an offer for a bad job, that she passes only to an unemployed worker $j (\neq i)$. We assume that among all unemployed workers connected with i by a social link, i chooses j randomly. Hence, the probability that worker j receives

⁵For instance β may represent the bargaining power of workers when wages are set by Nash bargaining, as is usual in search models. Clearly, profits are $(1 - \beta)y^s$.

⁶That is, each agent can receive both an offer for a good and a bad job.

the information by worker i is equal to $\frac{G_{ij}}{\sum_{k:s_k=u} G_{ik}}$. In the fourth case the worker i receives with probability a_g an offer for a good job when she is already employed in a good job, thus she passes the offer, with probability $\frac{G_{ij}}{\sum_{k:s_k \neq g} G_{ik}}$, to a worker connected to her who is either unemployed or employed in a bad job. Clearly, $p_{ij} = 0$ in all remaining cases.

To sum up, a worker who receives an offer makes direct use of it if the new job opportunity increases her wage. Otherwise, she passes the information to someone who is connected to her. The choice of the worker to whom pass the information is “selective”, in the sense that the information is never passed to someone who does not need it,⁷ but it is random with respect to the subset of the connected workers who improve their condition (wage) exploiting such information (for example, a worker receiving a good job offer is indifferent to pass it to an unemployed contact or a contact employed in a bad job). Finally, we assume that a worker receiving both an offer for a bad and a good job when she does not need them, decides to transmit first the information about the bad job and then, possibly to the same agent, the information about the good job, and we exclude that each job information may be transmitted to more than one (connected) worker.⁸

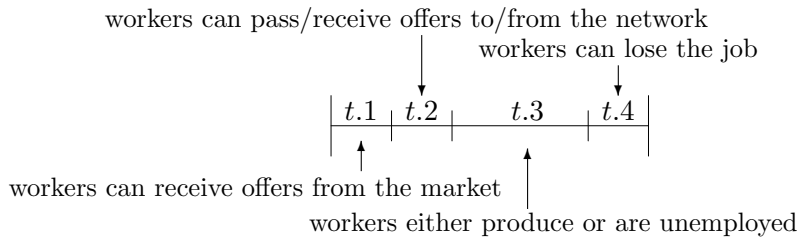


Figure 1: Timing

Figure 1 shows the timing of the events for a generic period t (for convenience, the period has been represented as composed by four different consecutive sub-periods, with sub-periods $t.1$, $t.2$ and $t.4$ having negligible length).

⁷For the sake of simplicity, we assume that a worker observe the state of her connections at the end of the previous period to make a decision on passing information. In other words, she cannot observe if her connections have already received an offer from someone else. If all of the worker’s acquaintances do not need the job information, then it is simply lost.

⁸ Calvó-Armengol and Jackson (2003) provide various extensions on the process of transmission of job information.

3 On networks symmetry

In this and the following sections we present the results of our simulations.⁹ Our aim is to assess how the structure of social networks affects the dynamics of output and wage inequality in the long run, as well as the correlation of workers' wages. We measure output by averaging over time the average output of the n workers in every period. Inequality is measured by the average Gini index over time.

As a preliminary general remark, it is important to point out that in this framework the network structure basically affects the possibility for the system to be in a state of maximum output (*SMO* henceforth), that is a state in which all agents are employed in the good job. Given our assumptions, *SMO* would be a steady state if the probability of losing the job was zero, as workers would be in the best possible position and would turn down any offer they received, directly or indirectly. In other words, without the exogenous breakdown probability, *SMO* would be an absorbing state for the system. In this respect, the network structure regulates the possibility to attain *SMO* and the speed at which the system recovers to it, after the occurrence of stochastic perturbations given by breakdowns of job relationships. Therefore, as we shall see, high average levels of output and low levels of inequality obtain when the system, driven by the network structure, reaches faster and persistently remains in *SMO* (note that in *SMO* inequality is clearly absent).

We begin by considering a simple case of symmetric vs asymmetric networks and then a case in which agents, belonging to different social groups, interact locally but some of them have links with otherwise unconnected groups.¹⁰ As it will be clear, both cases, in which social links are fixed and exogenous, suggest that network symmetry has a positive effect on output and inequality.

3.1 Symmetric vs asymmetric networks

In this section we study how (a)symmetry of network geometry affects output and inequality, and the pattern of wages' correlation. Technically speaking, symmetry implies that all agents are connected to the same number of other agents.

Consider the network structures in Figure 2, G_A and G_B .¹¹

⁹Simulations are run for 500,000 periods and, if not stated otherwise, the parameters are: $a_b = 0.15$, $a_g = 0.10$, $d = 0.015$, $\beta = 0.4$, $y^g = 5$, $y^b = 1$. All simulations are programmed in *R* (<http://www.r-project.org/>). Codes are available upon request from the authors.

¹⁰In general, in our examples we will consider networks where not all possible links are formed as a simple way to consider the fact that link formation is costly. For a full treatment of network formation with costly links see Calvó-Armengol (2004).

¹¹This case corresponds to Example 1 in Calvó-Armengol (2004) where, differently

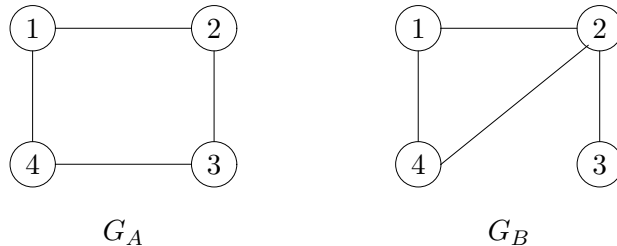


Figure 2: Networks G_A and G_B

Both networks have the same number of agents, n , and links, N ($N = n = 4$), and the same average number of links for each agent, that is $\mu = 2$.¹² However, they have a different geometry: network G_B is obtained from G_A by simply rewiring one link. This introduces an asymmetry, as in network G_B agent 2 has three links and agent 3 has one link, while agents 1 and 4 maintain the same number of links. In other words, agents 1, 2 and 4 form a cluster of interconnected agents, from which agent 3 is partially excluded.

In addition, there exists a difference in the number of links of the agents to whom every agent is connected. In network G_A any agent has two links with agents who have two links. Differently, in network G_B agents 1 and 4 have one link with an agent with two links (respectively agents 4 and 1), and one link with an agent with three links, agent 2. Agent 2 has two links with two agents, 1 and 4, who have two links, and one link with agent 3, who has one link. As we show in Table 1, this has consequences for both output and inequality.

Network	Output	Inequality	Av. wages [1, 2, 3, 4]	Var. wages [1, 2, 3, 4]
G_A	4.818	0.034	1.927, 1.927, 1.928, 1.928	0.122, 0.123, 0.121, 0.120
G_B	4.802	0.038	1.924, 1.945, 1.889, 1.924	0.126, 0.091, 0.183, 0.127

Table 1: Output, inequality and wages, networks G_A and G_B

We observe that, moving from G_A to G_B , output decreases and inequality increases. The emergence of a local cluster makes the network asymmetric, and affects both output and wage inequality. In particular, the decrease in output and the increase on inequality depend on the relative isolation of agent 3. Agent 3's average wage is sharply lower in network G_B . In this case the increase in the average wage of agent 2, due to an increase in the number of her connections, is not sufficient to counterbalance the decrease in the average wage of agent 3. Also notice that the variance of agent 2's wage is lower while the variance of agent 3's wage is higher in network G_B .

from here, workers and jobs are both homogeneous.

¹²The simple formula to obtain μ , the average number of links per agent, is $2N/n$.

Results are also different for agents 1 and 4 although the number of their connections is the same. In particular their average wage is lower and the variance is higher in network G_B . This can be explained by the fact that the number of links of their “connections” is different in network G_B , in particular they are both connected to agent 2 who has three links. This implies that their probability of receiving information on vacancies from agent 2 is lower in network G_B , as they have more “competitors” for information. This result is not so obvious since there could be also a positive effect deriving from a connection with an agent with many links, which should guarantee a more stable position in the state of employment and therefore have a higher propensity to transmit information on vacancies. We term the first effect as *competition effect*, and the second as *connection effect*, and note that the former dominates the latter in network G_B .

These results highlight the complexity of capturing the externalities produced by the structure of the network. In the present framework, the network exerts an externality on agents’ utilities as it affects their job opportunities. However, to put these network externalities in closed form is not an easy task, as they derive from a network stochastic process.¹³ Our numerical results, however, clearly show that such externalities differ across individuals depending on their location in the network. Moreover, switching from a symmetric to an asymmetric structure, it appears that the negative externalities that derive seem to prevail on positive externalities, since in symmetric networks aggregate results are better.

worker	1	2	3	4	worker	1	2	3	4
1	1	0.031	0.026	0.025	1	1	0.038	0.014	0.048
2	0.031	1	0.027	0.026	2	0.038	1	0.022	0.038
3	0.026	0.027	1	0.020	3	0.014	0.022	1	0.010
4	0.025	0.026	0.020	1	4	0.048	0.038	0.010	1

Table 2: Correlation of workers’ wages: G_A Table 3: Correlation of workers’ wages: G_B

The creation of a local cluster also affects the distribution of wage correlations across each pair of agents in the network. In detail, from Tables 2 and 3 we see that, as predictable, the values of the correlation of wages of the agents in the cluster (i.e. agents 1, 2 and 4) increase.¹⁴ In network G_B agent 3’s correlations with any other agent decreases. Note that the correlation between agent 3 and 2’s wages is lower, despite the fact that the two agents share a link as in network G_A . In network G_B , however, agent 2 has one extra link and, in practice, this weakens the connection between

¹³In a different setting, strategic and static, a recent paper of Ballester, Calvó-Armengol and Zenou (2005) studies analytically the variance of network externalities.

¹⁴All these numerical results are in accordance with the analytics of Calvó-Armengol and Jackson (2003).

2 and 3. Finally, the correlation between 3 and 4 is lower in network G_B because they are not directly connected.¹⁵

To sum up, the introduction of asymmetry in a network which preserves the same average number of links, causes output to decrease and inequality to increase. Next section takes a step further.

3.2 Symmetry and relational heterogeneity

In this section we go deeper on the role of social networks' symmetry in explaining economic outcomes. In fact, as remarked for instance by Ioannides and Loury (2004), p. 1064, there exist results related to social networks structure that may be explained by symmetry, while they have been often attributed to other network properties. Indeed, much of the initial sociological research on the effects of job networks properties mainly focused on *relational heterogeneity*, emphasizing that not all social relations (contacts) have the same role or strength in affecting employment outcomes. Here we aim to disentangle in our framework the effects of network symmetry on output and inequality with respect to some traditional concepts related to relational heterogeneity.

An important argument in the theory of social networks refers to the role of *structural holes*. As is well-known, Burt (1992) defines structural holes as the “gap” of non-redundant links: agents placed at structural holes of a network allow information to flow between otherwise unconnected groups of agents. The structural holes argument implies that networks with more non-redundant links (i.e. more agents placed at structural holes) can provide more information than network *of the same size*, but with more redundant links (see Ioannides and Loury (2004), p. 1063). Thus, networks in which (structural holes) agents link otherwise unconnected groups should be characterized by more efficient outcomes, since information in such networks circulates more widely.¹⁶

Consider the different network structures in Figure 3, with $n = 8$, $N = 12$ and $\mu = 3$.

¹⁵Other examples with larger networks, not reported here but available upon request, confirm these results.

¹⁶Note that the “structural hole effect” amplifies when information can be transmitted to indirect relationships (more than two-links away) by means of sequential passages, while in our simulations information may be transmitted only one time between direct contacts. However, in a long run perspective, since the transmission of information can improve the state of one (connected) agent in a given period and this allows her to be more prone to transmit information to others in future periods, the same effect should apply.

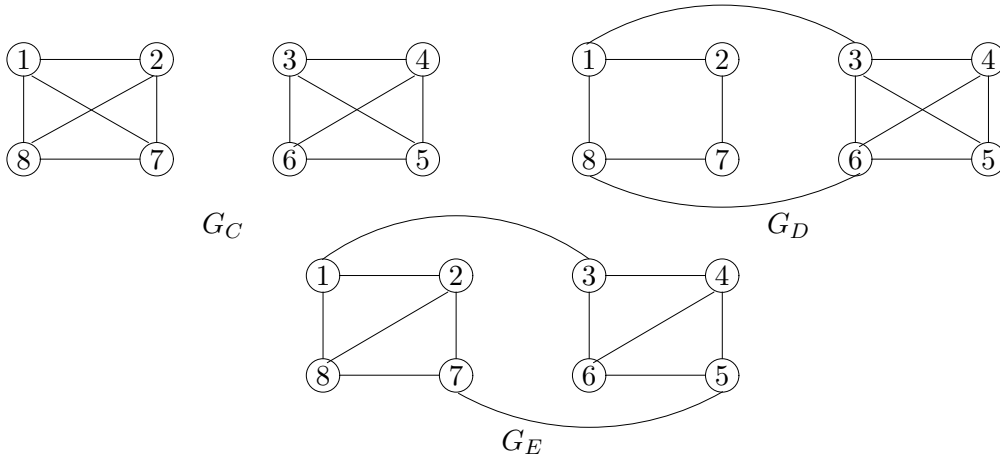


Figure 3: Networks G_C , G_D , G_E : symmetry and structural holes

In network G_C , there are two separated groups of four agents and, in each group, each agent is linked to each other. Clearly, this is a symmetric network, since all agents are connected to the same number of other agents (three, in this case). It is important to point out that, according to the theory of structural holes, some links in network G_C are, at least partially, redundant, since each pair of agents could be (indirectly) linked anyway via other agents in their group (e.g. agents 1 and 2 are linked via agent 7), and it would be more efficient to have some links to agents in the other group.

In Network G_D some agents become structural holes: 1 and 8 for the first subgroup, 3 and 6 for the second. The two groups are linked through a bridge provided by structural holes. The network is not symmetric, since there are agents with a different number of links: i.e. agents 3 and 6 have now four links, while agents 2 and 7 have just two links. Finally, network G_E is a symmetric network with the same number of “structural holes agents” (1, 3, 5 and 7 in this case) of G_D .

Running simulations for these networks, we obtain the following aggregate results:

Network	Output	Inequality
G_C	4.863	0.027
G_D	4.862	0.027
G_E	4.867	0.026

Table 4: Networks G_C , G_D , G_E : output and inequality

We note that the introduction of structural holes and asymmetry in network G_D slightly reduces output and leaves inequality unchanged, while output is higher and inequality slightly lower in G_E . Also, even if the results are fairly close, this example seems to suggest that, in the aggregate,

the positive effect on output which may derive from the introduction of “bridges” between different groups (as can be the case in a passage from G_C to G_E) could be counterbalanced if those bridges are created by rendering asymmetric the structure of the network and, consequently, the position of different agents.

This appears more transparent if we look at Table 5, in which we report individual (average) wages of three workers (1, 2 and 3)¹⁷ in G_C , G_D and G_E . Table 5 also shows the wage correlation of two workers with no direct connections (1 and 6) in the different networks.

Network	Av. wage [1]	Av. wage [2]	Av. wage [3]	Corr. wages [1;6]
G_C	1.944	1.944	1.944	0.000
G_D	1.947	1.929	1.959	0.010
G_E	1.947	1.947	1.948	0.012

Table 5: Networks G_C and G_D : individual wages [1,2,3] and correlation [1;6]

Wages of agents 1 and 3 increase in network G_D . While the increase of agent 3’s wage is largely due to the fact that she has now one extra link, the increase of agent 1’s wage is related to a “structural hole” effect: given that the number of her connections is unchanged, now she is linked to the other group and can take advantage, directly or indirectly, from the presence of all workers in the economy. Agent 2 loses one link in G_D . Her wage becomes lower than in G_C . In principle agent 2 could have benefited from the presence of a bridge connecting her group to the other, but this appears not sufficient to outweigh the negative effect of losing one link. Moreover, this negative effect appears so powerful that, although some agents become structural holes and have more links, aggregate output (which is proportional to wages) slightly decreases in network G_D .¹⁸

In network G_E agents maintain the same number of links as in G_C , but some agents (1, 3, 5 and 7) become structural holes. The wage of agents 1 and 3 increases with respect to G_C , indicating that these agents benefit from a better circulation of information. The wage of agent 2, who is not a structural hole, increases as well with respect to G_C even if the number of links is the same, and is much higher than in G_D . The latter effect clearly depends on 2 having more links in G_E than in G_D .

Finally, notice from Table 5’s last column that the presence of bridges between the two groups of workers affects the structure of wages correlations. While wages of workers 1 and 6 are not correlated in network G_C , where

¹⁷The situation of the chosen workers changes differently when we move from network G_C to network G_D ; in this sense, they have been chosen as representing typical cases. Of course, the same qualitative results hold also for other workers in analogous situations.

¹⁸This suggests the presence of decreasing returns in increasing the number of social links. This aspect is analyzed in more detail in Section 5.

they belong to two separated groups, the correlation becomes positive in networks G_D and G_E , even if those workers are not directly connected.

Overall, the positive effects of bridges creation can be better appreciated in network G_E in which symmetry is preserved. In such a case, output increases and inequality slightly decreases with respect to other networks. This happens because the advantages of a wider circulation of information can be exploited at no costs for agents, in the sense that they maintain the same number of links.

These results suggest that the effect of structural holes may depend on their being related to symmetry or asymmetry of the network. This remark can also apply to the relevance of “weak ties”, that is simple acquaintances in relation to “strong ties”, joining strong friends and relatives. Following Granovetter (1973), a “structural hole” is a weak tie (but a weak tie is not necessarily a “structural hole”!). Therefore, in our framework weak ties appear “stronger” if they are associated to symmetric geometries than to asymmetric ones. More generally, one dimension to define weak ties is the “amount of time” that two individuals spend together.¹⁹ We deal in more detail with this feature of social relationships in Section 6, where we consider random networks.

4 On social exclusion

In this section we consider the following issue: given a population of agents and a fixed number of social links, which effects on output and inequality does the exclusion of some agent from the network produce? To analyze this issue, we consider different possible network configurations in Figure 3 where there are two networks in which some agents are socially isolated, and one with no social exclusion.²⁰ For the three networks we have that $n = 8$, $N = 6$ and $\mu = 1.5$.

¹⁹See Granovetter (1973), p. 1361. The other dimensions to measure the strength of a tie are: “the emotional intensity, the intimacy..., and the reciprocal services”. Moreover, “the stronger the tie connecting two individuals, the more similar they are” (*ibidem*, p. 1362). Clearly, in our framework with identical agents we cannot consider the aspect of similarity of agents.

²⁰Network structures considered in this section are studied in Lavezzi and Meccheri (2005) for the case with heterogeneous workers and jobs.

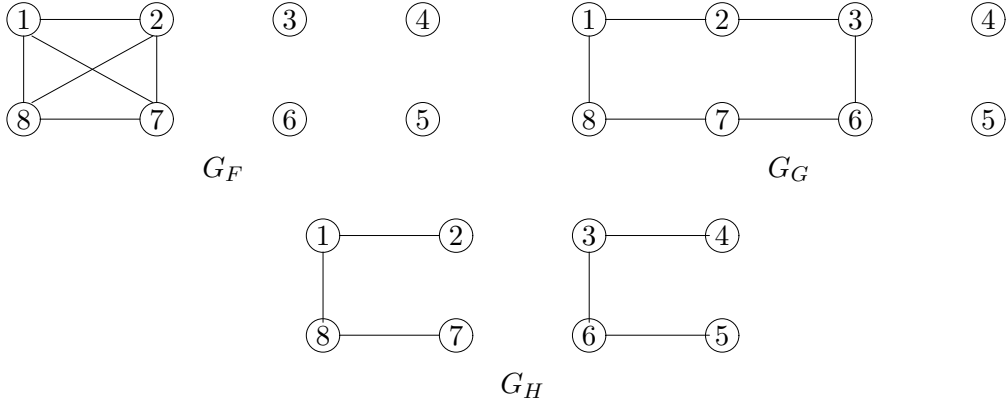


Figure 4: Networks G_F , G_G , G_H : exclusion vs no exclusion

In network G_F a cluster of agents enjoys a high level of social interaction, as each agent is directly connected to each other, while there are other agents completely isolated. With respect to network G_F , in network G_G some links are rewired in order to include two previously excluded agents. Finally, in network G_H no agent is excluded: with the same number of links, any agent is directly connected to at least another agent. In Tables 6 and 7 we present the results of the simulations.

Network	Output	Inequality
G_F	4.670	0.064
G_G	4.734	0.052
G_H	4.771	0.045

Table 6: Networks G_F , G_G , G_H : output and inequality

Network	Average wages [1, 2, 3, 4, 5, 6, 7, 8]
G_F	1.945, 1.946, 1.789, 1.793, 1.793, 1.789, 1.945, 1.945
G_G	1.928, 1.929, 1.926, 1.791, 1.789, 1.926, 1.930, 1.927
G_H	1.927, 1.889, 1.929, 1.887, 1.892, 1.928, 1.888, 1.927
	Variance of wages [1, 2, 3, 4, 5, 6, 7, 8]
G_F	0.092, 0.090, 0.331, 0.325, 0.325, 0.331, 0.093, 0.092
G_G	0.120, 0.118, 0.123, 0.328, 0.330, 0.123, 0.117, 0.121
G_H	0.122, 0.182, 0.119, 0.185, 0.179, 0.121, 0.184, 0.122

Table 7: Networks G_F , G_G , G_H : mean and variance of individual wages

We note that, as the inclusion of agents increases, output increases and inequality decreases. While the result on inequality is predictable, since the social inclusion of some agents involves that job opportunities spread more

evenly in the network, the result on output is not completely obvious, given that, in this perspective, links rewiring has potentially conflicting effects. In particular, considering the passage from G_F to G_G , there are some agents losing links, e.g. agents 1, 2, 7 and 8, who should consequently lose job opportunities, and some agents gaining links, e.g. 3 and 6. However, from Table 7 we can see that the increase in average wage (and output) of agents 3 and 6 is rather high, while the decrease in output of agents 1, 2, 7 and 8 is relatively lower in magnitude.

Similar results are obtained in the passage from G_G to G_H : the increase in wage (and output) of agents now included in the social environment, 4 and 5, more than offsets the decrease in wages (and output) of agents who lose some social contacts, 2 and 7. Wages' variances and correlations (the latter not reported) present behaviors in line with previous examples. In particular, Table 7 shows that an increase (decrease) in the number of links decreases (increases) the variability of wages.

Overall, we see that as social inclusion increases, there are no trade-offs between equality and efficiency. However, in this framework such result depends on the assumption of workers homogeneity. In our previous work (Lavezzi and Meccheri (2005)) we considered the more general case of heterogeneous workers and jobs, where workers are either skilled (more productive) or unskilled (less productive). We found that, for a given number of social links and starting from a case where links only connect skilled workers, the inclusion of some unskilled workers (at the cost of reducing social links for some skilled workers) *may* actually decrease output (besides reducing inequality). This is because the increase in total product given by the inclusion of some workers is outweighed by the decrease in output due to a worsening of the social connections of more productive workers. However, these results strictly depend on the productivity gap between skilled and unskilled workers. In particular, output decreases when this gap is high, while, when the gap is low, there is no trade-off between increasing output and reducing inequality.

5 On networks density: introducing random networks

In this section we aim to study the role of network *density* on output and inequality. Network density can be defined as the ratio of actual to the total number of possible links, the latter defined as $max = n(n - 1)/2$ (see, e.g. Granovetter (2005), p. 34), and it is simply given by p in random networks, where $0 \leq p \leq 1$ is the probability that a link is present in each period. Another possible measure for density is also the average number of links per agent, which is $(n - 1)p$ in random networks. Hence, in our case we redefine the average number of links $\mu = (n - 1)p$. Note that these measures

avoid any reference to network geometry and therefore are informative, as noted, on the “pure” effect of network density.

Random networks can be useful to investigate also the “strength of weak ties hypothesis”. Indeed, random links, if compared to fixed links, can be interpreted as “weak” ties, that is ties between two individuals which are not always “active”, as if the two individual only meet from time to time and are not always in touch with each other. In this section we can therefore consider all links as “weak ties”, while a deeper comparison of the effects produced by “weak” and “strong” ties is deferred to Section 6.

Given the networks density measures defined above, the relevant variables are n and p and we try first of all to disentangle their individual effect, if any, on output and inequality. In this perspective, we first consider the case of fixed μ in order to check whether there exist any difference between large networks with small probability of link formation or small networks with high probability of link formation. This because, in principle, certain properties of random networks depending on p are in turn dependent on the number of agents in the network (see, e.g. Albert and Barabasi (2002), p. 10).

In Tables 8 and 9 we consider different hypotheses for two constant values of μ .²¹

n	p	μ	output	inequality
3	1	2	4.810	0.035
5	0.5	2	4.813	0.036
9	0.25	2	4.811	0.037
17	0.125	2	4.810	0.038

Table 8: Random networks: $\mu = 2$

n	p	μ	output	inequality
5	1	4	4.894	0.020
9	0.5	4	4.893	0.021
17	0.25	4	4.894	0.021
33	0.125	4	4.893*	0.021*

Table 9: Random networks: $\mu = 4$

Results in Tables 8 and 9 seem to suggest that there is not a relevant difference between the relative effects produced by n or p : for given μ , independently of the relative values of its determinants, output and inequality do not change remarkably. Specifically, this is especially true for $\mu = 4$, while for $\mu = 2$ there is some evidence of a positive relation between inequality and n (or a positive relation between inequality and p).

Now we study in more detail the dynamics of output and inequality for changes in μ . We consider different values of μ by changing n , given p , and then by changing p , given n , in order to investigate the possible effect of the dimension of the population for a given level of social interaction, and vice versa. In principle, an increase in n , given p , and an increase in p , given n , should produce an increase in output, as any agent increases the expected number of contacts in each period and, by this way, the likelihood

²¹The symbol * in Table 9 indicates that the results are obtained with a 200,000 period simulation, due to computational constraints.

to receive more job offers. At the same time, it should decrease inequality, since each agent is, in each period, more likely to be employed in the good job; in other words, the system is more likely to be in the *SMO* over time, and this implies that individual wages' are identical.

In Table 10 and 11, results for different cases confirm these expectations.

n	p	μ	output	inequality
3	0.5	1	4.713	0.053
5	0.5	2	4.813	0.036
9	0.5	4	4.893	0.021
17	0.5	8	4.949	0.010

Table 10: Variable n , fixed p

n	p	μ	output	inequality
4	0	0	4.479	0.099
4	0.125	0.375	4.595	0.077
4	0.25	0.75	4.679	0.061
4	0.5	1.5	4.774	0.043
4	1	3	4.862	0.026

Table 11: Fixed n , variable p

Examples considered in this section permit us to infer that μ is clearly the relevant variable, while taking individually n and p does not convey conclusive indications on the effects on output and inequality. That is a variation of, say, n does not *per se* inform on the direction of the variations of output and inequality, as they may differ according to the corresponding level of p , and vice versa. In other words, an increase (decrease) of μ is necessary and sufficient to obtain an increase (decrease) in output and a decrease (increase) in equality, while an increase (decrease) in n or in p is neither necessary nor sufficient. In this sense, μ appears as to be a better measure to consider in order to analyze the effects of networks density on output and inequality.

Taking μ as the proper indicator for our purposes, we study now in more detail its relation with output and inequality and, in particular, the functional forms they assume in our framework.²² In Figures 5 and 6 we plot the values of output and inequality against our measure of network density, μ .²³

²²Additional work is required for more general frameworks and, in particular, for a continuum of job opportunities (wages).

²³The values in Figures 5 and 6 are from Tables 10 and 11, plus the values of output and inequality (respectively 4.980 and 0.004) for $\mu = 16$, obtained from a simulation with $n = 21$ and $p = 0.8$.

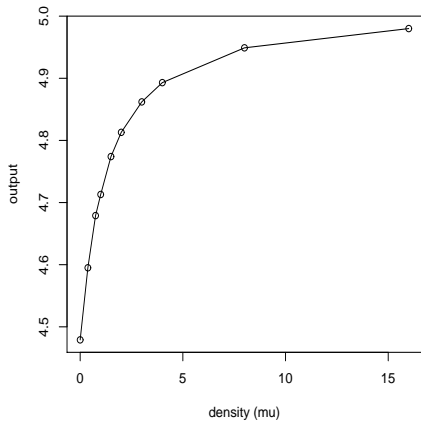


Figure 5: Output and network density μ

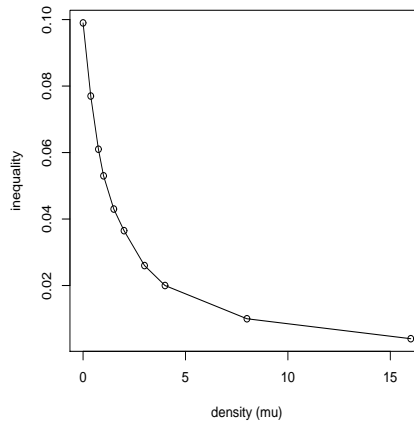


Figure 6: Inequality and network density μ

We observe that there is clear evidence of decreasing returns of network density on output and inequality, that is the biggest increases in output and decreases in inequality are obtained when density increases from low levels. Taking into account that, in general, link formation is costly, this result can acquire particular relevance from a policy viewpoint, as it will be discussed in Section 7.

6 Comparing fixed and random networks

In this section we are interested to identify possible differences between comparable networks in terms of density, i.e. with the same μ , but in which social links are fixed or random. Such a comparison can be interpreted as a comparison between network with *strong ties*, that is stable social relations, and networks with *weak ties*, that is networks where social relationships are occasional. Moreover, when links are fixed, we distinguish between symmetric and asymmetric structures, and between networks with and without social exclusion. This also allows us to synthesize and compare different scenarios and results analyzed in the previous sections.

We start by reproducing in Table 12 the results on output and inequality of Table 2, referred to the symmetric network G_A and the asymmetric network G_B , along with new results obtained for a random network, G_2^R , with the same number of agents, $n = 4$, and the same value of $\mu = 2$.²⁴

²⁴In network G_2^R , with $n = 4$, the value of $\mu = 2$ is obtained by setting $p = 2/3$.

Network	Output	Inequality
G_A	4.818	0.034
G_B	4.802	0.038
G_2^R	4.815	0.035

Table 12: Fixed (G_A and G_B) and random (G_2^R) networks: $\mu = 2$

As shown in Table 12, the value of output and inequality in G_2^R is intermediate between correspondent values related to networks G_A and G_B . Therefore, if we consider random contacts as weak ties, it seems that the hypothesis on the “strength of weak ties” is verified in our framework only if the comparison is made with a “fixed network” which displays an asymmetric geometry of social links. Once again, the (a)symmetric configuration of the social structure appears to play a crucial role in explaining economic outcomes.

In Table 13, instead, we compare a random network with the networks analyzed in Section 4, that is those with and without social exclusion. Again, we consider a random network, $G_{1.5}^R$, with the same number of agents ($n = 8$) and the same links density ($\mu = 1.5$).²⁵

Network	Output	Inequality
G_F	4.670	0.064
G_G	4.734	0.052
G_H	4.771	0.045
$G_{1.5}^R$	4.771	0.045

Table 13: Fixed (G_F , G_G , G_H) and random ($G_{1.5}^R$) networks: $\mu = 1.5$

We note that a network with “weak links” replicates the results of the network with “strong links” and no social exclusion. Therefore, we have another instance in which weak links can produce better results than strong links, but only when the latter are arranged with a certain degree of asymmetry or exclusion, that is when some agents have no social relationships with others. Finally, notice that even network G_H is asymmetric, since agents have different numbers of links, but in this case it seems that the degree of asymmetry is too low to make the strong links remarkably less effective than the weak links of a comparable random network.

7 Discussion and conclusions

In this paper we have provided an initial study of the effects of network symmetry, density and exclusion on output and inequality. In particular,

²⁵In network $G_{1.5}^R$, with $n = 8$, the value of $\mu = 1.5$ is obtained by setting $p = 0.2142857$.

our results allow for a first set of considerations.

First, the relevance of symmetric social architectures, which appeared in our examples, points to the relevance of having an “egalitarian” society in which individuals are relatively similar in their degree of social interaction. The importance of symmetry also appears in relation to weak ties in the form of structural holes: the importance of the latter is in fact related to the possibility of connecting two or more otherwise disconnected groups of individuals by establishing a symmetric geometry.

The elimination of social exclusion, as a social policy target, may also be supported in economic terms as it appears as a means to increase the productive efficiency of the system with no trade-offs in terms of inequality. As remarked, this may be invoked especially if agents do not display too high differences in terms of productivity.

Finally, the presence of strong decreasing returns for the effects of network density on output and inequality, as emerged in our framework, also helps to define the importance of possible policy interventions. Clearly, taking into account that building a social infrastructure is costly, investing in schools, residential areas, and other pro-socialization infrastructures, appear especially justified when the starting degree of network density is particularly low.

Our results can be checked by extending the theoretical framework in a number of ways: different hypotheses can be made on the job information transmission process, for instance by allowing some heterogeneity in the access to job information, and the dynamics of random networks can be enriched (see in particular the work of Marsili *et al.* (2004) for a possible route in this direction). These extensions are left for future work.

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