Tax Policies, Vintage Capital, and Entry and Exit of Plants

Shao-Jung Chang[†]   Dennis Jansen[††]

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Abstract

This paper studies the optimal combinations of tax rates in a vintage capital model with exit and entry of plants. Government expenditure is financed by three tax instruments including capital-income, labor-income, and property taxes. Our results show that in a stationary economy, given a fixed level of government expenditure and a zero property tax rate, the optimal capital-income tax rate may be negative, zero, or positive dependent on the level of government expenditure. We find that for many values of government spending the highest level of steady-state utility occurs with a subsidy to capital income and a tax on labor income. Finally, we find that when taxes on capital income, labor income, and property are available, a positive capital-income tax is generally the last resort to finance government expenditures.

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[†]*Corresponding Author:* Department of Economics, Texas A&M University, 4228 TAMU, College Station, TX 77843. Tel: 1-979-845-7351. Email: shao-jung-chang@neo.tamu.edu.

[††]Department of Economics, Texas A&M University, 4228 TAMU, College Station, TX 77843. Tel: 1-979-845-7375. Email: d-jansen@tamu.edu

1 Introduction

It is widely known that taxing capital income is taxing future consumption. Faced with taxes on capital income, consumers will consume more and save less to avoid being taxed on capital income in the future. But why is capital income still taxed? Chamley [4] discusses the optimal long-run capital-income tax rate, and argues that in an infinitely-lived representative agent model, even though the current (short-run) capital-income tax rate may be positive (because the current capital stock is fixed), the optimal capital-income tax rate should be zero in the long run. Lucas [17] advocates this view and shows that if the U.S. economy switches from the currently positive capital-income tax rate to a zero capital-income tax rate, and credibly commits to this rate of zero, the U.S. economy will experience a welfare gain. However, as Aiyagari [1] points out, the currently positive capital-income tax rate may be optimal. By also using an infinitely-lived representative agent model and adding idiosyncratic shocks to individual income and an uninsured market structure into the model, Aiyagari argues that, faced with a fluctuating income, consumers have the incentive to save more out of precautionary motives and thus overly accumulate capital. When capital is overaccumulated, the return on capital is depressed and falls below the rate of time preference. A positive capital-income tax rate can discourage consumers' willingness to save and adjust capital income so that the return on capital is equal to the rate of time preference.

1.1 The Aim of This Paper

This paper attempts to find out how capital-income tax rates affect welfare in the long run in a vintage capital model where embodied technology progresses exogenously and plants are retired if their current value becomes lower than their scrap value. To be more precise, we want to know if the optimal capital-income tax rate is zero or positive in a vintage capital model with the exit and entry of plants.

There are two elements that make this paper different from Chamley [4] and Aiyagari [1]: Vintage capital and the exit and entry of plants. The concept of vintage capital is that technology is embodied within capital (either equipment or machine) when capital is created, and the productivity of that capital cannot be changed after it is made. When the productivity of the capital becomes too low, firms invest in new capital. In Greenwood, Hercowitz, Krusell [9], the authors discuss if the idea of vintage capital is practical. They show that prices of equipment have been declining over the post-war years, and they conclude that the technology of producing equipment is making equipment either more productive or cheaper over time. They conclude that the idea of vintage capital is able to explain this phenomenon. Literature on vintage capital and investment behavior has been growing since then. For example, Cooley, Greenwood, and Yorukoglu [5] endogenize the rate of embodied technology growth by building a two-sector model with human capital accumulation and discuss how tax policies affect the compositions of the aggregate capital stock. Yorukoglu [21] argues why investment is lumpy when firms can choose to replace or upgrade

the current capital stock. On the other hand, Hopenhayn [11] extends a stochastic model for a competitive industry to account for exit, entry, sizes, and growth of plants. Campbell [3] sets up a model to simulate the correlations among exit and entry rates of plants and output over the business cycle when an exogenous technology shock hits an economy.

This paper is based on the model of Campbell [3], who observes the correlations among exit and entry rates of U.S. manufacturing plants and their output growth rates over the business cycle. He notes that the entry rate covaries positively with the contemporaneous output growth rate, and the exit rate leads both the entry rate and the output growth rate. To explain this pattern, his model is based on the phenomenon of exit and entry of plants and the premise of embodied technology growth. Business cycles occur when the embodied technology grows more or less than expected. While a positive shock to embodied technology growth speeds up the exit of marginal plants, more new plants are created and are embodied with higher technology. After some time, when these new plants enter the economy, output grows faster than the trend rate.

This paper departs from Campbell [3] to discuss how tax policies affect the economy in a vintage capital model and with exit and entry of plants. We introduce a government that engages in government expenditure and pays for this by taxing capital income, labor income, and/or property. The design of the exit and entry of plants makes it possible to discuss the effect of a property tax. In our model the plant value can be taxed when ownership of the plant is traded between consumers and firms. When plant value is taxed, it becomes lower and marginal plants whose value is now lower than the scrap value exit the industry. Cooley, Greenwood, and Yorukoglu [5] also discuss tax policies, but their main focus is on the effects of a capital-income tax and an investment credit on the compositions of the capital stock and welfare in the economy. In our model, government expenditure is assumed to be fixed in the steady state, and tax rates are chosen to satisfy the government budget constraint. There is no debt issued. The government attempts to maximize social welfare by finding optimal combinations of the three tax rates.

Our results show that in a stationary economy, given a fixed level of government expenditure and a zero property tax rate, the optimal capital-income tax rate may be negative, zero, or positive dependent on the level of government expenditure. We find that for many values of government spending the highest level of steady-state utility occurs with a subsidy to capital income and a tax on labor income. Finally, we find that when taxes on capital income, labor income, and property are available, a positive capital-income tax is generally the last resort to finance government expenditures.

One contribution of this paper is to further expand our knowledge about the optimal capital-income tax. Chamley [4] tells us that in the long run, the capital income tax rate should be zero. From Aiyagari [1], we know that with idiosyncratic shocks to income and a market structure of incomplete insurance, the capital-income tax rate may be positive because positive capital-income tax rates can prevent consumers from overly accumulating capital for precautionary motives. We use a vintage capital model of the exit and entry of plants to calculate the optimal capital-income tax rate and find that this rate is not

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generally zero. In addition, this paper also contributes to the knowledge of the optimal way to finance government expenditures by using capital-income, labor-income, and property taxes.

This paper is organized as follows: Section 2 describes the model economy; Section 3 discusses the optimal tax policies in the steady state by examining our simulation results; Section 4 concludes.

2 The Model Economy

2.1 The Story

In Campbell's [2] model, the private sector consists of consumers, firms, and plants. Consumers, whose size will later be normalized to one, are identical in ability and preference and are each endowed with same amount of initial wealth and one unit of time per period. In each period, consumers maximize utility by allocating their time between work and leisure and their wealth plus income between consumption and investment in the ownership of plants.

The ownership of plants are traded between consumers and firms. Ownership is purchased by consumers from firms at the end of the period, and is sold back to firms at the beginning of the next period. In other words, plants are held by consumers between periods, and by firms over periods. Plants are either operating plants or developing plants. Operating plants are capable of production, while developing plants are in need of more development (i.e. investment spending) before starting to produce. There are also two types of firms. At the beginning of the period, production firms purchase operating plants and manage them to produce the final good, while investment firms purchase developing plants and further develop them. In addition, production firms scrap unproductive plants after the production period, while investment firms create new plants. At the end of the period, all existing plants are sold to consumers. The timing of these events within a period is shown in Figure 1.

Operating plants differ in productivity. The initial productivity of an operating plant is partly decided by new capital created when the plant is built and is partly decided by the degree of success in using the capital with labor to produce the final good a few periods later when the development process is over.[1] The final good produced by an operating plant can then be used as consumption good or new capital. The idea of vintage capital in this model allows for new capital created in each period to be endowed with the most advanced technology available in that period. This frontier technology is assumed to grow exogenously. Capital, once equipped inside a plant, cannot be replaced or upgraded until the plant is scrapped [2], so only new plants are equipped with new capital which in turn

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[1]In this section, the length of this whole development process is assumed for illustrative purpose to take two periods and to be fixed across all new plants and over time. In our quantitative experiment in the following section, the development process takes five periods to complete, where one period can be thought of as one quarter.

[2]Yorukoglu [21] assumes that a plant can either replace the whole capital stock or upgrade part of it.
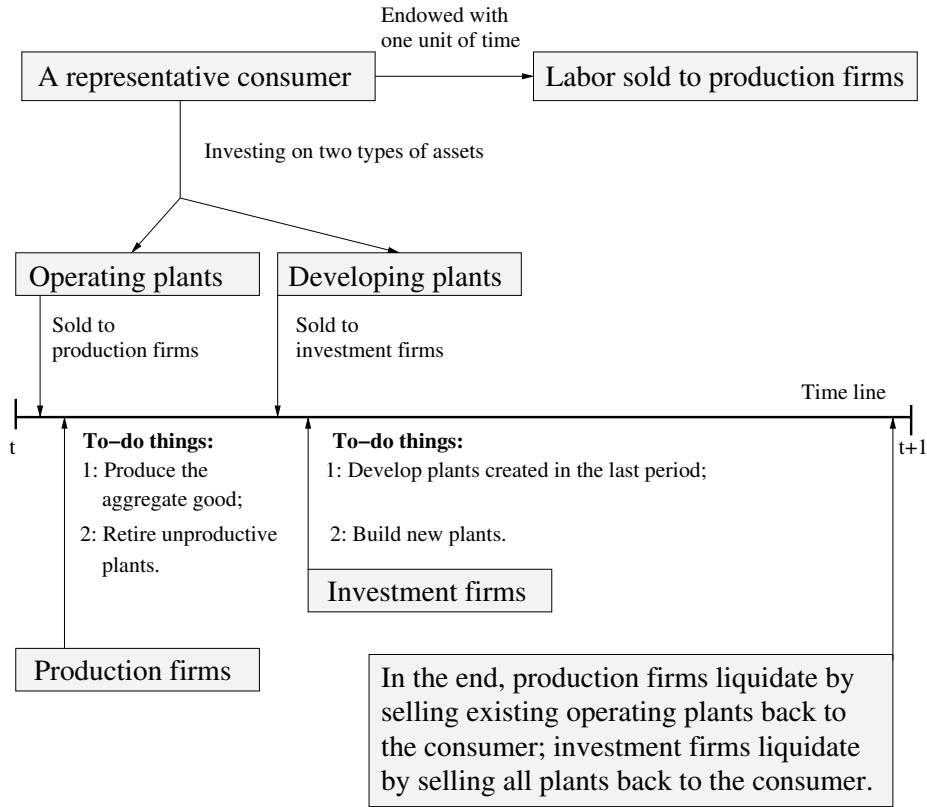
Figure 1: The Events in a Period

is embodied with the leading-edge technology. The subsequent evolution of productivity at an operating plant depends on the idiosyncratic productivity shock which hits the plant between periods, when consumers own the plant. [3] This shock is specific to individual plants, in contrast to the embodied technology shock which is specific to the whole economy and incorporated into all capital of a particular vintage. The average next-period productivity of an operating plant is expected to be as good as its current-period productivity, so the idiosyncratic productivity shock has mean zero.

In comparison with those in traditional models who earn capital income by renting their own capital to firms, consumers in this model earn capital income by selling their own operating plants to production firms. [4] What production firms earn from an operating plant is capital income from current production after paying labor and then the scrap value from plants to be retired and the resale value from the rest of the plants. Scrap value is measured in units of the final good, and in this model the units of final good per unit of capital in scrapped plants is assumed to be fixed. Because we assume free entry of firms, production firms compete with each other to the extent that each firm earns zero profit.

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How this assumption has an impact on the investment behavior of a plant over the plant's fixed lifetime is the focus in his paper.

[3] These shocks can be thought of as partly due to a random idiosyncratic depreciation.

[4] Consumers earn no capital income from a developing plant whose productivity remains undetermined until the plant enters the industry.

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This implies that the sum of all earnings from an operating plant at the end of the period should equal the purchase cost of the operating plant, the price that production firms pay to consumers for the right to operate the operating plant at the beginning of the period. In Campbell [3], trade in the ownership of plants is the mechanism for explicitly estimating the exit and entry of plants. This mechanism may look odd at first, but as a matter of fact, consumers in this model, just like those in traditional models, earn capital income in each period. Here that income is realized in the price paid and price received on the trade in operating plants. Note that investment firms also earn zero profit from developing plants, again because of free entry.

After adding the government sector to the original model, our focus is changed to study the effect of fiscal policy within this vintage-capital model with exit and entry of plants. [5] In this model the government is assumed to finance government expenditure by taxing capital income, labor income, and property. The last tax is collected through a tax on trading the ownership of operating plants. Since operating plants are traded twice between consumers and firms per period, it is assumed that the property tax is imposed on the production firms at the beginning of the period. The interesting part of the property tax lies on its influence on plant value, which was originally decided by the productivity of a plant, and on the exit threshold of productivity, which determines whether a plant survives through next period. Initially we conjecture that there might be a trade-off between the capital-income tax and the property tax for two reasons. First, compared to the tax base for the capital-income tax, the tax base for a property tax is large so that the property tax rate can be set at a relatively low rate compared to the capital-income tax rate. Second, a property tax may influence social welfare because it affects the exit threshold of productivity while capital-income tax does not. We assume the government budget constraint is balanced in each period, and there is no other instrument to pay for government expenditures. Our objective is to explore how capital-income, labor-income, and property taxes interact to decide the economy's long-run equilibrium. In the long run, the capital-augmenting economy grows at a fixed rate, and government expenditure is assumed to grow at the same rate as output. Therefore, after a growing economy is transformed to a stationary economy, government expenditure is again fixed at some level.

2.2 The Productivity of a Plant

This subsection explains the decision of the initial productivity of a plant and the evolution of its subsequent productivity until the plant is scrapped. The production of the final good needs capital and labor. An operating plant i faces a Cobb-Douglas production function with elements of effective capital and labor. Effective capital is the "real" unit of capital after its embodied technology is taken into account. At period t, the production function for the plant i is:

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[5]Cooley, Greenwood, and Yorukoglu [5] discuss a vintage-capital model in which capital has a fixed lifetime and firms decide the optimal time to retire a plant before the capital equipped inside of it reaches its end of lifetime.

$$y_{i,t} = (k_{i,t} \; e^{v_{i,t}})^{1-\alpha} \, n_{i,t}^{\alpha} \tag{1}$$

The capital stock of the plant, $k_{i,t}$, is fixed over the plant's lifetime, and its size is normalized to one because the plant's size is irrelevant. There is no limit on a plant's lifetime as long as it survives. The productivity of the plant is given by $e^{v_{i,t}}$, where e is the exponential function and $v_{i,t}$ is a random variable indicating the plant i's productivity at period t. The labor employed by the plant is $n_{i,t}$, and its output is $y_{i,t}$. The share of labor income is α and is the same across plants and over time. Effective capital is calculated as the product of the plant's capital, $k_{i,t}$, and the plant's productivity, $e^{v_{i,t}}$. Suppose that the productivity of the plant i at period t is $e^{v_{i,t}}$. Its productivity at period $t+1$ is described by Equation 2. Note that $v_{i,t}$ follows a random walk process. The innovation, $\varepsilon_{i,t+1}$, is one of the two idiosyncratic shocks in this model. It symbolizes the fluctuation of a plant's productivity. This shock is realized between periods, when consumers hold the ownership of plants.

$$v_{i,t+1} = v_{i,t} + \varepsilon_{i,t+1} \tag{2}$$

where

$$\varepsilon_{i,t+1} \sim N(0, \sigma^2) \tag{3}$$

To decide the initial productivity of a plant, we first denote the leading-edge technology as z and assume that this technology accumulates according to the following equation:

$$z_t = \mu + z_{t-1} + \varepsilon_t^z \tag{4}$$

where

$$\varepsilon_t^z \sim N(0, \sigma_z^2) \tag{5}$$

Intuitively, the drift, μ, should be positive. The shock to the leading-edge technology is ε_t^z. There are two factors deciding the initial productivity of a plant. One is the leading-edge technology, and the other one is the degree of success in operating the leading-edge technology, which is the second type of idiosyncratic shocks. For a plant being created at period t and entering the industry at period $t+2$, its initial productivity is a random draw from the following normal distribution with mean z_t (the leading-edge technology at t) and standard deviation σ_e.

$$v_{i,t+2} \sim N(z_t, \sigma_e^2) \tag{6}$$

All types of shocks, including the aggregate embodied technology shock and the two idiosyncratic shocks, are revealed between periods. Note that when the ownership of plants is traded at the beginning of the period, their new productivity is observable to both consumers and firms, while at the end of the period, it is not observable so that the resale price of plants is based on the expectation of the plant's productivity in the next period. Figure 2 illustrates the creation of a new plant before it enters the industry.
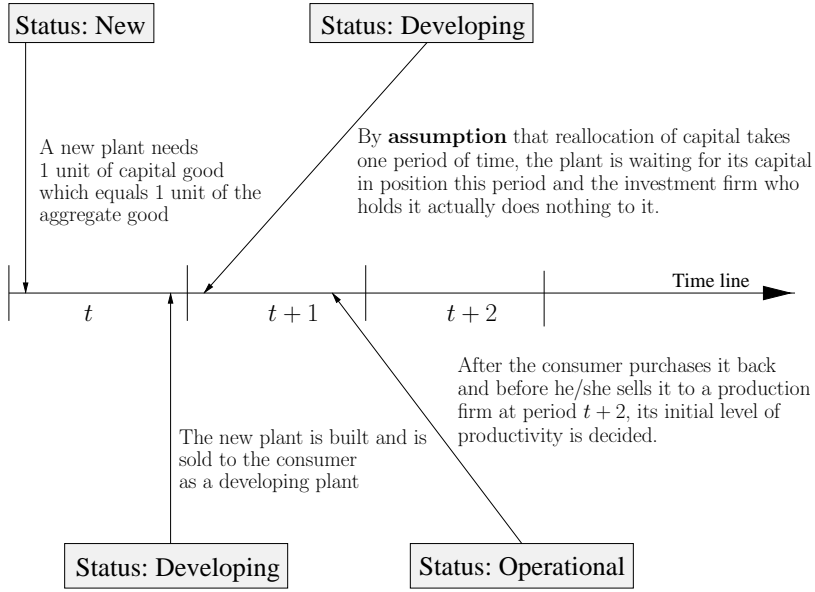
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Status: New          Status: Developing

                     By assumption that reallocation of capital takes
A new plant needs    one period of time, the plant is waiting for its capital
1 unit of capital good  in position this period and the investment firm who
which equals 1 unit of the  holds it actually does nothing to it.
aggregate good

                                                        Time line
         |          |          |          |
         t         t + 1      t + 2

                                    After the consumer purchases it back
                                    and before he/she sells it to a production
                                    firm at period t + 2, its initial level of
         The new plant is built and is   productivity is decided.
         sold to the consumer
         as a developing plant

         Status: Developing        Status: Operational
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Figure 2: The Creation of a New Plant

2.3 Decisions by Production Firms

Production firms have several decisions to make in each period. The first one is to decide the number of operating plants of different levels of productivity to purchase at the beginning of the period. Once this decision is made, these firms hire and allocate labor at the purchased plants, and after the production, they decide if any of the purchased plants should be retired and sell the rest back to consumers. A production firm's profit-maximizing objective function at period t is shown as follows:

$$
\max_{k_t(v_t), n_t(v_t), s_t(v_t)} \quad -(1 + \tau_t^p) \int_{-\infty}^{\infty} q_t^0(v_t) \ k_t(v_t) \ \mathrm{d}v_t
$$
$$
+(1 - \tau_t^c) \int_{-\infty}^{\infty} k_t(v_t) \ [\ e^{v_t(1-\alpha)} \ n_t(v_t)^\alpha - w_t \ n_t(v_t) \ ] \ \mathrm{d}v_t
$$
$$
+ \int_{-\infty}^{\infty} \eta \ s_t(v_t) \ k_t(v_t) \ \mathrm{d}v_t
$$
$$
+ \int_{-\infty}^{\infty} q_t^1(v_t) \ [\ 1 - s_t(v_t) \ ] \ k_t(v_t) \ \mathrm{d}v_t
$$

$$(7)$$

The first term is the total cost of purchasing operating plants of different productivity v_t that is paid to consumers at the beginning of period t. The value of a plant with a certain v_t at the beginning of period t is $q^0(v_t)$, and $k_t(v_t)$ is the number of such plants owned by the firm. The property tax that the firm pays for a plant is proportional to the plant's value and is τ_t^p. The second term is the net capital income that the firm earns after paying labor income, $w_t n_t(v_t)$. The real wage is w_t, and $n(v_t)$ is the labor allocated at the plant with v_t. The output of the plant with v_t is $e^{v_t(1-\alpha)} n_t(v_t)^\alpha$, where the plant's capital stock

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is normalized to one. The tax rate on capital income is τ_t^c. The third term is the scrap value earned from scrapped plants. The scrap value is assumed to be η per unit of capital and to be less than one. The proportion $s(v_t)$ means the percentage of plants with v_t to be scrapped after current production. The last term is the resale value earned from the existing plants. The value of the plant with v_t at the end of the period is $q^1(v_t)$. Note that except the first term, the other terms are income earned after production.

A production firm solves its problem in two steps. First, taking the labor hired by the firm, n_t, as fixed, the firm decides how to allocate its labor into their purchased plants of different productivity. The firm's problem is as follows:

$$\max_{n_t(v_t)} \int_{-\infty}^{\infty} k_t(v_t) \left[e^{v_t(1-\alpha)} \, n_t(v_t)^{\alpha} \right] dv_t \tag{8}$$

subject to

$$\int_{-\infty}^{\infty} k_t(v_t) \, n_t(v_t) \, dv_t \leq n_t \tag{9}$$

As shown in Solow [18], after the firm's effective capital is defined as $\bar{k}_t = \int_{-\infty}^{\infty} k_t(v_t) \, e^{v_t} \, dv_t$, a solution for $n_t(v_t)$ can be easily achieved as follows:

$$n_t(v_t) = \frac{n_t \, e^{v_t}}{\bar{k}_t} \tag{10}$$

The solution for $n_t(v_t)$, which decides the amount of labor hired at a plant with a certain v_t, depends on two factors, the labor-effective capital ratio and the plant's productivity. Because production firms are identical, each firm's labor-effective capital ratio is the same as that for the economy.

At the second step, after $n_t(v_t)$ is replaced with Equation 10, the firm's aggregate output can be simplified as the Cobb-Douglas production function, and its objective function becomes:

$$\max_{k_t(v_t), n_t, s_t(v_t)} \quad -(1 + \tau_t^p) \int_{-\infty}^{\infty} q_t^0(v_t) \, k_t(v_t) \, dv_t + (1 - \tau_t^c) [\, \bar{k}_t^{1-\alpha} \, n_t^{\alpha} - w_t \, n_t \,]$$
$$+ \int_{-\infty}^{\infty} \eta \, s_t(v_t) \, k_t(v_t) \, dv_t + \int_{-\infty}^{\infty} q_t^1(v_t) \left[\, 1 - s_t(v_t) \, \right] k_t(v_t) \, dv_t \tag{11}$$

Next, the firm derives its first-order conditions with respect to $k_t(v_t)$, n_t, and the proportion of plants with productivity v_t to be scrapped, $s_t(v_t)$.

$$w_t = \alpha \, [\, \frac{\bar{k}_t}{n_t} \,]^{1-\alpha} \tag{12}$$

$$q_t^1(\underline{v}_t) = \eta \tag{13}$$

$$(1+\tau_t^p)\, q_t^0(v_t) = (1-\tau_t^c)\,(1-\alpha)\, e^{v_t}\, (\frac{\bar{k}_t}{n_t})^{-\alpha} + 1\,\{\, v_t < \underline{v}_t \,\}\, \eta + 1\,\{\, v_t \geq \underline{v}_t \,\}\, q_t^1(v_t) \quad (14)$$

The first equation decides the labor hired by the firm. The second equation sets the exit threshold of productivity, $\underline{v}_t$, such that plants of this specific productivity are indifferent between operation and shutdown after the production. The plant value, which equals the expected sum of a stream of revenue earned from output net of wages paid to labor, rises with the plant's productivity, so a plant with productivity higher than $\underline{v}_t$ survives through the next period and exits otherwise. The third equation implies that the optimal purchase of a plant with productivity v_t yields zero profit. The term on the LHS is the purchase cost of the plant with v_t, and the three terms on the RHS are the after-tax capital income, the scrap value if the plant is scrapped, and the resale value if the plant is sold back to consumers, respectively. $1\,\{\cdot\}$ is an indicator function, which takes the value of 1 when the plant with v_t survives and 0 otherwise.

2.4 Decisions by Investment Firms

Investment firms make two decisions each period. The first one is to decide the number of developing plants to purchase at the beginning of the period. Investment firms purchase developing plants and further develop them. Meanwhile, these firms build new plants. At the end of the period, investment firms sell their plants to consumers. An investment firm's objective function at period t is presented as (15):

$$\max_{x_t(0),x_t(1)} [\, q_t^{1i}(1) - q_t^{0i}(0) \,]\, x_t(0) + [\, q_t^{1i}(2) - q_t^{0i}(1) \,]\, x_t(1) \quad (15)$$

The number of plants, $x_t(0)$, is created at period t, and $x_t(1)$ is the number of plants created at period $t-1$. The first term is the profits earned from creating new plants. The second term is the profits earned from further developing plants created in the last period. The resale value of a plant developed for j periods is $q_t^{1i}(j)$. The cost of creating a new plant is $q_t^{0i}(0)$, and equals one because the capital stock of a plant is normalized to one and the transformation from consumption good to capital is one-to-one. The purchase cost of a plant created in the last period is $q_t^{0i}(1)$. The zero-profit condition implies the following conditions:

$$q_t^{1i}(1) = q_t^{0i}(0) = 1 \quad (16)$$

$$q_t^{1i}(2) = q_t^{0i}(1) \quad (17)$$

2.5 Decisions by Consumers

Consumers are identical in preference. They consume the final good and invest in the portfolio consisting of the ownership of plants with different productivity. They are each

endowed with one unit of time to allocate between leisure and work. Utility comes from consumption and leisure. The problem for a consumer j at period s is shown as follows:

$$\max_{c_t^j, n_t^j, i_t^{1j}, i_t^{2j}, k_t^j(v_t)} E_s \left(\sum_{t=s}^{\infty} \beta^t u(c_t^j, 1 - n_t^j) \right) \tag{18}$$

where

$$u(c_t^j, 1 - n_t^j) = \ln c_t^j + \kappa (1 - n_t^j) \tag{19}$$

subject to

$$c_t^j + q_t^{1i}(1) i_t^{1j} + q_t^{1i}(2) i_t^{2j} + \int_{-\infty}^{\infty} q_t^1(v_t) k_t^{1j}(v_t) dv_t =$$
$$(1 - \tau_t^w) w_t n_t^j + q_t^{0i}(1) i_{t-1}^{1j} + \int_{-\infty}^{\infty} q_t^0(v_t) k_t^{0j}(v_t) dv_t \tag{20}$$

$$k_{t+1}^{0j}(v_{t+1}) = \int_{-\infty}^{\infty} \frac{1}{\sigma} \phi(\frac{v_{t+1} - v_t}{\sigma}) k_t^{1j}(v_t) dv_t + \frac{1}{\sigma_e} \phi(\frac{v_{t+1} - z_{t-1}}{\sigma_e}) i_t^{2j} \tag{21}$$

β is the discount factor. There are two constraints. The first one is the budget constraint. At the beginning of period t, the consumer sells his/her plants including operating plants and developing plants created in the last period, i_{t-1}^{1j}, to firms. The number of plants with a certain productivity v_t owned by the consumer j is denoted as $k^{0j}(v_t)$. At the end of period t, the consumer earns labor income, $w_t n_t^j$, and invests in new plants that are created in the current period, i_t^{1j}, the developing plants that are created in the last period and are going to enter the market in the next period, i_t^{2j}, and the surviving operating plants of different productivity, $k^{1j}(v_t)$. The second constraint shows the evolution of the next-period productivity of the plants invested by the consumer. The first term on the RHS is the expected number of the purchased operating plants having a certain productivity in the next period, where σ is the standard deviation of v and $\frac{1}{\sigma}\phi(\cdot)$ is the normalized standard deviation of v. The second term on the RHS is the number of plants that are ready to enter the market and will have the same productivity in the next period, where σ_e is the standard deviation of z and $\frac{1}{\sigma_e}\phi(\cdot)$ is the normalized standard deviation of z. The first-order conditions for the consumer j are:

$$q_t^1(v_t) = E_t [\beta \frac{c_t^j}{c_{t+1}^j} \int_{-\infty}^{\infty} \frac{1}{\sigma} \phi(\frac{v_{t+1} - v_t}{\sigma}) q_{t+1}^0(v_{t+1}) dv_{t+1}] \tag{22}$$

$$q_t^{1i}(2) = E_t [\beta \frac{c_t^j}{c_{t+1}^j} \int_{-\infty}^{\infty} \frac{1}{\sigma_e} \phi(\frac{v_{t+1} - z_{t-1}}{\sigma_e}) q_{t+1}^0(v_{t+1}) dv_{t+1}] \tag{23}$$

$$q_t^{1i}(1) = E_t [\beta \frac{c_t^j}{c_{t+1}^j} q_{t+1}^{0i}(1)] \tag{24}$$

$$\kappa = (1 - \tau_t^w) \ \frac{w_t}{c_t^{\bar{j}}} \tag{25}$$

The first equation decides $k^{1j}(v_t)$, the number of surviving operating plants with productivity v_t. The second equation decides i_t^{2j}, the number of plants created in the last period and starting to produce in the next period. The third equation decides i_t^{1j}, the number of plants created in the current period. The fourth equation decides the labor supply. The labor-income tax rate is τ_t^w.

2.6 Decisions by Government

The government taxes capital income, labor income, and property to finance government expenditure. Government expenditure does not enter the utility function. [6] There is no borrowing or transfer payment from the government to consumers. In equilibrium, government expenditure grows as fast as output so that it remains a constant proportion of economy-wide output. The government budget constraint is shown in Equation 26.

$$G_t = \tau_t^w \ w_t \ N_t + \tau_t^c \ (\ \bar{K}_t^{1-\alpha} \ N_t^\alpha - w_t \ N_t \ ) + \tau_t^p \ \int_{-\infty}^{\infty} q_t^0(v_t) \ K_t(v_t) \ \mathrm{d}v_t \tag{26}$$

where

$$Y_t = \bar{K}_t^{1-\alpha} \ N_t^\alpha \tag{27}$$

Denote government expenditure as G. The N, $\bar{K}$, $K(v)$, and Y are labor, effective capital, number of plants with productivity v, and output at the aggregate level, respectively. The government investigates the effect of various combinations of taxes on capital income, labor income, and property on the economy's long-run equilibrium given a fixed path of government expenditure.

This economic system includes the first-order conditions of production firms (Equations 12, 13, 14), investment firms (Equations 16, 17), and consumers (Equations 22, 23, 24, 25). In addition, there are two constraints of consumers (Equations 20, 21), one definition of the effective capital, one government budget constraint (Equation 26), and two aggregate resource constraints (Equations 28, 29) as follows:

$$C_t + I_t^1 + G_t = Y_t + \eta \ \int_{-\infty}^{v_t} K_t(v_t) \ \mathrm{d}v_t \tag{28}$$

$$I_{t+1}^2 = I_t^1 \tag{29}$$

Aggregate consumption is C and aggregate investment in the creation of new plants is I^1. The second term on the RHS of the first equation is the units of the final good that is

transformed from scrapped capital. Note that investment defined in the traditional way is either $Y_t - C_t - G_t$ or, equivalently, $I_t^1 - \eta \int_{-\infty}^{v_t} K_t(v_t) \, dv_t$. The second constraint ensures that all new plants enter the industry after the development process is completed. All markets are cleared. In the next section, we examine our simulation results and discuss how tax policies affect the economy's long-run equilibrium.

3 The Steady-state Economy

3.1 Parameters

Given government expenditure and tax rates on capital income, labor income, and property, there are some parameters left to be defined. These parameters include the labor-income share, α, the subjective discount factor, β, the scrap value, η, the average growth rate of embodied technology, μ, the standard deviation of the shock to the productivity of an incumbent plant, σ, the standard deviation of the shock to the initial productivity of a new plant, σ_e, the standard deviation of the shock to embodied technology growth rate, σ_z, and the marginal utility of leisure, κ. Table 1 lists the values for these parameters as used in Campbell [3]. Note that because the labor, N, is estimated as 0.26 from the real data, the marginal utility of leisure, κ, can be derived with other steady-state values under no-government scheme. Following that, we perform our experiment on the effects of various combinations of tax rates on the economy by fixing this derived κ and treating N as a variable.

Table 1: Parameter Values

α	β	η	μ	σ	σ_e	σ_z
0.66	0.9926	0.835	0.0066	0.03	0.36	0.0059

3.2 Optimal Tax Policies in the Transformed Model Economy

The goal of this section is to derive some interesting observations on social welfare in the steady state [7] from the simulation results of the quantitative experiment. This experiment will proceed in three steps, involving various combinations of capital-income, labor-income, and property tax rates. The first step, by assuming zero property tax rate, the government looks for the optimal combination of capital-income and labor-income tax rates that maximizes consumers' utility, taking government expenditure as fixed. This part of the experiment will provide information on the optimal tax rate on capital for this model. Next, we are curious to know if using a property tax instead of a capital-income tax is preferable for the economy. Following that, we allow all three types of tax rates to change in order to examine the combination that yields maximum utility for consumers.

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[7]The appendix shows how the model economy is transformed into a stationary one.

As a basis for comparison, we derive the steady-state values for our economy without government. This provides a baseline economy in which the resource allocation is not distorted by any policy instrument. Provided the above parameters, the distribution of plant productivity in the steady state is illustrated in Figure 3, where the x-axis is labeled as percentage difference between plant productivity and productivity brought by the leading-edge technology and the y-axis is labeled as units of plants of different levels of productivity. [8] As seen in Figure 3, this distribution of plant productivity is right-skewed, and the corresponding mode is around negative 25-30 percent. The average plant productivity is 6.6 percent less than the leading-edge level.
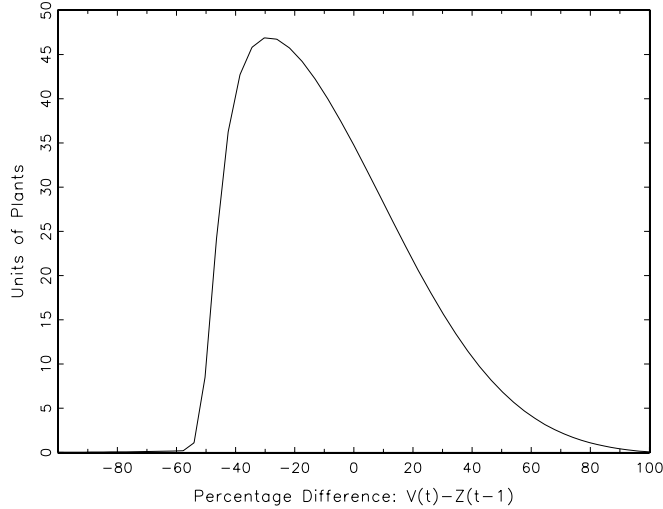

Figure 3: Distribution of Plant Productivity in the Steady State

The derived steady-state aggregate and other relevant values are listed in Table 2. Given the labor income share weight of $\frac{2}{3}$, output is 1.2649, created by the Cobb-Douglas production technology with inputs of effective capital, 29.9368, and labor, 0.26. Investment in new plants, 0.5077, net of scrapped capital, is net investment, 0.1762, so that consumption is 1.0887. Utility, which comes from consumption and leisure, is 2.2895. Without considering embodied technology, K^* is aggregate capital, the sum of all physical units of capital, and is 31.0523, which is larger than effective capital. The exit threshold, denoted as $\underline{u}$, provides a guide for plants with lower productivity to exit the market and is -49.51 percent. From this we derive the exit and entry rates. The exit rate is the ratio of scrapped capital over aggregate capital, while the entry rate is the ratio of new capital entering into the market over aggregate capital. These values are 1.07 percent and 1.61 percent, respectively. Represented as κ, the marginal utility of leisure is treated as a parameter. Given the parameters in Table 1 and N of 0.26, the parameter κ is equal to 2.9790.

The ranges for government expenditure and tax rates are to be decided before this quan-

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[8]This distribution is approximated by 71 abscissas, and the reason to choose this particular number is that singularity occurs with numbers larger than 71 in the process of deriving the structure form of this model.

Table 2: Values in the Steady State without Government

$\bar{K}^*$	N	Y^*	I^*	C^*	U^*
29.9368	0.2600	1.2649	0.1762	1.0887	2.2895

K^*	I^{*1}	$\underline{u}$	Exit Rate	Entry Rate	κ
31.0523	0.5077	-49.51%	1.07%	1.61%	2.9790

titative experiment begins. Because the government fixes its expenditure before collecting tax revenue, we set government expenditure first. Since the steady-state output in the case of no government is 1.2649, we chose the set of potential government expenditure to be $\{$ 10^{-9}, 0.1, 0.2, 0.3, 0.4 $\}$. In the subsequent grid search for the optimal combinations of capital-income, labor-income, and property tax rates, subsidies on capital income and property are also considered, again subject to the balanced budget constraint. It is of interest to see if there are benefits to any of these subsidies in the long run. Tax rates on capital income, labor income, and property arbitrarily range from [-50%, 50%], [0%, 68%], and [-1%, 1%], respectively. The reason of property tax rates being narrowed between positive/negative 1 percent is because of the relative large tax base on plant value compared to firms' capital income. The increment searched for all three taxes is 0.01 percent.

To be able to estimate excess burden later, a lump-sum tax is also considered, and consumption and utility are derived accordingly. Since the lump-sum tax is a non-distorting tax instrument that doesn't distort the decisions of investment and work hours, it is simply collected via consumption. Table 3 shows these values. Consumption, 1.0887, is directly reduced for different levels of government expenditure.

Table 3: Consumption and Utility in the Steady State with Lump-sum Tax

G^*	10^{-9}	0.1	0.2	0.3	0.4
C^*	1.0887	0.9887	0.8887	0.7887	0.6887
U^*	2.2895	2.1931	2.0865	1.9671	1.8315

Enter the first part of this experiment, where property tax rate is set to zero. Results for five different levels of government expenditure are illustrated respectively in Figures 4, 5, 6, 7, and 8. Each figure contains eight small figures, where the x-axis is capital-income tax rate shown in percentage term. In the left top figure is the corresponding labor-income tax rate with which the capital-income tax rate is used to derive steady-state values. Among others, except the left figure in the second row, the y-axis represents the percentage difference of variables derived with distortionary taxes from those derived with lump-sum tax. As for the figure left, because the ratio of government expenditure to output is itself percentage, its y-axis is simply the difference between the ratio derived with distortionary taxes and

that derived with lump-sum tax. Besides this, only consumption and utility are influenced by lump-sum tax, therefore most comparisons at this stage are made between steady states with distortionary taxes and without tax except the figures in the bottom row.

Figure 4 shows the results derived under different combinations of capital-income and labor-income tax rates given government expenditure at 10^{-9}. In the top left figure, when capital-income tax rate rises from -50 percent to a zero percent, labor-income tax rate slides from 25 percent to a near-zero percent. The x-axis stops at zero because we do not consider a subsidy on labor income. The relationship between these two rates is almost linear, implying that they are nearly perfectly substitutable when government expenditure is near zero. Next, output in the top right figure shapes concavely with both its starting and ending points staying close to zero (from below). The concavity comes from the opposite movement of effective capital and labor. In this case, when capital-income tax rate is negative, output (almost) always performs better than that derived without tax and reaches its peak about 1.95 percent more as capital-income tax rate is about -25 percent.

Because government expenditure is fixed and output is concave in the capital-income tax rate, the ratio of government expenditure to output in the left figure in the second row is convex. It starts at a near-zero point, falls to -1.5×10^{-9} percent when capital-income tax rate hits -25 percent, and then rises back to a near-zero percent. The curves of effective capital and (net) investment are identical, and their relationship with capital-income tax is negative. As the capital-income tax rate falls by one percent, effective capital and investment grow by the same magnitude. Normalized labor in the left figure in the third row drops from a near-zero percent to almost 19 percent as capital-income tax rate falls from a zero percent. This illustrates that, if capital-income subsidy rate is up by 2 percent, the corresponding 1-percent increase in the labor-income tax rate brings about a 0.75-percent reduction of labor force. The bottom two figures show that, with a negative capital-income tax rate, consumption is always worse compared to the situation when no government exists. On the contrary, utility is always higher because the increase in leisure makes up for the loss in consumption. This result is of course dependent in part on the assumed utility function.

Observation 1: When government expenditure is close to zero, a capital-income subsidy may be beneficial to the economy in the steady state.

Figures 5, 6, 7, and 8 show that the higher the government expenditure, the more convex the relationship between capital-income and labor-income taxes. This convexity implies that, when G^* is 0.2 or more, tax revenue lost by lowering the capital-income tax rate 1 percent needs to be filled in by using a larger increase in the labor-income tax rate than was necessary when the capital-income tax rate was originally lower. The output effect is positive only when government expenditure is infinitesimally small, and it turns out to be negative when government expenditure grows high enough. This change is due to the downward shifts in both effective capital and labor. Lowering the capital-income tax rate stimulates investment when G^* is 0.3 or less. Utility shifts downward with higher

government expenditure although leisure does increase to offset part of the effect of reduced consumption. When G^* is 0.3 or more, a negative capital-income tax rate harms social welfare. Lastly, because output decreases with higher government expenditure, the ratio of government expenditure becomes positive.

The following table lists the optimal values in the steady state derived with different levels of government expenditure, zero property tax rate, and a specific combination of capital-income and labor-income tax rates. Note that while the above figures illustrate differences from baseline steady-state values, Table 4 presents steady-state values for the economy and not differences from a baseline.

Table 4: Values in the Steady State with Distortionary Taxes ($\tau_p = 0$)

G^*	$\tau_c(\%)$	$\tau_w(\%)$	Y^*	$\frac{G^*}{Y^*}(\%)$	$\bar{K}^*$	N	I^*	C^*	U^*
10^{-9}	-45.78	22.89	1.2719	7.86E-08	43.8822	0.2165	0.2582	1.0136	2.3475
0.1	-31.23	27.88	1.2230	8.18	37.9853	0.2194	0.2235	0.8995	2.2193
0.2	-15.83	33.66	1.1653	17.16	31.9467	0.2226	0.1880	0.7773	2.0641
0.3	0.99	40.64	1.0938	27.43	25.6325	0.2260	0.1508	0.6430	1.8643
0.4	20.82	49.86	0.9955	40.18	18.6566	0.2300	0.1098	0.4858	1.5719

Table 4 reveals several things about maintaining optimality with higher government expenditure. First, when government expenditure, G^*, is low (0.2 or less in our simulation), the optimal capital-income tax rate, τ_c, is negative. As G^* rises (here 0.3 or more), τ_c is positive. Second, both τ_c and the optimal labor-income tax rate, τ_w, increase with G^*. Third, the increase in τ_w results in more labor as G^* rises. Note that as G^* is fixed, rising labor-income tax rate still discourages labor. Fourth, among other variables, compared with steady-state values derived without government, effective capital, $\bar{K}^*$, and investment, I^*, are higher when G^* is 0.2 or less due to capital subsidies, and labor, N, derived with any level of government expenditure, is lower due to positive labor-income tax rate. Fifth, consumption in this table is lower than that derived with lump-sum tax in Table 3. Lastly and the most interestingly, utility derived when G^* is 0.1 or less is greater than that when G^* is financed via lump-sum tax.

What causes this effect in this model? This effect seems to be a manifestation of intertemporal optimization resulting in a steady-state capital stock below the Golden Rule. A set of taxes and subsidies can generate higher steady-state capital-labor ratios and greater steady-state utility, but it is not optimal for this to occur starting from an initial steady state without taxes. Because of discounting, the initial decline in consumption and utility necessary to support the increased investment required to accumulate the increased capital stock is too costly relative to the gain in steady-state utility.

Observation 2: Capital subsidies can improve steady-state social welfare when govern-

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ment expenditure is low enough.

The goal of this second part of the experiment is to provide a policy comparison for the government who considers the possibility of choosing one of either a capital-income tax or a property tax as the policy instrument in addition to a labor-income tax. Like Table 4, Table 5 summarizes the property and labor-income tax rates that maximize utility and the corresponding steady-state values derived with different government expenditure.

Compare Table 5 with Table 4. As G^* rises, τ_w rises as does property tax rate, τ_p, which is much smaller than τ_c. In all cases, the optimal way for the government to finance its expenditure is to subsidize capital income or property when G^* is low and raise taxes on labor income and on capital income or property as G^* rises until eventually the tax rate on capital income or property becomes positive. Behavior of other variables are similar as G^* rises. Utility is also higher when G^* is 0.1 or less than that derived with lump-sum tax. Subsidizing property lowers the purchase cost paid by production firms at the beginning of the period and therefore raises these firms' incentive to purchase more plants to produce. Lastly, it is not obvious that choosing one of either a capital-income tax or a property tax creates more utility than choosing the other.

Table 5: Values in the Steady State with Distortionary Taxes ($\tau_c = 0$)

G^*	$\tau_p(\%)$	$\tau_w(\%)$	Y^*	$\frac{G^*}{Y^*}(\%)$	$\bar{K}^*$	N	I^*	C^*	U^*
10^{-9}	-0.41	22.89	1.2557	7.96E-08	42.9436	0.2147	0.2466	1.0092	2.3485
0.1	-0.31	27.74	1.2132	8.24	37.3607	0.2186	0.2158	0.8974	2.2195
0.2	-0.18	33.66	1.1605	17.23	31.6973	0.2220	0.1846	0.7759	2.0638
0.3	0.02	40.38	1.0946	27.41	25.5321	0.2267	0.1502	0.6444	1.8644
0.4	0.36	49.50	1.0013	39.95	18.7749	0.2312	0.1120	0.4894	1.5755

The third part of this experiment relaxes the limits on capital-income and property tax rates by allowing both of them to move simultaneously and expands the set of capital-income tax rate to [-100%, 100%] to match that of the property tax rate as much as possible. The relevant results are illustrated in Figures 9, 10, and 11.

In Figure 9, the top figure features the change in the exit threshold as the property tax rate rises. As shown in Equation 14, capital-income tax affects consumers' willingness to invest, but its change does not affect the exit threshold of productivity. On the contrary, for a production firm, a higher property tax rate seemingly increases the purchase cost of a marginal plant at the beginning of the period, $(1 + \tau_p)q_t^0(\underline{v}_t)$, but it also reduces the plant' value, $q_t^0(\underline{v}_t)$, which in turn reduces the plant's resale value at the end of the period, $q_t^1(\underline{v}_t)$. According to Equation 13, when this marginal plant's resale value is less than η, the exit threshold must rise to satisfy Equation 14 again, which in turn raises exit and entry rates. In the top figure, when the property tax rate is negative, the increase in the exit threshold

is steep, but when the property tax is positive, the increase in the exit threshold is less steep. The bottom figure shows the same trend. From a tax rate of -1 percent to 0 percent, the exit rate rises by 0.54 percent and the entry rate rises by 0.65 percent. As the tax rate varies from 0 percent to 1 percent, the exit and entry rates rise by 0.32 and 0.39 percent, respectively. [9]

The negative relationship between property and capital-income taxes is shown in Figure 10, where government expenditure is 10^{-9}. The capital-income tax rate falls from 72.67 percent to -100 percent when the property rate rises from -1 percent to 0.52 percent. Output, effective capital, labor, investment, and consumption illustrate the trend inherited from the exit threshold. Unlike the above aggregate variables rising over the entire set of property tax rate, utility mildly falls after property tax rate hits -0.21 percent, compared to its previous rising. The corresponding capital-income and labor-income tax rates that maximize utility are, respectively, -22.78 percent and 23.02 percent. The figure of the ratio of government expenditure to output looks empty because of the extremely low ratio. Another figure to compare with Figure 10 is Figure 11, where government expenditure is 0.4. In Figure 11, the shape of the ratio of government expenditure to output is symmetric to that of output. Capital-income tax rate, ratio of government expenditure to output, and labor move upwards, while others move downwards. The rising trend for utility extends until property tax rate reaches 0.77 percent, where the corresponding capital-income and labor-income tax rates are -24.38 percent and 49.74 percent, respectively.

Table 6 records the optimal combinations of capital-income, labor-income, and property tax rates and the corresponding steady-state values. As before, the optimal labor-income tax rate, τ_w, increases with government expenditure, G^*. While the optimal property tax rate, τ_p, also shows the same pattern as that of τ_w, the optimal capital-income tax rate, τ_c, decreases from -22.78 percent to -28.33 percent and then bounces back to -24.38 percent when G^* is 0.4. This result tells several things. First, when the government can avail itself of all three taxes, τ_c is negative over a reasonable range of G^*. Second, although τ_c falls as G^* rises and starts to rise after G^* reaches some threshold, (somewhere between 0.3 and 0.4 in this case), τ_c stays negative and overall is relatively stable, compared with τ_p and τ_w. Third, the optimal way for the government to finance higher G^* is through τ_p and τ_w rather than τ_c unless G^* is too high. In other words, when more marginal plants are forced to exit the market by higher property tax rate, capital subsidy is used instead to encourage the incentive to invest. Fourth, as the previous cases, utility is higher than that derived with lump-sum tax again when G^* is 0.1 or less.

Observation 3: When capital-income, labor-income, and property taxes are available to

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[9]Note that the steps in the bottom figure come from the limited ability to use more abscissas caused by a singularity when the matrix decomposition is performed. With more abscissas, spans, created by the Gauss-Legendre N-point quadrature formula between negative/positive one to simulate the distribution of the percentage difference in plant's productivity and the leading-edge productivity, become narrower. With narrower spans, the exit threshold crosses one span to another more easily so that exit and entry rates rise more quickly until steps are finally replaced by a curve. However, since the problem of singularity exists, simply connecting the middle point of each step to make it a curve may reduce confusions.

the government, a positive capital-income tax rate should be the last resort for use when government spending is high.

Table 6: Values in the Steady State with Distortionary Taxes

G^*	$\tau_p(\%)$	$\tau_c(\%)$	$\tau_w(\%)$	$\frac{G^*}{Y^*}(\%)$	$\bar{K}^*$	N	C^*	U^*
10^{-9}	-0.21	-22.78	23.02	7.92E-08	43.4442	0.2151	1.0109	2.3492
0.10	-0.06	-25.33	27.93	8.20	37.8242	0.2188	0.8985	2.2203
0.20	0.14	-27.88	33.65	17.15	32.0056	0.2226	0.7778	2.0646
0.30	0.40	-28.33	40.59	27.30	25.8504	0.2266	0.6448	1.8652
0.40	0.77	-24.38	49.74	39.82	19.0033	0.2309	0.4892	1.5762

4   Conclusion

In this paper, given various combinations of transformed government expenditure and tax rates, a steady-state transformed economy where, except idiosyncratic shocks, there is no other types of shocks at all, is solved. A comparison is further made among different sets of optimal steady-state values, and the conclusions are reached to provide some guides of optimal taxation for the government. They are summarized as follows:

• In the case of zero property tax rate, when government expenditure is close to zero, raising the capital-income subsidy may be beneficial to the economy in the steady state. As a matter of fact, the optimal capital-income tax rate may be negative, zero, or positive, depending on the magnitude of government expenditure.

• Capital subsidies can improve steady-state social welfare when government expenditure is low enough.

• When capital-income, labor-income, and property taxes are available to the government, a positive capital-income tax rate should be the last resort for use when government spending is high.

A   Appendix

A.1   Approximating Integrals

To compute the steady-state values, we need to approximate the integrals appearing in various equations. These integrals mainly come from the distribution of the number of plants with different productivity and the corresponding distribution of the plant value. The method we use is the Gaussian quadrature approximation, which was also used in Campbell [2]. To illustrate the idea of Gaussian quadrature approximation, suppose there is a function as follows:

$$g(y) = b(y) + \int_a^b A(y, x)g(x)\mathrm{d}x \tag{30}$$

As there is no analytic expression for the integral on the right side of the equation, this method approximates the integral with a weighted sum as follows:

$$g(y) \approx b(y) + \sum_{i=1}^{N} \omega_i(a, b) A(y, x_i)g(x_i) \tag{31}$$

The abscissas x_i's and weights ω_i's are decided by the Gauss-Legendre N-point quadrature formula. Each abscissa, x_i, is located between a and b and is assigned its weight, ω_i. There are N chosen abscissas, $(x_1, x_2, \ldots, x_N)$ to replace y in the equation. With a system of N equations consisting of $g(x_1), g(x_2), \ldots, g(x_N)$, the distribution of $g(y)$ can be approximated through simple matrix operations.

A.2 Building and Solving a Stationary System

Because the embodied technology grows constantly over time, such a non-stationary economy needs a transformation to stationarity before steady-state values can be derived. The original economy is transformed in two steps. The first step is to transform the economy into a labor-augmenting economy by defining:

$$u_t = v_t - z_{t-1} \tag{32}$$

$$\underline{u}_t = \underline{v}_t - z_{t-1} \tag{33}$$

$$K_t^T(u_t) = K_t(u_t + z_{t-1}) \tag{34}$$

u_t is defined as the difference between a plant's productivity at period t and the leading-edge technology at period $t-1$, and $\underline{u}_t$ is defined as the difference between the exit threshold of productivity at period t and the leading-edge technology at period $t-1$. The second step is to transform the labor-augmenting economy into a stationary economy by defining:

$$C_t^* = (e^{-\frac{1-\alpha}{\alpha}z_{t-1}})\, C_t \tag{35}$$

$$I_t^{*j} = (e^{-\frac{1-\alpha}{\alpha}z_{t-1}})\, I_t^j \ \forall\ j = 1, 2 \tag{36}$$

$$K_t^*(u_t) = (e^{-\frac{1-\alpha}{\alpha}z_{t-1}})\, K_t^T(u_t) \tag{37}$$

$$\bar{K}_t^* = \int_{-\infty}^{\infty} e^{u_t} K_t^*(u_t)\mathrm{d}u_t \tag{38}$$

$$G_t^* = (e^{-\frac{1-\alpha}{\alpha}z_{t-1}})\, G_t \tag{39}$$

After the second transformation, the new variables are consumption, C_t^*, investment on plants undergoing different periods of development process, I_t^{*j}, distribution of number of plants with different productivity, $K_t^*(u_t)$, effective capital, $\bar{K}_t^*$, and government expenditure, G_t^*. All are stable in the steady state. The transformed equations are listed below, followed by the explanation of the solution method for deriving steady-state values.

Equations are divided into two groups in order to solve the system. The first group includes the following equations:

If $u_t \leq \underline{u}_t$, then

$$(1 + \tau_t^p) \, Q_t(u_t) = (1 - \tau_t^c) \, D_t \, e^{u_t} + \eta \tag{40}$$

If $u_t > \underline{u}_t$, then

$$
\begin{aligned}
&(1 + \tau_t^p) \, Q_t(u_t) \\
&= (1 - \tau_t^c) \, D_t \, e^{u_t} \\
&+ E_t \left\{ \beta \, \frac{M_{t+1}}{M_t} \, e^{-\frac{1-\alpha}{\alpha} \, (\mu_z + \varepsilon_t^z)} \int_{-\infty}^{\infty} \frac{1}{\sigma} \phi(\frac{u_{t+1} - u_t + \mu_z + \varepsilon_t^z}{\sigma}) \, Q_{t+1}(u_{t+1}) \mathrm{d}u_{t+1} \right\}
\end{aligned}
\tag{41}
$$

$$M_t = E_t \left\{ \beta \, M_{t+1} \, Q_{t+1}^{0i}(1) \, e^{-\frac{1-\alpha}{\alpha} \, (\mu_z + \varepsilon_t^z)} \right\} \tag{42}$$

where

$$Q_t^{0i}(1) = Q_t^{1i}(2) \tag{43}$$

$$M_t \, Q_t^{1i}(2) = E_t \left\{ \beta \, M_{t+1} \, e^{-\frac{1-\alpha}{\alpha} \, (\mu_z + \varepsilon_t^z)} \int_{-\infty}^{\infty} \frac{1}{\sigma_e} \phi(\frac{u_{t+1} + z_t - z_{t-1}}{\sigma_e}) \, Q_{t+1}(u_{t+1}) \mathrm{d}u_{t+1} \right\} \tag{44}$$

$$\eta = E_t \left\{ \beta \frac{M_{t+1}}{M_t} e^{-\frac{1-\alpha}{\alpha} \, (\mu_z + \varepsilon_t^z)} \int_{-\infty}^{\infty} \frac{1}{\sigma} \phi(\frac{u_{t+1} - \underline{u}_t + \mu_z + \varepsilon_t^z}{\sigma}) \, Q_{t+1}(u_{t+1}) \mathrm{d}u_{t+1} \right\} \tag{45}$$

Before computation starts, the technology shock, ε_t^z, and the inverse of consumption, M_t, can be omitted from these equations because there is no technology shock in the steady state, and the steady-state consumption is constant. Given tax rates on capital income and property, there are two unknown variables, D_t and $\underline{u}_t$, and one unknown distribution of plant value, $Q_t(u_t)$, in this group. To determine $\underline{u}_t$, a number is chosen for D_t to derive the corresponding distribution of $Q_t(u_t)$, and the chosen D_t is the correct number if it satisfies the Equation 45. The second group includes the rest of the equations:

$$D_t = (1 - \alpha) \, (\frac{\bar{K}_t^*}{N_t})^{-\alpha} \tag{46}$$

$$M_t \, (1 - \tau^w) \, \alpha \, (\frac{\bar{K}_t^*}{N_t})^{1-\alpha} = \kappa \tag{47}$$

$$\frac{\bar{K}_t^*}{N_t} = \int_{-\infty}^{\infty} \frac{K_t^*(u_t)}{N_t} e^{u_t} \mathrm{d}u_t \tag{48}$$

$$\frac{K_{t+1}^*(u_{t+1})}{N_t} = e^{-\frac{1-\alpha}{\alpha}(\mu_z+\varepsilon_t^z)}\{\int_{\underline{u}_t}^{\infty}\frac{1}{\sigma}\phi(\frac{u_{t+1}-u_t+\mu_z+\varepsilon_t^z}{\sigma})\frac{K_t^*(u_t)}{N_t}\mathrm{d}u_t+\frac{1}{\sigma_e}\phi(\frac{u_{t+1}+\mu_z+\varepsilon_t^z}{\sigma_e})\frac{I_t^{*2}}{N_t}\}$$

<div align="right">(49)</div>

$$e^{-\frac{1-\alpha}{\alpha}(\mu_z+\varepsilon_t^z)}\frac{I_{t+1}^{*2}}{N_t}=\frac{I_t^{*1}}{N_t}$$

<div align="right">(50)</div>

$$\frac{G_t^*}{N_t}=(\tau_t^w\alpha+\tau_t^c(1-\alpha))(\frac{\bar{K}_t^*}{N_t})^{1-\alpha}+\tau^p\int_{-\infty}^{\infty}Q_t(u_t)\frac{K_t^*(u_t)}{N_t}\mathrm{d}u_t$$

<div align="right">(51)</div>

$$\frac{C_t^*+I_t^{*1}+G_t^*}{N_t}=(\frac{\bar{K}^*}{N_t})^{1-\alpha}+\eta\int_{-\infty}^{u_t}\frac{K_t^*(u_t)}{N_t}\mathrm{d}u_t$$

<div align="right">(52)</div>

Since D_t is known now, $\frac{\bar{K}_t^*(u_t)}{N_t}$, $\frac{K_t^*(u_t)}{N_t}$, $\frac{I_t^{*2}}{N_t}$, and $\frac{I_t^{*1}}{N_t}$ can be derived. There are three unknown variables left, and they are τ_t^w, M_t, and $\frac{G_t^*}{N_t}$. Given government expenditure, the correct τ_t^w is the value that satisfies Equations 51 and 52.

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Figure 4: Values in the Steady State When $G^* = 10^{-9}$ and $\tau^p = 0$
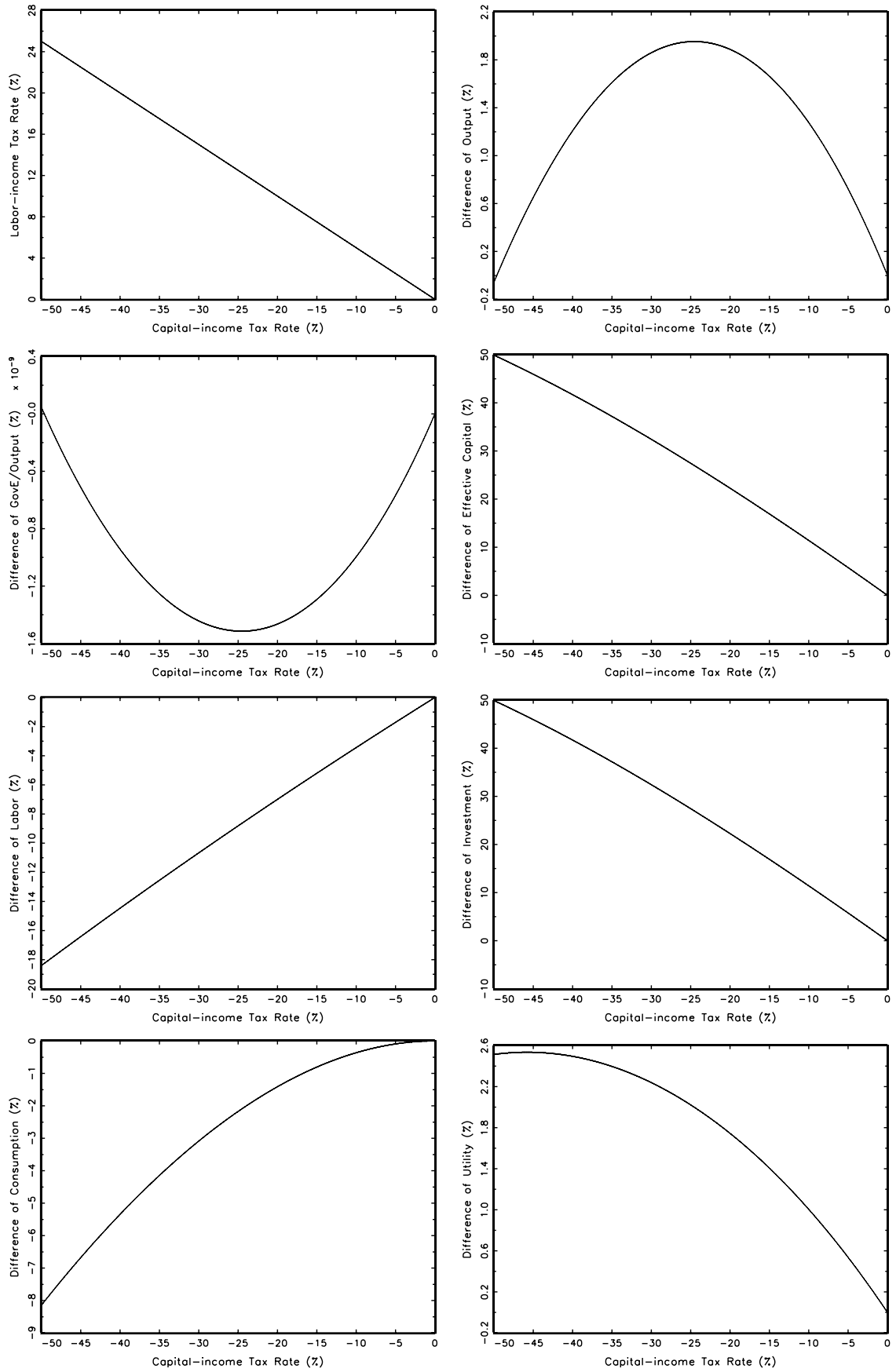
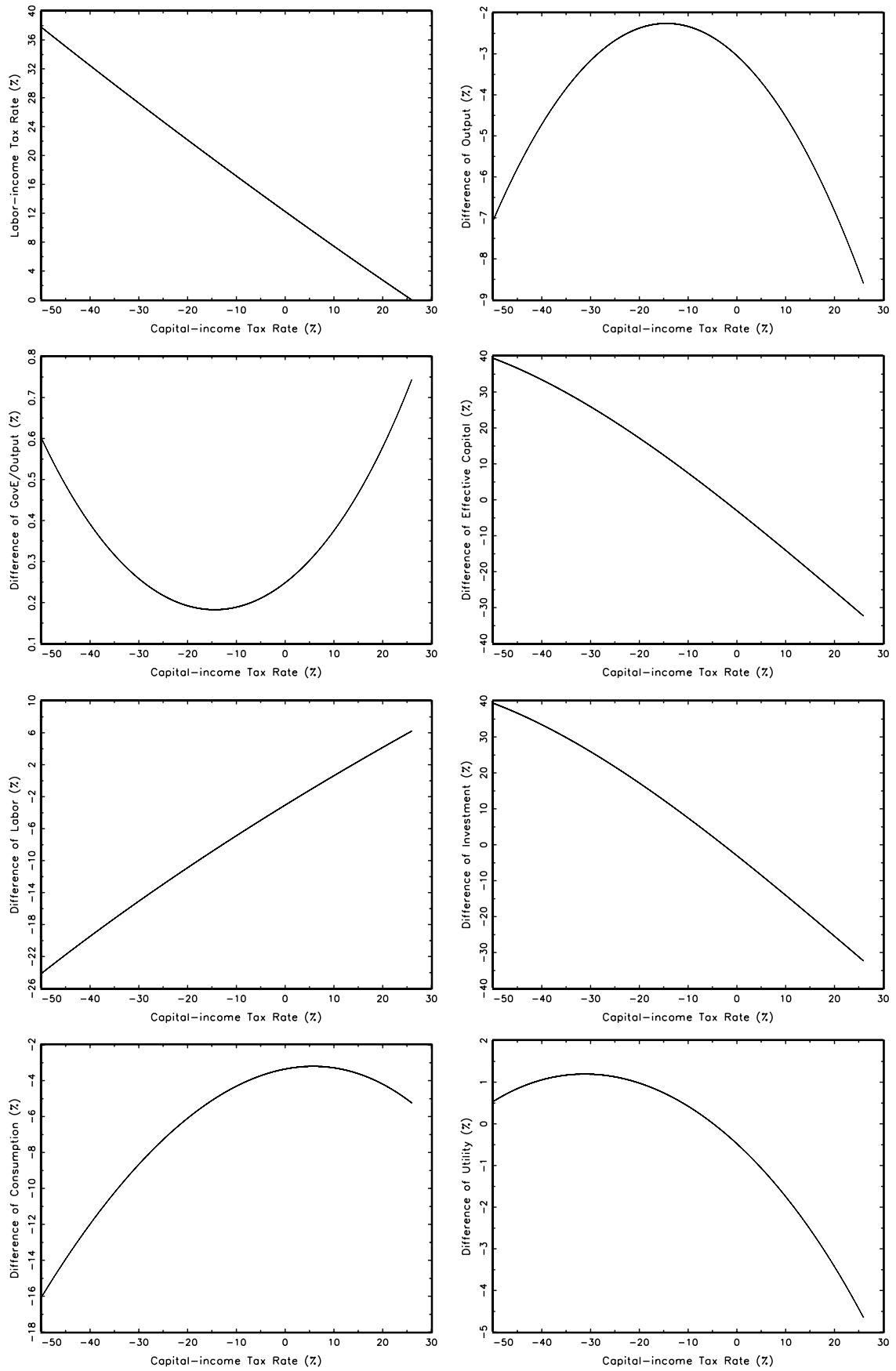Figure 5: Values in the Steady State When $G^* = 0.1$ and $\tau^p = 0$

Figure 6: Values in the Steady State When $G^* = 0.2$ and $\tau^p = 0$
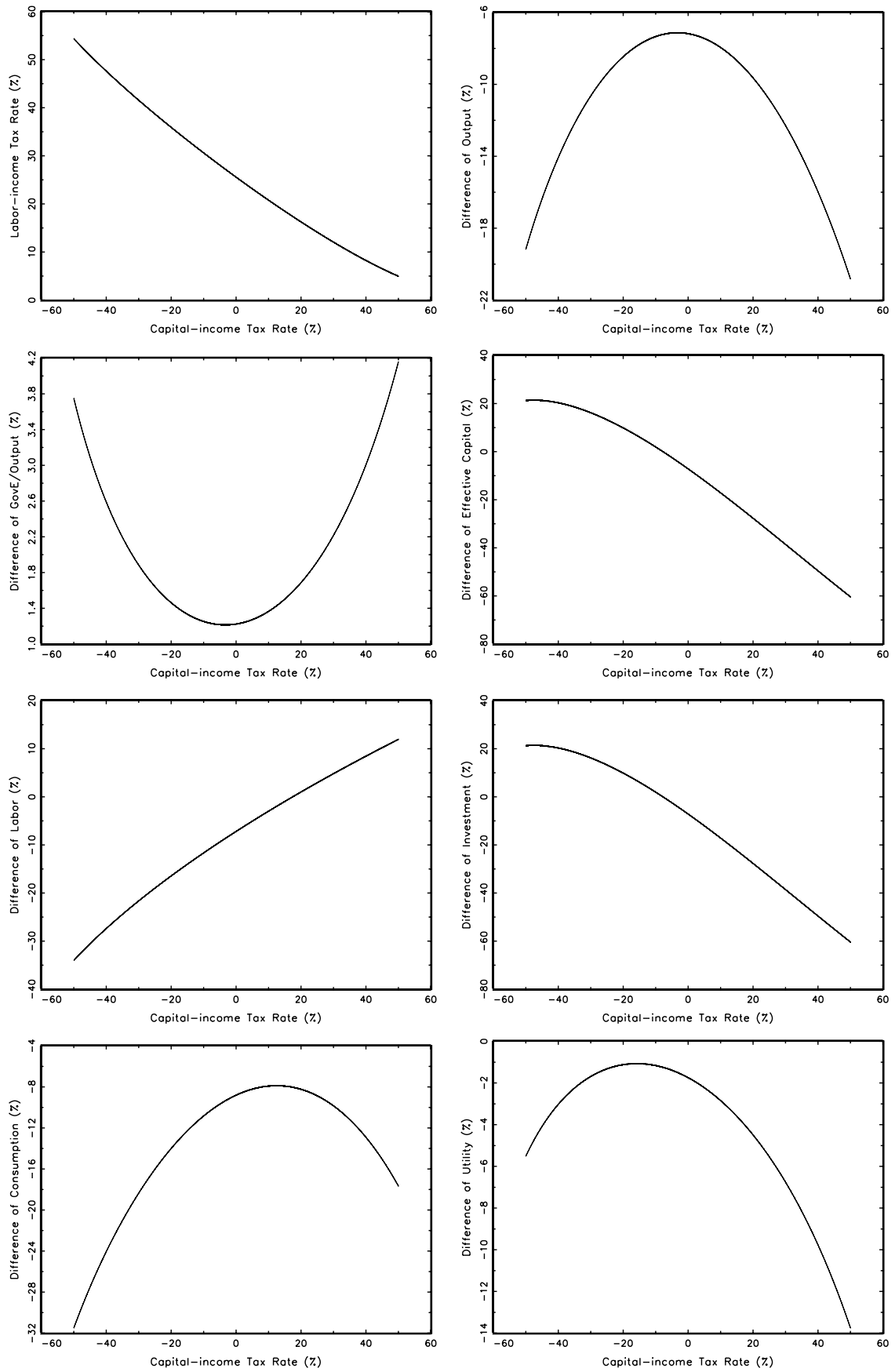
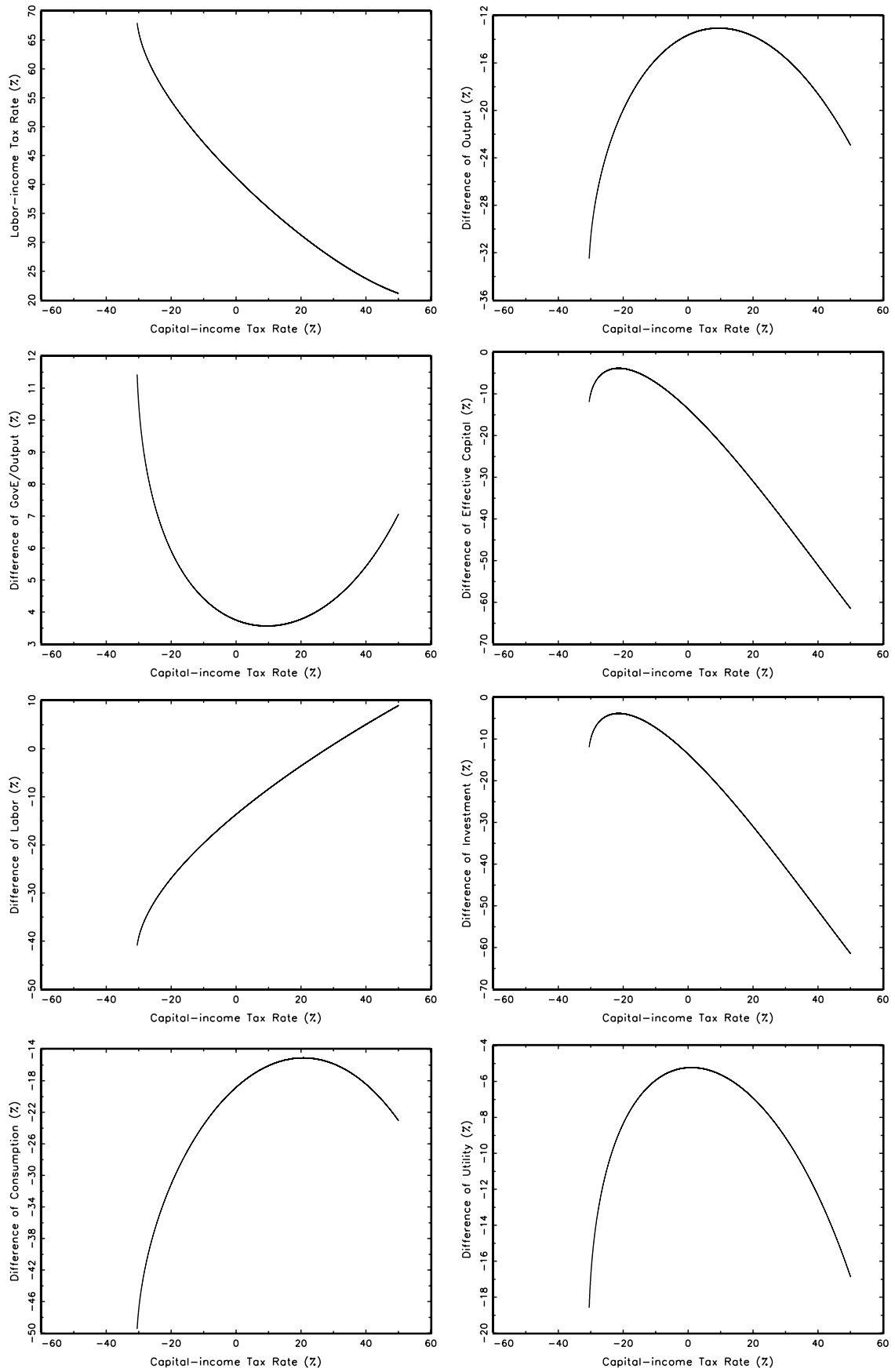Figure 7: Values in the Steady State When $G^* = 0.3$ and $\tau^p = 0$

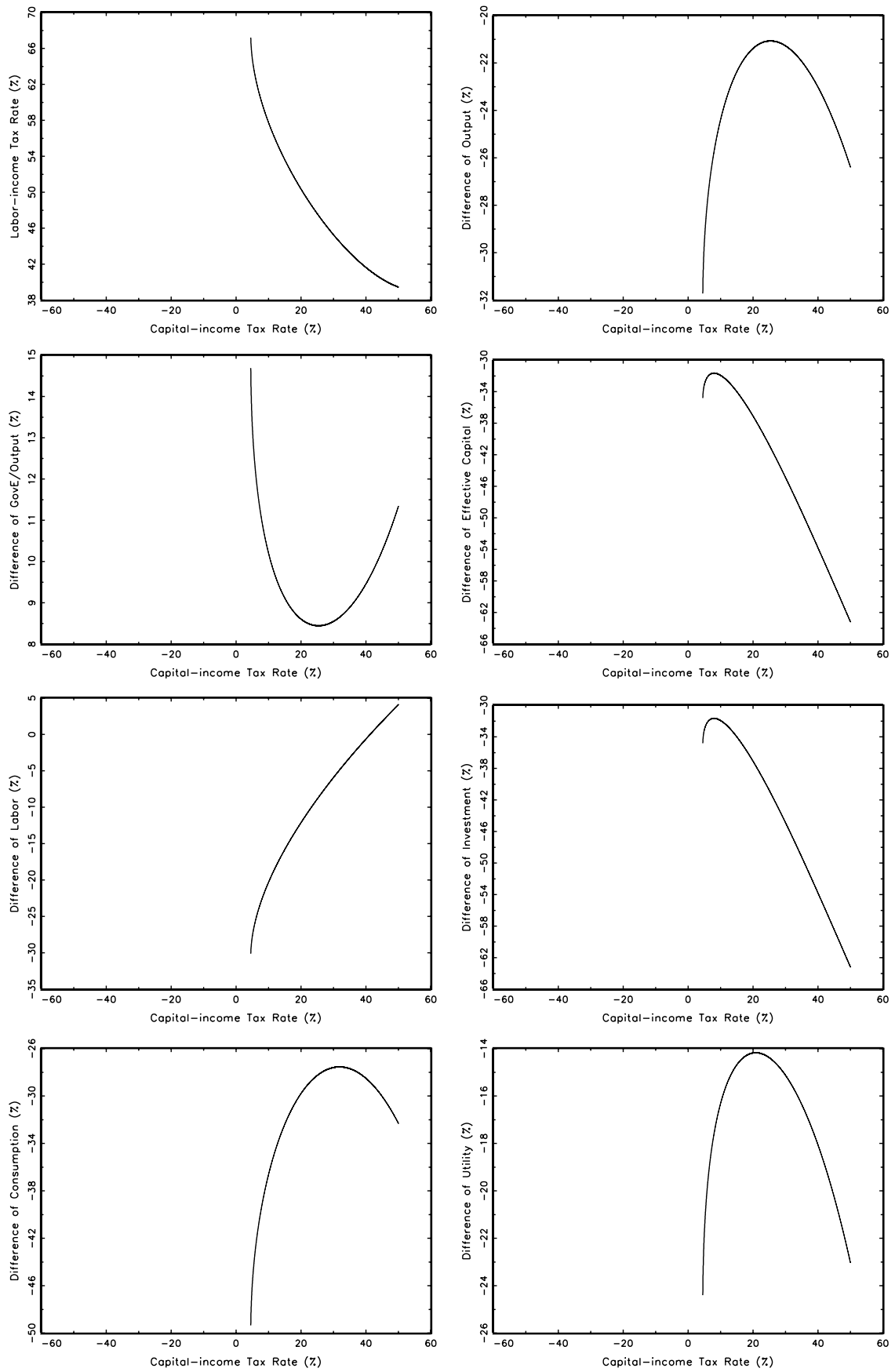Figure 8: Values in the Steady State When $G^* = 0.4$ and $\tau^p = 0$

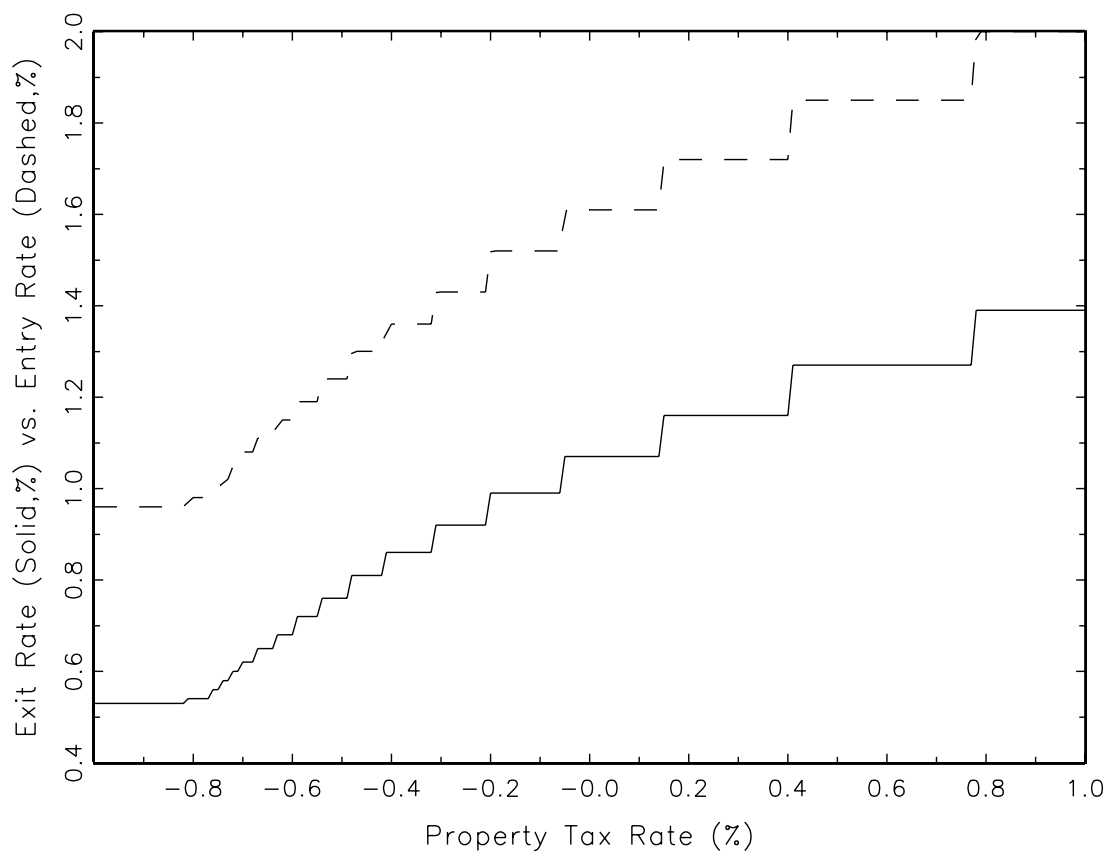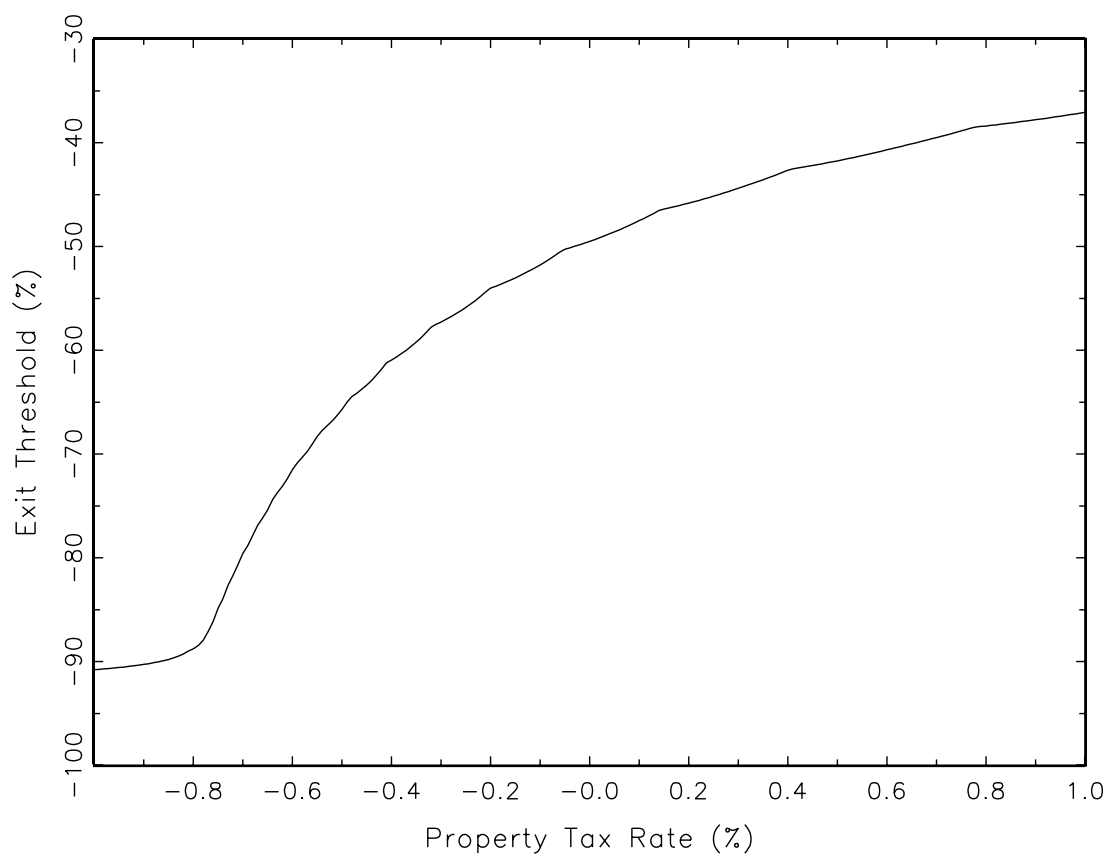Figure 9: Exit Threshold, Exit Rate, and Entry Rate
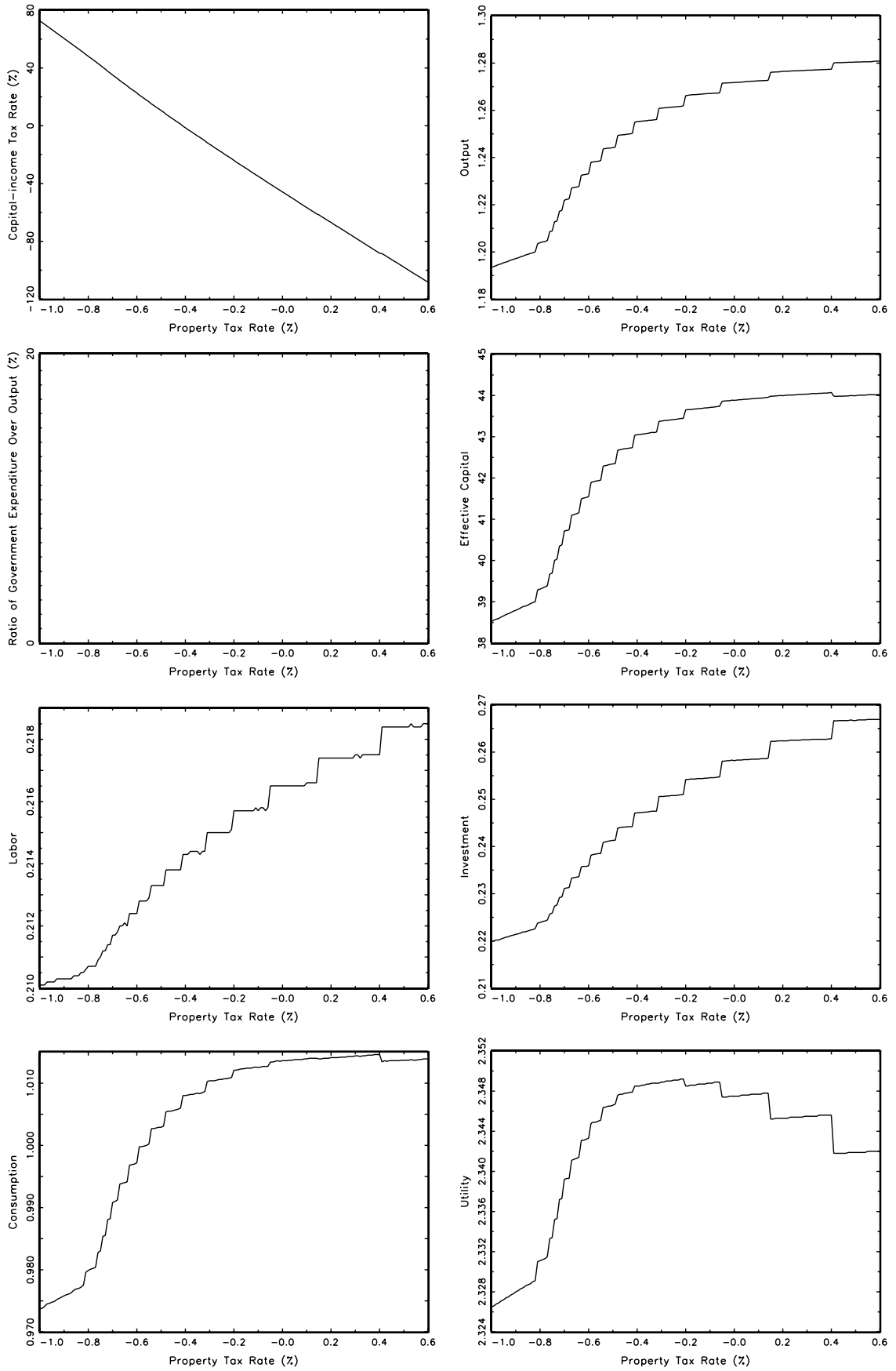
Figure 10: Values in the Steady State When $G^* = 10^{-9}$

Figure 11: Values in the Steady State When $G^* = 0.4$