What do robust policies look like for open economy inflation targeters?*

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Abstract

This paper examines the role of the open economy in determining robust rules when the central bank fears various model misspecication errors. A new Keynesian model is calibrated to fit the economies of three archetypal open economy inflation targeters — Australia, Canada and New Zealand. Robust policies respond more aggressively to not only the exchange rate, but also inflation, the output gap and their associated shocks. This result generalizes to the context of a flexible inflation targeting central bank that cares about the volatility of the real exchange rate. However, when the central bank places only a small weight on interest rate smoothing and fears misspecification in only

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exchange rate determination, a more aggressive response to the lag of the exchange rate is not warranted. It is shown that the benefits of an exchange rate channel far outweigh the concomitant costs of uncertain exchange rate determination.

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1 Introduction

Thinking about best practice monetary policy for policymakers means thinking about uncertainty. Alan Greenspan, Governor of the Federal Reserve Board exemplifies this perspective:

> The Federal Reserve's experiences over the past two decades make it clear that uncertainty is not just a pervasive feature of the monetary policy landscape; it is the defining characteristic of that landscape.[1]

Monetary policymakers operating in an open economy face an additional source of uncertainty — the exchange rate.[2] The goal of this paper is to identify policy rules robust to the uncertainty around model dynamics open economy inflation targeters face in practice. An open economy model with an explicit inflation targeting framework is calibrated to match the data for three archetypal small open inflation targeters, Australia, Canada and New Zealand. These countries are among the earliest inflation explicit inflation targeters and now form a useful dataset for identifying the open economy

[1] Address to the Meetings of the American Economic Association, January 3, 2004.**?**

[2] Meese and Rogoff (1983)**?** note the exchange rate is extremely difficult to distinguish from a random walk. West (2003)**?** aptly relabels UIP "Uncertain Interest rate Parity".

dynamics inflation targeters face in practice and rules robust to uncertainty around these dynamics.

Thinking about uncertainty has a long history in economics dating back at least as far as Knight (1924). For Knight (1924)**?**, uncertainty differs from risk because the policymaker does not know the nature of the uncertainty and is unable to form a probability distribution or risk statement, over different possible models. Hansen and Sargent (2004) apply Knight's (1924) philosophy to the linear-quadratic control framework, recognizing that policymakers work with models which are approximations to some true, unknown model and seek a rule that is robust to models close to the policymaker's best approximation.

Several researchers seek rules robust to dynamics in the neighbourhood of a single specific model.[3] Tetlow and von zur Muehlen (2001) study robust policies within the context of a forward-looking closed economy model, similar to the wage-contracting model of Fuhrer and Moore (1995)**?**. They conclude that under unstructured uncertainty, where model misspecification arises in the local vicinity of a single model, the implied policy rule is more aggressive than the case where the estimated model is assumed to be the true model. Onatski and Stock (2002)**?** who use the Rudebusch and Svensson (1999) model to compare generalized Taylor-type rules that are robust to specifications of uncertainty. Within this model, the robust rule is more aggressive than the standard case with no model uncertainty.

[3] See for example, Onatski and Stock (2002)**?**, Tetlow and von zur Muehlen (2001) **?**, Hansen *et al.* (1999) **?** or the macroeconomic models in Hansen and Sargent (2002), *inter alia.*

While there exist some open economy robust control policy experiments most of the literature focuses on the closed economy.[4] This paper calibrates an open economy model to capture the key features of the open economy dynamics for three inflation targeters and thus forms the laboratory for identifying monetary policy rules for open economy inflation targeters robust to model uncertainty. Section 2 specifies the linear-quadratic robust control framework. Section 3 gives the model and reveals the match of the model to the data. Section 4 presents optimal rules under a range of assumptions about the model and the policymaker's preference for robustness. Finally, section 5 concludes the paper.

2 The Robust Control Framework

Throughout the paper, monetary policy is examined from the perspective of the linear-quadratic optimal control framework. The central bank is assumed to possess a set of goals or objectives for monetary policy. These goals are achieved by setting the interest rate using a rule that responds to the variables in the model of the economy. The behaviour of the economy acts as a constraint on the ability of the central bank in achieving its goals. It is assumed that central bank preferences can be approximated by a quadratic function and further, that the economy can be approximated by a linear model. Under this set of assumptions, the optimal interest rate rule will be unique.

[4] Open economy robust control experiments include Sargent's (1999)**?** analysis of the Ball (1999)**?** model and Leitemo and Söderström (2004)**?**, who attain analytical solutions to the robust control problem for the purely forward-looking open economy new Keynesian model developed by Clarida et. al. (2001)**?** and Galí and Monacelli (2004)**?**.

Giordani and Söderlind (2004) provide a convenient exposition of solution methods for the robust control problem under commitment, discretion and simple monetary policy rules. Here we represent an outline of their solution method, assuming that the central bank implements policy under discretion. The specification of the problem hinges on the addition of the mechanism of an "evil agent", that represents nature and introduces feared misspecification dynamics to maximise the loss of the central bank. This *minimax* problem is:

$$\min_{\{u\}_0^\infty} \max_{\{v\}_1^\infty} E_0 \sum_{t=0}^{\infty} \beta^t (x_t' Q x_t + u_t' R u_t + 2 x_t' U u_t) \tag{1}$$

$$s.t. \quad A_0 x_{t+1} = A_1 x_t + B_1 u_t + C_1(\varepsilon_{t+1} + \upsilon_{t+1}) \tag{2}$$

$$E_0 \sum_{t=0}^{\infty} \beta^t \upsilon_{t+1}' \upsilon_{t+1} \leq \eta_0. \tag{3}$$

The matrix Q captures the central banks preference for minimising the variance of particular state variables; the matrix R represents the central bank's preferences for minimising the variance of the policy instrument (most typically the nominal interest rate); and the matrix U captures any preference over the covariance between state variables and the instrument. In practice, this proves useful for capturing a preference for interest rate smoothing. The matrix A_0 captures contemporaneous relationships between the state variables, A_1 encapsulates lagged relationships while the matrix B_1 gives the impact of the policy instrument on the state variables. It is assumed that the initial state vector, x_0, is given.

Relative to standard linear-quadratic control, the key introduction is the sequence of misspecification errors υ_{t+1} that the evil agent sets as a rule. These misspecification errors apply to equations that have the usual errors attached and represent relationships the policymaker regards with uncertainty. The matrix C_1 determines the magnitude of the errors attached to each equation while the evil agent determines the dynamics of the misspecification errors, constrained by equation (3) which states that these misspecification errors must be bounded in magnitude by η_0.

Hansen and Sargent (2004)**?** and Giordani and Söderlind (2004) show that the problem specified in equations (1), (2) and (3) can be represented in a rational expectations, linear quadratic state space form, so that the problem can be treated with standard techniques. With the constraint on the evil agent substituted into the model of the economy that representation is the following:

$$\min_{\{u\}_0^\infty} \max_{\{v\}_1^\infty} E_0 \sum_{t=0}^{\infty} \beta^t (x_t' Q x_t + u_t^{*\prime} R^* u_t^* + 2x_t' U^* u_t^* - \theta v_{t+1}' v_{t+1}) \qquad (4)$$

$$s.t. \quad x_{t+1} = A x_t + B^* u_t^* + C\epsilon_{t+1}, \quad where \qquad (5)$$

$$R^* = \begin{bmatrix} R & \mathbf{0}_{k\times n} \\ \mathbf{0}_{k\times n} & -\theta I_{n_1} \end{bmatrix}, u^* = \begin{bmatrix} u_t \\ v_{t+1} \end{bmatrix},$$

$$B^* = \begin{bmatrix} B & C \end{bmatrix}, \quad and\; U^* = \begin{bmatrix} U & \mathbf{0}_{n\times n} \end{bmatrix}. \qquad (6)$$

where $A = A_0^{-1}A_1$, $B = A_0^{-1}B_1$ and $C = A_0^{-1}C_1$. The parameter θ represents the policymakers preference for robustness. A low value for θ represents a large concern for model uncertainty while the standard case of no concern for uncertainty is recovered for $\theta = \infty$. This parameter maps directly to η_0, effectively specifying the bounds on the behaviour of the evil agent — a low value of θ translate to a relatively high value of η_0 implying the evil agent is less constrained.

Applying standard rational expectations solution techniques to the problem yields a policy rule for the central bank and a rule for the misspecification errors induced by the evil agent, both of which are expressed as a linear function of the state variables. That is,

$$u_t^* = -Fx_t \tag{7}$$

where u_t^* can be expanded as:

$$\begin{bmatrix} u_t \\ \upsilon_{t+1} \end{bmatrix} = - \begin{bmatrix} F_u \\ F_\upsilon \end{bmatrix} x_t \tag{8}$$

where F_u represents the policy rule for the central bank and F_υ gives the rule used by the evil agent. In summary, under robust control, the evil agent chooses a rule for implementing worst case dynamics, given the central bank's preference for robustness, while the central bank chooses a rule that minimises its loss function assuming the evil agent implements the plausible worst case dynamics.

Under the robust control framework there are two sets of model dynamics that should be considered: (i) the *worst* case dynamics, whereby the central bank slants their rule against feared misspecification errors that occur; and (ii) the *approximating* dynamics, where the central bank slants their rule against feared misspecification dynamics that are unfounded and do not eventuate. The dynamics of the model under the worst case model can be expressed as:

$$x_{t+1} = M_w x_t + C\epsilon_{t+1} \tag{9}$$

where:

$$M_w = A - BF_u - CF_\upsilon. \tag{10}$$

The dynamics of the approximating model are not affected by the machinations of the evil agent and can be represented as:

$$x_{t+1} = M_a x_t + C\epsilon_{t+1} \tag{11}$$

where:

$$M_a = A - BF_u. \tag{12}$$

Of course, the extent of the material difference between the two sets of dynamics depends on the extent to which the policymaker fears misspecification dynamics and desires a rule that is robust to this misspecification. The following section details a useful method for parameterising the policymaker's concerns.

2.1 How much robustness?

What is the appropriate choice of θ for the policymaker? The policymaker desires a rule that is robust to models that are difficult to distinguish from the policymaker's approximate model of the economy. The central bank can be over-insured — if it adopts a rule robust to misspecification errors so unlikely to occur as to warrant discarding these processes from the realm of possible models. Effectively, the econometrician first chooses an error detection probability, the probability of making an error in distinguishing the alternative model from the true model, which reflects the concern of the policymaker not to overinsure by using rules robust against implausible models.

In order to map a sequence of error detection probabilities to a sequence of robustness parameters, Hansen and Sargent (2004) advocate using log-likelihood ratios of the approximating model against the worst case model. For a fixed sample of observations, Hansen and Sargent (2004) define L_{ij} as the likelihood of that sample for model j under the assumption that model i generates the data. The log likelihood ratio for a given sample can then be expressed as:

$$r_i \equiv \frac{L_{ii}}{L_{ij}}. \tag{13}$$

Define the approximating model, equation (12), as model A, and the worst-case model, equation (10), as model B. Consider drawing repeated samples. There are two kinds of mistakes that can be made in attempting to determine which model generated the sample data. Firstly, model A could be the true data-generating process yet for a given sample, the log likelihood may be negative. It is possible to calculate the probability of making this mistake in repeated sampling:

$$p_A = \Pr(mistake/A) = freq(r_A \leq 0) \tag{14}$$

i.e., the frequency of generating *negative* log-likelihood ratios is the probability of mistaking model B for model A, when model A is the true data generating process. Secondly model A may be mistaken for model B such that:

$$p_B = \Pr(mistake/B) = freq(r_B \leq 0). \tag{15}$$

The probabilities of a mistake, p_A and p_B, are functions of the difference between the approximating model, equation (12), and the worst-case model, equation (10), which is a function of the robustness parameter, θ. The probability of detecting a difference between the approximating model and the worst-case model can thus be expressed as:

$$p(\theta) = \frac{1}{2}(p_A + p_B). \tag{16}$$

The next step is to calculate the map between the error detection probabilities and the robustness parameter. Firstly choose an appropriate value for the error detection probability.[5] Given the error detection probability, calculate the preference for robustness θ using the map. Following Hansen and Sargent (2004) a risk sensitivity parameter is defined where the risk sensitivity parameter $\sigma = -\theta^{-1}$. When $\sigma = 0$ the robustness parameter is infinite and the model conforms to the standard case. When the risk sensitivity parameter is negative, there exists a preference for a robust rule.

3 The Model

3.1 Theoretical model

McCallum and Nelson (1999)**?** show how an IS equation, derived from a consumption Euler equation, implies that the output gap is a function of agents' expectations of the output gap in addition to the real interest rate. However, this view of the output gap process is generally inconsistent with the finding that the output gap displays substantial persistence in the data. Fuhrer (2000)**?** shows that the addition of habit formation to the utility function for consumers implies that the lag of output enters the optimizing IS equation. If we appeal to inertia on the part of decision making on the part of consumers and lag the real interest rate, we obtain a closed economy output gap equation largely constructed from structural parameters yet sufficiently flexible to replicate the persistence in output gap data.

In addition, McCallum and Nelson (1999) derive an open economy version of their optimizing closed economy IS equation that implies the output

[5]Typically the literature has settled on error detection probabilities of 10 and 20 per cent (see Hansen and Sargent (2002)**?** and Giordani and Söderlind (2004)**?**, for example).

gap is a function of the real exchange rate, foreign output, the expectation of the real exchange rate and the expectation of foreign output gap. Simplifying open economy effects to the lag of the real exchange rate, the output gap equation takes the following form:

$$\tilde{y}_t = \beta_1 E_t \tilde{y}_{t+1} + (1 - \beta_1)\tilde{y}_{t-1} - \beta_2 r_{t-1} - \beta_3 q_{t-1} + \varepsilon_{\tilde{y}t} \tag{17}$$

where $\tilde{y}_t$ represents the output gap, r_t is a long term real interest rate and q_t represent the real exchange rate — an increase in q_t represents an exchange rate appreciation. All the coefficients are positive according to theory. The long term *ex ante* real interest rate is defined using a risk neutral arbitrage condition so that the long rate is the sum of the sequence of expected short term interest rates, that is:

$$r_t = \frac{1}{d} \sum_{s=0}^{\infty} \left(\frac{d}{1+d} \right)^s E_t(i_{t+s} - \pi_{t+1+s}) \tag{18}$$

where d defines the number of quarters for the effective long term real interest rate (see Söderlind (1999)**?** for an empirical example of this definition of the long term real interest rate).

A hybrid new-Keynesian Phillips curve is used to model domestic inflation. Structural models of the Phillips curve can be derived from wage-contracting behaviour on the part of firms and workers (see Fuhrer (1997)**?**, for example). These models suggest that workers form wage demand as an average of the expected real wage and observed past real wages with a mark-up in good times and a lower real wage in bad times, based on the realization of the output gap. Alternatively, pricing behaviour on the part of firms (see Calvo (1983)**?** and Galí and Gertler (1999) **?**) can be used to derive structural equations for inflation that contain forward and backwardlooking components. These behavioural assumptions generate hybrid domestic inflation equations similar to:

$$\pi_t^d = \alpha_1 E_t \pi_{t+1}^d + (1-\alpha_1)\pi_{t-1}^d + \alpha_2 \tilde{y}_{t-1} + \varepsilon_{\pi^d t} \tag{19}$$

where π_t^d represents domestic inflation. Inflation equations developed from strict microfoundations predict a contemporaneous relationship between inflation and the output gap but this is difficult to reconcile with the data and policy practitioners views of the transmission mechanism.

The foreign good component of inflation is assumed to be a direct markup over the change in the exchange rate with incomplete pass-through:

$$\pi_t^f = \kappa \pi_{t-1}^f + (1-\kappa)\Delta q_t \tag{20}$$

where π_t^f is foreign inflation and the parameter κ calibrates the degree of exchange rate pass-through. Finally, consumer price inflation is a combination of domestic price inflation and foreign good inflation, weighted according to ϕ, the proportion of foreign goods in the consumer price index:

$$\pi_t = \phi \pi_t^f + (1-\phi)\pi_t^d. \tag{21}$$

The no arbitrage condition that is the basis of Uncovered Interest rate Parity (UIP) forms a theoretically appealing structural relationship for modelling the real exchange rate. However, this condition does not appear to capture the predilection of the exchange rate to move through large, persistent cycles and we allow for autocorrelated exchange rate errors. Thus the real exchange rate equation is modelled by UIP:

$$q_t = E_t q_{t+1} + (i_t - E_t \pi_{t+1}) - (i_t^f - E_t \pi_{t+1}^f) + \varepsilon_{qt}. \tag{22}$$

while the exchange rate errors are modelled as AR(1) process:

$$\varepsilon_{qt} = \rho\varepsilon_{qt-1} + \xi_t \tag{23}$$

where ξ_t is a standard normal error process.[6]

It remains to choose an appropriate calibration of the model to serve as the laboratory for the robust control experiments to follow. The next section calibrates the model, presenting the fit of the model to the data and subsequently the calibration.

3.2 Model fit

The calibration is designed as a loose description of the data. In fact, a single generic model calibration is used to broadly match the key features of three datasets. Although, New Zealand was the first country in the world to adopt an explicit inflation targeting framework in February 1990, the model is calibrated to data from the period 1992q1 to 2003q4.[7] Australia, Canada and New Zealand underwent disinflationary periods that had largely ended by the beginning of 1992. Asking the model to explain the disinflationary period is misleading because these data points are generated from an alternative policy regime.

One criterion for model fit is the second moments implied by the model. Rows 2 to 4 in table 1 below depict the standard deviations for inflation, HP-filtered output gap, real exchange rate and the nominal interest rate

[6] The autocorrelation can be interpreted as autocorrelation in a risk premium term.

[7] Canada adopted explicit inflation targets in February 1991 and Australia two to three years later. The Australian approach was more gradual — Bernanke *et al.* (1999)**?** characterize the Reserve Bank of Australia's view as dating the adoption of inflation targeting in early 1993.

Table 1: Model versus Data Standard Deviations

	$\sigma_{\tilde{y}}$	σ_{π}	σ_q	σ_i
Model	1.068	0.840	1.447	1.268
Australia	0.801	1.014	1.415	1.094
Canada	1.117	0.508	1.468	1.546
New Zealand	1.583	0.901	1.547	2.215

for Australia, Canada and New Zealand.[8] This is compared to standard deviations based on a time series of 9,000 observations of model generated data.[9] The next three rows show the empirical standard deviations of these variables observed over the period 1992q1 to 2003q4.

Looking at the first column of the table, it appears that the baseline model gets the standard deviation about right — the standard deviation of the output gap implied by the baseline model is very close to the standard deviation observed in Canadian data. The New Zealand output gap is about 50% more volatile than both the model and the Canadian data. The standard deviation of inflation implied by the model is close to that of Australia and New Zealand, but overstates the volatility evident in the Canadian data series. However, this is probably due to differences in core

[8] The HP filter smoothing parameter is set to 1600 and the filter applied to data 1985q1 to 2003q4 to mitigate some of the effects of the end point problem. The standard deviation in the table are for the subsample 1992q1 to 2003q4. Inflation is annualized quarterly consumer price inflation in core inflation measures, excluding volatile items for Australia and weighted median measures for Canada and New Zealand. The real exchange rate standard deviations are trade-weighted CPI based measures expressed in percentage terms. The interest rate series are quarterly averages of monthly ninety day series.

[9] This was based on simulating 10,000 observations from the reduced form and discarding the initial 1,000 observations. Initial period values are set to 0. The variance of the domestic inflation, output gap and exchange rate shocks are set to 0.5, as part of the model calibration, detailed in the following subsection.

measures of inflation. The model appears to mimic the observed volatility in the real exchange rate particularly well. Finally, the volatility in the nominal interest rate implies by the model gives a good match to the data and is nested by the lower volatility in the Australian dataset and the slightly higher volatilities for Canada and New Zealand.

The implied persistence of key state variables provides a second criterion for model fit. Autocorrelation functions for observed data are compared to autocorrelation functions for the same set of simulated data used to construct table 1. The autocorrelation functions are shown in table 2.

The model matches the persistence observed in the Australian output gap remarkably well. However, the corresponding autocorrelation functions for Canadian and New Zealand data show slightly more persistence than the model. Possibly the model appears to understate the degree of output gap persistence. In addition, the persistence in the model is slightly higher than the observed persistence in inflation for all three countries. The model does not match the strong persistence observed in the New Zealand exchange rate but is broadly similar to the persistence for the Australian and Canadian exchange rates. The model appears to slightly overstate the persistence in the nominal interest rate for Australia and Canada but is representative of the persistence observed in the New Zealand ninety day interest rate.

3.3 Model Calibration

The calibration that supports the model properties presented in tables 1 and 2 is relatively standard with one exception — the lack of persistence in the inflation data implies a large role for expectations (a coefficient of 0.7, more than double the weight on the coefficient on the lag of domes-

Table 2: Model versus Data Autocorrelation Functions

Lag length	1	2	3	4	5	6	7	8
Panel 1 AC function for the output gap								
Model	0.72	0.45	0.26	0.14	0.06	0.03	0.00	-0.02
Australia	0.72	0.44	0.24	0.17	0.20	0.14	-0.06	-0.20
Canada	0.87	0.64	0.44	0.19	-0.01	-0.17	-0.31	-0.37
New Zealand	0.86	0.71	0.47	0.26	0.09	0.04	-0.12	-0.20
Panel 2 AC function for inflation								
Model	0.54	0.23	0.03	-0.07	-0.09	-0.04	-0.03	-0.03
Australia	0.48	0.16	0.04	0.01	-0.03	0.13	0.08	-0.09
Canada	0.45	0.25	0.26	0.03	0.17	0.06	-0.00	0.02
New Zealand	0.40	0.16	0.18	0.10	-0.01	0.03	0.01	-0.14
Panel 3 AC function for the exchange rate								
Model	0.82	0.56	034	0.19	0.09	0.03	0.00	-0.04
Australia	0.75	0.60	0.45	0.34	0.15	0.05	0.00	-0.01
Canada	0.83	0.68	0.57	0.46	0.35	0.24	0.13	0.06
New Zealand	0.93	0.83	0.75	0.64	0.52	0.36	0.21	0.07
Panel 4 AC function for the nominal interest rate								
Model	0.88	0.69	0.52	0.38	0.29	0.23	0.17	0.12
Australia	0.83	0.61	0.43	0.21	0.01	-0.12	-0.20	-0.25
Canada	0.77	0.56	0.39	0.08	-0.13	-0.18	-0.17	-0.07
New Zealand	0.83	0.58	0.38	0.24	0.17	0.16	0.15	0.07

tic inflation (0.3)) in the Phillips equation relative to the empirical literature. As Dennis and Söderström (2002)**?** note, the literature has not settled on an appropriate calibration for the forward-looking component in the Phillips equation. Completely forward-looking inflation equations have difficulty explaining the persistence in US inflation data. Ball (1999)**?** and Rudebusch and Svensson (1999) assume that the Phillips equation has no forward-looking component. Other researchers, for example Fuhrer (1997)**?**, Galí and Gertler (1999) **?**, Roberts (1997)**?** and Lindé (2001)**?**, suggest estimates on the forward-looking component to be in the range 0.1–0.7.

In addition, the effect of the output gap on inflation is calibrated to 0.1; incomplete pass-through from the exchange rate to the price of foreign goods is modelled by setting $\kappa = 0.8$, and domestic and foreign goods are weighted equally within the consumer price index, so $\mu = 0.5$. Calibrating serial correlation in the exchange rate errors, by setting $\rho = 0.8$, is necessary to explain the persistent deviations of the real exchange rate from UIP.

Within the IS equation, consumption has a high degree of persistence from a large role for habit formation — $\beta_1 = 0.1$. This calibration is much lower than the calibration of 0.5 in Söderström *et al.* (2002)**?** and the estimate of 0.3 in Fuhrer (2000)**?**, yet several researchers (Söderlind (1999)**?**, Rudebusch (2002) **?**, Ball (1999)**?**) specify a zero weight on the forward-looking component.

The sensitivity of the output gap to the real interest rate is set so that $\beta_2 = 0.1$. This is half the calibrated value in the open economy of Ball (1999), lower than the open economy calibration of 0.5 in Batini and Haldane (1999) but slightly higher than the parameter estimated in Söderlind (1999) and Rudebusch and Svensson (1999) — both closed economy models. The coefficient on the lag of the real exchange rate, β_3, is set to 0.2, twice the value in Ball (1999).

The variance of the shocks to the inflation, output gap and exchange rate error equations are set to 0.5. The baseline loss function for the central bank is calibrated such that the central bank is twice as concerned with stabilizing inflation relative to stabilizing the output gap such that $\lambda_1 = 0.5$, and desires interest rate smoothing — $\lambda_2 = 0.5$. This is required to mimic the volatility and persistence in the nominal interest rate series. No weight on minimising the real exchange rate was required to match the data.

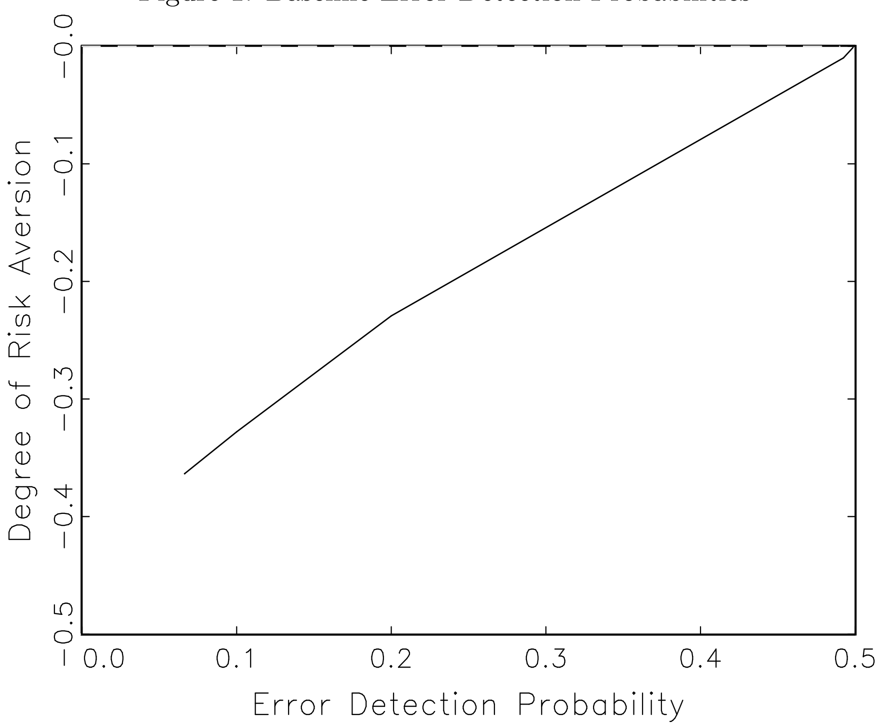
4 Robust policies

4.1 Robust rules

Prior to constructing robust rules, an appropriate preference for robustness must be obtained via the calculation of error detection probabilities. Figure 1 presents the map from error detection probabilities to the risk sensitivity parameter under the baseline model. Reading the solid black line to the x-axis, we see that an error detection probability of 0.1 is associated with a degree of risk aversion of -0.328 which maps to a robustness parameter of 3.05, under the assumption that the observed data sample contains sixty time periods.

When the policymaker desires a rule robust against models that have at least a 20% chance of generating the observed data, the error detection probability of 0.2 is associated with a degree of risk aversion of -0.229 and a robustness parameter of 4.36. Thus a policymaker that demands rules robust to a smaller set of models, is less risk averse and has a higher robustness parameter that bounds the nature of the worst case dynamics the evil agent can generate.

Figure 1: Baseline Error Detection Probabilities



Robust rules for the baseline model are presented in the first section of table 3. The standard rule — with no preference for robustness — is labelled “F” and presented in the first row of the table. The appropriate interest rate response (in percentage point terms) to the state variables in the model, labelled at the head of each column, are contained within each cell of the table. For example, the baseline rule indicates that the nominal interest rate should be increased 0.157 percentage points in response to a unit shock to domestic inflation.

Sensibly, the baseline rule indicates that the nominal interest rate should be lowered in response to a positive real exchange shock, the lag of the real exchange rate and lowered a little if the lag of the long term real interest rate is above its equilibrium value. The interest rate should be increased in response to positive shocks to the output and domestic inflation. The response to the lag of the output gap is substantially more aggressive than the corresponding response to the lag of domestic inflation, reflecting the large role for inflation expectations within the Phillips equation. Finally, the coefficient on the lag of the nominal interest rate is 0.33 — noticeably lower than many other studies.[10]

The second row of the table, labelled “F(20$^{\%}$)”, presents the robust rule associated with an error detection probability of 20%. That the rule recommends more aggressive policy is immediately apparent. The response to a domestic inflation shock approximately doubles to 0.340, the response to the exchange rate and output gap shocks increase substantially. This attenuated policy response is consistent across each of the state variables with one exception — the rule suggests less interest rate smoothing. Robust poli-

[10] See for example, Lansing (2000)**?**, Sack and Wieland (2000)**?**, and Rudebusch (2002)**?**.

cies for open economy inflation targeters are more aggressive than standard rules.

That robust policy calls for attenuated policy within the model, is underlined by the rule associated with an error detection probability of 10%. Interest rate smoothing is reduced and the response to the other state variables increases markedly. For example, the response to the domestic inflation shock is about triple the response under the standard rule.

The rules F(ε_π), F(ε_q) and F($\varepsilon_{\tilde{y}}$) show the result of alternative sets of restrictions on the misspecification dynamics that sets the variance of all but the bracketed shock equal to zero. For example rule F(ε_π) sets the variance of the output gap and exchange rate shocks to zero. This restricts the evil agent to inducing misspecification in only the inflation equation. Of course, under standard policy with no preference for robustness, certainty equivalence applies and the policy rule will be identical to the baseline rule. However, within the robust control framework, certainty equivalence does not hold because restricting the dimensions of the shocks appended to each equation acts as an additional constraint on the behaviour of the evil agent.

The set of rules obtained under restricted misspecification dynamics are generally more aggressive than the baseline although there are exceptions. For example, when the central bank is assumed to have a relatively small weight on interest rate smoothing (loss function (ii), in table 3), the model that restricts the evil agent to misspecification in the real exchange rate equation indicates that policy should respond less aggressively to all but the real exchange shock. This result echoes the finding of Leitemo and Söderström (2004)**?** who find that the policy response should be mitigated when the misspecification dynamics originate in the real exchange rate equation only.

Table 3: Optimal and Robust Rules: Baseline Model

Rule	$e_{\pi^d t}$	e_{qt}	$e_{\tilde{y}t}$	$\tilde{y}_{t-1}$	π^d_{t-1}	q_{t-1}	i_{t-1}	π^f_t	R_{t-1}
Loss function (i): $L_t = \pi_t^2 + 0.5\tilde{y}_t^2 + 0.5\Delta i_t^2$									
F	0.157	-0.575	0.548	0.509	0.047	-0.186	0.330	0.051	-0.055
F(20%)	0.340	-0.640	0.825	0.777	0.102	-0.359	0.246	0.131	-0.083
F(10%)	0.457	-0.683	1.005	0.950	0.137	-0.471	0.186	0.181	-0.101
F(ε_π)	0.711	-0.702	1.071	1.035	0.213	-0.623	0.058	0.283	-0.107
F(ε_q)	0.186	-0.727	0.655	0.608	0.056	-0.228	0.168	0.062	-0.065
F($\varepsilon_{\tilde{y}}$)	0.230	-0.580	0.755	0.703	0.069	-0.289	0.316	0.084	-0.076
Loss function (ii): $L_t = \pi_t^2 + 0.5\tilde{y}_t^2 + 0.1\Delta i_t^2$									
F	0.231	-0.745	0.755	0.703	0.069	-0.238	0.192	0.077	-0.076
F(20%)	0.553	-0.757	1.137	1.079	0.166	-0.511	0.160	0.224	-0.114
F(10%)	0.714	-0.763	1.303	1.244	0.214	-0.643	0.141	0.294	-0.130
F(ε_π)	0.808	-0.774	1.164	1.128	0.242	-0.649	0.124	0.329	-0.116
F(ε_q)	0.224	-0.809	0.749	0.696	0.067	-0.230	0.123	0.076	-0.075
F($\varepsilon_{\tilde{y}}$)	0.302	-0.731	0.949	0.885	0.091	-0.336	0.197	0.111	-0.095
Loss function (iii): $L_t = \pi_t^2 + \tilde{y}_t^2 + \Delta i_t^2$									
F	0.126	-0.519	0.533	0.492	0.038	-0.181	0.376	0.038	-0.053
F(10%)	0.253	-0.596	0.738	0.689	0.076	-0.304	0.277	0.090	-0.074
F(20%)	0.338	-0.638	0.860	0.808	0.101	-0.382	0.208	0.125	-0.086
F(ε_π)	0.440	4.091	0.650	0.629	0.132	-0.377	0.160	0.161	-0.065
F(ε_q)	0.144	-0.620	0.603	0.557	0.043	-0.208	0.266	0.043	-0.060
F($\varepsilon_{\tilde{y}}$)	0.159	-0.533	0.656	0.606	0.048	-0.235	0.358	0.052	-0.066
Loss function (iv): $L_t = \pi_t^2 + y_t^2 + \Delta i_t^2 + q_t^2$									
F	0.184	-0.669	0.646	0.600	0.055	-0.296	0.243	0.056	-0.065
F(20%)	0.235	-0.685	0.942	0.871	0.070	-0.388	0.204	0.078	-0.094
F(10%)	0.262	-0.689	1.135	1.048	0.079	-0.445	0.184	0.090	-0.113
F(ε_π)	0.498	-0.795	1.743	1.619	0.150	-0.716	-0.017	0.183	-0.174
F(ε_q)	0.194	-0.756	0.067	0.619	0.058	-0.305	0.159	0.057	-0.067
F($\varepsilon_{\tilde{y}}$)	0.203	-0.665	0.838	0.775	0.061	-0.347	0.232	0.064	-0.084

NB.Under the baseline rule, there is no preference for robustness and $\theta = \infty$.

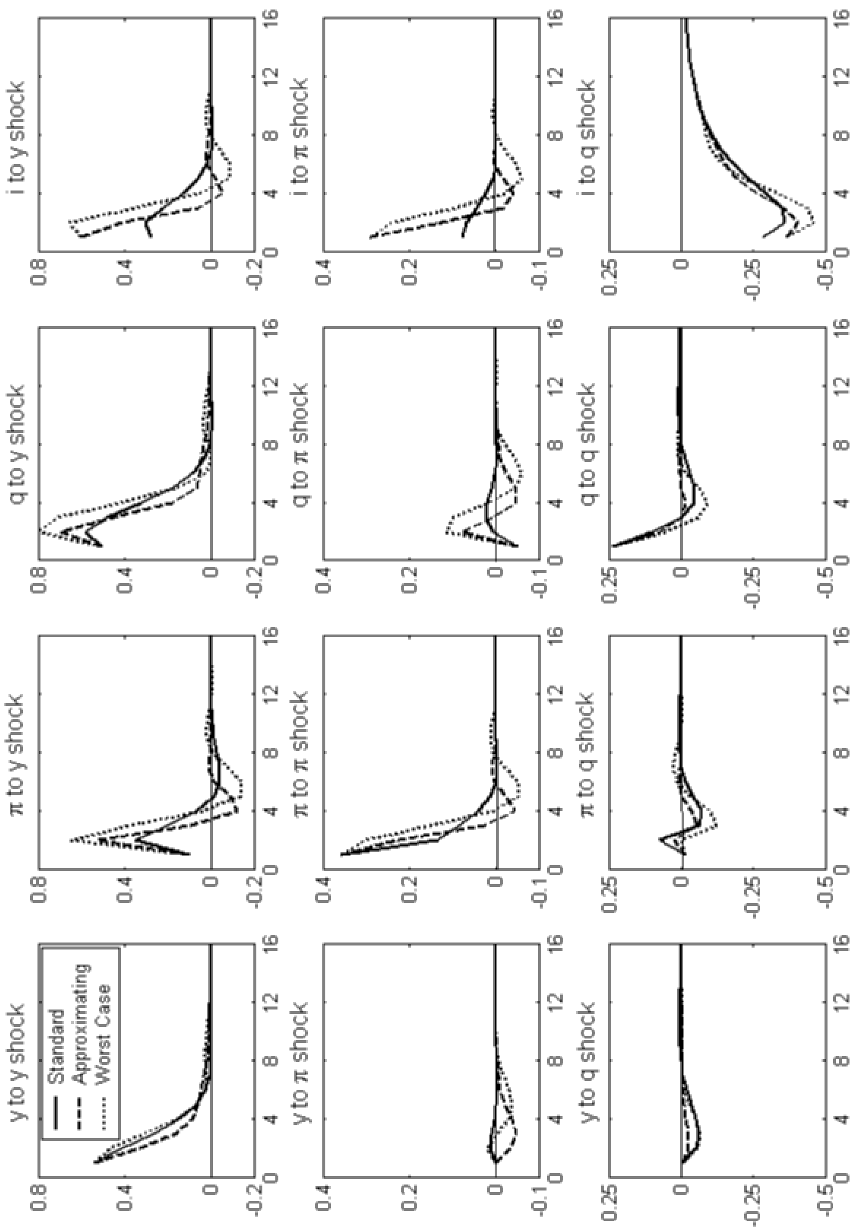
4.2 Robust Control Dynamics

To illuminate the differences in model dynamics when the policymaker adopts typically more aggressive robust rules against unknown misspecification errors, the dynamics of the model are depicted in figure 2 for three alternative scenarios. The standard case, where the policymaker does not in fact slant their rule against misspecification is depicted with a solid black line. The case where the policymaker slants the rule against unknown errors that do not eventuate, such that the underlining model of the constraint is the approximating model, is depicted with a dashed line. The worst case scenario, where the misspecification errors that are feared by the policymaker eventuate and the evil agent's rule for nature is incorporated into the underlying model of the economy, is depicted with the dotted line.

Firstly, turn to the first row of figure 2 and examine the response of the key macroeconomic variables to an output gap shock. The behaviour of the output gap following the output gap shock is broadly similar across the three alternative scenarios. After the initial output gap shock, the output gap decreases, falling to zero approximately four to six quarters after the shock. Under the worst case scenario, when the feared misspecification errors occur, the output gap remains above the baseline case for a period of time but this difference is barely discernible. Under the approximating scenario, where the worst-case misspecification dynamics do not occur, the output gap is returned to zero slightly more quickly than the standard case.

For the standard case, domestic inflation increases relatively sharply initially before returning towards zero after approximately four quarters. The initial increase in domestic inflation is more pronounced under the worst case model and domestic inflation remains substantially higher for a number of

Figure 2: Impulse Response Functions



periods than under the standard case.

This is because the evil agent, aiming to maximise the loss of the central bank, delivers dynamics that increase the persistence of both the output gap shock and domestic inflation. To protect against these feared, misspecification dynamics the policymaker slants their rule. If these misspecification errors do not in fact occur, yet the policymaker uses a rule slanted against feared misspecified dynamics a third permutation arises, depicted with a dashed line. Under this scenario, domestic inflation increases initially yet is returned towards target rapidly and actually falls below the path of domestic inflation for the standard case.

Turning to the behaviour of the real exchange rate, in response to the output gap shock, there is little discernible difference in the behaviour of the real exchange rate across the three alternative scenarios. Recall that the baseline loss function does not include the real exchange rate. Given a limited effective budget to manipulate the model's dynamics, the evil agent focuses their activities on manipulating the persistence of the process that affect the paths of the key macroeconomic variables that enter the central bank's loss function.

The response of the nominal interest rate is revealing about how the aggressiveness of the robust policy rule begins to translate into the three alternative dynamic structures. Under the standard model, the nominal interest rate ticks up approximately 30 basis points in response to the output gap shock before decreasing close to zero after about eight quarters. The initial response of the nominal interest rate under the approximating and worst case model is stronger — an increase of approximately 50 basis points. Note that the initial increase in the nominal interest rate is identical under both the approximating and worst case models because the policy rule is identical and the misspecification dynamics take time to impact on the paths of the variables. After the initial increase, the nominal interest rate decreases particularly rapidly under the approximating model, passing under the path of the standard model between three and four quarters. Under the worst case model, the machinations of the evil agent results in dynamics that force the nominal interest rate to remain about 25 basis points higher than the approximating model until about six quarters when this implied differential in the interest rate path begins to dissipate.

That the worst case dynamics map into a higher loss for the central bank can be seen in table 4. The table depicts the losses under the approximating and worst case models for a range of central bank preferences, robustness preferences and a range of constraints on the nature of the misspecification dynamics.

Table 4: Loss Comparison under Robust Policy: Baseline Model

	20%	10%	e_π	$e_{\tilde{y}}$	e_q
Loss function (i): $L_t = \pi_t^2 + 0.5\tilde{y}_t^2 + 0.5\Delta i_t^2$					
M$_s$	72.82	72.82	15.73	49.99	7.10
M$_a$	88.45	97.37	27.48	63.82	9.05
M$_w$	89.54	102.57	37.28	58.44	9.08
Loss function (ii): $L_t = \pi_t^2 + 0.5\tilde{y}_t^2 + 0.1\Delta i_t^2$					
M$_s$	62.27	62.27	15.46	44.85	1.96
M$_a$	80.72	75.66	24.53	50.49	2.14
M$_w$	86.35	77.96	29.28	53.22	2.15
Loss function (iii): $L_t = \pi_t^2 + \tilde{y}_t^2 + \Delta i_t^2$					
M$_s$	108.79	108.79	16.06	80.20	12.52
M$_a$	124.45	135.98	26.55	84.70	14.71
M$_w$	126.64	136.40	32.32	92.85	15.08
Loss function (iv): $L_t = \pi_t^2 + \tilde{y}_t^2 + \Delta i_t^2 + q_t^2$					
M$_s$	210.46	210.46	16.78	176.38	17.30
M$_a$	258.21	300.85	25.68	209.47	19.46
M$_w$	284.16	336.26	32.26	240.12	19.62

Firstly, the table shows that the loss the central bank incurs is always higher when the evil agent is able to implement the worst case dynamics. This of course is unsurprising because the task of the evil agent is to induce misspecified dynamics that impact negatively on the loss the central bank occurs.

Secondly, we can observe an increase in the loss when the central bank begins to slant their rule against misspecified dynamics — even when the worst case dynamics do not eventuate. This can be observed in the second row of table 4 where the loss increases by about 20% when the central bank slants their rule against misspecified dynamics that have an error detection probability of 20%.

4.3 The exchange rate transmission channel

A key feature of the small open economy new Keynesian model is that the policymaker can no longer perfectly offset demand shocks. Interest rate changes alter the exchange rate which plays a direct role in determining inflation via the price of foreign good component of the consumer price index. Although the link between the interest rate and exchange rate may hinder the central bank in the face of demand shocks, this link opens the exchange rate channel to the central bank, enhancing the effectiveness of policy via the impact of the exchange rate on both the output gap and inflation.

This section explores a particularly simple experiment for examining the role of the exchange rate channel for inflation targeters, operating under uncertainty. The exchange rate channel is closed off by setting the parameters of the model in a manner that allows no role of the exchange rate

Table 5: Optimal and Robust Rules: Baseline Model

Rule	$e_{\pi^d t}$	e_{qt}	$e_{\tilde{y}t}$	$\tilde{y}_{t-1}$	π^d_{t-1}	q_{t-1}	i_{t-1}	π^f_t	R_{t-1}
Loss function (i): $L_t = \pi_t^2 + 0.5\tilde{y}_t^2 + 0.5\Delta i_t^2$									
Baseline*	0.252	0	1.429	1.312	0.076	0	0.778	0	-0.143
Robust (i)	0.507	0	2.549	2.345	0.152	0	0.678	0	-0.255
Robust (ii)	0.676	0	3.332	3.066	0.203	0	0.606	0	-0.333
Robust (ε_π)	1.087	0	4.954	4.457	0.326	0	0.455	0	-0.495
Robust ($\varepsilon_{\tilde{y}}$)	0.378	0	2.021	1.857	0.113	0	0.727	0	-0.202
Loss function (ii): $L_t = \pi_t^2 + 0.5\tilde{y}_t^2 + 0.1\Delta i_t^2$									
Baseline*	0.618	0	2.458	2.274	0.185	0	0.743	0	-0.246
Robust (i)	1.299	0	4.324	4.021	0.390	0	0.632	0	-0.432
Robust (ii)	1.788	0	5.779	5.380	0.536	0	0.544	0	-0.578
Robust (ε_π)	2.300	0	6.930	6.467	0.690	0	0.466	0	-0.693
Robust ($\varepsilon_{\tilde{y}}$)	0.881	0	3.264	3.026	0.264	0	0.700	0	-0.326
Loss function (iii): $L_t = \pi_t^2 + \tilde{y}_t^2 + \Delta i_t^2$									
Baseline*	0.166	0	1.193	1.090	0.050	0	0.790	0	-0.119
Robust (i)	0.331	0	2.096	1.919	0.099	0	0.695	0	-0.210
Robust (ii)	0.424	0	2.627	2.407	0.127	0	0.639	0	-0.263
Robust (ε_π)	0.781	0	4.243	3.897	0.234	0	0.452	0	-0.424
Robust ($\varepsilon_{\tilde{y}}$)	0.251	0	1.703	1.558	0.075	0	0.739	0	-0.170

NB. *Under the baseline rule, there is no preference for robustness and $\theta = \infty$.
**The rules robust to e_π shocks only is robust at the 33% error detection probability.

— shifting the model from an open economy paradigm to a closed economy world.[11] Clearly this model will no longer approximate the data for Australia, Canada and New Zealand. The exercise is simply a hypothetical experiment to facilitate comparison between closed economy and open economy worlds.

[11] Practically, the component of foreign goods in the consumer price index is set to zero, $\mu = 1$; the role of the exchange rate in determining the output gap removed, $\beta_3 = 0$; and exchange rate shocks completely removed from the model.

That the exchange rate channel is useful is evident in table 6, which depicts the expected loss under the closed economy model for a range of loss function specifications and restrictions on the nature of the misspecification dynamics. Clearly, the central bank prefers to operate within the open economy world relative to its closed economy counterpart. The loss under the standard case is 289.43 — about 4 times the loss under the open economy model (72.43).

Turning to the baseline reaction function for the closed economy model (row 1 in table 6), there is no response to the lag of the exchange rate, the exchange rate shock, and the foreign good component of domestic inflation because these variables no longer affect the transmission mechanism. The coefficients on the remaining state variables have identical signs but are larger than their counterparts in the small open economy model The policymaker must respond more aggressively to the state variables to stabilise the economy in the absence of an exchange rate transmission channel.

A preference for robustness enhances this relative aggression observed in the closed economy rules. Relatively extreme responses for the nominal interest rate are implied. This result holds when misspecification dynamics are included in the analysis. Similar increases in the loss can be observed when the policymaker slants their rule when they fear — to different degrees — model misspecification. There is a substantial increase in the loss the central bank incurs when operating within the closed economy world both for the standard and robust cases.

Table 6: Loss Comparison under Robust Policy: Baseline Model

	20%	10%	e_π	$e_{\tilde{y}}$
Loss function (i): $L_t = \pi_t^2 + 0.5\tilde{y}_t^2 + 0.5\Delta i_t^2$				
M_s	289.43	289.43	59.91	229.52
M_a	418.86	510.98	85.20	268.21
M_w	453.46	561.59	101.73	313.84
Loss function (ii): $L_t = \pi_t^2 + 0.5\tilde{y}_t^2 + 0.1\Delta i_t^2$				
M_s	189.72	189.72	58.85	130.87
M_a	269.73	334.19	82.24	144.46
M_w	279.24	343.95	97.56	167.99
Loss function (iii): $L_t = \pi_t^2 + \tilde{y}_t^2 + \Delta i_t^2$				
M_s	410.23	410.23	60.46	349.76
M_a	568.06	665.74	86.27	404.03
M_w	636.34	764.24	103.05	478.99

5 Concluding Remarks

The calibration of the small open economy new Keynesian model to data from Australia, Canada and New Zealand proves a useful testing ground for identifying robust policies for open economy inflation targeters. The calibrated model appears capable of capturing key features of the Australian, Canadian and New Zealand data over the majority of the inflation targeting period in these countries. A comparatively high weight on inflation expectations within the Phillips equation appears necessary to capture the relative lack of persistence in inflation compared to US data. This results in policy responding particularly strongly to the output gap relative to domestic inflation.

Robust policies are found to tend to translate into responding more aggressively to the macroeconomy and reduced interest rate smoothing. This finding broadly holds when the nature of model uncertainty is restricted to particular equations within the model. But, this is not a general result — when the central bank places a relatively low weight on interest smoothing and misspecification is restricted to the exchange rate equation, mitigation of the policy response is suggested.

Finally, the role of the exchange rate channel is explored within a closed economy counterpart of the baseline model. For the parameterisation of uncertainty within the paper, the uncertainty that surrounds the UIP condition is not sufficient to offset the benefit of enhanced effectiveness of monetary policy. According to the expected sum of discounted future losses, the central bank prefers the open economy, with its concomitant UIP uncertainty, over the closed economy environment. The closed economy consistently suggests more aggressive policy.

References