An Empirical Analysis of Permanent Income Hypothesis Applied to Italy using State Space Models with non zero correlation between trend and cycle

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Abstract

This article explores by an econometric approach the permanent income hypothesis. The classical cointegration analysis applied by Cochrane and the Kalman filter technology with correlated error components are used. The latter approach compared with the former reveals a clear rejection of PIH for USA.These conclusions are reversed for Italy.

Keywords:Kalman filter, trend, cycles, cointegration, correlated components,state space models, unobserved components

JEL classification:C22,C32,E32

1. Introduction

In this paper we shall talk about permanent income hypothesis (PIH). It is obvious to think that households utilize their savings when they consume more than their income may allow. It is even more obvious to think that it will be impossible to continue to sustain their expenditures by their savings. Late or early they will have to spend less. On the other hand there is another way of thinking opposite to this. The main reason to this contraposition has to be searched for instance on the failure of the first view to predict events like the continuous growth of US GDP from 1994.

This paper will focus on the alternative: the permanent income theory of Friedman (1957) . Friedman argues that a saving decline, like the one happened in 1993, was caused by the expectations of an increment of their future income. The households save less when they expect that their income will improve. The history says that Friedman developed his

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theory in a book of 1957. We shall briefly review the source of his speculations. After this we shall focus (as in Morley (2004)) on how is possible to test PIH by vector error correction approach. Nevertheless this approach will be misleading because doesn't take into account all the causal relationships hidden in the movements of time series. This approach differs from the typical one described in details in Harvey (1989), and we shall motivate this choice on the basis of economic intuition.

2. Fisher's theory of interest and the Permanent Income Hypothesis

Ireland (1995) reviews Fisher's theory of interest (see Fisher (1907)) using the diagram shown in figure 1.

To simplify the things Fisher considers only two periods. The horizontal axis measures the goods at time 0, the vertical axis measures goods at time 1. The household theorized by Fisher receives an income y_0 during period 0 and an income y_1 during the period 1. The household regulates his consumption choices on the basis of the interest rate. This one measures the rate at which the market allows the household to exchange goods at time 1 for goods at time 0. If, for istance, the household lends one unit of the good at time 0, (1+r) unit of that good will be borrowed at time 1. In the same way, if the household receives one unit of the good at time 0, will be have to borrow (1+r) unit of that at time 1.

The line AA' in figure 1 represents the budget constraint that which passes trought the points y_0 e y_1 and has a slope of $-(1+r)$.

The indifference curves U_0 and U_1 represent the household's preferences in the two distinct periods. For a utility level and a budget constraint we shall have a consumption level c_0 for the first period and c_1 for the second period. The utility goes up with consumption in both periods. The slope of indifference curve represents the intertemporal marginal rate of substitution in the two period. This rate also represents the rate at which the household is keen on exchanging goods between period 1 and period 2. In order to maximize their utility, the households choose the couple of levels of consumption (c_0, c_1) where the indifference curve U_0 is tangent to the budget constrain AA'. At (c_0, c_1) the intertemporal marginal rate of substitution is equal to $(1+r)$.

The saving during the first period are equal to $s_0 = y_0 - c_0$. We suppose now that the couple satisfying the budget constrain is $(y_0^{'}, y_1^{'})$ (like shown in figure 1). Since this new point is on the same budget constrain than the former the household will continue to choose (c_0, c_1) for every combination of points of the line AA'. The present value of the couple $(y_{0,}, y_1)$ is

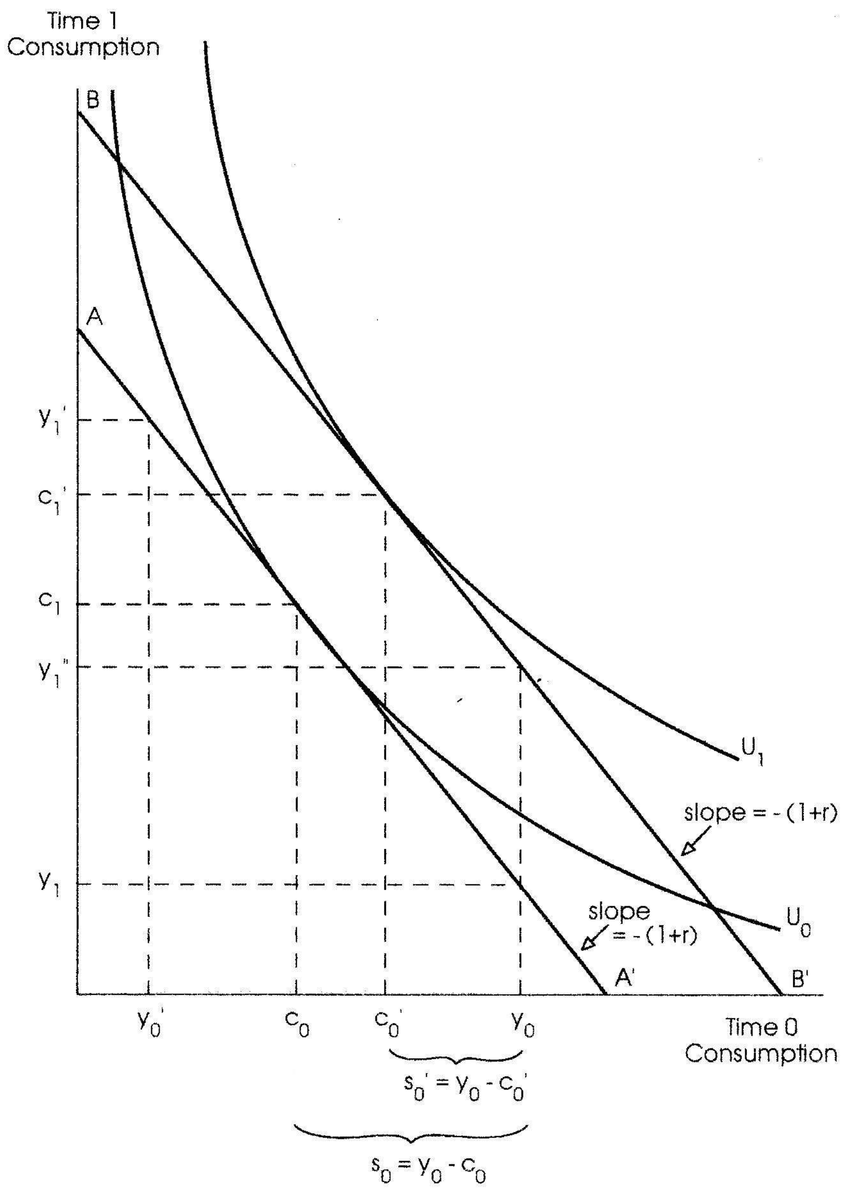
$$PV = y_0 + \frac{y_1}{(1+r)} \qquad (1)$$

The equation 1 shows how for Fisher the household's consumption choice depends exclusively on the total income flow and not from y_0 and y_1 seperately.

There is an another outstanding implication of Fisher diagram.

Suppose now that y_0 remains constant and that the income at time 1 becomes $y_1^{''}$. This change increases the present value (PV) of the new couple $(y_0, y_1^{''})$ from AA' to BB' and pushes the household to choose the new couple $(c_0^{'}, c_1^{'})$. Since $c_0 > c_1$, the income's increment at time 1 allows to the households to reduct their savings at time 0. More exactly we have a reduction from $s_0 = y_0 - c_0$ a $s_0^{'} = y_0 - c_0^{'}$.

Figure 1: Fisher's Diagram



This second implication is crucial because the households save less when they expect they future income will increase. On the other hand they will save more when they expect that their future income will decrease. This second implication contradicts the popular intuition by which a saving's reduction is strictly related with a reduction of income.

3. The PIH of Milton Friedman and Robert Hall

Friedman (1957) extended Fisher's speculations to a multiperiodal model in which households may be uncertain about their future income. Equation 1 may be rewritten for the case of an infinite living agent

$$PV = \sum_{t=0}^{\infty} \frac{Ey_t}{(1+r)^t} \tag{2}$$

where Ey_t denotes the expected income from the household at time t. Friedman defines the permanent income y_p as the costant level of income that, if received with certainty every period t , has the same present value of the present value of the expected income 's flow. This concept may be rewritten as

$$\sum_{t=0}^{\infty} \frac{y_p}{(1+r)^t} = PV = \sum_{t=0}^{\infty} \frac{Ey_t}{(1+r)^t} \tag{3}$$

From maths we know that

$$\sum_{t=0}^{\infty} \frac{1}{(1+r)^t} = \frac{1+r}{r} \tag{4}$$

and we may rewrite equation 4 as

$$y^p = \frac{r}{1+r} PV \tag{5}$$

The first implication of Fisher's research is extended to multiperiodal case. The consumption's choices of the initial period are a function of permanent income

$$c_0 = f(y_p) \tag{6}$$

Even the second implication may be easily extended. The households save less when they expect that their permanent income will increase. On then other hand they will save more when they expect they permanent income will decrease.

Hall (1978) developed a mathematical version of the same model that gives a more precise relation between savings and expected future income. As a matter of the fact, Hall's model indicates exactly how saving's data may be utilized to forecast future changes in the income. Also Hall assumes that representative agent lives many periods and that he is uncertain about future income's prospective. The representative agent has an expected utility of the following form:

$$E \sum_{t=0}^{\infty} \beta^t u(c_t) \tag{7}$$

where E measures the household's expectation, $u(c_t)$ measures his utility caused by consuming a certain quantity c_t at time t, and β is the discount factor that has value between zero and one. The household has at time t some assets A_t, and earns a costant interest rate r on these assets. His capital income at time t is $y_{kt} = rA_t$. His labour income at time t will be y_{lt}. At the end of time time household's total income will be divided between consumption c_t and saving $s_t = y_t - c_t$. The total ammount of the assets will be

$$A_{t+1} = A_t + s_t = (1+r)A_t + y_{lt} - c_t \tag{8}$$

To the households is allowed to borrow at a costant rate r. As a consequence of this the assets may have a negative value. In the long run the agents will have to bring back all the money

$$lim_{t \longrightarrow \infty} \frac{A_t}{(1+r)^t} = 0 \tag{9}$$

The effects of this last equation on the preceding one is straightforward solving 8 iteratively forward to obtain

$$A_t = \sum_{t=0}^{\infty} \frac{c_{t+j} - y_{lt+j}}{(1+r)^{j+1}} \tag{10}$$

Equation 10 shows how every debt at time t is paid plugging future consumption below future income.

Another implication of equation 10 is the following

$$A_t = \sum_{t=0}^{\infty} \frac{E_t c_{t+j}}{(1+r)^{j+1}} - \sum_{t=0}^{\infty} \frac{E_t y_{lt+j}}{(1+r)^{j+1}} \tag{11}$$

Equation 11 states the total present values of assets has to be enough strong to counterbalance every discrepancy between the present value of expected future consumption and the present value of expected future labour income. The representative agent chooses consumption c_t and A_{t+1} for every period in order to maximize utility function in equation 7 subject to constraints in equation 8 and 9. The solution to the problem is the following intertemporal euler equation

$$u^{'}(c_t) = \beta(1+r)E_t u^{'}(c_{t+1}) \tag{12}$$

If we further assume the following identities holds $\beta = \frac{1}{1+r}$ and $u(c) = u_0 + u_1 c - (u_2/2)c^2$ where u_0, u_1, u_2 are positive constants. Under these peculiar conditions, equation 12 becomes

$$c_t^{'} = E_t c_{t+1} \tag{13}$$

This synthesizes Hall's idea of consumption's random walk under PIH.

More exactly we may observe that $E_t c_{t+j} = c_t$, for every j=0,1,2,.... When this latter result is substitued in 11 we obtain

$$c_t = rA_t + \frac{r}{(1+r)} \sum_{t=0}^{\infty} \frac{E_t y_{lt+j}}{(1+r)^t} \tag{14}$$

The right part of 14 tells us how current consumption is function present value of current capital income plus $\frac{r}{(1+r)}$ present value of expected labour income. In other words consumption is function of permanent income. Moreover if we pose $y_{kt} = rA_t$ and $s_t = y_{kt} + y_{lt} - c_t$, and $\Delta y_{lt} = y_{lt} - y_{lt-1}$, equation 14 becomes

$$s_t = -\sum_{t=0}^{\infty} \frac{E\Delta y_{lt+j}}{(1+r)^j} \tag{15}$$

Equation 15 clearly states that household's saving decreases when expected future income will increase. Vivecersa saving increases when expected household's future income decreases.

4. Further developments

An important feature of Hall's model is that agents maximize their utility consuming their permanent income each period. As consequence of this if consumption is too volatile than permanent income, PIH is rejected. Deaton (1987) finds that consumption is excessively smooth. In order to reconcile empirical evidence with theory, alternative models were build (see Flavin (1981) , Flavin (1993), Campbell and Mankiw (1989, 1990)). How Vahid and Engle (1993) note, these type of models use the following unobserved component form:

$$\begin{bmatrix} y_t \\ c_t \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} y_t^p + \begin{bmatrix} 1 \\ \lambda \end{bmatrix} y_t^T$$

Schleicher (2003) points out: " *In Campbell and Mankiw's model the economy is populated by two type of agents. Rational individuals consume their permanent income in each period, while their myopic counterparts consume their present income. λ is defined as the ratio of income that belongs to the myopic rule of thumb consumers. In Flavin's model λ has a conceptually similar interpretation as the marginal propensity to consume out of transitory income.*"

Due to the failure of the PIH to explain the smoothness of aggregate consumption, althernative theories of consumption behaviour were build. They are habit formation in consumer preferences (see Deaton (1987), Abel (1990), Costantinides (1990) ,Carroll and Weil (1994) ,Heaton (1995) ,Campbell and Cochrane (1999) ,Carroll et al. (2000) ,Dynan (2000) ,Fuhrer (2000), Boldrin et al. (2001)) and precautionary savings under uncertainty about future income (see Leland (1968), Caballero (1990), Carroll (1994) ,Hubbard et al. (1994) ,Normandin (1994) ,Carroll and Samwick (1998) ,Carroll et al. (2003)).

5. VECM and common trends representation

We begin our discussion focusing on this set of equations

$$y_t = y_t^p + v_t \tag{16}$$

$$y_t^p = \mu_y + y_{t-1}^p + u_t \tag{17}$$

$$c_t = y_t^p \tag{18}$$

Permanent income is the stochastic trend of income, which is made of the permanent component and of a transitory component, v_t and u_t are the shocks to the transitory and the permanent component of income. They should be orthogonal and normally and independently distributed. Consumption and income are cointegrated because they share the single unobservable common stochastic trend in this system. Intuitively by some mathematical passages the system 1-3 may be rewritten for a VAR(1) process as

$$\begin{pmatrix} y_t \\ c_t \end{pmatrix} = \begin{pmatrix} \mu_y \\ \mu_y \end{pmatrix} + \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} y_{t-1} \\ c_{t-1} \end{pmatrix} + \begin{pmatrix} w_t \\ u_t \end{pmatrix},$$

$$w_t = u_t + v_t,$$

from which we obtain the VECM representation:

$$\begin{pmatrix} \Delta y_t \\ \Delta c_t \end{pmatrix} = \begin{pmatrix} \mu_y \\ \mu_y \end{pmatrix} + \begin{pmatrix} -1 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} y_{t-1} \\ c_{t-1} \end{pmatrix} + \begin{pmatrix} w_t \\ u_t \end{pmatrix}$$

where

$$\Pi = \begin{pmatrix} -1 & 1 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \begin{pmatrix} -1 & 1 \end{pmatrix} = \alpha\beta'$$

The common trend representation is derived by considering that, as $y_t - c_t = v_t$, the MA representation for consumption and income growth is:

$$\begin{pmatrix} \Delta y_t \\ \Delta c_t \end{pmatrix} = \begin{pmatrix} \mu_y \\ \mu_y \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} w_t \\ u_t \end{pmatrix} + \begin{pmatrix} -1 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} w_{t-1} \\ u_{t-1} \end{pmatrix}$$

from which:

$$\begin{pmatrix} y_t \\ c_t \end{pmatrix} = \begin{pmatrix} \mu_y \\ \mu_y \end{pmatrix} t + C*(L) \begin{pmatrix} w_t \\ u_t \end{pmatrix} + C(1) z_t,$$

and $C(1) = \beta_\perp (\alpha'_\perp \beta_\perp)^{-1} \alpha'_\perp$

$$\begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \left[\begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right]^{-1} \begin{pmatrix} 0 & 1 \end{pmatrix}$$

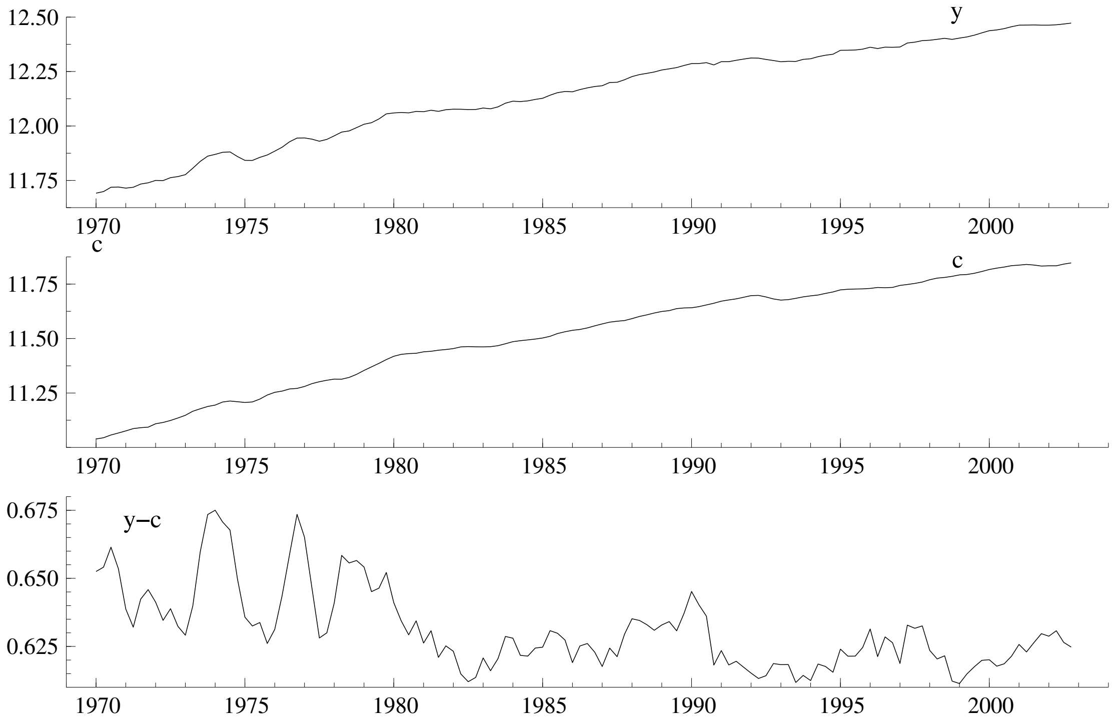
How Favero (2001) points out : "... *since in this application* $(\alpha'_\perp \beta_\perp)^{-1} = 1$, *consumption and income share a single common stochastic trend. Such trend can be represented as*

$$\alpha'_\perp \left(\begin{pmatrix} \mu_y \\ \mu_y \end{pmatrix} t + \begin{pmatrix} \sum_{i=1}^t w_t \\ \sum_{i=1}^t u_t \end{pmatrix} \right)$$

and only shocks to the permanent component of income enter the trend ."

We may clearly extend this way of reasoning to any Var of order p model (see Banerjee et al. (1993) for further details). This discussion spin on the orthogonalization among the various stochastic components. A clear limit of this type of model is on these restrictions. By Kalman filter technology and by a proper state space form we may relax these orthogonality strong assumptions and let the system evolves as it likes during the

Figure 2: Log of Real Italian GDP and log of consumption of non durable goods and service sector from 1970:1 to 2002:4



maximum Likelihood estimation (see Morley (2004)).

6. The VECM for Italy and US

As Cochrane (1994) and Morley (2004) [1] we will use quarterly time data about US real GDP, real private consumption expenditure in non durable goods plus private consumption expenditure in the service sector available from St.Louis Fed website [2] from 1953:1 up to 2003:3 in chained 2000 US dollars. A similar quarterly data set will be used for Italy from ISTAT the National Italian Bureau of Statistics [3].
For our experiment we shall use the log of real GDP and the log of real private consumption expenditure of non durable goods plus consumption expenditure in the service sector.

Figure 2 and figure 3 plot the log of income, the log of consumption and the log of their ratio for Italy and USA. The latter series appears clear stationary for both countries while the others are clearly non stationary. This straightforward intuition is confirmed by the Augmented Dickey Fuller tests applied to these rime series. As in Morley (2004) I include a time trend and an intercept for income and consumption, while I use only an intercept for their ratio. Table 1 and Table 2 show the results respectively for the USA

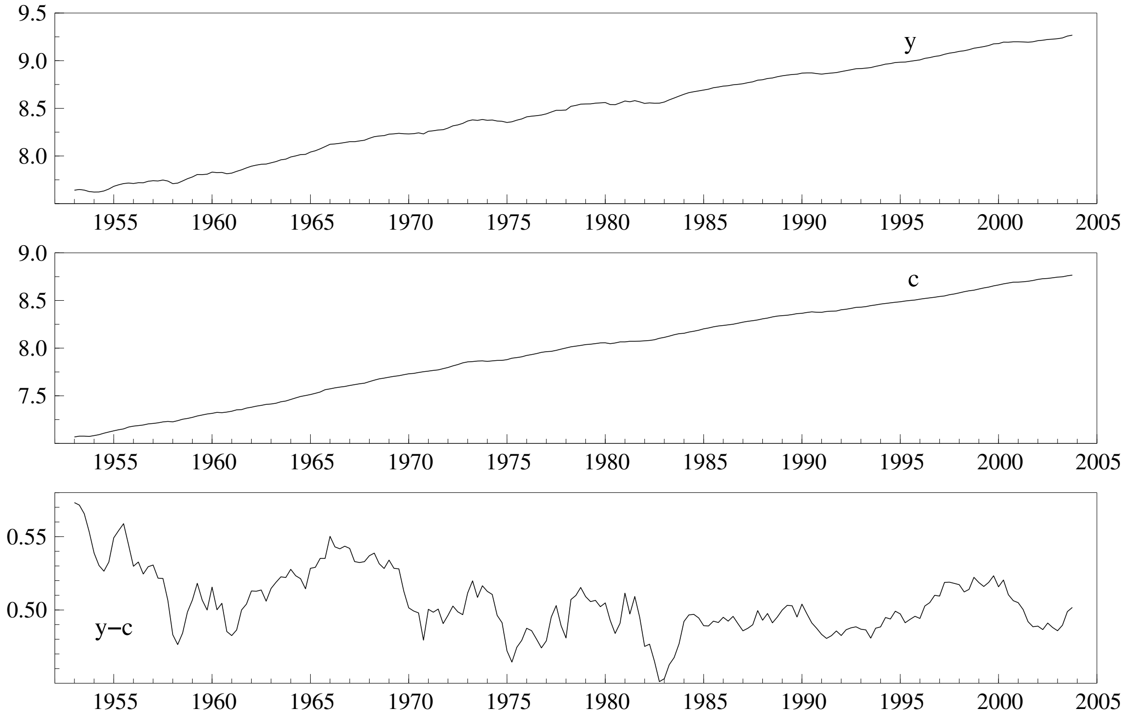
[1] Morley's analysis of VECM differs from Cochrane mainly because he uses quarterly updated US GDP time series instead of GNP's one

[2] http://www.stls.frb.org/fred

[3] http://con.istat.it/default.asp?lg=E

For further queries regarding these particular datasets please contact dipalma@istat.it

Figure 3: Log of Real US GDP and log of consumption of non durable goods and service sector from 1953:1 to 2003:4



and Italian time series.

The ADF is based on the following equation

$$\Delta y_t = \alpha + (\beta - 1)y_{t-1} + \Sigma_{i=1}^{s}\gamma_i \Delta y_{t-1} + \epsilon_t \tag{19}$$

Hendry and Doornik (2001a) show that the first column is the number of lagged differences, so the first line gives the results for the ADF(4) test.The second column is the t-value, which is the ADF test statistic, the third column is the coefficient on y_{t-1} (the coefficient used in t-adf), the next column gives the equation standard error. The next two columns give the t-value of the highest lag (of γ_s =4,3,2,1), followed by the p-value of the lag. The suggested strategy is to select the highest s with a significant last γ_s (the distribution of $\hat{\gamma_s}$ is the conventional student-t distribution). So for y_t and c_t we have for both countries an ADF(1). The latter time series are clearly stationary for both countries.

Exactly as in Morley (2004) I use the VECM representation of the VAR(p) model

$$\Delta c_t = \delta_c + \pi_c(y_{t-1} - c_{t-1}) + \sum_{j=1}^{p-1}(\beta_{cc,j}\Delta c_{t-j} + \beta_{cy,j}\Delta y_{t-j}) + \varepsilon_{ct}$$

$$\Delta y_t = \delta_y + \pi_y(y_{t-1} - c_{t-1}) + \sum_{j=1}^{p-1}(\beta_{yc,j}\Delta c_{t-j} + \beta_{yy,j}\Delta y_{t-j}) + \varepsilon_{yt}$$

and I report Johansen cointegration test and OLS estimation for USA and Italy.

Table 3 and Table 4 show the results of Johansen Cointegration analysis for a Var(3) and a Var(2) applied respectively to USA and to Italy. For the US case, we observe that

Table 1: ADF test on USA dataset

y_t		ADF tests Costant + Trend		cv5=-3.43		cv1=-4.01	
D-lag	t-adf	$(\beta)y_{t-1}$	σ	t-Δy_{t-1}	t-prob	AIC	F-prob
4	-2.546	0.95472	0.008755	-0.5261	0.5994	-9.442	
3	-2.706	0.95289	0.008739	-0.4500	0.6532	-9.450	0.5994
2	-2.870	0.95125	0.008721	1.737	0.0839	-9.459	0.7874
1	-2.575	0.95684	0.008766	4.762	0.0000	-9.454	0.3271
0	-1.900	0.96669	0.009237			-9.354	0.0001
c_t		ADF tests Costant + Trend		cv5=-3.43		cv1=-4.01	
D-lag	t-adf	$(\beta)y_{t-1}$	σ	t-Δy_{t-1}	t-prob	AIC	F-prob
4	-1.931	0.98370	0.004138	-0.7872	0.4322	-10.94	
3	-2.038	0.98293	0.004134	2.430	0.0160	-10.95	0.4322
2	-1.786	0.98493	0.004186	1.107	0.2696	-10.93	0.0406
1	-1.698	0.98571	0.004188	4.394	0.0000	-10.93	0.0542
0	-1.564	0.98624	0.004379			-10.85	0.0000
$y_t - c_t$		ADF tests Costant		cv5=-2.88		cv1=-3.46	
D-lag	t-adf	$(\beta)y_{t-1}$	σ	t-Δy_{t-1}	t-prob	AIC	F-prob
4	-3.154*	0.90573	0.007903	-0.7292	0.4667	-9.651	
3	-3.299*	0.90256	0.007893	-0.8422	0.4007	-9.659	0.4667
2	-3.495**	0.89834	0.007887	0.9263	0.3554	-9.665	0.5392
1	-3.389*	0.90285	0.007884	1.283	0.2009	-9.671	0.5543
0	-3.215*	0.90901	0.007897			-9.672	0.4457

Table 2: ADF test for Italian dataset

y_t		ADF tests Costant + Trend		cv5=-3.43		cv1=-4.01	
D-lag	t-adf	$(\beta)y_{t-1}$	σ	t-Δy_{t-1}	t-prob	AIC	F-prob
4	-2.284	0.95663	0.007116	-0.7021	0.4840	-9.837	
3	-2.394	0.95499	0.007101	-1.472	0.1437	-9.849	0.4840
2	-2.615	0.95110	0.007135	0.4404	0.6604	-9.847	0.2697
1	-2.586	0.95255	0.007111	5.143	0.0000	-9.861	0.4196
0	-1.946	0.96095	0.007807			-9.682	0.0000
c_t		ADF tests Costant + Trend		cv5=-3.43		cv1=-4.01	
D-lag	t-adf	$(\beta)y_{t-1}$	σ	t-Δy_{t-1}	t-prob	AIC	F-prob
4	-1.391	0.98645	0.003831	-0.6949	0.4885	-11.08	
3	-1.454	0.98592	0.003823	-0.8708	0.3856	-11.09	0.4885
2	-1.511	0.98541	0.003819	-2.082	0.0394	-11.10	0.5402
1	-1.761	0.98290	0.003870	7.807	0.0000	-11.08	0.1420
0	-1.275	0.98492	0.004714			-10.69	0.0000
$y_t - c_t$		ADF tests Costant		cv5=-2.88		cv1=-3.46	
D-lag	t-adf	$(\beta)y_{t-1}$	σ	t-Δy_{t-1}	t-prob	AIC	F-prob
4	-2.545	0.88128	0.006517	-0.6459	0.5196	-10.02	
3	-2.760	0.87470	0.006502	-1.921	0.0571	-10.03	0.5196
2	-3.395*	0.85036	0.006572	-0.1871	0.8519	-10.02	0.1339
1	-3.614**	0.84799	0.006547	3.246	0.0015	-10.03	0.2538
0	-2.843	0.87928	0.006792			-9.968	0.0072

Table 3: Johansen Cointegration analysis form a Var(3) and a Var(2) applied to USA

I(1) cointegration analysis for VAR(3)				
		eigenvalue	loglik	rank
			1510.512	0
		0.11897	1523.242	1
		0.022389	1525.517	2
rank Trace test [Prob]	Max test [Prob]	Trace test (T-nm)	Max	test (T-nm)
0	30.01 [0.000]**	25.46 [0.000]**	29.11 [0.000]**	24.70 [0.001]**
1	4.55 [0.033]*	4.55 [0.033]*	4.42 [0.036]*	4.42 [0.036]*
Standardized β'				
c	y			
1.0000	-0.90497			
-1.0018	1.0000			
Standardized α				
c	y			
-0.012378	-0.015820			
0.12298	-0.017459			

I(1) cointegration analysis for VAR(2)				
		eigenvalue	loglik	rank
			1509.389	0
		0.10194	1520.195	1
		0.022285	1522.460	2
rank Trace test [Prob]	Max test [Prob]	Trace test (T-nm)	Max	test (T-nm)
0	26.14 [0.001]**	21.61 [0.002]**	25.62 [0.001]**	21.18 [0.003]**
1	4.53 [0.033]*	4.53 [0.033]*	4.44 [0.035]*	4.44 [0.035]*
Standardized β'				
c	y			
1.0000	-0.90097			
-1.0023	1.0000			
Standardized α				
c	y			
-0.012746	-0.014836			
0.10997	-0.017089			
F-test for joint null hypothesis Var(3) versus Var(2)				
F(4,386) = 1.4788 [0.2079]				
LR test of restrictions: = 0.042109 [0.8374]				

Table 4: Johansen Cointegration analysis for a Var(3) and a Var(2) applied to Italy

I(1) cointegration analysis for VAR(3)				
		eigenvalue	loglik	rank
			999.0253	0
		0.19931	1013.362	1
		0.052072	1016.811	2
rank Trace test [Prob]	Max test [Prob]	Trace test (T-nm)	Max	test (T-nm)
0	35.57 [0.000]**	28.67 [0.000]**	33.92 [0.000]**	27.34 [0.000]**
1	6.90 [0.009]**	6.90 [0.009]**	6.58 [0.010]*	6.58 [0.010]*
Standardized β'				
c	y			
1.0000	-0.74795			
-1.0295	1.0000			
Standardized α				
c	y			
-0.049627	-0.018324			
0.26093	-0.021131			

I(1) cointegration analysis for VAR(2)				
		eigenvalue	loglik	rank
		996.0095	0	
		0.19629	1010.104	1
		0.036047	1012.472	2
rank Trace test [Prob]	Max test [Prob]	Trace test (T-nm)	Max	test (T-nm)
0	32.92 [0.000]**	28.19 [0.000]**	31.90 [0.000]**	27.31 [0.000]**
1	4.74 [0.030]*	4.74 [0.030]*	4.59 [0.032]*	4.59 [0.032]*
Standardized β'				
c	y			
1.0000	-0.68490			
-1.0305	1.0000			
Standardized α				
c	y			
-0.049944	-0.011768			
0.23045	-0.013884			
F-test for joint null hypothesis Var(3) versus Var(2)				
F(4,242) = 2.0698 [0.0854]				
LR test of restrictions: = 5.2761 [0.0216]*				

Table 5: VECM Estimates for USA - Consumption Growth

	$Coefficient$	$std-err$	$tval$	$t-prob$
δ_c	0.002	0.007	0.331	0.741
π_c	0.006	0.014	0.439	0.661
$\beta_{cc,1}$	0.254	0.077	3.29	0.001
$\beta_{cy,1}$	0.068	0.037	1.82	0.071

Table 6: VECM Estimates for USA - Income Growth

	$Coefficient$	$std-err$	$tval$	$t-prob$
δ_y	0.056	0.0139	4.05	0.000
π_y	-0.111	0.027	-4.04	0.000
$\beta_{yc,1}$	0.816	0.146	5.59	0.000
$\beta_{yy,1}$	0.160	0.071	2.27	0.025

Table 7: VECM Estimates for Italy - Consumption Growth

	$Coefficient$	$std-err$	$tval$	$t-prob$
δ_c	-0.039	0.017	-2.31	0.022
π_c	0.067	0.027	2.46	0.015
$\beta_{cc,1}$	0.649	0.091	7.13	0.000
$\beta_{cy,1}$	0.040	0.050	0.795	0.428
$\beta_{cc,2}$	-0.201	0.090	-2.22	0.028
$\beta_{cy,2}$	0.012	0.052	0.231	0.818

Table 8: VECM Estimates for Italy - Income Growth

	$Coefficient$	$std-err$	$tval$	$t-prob$
δ_y	0.093	0.031	2.95	0.004
π_y	-0.146	0.050	-2.89	0.005
$\beta_{yc,1}$	0.464	0.168	2.75	0.007
$\beta_{yy,1}$	0.352	0.093	3.77	0.000
$\beta_{yc,2}$	-0.004	0.168	-0.028	0.977
$\beta_{yy,2}$	0.039	0.097	0.405	0.687

null hypothesis of no cointegrating vector is strongly rejected. The null hypothesis of at least one cointegrating vector is not rejected at five per cent for both lag (see Hendry and Doornik (2001b, Chap. 12) and Favero (2001, Chap. 2)). The F-test of significance of the third lag gives a strong acceptance of the null hypothesis. The PIH cointegrating vector (-1,1) is strongly accepted by Likelihood Ratio test. For Italy, we find, looking at table 4, a strong significance of the third lag. The two test on the right column (respectively the trace test and the trace test corrected for a small sample, i.e. using *T-k* instead of *T*) give conflicting results about statistical significance of the hypothesis of one cointegrating vector. In addition to this the restrictions about the cointegrating vector (-1,1) are hardly accepted. Table 5 shows clearly how for the adjustment coefficient π_c the null hypothesis is strongly accepted for the consumption growth equation for USA. This result is not so evident for consumption growth equation for Italy. According to Cochrane (1994) and Morley (2004) , when one of the adjustment coefficient is equal to zero the corresponding variable will be weekly exogenous with respect the long run equilibrium. Under PIH consumption will be weekly exogenous. By VECM analysis PIH may be accepted.

7. The unobserved component model

Morley et al. (2003) show for the univariate case that the Beveridge and Nelson decomposition will be identical to State Space decomposition with non zero correlation between trend and cycle. Moreover they found that this zero correlation is strongly rejected by USA GDP data. Proietti (2002) shows that correlation may be explained by many other models, and moreover when the correlation is negative the future is more informative than the past. In this case the Kalman's filter smoother[4] is the proper tool.

Even in the context of state space space model me could apply particular unit root test and common trend test as described in Harvey (2000). What we want to point out here is : 1) they do not require any Kalman filter's likelihood parameter estimation, 2) for the univariate case the null hypothesis is that we have a stationary time series (see Nyblom and Mäkeläinen (1983)), and 3) for the bivariate case we have two null hypothesis. The first is about the existence of no common trend, while the second is about the existence of at least one common trend (see Nyblom and Harvey (2000)). In order to take into account of the non orthogonality among the stochastic components we use a lag correction of eight and nine lags (respectively for Italy and USA) as described in Harvey (2000) and Kwiatkowski et al. (1992). All the tests presented from table 9 to table 11 (for the italian case) and from table 12 to table 14 (for the US case) reveal, even in the unobserved component framework, that the two series are not stationary and they do share a common stochastic trend.

Our first model will be very general, allowing the two series having a different trend. For log of GDP we specify a stochastic component with a AR(2) cycle. This is choice is motivated by the fact that this type of cycle has a peak in its spectral density, and its parameters may be easily forced to be stationary during the likelihood estimation. For log of aggregate consumption the specification of an AR(2) cycle seems redundant, and we shall follow here the principle of parsimony allowing only a white noise in log consumption's equation. In other words

$$y_t = \tau_t^y + \phi_1\psi_{t-1} + \phi_2\psi_{t-2} + \varepsilon_t^y \tag{20}$$

[4] See appendix for details

Table 9: Non parametric unit root test and common stochastic trend using Random Walk model applied to Italy

Non-parametric Tests:				
vRW				
N=1			N=2	
c.v.5 per cent				
			K=1	K=0
0.463			0.218	0.748
	on y_t	on c_t	on both	
0	12.841	12.875	0.330	13.205
1	6.500	6.515	0.184	6.699
2	4.376	4.385	0.138	4.522
3	3.312	3.318	0.117	3.435
4	2.673	2.677	0.107	2.784
5	2.248	2.250	0.101	2.351
6	1.944	1.945	0.097	2.042
7	1.716	1.717	0.094	1.811
8	1.540	1.540	0.092	1.632

Table 10: Non parametric unit root test and common stochastic trend using Random Walk with Drift model applied to Italy

Non-parametric Tests:				
vRWD				
N=1			N=2	
c.v.5 per cent				
			K=1	K=0
0.146			0.105	0.247
	on y_t	on c_t	on both	
0	2.499	2.889	0.182	3.096
1	1.281	1.471	0.101	1.587
2	0.874	0.997	0.076	1.085
3	0.672	0.761	0.065	0.836
4	0.551	0.620	0.059	0.688
5	0.471	0.526	0.056	0.590
6	0.414	0.458	0.054	0.520
7	0.372	0.408	0.053	0.468
8	0.338	0.369	0.052	0.427

Table 11: Non parametric unit root test and common stochastic trend using Integrated Random Walk model applied to Italy

Non-parametric Tests:				
vIRW(*100)				
N=1			N=2	
c.v.5 per cent				
			K=1	K=0
0.808			0.1278	
	on y_t	on c_t	on both	
0	19.917	23.102	0.145	23.422
1	10.207	11.763	0.081	11.948
2	6.970	7.976	0.061	8.117
3	5.357	6.086	0.052	6.208
4	4.394	4.955	0.047	5.066
5	3.757	4.202	0.045	4.304
6	3.303	3.665	0.043	3.759
7	2.962	3.263	0.042	3.350
8	2.695	2.951	0.042	3.032

Table 12: Non parametric unit root test and common stochastic trend using Random Walk model applied to USA

Non-parametric Tests:				
vRW				
N=1			N=2	
c.v.5 per cent				
			K=1	K=0
0.463			0.218	0.748
	on y_t	on c_t	on both	
0	20.222	20.251	1.170	21.422
1	10.183	10.201	0.620	10.821
2	6.827	6.840	0.435	7.275
3	5.147	5.158	0.343	5.502
4	4.139	4.149	0.288	4.438
5	3.467	3.476	0.252	3.728
6	2.987	2.995	0.225	3.222
7	2.628	2.634	0.205	2.842
8	2.348	2.354	0.190	2.546
9	2.124	2.130	0.177	2.309

Table 13: Non parametric unit root test and common stochastic trend using Random Walk with Drift model applied to USA

Non-parametric Tests:				
vRWD				
N=1			N=2	
c.v.5 per cent				
			K=1	K=0
0.146			0.105	0.247
	on y_t	on c_t	on both	
0	2.501	4.267	0.480	5.030
1	1.272	2.150	0.253	2.558
2	0.864	1.444	0.177	1.729
3	0.661	1.091	0.140	1.315
4	0.541	0.880	0.118	1.067
5	0.461	0.740	0.103	0.901
6	0.405	0.640	0.093	0.783
7	0.362	0.566	0.085	0.694
8	0.329	0.508	0.079	0.625
9	0.303	0.462	0.074	0.570

Table 14: Non parametric unit root test and common stochastic trend using Integrated Random Walk model applied to USA

Non-parametric Tests:				
vIRW(*100)				
N=1			N=2	
c.v.5 per cent				
			K=1	K=0
0.808			0.1278	
	on y_t	on c_t	on both	
0	15.666	31.020	0.426	34.639
1	7.968	15.630	0.224	17.583
2	5.410	10.495	0.157	11.861
3	4.142	7.932	0.124	8.993
4	3.388	6.399	0.104	7.272
5	2.889	5.381	0.091	6.123
6	2.534	4.656	0.082	5.300
7	2.269	4.113	0.075	4.682
8	2.063	3.692	0.070	4.200
9	1.898	3.355	0.066	3.814

$$\tau_t^y = \mu_y + \tau_{t-1}^y + \eta_t^y \tag{21}$$

$$c_t = \tau_t^c + \varepsilon_t^c \tag{22}$$

$$\tau_t^c = \mu_c + \tau_{t-1}^c + \eta_t^c \tag{23}$$

$$\begin{pmatrix} \eta_t^y \\ \eta_t^c \\ \varepsilon_t^y \\ \varepsilon_t^c \end{pmatrix} \sim WN \left[\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma^2_{\eta_t^y} & \sigma_{\eta_t^y \eta_t^c} & \sigma_{\eta_t^y \varepsilon_t^y} & \sigma_{\eta_t^y \varepsilon_t^c} \\ \sigma_{\eta_t^y \eta_t^c} & \sigma^2_{\eta_t^c} & \sigma_{\eta_t^c \varepsilon_t^y} & \sigma_{\eta_t^c \varepsilon_t^c} \\ \sigma_{\eta_t^y \varepsilon_t^y} & \sigma_{\eta_t^c \varepsilon_t^y} & \sigma^2_{\varepsilon_t^y} & \sigma_{\varepsilon_t^y \varepsilon_t^c} \\ \sigma_{\eta_t^y \varepsilon_t^c} & \sigma_{\eta_t^c \varepsilon_t^c} & \sigma_{\varepsilon_t^y \varepsilon_t^c} & \sigma^2_{\varepsilon_t^c} \end{pmatrix} \right],$$

$$\sigma_{\eta_t^y \eta_t^c} = \rho_{\eta_t^y \eta_t^c} \sigma_{\eta_t^y} \sigma_{\eta_t^c}, \quad \sigma_{\eta_t^y \varepsilon_t^y} = \rho_{\eta_t^y \varepsilon_t^y} \sigma_{\eta_t^y} \sigma_{\varepsilon_t^y}, \quad \sigma_{\eta_t^c \varepsilon_t^y} = \rho_{\eta_t^c \varepsilon_t^y} \sigma_{\eta_t^c} \sigma_{\varepsilon_t^y}, \quad \sigma_{\eta_t^y \varepsilon_t^c} = \rho_{\eta_t^y \varepsilon_t^c} \sigma_{\eta_t^y} \sigma_{\varepsilon_t^c},$$

$$\sigma_{\eta_t^c \varepsilon_t^c} = \rho_{\eta_t^c \varepsilon_t^c} \sigma_{\eta_t^y} \sigma_{\varepsilon_t^c}, \quad \sigma_{\varepsilon_t^y \varepsilon_t^c} = \rho_{\varepsilon_t^y \varepsilon_t^c} \sigma_{\varepsilon_t^y} \sigma_{\varepsilon_t^c}$$

This model with two trends is identified since rewriting equations 21 and 23 as

$$\Delta \tau_t^y = \mu_y + \eta_t^y \tag{24}$$

$$\Delta \tau_t^c = \mu_c + \eta_t^c \tag{25}$$

and plugging 24 and 25 into 22 and 20, we obtain

$$\phi(L) \Delta y_t = \mu_y + \phi(L) \eta_t^y + \Delta \varepsilon_t^y \tag{26}$$

$$\Delta c_t = \mu_c + \eta_t^c + \Delta \varepsilon_t^c \tag{27}$$

On the right side of equation 26 we have an AR(2) plus an MA(1). By lemma of Granger and Newbold (1986) this is equal to an ARMA(2,2). So we have in this particular reduced form 13 parameters (7 of base plus 6 correlations), while we have only 10 for its stace space model.

Table 15 and table 16 report Likelihood estimation by BFGS maximization algorithm for USA and Italy[5].

In Figure 4 and 5 we see the real time cycle and the potential cycle for log of GDP. We also show the two trends estimated by one step-prediction error decomposition and the smoothed flow of the trends.

It is easy to check for both cases that (as shown by Proietti (2002) for the univariate case) the future carries more information than the past for the cycle. The potential output gap is quite different from the real time one for USA .

The model with a common trend is crucial for economic interpretation. If we associate the idea of a common trend given by the smoothed estimate of Kalman filter technology,

[5] All computations presented here are performed by Ox 3.3 matrix programming language with the package Ssfpack 2.2 on a Linux Mandrake 9.2 platform. Computer programs are available from the author upon request.

Table 15: Kalman filter Likelihood estimation for the model with two trends applied to USA

M.L.E	$L.E$	$Coeff.$	$Std.err.$	$t-stat.$
$\sigma^2_{\eta^y}$	0.841	-0.087	0.051	-1.706
$\sigma^2_{\eta^c}$	0.194	-0.821	0.047	-17.426
$\sigma^2_{\varepsilon^y}$	0.118	-1.068	0.614	-1.740
$\sigma^2_{\varepsilon^c}$	0.046	-1.542	0.240	-6.438
$\rho_{\eta^y\eta^c}$	0.500	1.000	0.248	4.035
$\rho_{\eta^y\varepsilon^y}$	-0.500	-1.000	0.615	-1.625
$\rho_{\eta^c\varepsilon^y}$	-0.458	-0.655	1.127	-0.581
$\rho_{\eta^y\varepsilon^c}$	0.500	1.000	1.351	0.740
$\rho_{\eta^c\varepsilon^c}$	0.500	1.000	1.451	0.689
$\rho_{\varepsilon^y\varepsilon^c}$	-0.197	-0.206	0.771	-0.267
ϕ_1	1.000	1.000	0.422	2.370
ϕ_2	-0.500	0.000	0.342	0.000
μ_y	0.804	0.804	0.064	12.477
μ_c	0.838	0.838	0.031	27.130
$Loglik$	-359.783			
$Normality$	16.143			

Note: L.E stands for Likelihood estimation. Coefficient stands for the costrained parameters. On the two remaining columns there are standard errors and t-student's critical value

Table 16: Kalman filter Likelihood estimation for the model with two trends applied to Italy

M.L.E	$L.E$	$Coeff.$	$Std.err.$	$t-stat.$
$\sigma^2_{\eta^y}$	0.586	-0.268	0.111	-2.410
$\sigma^2_{\eta^c}$	0.240	-0.713	0.059	-11.992
$\sigma^2_{\varepsilon^y}$	0.100	-1.151	2.394	-0.481
$\sigma^2_{\varepsilon^c}$	0.032	-1.727	2.723	-0.634
$\rho_{\eta^y\eta^c}$	0.500	1.000	0.356	2.807
$\rho_{\eta^y\varepsilon^y}$	-0.500	-1.016	2.407	-0.422
$\rho_{\eta^c\varepsilon^y}$	-0.333	-0.381	1.267	-0.301
$\rho_{\eta^y\varepsilon^c}$	0.498	1.098	5.401	0.203
$\rho_{\eta^c\varepsilon^c}$	0.255	0.274	0.882	0.311
$\rho_{\varepsilon^y\varepsilon^c}$	0.401	0.502	9.208	0.055
ϕ_1	1.000	1.000	0.390	2.565
ϕ_2	-0.500	0.000	1.154	0.000
μ_y	0.597	0.597	0.067	8.910
μ_c	0.617	0.617	0.043	14.425
$Loglik$	-224.949			
$Normality$	6.334			

Note: L.E stands for Likelihood estimation. Coefficient stands for the costrained parameters. On the two remaining columns there are standard errors and t-student's critical value

Figure 4: Kalman filter real time and potential estimates of cycle and trends for USA

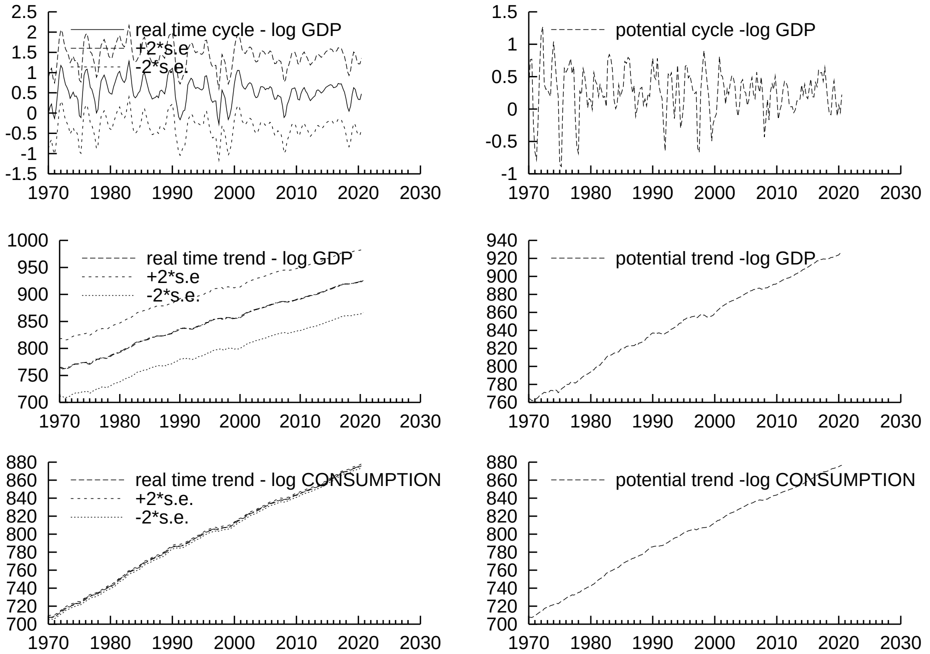


Figure 5: Kalman filter real time and potential estimates of cycle and trends for Italy

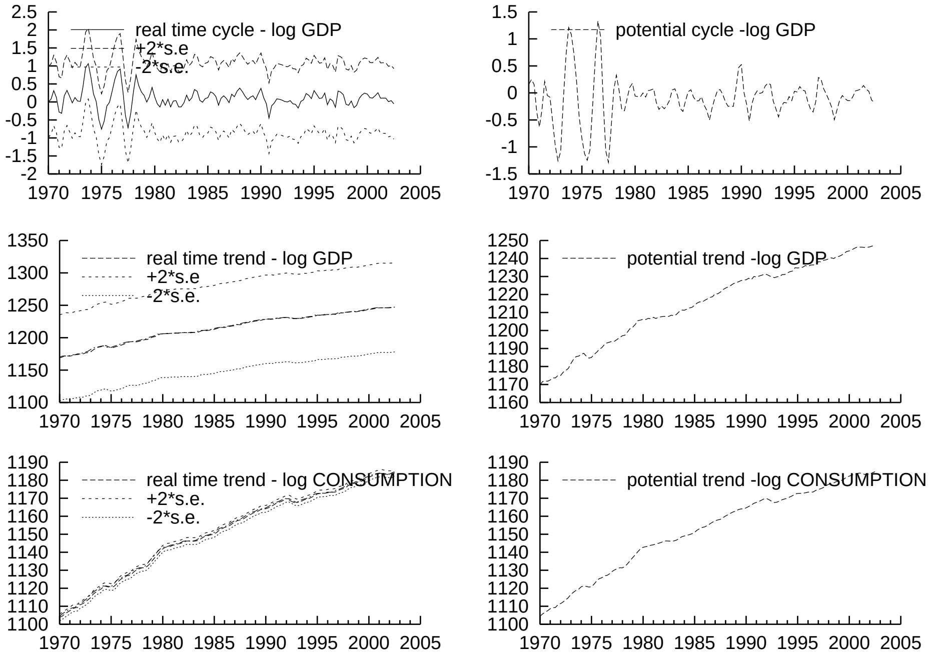
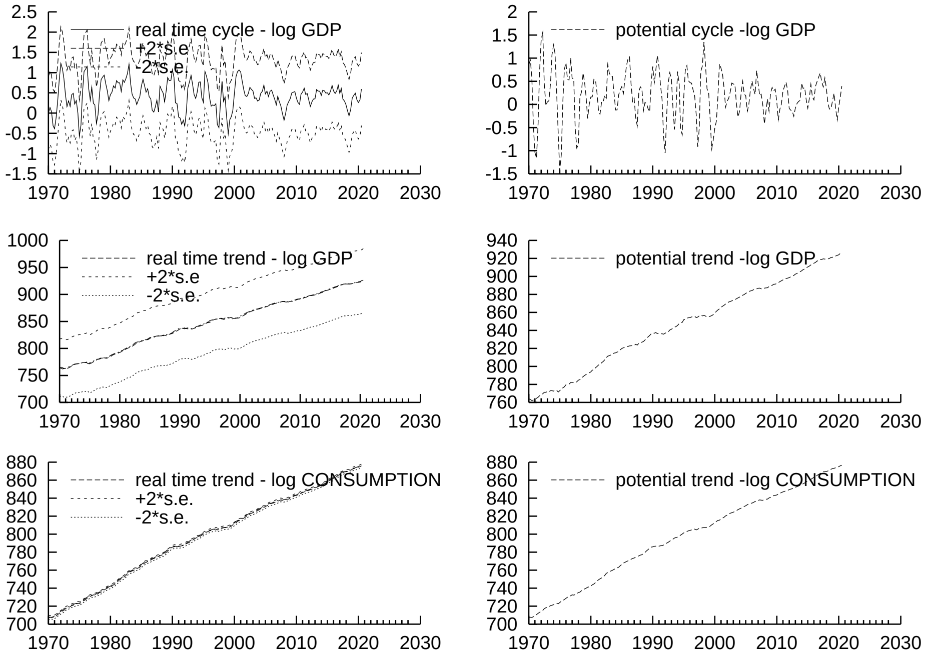


Figure 6: Kalman filter real time and potential estimates of cycle and common trend for USA



with the one given in preceding sections about permanent income, we may test PIH by focusing on the various correlation estimates.The PIH model has the following structure

$$y_t = \tau_t^y + \phi_1 \psi_{t-1} + \phi_2 \psi_{t-2} + \varepsilon_t^y \tag{28}$$

$$\tau_t^y = \mu_y + \tau_{t-1}^y + \eta_t^y \tag{29}$$

$$\tau_t^c = \mu_y + \tau_{t-1}^c + \eta_t^y \tag{30}$$

$$c_t = \tau_t^c + \varepsilon_t^c \tag{31}$$

$$\tau_t^c = \gamma \tau_t^y \tag{32}$$

where γ is the marginal propensity to consume out of permanent income.

Even this model with a common trend is identified since we have 12 parameters in the reduced form, while we have only 9 for its space model.

The first thing we notice, giving a quick glance at table 17 and 18, is that the correlation coefficients are quite similar for both countries. The two outstanding execptions are for coefficients $\rho_{\eta^y \varepsilon^c}$(0.135,0.027 respectively) and $\rho_{\varepsilon^y \varepsilon^c}$ (0.014,0.362).

On one hand this last result is very interesting and may confirms the fact that Italians are more reactive to a shock to the transitory part of their income to their current consumption decisions. This is in favour of Keynesian way of thinking. This result has to be got with caution because its t-value induces us to accept the null hypothesis. On the other hand for USA these considerations may be well reversed, thinking about how US people

Figure 7: Kalman filter potential estimate of common trend (Permanent Income) for USA

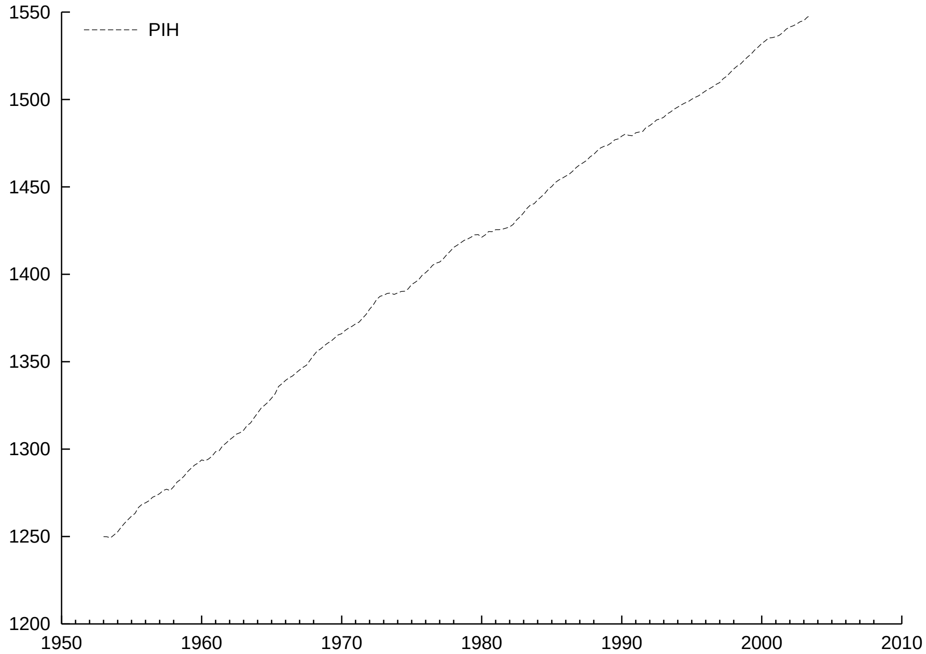


Figure 8: Kalman filter real time and potential estimates of cycle and common trend for Italy

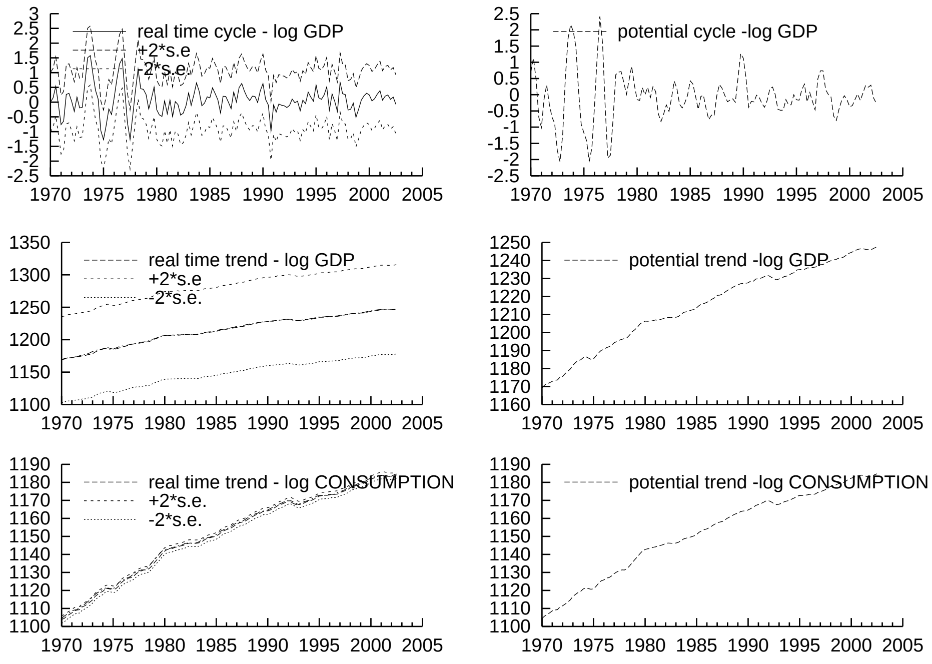


Figure 9: Kalman filter potential estimate of common trend (Permanent Income) for Italy

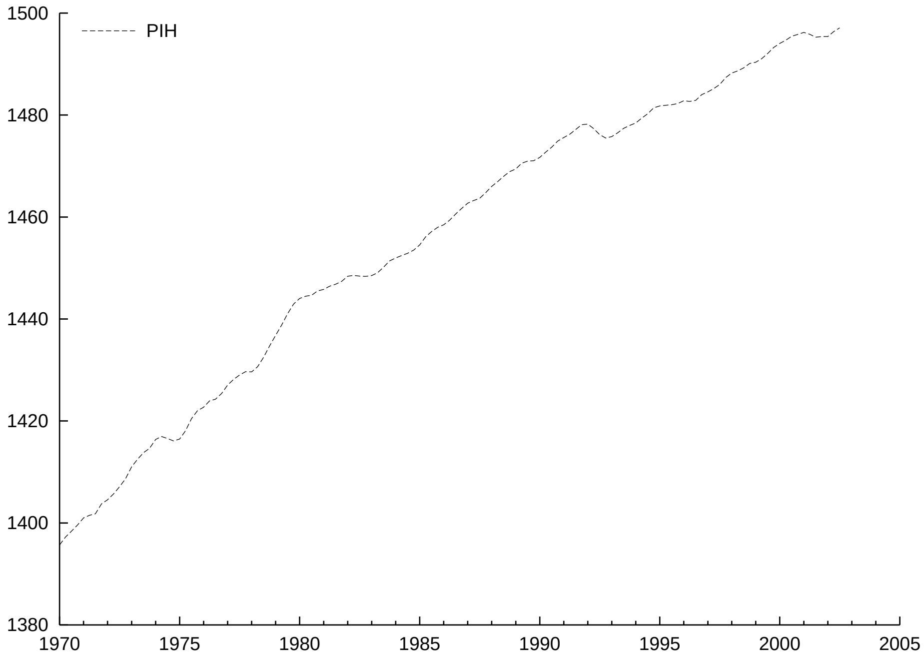


Table 17: Kalman filter Likelihood estimation for the model with one trend applied to USA

M.L.E	$L.E$	$Coeff.$	$Std.err.$	$t-stat.$
$\sigma^2_{\eta^y}$	0.632	-0.230	0.076	-3.022
$\sigma^2_{\eta^c}$	0.203	-0.798	0.049	-16.451
$\sigma^2_{\varepsilon^y}$	0.071	-1.320	0.321	-4.109
$\sigma^2_{\varepsilon^c}$	0.138	-0.990	0.198	-5.002
$\rho_{\eta^y\eta^c}$	1			
$\rho_{\eta^y\varepsilon^y}$	-0.500	-1.000	0.718	-1.394
$\rho_{\eta^c\varepsilon^y}$	-0.500	-0.985	2.567	-0.384
$\rho_{\eta^y\varepsilon^c}$	0.135	7.251	8.430	0.860
$\rho_{\eta^c\varepsilon^c}$	0.500	1.000	0.498	2.009
$\rho_{\varepsilon^y\varepsilon^c}$	0.014	72.652	7.316	9.930
ϕ_1	1.000	1.000	0.372	2.689
ϕ_2	-0.500	0.000	0.238	0.000
μ_y	0.804	0.804	0.056	14.374
μ_c	0.837	0.838	0.032	26.520
γ	0.566			
$Loglik$	-360.890			
$Normality$	17.283			

Note: L.E stands for Likelihood estimation. Coefficient stands for the costrained parameters. On the two remaining columns there are standard errors and t-student's critical value

Table 18: Kalman filter Likelihood estimation for the model with one trend applied to Italy

M.L.E	$L.E$	$Coeff.$	$Std.err.$	$t-stat.$
$\sigma^2_{\eta^y}$	0.401	-0.457	0.133	-3.440
$\sigma^2_{\eta^c}$	0.251	-0.691	0.062	-11.192
$\sigma^2_{\varepsilon^y}$	0.094	-1.185	14.655	-0.081
$\sigma^2_{\varepsilon^c}$	0.146	-0.961	4.509	-0.213
$\rho_{\eta^y\eta^c}$	1			
$\rho_{\eta^y\varepsilon^y}$	-0.494	-1.168	59.730	-0.020
$\rho_{\eta^c\varepsilon^y}$	-0.489	-0.808	56.607	-0.014
$\rho_{\eta^y\varepsilon^c}$	0.027	0.027	4.843	0.006
$\rho_{\eta^c\varepsilon^c}$	0.303	0.337	1.916	0.176
$\rho_{\varepsilon^y\varepsilon^c}$	0.362	0.428	13.894	0.031
ϕ_1	1.000	1.000	0.281	3.554
ϕ_2	-0.500	0.000	0.279	0.000
μ_y	0.598	0.598	0.056	10.759
μ_c	0.617	0.618	0.044	14.110
γ	0.791			
$Loglik$	-222.595			
$Normality$	5.052			

Note: L.E stands for Likelihood estimation. Coefficient stands for the costrained parameters. On the two remaining columns there are standard errors and t-student's critical value

are less eager to change their transitory consumption due to a shock in their transitory income.

The correlations between the common stochastic trends and their respectively transitory part of income are in line with the sign with the one shown by Morley et al. (2003) for USA and with Proietti (2002) for USA and Italy for the univariate case. This is remarkable since we are considering a bivariate model. We also observe that correlation coefficients of Italy (see last column of table 18 and of table 17) are less significant than US ones. Another interesting point is on the γ coefficients which results close to one (0.8) for Italy, and close to $\frac{1}{2}$ for US. This is not only in line with Carroll's intuition (see Carroll (2001)), but reveals us that Italians are less myopic than US people. The intuition that PIH may be strongly rejected for US is also given by figure 7, where we see that PI flow is very smooth. The same intuition is not confirmed giving a glance at figure 9. Figure 10 and 11 show the impulse response function for both countries based on Proietti (2002) 's univariate model [6]. Given habit formation in consumer preferences consumption will adjust slowly in response to a shock to income. The same could be said in presence of precautionary saving's behavior. It is straightforward to notice that in figure 11 consumption adjusts more quickly to an income shock for Italy than US does.

As pointed out by Morley (2004) and by Schleicher (2003) an important issue is the joint significance of correlation parameters: $\rho_{\eta^y\varepsilon^c}$, $\rho_{\eta^y\varepsilon^y}$, $\rho_{\varepsilon^c\varepsilon^y}$, $\rho_{\eta^c\varepsilon^y}$, $\rho_{\eta^y\varepsilon^c}$. A jointly rejection will lead us to reject PIH for both countries.

This is what we find for USA (see table 19). These results are well reversed for Italy (see table 20).

[6]The graphs in figure 10 and figure 11 are obtained from an extension of Proietti's program

Figure 10: Impulse Response Function for State Space Model with a common trend applied to USA

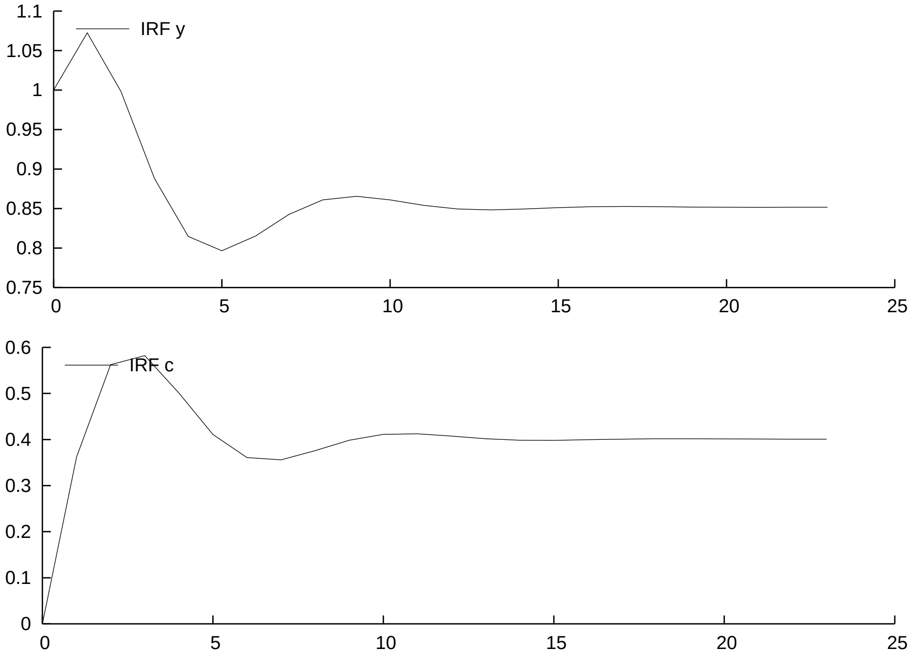


Figure 11: Impulse Response Function for State Space Model with a common trend applied to Italy

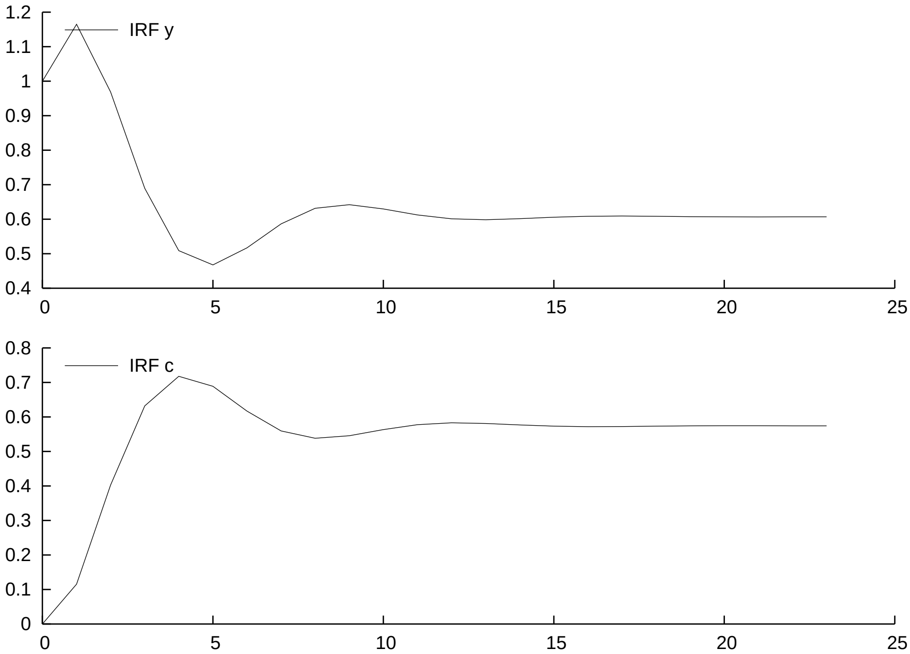


Table 19: Likelihood ratio test applied to USA

H_0	LR	$pv\chi_r$
$\rho_{\eta^y\varepsilon^c} = \rho_{\eta^y\varepsilon^y} = \rho_{\varepsilon^y\varepsilon^c} = \rho_{\eta^c\varepsilon^c} = \rho_{\eta^c\varepsilon^y} = 0$	25.2857	0.0003
$\rho_{\eta^y\varepsilon^c} = \rho_{\eta^c\varepsilon^c} = \rho_{\eta^y\varepsilon^y} = \rho_{\eta^y\varepsilon^c} = 0$	25.2855	0.0000

Table 20: Likelihood ratio test applied to Italy

H_0	LR	$pv\chi_r$
$\rho_{\eta^y\varepsilon^c} = \rho_{\eta^y\varepsilon^y} = \rho_{\varepsilon^y\varepsilon^c} = \rho_{\eta^c\varepsilon^c} = \rho_{\eta^c\varepsilon^y} = 0$	5.74388	0.547516
$\rho_{\eta^y\varepsilon^c} = \rho_{\eta^c\varepsilon^c} = \rho_{\eta^y\varepsilon^y} = \rho_{\eta^y\varepsilon^c} = 0$	5.65086	0.226784

8. Conclusions

The main goal of this paper was testing PIH by classical cointegration approach used by Cochrane (1994) and by cointegration analysis by unobserved correlated components recently suggested by Morley (2004) and by Schleicher (2003). Our approach differs from Morley's one mainly because it gives an estimate of γ, which is the marginal propensity of consume out of permanent income, and from Schleicher (2003) and Morley (2004) mainly because doesn't allow the exsistence of an AR(2) for consumption's cycle. The review of the theory was given in order to check the meaning of our speculations. A forward looking agent save less when expects that his PI will raise, save more when expects that his PI will decrease. Hall (1978)'s conclusion was used by Cochrane to state by a Vector Error Correction Model (VECM) that PIH was valid because consumption follows a random walk in USA. This conclusion has been reversed for the Italian case.

Morley argues that the limit of the VECM is that implies the same speed of adjustment for both variables to restore their long run ratio relationship, and also that if we have habit formation in consumer's preferences or the agents save for precautionary motive, the adjustment coefficient in the consumption growth equation may be near zero but not equal to zero. Following this suggestion we estimated two unobserved component models for both countries. The more important for us was clearly the one with a common trend.From a direct comparison of figure 10 (related to USA) and figure 11 (related to Italy) we find out that consumption adjusts slowly to an income shock while the opposite is valid for the Italian case. The reflections on likelihood ratio test applied to USA and to Italy conduct us to reject PIH for USA and retain it still valid for Italy.

Appendix

Maximum Likelihood estimation and the Kalman filter

Koopman et al. (1999) formulate the Gaussian state space form as

$$\alpha_{t+1} = d_t + T_t\alpha_t + H_t\varepsilon_t, \;\; \alpha_1 \sim N(a, P), \quad t = 1, \ldots, n \tag{33}$$

$$\theta_t = c_t + Z_t\alpha_t, \tag{34}$$

$$y_t = \theta_t + G_t\varepsilon_t, \;\; \varepsilon_t \sim NID(0, I) \tag{35}$$

where NID(0,Ψ) indicates an independent sequence of normally distributed random vectors with mean μ and variance matrix Ψ, and, similarly, N(.,.) a normallly distributed variable. The N observations at time *t* are placed in the vector y_t and the N x n data matrix is given by (y_1,...,y_n). The *m* x 1 state vector α contains unobserved stochastic processes and unknown fixed effects. For the two state space models discussed here we will neglect

c_t and consider no-time varying system matrixes. So our system may be considered more compactly as

$$\alpha_{t+1} = d + T\alpha_t + H\varepsilon_t, \quad \alpha_1 \sim N(a, P), \quad t = 1, \dots, n \tag{36}$$

$$y_t = Z\alpha_t + G_t\varepsilon_t, \quad \varepsilon_t \sim NID(0, I) \tag{37}$$

Equation 35 is the transition equation. Equation 36 is called measurement equation. In particular we have for the model with two distinct trends, the following system matrixes:

$$T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \phi_1 & 1 \\ 0 & 0 & \phi_2 & 0 \end{bmatrix}$$

$$Z = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$HH' = \begin{bmatrix} \sigma^2_{\eta^y_t} & \sigma_{\eta^y_t\eta^c_t} & \sigma_{\eta^y_t\varepsilon^y_t} & \sigma_{\eta^y_t\varepsilon^c_t} \\ \sigma_{\eta^y_t\eta^c_t} & \sigma^2_{\eta^c_t} & \sigma_{\eta^c_t\varepsilon^y_t} & \sigma_{\eta^c_t\varepsilon^c_t} \\ \sigma_{\eta^y_t\varepsilon^y_t} & \sigma_{\eta^c_t\varepsilon^y_t} & \sigma^2_{\varepsilon^y_t} & \sigma_{\varepsilon^y_t\varepsilon^c_t} \\ \sigma_{\eta^y_t\varepsilon^c_t} & \sigma_{\eta^c_t\varepsilon^c_t} & \sigma_{\varepsilon^y_t\varepsilon^c_t} & \sigma^2_{\varepsilon^c_t} \end{bmatrix}$$

$$d = \begin{bmatrix} \mu_y \\ \mu_c \\ 0 \\ 0 \end{bmatrix}$$

$$G = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

For the model with one common trend the innovation matrix $HH^{'}$ has a reduction's rank and is putted in the following form

$$HH' = \begin{bmatrix} \sigma^2_{\eta^y_t} & \gamma^2\sigma_{\eta^c_t} & \rho_{\eta^y_t\varepsilon^y_t}\sigma_{\eta^y_t}\sigma_{\varepsilon^y_t} & \rho_{\eta^y_t\varepsilon^c_t}\sigma_{\eta^y_t}\sigma_{\varepsilon^c_t} \\ \gamma^2\sigma_{\eta^c_t} & \sigma^2_{\eta^c_t} & \rho_{\eta^c_t\varepsilon^y_t}\gamma\sigma_{\eta^y_t}\sigma_{\varepsilon^y_t} & \rho_{\eta^c_t\varepsilon^c_t}\gamma\sigma_{\eta^y_t}\sigma_{\varepsilon^c_t} \\ \rho_{\eta^y_t\varepsilon^y_t}\sigma_{\eta^y_t}\sigma_{\varepsilon^y_t} & \rho_{\eta^c_t\varepsilon^y_t}\gamma\sigma_{\eta^y_t}\sigma_{\varepsilon^y_t} & \sigma^2_{\varepsilon^y_t} & \sigma_{\varepsilon^y_t\varepsilon^c_t}\sigma_{\varepsilon^y_t}\sigma_{\varepsilon^c_t} \\ \rho_{\eta^y_t\varepsilon^c_t}\sigma_{\eta^y_t}\sigma_{\varepsilon^c_t} & \rho_{\eta^c_t\varepsilon^c_t}\gamma\sigma_{\eta^y_t}\sigma_{\varepsilon^c_t} & \rho_{\varepsilon^y_t\varepsilon^c_t}\sigma_{\varepsilon^y_t}\sigma_{\varepsilon^c_t} & \sigma^2_{\varepsilon^c_t} \end{bmatrix}$$

The roots of the AR(2) polynomial in the transition matrix T are forced to be inside the unit disk following Morley et al. (2003). The Kalman filter (see Kalman (1960)) computes minimum mean squared estimates a_t of the state vector a_{t+1} and its mean square error matrix P_{t+1} conditional on available information at time *t* using the following recursive equations

$$v_t = y_t - Z\alpha_t, \quad F_t = ZP_tZ^{'} \tag{38}$$

$$K_t = TP_tZ^{'}F_t^{-1}, \quad L_t = T - K_tZ \tag{39}$$

$$a_{t+1} = d + Ta_t + K_tv_t, \quad P_{t+1} = TP_tL_t^{'} + K_tF_tK_t^{'} \tag{40}$$

The real time filtering equations give the state vector and its mean square error matrix

$$a_{t|t} = a_t + P_tZ_t^{'}F_t^{'}v_t, \quad P_{t|t} = P_t - Z^{'}F_t^{-1}ZP_t^{'} \tag{41}$$

If the two roots of the transition matrix T are inside the unit disk, the log-likelihood of the estimated model is given by

$$logL(y) = -\frac{NT}{2}log2\pi - \frac{1}{2}\sum_{t=1}^{T}(log|F_t| + v_t^{'}F_t^{-1}v_t) \tag{42}$$

Moment smoothing

The Kalman filter is a forward recursion which evaluates one-step ahead estimators. The associated moment smoothing algorithm is a backward recursion which evaluates the mean and variance of specific conditional distributions given the data set $Y_n = (y_1, \ldots, y_1)$ using the output of the Kalman filter; see Anderson and Moore (1979), Kohn and Ansley (1989), de Jong (1988), de Jong (1989), Koopman (1998). The backward recursion are given by

$$e_t = F_t^{-1}v_t - K_t^{'}r_t \quad D_t = F_t^{-1} + K_t^{'}N_tK_t \tag{43}$$

$$r_t = Z_t^{'}F_t^{-1}v_t + L_t^{'}r_t \quad N_t = Z_t^{'}F_t^{-1}Z_t + L_t^{'}N_tL_t \tag{44}$$

with $L_t = T - K_tZ$ and with the initialization $r_n = 0$ and $N_n = 0$, for $t = n, \ldots, 1$.

The exact initial Kalman filter

The covariance matrix of the initial state vector, $P_{t|0}$ can be split into an unbounded component kP_∞ pertaining to the stochastic trends and a bounded component associated with the stationary component P_*

$$P_{1|0} = kP_\infty + P_*, \quad k \to \infty \tag{45}$$

The stationary component P_* can be initialized at the steady state value

$$vec(P_*) = (I - T^* \otimes T^*)vec(K_tF_tK_t^{'}), \quad k \to \infty \tag{46}$$

where $T^* = T - P_\infty$ is the stationary component.

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