AN EQUILIBRIUM ASSET PRICING MODEL WITH NOISY EARNINGS REPORTS

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ABSTRACT. We examine the effect of the noisy earnings reports on the equity premium in an asset pricing model. In our model, consumers make their investment decisions based on preliminary announcements of earnings reports and after the revisions are made by the release of the actual earnings reports they make their consumption decisions. Consequently, the stochastic discount factor used for asset price determination is based on the preliminary announcements rather than the true earnings process. The variance of the revisions plays an important role in the decisions of the consumers. If the variance of revisions is high the agents will tend to ignore the announcements and rely on the mean of historical earnings realizations. This tends to smooth the stochastic discount factor in the pricing equation which has the impact of reducing the equity premium in the model. Therefore, the equity premium puzzle is even more severe than reported by Mehra and Prescott (1985) when imperfect earnings forecasts are accounted for and consumers face a signal extraction problem in earnings.

1. INTRODUCTION

In this paper we extend the standard general equilibrium asset pricing model of Lucas (1978) to include earnings reports that are observed with measurement error and are later revised. Our motivation is to determine the effect this has on the equilibrium pricing function and how this, in turn, effects the equity premium associated with risky assets. We envision some agency such as the Institutional Brokers' Estimate System (IBES) that releases preliminary announcements of earnings upon which agents base their investment decisions. After the investment decision is made the true earnings are reported and the agents choose their current consumption level.

The predictions of this model are interesting and, although quite intuitive in retrospect, not obvious at the outset. In an environment with noisy earnings reports the equity premium will fall as the level of noise in the preliminary announcements increases. Essentially, since forecasted earnings are less volatile than actual earnings, the stochastic discount factor used to make the investment decision with noisy earning reports is less volatile than that based upon true earnings reports. Additionally, the increased

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uncertainty makes the risky asset a less desirable investment so its price will be lower in order to drive up expected returns.

In section two we describe our model and discuss some specification issues. In the third section we examine the possibilities of finding an analytical solution for our model and, concluding that this is not possible, we then describe our numerical solution method. We explore the results and implications of our model in section four and discuss some possible extensions.

2. The Model

The model is an extension of the standard Lucas (1978) representative agent, asset pricing model without production. In order to compare our results to the existing literature, we use the particular specification of Burnside (1998) and the parameterization of Mehra and Prescott (1985). Our revision process is similar to that used by Bomfim (2001) and Aruoba (2004) who study a growth model where the productivity shocks are subject to revisions.

There is one risk free asset (b) that we interpret as a one period, pure discount bond and one risky asset (s), that we interpret as an equity, that pays off a random dividend each period. The pay offs of both assets are in terms of a single non-storable consumption good (c). The i^{th} agent begins each period t with a stock of bonds b_t^i and equities s_t^i . Preliminary earnings announcements of d_t^a are reported and the agents' make their investment decisions, b_{t+1}^i and s_{t+1}^i . The equilibrium bond and equity prices, q_t and p_t , are determined at this time. After the securities markets clear the revisions r_t to the preliminary reports are announced and the fundamental earnings, d_t^f , are revealed to the agents who then make their consumption decisions c_t^i .

Note that, in a world with no production and a non-storable consumption good, it is not feasible to reverse the order of the agents' decisions since the agents cannot credibly commit to a consumption plan prior to the revelation of the fundamental earnings. If the fundamental earnings turn out to be lower than the preliminary announcement then agents will be forced to consume the fundamental earnings no matter what they had previously committed to. Thus, when the agent choose consumption first, the model reduces to the conventional asset pricing model without noisy earnings reports.

We assume that there are a large number, N, of identical agents who are expected utility maximizers with time separable utility functions. The i^{th} agent's problem is:

$$\max_{\{c_t^i, b_{t+1}^i, s_{t+1}^i\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \, u(c_t^i), \tag{1a}$$

subject to

$$c_t^i + p_t s_{t+1}^i + q_t b_{t+1}^i \le (p_t + d_t^f) s_t^i + b_t^i,$$
(1b)

$$d_t^J = d_t^a + r_t, \tag{1c}$$

$$c_t^i \ge 0, \tag{1d}$$

and given

$$b_0^i \text{ and } s_0^i,$$
 (1e)

where $0 < \beta < 1$ is the discount rate and the utility function u is increasing and strictly concave. We assume that the stochastic process driving the fundamental earnings is known to all agents and is given by

$$d_{t+1}^f = d_t^f \exp(x_{t+1}^f) x_{t+1}^f \qquad = (1-\rho)\mu + \rho x_t^f + \varepsilon_{t+1} \qquad (2a)$$

$$\varepsilon_t \sim \text{i.i.d. } N(0, \sigma_{\varepsilon}^2)$$
 (2b)

$$x_t^f = x_t^a + r_t \tag{2c}$$

$$r_t \sim \text{i.i.d. } N(0, \sigma_r^2)$$
 (2d)

where μ is the mean growth rate and ρ is the persistence of the dividend process, and we assume that the stochastic process driving the revisions is public information and is given and is independent of the process for ε_t . We also assume that the agents' utility functions are of the constant relative risk aversion variety,

$$u(c_t^i) = \frac{(c_t^i)^{1-\gamma} - 1}{1-\gamma}, \ \gamma > 0.$$
(3)

Although these assumptions are more strict than absolutely necessary, they will simplify our exposition and solutions without seriously compromising our results.

Note that we do not specify how the reporting agency determines the announced earnings d_t^a , or equivalently, the announced dividend growth rate, x_t^a . Nor do we assume that the agent has any idea how these announcements are arrived at by the reporting agency. The agent only knows the fundamental process and the revision process and that the revisions are not predictable. It is possible to show that

$$x_{t+1}^a = (1-\rho)\mu + \rho(x_t^a + r_t) - r_{t+1} + \varepsilon_{t+1}$$
(4)

which is perhaps an easier way to view the problem from an agent's perspective. However, it is important to keep in mind that the true generating process is (2) and that (4) is simply a convenient way of expressing the announced dividend growth rate in terms of the underlying innovations.

Denote the i^{th} agent's individual state variables as $z_t^i = \{b_t^i, s_t^i\}$ and let $Z_t^1 = \{d_t^a, S_t, B_t\}$ denote the aggregate state after the preliminary announcement and let $Z_t^2 = \{d_t^f, S_t, B_t\} = \{Z_t^1, r_t\}$ denote the aggregate state after

the revision, where B_t and S_t are the aggregate supply of bonds and stocks, respectively. As is typical, we will assume that $B_t = 0$ and $S_t = 1$, for all t.

The competitive equilibrium for this economy is defined to be a set of demand functions $\{b^i(z_t^i, Z_t^1), s^i(z_t^i, Z_t^1), c^i(z_t^i, Z_t^2)\}_{i=1}^N$ and pricing functions $\{p(Z_t^1), q(Z_t^1)\}$ that solve each agent's optimization problem and such that all markets clear. What makes this problem distinct is that the aggregate states in the consumption function differ from the aggregate states in the investment and pricing functions. This subtlety greatly increases the complexity of the solution for this problem.

3. The Solution

Our goal is to solve for the equilibrium pricing functions $p(Z_t^1)$ and $q(Z_t^1)$. Writing the problem recursively and dropping the agent *i* superscripts to simplify notation, yields the Euler equations

$$E\left[u'\left(c(z_{t}, Z_{t}^{2})\right)\right]p(Z_{t}^{1}) = E\left[\beta u'\left(c(z_{t+1}, Z_{t+1}^{2})\right)\left(p(Z_{t+1}^{1}) + d_{t+1}^{f}\right)\right]$$
(5a)

and

$$E\left[u'\left(c(z_{t}, Z_{t}^{2})\right)\right]q(Z_{t}^{1}) = E\left[\beta \, u'\left(c(z_{t+1}, Z_{t+1}^{2})\right)\right].$$
(5b)

Note that the states of consumption process at time t, include the revision r_t which is unknown to the agent when the investment decision is made. Thus, unlike the usual Euler equations for the asset pricing model without the signal extraction problem, the Euler equations above include a conditional expectation on the left-hand-side term. This prevents us from dividing both sides by the marginal utility of consumption at time t to produce a simple expression of the stochastic discount factor. The right-hand-side conditional expectations are with respect to period t's revision r_t and period t+1's fundamental earnings d_{t+1}^f —which includes next period's announced earnings d_{t+1}^a .

Using (2) and (4), and imposing the equilibrium condition $c_t = d_t^f$, for all t, yields

$$c_t = d_{t-1}^f \exp\left(x_t^a + r_t\right) \tag{6a}$$

and

$$c_{t+1} = d_{t-1}^f \exp\left(x_t^a + r_t + (1-\rho)\mu + \rho\left(x_t^a + r_t\right) + \varepsilon_{t+1}\right).$$
(6b)

Substituting these expressions into the Euler equations (5) and defining the price-dividend ratio prior to the revision as $v_t = p(Z_t^1)/d_t^a$ allows us to rewrite the Euler equations as

$$E_{\{r_t\}}[\exp((x_t^a + r_t)(1 - \gamma))v_t] = E_{\{r_t, r_{t+1}, \varepsilon_{t+1}\}}[\beta \exp((x_t^a + r_t + (1 - \rho)\mu + \rho(x_t^a + r_t) + \varepsilon_{t+1})(1 - \gamma))(v_{t+1} + 1)],$$

$$E_{\{r_t\}}[\exp((x_t^a + r_t)(-\gamma))q_t] = E_{\{r_t, r_{t+1}, \varepsilon_{t+1}\}}[\beta \exp((x_t^a + r_t + (1 - \rho)\mu + \rho(x_t^a + r_t) + \varepsilon_{t+1})(-\gamma))],$$
(7a)

where the subscripts on the conditional expectations denote the stochastic processes over which the expectations are taken. The first of these Euler equations can be used to solve for the price-dividend ratio $v_t \equiv v(x_t^a)$ and the second will be used to compute the bond price and the risk free rate of return for our equity premium calculations.

3.1. Analytical Solutions. Ideally, we could find an analytical solution for the equilibrium price-dividend ratio $v(x_t^a)$ and there is some reason to hope that this might be possible. Burnside (1998) has found an analytical solution for this particular specification of asset pricing model for the case where there are no revisions ($\sigma_r^2 = 0$). Tsionas (2003), generalized Burnside's analytical solution to include any innovations to the dividend growth process that are iid from any distribution that admits a moment generating function and Bidarkota and McCulloch (2003) further generalized the solution to include the family of stable distributions—some of which do not admit moment generating functions. All of these solutions require an infinite order, recursive substitution of the stochastic process in the Euler equation. While this is possible when $\sigma_r = 0$, it is not possible when $\sigma_r \neq 0$ because the two-step nature of the decision process when revisions are present requires that both aggregate state vectors Z_t^1 and Z_t^2 be present in each period.

Calin, Chen, Himonas and Cosimano (2005) find analytical solutions to a much larger set of asset pricing problems. They use complexification of the Euler equation to establish that the integral, and thus the price-dividend ratio function within it, are analytic. Having established the analyticity of the price-dividend function as well as its radius of convergence, they are able to use a change of variables and write the solution to the integral equation as in infinite order Taylor series which they are then able to solve for the coefficients. Unfortunately, their method applies only to problems with a single state variable since establishing analyticity of multiple dimension complex functions is extremely difficult. We have two independent shocks in our model so there are two exogenous state variables and we are unable to use this solution method.

3.2. Numerical Solutions. Failing to find an analytical solution, we turn to numerical methods. Bomfim (2001) solves the noisy productivity signal version of his growth model by linearizing the Euler equations about the deterministic steady-state and then applying a Kalman filter method to solve the agent's signal extraction problem. This approach works well for Bomfim's problem because the variance of his state variable (the capital stock) is small relative to its mean, making the quadratic terms in the approximation to the Euler equation fairly insignificant. This is decidedly not the case, however, in asset pricing problems such as ours where the variance of the state variable may be larger than the mean. In this situation linear approximations to the Euler equations will produce very large pricing errors.

This suggests that we either use a perturbation method like Collard and Juillard (2001) or a projection method like Aruoba (2004). Both approaches

have merits but we find the perturbation method clumsy for problems such as ours with complicated state spaces so we opt for a projection method.

We approximate the price-dividend ratio as

$$\widehat{v}(x_t^a; \vec{\alpha}) = \sum_{i=0}^n \alpha_i T_i\left(g(x_t^a)\right) \tag{8}$$

where $T_i(y) = \cos(i \operatorname{arccos}(y))$ are the Chebyshev polynomials and $g(\cdot)$ is a function to transform its argument into the [-1, 1] interval. This approximation for v is substituted into (7a) and the sum of squared residuals is minimized with respect to the coefficients $\vec{\alpha}$. The Chebyshev polynomials make a useful set of basis functions for projection methods because their orthogonality leads to particularly simple first-order conditions for the minimization problem. We evaluate the integrals implied by the triple expectations in (7a) by Gauss-Hermite quadrature which is well-suited for Gaussian innovations.

Projection methods are by now well-understood by economists and have proven to yield reliable numerical solutions for a broad range of problems (Judd 1992, 1998). We validate our solution method by solving the version of our model when $\sigma_r = 0$ for which Burnside (1998) provides an analytical solution. For a broad range of parameter values our numerical solution is accurate to six decimal places even for only fourth order (n = 4) approximations. As expected, linear approximations (n = 1) produce very poor solutions except in special cases such as risk-neutral preferences, extremely small variances for the innovations, or iid dividend growth processes, all of which produce linear price-dividend ratio functions. For our signal extraction problem when $\sigma_r \neq 0$, we note that the rate of decay of the coefficients is sufficiently fast that we are confident in the precision of our solutions.

There is one issue with the projection method that is sufficiently interesting to deserve special mention. Since the orthogonality condition of the Chebyshev polynomials holds only on the [-1, 1] interval, we must fix a priori a range $[x_{min}^a, x_{max}^a]$ for the announced dividend growth rates. However, when the persistence parameter ρ in (4) is not zero, a value for x_t^a near the end points of this interval will produce a value of x_{t+1}^a that lies outside the interval. Judd (1992), observes in a similar context that a certain amount of experimentation with the range is required in order to minimize the effects of this problem. Often the range for the x_t^a 's is chosen to be several standard deviations from the mean so that the Gauss-Hermite weights are sufficiently small that we hope the problem can be ignored. This is perhaps risky, however, since high order Chebyshev polynomials are notoriously illbehaved outside of [-1, 1] support. One may either truncate the offending x_{t+1}^a points by forcing them back to the end points of the interval or simply ignore the points by setting the Chebyshev polynomial equal to zero at those points. We dealt with this issue by choosing the number of Hermite nodes in our integration such that, given the number of standard deviations for the x_t^a range, the x_{t+1}^a values never leave this range. For larger values of $|\rho|$ this means that fewer nodes are used in integral approximations with correspondingly larger approximation errors in our approximations. While this approach makes us at least aware of the approximation error that we are committing, there is a certain inelegance here that deserves some future attention.

4. The Results

To make comparisons with Burnside (1998) and others who have used this specification easier, we choose Mehra and Prescott's (1985) parameterization as our benchmark case. The mean growth rate of dividends is set to $\mu =$ 0.0179, with a persistence of $\rho = -0.139$ and the volatility of the innovations set at $\sigma_{\varepsilon} = 0.0348$. For the agents we choose the coefficient of risk aversion to be $\gamma = 2.5$ and a discount rate of $\beta = 0.95$. Note that the conditional variance of the revisions should not be larger than the conditional variance of the fundamental process, $\sigma_r \leq \sigma_{\varepsilon}$, otherwise the forecast of earnings would be more volatile than the true fundamental process for earnings (Aruoba 2004). To test the impact of different magnitudes of revisions, we calibrate the model for various values of $\sigma_r = k \sigma_{\varepsilon}$ where $k \in [0, 1]$.

For the benchmark parameterization, the computed equilibrium pricedividend function, $\hat{v}(x_t^a; \vec{\alpha})$, is positively sloped with a modest convex curvature. As the magnitude of the revision process increases, the price-dividend function shifts downward and becomes less convex. Thus, as the earnings announcements become noisier agents bid down the price of the risky asset. This result seems reasonably intuitive since the uncertainty in the earnings process perceived by the agents is increasing. However, it is not clear yet what is happening to the equity premium.

The risk free gross rate of return can be computed from (7b) as $R_{f,t} = 1/q_t$, where we are following the convention of dating this return with t since the return on this discount bond is known at time t when the bond prices are determined. This is a reasonably complicated computation since there is one integral in the numerator and a triple integral in the denominator of the expression of $R_{f,t}$. The gross return on the risky asset is computed as

$$R_{e,t+1} = \frac{p_{t+1} + d_{t+1}^a}{p_t} \\ = \left(\frac{\hat{v}(x_{t+1}^a; \vec{\alpha}) + 1}{\hat{v}(x_t^a; \vec{\alpha})}\right) \exp(x_{t+1}^a)$$
(9)

which is evaluated using the same triple integrals as in (7a).

The equity premium calculations for various model parameterizations are given in Table 1. The first column is for the case where k = 0 so that there are no revisions and the model reduces to the standard asset pricing model of Mehra and Prescott (1985). In the second column k = 1/3 and in the third column k = 2/3, so that the variance of the revisions is increasing as we move from left to right in the table. The top panel of the table corresponds

TABLE 1. I	Equity	Premiums
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		$\sigma_r = 0$	$\sigma_r = \sigma_{\varepsilon}/3$	$\sigma_r = 2\sigma_\varepsilon/3$
Benchmark	$E[R_e]$	1.1016	1.1015	1.1013
	R_{f}	1.0977	1.0977	1.0977
	$E[R_e - R_f]$	0.0039	0.0038	0.0036
$\gamma = 10$	$E[R_e]$	1.2314	1.2309	1.2296
	R_{f}	1.2027	1.2029	1.2034
	$E[R_e - R_f]$	0.0286	0.0280	0.0263
$\gamma = 0$	$E[R_e]$	1.0526	1.0526	1.0527
	R_{f}	1.0526	1.0526	1.0526
	$E[R_e - R_f]$	5.0414e-11	1.7354e-5	6.9419e-5
$\rho = 0.139$	$E[R_e]$	1.1003	1.1001	1.0996
	R_{f}	1.0977	1.0974	1.0967
	$\check{E[R_e-R_f]}$	0.0025895	0.0026699	0.0029108
$\rho = 0$	$E[R_e]$	1.1000	1.1000	1.1000
	R_{f}	1.0966	1.0966	1.0966
	$E[R_e - R_f]$	0.0033	0.0033	0.0033
$\sigma_{\varepsilon} = 0.06$	$E[R_e]$	1.1031	1.1029	1.1022
	R_{f}	1.0915	1.0915	1.0916
	$E[R_e - R_f]$	0.0116	0.0113	0.0106
$\sigma_{\varepsilon} = 0.0003$	$E[R_e]$	1.1008	1.1008	1.1008
	R_{f}	1.1008	1.1008	1.1008
	$E[R_e - R_f]$	2.8976e-7	2.8365e-7	2.6639e-7

Note: Benchmark case: ρ =-0.139, γ =2.5, σ_{ε} =0.0348

to the benchmark parameters taken from Mehra and Prescott (1985), and the subsequent panels vary one parameter at a time from the benchmark case.

Consider first the results in the $\sigma_r = 0$ column which correspond to the model without the signal extraction problem. The model responds precisely as we would anticipate. For risk neutral agents ($\gamma = 0$) the equity premium is zero and the equity premium increases as the agent's risk aversion increases; to 0.0039 when $\gamma = 2.5$ (benchmark case) and to 0.286 when $\gamma = 10$. The equity premium is also increasing with the volatility of the innovations to

the dividend growth process σ_{ε} ; from essentially zero when $\sigma_{\varepsilon} = 0.0003$, to 0.0039 when $\sigma_{\varepsilon} = 0.0348$ (benchmark case), and to 0.0116 when $\sigma_{\varepsilon} = 0.06$. Somewhat less intuitive is the result that the equity premium is decreasing in the persistence parameter ρ ; from 0.0039 in the benchmark case when $\rho = -0.139$, to 0.0033 when $\rho = 0$, and to 0.0026 when $\rho = 0.139$. Note that when $\rho = 0$ the dividend growth process is iid and the price-dividend function is constant with respect to the dividend announcements x^a . It is also worth noting that the price-dividend function is positively sloped when $\rho < 0$ and negatively sloped when $\rho > 0$.

Now consider the results as we increase the noise in the dividend announcements. In the top panel, the benchmark case, we see that the equity premium decreases as the noise in the announcements increases. This result is more pronounced for more risk averse agents ($\gamma = 10$) and when the fundamental dividend process is more variable ($\sigma_{\varepsilon} = 0.06$). Recall that the price-dividend ratio itself is falling as we increase σ_r . Now we can see that the expected return on the risky asset is decreasing in σ_r and that the risk free return is increasing in σ_r so that the equity premium is declining. Notice that this result breaks down for the risk neutral agents ($\gamma = 0$) and when the price-dividend function is constant ($\rho = 0$) or when we are in the very low risk case ($\sigma_{\varepsilon} = 0.0003$); all of which are consistent with our intuition.

The intuition for the decline of the equity premium as the noise in the earnings announcements increases is interesting. When the agents face the signal extraction problem they use different stochastic discount factors when making their investment decisions than when making their consumption decisions. After the preliminary announcement but before the revisions are announced, the agents base their investment decisions on the stochastic discount factor $\beta u'(d_{t+1}^a)/u'(d_t^a)$. After the revisions are released agents base their consumption decisions on the stochastic discount factor $\beta u'(d_{t+1}^f)/u'(d_t^f)$. As the volatility of the revisions (σ_r) increases the preliminary announcements become less reliable and agents respond by putting less weight on these preliminary announcements and more weight on the historical mean of the fundamental earnings reports, μ . This reduces the variance of the stochastic discount factor used in the investment decision which, in turn, decreases the covariance between the stochastic discount factor and the return on the risky asset. This is a nice example of the consumption-based CAPM in practice. As this covariance term decreases the risk premium decreases giving the result that we have observed. These agents are, above all, consumption smoothers. When faced with increased uncertainty in the revision process they will choose to smooth their investment process by ignoring the signal and relying more on the long run observable process.

As we observed at the outset, this result seems rather obvious in retrospect. We confess, however, that we were initially surprised by this result. Our *a priori* reasoning was that the increased variation in earnings reports caused by the revisions would drive up the return on the risky asset and increase the equity premium. Note that it is indeed the case that the price of the risky asset falls but the discounted, expected future pay off falls even faster, driving down the expected return on the risky asset.

If we believe that agents have imperfect forecasts of earnings when they make their investment decisions, then the equity premium puzzle is even more severe for the standard asset pricing model than Mehra and Prescott (1985) initially reported. Of course, this literature has gone in many different directions since then. It would be interesting to examine cases where agents have non-expected utility functions such as in Campell and Cochrane (1999), or participation constraints such as Guvenen (2004), or in the case with production and changing tax rates as in McGrattan and Prescott (2003). In any case, the equity premium puzzle continues to fascinate.

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