

# Structural Breaks between Determinacy and Indeterminacy in Estimated DSGE Models: A Bayesian Change-Point Approach

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## Abstract

Sub-sample estimates of dynamic general equilibrium models of the U.S. economy since 1959 suggest that monetary policy underwent structural breaks that moved the economy from a determinate equilibrium to indeterminacy and back. Dating these structural breaks in policy determinacy would facilitate an understanding of what caused monetary policy to go off course and would provide more accurate parameter estimates with which to study the welfare consequences of indeterminacy. We rely on methods from Sims (2002) and Lubik and Schorfheide (2003) to solve the DSGE model in the presence of indeterminacy and apply Chib's (1998) change-point algorithm to a state-space representation of Woodford's (2003) linearized DSGE model. To make the model applicable to a long sample period, we add, in addition to the structural breaks, a time-varying inflation target and interest rate smoothing to the monetary policy equation. The resulting empirical model provides a robust representation of U.S. monetary policy regimes since 1959.

*Keywords:* State-Space model, change-point, regime switching, Kalman Filter, MCMC, DSGE model, Indeterminacy, Metropolis-Hastings, Gibbs, Bayes

# 1 Introduction

Macroeconomic models that are linearized reduced forms of general-equilibrium optimizing (DSGE) models with sticky prices are now widely considered to be ready for prime time—in the sense that they can confront the data, yield sensible parameter estimates, and provide useful policy analysis [Smets and Wouters (2002); McCallum and Nelson (1999)]. With specific reference to monetary policy, two issues that estimated DSGE models have begun to address are whether policy rules are indeterminate and whether monetary policy rules include interest rate smoothing [Rotemberg and Woodford (2002); Lubik and Schorfheide (2004)]. The promise of estimated DSGE models is that one can take the parameter estimates, plug them into the underlying optimizing model, and perform welfare calculations. In this way, policymakers could get a handle on the welfare implications of key features of monetary policy reaction functions, such as the choice to smooth interest rates or steps to avoid policy indeterminacy.

Lubik and Schorfheide (2004) present a Bayesian estimation method for DSGE models that evaluates the likelihood function under monetary policy indeterminacy. Without having structural breaks, however, one has to assume either determinacy or indeterminacy for the entire sample period in question. For U.S. data from 1960-1979, Lubik and Schorfheide show that the posterior odds ratio that is overwhelmingly in favor of monetary policy indeterminacy. For the period 1983-1997, in contrast, the posterior odds ratio is greatly in favor of monetary policy determinacy. These ad hoc estimation periods strongly suggest that at least one structural break took place in U.S. monetary policy between 1960 and 1990. Further consideration of U.S. monetary policy—and stable long-term interest rates—from the 1950s through the early 1960s would suggest that monetary policy might well have been determinate until at least the late 1960s. Thus, we apply a change-point model to an estimated DSGE model, in which two breaks can occur: first from policy determinacy to indeterminacy and then back. Parameter estimates that correspond with the sub-sample periods implied by the break points will provide the best inferences of the welfare consequences of the form of monetary policy indeterminacy that took place in the United States in the late 1970s. Another advantage of our change-point modelling approach over ad hoc sub-sample estimation is that parameters that are not subject to the structural breaks are estimated using all of the available data. In macroeconometrics, it is not desirable to re-estimate deep macroeconomic parameters, such as the rate of time preference or relative risk aversion, from scratch in sub-samples that delineate changes in monetary policy.

## 2 Solving the DSGE Model

With the above interest rate representation and a time-varying target rate of inflation, we *linearize* the New Keynesian monetary DSGE model from Woodford (2003) and express variables as deviations from the steady state levels:

$$\begin{aligned}
 \widetilde{GDP}_t &= E_t \widetilde{GDP}_{t+1} - \tau(\tilde{R}_t - E_t \tilde{\pi}_{t+1}) + g_t \\
 \tilde{\pi}_t &= \beta E_t \tilde{\pi}_{t+1} + \kappa(\widetilde{GDP}_t - z_t) \\
 \tilde{R}_t &= \rho_R \tilde{R}_{t-1} + \\
 &\quad + (1 - \rho_R)[\tilde{\pi}_t^T + \psi_1(\tilde{\pi}_t - \tilde{\pi}_t^T) + \psi_2(\widetilde{GDP}_t - z_t)] + \varepsilon_{R,t} \\
 \tilde{\pi}_t^T &= \rho_\pi \tilde{\pi}_{t-1}^T + \varepsilon_{\pi,t}
 \end{aligned} \tag{1}$$

where  $z$  is a technology shock,  $g$  is a demand shock,  $\varepsilon_R$  is a monetary policy shock, and  $\nu_t$  is an inflation target shock. Note that *in the linearized model certainty equivalence holds*, so we can add the additional linear shock process for the inflation target, or introduce other changes in the monetary policy equation, without disturbing the basic structure of the reduced form of the model. The first 2 equations in our model emerge from the GE theory, and the third equation takes the form of a Taylor rule with interest rate smoothing.

In canonical econometric form, the above LRE model can be re-written as:

$$y_t = A + B s_t \tag{2}$$

$$\Gamma_0 s_t = \Gamma_1 s_{t-1} + \Psi \varepsilon_t + \Pi \eta_t, \tag{3}$$

where the vector of rational expectations forecast error

$$\eta_t = [\widetilde{GDP}_t - E_{t-1} \widetilde{GDP}_t, \tilde{\pi}_t - E_{t-1} \tilde{\pi}_t]'$$

We solve the resulting linear rational expectations model using the time-series techniques introduced by Sims (2002) and later improved upon by Lubik and Schorfheide (2003) and (2004) dubbed LS(03) and LS(04) for convenience in what follows. Essentially the econometric problem that they are addressing is closely related to the common-sense perception of the proliferation of the impact that the interest rate rules have on the economy. The fact that the forecast errors might be influenced not only by the structural shocks in the economy (determinacy) but also by sunspot fluctuations (indeterminacy) amounts to a simultaneous equations setting and precludes

direct estimation of the model. In order to proceed with estimation, an econometrician (even without any regard for macroeconomic implications) *must* find a way to express  $\eta_t$  as a function of  $\varepsilon_t$ . Sims (2002) provided precise necessary and sufficient conditions to distinguish between 3 possible cases, depending on the parameters in the transition equation: a) such a function might be a deterministic one-to-one map, which would correspond to determinacy; b) there might be multiple solutions (indeterminacy), in which case LS(03) and LS(04) suggest a simple (linear) model for  $\eta_t$  which would uniquely determine it as a function of both structural shocks  $\varepsilon_t$  and one sunspot shock for the case of one-degree indeterminacy; c) no solution.

In this paper, we will fit Woodford's 3-equation DSGE model enriched with time-varying inflation target and smoothing in monetary policy, while allowing for 1 or 2-degree indeterminacy for the sample period between 1959 and 2004. In solving the above LRE model, following Sims (2002), we used Generalized Schur (QZ) decomposition of  $(\Gamma_0, \Gamma_1)$  to avoid possible problems with inverting  $\Gamma_0$ , and used the column space spanning conditions to rule out the "no solution" parameter configurations, and then applied Singular Value Decomposition to the matrix formed using the rows corresponding to unstable eigenvalues resulting from Schur Decomposition<sup>1</sup>. The resulting canonical multidimensional linear Gaussian state-space model has the following form:

$$y_t = A + Bs_t \quad (4)$$

$$s_t = \Gamma_1^* s_{t-1} + \Gamma_2^* \varepsilon_t \quad (5)$$

where the measurement equation can be re-written in greater detail:

$$\begin{pmatrix} \widetilde{GDP}_t \\ \pi_t \\ R_t \end{pmatrix} = \begin{pmatrix} 0 \\ \pi^* \\ R^* + \pi^* \end{pmatrix} + \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \widetilde{GDP}_t \\ \tilde{\pi}_t \\ \tilde{R}_t \\ \tilde{\pi}_t^T \\ E_t[GDP_{t+1}] \\ E_t[\pi_{t+1}] \\ g_t \\ z_t \end{pmatrix} \quad (6)$$

The output measure does not require removal of a mean because we use Hodrick-Prescott filtered

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<sup>1</sup>see Sims (2002) or LS(04) for details

GDP data, which is mean-zero by construction. Matrix  $A$  and transition equation matrices  $\Gamma_1^*, \Gamma_2^*$  are respectively 2x1, 8x8, and 8x4 convoluted functions of the original LRE model's parameters

$$\Theta = \{\tau, \beta, k, \psi_1, \psi_2, \rho_\pi, \rho_R, \rho_g, \rho_z, \pi^*, r^*\}.$$

In case of determinacy,  $\varepsilon_t$  is a 4x1 vector of fundamental exogenous shocks:

$$\varepsilon_t = \begin{pmatrix} \varepsilon_{R^*,t} \\ \varepsilon_{g,t} \\ \varepsilon_{z,t} \\ \varepsilon_{\pi^*,t} \end{pmatrix} \sim N(\mathbf{0}, \Omega), \text{ where } \Omega = \begin{pmatrix} \sigma_{\varepsilon_R}^2 & 0 & 0 & 0 \\ 0 & \sigma_{\varepsilon_g}^2 & \rho_{gz}\sigma_{\varepsilon_z}\sigma_{\varepsilon_g} & 0 \\ 0 & \rho_{gz}\sigma_{\varepsilon_z}\sigma_{\varepsilon_g} & \sigma_{\varepsilon_z}^2 & 0 \\ 0 & 0 & 0 & \sigma_{\varepsilon_{\pi^*}}^2 \end{pmatrix}$$

In case of 1- or 2-degree indeterminacy, we will assume that  $\eta_t$  could be expressed as a linear combination of the 4-dimensional exogenous shock and one-dimensional sunspot shock<sup>2</sup>. Then,  $\varepsilon_t$  is a 5x1 vector of fundamentals and the sunspot shock, which is orthogonal by assumption:

$$\varepsilon_t = \begin{pmatrix} \varepsilon_{R^*,t} \\ \varepsilon_{g,t} \\ \varepsilon_{z,t} \\ \varepsilon_{\pi^*,t} \\ \zeta_{1,t} \end{pmatrix} \sim N(\mathbf{0}, \Omega^{Ind}), \text{ where } \Omega^{Ind} = \begin{pmatrix} \sigma_{\varepsilon_R}^2 & 0 & 0 & 0 & 0 \\ 0 & \sigma_{\varepsilon_g}^2 & \rho_{gz}\sigma_{\varepsilon_z}\sigma_{\varepsilon_g} & 0 & 0 \\ 0 & \rho_{gz}\sigma_{\varepsilon_z}\sigma_{\varepsilon_g} & \sigma_{\varepsilon_z}^2 & 0 & 0 \\ 0 & 0 & 0 & \sigma_{\varepsilon_{\pi^*}}^2 & 0 \\ 0 & 0 & 0 & 0 & \sigma_{\zeta_1}^2 \end{pmatrix}$$

Our quarterly data  $Y_n$  is given by 3x1 vector  $y_t$ , where  $t = 1959Q2, \dots, 2004Q3$ . Note that the first element of the state vector  $s_t$  is actually an observed data point<sup>3</sup>. The remaining 7 elements are latent, although the second and the third elements are latent only up-to  $\pi^*$  and  $R^*$  parameter values. The first three elements of  $s$  correspond to H-P filtered GDP, the deviation from steady-state quarterly PCE inflation, and the deviation of the average quarterly funds rate from its steady state level.

<sup>2</sup>We could easily allow for 2- or higher-dimensional vector of sunspot shocks, but we would expect to get poor identification of high-dimensional sunspot shocks given such relatively short data series for the period attributed to indeterminacy

<sup>3</sup>Kalman Filter can still be used for a linear Gaussian state-space model even if some elements of the state vector are observed (see Harvey p.109). In that case, Kalman filter will produce an updated forecast of the state vector with observed elements exactly matching the data and zeros in the rows and columns corresponding to such elements in the covariance matrix.

### 3 Structural Breaks

So far we have presented a fully consistent GE-based model that leads to a likelihood function of the data that depends on the *constant determinacy state* of the economy  $d_t = d (\forall t = 1, \dots, n)$ . However, in reality US economy was subject to several structural breaks in the past. In order to account for these regime changes, let's assume that our sample starts with determinacy *D1*:  $d_0 = 1$ , but then, at some time point  $t_1$  there is one structural break to indeterminacy *Ind*:  $d_{t_1} = 2$ , and then at time  $t_2$  there is a second break to a new determinacy *D2*:  $d_{t_2} = 3$ . As discussed above, because certainty equivalence holds in our model, neither I-S nor Phillips curve equations, derived as log-linearizations of (respectively) the consumer utility and firm profit maximization conditions, coupled with market clearing, are effected by the introduction of structural breaks - optimization conditions are still the same. The only change across the states occurs to the parameters (not the form) of the policy rule equation.

Define these regime-driving parameters as  $(\psi_1^{(1)}, \psi_2^{(1)})$ ,  $(\psi_1^{(2)}, \psi_2^{(2)})$ ,  $(\psi_1^{(3)}, \psi_2^{(3)})$  resulting in  $\Theta^{(i)} = \Theta_{\{\psi_1, \psi_2\}} \cup \{\psi_1^{(i)}, \psi_2^{(i)}\}$ ,  $i = 1, 2, 3$ . Also, as discussed above, indeterminacy regime has a "bigger" error covariance matrix  $\Omega^{(2)} = \Omega^{Ind}$ , while  $\Omega^{(1)} = \Omega^{(3)} = \Omega$ .

It is important to mention that indeterminacy in our model could be 1 or 2-dimensional<sup>4</sup>, depending on the parameter vector  $\Theta^{(2)}$  from the indeterminacy region, leading to a particular form of the likelihood function for that region. Essentially, there are only 3 states in our model, but the second state could produce two different forms of the likelihood function. Given *Ind*, one can think of the likelihood consisting of two distinct pieces corresponding to two possible degrees of indeterminacy, which will be constructed using the logic similar to LS(04) by introducing indicator functions for parameter vector lying in 1 or 2-degree indeterminacy region.

From GE prospective, at each time point there is uncertainty about the state of the economy driven by an exogenous to our model mechanism (transition from  $\psi_j^{(i)}$  to  $\psi_j^{(i+1)}$ ,  $i, j = 1, 2$ ) controlling the Federal Monetary Policy, which *is* in turn a part of our model. We do not attempt to model that transition mechanism in this paper. Instead, we take it as a given exogenous event that creates structural breaks in our economy at some two random time points  $t_1$  and  $t_2$  by changing the parameters of monetary policy rule. Conditional on such exogenous structural break occurring, we are back to the full GE paradigm. Timing of the structural breaks

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<sup>4</sup>The maximum degree of indeterminacy in our model with two expectational errors is 2. An alternative would be to follow LS(04) restricting the indeterminacy to be only one-dimensional, which is by no means a required condition for our model.

is the main focus of our paper. Namely, we attempt to deduce the probability of a structural break occurring at each point in the past conditional on the observed data.

## 4 Estimation of the State-Space Model with Structural Breaks

The state-space model with structural breaks estimated in this paper has the following form

$$y_t = A + Bs_t \quad (7)$$

$$s_t = \Gamma_1^*(d_t)s_{t-1} + \Gamma_2^*(d_t)\varepsilon_t \quad (8)$$

where  $\Gamma_1^*(d_t)$  and  $\Gamma_2^*(d_t)$  are determined according to the dynamics of the economy in the determinacy state  $d_t$  at each time  $t$ .

The prevalent estimation approach for the above model in the past was to draw from the joint posterior  $\pi(\Theta, \Omega, t_1, t_2|Y_n)$  using the following blocking scheme:

1.  $\pi(\Theta, \Omega|Y_n, t_1, t_2)$
2.  $\pi(t_1, t_2|\Theta, \Omega, Y_n) = \pi(t_1|\Theta, \Omega, Y_n)\pi(t_2|\Theta, \Omega, Y_n, t_1)$ .

However, in our paper we will follow a modern "multiple change-point" estimation approach originally introduced in Chib (1996) and further generalized in Chib (1998). The centerpiece of this method is a transformation in terms of a latent discrete state variable that indicates the regime from which a particular observation has been drawn. In other words, instead of using a single-move sampler to draw times  $t_1$  and  $t_2$  at which the structural break occurred, we will draw states  $D_n = \{d_t \in \{1, 2, 3\}\}_{t=1}^n$  in a multi-move sampler, which is by far more efficient than a single-move, because it groups highly correlated elements of a Markov Chain in one block drastically reducing autocorrelation of the draws.

Following Chib (1998), we construct Markov state transition probability matrix<sup>5</sup>

$$P = \begin{pmatrix} p_{11} & p_{12} & 0 \\ 0 & p_{22} & p_{23} \\ 0 & 0 & 1 \end{pmatrix} \quad (9)$$

with the joint prior of its elements given by

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<sup>5</sup>where  $p_{ij} = Pr(d_t = j|d_{t-1} = i)$

$$\pi(P) \propto \prod_{i=1}^2 p_{ii}^{(a-1)} (1 - p_{ii})^{(b-1)},$$

where  $a \gg b$  are selected to result in reasonable prior moments.

Our objective is to draw from the posterior  $\pi(\Theta, \Omega, P, D_n | Y_n)$  which can be accomplished by simulating the following full conditional distributions:

1.  $\pi(\Theta, \Omega, P, S_n | D_n, Y_n) = \pi(P | D_n) \pi(\Theta, \Omega | Y_n, D_n, P) \pi(S_n | \Theta, \Omega, P, D_n, Y_n)$
2.  $p(D_n | Y_n, \Theta, \Omega, P, S_n) = \prod_{t=1}^{n-1} p(d_t | Y_n, D^{t+1}, \Theta, \Omega, P, S_n),$

where we adapt the notation similar to Chib (1998):  $D_t = (d_1, \dots, d_t)$ ,  $D^{t+1} = (d_{t+1}, \dots, d_n)$ ,  $S_n = (s_1, \dots, s_n)$ ,  $Y_t = (y_1, \dots, y_t)$  and use  $p(\cdot)$  to denote a discrete mass function, while  $\pi(\cdot)$  denotes a density function of some random variable with one or more continuous components. To avoid ambiguity we would like to emphasize the difference between the latent state of determinacy  $d_t$  and the latent state variable  $s_t$  from the state-space model in equation 4.

#### 4.1 Parameter sampler

The first block is sampled using method of composition in three parts.

First, let  $n_{ii}$  be the number of one-state transitions from state  $i$  to state  $i$  (i.e. staying put). Then, a Bernoulli likelihood  $p(D_n | p_{ii})$  multiplied by the Beta prior  $\pi(p_{ii})$  given above results in  $p_{ii} | D_n \sim \text{Beta}(a + n_{ii}, b + 1)$ ,  $i = 1, 2$ .

Second, for convenience of notation, let  $t_0 = 0$ ,  $t_1$  and  $t_2$  be the time points of the first and second structural breaks respectively, and  $t_3 = n$ . As discussed above, the likelihood function will differ across states, although it will still be multivariate normal. Let  $f^{(i)}$  denote Gaussian density function given state  $i$ . Then, the density  $\pi(\Theta, \Omega | Y_n, D_n, P)$  could be sampled using the usual Kalman Filter recursion formula:

$$\pi(\Theta, \Omega | Y_n, D_n, P) \propto f(Y_n | \Theta, \Omega, D_n) \pi(\Theta, \Omega | D_n) = \tag{10}$$

$$\prod_{i=1}^3 \prod_{t=t_{i-1}+1}^{t_i} f^{(i)}(y_t | Y_{t-1}, \Theta^{(i)}, \Omega^{(i)}, D_n) \pi(\Theta, \Omega | D_n) \tag{11}$$



where

$$f^{(i)}(y_t|Y_{t-1}, \Theta^{(i)}, \Omega^{(i)}, D_n) = N(y_t|A + Bs_{t|t-1}^{(i)}, f_{t|t-1}^{(i)}) \quad (12)$$

such that the log-likelihood term is proportional to:

$$\begin{aligned} \log(f(Y_n|\Theta, \Omega, D_n)) &\propto -\sum_{i=1}^3 \sum_{t=t_{i-1}+1}^{t_i} [\log(\det(f_{t|t-1}^{(i)})) \\ &\quad + (y_t - A - Bs_{t|t-1}^{(i)})'(f_{t|t-1}^{(i)})^{-1}(y_t - A - Bs_{t|t-1}^{(i)})] \end{aligned}$$

where, suppressing the  $(i)$  superscript for transparency, the details of Kalman filter updates are as follows:

- 1) state forecast mean  $s_{t|t-1} = \Gamma_1^* s_{t-1|t-1}$
- 2) state forecast variance  $P_{t|t-1} = \Gamma_1^* P_{t-1|t-1} (\Gamma_1^*)' + \Gamma_2^* \Omega (\Gamma_2^*)'$
- 3) data forecast variance  $f_{t|t-1} = B P_{t|t-1} B' + \mathbf{0}$
- 4) Kalman gain  $K_t = P_{t|t-1} B' f_{t|t-1}^{-1}$
- 5) update state mean  $s_{t|t} = s_{t|t-1} + K_t (y_t - A - B s_{t|t-1})$
- 6) update state variance  $P_{t|t} = (I - K_t B) P_{t|t-1}$

The above equations require an initialization of  $s_{t=0|t=0}$  and  $P_{t=0|t=0}$ . We set all elements of  $s_{t=0|t=0}$  to equal 0 in order to ensure that the linear combination of the state variables associated with unstable roots of the LRE system is zero<sup>6</sup>.

The density  $\pi(\Theta, \Omega|Y_n, D_n, P)$  found in equation 10 serves as a target density for Tailored<sup>7</sup> Metropolis-Hastings (MH) algorithm originally proposed by Chib and Greenberg (1994).

Third, sampling of  $S_n|\Theta, \Omega, P, D_n, Y_n$  is straightforward, because conditional on  $D_n$ , we are faced with a simple linear Gaussian state-space model with time-varying coefficients. The standard approach is to draw  $S_n$  using one-period smoothing, which amounts to adding two more steps to the Kalman filter procedure above:

- 7)  $M_t = P_{t|t} (\Gamma_1^*)' (P_{t+1|t})^{-1}$
- 8)  $P_{t|t+1} = P_{t|t} - M_t P_{t+1|t} (M_t)'$

and then sampling the states backwards starting with

$$s_n \sim N(s_{n|n}, P_{n|n})$$

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<sup>6</sup>see LS(03)

<sup>7</sup>see Chib (2001) for discussion of various MH algorithms and their tuning

followed by ( $\forall t = n - 1, \dots, 1$ )

$$s_t \sim N(s_{t|t+1}, P_{t|t+1}), \text{ where } s_{t|t+1} = s_{t|t} + M_t(s_{t+1} - \Gamma_1^* s_{t|t}).$$

## 4.2 Determinacy state sampler

One important observation to make about our SSM in equation (7) is that determinacy state  $d_t$  only appears in the transition equation, which means that  $y_t$  is independent of  $d_t$  given  $s_t$  and  $\Theta$ . Therefore, we have:

$$p(D_n|Y_n, \Theta, \Omega, P, S_n) = p(D_n|S_n, \Theta, \Omega, P) = \prod_{t=1}^{n-1} p(d_t|S_n, D^{t+1}, \Theta, \Omega, P)$$

Effectively, for the purposes of sampling determinacy states  $D_n$ , the latent variables  $S_n$  have entirely replaced the actual data  $Y_n$ , thereby reducing this block to the linear regression model with multiple change-points. Following Chib (1998), we use method of composition to sample  $D_n|S_n, \Theta, \Omega, P$  by drawing  $d_t$  backwards from time  $t = n - 1$  conditional on  $D^{t+1}$ . Chib (1996) has shown that

$$p(d_t|S_n, D^{t+1}, \Theta, \Omega, P) \propto p(d_t|S_t, \Theta, \Omega, P)p(d_{t+1}|d_t, P),$$

where  $p(d_{t+1}|d_t, P) = p_{d_t d_{t+1}}$

Starting with  $t = 1$ , Chib (1998) utilizes a recursive forward calculation to find the mass function  $p(d_t|S_t, \Theta, \Omega, P)$  ( $\forall t = 1, \dots, n$ ) by recursively transforming  $p(d_{t-1}|S_{t-1}, \Theta, \Omega, P)$  through:

$$p(d_t = k|S_t, \Theta, \Omega, P) = \frac{p(d_t = k|S_{t-1}, \Theta, \Omega, P)f(s_t|S_{t-1}, \Theta^{(k)}, \Omega^{(k)})}{\sum_{l=1}^m p(d_t = l|S_{t-1}, \Theta, \Omega, P)f(s_t|S_{t-1}, \Theta^{(l)}, \Omega^{(l)})}$$

where  $p(d_t = k|S_{t-1}, \Theta, \Omega, P) = \sum_{l=k-1}^k p_{lk} \times p(d_{t-1} = l|S_{t-1}, \Theta, \Omega, P)$

and  $f(s_t|S_{t-1}, \Theta^{(l)}, \Omega^{(l)}) = N(\Gamma_1^*(d_t = l)s_{t-1}, \Gamma_2^*(d_t = l)\Omega^{(l)}(\Gamma_2^*(d_t = l))')$

## 5 Estimation Results

## 6 Conclusion

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