A dynamic model of a monetary production economy under the disequilibrium economics approach

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Abstract

This paper presents a model of a monetary production economy with non-Walrasian good, labor and money markets. In the non-Walrasian approach, transactions occur at non clearing prices and agents's demand and supply are affected by quantity constraints in the opposite side of the market. The model is characterized by a representative firm, which maximize profits subject to a production technology, a representative consumer, which maximize utility subject to a budget constraint, and by a central bank which provide liquidity. The consumer provides the labor force and owns all the equities of the firm. The main result of the model is the existence of non-Warlasian equilibria which are suboptimal with respect to Warlasian ones. Furthermore, non-Warlasian equilibria are characterized by money non-neutrality and proper monetary policies are found to be able to bring the system near to the Walrasian point.

Key words: disequilibrium economics, economic dynamics, monetary policy.

Introduction

Under the Walrasian approach [1,2], agents take as given a common perception of relative prices and send quantity signals (demand and supply) to the Walrasian auctioneer which provides to adjust the relative prices in order to equilibrate the system and set excess demands to zero. In the Walrasian framework, realized and expected quantity signals do not affect agents behavior. Indeed, the Walrasian framework is a good description of reality for the few real world markets, such as the stock market which inspired Walras,

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where the equilibrium between demand and supply is ensured institutionally by an actual auctioneer. Conversely, some markets, e.g., the good and labor markets, where no central auctioneer is present, often do no clear. The failure of a market clearing implies that, for at least some agents, actual quantities transacted diverge from the quantities that they supply or demand. Thus, an agent should take also into account quantity signals issued by other agents in addition to price signals.

The disequilibrium or non-Walrasian approach to economics has been pioneered in the '60 by Patinkin [3], Clower [4] and Leijonhufvud [5] and flourished in the seventies especially among European economists [6–9]. Within the non-Walrasian approach to economics, markets generally do not clear and agents engage in maximizing behavior facing quantity constraints in their buying or selling decision. Furthermore, demand-supply imbalances in one market influence the disequilibrium in another market, e.g., the well-known spillover effects between good market and labor market. The seminal paper by Barro and Herschel Grossman [10] examined how good and labor markets interact when prices are fixed at nonmarket clearing levels. Varian [11] showed that non-Walrasian equilibria can persist in dynamic models with flexible prices. Recently, Bénassy [12,13] endogenized the price setting mechanism within the framework of monopolistic competition where sellers are usually price makers and quantity takers, whereas buyers are price takers and quantity setters.

This paper is intended to study the connections between real economic activity, i.e., production, employment and growth, and the dynamics of some financial variables, i.e., money supply and interest rates. The disequilibrium approach is adopted. Thus, distinctive features of this model are non-clearing good, labour and money markets. The model is ruled by the fix-price assumption, that requires the quantities react faster than prices. Moreover, prices are assumed not exogenous. Consequently, the model adopts both a price-vector dynamics and a quantity dynamics. The quantity adjustment process is regulated by the interaction of notional demands, Clower demands and Drèze demands.

Results pointed out that the system is not neutral with respect to a monetary policy. The effect of the monetary policy is exposed and compared to the Walrasian case.

1 The model

The proposed model is populated by three agents: a representative firm, a representative consumer, and a central bank. Three classes of assets characterize the model: physical capital K, bank money M and labor N. Physical capital is owned by the firm and is employed with labor to produce output. Output is a single homogeneous good that can be used both for consumption and investment. The firm is endowed with a production technology characterized by decreasing returns and decides the optimal level of production and investment according to a profit maximizing behavior. The consumer provides the labor force and owns all the equities of the firm. The consumer decides the optimal level of consumption maximizing an utility function. Three markets are open at each time period, i.e., the good market, the labor market and the money market.

1.1 The representative firm

The representative firm is subject to a specific technology and has a production function given by:

$$Y^{(s)} = \gamma K^{\alpha} N^{\beta}\,, \tag{1}$$

where $Y^{(s)}$ is the amount of the supplied homogeneous good, K and N are the physical capital and labor employed, respectively. The production function is assumed to be characterized by positive through diminishing marginal products of capital and labour, i.e., $\alpha, \beta \in (0, 1)$. The constant γ is used for normalization purposes.

The firm sets the optimal level of desired N according to a profit maximization rule with given p, w, and r. In this framework, the firm is supposed to be a price taker in the three markets of goods, labour and capital. The firm economic profit $\Pi^{(e)}$ is defined as:

$$\Pi^{(e)} = p\,\gamma K^{\alpha} N^{\beta} - wN \tag{2}$$

where $p\,\gamma K^{\alpha} N^{\beta}$ represents the sale revenues, whereas wN represents the aggregate labour cost. The solution of the maximization problem for the firm's profit gives:

$$N_t^d(p, w) = (\beta\gamma p/w)^{1/(1-\beta)} K^{\alpha/(1-\beta)} \tag{3}$$

$$Y_t(p, w) = (\beta p/w)^{\beta/(1-\beta)} \gamma^{1/(1-\beta)} K^{\alpha/(1-\beta)} \tag{4}$$

$N_t^d(p, w)$ and $Y_t(p, w)$ are the notional demands of the firm, i.e., if the firm was free of constraints it would produce $Y_t(p, w)$ employing $N_t^d(p, w)$ work.

The representative consumer is characterize by an utility function that depends on consumption expenditures, real wealth and leisure.

$$U_t = a\log(C_t^d) + b\log\frac{M_t^e}{p_{t+1}^e} + c\log(1-\frac{N_t^s}{Nmax})\,, \tag{5}$$

where C_t^d represents the notional demand for consumption, M_t^e is the expected nominal wealth and p_{t+1}^e is the expected price for the next period. N_t^s is the notional supply of labor whereas $Nmax$ is the maximin number of workers supported by the system. Thus, $(1-N_t^s/Nmax)$ represents leisure.
Furthermore, households earn dividends from firms profits

$$d_t = \pi_{t-1}\,, \tag{6}$$

and the expected nominal wealth is

$$M_t^e = (1+r_t)M_{t-1} - p_tC_t^d + w_tN_t^s + d_t \tag{7}$$

The household has perfect foresight with respect to the profits that accrue to it in the current period. Money, in the model, is an instrument for transferring purchasing power from one period to the next. Maximizing the consumer utility function for the consumer gives:

$$C_t^d(p,w,r) = \frac{a}{a+b+c}\frac{w_t}{p_t}Nmax + \frac{a}{a+b+c}\frac{(1+r)M_{t-1}+d_t}{p_t} \tag{8}$$

$$N_t^s(p,w,r) = \frac{a+b}{a+b+c}Nmax - \frac{c}{a+b+c}\frac{(1+r)M_{t-1}+d_t}{w_t} \tag{9}$$

Therefore, savings at time step t are given by

$$\Delta M_t(p,w,r) = r_tM_{t-1} - p_tC_t^d + w_tN_t^s + d_t \tag{10}$$

2 Model dynamics

Our model works under the logic of *fix price assumption*, that means that the market participants fix prices by themselves, so that there are potentially situations of non-market clearance. Quantity constraints must arise first in order to induce the individuals to alter prices; and the quantities react faster than prices. a central problem in the fixed-price literature [14] is how agent's behavior is modified when they encounter additional constraints, i.e., quantity constraints in addition to budget constraints. In particular, it is crucial to know what demand they will express to the market under these circumstances. One straightforward suggestion for the behavior of agents when there are quantity constraints has been provided by Dreze [15]. Each agent chooses the most preferred trade vector, subject to budget constraints and all quantity constraints. This trade vector is called *Dreze demand*. In this model two different dynamics are combined: a *quantity adjustment* , i.e., a faster dynamic and a *price vector*, i.e., a slower dynamic. For the quantity adjustment dynamic we refer to the Clower demands [4] that, in general, a Clower demand is defined by the fact that an agent adheres to his notional plan in the market where he is constrained, but received his plans relating to all other markets.

The notional functions are determined by the price vector. If the notional demands are different on one market, at a given price vector, an agent will be rationed in that market, i.e., he will be unable to satisfy his demand on that market.

The method consists in comparing notional demands at the first step, i.e., comparing Equations 3 with 9 and 4 with 8. According to Clower's theory, one has to express a Clower demand on one market if he is rationed one other markets. Consequently, if households are rationed on the labor market, they will express a Clower demand consumption

$$\tilde{C}_t^d = \frac{a}{a+b}\frac{(1+r)M_{t-1} + w_t\bar{N}_t + d_t}{p_t}\,, \tag{11}$$

where $\bar{N}_t$ is the constraint on the labor market. In the same way, if households are rationed on the goods market, they will express a Clower supply of labor

$$\tilde{N}_t^s = \frac{b}{b+c}Nmax + \frac{c}{b+c}\frac{p_t\bar{C}_t - (1+r)M_{t-1} - d_t}{w_t} \tag{12}$$

where $\bar{C}_t$ is the constraint on the goods market. When firms are constrained in the labor or commodity market, they revise their plans according to the Clower effective demands, i.e.,

$$\tilde{N}_t^d = \left(\frac{\bar{Y}_t}{\gamma K^\alpha}\right)^{\frac{1}{\beta}} \tag{13}$$

$$\tilde{Y}_t^s = \gamma k^\alpha \bar{N}_t^\beta \tag{14}$$

The quantity adjustment process continues until the respective Drèze demands match. In this condition, all quantities are matched and the price adjustment process starts. Price p and wage w evolve exponentially due to the difference between Clower's demands and supplies

$$p_t = p_{t-1} \exp(g_p(\tilde{C}_t^d - \tilde{Y}_t^s)) , \tag{15}$$

$$w_t = w_{t-1} \exp\left(g_w(\tilde{N}_t^d - \tilde{N}_t^s)\right) . \tag{16}$$

3 Results

This paper is intended to study the connections between real economic activity, i.e., production, employment and growth, and the dynamics of some financial variables, i.e., money supply and interest rates. An interesting result is shown in Figures 1 and 2. It represents the temporal evolution of output and labor in three different cases. The first one is the Walrasian case, where no quantity adjustment is adopted. In this condition, only a price-vector dynamics is present and markets are cleared at every step, i.e, prices are such that no agent is rationed and supply always equals demand. The second and third curves represent the time evolution for a non-Walrasian system with quantity adjustments. It is worth noting that the output level in the Walrasian case is higher that in the other two cases, thus confirming that the Walrasian equilibrium is the most efficient point. Furthermore, the model shows that perturbation from the Walrasian equilibrium quickly converge again to the Walrasian equilibrium. In particular, a monetary policy, represented by money inflow, would be ineffective for the model, because the dynamics would rapidly return to the previous state. Conversely, if the system is on a non-Walrasian equilibrium, a monetary policy results in a strong effect on the regime dynamics. Figures 1 and 2 clearly show that a money inflow changes the equilibrium point of the system. In one case, at $t = 1000$ an inflow corresponding to five percent of the initial money is put in the system, whereas, in the second case, at time $t = 2000$ the effect of a 20 percent variation is represented. In both cases the output and the labor rise to a higher level equilibrium, and, in particular, the second case nearly reaches the Walrasian equilibrium point.

Acknowledgments

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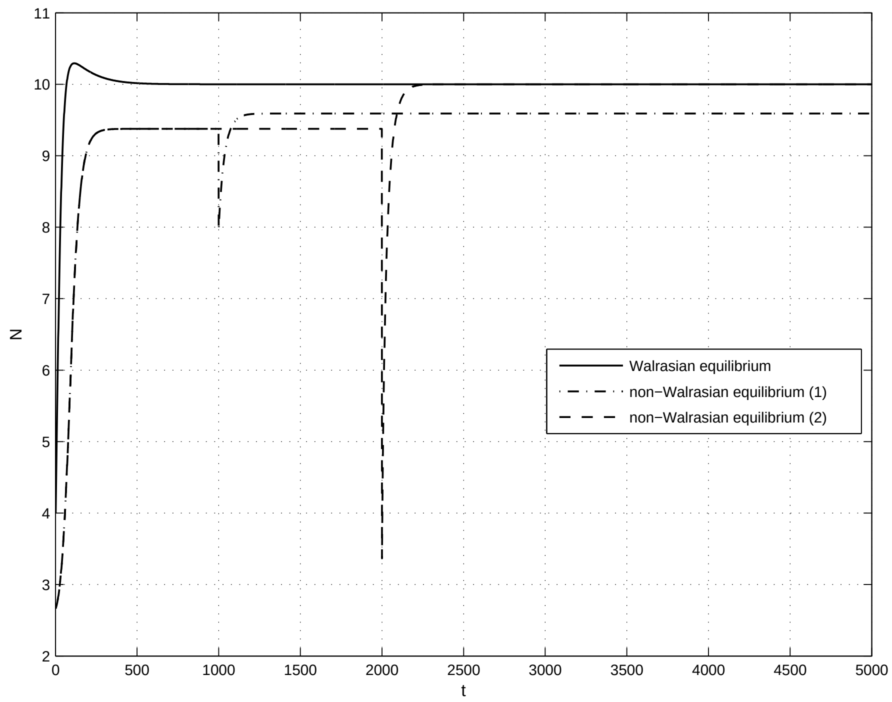

Fig. 1. Dynamics of employment in the Walrasian equilibrium case (continuous line) and in two different non-Walrasian equilibrium cases (dashed-dotted and dashed line). The two non-Walrasian equilibria are characterized by a sudden increase of 5% and 20% of the money supply, respectively.

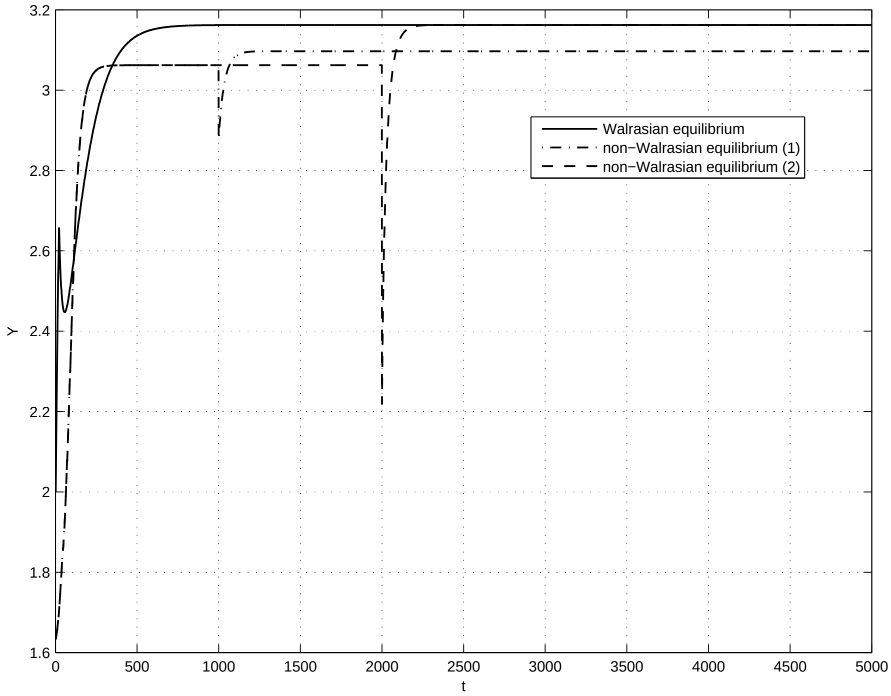

Fig. 2. Dynamics of output in the Walrasian equilibrium case (continuous line) and in two different non-Walrasian equilibrium cases (dashed-dotted and dashed line). The two non-Walrasian equilibria are characterized by a sudden increase of 5% and 20% of the money supply, respectively.