Testing for Stationarity and Cointegration in an Unobserved-Components Framework[1]

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Abstract

While tests for stationarity and cointegration have important econometric and economic implications, they do not always offer conclusive results. In this paper we suggest that exploiting the parametric structure of the multivariate correlated unobserved components framework can provide a more powerful way to test for stationarity and cointegration than the non-parametric, asymptotic tests currently available. The parametric test nests a partial or restricted unobserved components model within a more general unobserved components model. Then we estimate both the general and the restricted models and determine the likelihood ratio test statistic. The distribution of this likelihood ratio test statistic is nonstandard, but a Monte Carlo simulation provides proper error bands for use in inference. We then compare these results to the asymptotic, non-parametric KPSS test and the common trends test of Nyblom and Harvey (2000).

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Introduction

Beginning in the 1970s, a number of papers appeared which suggested that permanent movements in many major economic time series follow a variable, or stochastic, trend, instead of a smooth deterministic time trend. Granger and Newbold (1974) were among the first to argue that macroeconomic data as a rule contained stochastic trends, characterized by unit roots, and that using these series in traditional econometric models may lead to spurious regressions. In addition, it was found that some series appear to share a common stochastic trend. These series, referred to as cointegrated, provide one way to avoid spurious regressions.

Nelson and Plosser's seminar work (1982) could not reject the unit root hypothesis in favor of trend stationarity for 13 out of 14 major macroeconomic time series using statistical techniques developed by Dickey and Fuller (1979). Their results suggest that nonstationarity is indeed prevalent in macroeconomic time series data.

Tests for unit roots and cointegration do not always offer conclusive results, however. Rudebusch (1992; 1993) demonstrates that unit root tests have low power against estimated trend stationary alternatives. In addition, Perron (1989) shows that unit root tests cannot always distinguish unit root from stationary processes that contain segmented or shifted trends. He finds evidence of trend stationarity when he incorporates a single break in 1929 for 10 out of the 13 series found to contain unit roots by Nelson and Plosser.

Using an appropriate parametric model should provide more power than existing nonparametric tests. Recent research (Harvey 1993; Engel and Morley 2001; Morley, Nelson et al. 2003; Morley 2004; Sinclair 2004) suggests that unobserved components models are useful for representing economic time series which may contain unit roots and for those series which may be cointegrated. These series can be modeled as containing a permanent component, representing the stochastic trend, and a transitory component, representing the stationary component of the series.

The purpose of this paper is to suggest a new way to test for both stationarity and cointegration by exploiting the parametric structure of the correlated unobserved components framework. An unobserved components model may often be an appropriate way to model macroeconomic time series, and using the parametric model should provide additional power over a non-parametric test. The test nests a partial or restricted unobserved components model within a more general unobserved components model. Then the general and the restricted models are estimated which provides a likelihood ratio test statistic. The distribution of this likelihood ratio test statistic is nonstandard, but a Monte Carlo simulation can provide proper error bands for use in inference. The simulation uses data generated with the results from the partial unobserved components model as the values for the null. Consequently, the null hypothesis for this test is stationarity or cointegration, which is useful in many cases. In this sense the test is like the well-known KPSS test (Kwiatkowski, Phillips et al. 1992), but we propose a parametric test which should provide more power. In addition, using Monte Carlo simulations also corrects for size, whereas KPSS and other tests similar to KPSS such as Nyblom and Harvey's (2001) test for common trends, appeal to asymptotic results.

This test will thus provide a new and more powerful way to test for the presence of unit roots and cointegration, which will help in evaluating many important macroeconomic theories. The applications presented in this current draft include the variability of the permanent component of the unemployment rate and the permanent income hypothesis. Other potential applications include certain real business cycle theories and purchasing power parity.

A traditional unobserved components model (Harvey 1985; Clark 1987) imposes zero-correlation restrictions in order to achieve identification. In addition, these models are generally applied to a single series for univariate detrending. One exception to this is Clark (1989), which explores the relationship between output and the unemployment rate, but imposes ad-hoc restrictions on the correlations for identification. A recent paper by Morley, Nelson, and Zivot (2003, hereafter MNZ) shows that in some cases we can estimate an unobserved components (UC) model without restricting the correlations between the components. Their correlated unobserved components (UC-UR) model, extended to the multivariate case by Sinclair (2005) and by Schleicher (2003) allows for a new way to test for both stationarity and cointegration using a parametric model and information from correlated, but not necessarily perfectly correlated additional series. An example of this test appears in Sinclair (2005) where output and the unemployment rate are jointly decomposed into permanent and transitory movements with a general variance-covariance matrix. Including GDP as a second series allows for additional information about the permanent movements in the unemployment rate, which provides a much more powerful test for stationarity of the unemployment rate than the non-parametric test presented in Kwiatkowski et al (1992, hereafter KPSS).[2] The KPSS test weakly rejects stationarity of the unemployment rate (at the 10% level), but the test based on the parametric model clearly rejects stationarity. Similarly, the multivariate correlated unobserved components approach allows for a more powerful test of cointegration. Nyblom and Harvey (2000) have provided a test of cointegration in the uncorrelated UC framework, but their test appeals to asymptotics. In addition, their structure does not allow for the inclusion of additional series

[2] Alternative tests involving testing for a unit MA root, which appears when a series is over-differenced are discussed in Saikkonen and Luukkonen (1993) and Tanaka (1990).

which are correlated but not necessarily cointegrated in order to provide additional information in testing for stationarity and cointegration. For example, Nyblom and Harvey's common trends test requires stepwise testing: is x cointegrated with y? If so, is z also cointegrated with x and y? Within the correlated unobserved components framework we can instead test whether z is cointegrated with x and y or just correlated with them without it being a problem for estimation.

This paper proceeds as follows. Section 2 presents the correlated unobserved components (UC-UR) model. Section 3 discusses existing asymptotic tests for stationarity and cointegration in the unobserved components framework. Section 4 discusses our proposed test. Section 5 presents two preliminary applications. Application 1 applies the stationarity test to the unemployment rate within a bivariate model of output and the unemployment rate. Application 2 applies the cointegration test to consumption and income. Section 6 concludes and suggests some future directions.

Section 2: The Correlated Unobserved Components Model

Suppose we have n series (y) which may each be represented as the sum of two unobserved components: a "trend" component and a "cycle" component. The "trend" (τ), also called the permanent component, is the steady-state level after removing all temporary movements. The "cycle" (c), also called the transitory component, embodies all temporary movements and is assumed to be stationary:

$$y_{it} = \tau_{it} + c_{it}, \tag{1}$$

$t = 1,\ldots, T$ and $i = 1,\ldots, n$.

We begin by assuming that each series is I(1), thus a random walk for each of the trend components allows for permanent movements in the series. It is also possible to allow for a drift (μ) in the trend:

$$\tau_{it} = \mu_i + \tau_{it-1} + \eta_{it} \tag{2}$$

Following Sinclair (2004), Morley, Nelson, and Zivot(2003), and Morley (2004), we model each transitory component as an autoregressive process of order two (AR(2))[3]:

$$c_{it} = \phi_{1i} c_{it-1} + \phi_{2i} c_{it-2} + \varepsilon_{it} \tag{3}$$

We assume the innovations (η_{it}, and ε_{it}) are normally distributed random variables with mean zero and a general covariance matrix (allowing possible correlation between any of the innovations to the unobserved components). This model also nests the partial unobserved components model, where the variance of some of the η_i's may be zero, and the restricted

[3] Sinclair (2005) shows that the correlated unobserved components model is identified as long as each transitory component has at least AR(2) dynamics. Thus this analysis generalizes to any higher-order AR process. Morley (2004) shows that an AR(1) may be sufficient for identification in the case of cointegration, but for testing purposes we need both the null and the alternative to be identified, thus we require at least an AR(2) for the transitory component.

(cointegrated) unobserved components model, where some of the η_i's may be perfectly (or perfectly negatively) correlated with each other.

As shown in Appendix 1, the model can be cast into state-space form. Then it is possible to use the Kalman filter for maximum likelihood estimation of the parameters for both the restricted and unrestricted models.[4]

Section 3: Asymptotic Tests for Stationarity and Cointegration in the UC Framework

Bailey and Taylor (2002) provide us with a useful result: if the cycle error and the trend error are contemporaneously correlated then the test statistic used by KPSS, Nabeya and Tanaka (1988), and Nyblom and Harvey (2000) is still the locally best invariant test for a null of stationarity (or cointegration). These tests do not benefit, however, from the parameterization of the correlated model. In addition, they do not allow for the inclusion of series which are correlated, but not cointegrated with the series of interest.

Nyblom and Harvey (2000, hereafter NH) propose a test of common trends where the null hypothesis that there exists $k < n$ common trends (i.e. rank ($\mathbf{\Sigma_\eta}$) = k), and the alternative is that there exists more than k common trends (i.e. rank ($\mathbf{\Sigma_\eta}$) > k). If **A**, the r x n matrix of cointegrating vectors, is known, then their test statistic can be written as:

$$\xi_r(\mathbf{A}) = \mathrm{tr}(\mathbf{ASA'})^{-1}\mathbf{ACA'},$$

where **S** is the nonparametric estimator of the spectral density at frequency zero using a Bartlett Window following KPSS[5]:

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[4]See chapter 3 of Kim and Nelson (1999) or chapter 4 of Harvey (1993) for a discussion of the implementation of the Kalman filter. The estimation was done in GAUSS.

[5] Harvey and Streibel (1997) show that if the process generating the stationary part of the model were known, then the locally best invariant (LBI) test for the presence of a random walk component could be constructed with a parametric estimator of the long-run variance. The also provide a not strictly LBI test which is more reliable in terms of size in small samples using the standardized one-step ahead prediction errors calculated assuming that the initial value of the random walk (τ_0) is fixed.

$$\mathbf{S} = \hat{\mathbf{\Gamma}}_{\mathbf{0}} + \sum_{j=1}^{m}\left[1 - \frac{j}{m+1}\right]\left[\hat{\mathbf{\Gamma}}_{\mathbf{j}} + \hat{\mathbf{\Gamma}}_{\mathbf{j}}'\right],$$

where m is the number of lags in the transitory component[6] and

$$\hat{\mathbf{\Gamma}}_{\mathbf{j}} = \frac{1}{T}\sum_{t=j+1}^{T}(\mathbf{y}_{\mathbf{t}} - \overline{\mathbf{y}})(\mathbf{y}_{\mathbf{t-j}} - \overline{\mathbf{y}})'.$$

And **C** is an estimator of the second moments of partial sums of the time series:

$$\mathbf{C} = \frac{1}{T^2}\sum_{i=1}^{T}\left[\sum_{t=1}^{i}(\mathbf{y}_{\mathbf{t}} - \overline{\mathbf{y}})\right]\left[\sum_{t=1}^{i}(\mathbf{y}_{\mathbf{t}} - \overline{\mathbf{y}})\right]'.$$

This test is more specifically a test of the pre-specified cointegrating vectors, i.e. a test of **A**. In many cases, however, we do not know the correct matrix **A**, but we may still be interested the testing for common trends. When we do not know **A**, NH propose the following modification:

$$\zeta_{k,n} = \min_{\mathbf{A}} tr[(\mathbf{ASA'})^{-1}\mathbf{ACA'}].^{7}$$

This allows us to estimate **A** and test for common trends.

The univariate version of this test was shown by Nyblom and Mäkeläinen (1983) to be the locally best invariant test of the null hypothesis that $\sigma_\eta^2 = 0$, i.e. that the series is stationary. Note that this test can also be interpreted as a one-sided Lagrange multiplier (LM) test. The test statistic in this case is:

$\zeta_1 = C/S$, since C and S will both be scalars when $n = 1$.

[6] In practice, more lags are generally included in the window than are actually modeled in the transitory component because there is a size/power tradeoff. For example, in Schleicher (2003), he includes 2 lags in the transitory component, but he uses m=8 for the test.

[7] Although asymptotically AS(y)A′ and S(Ay) return the same result, it may be appropriate to use S(Ay) in small samples and use the stationarity test described below. Here we simply present what NH reported as the test statistic.

Nyblom and Harvey also suggest a multivariate joint test for unit roots, a test for $\Sigma_\eta = \mathbf{0}$. The test statistic in this final case is:

$$\zeta_n = \mathrm{tr}[\mathbf{S}^{-1}\mathbf{C}].$$

The alternative is $\mathbf{\Sigma_\eta} = q\mathbf{\Sigma_\varepsilon}$. The test maximizes the power against homogenous alternatives, but it is consistent against all non-null $\mathbf{\Sigma_\eta}$'s.

Section 4: The Proposed Test

One advantage of the parametric test is that it allows for us to include variables which may be correlated with the variables of interest, thus they provide additional information, but they may not be cointegrated with the variables of interest. This is not a possibility in the NH world because the vector $\boldsymbol{A}$ is required in order to work with a spectral density at frequency zero. From the state-space representation of the correlated unobserved components model (see appendix), we have the following variance-covariance matrix:

$$E\left(\begin{bmatrix}\eta_t \\ \varepsilon_t\end{bmatrix}\begin{bmatrix}\eta_t & \varepsilon_t\end{bmatrix}\right) = \begin{bmatrix}\Sigma_\eta & \Sigma_{\eta\varepsilon} \\ \Sigma_{\varepsilon\eta} & \Sigma_\varepsilon\end{bmatrix}.$$

Recall that η_{it} represents the innovation to the permanent component of series i in our model, thus the submatrix of interest from the variance-covariance matrix is Σ_η. If this matrix is of full rank then the series are all integrated but there does not exist any cointegrating vector. If it is of less than full rank, then either one or more of the series is stationary or there exists at least one common trend.

Section 4.1: The Proposed Test for Stationarity

The general correlated unobserved components model nests the partial unobserved components model with one or more of the diagonal elements of Σ_η set equal to zero. The distribution of the likelihood ratio test statistic is nonstandard, but a Monte Carlo simulation can be used to establish appropriate confidence bands. The data for the Monte Carlo simulation can be generated under the assumption that the partial unobserved components model is the true model.

Section 4.2: The Proposed Test for Cointegration

The general correlated unobserved components model nests in this case the restricted unobserved components model with at least one of the innovations to the permanent component of one series is equal to a scale constant λ times the innovation to the permanent component of another series.[8] The distribution of the likelihood ratio test statistic is once again nonstandard, but a Monte Carlo simulation can again be used to establish appropriate confidence bands. The data for the Monte Carlo simulation can be generated under the assumption that the restricted unobserved components model is the true model. Consider the two-series example under the null of cointegration:

$$\Sigma_\eta = \begin{bmatrix} \sigma_\eta^2 & \lambda\sigma_\eta^2 \\ \lambda\sigma_\eta^2 & \lambda^2\sigma_\eta^2 \end{bmatrix}.$$

Note that the correlated unobserved components model also finds information relevant for the test of cointegration from the variance-covariance submatrix of permanent-transitory covariances. If under the null $\eta_1 = \eta_2$ and $\eta_2 = \lambda\eta_1 = \lambda\eta$, then we have:

$$\Sigma_{\eta\varepsilon} = \begin{bmatrix} \sigma_{\eta 1\varepsilon 1} & \sigma_{\eta 1\varepsilon 2} \\ \sigma_{\eta 2\varepsilon 1} & \sigma_{\eta 2\varepsilon 2} \end{bmatrix} = \begin{bmatrix} \sigma_{\eta\varepsilon 1} & \sigma_{\eta\varepsilon 2} \\ \lambda\sigma_{\eta\varepsilon 1} & \lambda\sigma_{\eta\varepsilon 2} \end{bmatrix}$$

[8] Or, in a larger system, we may have a weighted sum of other innovations. For example, in a three-series system we may have $\eta_{1t}=\lambda_1\eta_{2t} + \lambda_2\eta_{3t}$. We could alternatively have $\eta_{1t}=\lambda\eta_{2t}$ where η_{3t} remains an integrated series which is not cointegrated with the other two series (of course other combinations are also possible). Note that for cointegration there can be no additional error term. That would instead be imperfect correlation, not cointegration.

Section 5: Applications

Allowing for correlation between the components opens up the possibility of useful multivariate correlated unobserved components models. In particular, for permanent-transitory decomposition, there are many series which may be correlated with each other, thus providing useful information for the decomposition, but they may not be cointegrated. For example, Okun's Law suggests that output and the unemployment rate are correlated, but it does not suggest they are cointegrated. Estimating the permanent and transitory movements jointly, however, provides better estimates of these movements than univariate decomposition. With the additional information from output, we can have more power for a test of whether unemployment should be modeled as stationary. Previously multivariate UC models depended upon imposing the correlation, hence they were primarily used to impose cointegration, but we can now test for cointegration in this framework, as is shown in application 2 where we consider consumption and income.[9]

Application 1: Is the Unemployment Rate Stationary?

Sinclair (2005) jointly estimates the components of the unemployment rate and output in a correlated unobserved components model. Her paper provides new estimates of the relative importance of permanent versus transitory movements in the U.S. unemployment rate and contributes to the debate about the variability in the natural rate of unemployment, or the NAIRU,[10] by finding support for a variable permanent component in the unemployment rate. In addition, this model provides a new way to test if the unemployment rate is stationary, which is

[9] There will be additional and hopefully more novel examples in a future version of this paper.

[10] There has been much discussion about the different nuances of these terms. We use them here to represent the permanent component of the unemployment rate. For the different sides of the debate on the variability of trend unemployment, see Weiner (1993), Gordon (1997), Salemi (1999), Grant (2002), and King and Morley (2003).

rejected for the U.S. in favor of a model with significant permanent movements in the unemployment rate.

The unemployment rate is bounded between zero and one, but it can undergo permanent shocks. For example, the random walk will capture frequent structural breaks. Some models (for example Blanchard and Quah 1989), assume the unemployment rate does not contain a stochastic permanent component, so it is important to test this assumption of the UC-UR model used in Sinclair (2005). The distribution of the likelihood ratio test statistic is nonstandard, but a Monte Carlo simulation can be used to establish appropriate confidence bands. The data was generated under the assumption that the unemployment rate can be represented as a stationary AR(2). The largest likelihood ratio test statistic generated from 999 draws was 39.8. The likelihood ratio test statistic for this restriction in the UC-UR model is 67.2, which implies that the null of stationary unemployment can clearly be rejected in favor of the alternative of permanent movements in the unemployment rate. In addition, recall that the UC-UR model finds a large variance for the permanent component of the unemployment rate, when it is not constrained to zero.[11] These results presented in Table 1 and Figure 1 suggest that the unemployment rate should be modeled with a permanent component. In comparison, the KPSS test only weakly rejects stationarity of the same unemployment rate data (at the 10% level), but the test based on the parametric model clearly rejects stationarity.

[11] Unemployment may experience a few large permanent movements which might be difficult to capture with the random walk and might better be modeled as structural breaks. Another possibility is that unemployment responds asymmetrically to shocks. Caner and Hansen (2001) find that allowing asymmetric responses in unemployment results in the rejection of a unit root.

Application 2: Are Consumption and Income Cointegrated?

Both Morley (2004) and Schleicher (2003) explore the relationship between consumption and income in a bivariate correlated unobserved components model. Morley assumes cointegration for his analysis, whereas Schleicher tests for cointegration (and is unable to reject it) using the test of Nyblom and Harvey (2000). Here we employ the parametric test. Table 2 presents the unrestricted and restricted estimates of the model. Using the common trends test of Nyblom and Harvey, Schleicher finds a test statistic of 0.136 where the critical value at the 10% level is 0.162, thus he cannot reject cointegration.

In our example, as shown in Table 2, it is clear even from the unrestricted UC-UR estimate that consumption and income are cointegrated since the point estimate of the correlation of η_y and η_u is exactly 1, but the t-statistic on the estimate of this correlation is nonstandard, so we turn to a likelihood ratio test to confirm that these two series are cointegrated. The likelihood ratio test statistic is 1.4674. This test statistic does not have the standard chi-squared distribution, so we use a Monte Carlo simulation to establish the correct distribution.[12] This also corrects for small-sample issues.

[12] Monte Carlo simulation is not available at this time, but it will be available shortly in a more complete version of this paper.

Section 6: Conclusions and Extensions

The problem with any parametric test is that it does depend critically on the assumed structure of the model. For applications employing an unobserved components model for analysis, however, we believe that it appropriate to test for stationarity and for cointegration within the model being assumed for the analysis. This provides a more powerful test and is consistent with the assumptions made for analysis. For the small samples available for many macroeconomic time series asymptotic tests are also questionable on size issues.

In future work we intend to extend the test to an asymmetric UC-UR model such as that of Sinclair (2005). We also need to develop a grid search to confirm that we are not counting any local maxima in the Monte Carlo simulation. Finally, additional desirable applications include applying the test to the data of King, Plosser, Stock, and Watson (1991), purchasing power parity, and the inclusion of additional variables which may be correlated but not cointegrated with important macroeconomic time series.

Appendix 1: State Space Form

We can cast the model into state-space form and apply the Kalman filter for maximum likelihood estimation of the parameters and the permanent and transitory components for both the restricted and the unrestricted UC-UR models. For the permanent components the Kalman filter requires initial values. We use diffuse priors for the initial values, but the results are robust to instead estimating them as parameters.

Observation Equation: $\mathbf{y_t} = \begin{bmatrix} \mathbf{I} & \mathbf{I} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \boldsymbol{\tau}_t \\ \mathbf{c}_t \\ \mathbf{c}_{t-1} \end{bmatrix}$,

State Equation: $\begin{bmatrix} \boldsymbol{\tau}_t \\ \mathbf{c}_t \\ \mathbf{c}_{t-1} \end{bmatrix} = \begin{bmatrix} \boldsymbol{\mu} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix} + \begin{bmatrix} \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Phi}_1 & \boldsymbol{\Phi}_2 \\ \mathbf{0} & \mathbf{I} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \boldsymbol{\tau}_{t-1} \\ \mathbf{c}_{t-1} \\ \mathbf{c}_{t-2} \end{bmatrix} + \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \boldsymbol{\eta}_t \\ \boldsymbol{\varepsilon}_t \end{bmatrix}$

where $\mathbf{y_t}$, $\boldsymbol{\tau}_t$, $\mathbf{c}_t$, $\boldsymbol{\eta}_t$, $\boldsymbol{\varepsilon}_t$, $\boldsymbol{\Phi}_1$, and $\boldsymbol{\Phi}_2$ are each vectors with *n* rows and the identity and zero matrices are each *n* x *n*, with zero vectors being *n* x 1.

Variance-Covariance Matrix: $E\left(\begin{bmatrix} \boldsymbol{\eta}_t \\ \boldsymbol{\varepsilon}_t \end{bmatrix} \begin{bmatrix} \boldsymbol{\eta}_t & \boldsymbol{\varepsilon}_t \end{bmatrix}\right) = \begin{bmatrix} \boldsymbol{\Sigma}_\eta & \boldsymbol{\Sigma}_{\eta\varepsilon} \\ \boldsymbol{\Sigma}_{\varepsilon\eta} & \boldsymbol{\Sigma}_\varepsilon \end{bmatrix}$,

where $\boldsymbol{\Sigma}_\eta$ is the *n* x *n* variance-covariance matrix for the innovations to the permanent components, $\boldsymbol{\Sigma}_\varepsilon$ is the *n* x *n* variance-covariance matrix for the innovations to the transitory components, and $\boldsymbol{\Sigma}_{\eta\varepsilon} = \boldsymbol{\Sigma}_{\varepsilon\eta}'$ represent the cross-covariance terms between the permanent and transitory innovations. For this paper we focus on the rank of $\boldsymbol{\Sigma}_\eta$. If it is of full rank then the series are all integrated but there does not exist any cointegrating vector. If it is of less than full rank, then either one or more of the series is stationary or there exists at least one common trend.

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Table 1: Comparing the UC-UR Results to Stationary Unemployment

Description	**Parameter**	**UC-UR Estimate (Standard Error)**	**Partial UC-UR Estimate (Standard Error)**
	Log Likelihood	-327.7777	-361.354939
S.D. of Permanent Innovation to GDP	$\sigma_{\eta y}$	1.5689 (0.2334)	0.9207 (0.1350)
S.D. of Temporary Innovation to GDP	$\sigma_{\varepsilon y}$	1.1406 (0.3407)	0.6402 (0.2231)
Correlation between GDP Components	$\rho_{\eta y \varepsilon y}$	-0.8666 (0.0500)	-0.3005 (0.2784)
GDP 1st AR parameter	ϕ_{1y}	0.7576 (0.0698)	1.2382 (0.0946)
GDP 2nd AR parameter	ϕ_{2y}	-0.2047 (0.1008)	-0.4092 (0.1137)
GDP Drift	μ	0.8570 (0.0364)	0.8588 (0.0625)
S.D. of Permanent Innovation to Unemployment	$\sigma_{\eta u}$	0.7109 (0.1037)	= 0 by assumption
S.D. of Temporary Innovation to Unemployment	$\sigma_{\varepsilon u}$	0.6860 (0.1512)	0.3818 (0.0182)
Correlation between Unemployment Components	$\rho_{\eta u \varepsilon u}$	-0.9286 (0.0371)	Does not exist by assumption
Unemployment 1st AR parameter	ϕ_{1u}	0.7416 (0.0628)	1.4192 [13] (0.0567)
Unemployment 2nd AR parameter	ϕ_{2u}	-0.1789 (0.0621)	-0.4822 (0.0567)
Correlation: Permanent GDP/Permanent Unemp.	$\rho_{\eta y \eta u}$	-0.9395 (0.0249)	Does not exist by assumption
Correlation: Permanent Unemp./Transitory GDP	$\rho_{\eta u \varepsilon y}$	0.6644 (0.1289)	Does not exist by assumption
Correlation: Transitory GDP/Transitory Unemp.	$\rho_{\varepsilon y \varepsilon u}$	-0.6583 (0.3688)	-0.9779 (0.0363)
Correlation: Permanent GDP/Transitory Unemp.	$\rho_{\eta y \varepsilon u}$	0.9939 (0.0206)	0.0961 (0.2820)

[13] Note that the model used constrains the AR parameters to be within the unit circle.

Table 2: Comparing the UC-UR Results to Cointegrated UC-UR

Description	**Parameter**	**UC-UR Estimate (Standard Error)**	**Cointegrated UC-UR Estimate (Standard Error)**
	Log Likelihood	1431.3265	1430.5928
S.D. of Permanent Innovation to Income	$\sigma_{\eta y}$	0.0191 (0.0020)	0.0198 (0.0021)
S.D. of Temporary Innovation to Income	$\sigma_{\varepsilon y}$	0.0245 (0.0018)	0.0251 (0.0020)
Correlation between Income Components	$\rho_{\eta y \varepsilon y}$	-0.9964 (0.0054)	-0.9976 (0.0039)
Income 1st AR parameter	ϕ_{1y}	0.8440 (0.0325)	0.8387 (0.0369)
Income 2nd AR parameter	ϕ_{2y}	-0.0360 (0.0236)	-0.0320 (0.0257)
Income Drift	$\mu 1$	0.0080 (0.0016)	0.0080 (0.0015)
S.D. of Permanent Innovation to Consumption	$\sigma_{\eta c}$	0.0211 (0.0021)	$= \sigma_{\eta y}$ by assumption
S.D. of Temporary Innovation to Consumption	$\sigma_{\varepsilon c}$	0.0201 (0.0023)	0.0194 (0.0023)
Correlation between Consumption Components	$\rho_{\eta c \varepsilon c}$	-0.9849 (0.0043)	$= \rho_{\eta y \varepsilon c}$ by assumption
Consumption 1st AR parameter	ϕ_{1c}	0.9511 (0.0154)	0.9449 (0.0130)
Consumption 2nd AR parameter	ϕ_{2c}	-0.0219 (0.0134)	-0.0218 (0.0141)
Consumption Drift	$\mu 2$	0.0080 (0.0017)	0.0080 (0.0015)
Correlation: Permanent Income /Permanent Consump.	$\rho_{\eta y \eta c}$	1.0000 (0.0000)	= 1 by assumption
Correlation: Permanent Consump./Transitory Income	$\rho_{\eta c \varepsilon y}$	-0.9965 (0.0062)	$= \rho_{\eta y \varepsilon y}$ by assumption
Correlation: Transitory Income /Transitory Consump.	$\rho_{\varepsilon y \varepsilon c}$	0.9960 (0.0053)	0.9934 (0.0040)
Correlation: Permanent Income /Transitory Consump.	$\rho_{\eta y \varepsilon c}$	-0.9848 (0.0075)	-0.9831 (0.0052)

Figure 1: Distribution of LR Test Statistic Under the Null of Stationarity for Application 1