Housing Wealth and Mortgage Contracts

Joseph B. Nichols*

University of Maryland

Department of Economics

JEL Classification Codes: E21, G11, G21, R21, C61

August 2004

Abstract

This paper develops a detailed partial equilibrium model of housing wealth's role over the life-cycle to explore (1) housing's dual role as a consumption and investment good; (2) the significance of the mortgage contract being in nominal and not real terms; and (3) the tax benefits associated with owner-occupied housing. The stochastic dynamic programming problem is solved using parallel processing. The baseline model is then compared with a set of alternate scenarios to explore these three key aspects of housing wealth. The results show that the "over-investment" in housing is not just a function of consumption demand but also can be driven by the benefits inherent in the mortgage contract. It also shows that the nominal mortgage contract results in the non-neutrality of perfectly expected inflation. Finally, the paper documents the effect of preferential tax treatment on housing demand.

*Comments are welcome at: nichols@econ.bsos.umd.edu. I would like to thank John Rust, John Shea, Anthony Yezer, Michael Pries, Charles Capone, Robert Martin, Andreas Lehnert, Francois Ortalo-Mange, Carol Bertaut, and Julie Nichols for their comments. I would also like to thank the participants at the Yale Inter-University Student Conference, the American Real Estate and Urban Economics Association, Econometric Society, and the Bureau of Labor Statistics, Board of Governors of the Federal Reserve, and SUNY-Stony Brook seminars. Financial support from the University of Maryland Population Center Seed Grant, Economic Club of Washington Doctoral Research Fellowship, and Department of Housing and Urban Development Doctoral Dissertation Grant is gratefully acknowledged, as is computing resources from the Pittsburg Supercomputing Center. All errors and omissions are the sole responsibility of the author.

Contents

1 Introduction.

This paper develops a detailed model of housing wealth's role over the life-cycle. Three key issues are explored: (1) housing's dual role as a consumption and investment good; (2) the significance of the mortgage contract being in nominal and not real terms; and (3) the tax benefits associated with owner-occupied housing. The paper then demonstrates how each of these unique aspects of housing wealth affect consumption, savings, housing demand, and portfolio allocation over the life-cycle. This paper takes an initial step toward integrating a realistic model of housing wealth into the larger literature of life-cycle wealth accumulation and asset pricing.

1.1 The Importance of Housing Wealth

Housing wealth is a vitally important but understudied component of household wealth. The single most significant asset for many households in the United States is the equity held in their home. Flavin and Yamashita (2002) used the Panel Study of Income Dynamics to show that among homeowner households with a head between 18 and 30 years old, 67.8% of their portfolio is in their home. In the same article the authors documented how a household's exposure to risk through their housing wealth could impact the portfolio allocation of their financial wealth. Fernández-Villaverde and Krueger (2001) document how housing could be used as collateral to relax lending constraints. These papers, among others, demonstrate that to understand the accumulation and composition of household wealth, one must first understand housing wealth.

The inclusion in a model of a simple consumption good, even one that is durable, or a simple investment good, even one with significant transaction costs, is relatively straight forward. Housing's unique dual role as a consumption and an investment good makes it a far more interesting challenge to model. This unique aspect of housing is the first addressed in this paper. In general, the demand for a pure consumption good is simply determined by the marginal utility that it generates. On the other hand, the demand for an investment good is simply determined by its riskiness and correlation with the total portfolio. The demand for a composite good that acts as both an investment and consumption good and provides both utility and returns on investment is more complicated to determine. Therefore, it is necessary to explore the interaction between the consumption and investment motive for the good. The question becomes even more complex when the market for the good in question contains many frictions. The two most significant frictions in the housing market are the use of mortgage contracts and the tax treatment of housing.

The second aspect of housing wealth addressed in this paper is that the mortgage balance and payments are set to nominal and not real values. The mortgage payment is not adjusted to reflect changes in the underlying cost of housing. Homeowners with existing mortgages see the real value of their mortgage balance and payment decline during periods of high inflation. This could have a direct impact on the behavior of households even if the inflation was perfectly anticipated.

The final aspect of housing wealth addressed is the three main ways owner-occupied housing benefits from preferental tax treatment: (1) the implicit rental from owner-occupied housing is taxed as regular income, (2) the capital gains from housing is taxed, and (3) mortgage interest is no longer tax-deductable. The tax-free implicit rent is perhaps the most significant tax benefit associated with owner-occupied housing. Specifically, the homeowner is not taxed on the implicit rent generated by the housing. An investor who purchases and then rents a home must pay taxes on the rental income generated. However, a household that purchases and then occupies the same home directly consumes the stream of housing services, but pays no tax on the economic value of this stream of housing services. The implicit rent is equivalent to an untaxed dividend from a traditional financial asset. Models that do not address these aspect of housing may be significantly understating one of the key advantages of housing as an investment good.

These unique aspects of housing wealth are not merely interesting, they can also have a profound effect on household behavior. It is impossible to develop a realistic model of housing wealth without explicitly addressing these issues. A traditional model of financial assets cannot explain the portion of housing demand driven by the desire to consume housing services. Likewise, a traditional model of durable consumption goods cannot explain the portion of housing demand driven by investment motives. A model that has real instead of nominal mortgage contracts is overstating the costs of the mortgage. The tax-free status of the implicit rent is perhaps the single biggest tax advantage housing has over other financial assets. Given the importance of these issues, it is vital to explicitly include them in a model of housing wealth.

1.2 Challenges of Modeling Housing Wealth

The two approaches to modeling housing wealth's role in the life-cycle each have advantages and disadvantages. The first approach is to develop an abstract model that captures only a few of the most important aspects of housing as an investment good. This type of model's advantages are that many can be solved analytically, or embedded in a general equilibrium framework and solved numerically. The primary disadvantage is the relatively narrow scope of such a model. The second approach is to sacrifice simplicity for a more complicated partial equilibrium model that can be solved numerically using stochastic dynamic programming. Its advantage is that as a more complex model it presents a more realistic picture of the role of housing wealth over the life-cycle. The downside is an upper limit on the model's complexity level, beyond which the solution times are no longer tractable. Parallel processing can extend this upper limit in a grid-cluster or super-computer environment. The greater complexity of the model requires great care in presenting the results and currently precludes the option of embedding the model in a general equilibrium framework.

Both of the approaches described above are important and legitimate. Many of the questions the more detailed partial equilibrium models can address are outside the scope of the general equilibrium models. By explicitly including so many different aspects of housing wealth simultaneously, the partial equilibrium model is extraordinarily flexible. For example, by incorporating a few simple changes to the mortgage balance transition rule, the model can be used to simulate the effects of alternate mortgage contracts on housing demand and portfolio allocation. The same is true for changes in the tax treatment of housing or the success of alternative preferences in explaining the role of housing wealth of the life-cycle. The detailed partial equilibrium model in this paper provides important insight into how to develop an more abstract general equilibrium model that can still capture the complexities associated with modeling housing wealth.

1.3 A Detailed Partial Equilibrium Model of Housing Wealth

Housing wealth plays an important role, both as a significant component of a household's portfolio and through its indirect effects on the demand for other types of investment assets. Housing wealth is also very different from other types of financial assets, calling for a different modeling approach. The model developed in this paper is used to demonstrate the importance of three unique aspects of housing wealth: (1) the dual role as a consumption and investment good; (2) the significance of the mortgage contract being in nominal and not real terms; and (3) the tax benefits associated with owner-occupied housing. The paper shows how each of these unique aspects of housing wealth has profound effects on the demand for housing and the composition of household portfolios.

The model's design allows households to choose their current consumption, their savings, their savings allocated to risky assets, which type of housing to occupy, and whether to refinance their mortgages. The housing tenure choice includes a rental unit, a small home, and a large home. Households may increase their mortgage balance through the use of a cash-out refinance. Renters choose the size of the rental unit so that the intra-period marginal utility of housing is equal to the marginal utility of non-durable consumption. The size of the large and small homes are fixed in terms of the number of housing units they represent. Households face uncertainty in the returns on risky assets and housing, the probability of survival, and a transitory shock to income; which is otherwise a deterministic function of age. The model includes moving, maintenance, and transaction costs. Both the ability to and the costs of defaulting on a mortgage are also included in the model. The model is solved given the terms of a traditional 30-year fixed rate mortgage contract. The values of non-structural parameters, such as returns on different types of assets, the survival probability, mortgage terms, and income process, are taken from historical data.

The model's solution is then used to demonstrate the importance of each of the three unique aspects of housing wealth being examined. Two different versions of the model are solved differentiating the demands for housing as either an investment good or a consumption good. In the first version housing is treated as only an investment good and there exists a perfect rental market. Households may always rent the appropriate number of housing units such that the intra-period marginal utility of renting a house is equal to the marginal utility of all other consumption. Housing merely represents an unusual investment good that must be purchased using a traditional mortgage contract but generates no direct utility. The second version of the model treats housing as only a consumption good. The downpayment and the mortgage payments by the household are sunk costs and are not recouped when the home is sold. Instead households walk away from home sales with no gain or loss from the transaction. The paper demonstrates the significance of the other two unique aspects of housing wealth in a similar way. A version of the model is solved where the mortgage contract is in real and not nominal terms. Additionally, a version with a historically high rate of inflation, as well as the mortgage contract is still in nominal terms. Finally, versions of the model are solved where each of the three major tax benefits of owner-occupied housing are removed: (1) the implicit rental from owner-occupied housing is taxed as regular income, (2) the capital gains from housing is taxed, and (3) mortgage interest is no longer tax-deductable.

2 Literature Review.

Many of the existing papers on the role of housing wealth tend to focus on different factors behind housing demand in isolation. Often housing was treated as either an investment good or as a consumption good. Often the models that explicitly captured housing's dual role did not include mortgage financing, or at most included only an abstract version of the mortgage contract. One of the innovations of this paper is to model housing as both an investment good and consumption good. The other key innovation is to explicitly model the mortgage contract.

In several papers that explored in detail the role of housing wealth the actual decision of when and how much housing to consume was not endogenous. Fratantoni (1997) solved a finite-horizon model with exogenous housing consumption and showed that introducing housing in the model reduced the share of risky assets held by households. The author extended the model in Fratantoni (2001) to show that the commitment to make future mortgage payments resulted in a lower level of equity holdings. In Fernández-Villaverde and Krueger (2001) the authors observed that young consumers have portfolios with little liquid assets but a significant amount invested in durables. Fernández-Villaverde and Krueger hypothesized that young consumers can only borrow against future income by using their durable assets as collateral for loans. They then developed a structural life-cycle model with endogenous borrowing constraints and interest rates. Each of these papers explored an important aspect of housing wealth. However, by making the actual demand for housing exogenous they are unable to explore what exactly drives the demand for housing wealth. In this paper housing demand is endogenous and it is possible to determine how some of the unique aspects of housing wealth can drive the demand for housing.

Cocco (2000) developed a model with endogenous tenure choice to explore the effect of labor income, interest rate, and house price risk on both housing choices and investor welfare. Cocco utilized an abstract version of the mortgage contract where the level of mortgage debt adjusts in each period so that the loan-to-house value ratio remains fixed. Cocco's automatically-refinancing mortgage precludes the opportunity to pay down or pay off a mortgage, two very common strategies among households. The more realistic mortgage model in this paper makes both of these strategies available to households.

Martin (2001) argued that consumers have an inaction region in the purchase of durable goods caused by transaction costs. Martin then argued that the inaction region in durable goods induces variation in the consumption of non-durable goods. Martin's model is a general equilibrium model and includes only a risk-free financial asset. It does not address the interaction between housing investment and the household's portfolio allocation problem. Martin also models the mortgage used to purchase a house as simply a negative position in the risk free bonds. In Martin, the inability of a single household to hold both a mortgage and a financial asset prevents any discussion of household level portfolio allocation, one of the key topics of this paper.

Hu (2002) developed a model very similar to the one in this paper where housing is endogenous. Hu solved a finite-horizon model that allowed for households to hold a risk-free asset, a risky asset, or risky owner-occupied housing. Of the papers discussed here, Hu has the most detailed and realistic treatment of the mortgage contract. Hu's model reflects the composite nature of home equity and includes both the current value of the house and the current balance of the mortgage. Hu also allows for the mortgage payment to be fixed at the time of purchase. This model differs from Hu's by fixing the nominal mortgage payment while inflation reduces the real value of the mortgage payment. Additionally, Hu does not allow cash-out refinancing, a significant aspect of this model.

Li and Yao (2004) explore the housing and mortgage decisions of a household over the life-cycle. As this paper does, they utilize stochastic dynamic programming and parallel processing to solve an extremely detailed model. Many of their results are broadly consistent with this paper. However, this paper differs significantly from Li and Yao in the treatment of the mortgage contract. Li and Yao make two key assumptions when modeling the mortgage contract in order to make their model tractable. First, they assume that mortgages are amortized over the remainder of the household's life. Secondly, they assume that the mortgage payment is indexed to the current value of the house. These simplifications allow them to introduce permanent income shocks; which are absent from this paper. The cost of this simplification is to ignore the ability of a household to lock in its mortgage payments at a constant nominal value. The result from Li and Yao's approach is that mortgage payments are significantly lower, understating the cost of housing, and mortgage payments fluctuate with the value of the home, providing a form of insurance.

The main source of methodology used in this paper is Rust and Phelan (1996). Rust and Phelan set up and solve a dynamic programming problem of labor supply with incomplete markets, Social Security, and Medicare. The dynamic programming problem in their paper is solved by discretizing the continuous state spaces and then using backward recursion to solve for the optimal value of the continuous choice variable at each point on the state space grid. The detailed rules governing the Social Security and Medicare application processes and benefits are imbedded in the income transition matrix. The model in this paper has a similar structure, but instead imbeds the detailed characteristics of the mortgage market contract in the income transition matrix.

All of these papers represent important work on interesting aspects of housing wealth. However, none of these papers attempts to develop a model that addresses all the unique characteristics of housing wealth. The key issue in modeling housing wealth is the treatment of the mortgage contract. More than any other single issue, it is mortgage financing that complicates realistically modeling housing wealth. The main contribution of this paper is to include an unprecedentedly detailed model of the mortgage contract.

3 A Partial Equilibrium Model of Housing Wealth with Mortgage Contracts.

This section describes the structure of the finite-horizon life-cycle model of a household's savings, investment, and housing decisions. The section concludes with a discussion of the method used to solve the household's optimization problem. The structure of the model is actually quite straight forward, with most of the complexities embedded in the wealth transition rules. Households receive utility from the consumption of both a non-durable good and the stock of housing that they own. Each period in the model represents a single year. Their optimization problem is to maximize their lifetime utility, defined as:

$$E\sum_{t=20}^{80}\beta^t\rho_t U(c_t,h(i_t))+\beta^t(1-\rho_t)(\theta_A U_B(A_t)+\theta_H U_B(H_t)-\theta_D U_B(D_t)) \quad c_t>0,\forall t \tag{3.1}$$

$$U(c_t,i_t)=\frac{(c_t^{1-\phi}h(i_t)^{\phi})^{1-\lambda}}{1-\lambda} \tag{3.2}$$

$$U_B(b)=\frac{b^{1-\lambda}}{1-\lambda} \tag{3.3}$$

where,

- c_t represents the consumption of non-durables;
- $h(i_t)$ represents the number of housing units consumed given the housing tenure choice in period t;
- A_t, H_t, and D_t are respectively the value of the financial assets, home, and mortgage debt left as bequests;
- β represents the discount rate;
- ρ_t is the survival probability;
- ϕ represents the measure of preference between of housing and consumption;

- λ represents a measure of risk aversion; and,

- θ_A, θ_H, and θ_M represent bequest parameters.

A household lives at most 80 years. It faces uncertainty about its survival, temporary income shocks, and the rate of return on both housing and risky assets. In addition to the stochastic elements for income and the rate of return on risky assets, the households may experience an additional shock. A small probability exists that the household will experience unemployment in one period, reducing income to zero. Also, a small independent probability exists of a stock market crash where the household will lose 100% of its investment in the risky financial asset. Households also are not allowed to consume negative amounts of non-durable goods.

3.1 Consumption of Housing

While consumption of the non-durable good in the model is continuous, the choices for housing consumption are partially discrete. The model has three different alternatives for housing: a rental unit, a small home, and a large home, represented by the corresponding symbols i_r, i_s, and i_l. The number of housing units available to rent is continuous while the number of housing units provided by a small or large home is fixed. Renter households are able to choose the number of housing units that equalizes their intra-period marginal utility from housing to their intra-period marginal utility from non-durable consumption.

$$\frac{\delta U(c_t, h(i_t))}{\delta c_t} = \frac{\delta U(c_t, h(i_t))}{\delta h(i_t)} \tag{3.4}$$

Optimal rental units may now be defined as a function of consumption,

$$h(i_r) = (\phi/(1-\phi))c_t \tag{3.5}$$

Many other factors in the model are conditional on current housing tenure, including rent or mortgage payments, maintenance costs, level of utility derived from housing, and the rate of appreciation in home value. The size of a small home is set equal to that of a median priced home, while the size of a large home is set to be twice that of a median priced home.

3.2 Accumulation of Financial Wealth and the Income Process

A household is “born” at age 20 with zero financial and housing wealth. It starts off as a renter with no savings. In each period it receives a draw from an age-dependent income process. The model contains no permanent income shock, only transitory shocks. In retirement, pension income is set to 60% of the deterministic portion of age 65 income. Pension income is still subject to transitory shocks, representing uncertainty regarding medical costs. Households can store their wealth in two different classes of assets, financial and real. The household’s financial assets are held in a portfolio of risk free and risky assets. The household can, at no cost, rebalance its financial portfolio between risk free and risky assets every period. Households with zero wealth face a binding liquidity constraint for financial assets in that they cannot borrow against their future income. Households also cannot purchase leveraged portfolios, where they borrow at the risk free rate to invest more in the risky asset. In addition to moving to one of the three types of housing, $\{i_r, i_s, i_l\}$, the household can also decide to stay in its current home, $\{i_{t+1} = i_n\}$. Households may also either add to their mortgage balance through a cash-out refinance or reduce their mortgage balance through a pre-payment refinance.

The transition rule for the level of financial wealth is defined as:

$$\begin{aligned} A_{t+1} = {} & (1 + (1-\gamma)(\alpha_t \widetilde{r_{s_t}} + (1-\alpha_t) r))(A_t - c_t - X_t(i_t, \kappa_t) + \\ & G_t(i_t, i_{t+1}, \kappa_t) + Z_t(\kappa_t, \kappa_{t+1})) + (1-\gamma)\widetilde{w_{t+1}} + \gamma I_t(i_t, \kappa_t) \\ & s.t. \quad A_{t+1} \geq 0 \ \& \ 0 \leq \alpha_t \leq 1 \end{aligned} \tag{3.6}$$

where,

- A_t is the level of financial assets in period t;
- A_{t+1} is a random variable that depends on the stochastic rate of return on risky assets ($\widetilde{r_{s_t}}$) in period t and the realizations of wages ($\widetilde{w_{t+1}}$) in period $t+1$;

- α_t is the share invested in risky assets in time t;
- r is the deterministic rate of return on risk-free assets;
- $G_t(i_t, i_{t+1}, \kappa_t)$ is the net gain for a household choosing i_t this period and i_{t+1} next period;
- $Z_t(\kappa_t, \kappa_{t+1})$ is the net gain from refinancing;
- γ is the tax rate on income and capital gains;
- $I_t(i_t, \kappa_t)$ is the mortgage interest paid; and
- $X_t(i_t, \kappa_t)$ is the housing costs incurred in period t for a household currently choosing tenure type i_t with a mortgage κ_t years old.

The net gain from a home sale is tax-free and the mortgage interest paid is deducted from taxable income. Both the housing expenses and the amount of the mortgage interest deduction are functions of the current housing choice and age of mortgage. Refinancing is modeled as a choice to lengthen the remaining number of years on the mortgage, or inversely, to shorten the current age of the mortgage. The age of a mortgage for a rental unit or a mortgage that has been paid off is zero. Households receive their wages at the same time they realize the returns on their investment from the previous period. As a result, the state variable A_t represents all available cash on hand, consisting of previous financial wealth and current income.

The income process is defined as a deterministic function of age plus a transitory shock, as shown below in log form:

$$\log(w_t) = \psi_0 + \psi_1 t + \psi_2 t^2 + \varepsilon_w \tag{3.7}$$

$$\varepsilon_w \sim N(0, \sigma_w)$$

The rate of return on risky assets is a random variable with the distribution:

$$r_{s_t} \sim N(\eta_s, \sigma_s^2) \tag{3.8}$$

where η_s is the expected rate of return on the risky asset and σ_s^2 is the variance.

3.3 Price of Housing

In addition to the portfolio of financial assets, households can also store their wealth in real assets by purchasing a house. It is only through the purchase of a house, and the acquisition of a mortgage loan, that households can borrow against their future income. The use of durable goods as collateral is in the same spirit as Fernández-Villaverde and Krueger (2001). The only mortgage contract available to the household in this model requires a 20% down payment; has a term of 30 years; and requires mortgage payments based on a fixed interest rate and the size of the original mortgage. The mortgage balance and the mortgage payment are both in nominal terms while the rest of the model is in real terms. Households selling their home are also required to pay a transaction cost equal to 10% of the value of the home that they are purchasing. This represents realtors' fees, credit checks, and other expenses associated with the purchase.

The real price of housing has a positive trend over time. The purchase price of either a small or large home increases non-stochastically by the average market price increase in each period. The value of homes that have already been purchased changes according to a stochastic process, with the expected increase equal to the non-stochastic market price increase. A household that has had a series of excellent draws in home price appreciation will own a home worth relatively more than a comparable home on the market. A household that has had a series of poor draws in home price appreciation will own a home worth relatively less than a comparable home on the market.

The price per housing unit is the same across all types of housing. Large homes cost more than small homes because they provide more units of housing for the homeowner to consume. Renters may choose as small or as large a home to rent as they wish. Their rent is proportional to the current value of their chosen home. The formulas for the market price of home type i_t ($P_t(i_t)$) and the housing wealth (H_{t+1}) transition rule are:

$$P_t(i_t) = (1+\eta_h)^t P_0 h(i_t) \tag{3.9}$$

$$H_{t+1} = \begin{cases} H_t(1+\widetilde{r_{h_t}}), & i_{t+1} \in i_n \\ P_t(i_t), & i_{t+1} \in i_s, i_l \\ 0, & i_{t+1} = i_r \end{cases} \tag{3.10}$$

$$r_{h_t} \sim N(\eta_h, \sigma_h^2) \tag{3.11}$$

where P_0 is the price of a single unit of housing in period 0; $\widetilde{r_{h_t}}$ is the realized rate of appreciation on housing in period t; η_h is the expected rate of appreciation on housing; and σ_h^2 is the variance of the house price growth.

3.4 The Mortgage

A significant source of the complexity in the model is the need to include the age of the mortgage in the state space. In the model this adds a discrete state variable with thirty-one discrete values, resulting in over 1.7 million points in the final state space. The computational techniques used to solve a problem of this scope are discussed briefly at the end of this section. The reason for including the age of the mortgage in the state space is the nature of the 30-year self amortizing mortgage. First, the actual equity households hold in their home is the difference between the value of the home minus the remaining balance on the outstanding mortgage. To accurately track the value of the household's home equity, it is necessary to track both the value of the home and the mortgage balance independently. The nature of the mortgage contract further complicates what would be a logical solution, the addition of a third continuous state variable for mortgage debt. The principal paid on a self amortizing mortgage is not constant over the life of the mortgage. Initial payments are almost completely composed of interest, with very little principal being paid. The final payments on a 30-year mortgage on the other hand are almost completely principal, with very little interest being paid. Therefore, the transition rule for mortgage debt is a function of the age of the mortgage. The fact that the mortgage balance and mortgage payment are in nominal terms provides an additional motivation for including the age of the mortgage in the state space. The real values of the mortgage balance and payment decline steadily over the life of the mortgage due to inflation.

The mortgage payment is based on the home price when purchased, and only changes when the household refinances the mortgage or sells the house. A cash-out refinance increases the number of years left on the mortgage. The formula for the real value of a mortgage payment at time t after κ_t years on a house of type i_t is:

$$M_t(i_t,\kappa_t) = \frac{\pi(1-\mu)P_{t-\kappa_t}(i_t)}{(1-\pi)^{-30}(1+\nu)^{\kappa_t}} \tag{3.12}$$

where π is the mortgage interest rate; ν is the inflation rate; and μ is the required down payment.

The cost of housing services also reflects the maintenance costs paid by homeowners. As a result, the formula for the real cost of housing services is:

$$X_t(i_t,\kappa_t) = \begin{cases} M_t(i_t,\kappa_t)+\delta H_t, & i_t \in i_s, i_l \\ 0.06P_t(i_r), & i_t = i_r \end{cases} \tag{3.13}$$

where δ is the percent of current home value required in maintenance costs. Rent is equal to 6% of the current market value of the unit being rented and renters pay none of the maintenance costs for the property.

The present value of the household's home equity is the current value of the house minus the amount of the outstanding mortgage balance. While the value of the house increases or decreases according to the stochastic return on housing, the outstanding mortgage balance is a monotonically declining function of the age of the mortgage. The formula for the real value of the mortgage balance at time t after κ_t years on a house of type i_t is:

$$D_t(i_t,\kappa_t) = \begin{cases} M_t(i_t,\kappa_t)\frac{1-(1-\pi)^{\kappa-30}}{\pi}, & i_t \in i_s, i_l \ \& \ \kappa \leq 30 \\ 0, & (i_t \in i_s, i_l \ \& \ \kappa > 30) \quad \text{or} \quad (i_t = i_r) \end{cases} \tag{3.14}$$

The formulas for the remaining mortgage balance and the mortgage payment are used to calculate the amount of mortgage interest paid for tax purposes. The values must be adjusted back from the real terms since this deduction is in nominal terms. The formula for the mortgage interest deduction is:

$$I_t(i_t,\kappa_t) = (1+\nu)^{\kappa_t} M_t(i_t,\kappa_t) - ((1+\nu)^{\kappa} D_t(i_t,\kappa_t) - (1+\nu)^{\kappa_{t+1}} D_t(i_t,\kappa_{t+1})) \tag{3.15}$$

3.5 Gains from Sale or Refinancing

The net gain after paying transaction costs and down payments for a household choosing next period's tenure $i_{t+1} \in \{i_r, i_s, i_l, i_n\}$ is given by:

$$G_t(i_t,i_{t+1},\kappa_t) = \begin{cases} H_t - D_t(i_t,\kappa_t) - \mu P_t(i_{t+1}) - \tau H_t - \chi, & i_{t+1} \in i_r, i_s, i_l \\ 0, & i_{t+1} = i_n \end{cases} \tag{3.16}$$

where τ is the transaction cost; μ is the downpayment rate; and χ is a fixed moving cost paid regardless of which type of housing is being purchased. When the household chooses not to move, $i_{t+1} = i_n$, it has zero net gain.

The net gain after choosing to refinance a mortgage is defined as the sum of the difference between the mortgage balances before and after the refinance and a fee for the transaction. When no refinance occurs $\kappa_{t+1} = \kappa_t + 1$ and the net gain is zero.

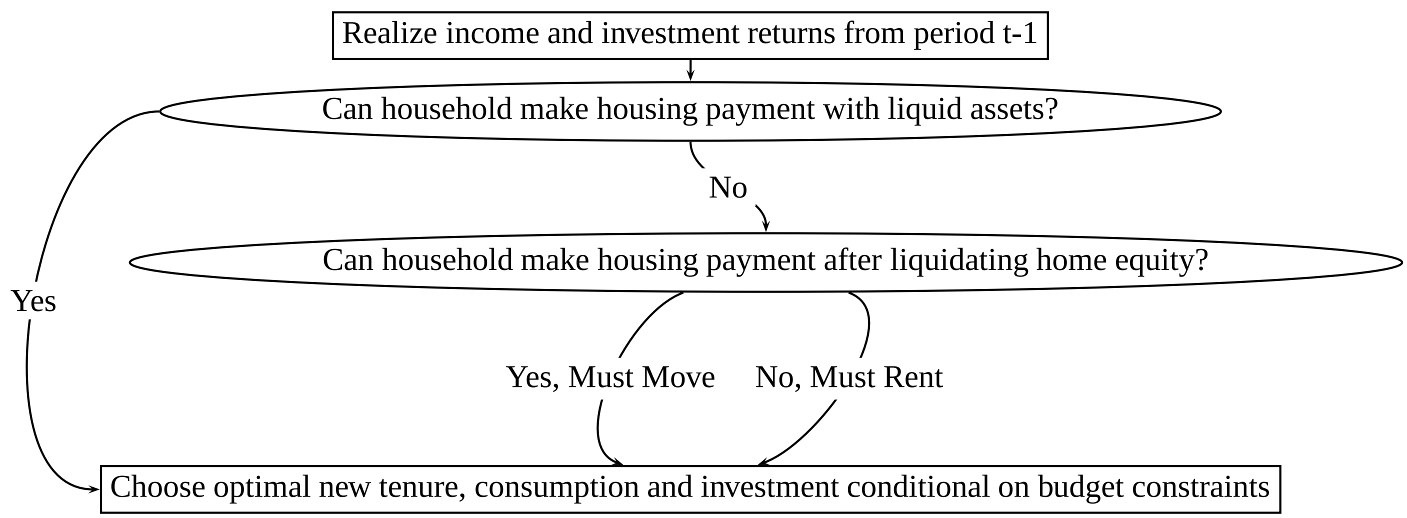
$$Z_t(\kappa_t,\kappa_{t+1}) = \begin{cases} (1-\zeta) D_t(i_t,\kappa_{t+1}) - D_t(i_t,\kappa_t), & \kappa_{t+1} \neq \kappa_t + 1 \\ 0, & \kappa_{t+1} = \kappa_t + 1 \end{cases} \tag{3.17}$$

where ζ represent the transaction costs associated with refinancing. When there is a cash-out refinance the household is increasing the number of years left on the mortgage, $\kappa_{t+1} < \kappa_t + 1$ and $Z_t(\kappa_t,\kappa_{t+1}) > 0$. Only households who choose not to move in a given period may choose to refinance.

The effect of steadily increasing home prices provides another argument for the inclusion of the age of the mortgage as a state variable. Due to the steady increase in home prices, the initial mortgage on a given home today would be significantly greater than the mortgage on a similar home twenty years ago. The current mortgage payments on these two similar homes would reflect this, with the mortgage payment for the home with the twenty-year old mortgage being significantly less than the payment for the home with the new mortgage. The implication is that there might be some economic value to the ability to lock-in the recurring housing expense at a fixed level while the market price of housing fluctuates. This allows the model to capture the role of housing as a hedge against variability in rents, as argued by Sinai and Souleles (2003).

3.6 Default Penalties

Figure 3.1: Timing of Decisions



The model also contains a default penalty. In any period the household must be able to cover its housing expenses, including the rent or mortgage and maintenance costs. If it fails to do so, it must move the next period into rental housing, forfeiting all its home equity and all its financial equity above some small nominal amount. Households that can cover their expenses by selling their current house and extracting their home equity are allowed to do so. Households that can afford the associated transaction costs may also avoid defaulting through a cash out refinance. The advantage of this for the household is the ability to keep its housing equity. Current consumption is also constrained to equal that same small nominal amount. The first constraint, shown in equation

(3.18), affects those households that are forced to move but can avoid defaulting and the second constraint affects those households that default. The restriction that A_{t+1} may not be negative, combined with the definitions of $X_t(i_t,\kappa_t)$, $Z_t(\kappa_t,\kappa_{t+1})$, and $G_t(i_t,i_{t+1},\kappa_t)$, along with the budget constraint, create an upper bound on possible levels of non-durable consumption, and also rule out some possible choices of housing tenure. If the household cannot afford the down payment for a large home without incurring negative wealth, it is not allowed to move to such a home. The flow chart above shows how the default penalties affect the household's decisions.

3.7 Optimization Problem and Value Functions

The household's optimization problem is to choose variables $c_t,\alpha_t,i_{t+1},\kappa_{t+1}$ given a series of state variables t,κ_t,i_t,A_t,H_t to optimize equation (3.1) given equations (3.2) (3.16).

The value function of the household is the maximum utility, subject to the default constraints of the value functions for the households that choose next period tenure type $i_{t+1}\in\{i_r,i_s,i_l,i_n\}$:

$$\begin{aligned}&(A_t-X_t(i_t,\kappa_t)<0)\ \ \&\ \ (A_t-X_t(i_t,\kappa_t)+\max_{i_{t+1},\kappa_{t+1}}(G_t(i_t,i_{t+1})+Z_t(\kappa_t,\kappa_{t+1})))>0)\Rightarrow\\&V_t(i_t,A_t,H_t,\kappa_t)=\max_{i_{t+1}\neq i_n,c_t,\alpha_t}V_t^{i_{t+1}}(i_{t+1},A_t,H_t,\kappa_t)\end{aligned}\tag{3.18}$$

$$\begin{aligned}&(A_t-X_t(i_t,\kappa_t)<0)\ \ \&\ \ (A_t-X_t(i_t,\kappa_t)+\max_{i_{t+1},\kappa_{t+1}}(G_t(i_t,i_{t+1})+Z_t(\kappa_t,\kappa_{t+1})))>0)\Rightarrow\\&V_t(i_t,A_t,H_t,\kappa_t)=U(\omega,h(i_t))+\beta\rho_tV_t(i_r,\omega,0,0)+\beta(1-\rho_t)\theta_AU_B(\omega)\end{aligned}\tag{3.19}$$

$$\begin{aligned}&(A_t-X_t(i_t,\kappa_t)>0)\Rightarrow\\&V_t(i_t,A_t,H_t,\kappa_t)=\max_{i_{t+1}\in\{i_r,i_s,i_l,i_n\},c_t,\alpha_t,\kappa_{t+1}}V_t^{i_{t+1}}(i_m,A_t,H_t,\kappa_t)\end{aligned}\tag{3.20}$$

where the three different default states result in three different possible definitions of the value function and where ω is the amount of consumption and wealth protected in default from creditors.

The value function conditional on next period's tenure choice i_{t+1} is:

$$V_t^{i_{t+1}}(i_t, A_t, H_t, \kappa_t) = \begin{cases} \max\limits_{c_t, \alpha_t} U(c_t, h(i_t)) + \beta\rho_t V_t(i_{t+1}, A_{t+1}, H_{t+1}, 1) + & \\ \beta(1-\rho_t)(\theta_A U_B(A_t) + \theta_H U_B(H_t) - \theta_D U_B(D_t)), & i_{t+1} \in \{i_r, i_s, i_l\} \\ \max\limits_{c_t, \alpha_t, \kappa_{t+1}} U(c_t, h(i_t)) + \beta\rho_t V_t(i_{t+1}, A_{t+1}, H_{t+1}, 1) + & \\ \beta(1-\rho_t)(\theta_A U_B(A_t) + \theta_H U_B(H_t) - \theta_D U_B(D_t)), & i_{t+1} = i_n \end{cases} \tag{3.21}$$

such that equations (3.2) to (3.20) hold.

The structure of this problem contains several significant sources of non-continuity. The first is the discrete nature of housing tenure, which functions as both a choice and a state variable. The second main source of the non-continuity is the structure of the value function, which is defined as the maximum of over sixty-six different value functions, one for each possible combination of four tenure choices or two refinance options and eleven portfolio allocations. This non-continuity of the model prevents the use of analytical methods to derive a solution. It also prevents the derivation of Euler equations. The model is instead solved using computational methods based on the methods used in Rust and Phelan (1997).

The code used to solve this problem is in C. One solution of the problem initially took roughly two weeks on a dual processor Pentium Xeon 1.8GHz with 512K L2 cache and 1GB of RAM running Linux. In order to improve the run-time, the code was re-written to take advantage of parallel processing, using the Message Passing Interface (MPI) standard. In this version of the code one processor is designated the master while a pool of other processors are designated slaves. As the model is solved recursively by year, the master distributes the current value function for all previous years to the slaves. Each slave then solves for the optimal value function for a sub-set of state spaces for the given year. The slaves then return the new value function values to the master. The master then combines the new values with the value function for the previous year, completing the recursion for one year. The problem was solved using 61 high-performance Digital Alpha 64-bit microprocessors running at 450MHz each on a scalable parallel Cray T3E at the

Pittsburgh Supercomputing Center. One solution involved roughly 1.3 billion evaluations of the value function and took roughly eight and a half hours.

4 Baseline Model Results.

The parameter values for the model calibration are chosen to be consistent with other models in the relevant literature. The parameter values for the size of small and large homes are set so that they represent, respectively, a home 80% and 120% the size of a median priced home. The ϕ value of 0.2 reflects the share of total household expenditures allocated to housing expenditures in the 2001 Consumer Expenditure Survey from the U.S. Department of Labor. This paper does not represent a serious attempt to calibrate a model of housing wealth or to estimate the maximum likelihood parameters of such a model. The goal is to see how closely the model can match certain stylized facts while using fairly standard and common parameter values. Appendix A contains more information on the values of the market and preference parameters chosen. A series of graphs of the policy functions, from one of the calibrated models, for households receiving different series of shocks are then presented, to illuminate the factors driving the economic decisions of the household. Finally, some results from simulations based on the baseline model are given. The baseline model matches several patterns seen in the empirical data.

4.1 Policy Functions

Figures 4.1 through 4.4 report sample policy functions for a range of households. Each figures contains the policy functions for three different types of households over the life-cycle, based on the type of shocks to income and the returns to both housing and risky assets. In each figure, the top panel reports the policy function for a household that receives in each period above average shocks, the middle panel reports the policy functions for a household receiving average shocks, and the final panel reports the policy function for a household that receives in each period below average shocks. It is important to note that these households do not realize that their future shocks have been artificially pre-ordained. They each believe that the shocks each period are independent from those in other periods, just as was the case when the model was solved.

The three panels in Figure 4.1 shows the tenure choices for each of our three sample households as a function of age. Naturally the household with the above average shocks is the first to purchase

Figure 4.1: Housing Tenure Policy Functions

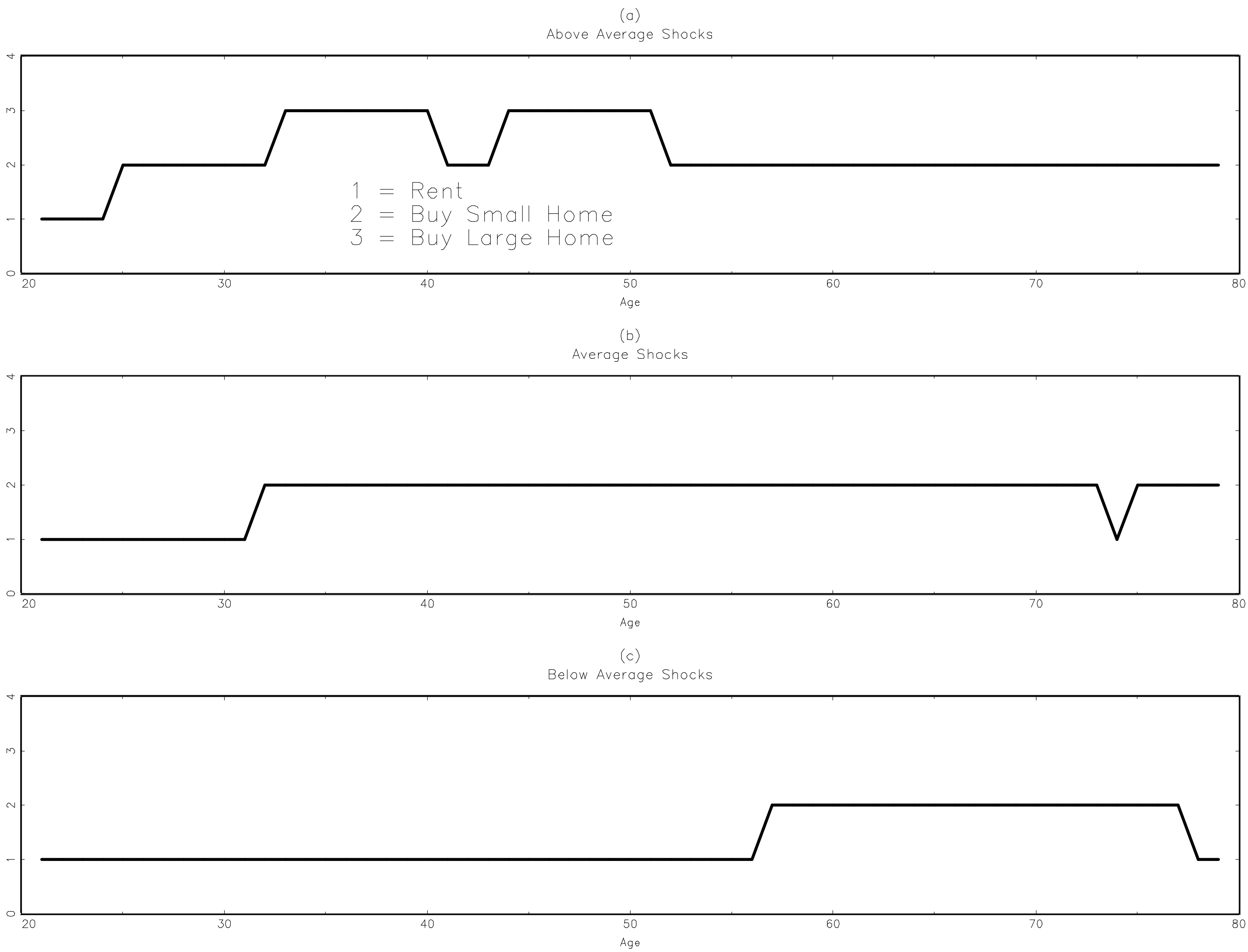


Figure 4.2: Consumption Policy Functions and Realized Wages

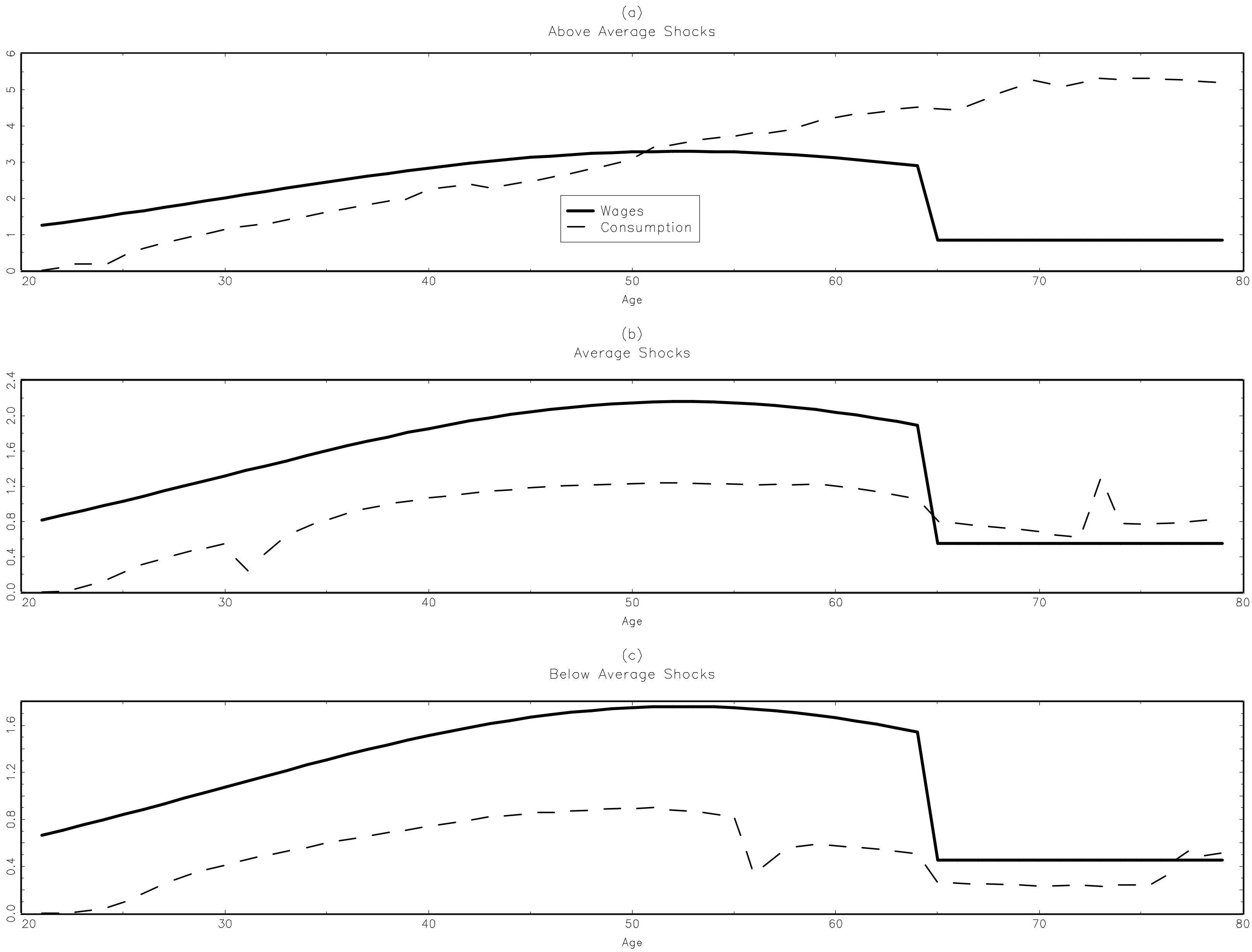
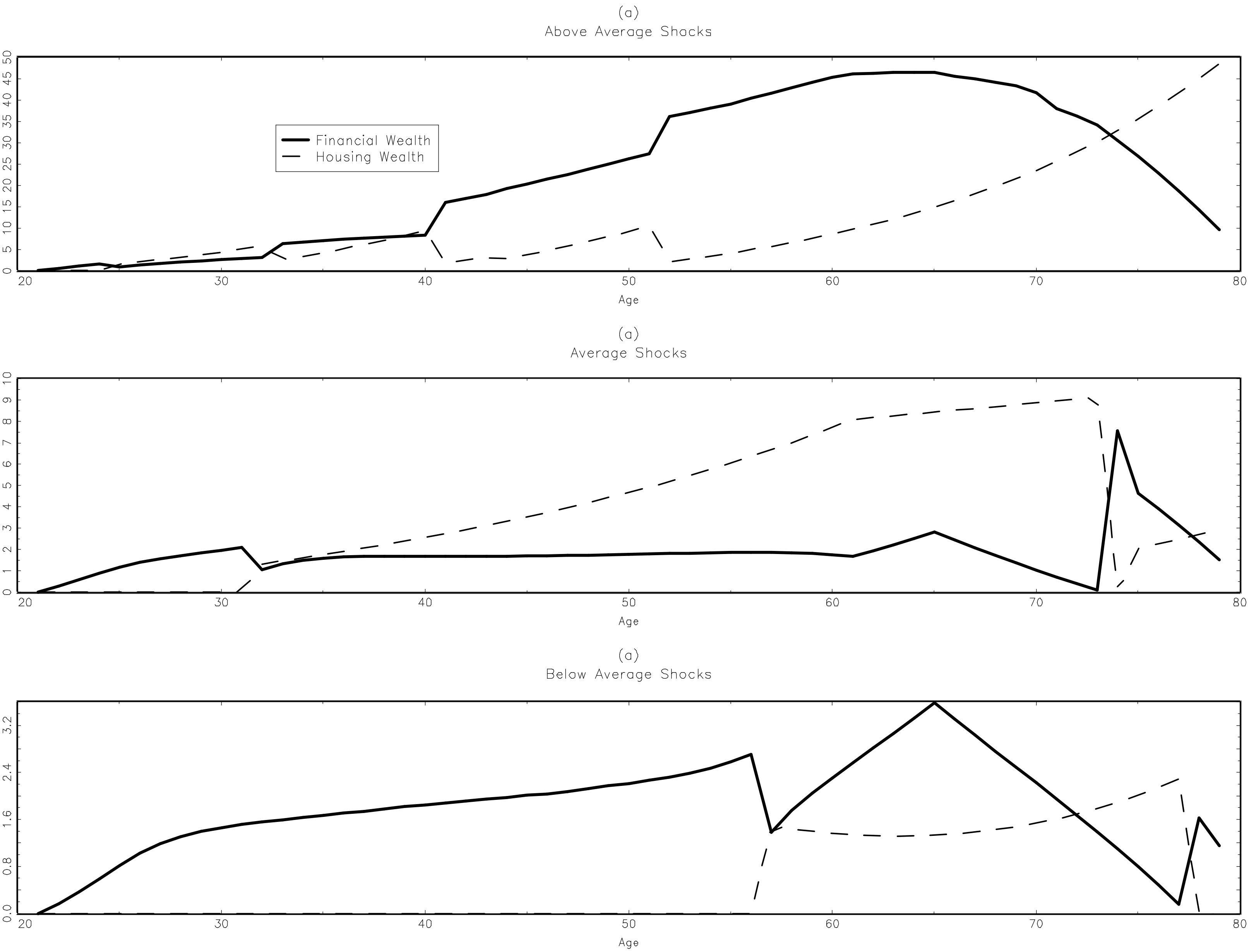


Figure 4.3: Home Equity and Financial Wealth Policy Functions

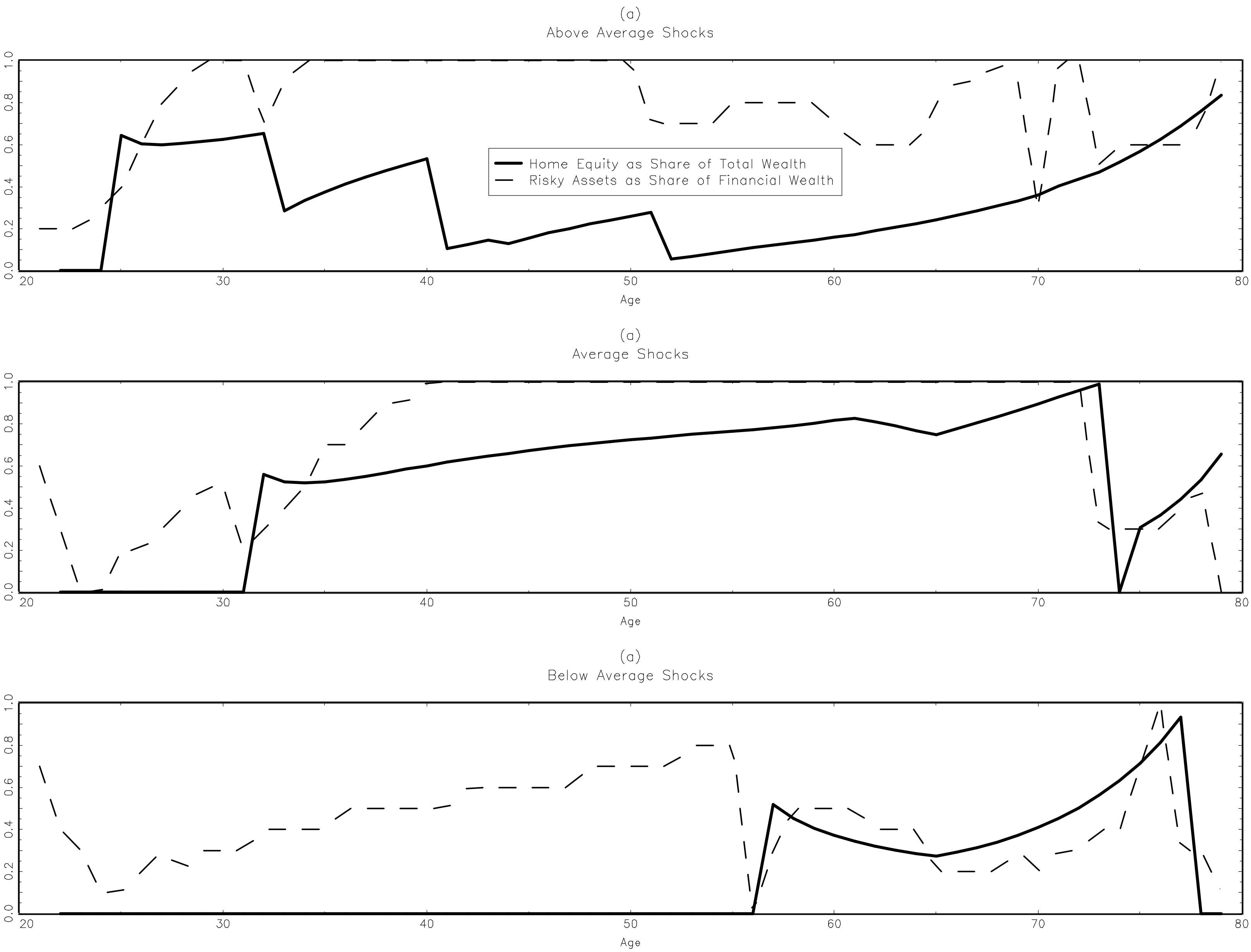


a home in their mid-twenties. The average household is only able to afford this transition in their earlier thirties while the below average household is forced to wait until their mid-fifties. The above average household is also able to trade-up to a larger home in their earlier thirties. In about ten-years they trade back down to a small home, shifting a significant portion of their wealth from housing to financial assets and reducing their mortgage payment. After a few more years of above average returns, they trade-up again, only to trade-down again after age 50. Once they reach this age, they can lock in their nominal mortgage payments for the rest of their life by purchasing a smaller home. The average household stays in their home until the mid-seventies when they spend a brief time renting, before buying another small home. The below average household sells their home in their late-seventies and rents for the rest of their life.

Figure 4.2 shows the consumption policy functions and realized wages for each of the three households. The higher realized investment returns allows the above average household to consume more than their annual wage by the time they are fifty. As they continue to receive above average shocks, they continue to increase their consumption. One interesting results is that each of these three households reduce their consumption immediately prior to purchasing a home. They also increase their consumption when they trade-down. Households who choose not to move also have higher levels of consumption. Since they are not adjusting their housing consumption, they compensate by increasing their consumption of non-durables. This pattern of behavior is similar to that described in Martin (2001)

Figure 4.3 shows how the housing and financial wealth policy function for the three households. It shows how financial wealth falls when households purchase homes, representing the effect of the downpayment and transaction costs. The figure also shows how households shift wealth back from housing to financial assets. Figure 4.4 reports the portfolio allocation policy functions for the three households. Households who are remaining renters invest a smaller amount of their portfolio in risky assets. They are focused on saving for a downpayment as quickly as possible, giving them a fairly short time horizon. The renters therefore choose a conservative portfolio that is more tilted towards asset protection than asset growth. Those households purchasing homes now own

Figure 4.4: Portfolio Allocation Policy Function



a second risky asset, their house, that is uncorrelated with the risky financial asset. In response to their increased diversification, they increase their investment in the risky asset. In the period in which households purchase their home they also sharply reduce their holdings in the risky asset.

4.2 Simulation Results

To better explore the implications of the model, 1,000 simulations are generated using the calibrated model. The table and figures below contain the results from these simulations. Households begin at age 20 as renters with no assets. Households retire at age 65 and live to at most 80 years of age. The simulations track their accumulation of housing and financial wealth over their lifetime. Figures 4.5 and 4.7 present the simulation results across the life cycle. These figures show the role of housing over the life cycle, and how consumption and investment decisions are linked to housing decisions.

Figure 4.5 shows the consumption and income paths over the life-cycle. The sharp drop in income in retirement can be seen in panel (a), while consumption is much smoother. Panel (b) shows the path of consumption as a share of total wealth. Younger households who are aggressively saving for a downpayment consume the smallest share of their wealth. Once households become homeowners, their consumption as a share of total wealth climbs, peaking near 16% around the age of 30. As households approach retirement, they start to accumulate more wealth, and consumption as a share of total wealth starts to fall reaching a low point of 9% at age 65. In retirement households draw down their savings and consumption as a share of total wealth climbs again. At retirement the average household has roughly forty-five times their annual income saved in both housing and financial wealth.

The importance of housing wealth in retirement is emphasized by the next set of figures. Figure 4.6 (a) shows that housing wealth has a hump over the life cycle, reaching a peak at 60 and starting to decline as households approach retirement. The sharp downward movement in housing wealth at age 50 is caused by many households either trading down to smaller homes or refinancing their existing mortgage in order to lock in nominal mortgage payments for the rest of their expected

Figure 4.5: Consumption and Income

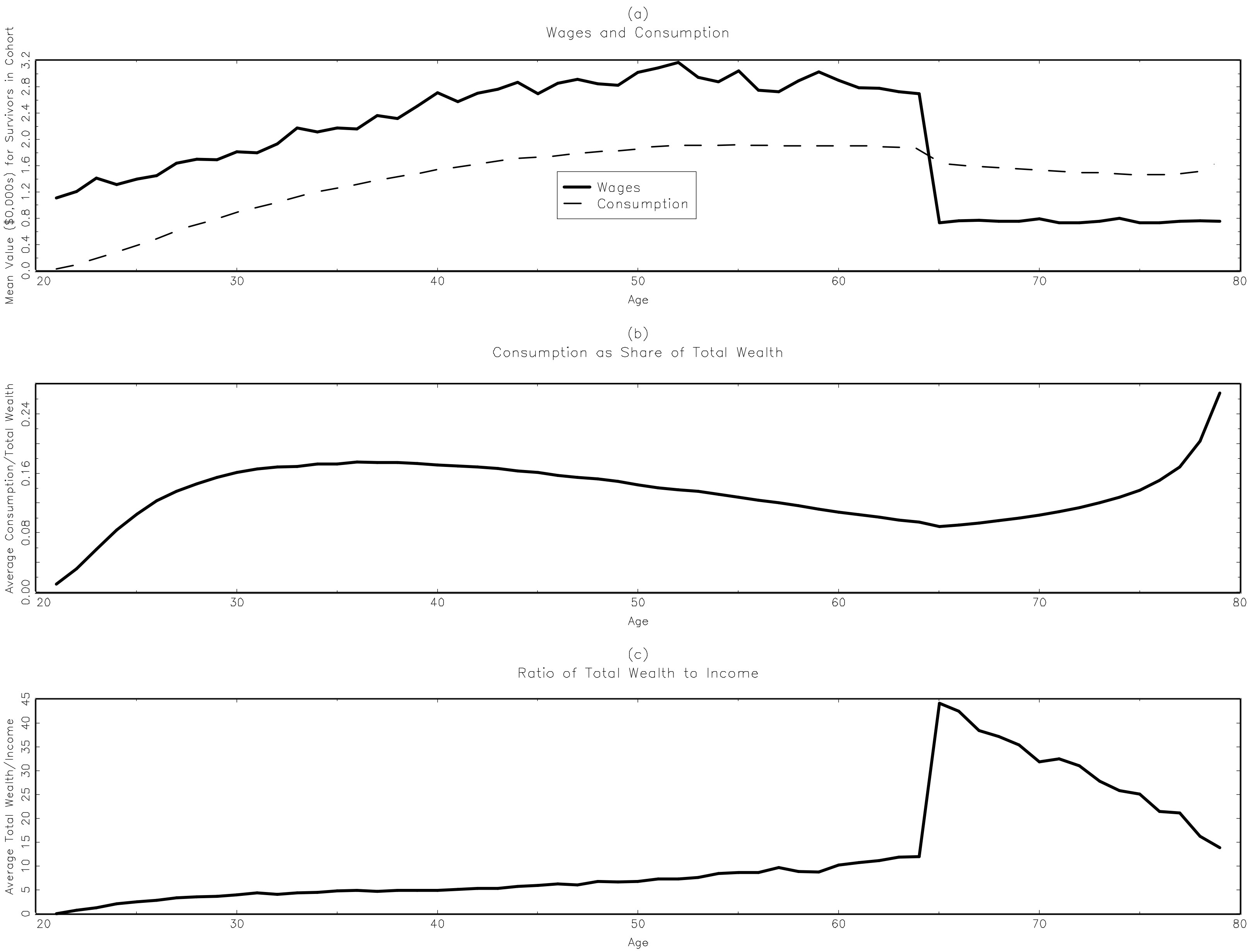
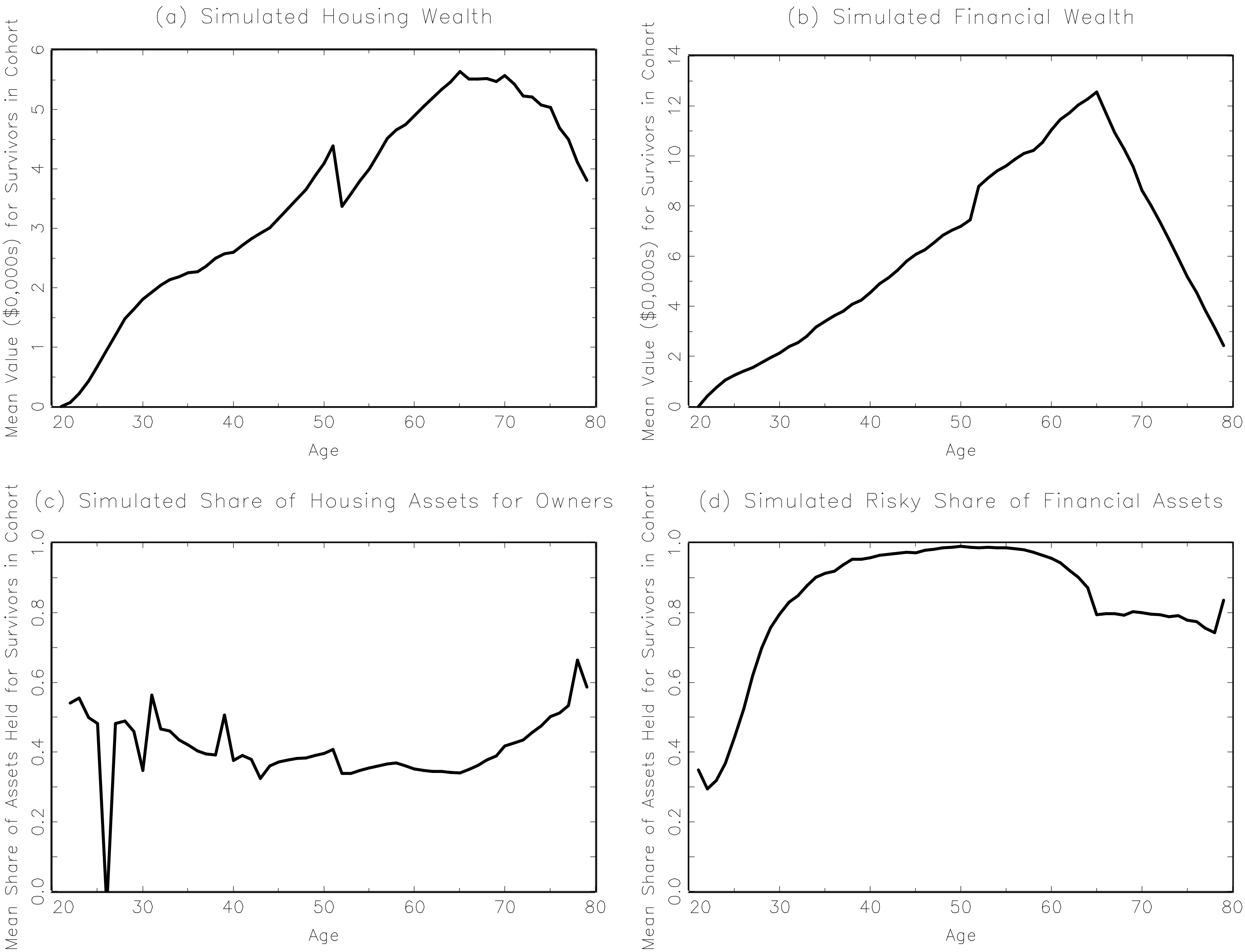


Figure 4.6: Wealth and Portfolio Choice

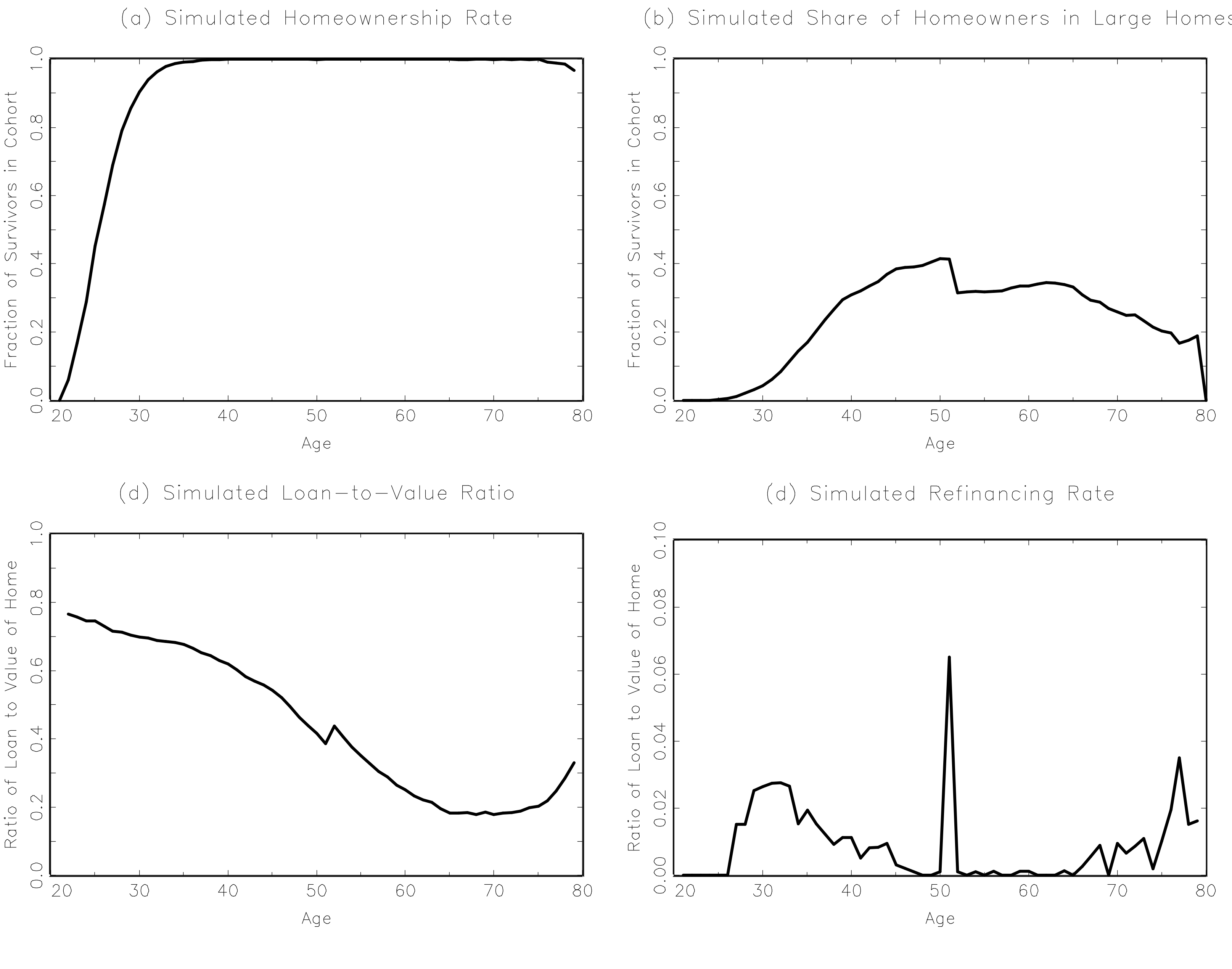


life. Financial wealth, shown in Figure 4.6 (b), is more sharply humped and peaks at age 65. As their first homes continue to increase in value, households start shifting their wealth to the financial portfolio. This can be seen in the sharp up-tick in financial wealth at age 50 that corresponds with the downtick seen in housing wealth.

One implication of the model is that accumulated home equity is used to finance the consumption of non-durables in only late in retirement. The actual role of housing wealth among the elderly is a bit more complicated. Venti and Wise (2000) found that housing wealth was not in fact used to support non-housing consumption. They find that households resort to their home equity only when faced by a significant shock such as the death of a spouse or a serious illness. This is similar to the finding in Sheiner and Weiss (1992) that anticipation of death and illness significantly increases the probability that households reduce their home equity. These conclusions find additional support in the results of this model.

Figures 4.6 (c) and 4.6 (d) provide the most significant results of the model. As Figure 4.6 (c) shows, the simulated share of assets held in housing is consistently near 40%, a bit below the empirical average of 67%. The housing share is high among young households who must invest a large portion of their savings in a downpayment. As financial wealth grows faster than housing wealth this share falls initially. The jagged nature of the curve reflects a combination of refinancing and trading up as younger households try to keep their portfolios balanced. The sharp drop at age fifty reflects the desire to lock-in nominal mortgage payments during retirement. The housing share starts to grow after that point as the growth of housing wealth outpaces that of financial wealth in part due to the power of leveraging. The return on the house is based on the total value of the house, not just the downpayment made by the household. The rate of increase in the share climbs in retirement, as households draw down financial wealth prior to extracting home equity. Household's face significant transaction costs, due in part to the nature of the mortgage contract, to access their home equity. As a result, households turn to financial equity initially to fund consumption in retirement. This partially matches the ”over-investment” in housing seen in the empirical data, as reported by Flavin and Yamashita, using a model of rational, forward

Figure 4.7: Housing Tenure Choice



looking agents. The implication is that while some degree ”over-investment” in housing is the result of something innate in the nature of the housing good or the mortgage contract used to purchase it and not the result of sub-optimal behavior by non-rational consumers, the actual level of ”over-investment” in housing seen empirically cannot be fully explained.

Figure 4.6 (d) shows the pattern of allocation in the financial portfolio over the life cycle. Young households who are aggressively saving for or already have large shares of their wealth tied up in downpayments invest less in the risky asset, as do older households who have drawn down their financial wealth relative to their housing wealth. The risky portfolio share peaks around age 50, just when the households start to actively shift their total portfolio away from home equity.

The final set of figures from the simulations document the role of housing over the life-cycle. Figure 4.7 (a) shows home-ownership increasing rapidly for younger households and declining very slightly in retirement. The share of homeowners living in larger homes has a similar hump, as seen in Figure 4.7 (b), with a sharp drop at age 50. Both of these charts document the strategy of households trading down in retirement to access housing wealth to finance consumption. Figure 4.7 (c) documents an interesting pattern. Households who have recently purchased their homes are required to have an initial loan-to-value ratio of 80%. They are then able to pay down their mortgage through the regular amortization schedule and the average loan-to-value ratio falls. The average loan-to-value ratio seems to stabilize at 20%. Figure 4.7 (d) reports the level of refinancing activity over the life-cycle. Younger households and those who have just purchased their homes take advantage of refinancing to re-balance their portfolios and smooth their income. Their is a sharp jump in refinancing at age 50 as households lock in their mortgage payments for retirement. Older households start to use cash-out refinances to access their equity.

Table 4.1 below shows some of the sample statistics from the simulation results. The share of the total portfolio held in home equity for large homeowners is 38.7% and for small homeowners is 40.3%. These numbers show that the model is partially successful in capturing the ”over-investment” in housing. The model also captures how wealthier, better diversified households tend to own larger homes. These results also show how renters, who are aggressively saving for a down payment, have the smallest risky asset portfolio share.

Table 4.1: Simulation Results - Baseline Model

	Total	Rental Units	Small Homes	Large Homes
Percent	100%	10.8%	65.3%	23.9%
Consumption	13,910	1,760	14,520	17,770
	(7,310)	(2,240)	(6,180)	(5,880)
Financial Assets	59,290	7,490	58,080	86,030
	(60,620)	(7,410)	(59,130)	(62,170)
Risky Asset Share	83.5%	28.5%	89.7%	91.5%
	(25.6%)	(24.2%)	(16.3%)	(14.5%)
Tenure Length	8.6	1.0	9.1	10.5
	(8.5)	(0.0)	(8.5)	(8.6)
Net Equity in Home			31,110	50,160
			(22,940)	(34,480)
Home Equity Share			40.3%	38.7%
			(200.4%)	(45.0%)

Note: The standard deviations are presented in parentheses.

4.3 Tenure Transitions

Table 4.2 more fully explores the role of housing tenure decisions in the model. It demonstrates that households are eager to move out of rental housing with over 20% of renters purchasing homes. Households that have saved enough money by their mid-twenties are able to move into small homes. In a few cases first-time home buyers move straight into larger homes. Households that are still saving for a downpayment tend to have the least held in risky assets, only 26.9% of the financial portfolio. Households that have already saved enough to purchase a home hold more in risky assets.

The transition out of small homes seldom occurs. Almost 96.5% of small home-owners remain in small homes. Most of the 3.5% that are moving are trading up to larger homes. Households who manage to trade-up, do so fairly quickly, averaging less than eight years in their current home and being in their earlier forties. Households with low loan-to-value ratios but high share of housing in their portfolio are the most likely to take cash-out refinances. Households who are staying put and not refinancing have the largest risky asset share in their financial portfolio, while those being forced to move to rental housing have the lowest. Households trading up also hold lower shares of risky assets in their financial portfolios. This is another example of the inverse relationship between demand for home equity and for risky assets. Recent negative income and investment shocks contribute to the decision to return to rental housing while very strong positive shocks to housing encourage the households to trade-up. Households also respond to negative income shocks with cash-out refinancing, providing an example of the role of housing wealth in smoothing income shocks.

Table 4.2: Tenure Transitions - Baseline Model

Current Status		Not Moving	Rent	Buy Small Home	Buy Large Home	Cash-Out Refi
Renter	Transition Probability		80.6%	19.2%	0.2%	
	Financial Wealth		6,190	12,920	12,980	
	Risky Portfolio Share		26.9%	35.0%	77.8%	
	Age		23.8	26.2	27.0	
	Risky Asset Shock Last Period		5.1%	6.3%	-1.6%	
	Wage Shock Last Period		-1,300	190	-1,620	
Small Home	Transition Probability	96.5%	0.3%		1.9%	1.3%
	Financial Wealth	58,930	4,610		48,670	19,680
	Housing Wealth	30,280	51,840		59,030	47,490
	Risky Portfolio Share	90.3%	45.6%		79.6%	64.4%
	Portfolio Share of Housing	39.3%	92.9%		61.3%	74.3%
	Loan-To-Value Ratio	45.0%	24.1%		38.1%	39.0%
	Age	48.5	58.5		41.6	43.4
	Tenure Length	9.1	5.2		7.8	10.1
	Housing Shock Last Period	0.7%	6.4%		14.3%	4.7%
	Risky Asset Shock Last Period	5.2%	-1.1%		6.7%	2.0%
	Wage Shock Last Period	5,950	-1,440		8,570	-880
Large Home	Transition Probability	96.3%	0.1%	3.5%		0.1%
	Financial Wealth	87,270	660	53,110		16,030
	Housing Wealth	48,591	39,560	93,120		121,490
	Risky Portfolio Share	92.1%	57.5%	76.4%		63.3%
	Portfolio Share of Housing	37.6%	98.3%	67.6%		89.5%
	Loan-To-Value Ratio	46.1%	7.2%	31.5%		11.1%
	Age	53.1	79.0	56.6		63.0
	Tenure Length	10.6	6.0	7.5		15.0
	Housing Shock Last Period	0.7%	-3.6%	10.0%		2.9%
	Risky Asset Shock Last Period	5.7%	8.1%	-2.5%		24.9%
	Wage Shock Last Period	7,970	1,450	3,560		2,930

Once households have managed to move into large homes, they naturally prefer to stay there. Over 96% of large homeowners do not move. Large homeowners with very small loan-to-value ratios and high shares of housing in their portfolio tend to utilize cash-out refinancing to rebalance their portfolio. Those with lower values of housing wealth and higher loan-to-value ratios will instead trade down into a smaller house. Large homeowners with very little home-equity who face a cash crunch must return to rental housing. Again, these homeowners take the most conservative positions in the financial portfolio.

This section has established the most significant accomplishment of the model the ability to partially match the ”over-investment” in housing seen in the data within a framework of rational, forward-looking agents. The next section will build on these results with a detailed examination of how the unique nature of housing wealth affects the demand for housing and the allocation of household portfolios.

5 Alternative Scenarios.

The previous section provided evidence that the baseline model can match certain stylized facts about housing wealth over the life-cycle. This section determines exactly in what way does these three specific aspects of housing wealth affect the demand for housing and financial portfolio allocation over the life-cycle. This is accomplished through a series of comparative static exercises. The three aspects of housing wealth being investigated are in each turn excluded from the model. Each alternative model is then re-solved and the simulations regenerated. The levels of wealth accumulation, housing demand, refinance activity, and portfolio allocation under each alternative assumption are then compared to the base case. Table 5.1 summarizes how the model is altered for each of the alternative scenarios.

Table 5.1: Alternative Scenarios

Alternative Assumption	Model Effects
Housing's Dual Investment/Consumption Role	
Consumption Only	Replace (3.16) with (5.1)
	Replace (3.17) with (5.2)
Investment Only	Replace (3.1) with (5.3)
	Replace (3.13) with (5.4)
Nominal Mortgage Contract	
High Inflation	$\nu = 6\%$
Real Mortgage Contract	$\nu = 0\%$
Taxes	
Tax Implicit Rent	Replace (3.13) with (5.5)
Tax Capital Gains	Replace (3.16) with (5.6)
No Mortgage Int. Deduction	$I_t(i_t, \kappa_t) = 0, i_t \in i_r, i_s, i_l, i_n$

5.1 Housing as an Investment and Consumption Good

The first set of alternate scenarios are used to examine the consumption and investment motives for housing demand in isolation. To redefine housing as only a consumption good, the equation for the net gain from a home sale is redefined. Households face the same downpayment constraints and transaction costs for selling a home. They also make the same mortgage payments and still must repay the outstanding mortgage balance when they sell the home. The difference is that these expenses merely purchase the stream of housing services associated with the home, not the home itself. Under this alternative, no such thing as housing wealth exists. Households have no home equity to access either by trading down or through cash out refinances. The new equation for the net gain from selling a home is:

$$G_t(i_t, i_{t+1}, \kappa_t) = \begin{cases} -\max(D_t(i_t, \kappa_t) - H_t) - \mu P_t(i_{t+1}) - \tau H_t - \chi, & i_{t+1} \in i_r, i_s, i_l \\ 0, & i_{t+1} = i_n \end{cases} \tag{5.1}$$

and the new equation for the net gain from refinancing is:

$$Z_t(\kappa_t, \kappa_{t+1}) = 0 \tag{5.2}$$

The flip side of this scenario is one in which housing acts only as an investment good and does not directly enter the utility function. Under this scenario all households rent in every period of their life. The households always choose the number of housing units to rent such that intra-period marginal utility of housing is equal to the marginal utility of non-durable consumption. Households may also purchase small or large homes. However, now they do not consume the housing services associated with these homes. Rather, they rent these homes and receive a stream of income based on the market rental rates. Homes are still purchased with a mortgage and still have all their previous tax advantages. Now the utility function assumes that households always rent:

$$E\sum_{t=20}^{80} \beta^t \rho_t U(c_t, h(i_r)) + \beta^t(1-\rho_t)(\theta_A U_B(A_t) + \theta_H U_B(H_t) - \theta_D U_B(D_t)) \quad c_t > 0, \forall t \tag{5.3}$$

The housing costs for home owners include their new rent payment for the housing that they consume, $0.06P_t(i_r)$, and the rent payment they receive as landlord for the housing that they own, $0.06P_t(i_t)$;

$$X_t(i_t, \kappa_t) = \begin{cases} 0.06P_t(i_r) - 0.06P_t(i_t) + M_t(i_t, \kappa_t) + \delta H_t, & i_t \in i_s, i_l \\ 0.06P_t(i_r), & i_t = i_r \end{cases} \tag{5.4}$$

Figures 5.1 and 5.2 show the simulation results for the alternate scenarios where housing is a consumption good only and for when it is an investment good only. Note that when housing is a consumption good only, no housing wealth results and naturally no share of the total portfolio is held in housing. When housing is a consumption good only, households hold slightly more financial wealth. They also change the allocation of their financial portfolio, with the younger and older households holding fewer risky assets. Young households no longer are able to diversify across both risky financial assets and risky housing equity and invest less of their portfolio in risky

Figure 5.1: Wealth and Portfolio Choice

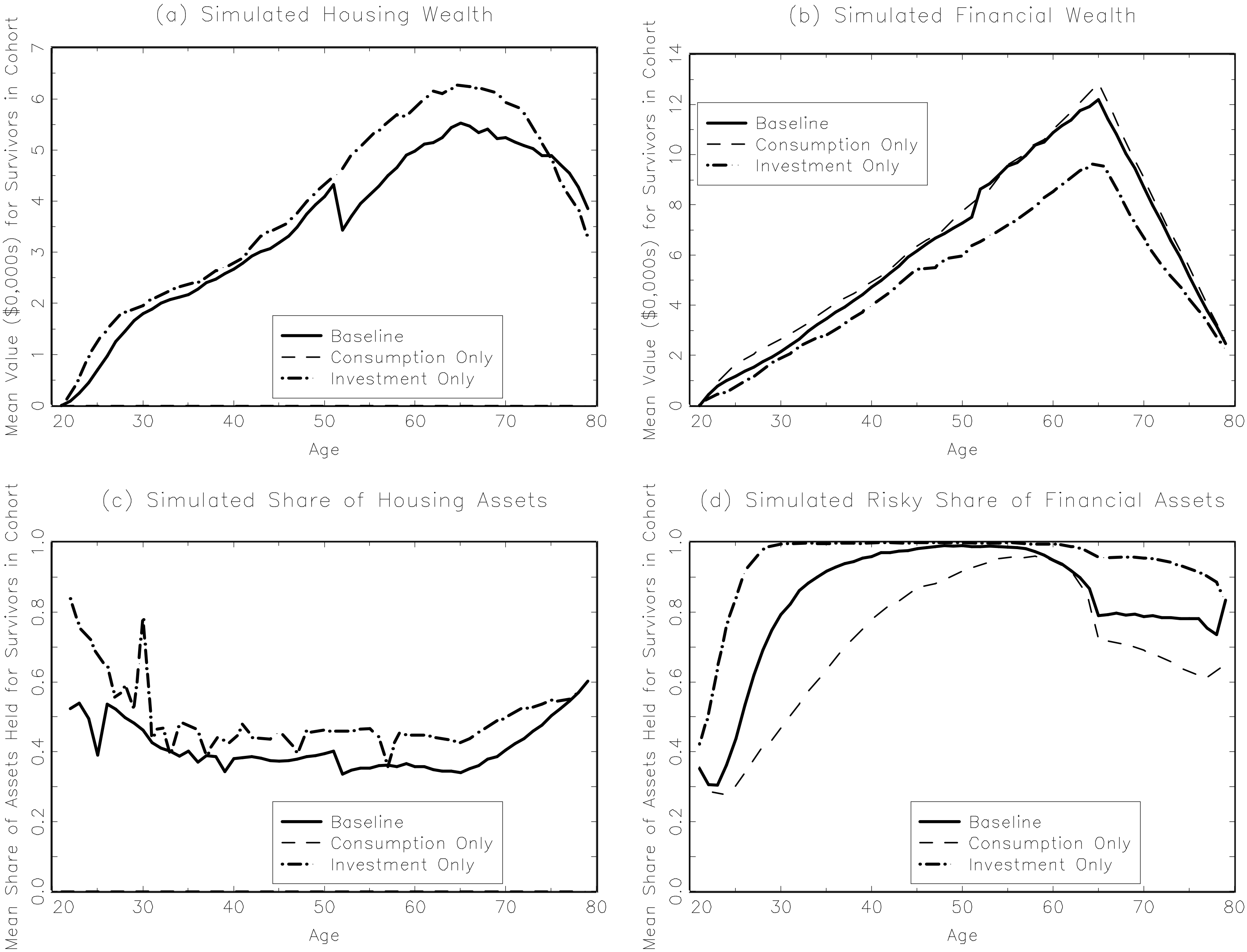
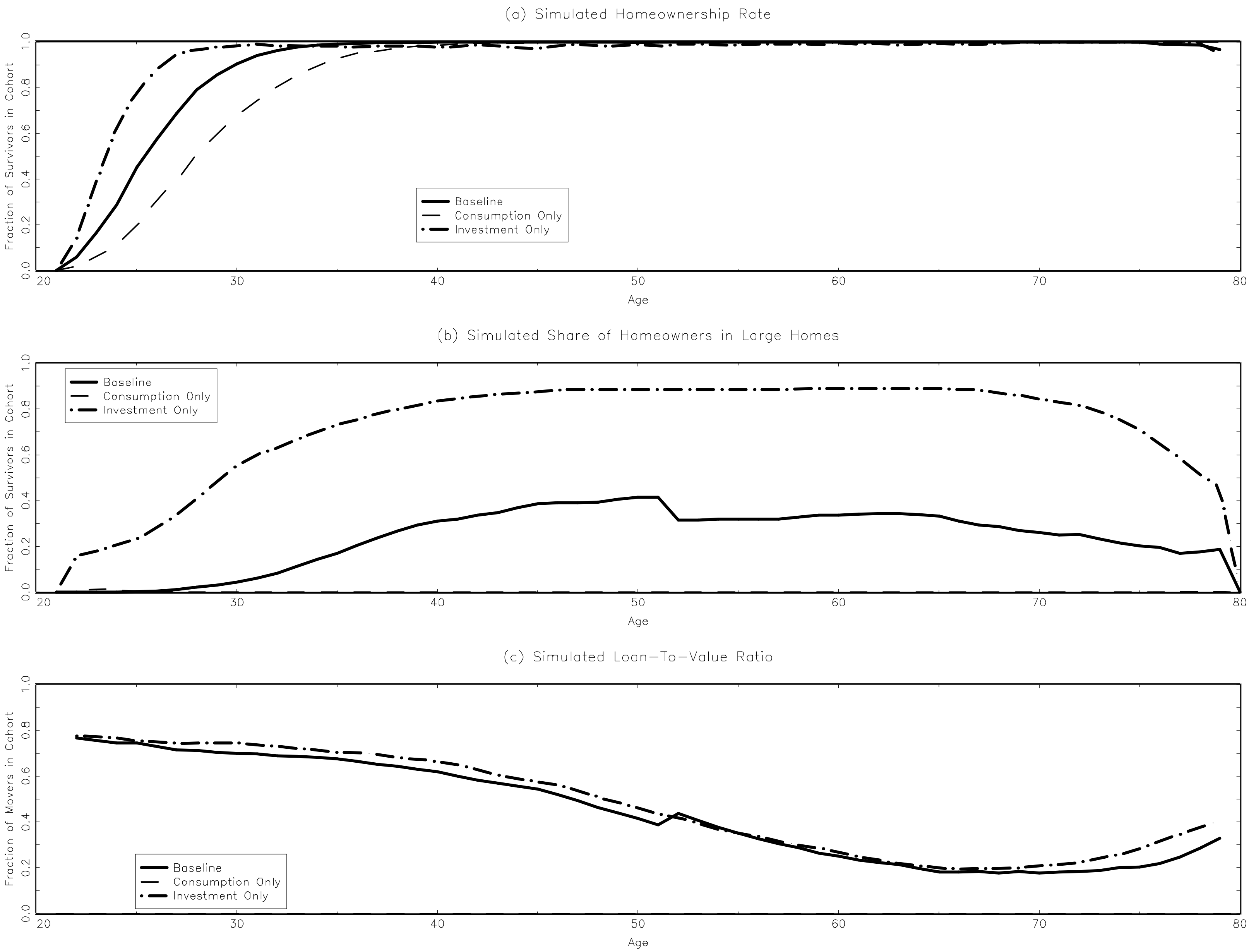


Figure 5.2: Housing Tenure Choice



assets. Older households no longer have a store of home equity that they may replenish their financial portfolio with through cash-out refinances. As a results, they too invest a small share of their portfolio in risky assets. The demand for housing also drops, with households buying their first home later in life, and almost no households purchasing large homes. This result implies that the demand for large homes, absent any demographic factors, is driven primarily by the investment motive.

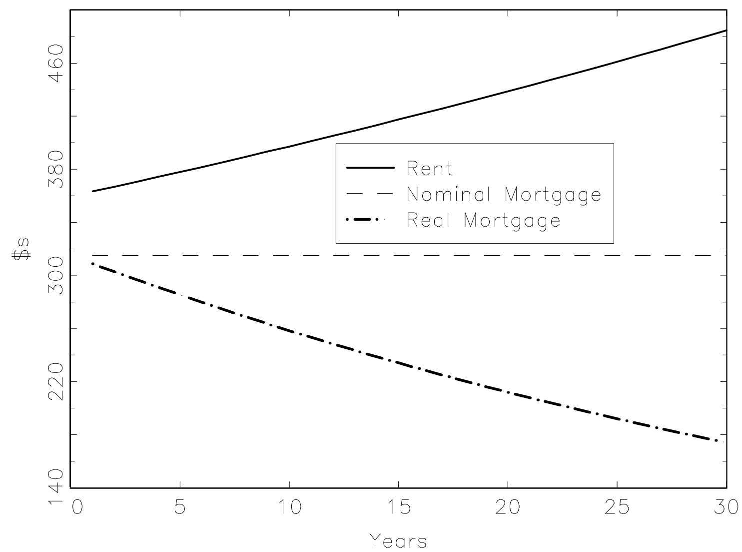
The results from the investment good only scenario are especially enlightening. A common explanation for the "over-investment" in housing is that household's simply desire to consume morehouse than they wish to invest in. Therefore, the consumption motive is often fingered as the culprit for the large share of the household's total portfolio held in home equity. Interestingly enough, when the consumption motive is removed the demand for housing increases. As can be seen in Figure 5.1(a) the level of housing wealth increases, at the expense of financial wealth. The share of the total housing portfolio held in home equity also increases as does the share of the financial portfolio held in risky assets. This increase in exposure to risk from both housing and risky assets is in response the homeowners now having the risk-free stream of rent payments from their housing investment. The presence of this risk free income stream increases their tolerance of risk elsewhere in their portfolio. The demand for housing increases as well with both the home-ownership rate and the share of homeowners buying larger houses increasing.

It is important when comparing the results of these two scenarios to keep in mind that the consumption and investment motives of housing are not being examined in isolation. The advantage of using the detailed partial equilibrium model is the richness of the detail. The benefits from the tax treatment of housing and most importantly the mortgage contract are held constant across the scenarios. Investors are drawn to the ability to create a leveraged portfolio through the mortgage and receive a steady tax-free stream of dividends. Consumers appreciate the ability to lock in their housing costs at a fixed amount, even though they will never see the money they put into the house again. The results emphasize the fact that the mortgage contract itself has significant economic value to the household, above and beyond the attraction of housing as a consumption or investment good.

5.2 Effects of Inflation on Housing Wealth

The next set of scenarios are relatively more simple to model. All that is needed in order to simulate the effects of high inflation, or of mortgage contracts that are real and not nominal is to set the inflation parameter respectively to a higher level or to zero. The presence of nominal mortgage contracts effectively reduces the costs of home-ownership. The higher the inflation rate,

Figure 5.3: Rent and Mortgage Payments



the lower the costs of home-ownership, as can be seen in Figure 5.3 which documents how the real value of the mortgage payment declines over the life of the mortgage. This is of course factored into the rate of the orginal mortgage and partially explains the gap between the mortgage and risk-free rate. Figures 5.4 and 5.5 show how housing demand and portfolio allocation differs under different inflation rates. High inflation increases the rate at which the nominal mortgage payments are discounted over time. As a result, there is a much more pronounced move from large to small homes at age 50 under the high inflation scenario. Households are eager to purchase small homes at age 50 and lock in their nominal mortgage payments for the rest of their life. In fact they almost never move or refinance after age 50, due to the increase value of the inflation discounting of the nominal mortgage payment. The main effect of zero inflation is to reduce the demand for large homes and remove the tendency to lock in nominal mortgage payments at age 50 as the real mortgage payments no longer decline with the age of the loan.

None of these results should come as a surprise. A reduction in the cost of housing will increase demand. Reduced housing costs will also have a wealth effect, increasing the amount of housing and financial wealth households can accumulate. What is of interest is the non-neutrality of inflation even when the inflation is perfectly anticipated. As was the case previously, this result is dependent on the nature of the mortgage contract, again highlighting the importance of including

Figure 5.4: Wealth and Portfolio Choice

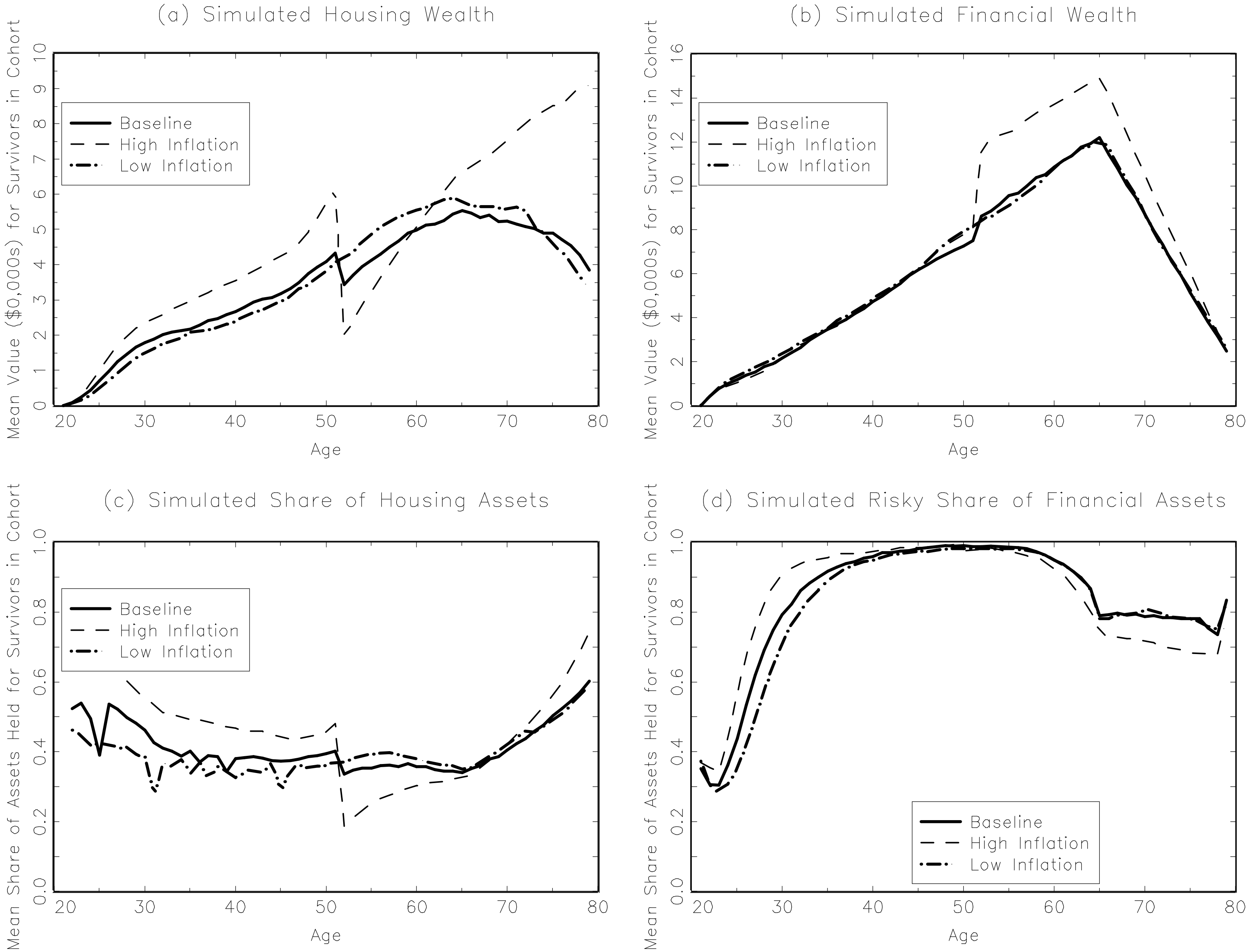
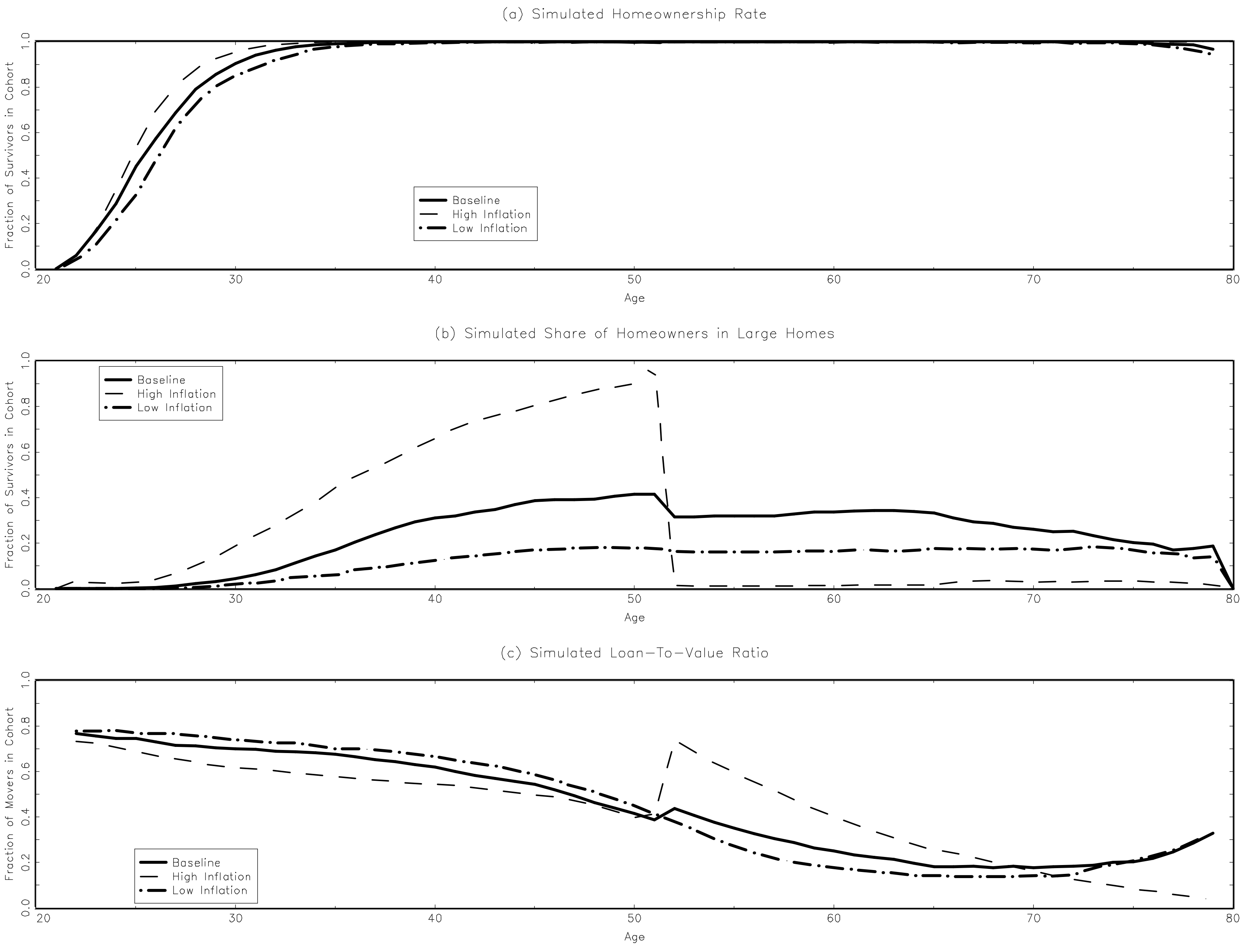


Figure 5.5: Housing Tenure Choice



mortgage contracts in models of housing wealth.

5.3 Tax Implications

The final aspect of housing wealth explored in this paper is three aspects of the tax treatment of housing: (1) the implicit rental from owner-occupied housing is taxed as regular income, (2) the capital gains from housing is taxed, and (3) mortgage interest is no longer tax-deductable. These final scenarioa explore to what extent this beneficial tax treatment contributes to the demand for housing and distorts household portfolio allocation. Under the first scenario, the household must pay tax on the level of implicit rent they receive from their home. Renters naturally receive no implicit rent, they are paying explicit rent out of after-tax income. The implicit rent is defined based on the market rent for the type of home and not the actual value of the individual home. The equation for the housing costs now is:

$$X_t(i_t,\kappa_t) = \begin{cases} M_t(i_t,\kappa_t) + \delta H_t + \gamma 0.06 P_t(i_t), & i_t \in i_s, i_l \\ 0.06 P_t(i_r), & i_t = i_r \end{cases} \tag{5.5}$$

The second version of this scenario imposes a tax on the capital gains from home sale. Recent tax legislation as progressively increase the amount of capital gains from home sales that can be shielded from tax.

$$G_t(i_t,i_{t+1},\kappa_t) = \begin{cases} (1-\gamma)(H_t - D_t(i_t,\kappa_t)) - \mu P_t(i_{t+1}) - \tau H_t - \chi, & i_{t+1} \in i_r, i_s, i_l \\ 0, & i_{t+1} = i_n \end{cases} \tag{5.6}$$

The final version of this scenario simple suppresses the mortgage interest tax deduction. Figures 5.6 and 5.7 show the simulation results from this scenario. In the case of the repeal of the mortgage interest tax deduction, there is almost no change in the economic behavior of the households, suggesting that this policy's contribution to increasing home ownership is questionable at best. The other two tax advantages have a far more significant effect. In both cases the additional costs to home ownership reduce the level of housing wealth and slightly increases the level of financial wealth. Households also hold less of their portfolio in housing and less of their financial portfolio in risky assets. They also purchase homes later in life and purchase fewer large homes. Interestingly the capital gains provision seems to have the greatest effect on behavior, expect in the case of the demand for large homes, where the tax on implicit rent seems to have the greatest effect. These results make it clear that the tax treatment of housing has significant and important

Figure 5.6: Wealth and Portfolio Choice

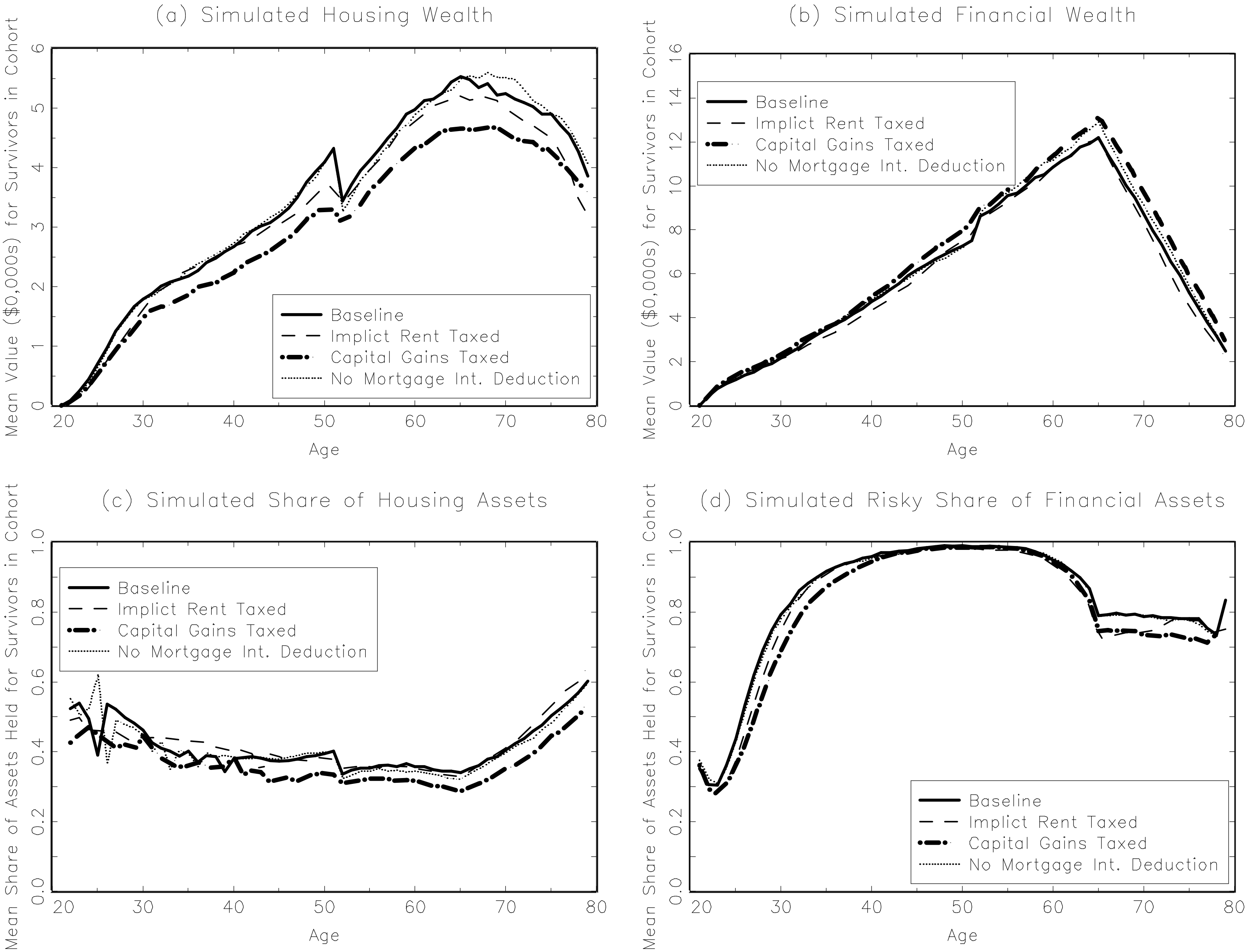
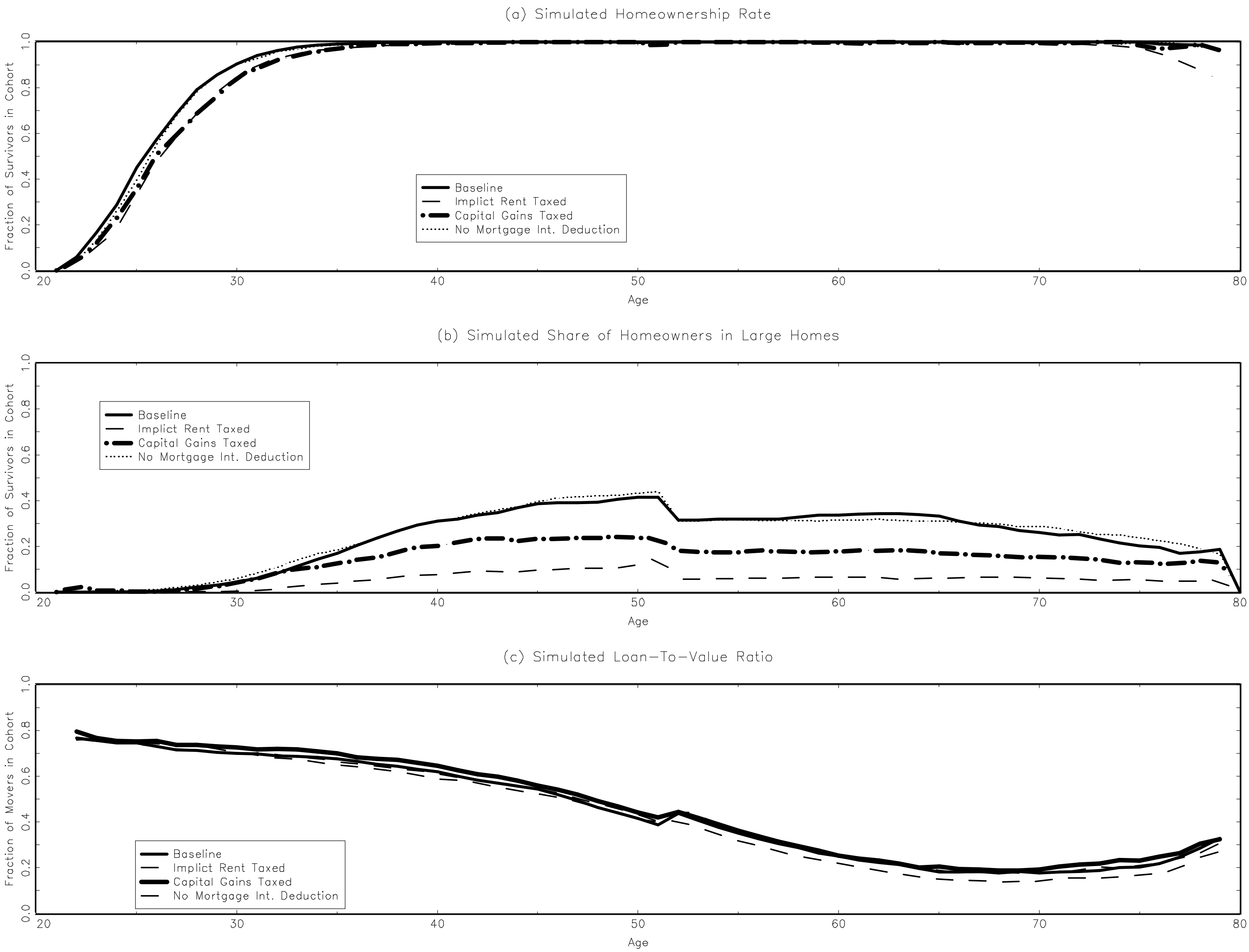


Figure 5.7: Housing Tenure Choice



impacts on economic behavior over the life-cycle.

6 Conclusion.

One of the goals of this paper is to explain the life-cycle patterns in both home ownership and portfolio allocation using a model of rational agents. Two key innovations are incorporated in the model. First, housing is explicitly modeled as both a consumption and investment good, as opposed to examining just one aspect in isolation from the other. Second, mortgage contracts are explicitly introduced into the model. The result is a more realistic treatment of the role of housing in an agent's economic decision-making over its lifetime. The model is then used to explore the relationship between the housing share and risky asset share of household portfolios.

The baseline model succeeds in partially replicating home equity's large position in household level portfolios. The implication is that while some degree ”over-investment” in housing is the result of something innate in the nature of the housing good or the mortgage contract used to purchase it and not the result of sub-optimal behavior by non-rational consumers, the actual level of ”over-investment” in housing seen empirically cannot be fully explained. The transitory nature of housing, as households react to wealth and income shocks by trading up and down, is also captured by the model. Other key results are the importance of housing wealth in retirement and the role of cash-out refinances. Finally, the model shows how the allocation of the financial portfolio varies in response to the position in housing wealth and tenure decisions.

The baseline model is then compared with a set of alternate scenarios to explore three key aspects of housing wealth: (1) housing's dual role as a consumption and investment good; (2) the significance of the mortgage contract being in nominal and not real terms; and (3) the benefits of the tax treatment of owner-occupied housing. The results show that the “over-investment” in housing is not just a function of consumption demand but can also be driven by the benefits inherent in the mortgage contract. It also shows that the nominal mortgage contract results in the non-neutrality of perfectly expected inflation. Finally, the contribution of the favorable tax treatment on housing demand is documented.

Appendix A - Baseline Model Parameter Values

The parameter values for the baseline model are chosen to be consistent with other models in the relevant literature. As was discussed in the Section 3, the income process consists of a deterministic and a transitory factor. The income process is based on the results of regressions of Social Security earnings on age and age-squared. The dependent variable is the log of the wage income in constant 1990 dollars. The transitory factor of wage is reflected in the estimated standard error of the regression. The wage is converted from log to level terms in the model. At age 65 the level of the deterministic wage falls to a flat level equal to 60% of the last period's income before any transitory shocks, representing a system of forced retirement and a defined benefit pension plan. The coefficients and standard deviation used in this version of the model are shown in Table A-1 below.

Table A-1: Log Income Regression Results

Constant	ψ_0	7.28626
Coefficient Age	ψ_1	0.10278
Coefficient of Age^2	ψ_2	-0.00098
Std. Dev.	σ_w	0.80778
R^2		15.5%
Probability of Unemployment	υ	1%

The market price of a housing unit is the result of setting the deterministic home price at age 60 with the National Association of Realtors' 1990 median home price. It is assumed that a median home consists of 10 housing units. The home prices are converted to constant 1990 dollars and the deterministic home price series are calculated using the historical average return. The average and standard deviation of the return on housing are at taken from Li and Yao (2004) and are consistent with Campbell Cocco (2003). The mortgage interest rate used is the average rate on loans with 80% loan-to-value ratios as reported by Freddie Mac from 1969 to 2001, adjusting for the inflation rate. The percent required for downpayment represents the minimum needed to avoid paying mortgage insurance. The transaction, maintenance, and moving costs are based on survey data provided by the National Association of Realtors. The values chosen for the current version of the model are presented in Table A-2 below. The risk and return on risky assets follows Yao and Zhang (2004).

The values for the preference parameters shown in Table A-3 below were chosen to replicate certain stylized facts about the role of owner-occupied housing in portfolios, specifically the large share of total wealth held in home equity. An λ value of 2 represents a relatively low, but realistic, level of risk aversion. An β value of 0.96 is a commonly used discount rate. The ϕ value of 0.2 reflects the share of total household expenditures allocated to housing expenditures in the 2001

Table A-2: Values of Market Parameters

Parameter Name and Definition	Symbol	Value
Real risk free rate of return	r	2%
Price of 1 housing unit, at age 60	$P_{60}(1)$	1.003
Size of small homes	$h(i_s)$	8
Size of large homes	$h(i_l)$	12
Mean of real return on housing	η_h	1%
Standard deviation of housing return	σ_h	11.5%
Mean of real return on risky asset	η_s	6%
Standard deviation of risky asset return	σ_s	15.7%
Probability of 100% loss on risky asset	ς	1%
Mortgage interest rate	π	5%
Percent required as downpayment	μ	20%
Percent of home price lost to transaction costs	τ	10%
Maintenance costs	δ	0.7%
Moving costs	χ	0.3
Tax Rate	γ	30%
Refinancing Costs	ζ	3%
Inflation	ν	2%

Note: Units are in $10,000s or percent.

Consumer Expenditure Survey from the U.S. Department of Labor. The discount rate for bequests are 0.8 for θ_A, 0.8 for θ_H, and 0.8 for θ_M. They are chosen to imply that households would rather consume one additional dollar than leave an additional dollar as a bequest and that households place a premium on leaving their homes as bequests relative to other assets.

Table A-3: Values of Structural Parameters in Calibrated Model

λ	β	ϕ	θ_A	θ_H	θ_M
2	0.96	0.2	0.8	0.8	0.8

References

[1] Attanasio, Orazio, P.K. Goldberg and E. Kyriazidou. “Credit Constraints in the Market for Consumer Durables; Evidence from Mircodata on Car Loans.” NBER Working Paper No. 7694. 2000.

[2] Balvers, Ronald J. and Lszl Szerb. “Precaution and Liquidity in the Demand for Housing.” *Economic Inquiry*. Vol. 38. No. 2. 2000.

[3] Bar-Ilan, Avner and Allan S. Blinder. “Life-Cycle Permanent-Income Model and Consumer Durables.” NBER Working Paper No. 2149. 1987.

[4] Berkovec, James and Don Fullerton. “A General Equilibrium Model of Housing, Taxes, and Portfolio Choice.” *The Journal of Political Economy*. Vol. 100. Issue 2. 1992. pp. 390-429.

[5] Bertola, Giuseppe, Luigi Guise, and Luigi Pistaferri. “Uncertainty and Consumer Durables Adjustment.” Working Paper. European University Institute. 2002.

[6] Buist, Henry and Tyler T. Yang. “Housing Finance in a Stochastic Economy: Contract Pricing and Choice.” *Real Estate Economics*. Vol. 28. No. 1. 2000. pp. 117-139.

[7] Caplin, Andrew and Sewin Chan, Charles Freeman and Joseph Tracy. “Household Asset Portfolios and the Reform of the Housing Finance Market.” *TIAA-CREF Research Dialogues* 59. February 1999. pp. 1-12.

[8] Case, Karl E., and Robert J. Shiller, and Allan N. Weiss. “Index-Based Futures and Options Trading in Real Estate.” *Journal of Portfolio Management* (Winter 1993).

[9] Chah, Eun Young, Valerie A. Ramey, and Ross M. Starr. “Liquidity Constraints and Intertemporal Consumer Optimization: Theory and Evidence from Durable Goods.” *Journal of Money, Credit, and Banking*. Vol. 27. issue 1. 1995. 272-287.

[10] Cocco, Joo F. “Hedging House Price Risk With Incomplete Markets.” Working Paper. 2000.

[11] Engelhardt, Gary. “House Prices and Home Owner Saving Behavior.” NBER Working Paper No. 5183. 1995.

[12] Ermisch, John. “The Demand for Housing in Britain and Population Aging: Microeconomic Evidence.” *Econometrica*. Vol. 63. Issue 251. 1996. pp. 383-404.

[13] Fernndez-Villaverde, Jess and Dirk Krueger. “Consumption and Saving over the Life-cycle: How Important are Consumer Durables?” Working Paper. 2001.

[14] Flavin, Marjorie and Takashi Yamashita. “Owner-Occupied Housing and the Composition of the Household Portfolio Over the Life-cycle.” University of California, San Diego Discussion Paper 98-02. 1998.

[15] Flavin, Marjorie and Takashi Yamashita. “Owner-Occupied Housing and the Composition of the Household Portfolio Over the Life-cycle.” *The American Economic Review*. Vol. 92. 2002. pp. 345-362.

[16] Fratantoni, Michael C. “Housing Wealth, Precautionary Savings and the Equity Premium.” Dissertation. Johns Hopkins University. 1997.

[17] Genesove, David and Christopher Mayer. “Equity and Time to Sale in the Real Estate Market,” *The American Economic Review*. Vol. 87. No. 3. 1997. pp. 255-269.

[18] Gourinchas, P.O. and J.A. Parker. “Consumption over the Life-cycle.” NBER Working Paper No.7271. 1999.

[19] Green, Richard and Patric Hendershott. “Age, Housing Demand, and Real House Prices.” *Regional Science and Urban Economics*. Vol. 26. 1996. pp. 465-480.

[20] Guisio, Luigi, Tullio Jappelli, and Daniele Terlizzese. “Income Risk, Borrowing Constraints, and Portfolio Choice.” *The American Economic Review*. Vol. 86. Issue 1. pp. 158-172. 1996.

[21] Guisio, Luigi and Tullio Jappelli. “Private Transfers, Borrowing Constraints, and the Timing of Homeownership.” CSEF Working Paper No. 17. 1999.

[22] Grossman, Sanford and Guy Laroque. “Asset Pricing and Optimal Portfolio Choice in the Presence of Illiquid Durable Consumption Goods.” *Econometrica*. Vol. 58. No. 1. 1993. pp.25-51.

[23] Haurin, Donald R. “Income Variability, Homeownership, and Housing Demand.” *Journal of Housing Economics*. Vol. 1. 1991. pp. 60-74.

[24] Henderson, J. Vernon and Yannis M. Ioannides. “A Model of Housing Tenure Choice.” *The American Economic Review*. Vol. 73. Issue 1. 1983. pp. 98-113.

[25] Henderson, J. Vernon and Yannis M. Ioannides. “Dynamic Aspects of Consumer Decision in Housing Markets.” *Journal of Urban Economics*. Vol. 26. 1989. pp. 212-230.

[26] Hoyt, William H. and Stuart Rosenthal. “Capital Gains Taxation and the Demand for Owner-Occupied Housing.” *The Review of Economics and Statistics*. Vol. 72. Issue 1. 1990. pp. 45-54.

[27] Hu, Xiaoqing. “Essays on Portfolio Choice for Homeowners.” Dissertation. Northwestern University. 2002.

[28] Hurst, Erik and Frank Stafford. “Home is Where the Equity Is: Liquidity Constraints, Refinancing and Consumption.” Working Paper. 2002.

[29] LaFayette, William C., Donald R. Haurin, and Patric H. Hendershott. “Endogenous Mortgage Choice, Borrowing Constraints and The Tenure Decision.” NBER Working Paper No. 5074. 1995.

[30] Li, Wenli and Rui Yao. “A Life-Cycle Model of Housing and Mortgage Choices with Labor Income and House Price Risks.” Working Paper. 2004.

[31] Linneman, Peter and Susan Watcher. “The Impacts of Borrowing Constraints on Homeownership.” *AREUEA Journal*. Vol. 17. No. 4. 1989. pp.389-402

[32] Maki, Atsushi. “Liquidity Constraints: A Cross-Section Analysis of the Housing Purchase Behavior of Japanese Households.” *The Review of Economics and Statistics*. Vol. 75. Issue 3. 1993. pp. 429-437.

[33] Martin, Robert F. “Consumption, Durable Goods, and Transaction Costs.” Working Paper. 2001.

[34] Nichols, Joseph. “Explaining the Housing “Over-Investment” Puzzle.” Working Paper. 2004.

[35] Nichols, Joseph. “A Life-Cycle Model with Housing, Mortgage Contracts, and Portfolio Allocation” Working Paper. 2004.

[36] Nordvik, Viggo. “Moving Costs and the Dynamics of Housing Demand.” *Urban Studies*. Vol. 38. No. 3. 2001. pp. 519-523.

[37] Ortalo-Mange, Francois and Sven Rady. “Housing Market Fluctuation in a Life-Cycle Economy with Credit Constraints.” Stanford University, Graduate School of Business Research Paper No. 1501. 1998.

[38] Phelan, Christopher and John Rust. "How Social Security and Medicare Affect Retirement Behavior in a World of Incomplete Markets." *Econometrica*. Vol. 65. Issue 4. pp. 781-832. 1997.

[39] Poterba, James M. "Tax Subsidies to Owner-Occupied Housing: An Asset-Market Approach." Quarterly Journal of Economics. Vol. 99. issue 4. 1984. pp.729-752.

[40] Sheiner, Louise and David Weil. "The Housing Wealth of the Aged." NBER Working Paper No. 4115. 1992.

[41] Sinai, Todd and Nicholas S. Souleles. "Owner-Occupied Housing as a Hedge Against Rent Risk." Working Paper. 2003.

[42] Stein, Jeremy. "Prices and Trading Volume in the Housing Market: A Model with Downpayment Effects." *Quarterly Journal of Economics*. Vol. 110. No. 2. 1995. pp. 379-406.

[43] Smith, Lawrence B., Kenneth T. Rosen, and George Fallis. "Recent Developments in Economic Models of Housing Markets." *Journal of Economic Literature*. Vol. 26. Issue 1. 1988. pp. 29-64.

[44] Venti, Steven F. and David A. Wise. "Aging and Housing Equity." NBER Working Paper No. 7882. 2000.

[45] Yao, Rui and Harold H. Zhang. "Optimal Consumption and Portfolio Choices with Risky Housing and Borrowing Constraints." Forthcoming in *The Review of Financial Studies*. 2004.