

# Measuring inflation persistence: a structural time series approach\*

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March 2005

## Abstract

Time series estimates of inflation persistence incur an upward bias if shifts in the inflation target of the central bank remain unaccounted for. Using a structural time series approach we measure different sorts of inflation persistence allowing for an unobserved time-varying inflation target. Unobserved components are identified using Kalman filtering and smoothing techniques. Posterior densities of the model parameters and the unobserved components are obtained in a Bayesian framework based on importance sampling. We find that inflation persistence, expressed by the half-life of a shock, can range from 1 quarter in case of a cost-push shock to several years for a shock to long-run inflation expectations or the output gap.

JEL classification: C11, C13, C22, C32, E31

Keywords: Inflation persistence, inflation target, Kalman filter, Bayesian analysis.

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\*This study was developed in the context of the "Eurosysteem Inflation Persistence Network". We are grateful for helpful comments of participants of the ECB Conference on "Inflation Persistence in the Euro Area", 10-11 December 2004, Frankfurt and the CFS International Macroeconomics Summer School, 24-30 August 2004, Eltville. We particularly thank Luc Aucremanne, Reinout De Bock, Freddy Heylen, Andrew Levin, Philippe Moës, Benoît Mojon, Gert Peersman, Raf Wouters and an anonymous referee for useful comments and suggestions. The views expressed in this paper are those of the authors and do not necessarily reflect the views of the National Bank of Belgium. We acknowledge support from the Interuniversity Attraction Poles Program - Belgian Science Policy, contract no. P5/21. All remaining errors are the authors'.

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## Non-technical summary

It is generally accepted that over the medium to long run inflation is a monetary phenomenon, i.e. entirely determined by monetary policy. Over shorter horizons, though, various macroeconomic shocks, including variations in economic activity or production costs, will temporarily move inflation away from the central bank's inflation target. Therefore, a profound understanding of the process generating inflation, in particular the speed of inflation adjustment in response to such shocks is of crucial importance for an inflation targeting central bank. Inflation persistence then refers to the tendency of inflation to converge slowly towards the central bank's inflation target in response to these shocks.

With respect to measuring historical inflation persistence, a common practice in empirical research is to estimate univariate autoregressive (AR) time series models and measure persistence as the sum of the estimated AR coefficients (Nelson and Plosser, 1982; Fuhrer and Moore, 1995; Pivetta and Reis, 2004). In most of these studies, inflation is found to exhibit high to very high persistence over the post-WW II period, i.e. persistence is found to be close to that of a random walk. This suggests that, in order to bring inflation back to its target level, a central bank should react more vigorously than if persistence were low.

This paper aims at measuring different sorts of inflation persistence, i.e. the sluggish response of inflation in response to different macroeconomic shocks. The main point stressed in this paper is that unconditional estimates of high post-WW II inflation persistence are hard to interpret as the data generating process of inflation can be decomposed in a number of distinct components, each of them exhibiting its own degree of persistence. First, shifts in the central bank's inflation target can induce permanent shifts in the mean inflation rate. Second, imperfect or sticky information implies that private agents have to learn about the true central bank's inflation target. As such, the inflation target perceived by private agents can persistently differ from the true central bank's inflation target. Third, persistence in the various determinants of inflation also introduces persistence in the observed inflation rate. As the first three sources of persistence typically show relatively high inertia, ignoring one of them might create an upward bias in measured intrinsic inflation persistence.

Therefore, we measure inflation persistence in a structural time series model which explicitly models the various components driving inflation. We pursue both a univariate and a multivariate approach. Extracting information from the central bank's key interest rate we find confirmation that shifts in the central bank's inflation target induce a non-stationary component in the inflation rate. In addition, slow adjustment of inflation expectations in response to changes in the central bank's inflation target and persistence of shocks hitting inflation are important factors determining the observed inflation persistence. These com-

ponents explain a large fraction of the high degree of persistence observed in the post-WW II inflation rate. Taking these components into account, intrinsic inflation persistence is found to be lower than the persistence of a random walk, i.e. the sum of the AR coefficients in the data generating process of inflation is estimated to range from 0.45 in the euro area to 0.8 in the United States.

The implications for monetary policy are as follows. Our evidence indicates that in a stable inflation regime, where the central bank's inflation target does not change and where the public perception about this inflation target is well anchored, inflation persistence is relatively lower. The results also imply that in the case monetary policy would again give rise to unstable inflation, it would afterwards be very hard to disinflate due to the slow adjustment of inflation expectations in response to changes in the inflation target. In the case of natural rate misperceptions (Orphanides and Williams, 2004) this might however not be straightforward to avoid.

# 1 Introduction

It is generally accepted that over the medium to long run inflation is a monetary phenomenon, i.e. entirely determined by monetary policy. Over shorter horizons, though, various macroeconomic shocks, including variations in economic activity or production costs, will temporarily move inflation away from the central bank's inflation target. Therefore, a profound understanding of the process generating inflation, in particular the speed of inflation adjustment in response to such shocks is of crucial importance for an inflation targeting central bank. Inflation persistence then refers to the tendency of inflation to converge slowly towards the central bank's inflation target in response to these shocks.

With respect to measuring historical inflation persistence, a common practice in empirical research is to estimate univariate autoregressive (AR) time series models and measure persistence as the sum of the estimated AR coefficients (Nelson and Plosser, 1982; Fuhrer and Moore, 1995; Pivetta and Reis, 2004). In most of these studies, inflation is found to exhibit high to very high persistence over the post-WW II period, i.e. persistence is found to be close to that of a random walk. This suggests that, in order to bring inflation back to its target level, a central bank should react more vigorously than if persistence were low.

Important to note, though, is that this estimated high persistence should be interpreted as a measure of unconditional inflation persistence as this literature does not take into account that the data generating process of inflation is composed of a number of distinct components, each of them exhibiting its own level of persistence. As such, there are various factors underlying measured historical inflation persistence. First, over the last four decades large changes in the monetary policy strategy of industrialised economies have occurred. This has led to shifts in the inflation target<sup>1</sup> of central banks, which introduces a non-stationary component in the observed inflation series. Second, due to asymmetric information, sticky information or imperfect credibility, private agents' perceptions about the central bank's inflation target can differ from the true inflation target. The persistence of such deviations can be called expectations-based persistence (see Angeloni et al., 2004). Third, the sluggish response of inflation to various macroeconomic shocks is likely to be related to the wage- and price-setting mechanism. If wages and prices are adjusted infrequently, they will only gradually incorporate the effects of these shocks and therefore deviations of the observed inflation rate from the perceived inflation target will persist during several consecutive periods. This kind of inflation persistence can be called intrinsic inflation persistence (see Angeloni et al., 2004). Also price and wage indexation, which in-

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<sup>1</sup>Although inflation targeting is a monetary policy strategy that only emerged in the 1990s, we will still use this framework for the 1970s and 1980s. It enables us to identify the implicit inflation target of central banks from their policy choices as well as subsequent economic outcomes.

troduces backward-lookingness into inflation, add to intrinsic inflation persistence. Fourth, inflation persistence is determined by the persistence of the various macroeconomic shocks hitting inflation, e.g. persistent deviations of output from its potential level. This type of inflation persistence can be called extrinsic inflation persistence (see Angeloni et al., 2004).

In order to get a reliable estimate of the various types of inflation persistence, each of the above mentioned components should be taken into account explicitly when constructing the data generating process of inflation. First, permanent shifts in the central bank's inflation target lead to permanent changes in inflation. As standard AR models assume that inflation has a stable mean, these shifts induce an upward bias on measured inflation persistence (Levin and Piger, 2004). In fact, this argument goes back to Perron (1990) who pointed out that the standard Dickey-Fuller unit root test is biased towards non-rejection of the unit root hypothesis if the true data generating process includes breaks in its deterministic components. Taking historical changes in the central bank's inflation target into account might not be straightforward, though. Contrary to the current conduct of monetary policy, most countries typically did not directly communicate their inflation target to the public. Second, if the central bank's inflation target is not known to private agents or if it is not fully credible, the inflation target perceived by economic agents might differ from the central bank's inflation target. In this case intrinsic and extrinsic inflation persistence should be measured as the persistence in the deviations of the actual inflation rate from the perceived inflation target rather than from the central bank's inflation target. Third, in order to disentangle intrinsic and extrinsic persistence, the persistence in macroeconomic shocks hitting inflation should be modelled as well.

In the recent literature, shifts in the central bank's inflation target are accounted for in three different ways. First, O'Reilly and Whelan (2004) and Pivetta and Reis (2004) use rolling regressions to allow for shifts in the mean of inflation over different sub-samples. By lowering the sub-sample size, the number of breaks that can occur is reduced. Still, the authors cannot reject the hypothesis that the sum of the AR coefficients equals 1. Second, Levin and Piger (2004), Gadzinski and Orlandi (2004) and Bilke (2004) estimate an AR process allowing for discrete breaks in the mean of the inflation process. Without accounting for possible shifts, Levin and Piger (2004) report a persistence parameter for the United States GDP deflator of 0.92 over the period 1984Q1-2003Q4. Once a structural break is allowed for, persistence drops to 0.36. Third, Cogley and Sargent (2001, 2003), and Benati (2004) estimate time-varying AR coefficients conditional on a time-varying mean, which is specified as a random walk process. They find evidence that the AR coefficients of inflation have dropped considerably over the last decade.

With respect to these recent contributions to the literature, the following drawbacks

should be stressed. First, rolling regressions do not entirely rule out the possibility that a shift occurred in a specific sub-sample, especially when shifts are frequent. Moreover, this approach has limits in terms of degrees of freedom. Second, capturing shifts in monetary policy by allowing for a time-varying mean inflation rate, either by adding discrete breaks or a random walk process to the AR model, is inappropriate if the perceived inflation target differs from the central bank's inflation target. As this difference is not accounted for in these models, the persistence in the deviation of the perceived inflation target from the central bank's inflation target is implicitly restricted to equal the average of intrinsic and extrinsic inflation persistence.

This paper uses a structural time series approach to model the data generating process of inflation in the euro area<sup>2</sup> and the United States. Given the various sources of inflation persistence, structural time series models are particularly suited as in these models a time series can be decomposed into a number of distinct components, each of them being modelled explicitly. We pursue both a univariate and a multivariate approach. In both approaches, intrinsic and extrinsic inflation persistence are measured as the persistence of the deviations of inflation from the perceived inflation target. In contrast to the current literature, this allows for expectations-based persistence in response to shocks to the inflation target. Expectations-based persistence is incorporated by modelling the perceived inflation target as an AR process around the central bank's inflation target, the latter being modelled as a random walk. Kozicki and Tinsley (2003) use a similar model to disentangle permanent and transitory monetary policy shifts. Contrary to these authors, in the multivariate model we explicitly decompose output into potential output and a business cycle component. In this way we can consistently disentangle intrinsic and extrinsic inflation persistence in response to shocks to the business cycle.

As the univariate and the multivariate model both include a number of unobserved components, they are cast in a linear Gaussian state space representation. This enables the identification of the unobserved components from the observed data using Kalman filtering and smoothing techniques. The unknown parameters are estimated in a Bayesian framework, exploiting information both from the sample data and from previous studies estimating similar models. Posterior densities of the model parameters and the unobserved components are obtained using importance sampling.

The results indicate that intrinsic inflation persistence is not close to that of a random walk, i.e. the sum of the AR coefficients ranges from 0.45 in the euro area to 0.80 in the United States. Considerable extrinsic persistence explains why inflation deviates from the

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<sup>2</sup>Although the euro area did not exist for the larger part of our data sample (1970Q2-1998Q4), we use synthetic data aggregating the respective national data (Fagan et al, 2005). We thus implicitly assume that the euro area was an economy with a homogeneous monetary policy over the entire sample.

perceived inflation target during several consecutive periods. This source of persistence corresponds to the persistence in the output gap that drives inflation. Expectations-based persistence is estimated to be at least as high as intrinsic persistence, indicating that the dissipation of changes in the policy target is typically slower than in case of temporary shocks. Next to permanent changes in the central bank’s inflation target, this explains the observed high degree of aggregate post war inflation persistence.

The implications for monetary policy are as follows. Our evidence indicates that in a stable inflation regime, where the central bank’s inflation target does not change and where the public perception about this inflation target is well anchored, inflation persistence is relatively lower. The results also imply that in the case monetary policy would again give rise to unstable inflation, it would afterwards be very hard to disinflate due to the slow adjustment of inflation expectations in response to changes in the inflation target. In the case of natural rate misperceptions (Orphanides and Williams, 2004) this might however not be straightforward to avoid.

## 2 A structural time series approach

In this section, we present a structural time series model for inflation which takes into account (i) possible shifts in the central bank’s inflation target, (ii) expectations-based persistence, (iii) intrinsic persistence and (iv) extrinsic persistence. The model is identified both in a univariate and a multivariate set-up. The univariate approach relies on time series data for inflation only. In the multivariate model, we add information contained in real output and the central bank’s key interest rate. Using a variant of the macroeconomic model of Rudebusch and Svensson (1999), this allows us to impose more economic structure on the identification process. The advantage of the univariate over the multivariate model is that its relative simplicity reduces the risk of specification errors. The state space representation of both models is given in section 3.

### 2.1 Baseline structural model

The baseline structural model is given by:

$$\pi_{t+1}^T = \pi_t^T + \eta_{1t}, \quad (1)$$

$$\pi_{t+1}^P = E_{t+1}\pi_{t+1}^T, \quad (2)$$

$$\pi_t = (1 - \sum_{i=1}^q \varphi_i)\pi_t^P + \sum_{i=1}^q \varphi_i L^i \pi_t + \beta_1 z_{t-1} + \varepsilon_{1t}, \quad \sum_{i=1}^q \varphi_i < 1, \quad (3)$$

where  $\pi_t^T$  is the central bank’s inflation target,  $\pi_t^P$  is the perceived inflation target,  $\pi_t$  is the observed inflation rate and  $z_t$  is the output gap, i.e. the percentage deviation of real output

from potential output.  $L$  is the lag operator so that  $L^i \pi_t = \pi_{t-i}$ .  $\varepsilon_{1t}$  and  $\eta_{1t}$  are mutually independent zero mean white noise processes.

Equation (1) specifies  $\pi_t^T$  as a random walk process, i.e. shifts in the central bank's inflation target are assumed to be permanent. These shifts can be thought of as representing (i) changes in the central bank's preferences over alternative inflation outcomes (see Andolfatto et al., 2002) or (ii) an implicit change in the inflation target of the central bank created by misperceptions about the natural rate of different real variables (Orphanides and Williams, 2004)

Shifts in  $\pi_t^T$  are unlikely to be passed on to inflation expectations immediately. Castelnovo et al. (2003) present data on long-run inflation expectations. These suggest that in the aftermath of shifts in monetary policy, convergence towards the new equilibrium evolves smoothly over time. In the literature, this is often attributed to asymmetric information and signal extraction, sticky information or imperfect credibility. The source of asymmetric information on behalf of the private agents can be due to a lack of knowledge about the central bank's inflation target (Kozicki and Tinsley, 2003) or uncertainty about the central bank's preferences of inflation over real activity (Cukierman and Meltzer, 1986; Tetlow and von zur Muehlen, 2001). If private agents have to extract information about the central bank's inflation target from a monetary policy rule, the signal-to-noise ratio of this policy rule determines the uncertainty faced by private agents in disentangling transitory and permanent policy shocks and therefore also the speed at which they recognise permanent policy shocks (Erceg and Levin, 2003). Further, even if the central bank clearly announces a new inflation target, it can take quite some time before the new policy target is incorporated into long-run inflation expectations of private agents (for evidence see Castelnovo et al., 2003). This might be due to costs of acquiring information and/or re-optimisation (Mankiw and Reis, 2002). Summing up, private agents must form expectations about the inflation target  $\pi_t^T$ . Therefore, equation (2) introduces the perceived inflation target  $\pi_t^P$ , which captures the private agents' beliefs about the central bank's inflation target  $\pi_t^T$ .

The expectations operator in equation (2) is operationalised by modelling  $\pi_{t+1}^P$  as a weighted average of  $\pi_t^P$  and  $\pi_{t+1}^T$ ,

$$\pi_{t+1}^P = (1 - \delta)\pi_t^P + \delta\pi_{t+1}^T + \eta_{2t}, \quad 0 < \delta \leq 1, \quad (4)$$

where  $\eta_{2t}$  is a zero mean white noise process. The weighting parameter  $\delta$  can be interpreted as being the information updating parameter  $\lambda$  in a variant of the sticky-information model of Mankiw and Reis (2002) or as being proportional to the Kalman gain parameter  $k_g$  in the signal extraction problem of Erceg and Levin (2003) and Andolfatto et al. (2002).<sup>3</sup>

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<sup>3</sup>See appendix 1 for more details on how equation (4) can be derived from these two models.



Consequently,  $\delta$  measures the speed at which changes in the central bank's inflation target affect long-run inflation expectations of private agents, i.e.  $\delta$  measures expectations-based persistence. If  $\delta$  is one, a shift in the central bank's inflation target is immediately and completely passed on to inflation expectations. This would be the case if the central bank's inflation target is perfectly known to all private agents and immediately credible. The smaller  $\delta$ , the slower expectations respond to a shift in the central bank's inflation target.<sup>4</sup> In the sticky-information model of Mankiw and Reis (2002),  $\delta$  decreases in the cost of acquiring information and/or the cost of re-optimising prices in response to a shift in the central bank's inflation target. In the signal extraction problem of Erceg and Levin (2003) and Andolfatto et al. (2002),  $\delta$  increases in the signal-to-noise ratio of the monetary policy rule, i.e. the lower the uncertainty about whether monetary policy signals reflect transitory rather than permanent policy changes, the faster private agents will react to these signals by updating their inflation expectations.<sup>5</sup>

Note that shocks to the perceived inflation target,  $\eta_2$ , only have a short-run impact on  $\pi^P$ . These shocks should be interpreted as misperceptions of private agents about the central bank's inflation target, due to for instance noise in the signal extraction problem of Erceg and Levin (2003) and Andolfatto et al. (2002). Shocks to the central bank's inflation target,  $\eta_1$ , have a unit long-run impact on  $\pi^P$ , i.e.  $\pi^T$  is the long-run equilibrium inflation rate. This is consistent with the generally accepted feature that long-run inflation is a purely monetary phenomenon.

Equation (3) is a Phillips curve, relating the observed inflation rate  $\pi_t$  to the perceived inflation target  $\pi_t^P$ ,  $q$  lags of inflation and the lagged output gap  $z_{t-1}$ . The perceived inflation target  $\pi_t^P$  is the inflation rate consistent with the private agents' inflation expectations. Therefore, it serves as the medium-run inflation anchor. Both business cycle shocks, reflected in the output gap  $z_{t-1}$ , as well as cost-push shocks, measured by  $\varepsilon_{1t}$ , hitting inflation induce temporary deviations of  $\pi_t$  from  $\pi_t^P$ . The sluggish adjustment of  $\pi_t$  in response to cost-push shocks  $\varepsilon_{1t}$  is measured by the sum of the AR coefficients,  $\sum_{i=1}^q \varphi_i$ . This intrinsic inflation persistence is likely to be related to price- and wage-setting mechanisms, e.g. price and wage indexation. The sluggish adjustment of  $\pi_t$  in response to business cycle shocks is determined, besides the intrinsic inflation persistence, by the persistence of the output gap  $z_t$  in response to business cycle shocks. The latter source of inflation persistence can be called extrinsic inflation persistence.

Note that equation (3) does not impose that the observed inflation series is additively

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<sup>4</sup>We do not allow  $\delta$  to take a value of 0, as in this case  $\pi_t^P$  does not react to monetary policy shocks, i.e. monetary policy is not credible. Note that this restriction does not imply that all monetary policy actions are fully credible. Rather, only credible shifts in the central bank's inflation target are included in  $\eta_{1t}$ .

<sup>5</sup>Equation (4) does not distinguish between these two theories, neither excludes that  $\delta$  is a weighted average of  $k_g$  and  $\lambda$ , which could be the case if reality is a mixture of both theories.

composed of the perceived inflation target and a temporary component. Rather, shifts in  $\pi_t^P$  are only slowly passed on to observed inflation, with the speed of convergence being determined by the degree of intrinsic inflation persistence. In this way, we assume that in case of a shift in the perceived inflation target the structural determinants for intrinsic persistence, e.g. price and wage indexation, are present in addition to the determinants of expectations-based persistence, e.g. sticky or imperfect information.

## 2.2 Univariate identification

In a first step, we use time series data on inflation only to estimate the model specified in equations (1)-(4). Given the limited information set, the baseline model is simplified in two respects. First, we set  $\beta_1 = 0$  in equation (3). This restriction stems from the fact that we do not include any information about real output and therefore cannot disentangle intrinsic from extrinsic inflation persistence in response to business cycle shocks. Second, we exclude the possibility of shocks to  $\pi_t^P$ , i.e.  $\eta_{2t} = 0 \forall t$ . This restriction is motivated from the concern to keep, given the limited information set, the identification of  $\pi_t^P$  and  $\pi_t^T$  as simple as possible. Under this restriction, equation (4) can be rewritten, using equation (1), as:

$$\pi_{t+1}^P = (2 - \delta) \pi_t^P + (\delta - 1) \pi_{t-1}^P + \delta \eta_{1t} \quad (5)$$

This way of writing equation (4) shows that the univariate identification scheme boils down to the empirical restriction that (i) shocks to the central bank's inflation target,  $\eta_{1t}$ , have a unit long-run impact on observed inflation, (ii) inflation expectations can deviate from the central bank's inflation target over a long period of time and (iii) observed inflation is a stationary AR process around the perceived inflation target. Note that equation (5) is broadly consistent with the idea advocated by, among others, Young et al. (1991), that in order to introduce enough smoothness in estimates of unobserved trend components, they are best modelled as an integrated random walk process. Although strictly speaking the data generating process for  $\pi_t^P$  is not allowed to be an integrated random walk process, as  $\delta > 0$ ,  $\pi_t^P$  will exhibit a similar smoothness in response to monetary policy shocks provided that  $\delta$  is sufficiently close to 0. A similar specification of the data generating process of inflation expectations can be found in Doménech and Gomez (2003).

## 2.3 Multivariate identification

The univariate model exhibits two main drawbacks. First, identification of shocks to the central bank's inflation target stems from the purely statistical restriction that these shocks should have a unit long-run impact on inflation. Second, intrinsic and extrinsic inflation

persistence cannot be disentangled. Therefore, we add data on the central bank's key interest rate and real output. We use a variant of the widely used macroeconomic model of Rudebusch and Svensson (1999) to (i) identify the central bank's inflation target from information contained in the central bank's key interest rate and (ii) to measure extrinsic inflation persistence in response to shocks to the output gap from information contained in real output. Therefore, the baseline specification in equations (1)-(4) is extended with the following equations:

$$i_t = \rho_2 i_{t-1} + (1 - \rho_2)(r_t^* + \pi_t^P) + \rho_1(\pi_{t-1} - \pi_t^T) + \varepsilon_{2t} \quad (6)$$

$$y_t^r = y_t^P + z_t \quad (7)$$

$$z_t = \beta_2 z_{t-1} + \beta_3 z_{t-2} - \beta_4 (i_{t-1} - \pi_{t-1}^P - r_{t-1}^*) + \varepsilon_{3t} \quad (8)$$

$$y_{t+1}^P = \lambda_{t+1} + y_t^P + \eta_{3t} \quad (9)$$

$$\lambda_{t+1} = \lambda_t + \eta_{4t} \quad (10)$$

$$r_{t+1}^* = \gamma \lambda_{t+1} + \tau_{t+1} \quad (11)$$

$$\tau_{t+1} = \theta \tau_t + \eta_{5t} \quad (12)$$

where  $\varepsilon_{2t}$ ,  $\varepsilon_{3t}$ ,  $\eta_{3t}$ ,  $\eta_{4t}$  and  $\eta_{5t}$  are mutually independent zero mean white noise processes.

The interest rate rule in equation (6) infers on the stance of monetary policy through comparing the central bank's key nominal interest rate,  $i_t$ , with a measure for the neutral stance of monetary policy. Following Laubach and Williams (2003), this measure is assumed to be the natural short-run nominal interest rate ( $r_t^* + \pi_t^P$ ), where  $r_t^*$  is the time-varying real short-term interest rate consistent with output equal to potential (cf. below). As the perceived inflation target  $\pi_t^P$  is the medium-run inflation anchor consistent with long-run inflation expectations,  $r_t^* + \pi_t^P$  is the medium-run nominal interest rate anchor for monetary policy. The term  $(\pi_{t-1} - \pi_t^T)$  captures the reaction of the central bank to deviations of inflation from its target, i.e. monetary authorities will increase the nominal interest rate  $i_t$  when observed inflation  $\pi_{t-1}$  lies above the inflation target  $\pi_t^T$ . The lagged interest rate  $i_{t-1}$  introduces a degree of nominal interest rate smoothing or policy inertia (Amato and Laubach, 1999; English et al., 2003; Erceg and Levin, 2003). We assume that the policy parameters  $\rho_1$  and  $\rho_2$  are time-invariant. Although Clarida et al. (1998) find that the policy parameters are unstable in a number of countries, this assumption is not in contradiction with their results. They estimate the parameters conditional on a constant inflation target, whereas we estimate the inflation target conditional on constant policy parameters. Both strategies are to a high degree observationally equivalent. The reason why we do so is that we are interested in the time-varying inflation target and less in the policy parameters. For examples of the same approach see e.g. Kozicki and Tinsley (2003) or Smets and Wouters

(2005).

The interest rate rule enables us to extract information on shifts in the monetary policy regime contained in the key nominal interest rate  $i_t$ . Figures 1 and 2 present data for key nominal interest rates and inflation in the euro area and the United States since 1970. For a given fully credible central bank inflation target, inflation and the key nominal interest rate  $i_t$  should, over an entire business cycle, move around a fixed point on a 45 degree line with an intercept equal to the equilibrium real interest rate. This 45 degree line corresponds to the sum of the natural real interest rate and the perceived inflation target  $\pi_t^P$ , that equals the credible central bank inflation target  $\pi_t^T$ . However, the seven year moving average line of the data, which approximately filters out business cycle fluctuations, shows that from the 1970s until now inflation and interest rates did not move around a fixed point. This suggests that there have been substantial shifts in the central bank's inflation target.

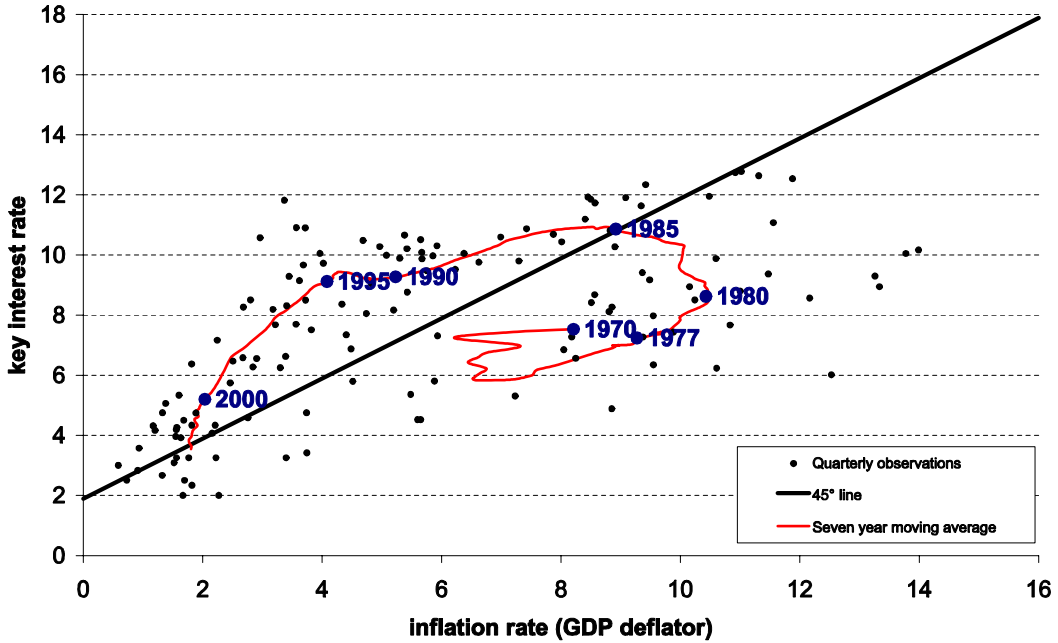


Figure 1: Shifts in the inflation target (euro area). Notes: a) The intercept is the mean of the real interest rate in the sample 1970Q2-2003Q4. b) As the sample begins in 1970Q2, the moving average will only start to contain seven years of data from 1977Q2. Therefore, the average is a slightly more volatile in the beginning of the sample.

The same figures also reveal to what extent the perceived inflation target differed from the central bank's inflation target at a certain point in time. Suppose we start from a point on the 45 degree line, e.g. a high inflation rate and a high key interest rate in the early 1980s. Now consider a central bank that wants to disinflate, i.e. the central bank reduces its target  $\pi_t^T$ . If the shift in  $\pi_t^T$  immediately feeds through into  $\pi_t^P$ , we would observe

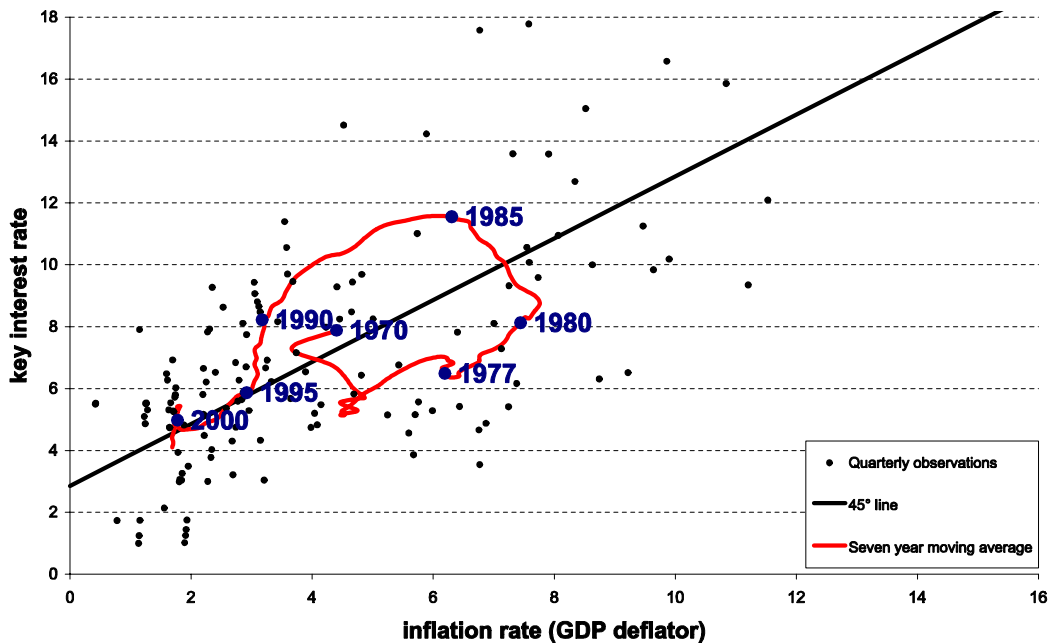


Figure 2: Shifts in the inflation target (United States). Notes: See figure 1.

a contemporaneous decrease in the key interest rate. Graphically, this would correspond to a downward shift *along* the 45 degree line. As this is neither the case for the United States nor for the euro area in most of the sample, this shows that changes in the central bank's inflation target are usually only slowly reflected in the perceived inflation target. The only time this observation seems not to hold is for the period between 1994 and today in the United States. It suggests that during the last decade, the Federal Reserve was able to disinflate in a credible way by about 2 percentage points<sup>6</sup>. Note that, as Laubach and Williams (2003) point out, shifts in the natural real rate of interest could mislead our judgement of the stance of monetary policy if we would assume that the natural rate remains constant. Time variation in the natural rate implies that the intercepts in Figures 1 and 2 are also time-varying. Still it is hard to believe that the natural rate of interest was persistently lower in the seventies than in the eighties and nineties, which lets us conclude that the interest rate rule indeed contains information about the timing and magnitude of shifts in the central bank's inflation target.

Equation (7) decomposes the log of real output  $y_t^r$  into potential output  $y_t^P$  and the output gap  $z_t$ . Equation (8) is an aggregate demand equation, relating the output gap  $z_t$  to

<sup>6</sup>This seems to be confirmed by narrative evidence. Goodfriend (2002) writes: "... in February 1994, the Fed started to announce its current intended federal funds rate target immediately after each FOMC meeting. This new practice made Fed policy more visible than ever. Every increase in the federal funds rate since then has attracted considerable attention."

its own lags and a term  $(i_{t-1} - \pi_{t-1}^P - r_{t-1}^*)$  which captures monetary policy transmission. Following Harvey (1985), Stock and Watson (1998) and Laubach and Williams (2003), equations (9)-(10) model potential output as a random walk with drift, where the drift term  $\lambda_t$  varies over time according to a random walk process. The time-variation in  $\lambda_t$  allows for the possibility of permanent changes in the trend growth of real output, e.g. the productivity slowdown of the early 1970s.<sup>7</sup>

Laubach and Williams (2003) argue that the natural real rate of interest varies over time due to shifts in the trend growth of output and other factors such as households' rate of time preference. Therefore, equation (11) relates the real short-term interest rate  $r_t^*$  to the trend growth in potential output  $\lambda_t$  and a component  $\tau_t$  that captures other determinants like time preferences.  $\tau_t$  is assumed to be an AR process that, depending on the value for  $\theta$ , can be either stationary or non-stationary.

Because we want to measure inflation persistence as the sum of the coefficients on the lagged inflation terms, the non-expectational autoregressive model presented above suits our purpose very well. In the case the economy is characterised by forward looking rational expectations, it can be considered as its reduced form representation. Rudebusch (2005), however, shows that in that case the reduced form representation of a simple forward looking monetary policy model would be subject to the Lucas critique. In this context Lansing and Trehan (2003) analytically show that the reduced form parameters depend on the policy parameters  $\rho_1$  and  $\rho_2$ . This is not relevant for our extension, though, as we model the economy in a reduced form around a time varying steady state inflation rate. The policy parameters  $\rho_1$  and  $\rho_2$  remain constant and therefore the reduced form parameters are not affected by policy changes.

## 3 Estimation methodology<sup>8</sup>

### 3.1 State space representation

The structural time series models outlined in section 2 both include a number of unobserved components  $(\pi_t^P, \pi_t^T, \dots)$ . In order to estimate these models, it is necessary to write them into state space form<sup>9</sup>. In a state space model, the development over time of the system under study is determined by an unobserved series of vectors  $\alpha_1, \dots, \alpha_n$ , which are associated with

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<sup>7</sup>Note that the random walk in equation (10) implies that  $y_t^P$ , and therefore also  $y_t$ , is an I(2) process. This seems inconsistent with the empirical evidence from Dickey-Fuller (DF) unit root tests that real output is I(1). Stock and Watson (1998) argue, though, that when the variance of  $\eta_{4t}$  is small relative to the variance of  $\eta_{3t}$ ,  $\Delta y_t^P$  has a moving average (MA) root close to unity. Schwert (1989) and Pantula (1991) show that the size of the standard DF unit root test is severely upwards biased in the presence of a large MA root. In this case, the standard DF unit root test is inappropriate to pick up a possible I(2) component in real output.

<sup>8</sup>The methodology outlined in this section was implemented using a set of GAUSS procedures. The code of these procedures is available from the authors on request.

<sup>9</sup>See e.g. Durbin and Koopman (2001) for an extensive overview of state space methods.

a series of observed vectors  $y_1, \dots, y_n$ . A general linear Gaussian state space model can be written in the following form:

$$y_t = Z\alpha_t + Ax_t + \varepsilon_t, \quad \varepsilon_t \sim N(0, H), \quad (13)$$

$$\alpha_{t+1} = T\alpha_t + R\eta_t, \quad \eta_t \sim N(0, Q), \quad t = 1, \dots, n, \quad (14)$$

where  $y_t$  is a  $p \times 1$  vector of observed endogenous variables, modelled in the observation equation (13),  $x_t$  is a  $k \times 1$  vector of observed exogenous variables and  $\alpha_t$  is a  $m \times 1$  vector of unobserved states, modelled in the state equation (14). The disturbances  $\varepsilon_t$  and  $\eta_t$  are assumed to be independent sequences of independent normal vectors. The matrices  $Z, A, T, R, H$ , and  $Q$  are parameter matrices.<sup>10</sup>

### 3.2 Kalman filter and smoother

Assuming that  $Z, A, T, R, H$ , and  $Q$  are known, the purpose of state space analysis is to infer the relevant properties of the  $\alpha_t$ 's from the observations  $y_1, \dots, y_n$  and  $x_1, \dots, x_n$ . This can be done through the subsequent use of two recursions, i.e. the Kalman filter and the Kalman smoother. The objective of filtering is to obtain the distribution of  $\alpha_t$ , for  $t = 1, \dots, n$ , conditional on  $Y_t$  and  $X_t$ , where  $Y_t = \{y_1, \dots, y_t\}$  and  $X_t = \{x_1, \dots, x_t\}$ . In a linear Gaussian state space model, the distribution of  $\alpha_t$  is entirely determined by the filtered state vector  $a_t = E(\alpha_t | Y_t, X_t)$  and the filtered state variance matrix  $P_t = Var(\alpha_t | Y_t, X_t)$ . The (contemporaneous) Kalman filter algorithm (see e.g. Hamilton, 1994, or Durbin and Koopman, 2001) estimates  $a_t$  and  $P_t$  by updating, at time  $t$ ,  $a_{t-1}$  and  $P_{t-1}$  using the new information contained in  $y_t$  and  $x_t$ . The Kalman filter recursion can be initialised by the assumption that  $\alpha_1 \sim N(a_1, P_1)$ . In practice,  $a_1$  and  $P_1$  are generally not known though. Therefore, we assume that the distribution of the initial state vector  $\alpha_1$  is

$$\alpha_1 = V\Gamma + R_0\eta_0, \quad \eta_0 \sim N(0, Q_0), \quad \Gamma \sim N(0, \kappa I_r), \quad (15)$$

where the  $m \times r$  matrix  $V$  and the  $m \times (m - r)$  matrix  $R_0$  are selection matrices composed of columns of the identity matrix  $I_m$ . They are defined so that, when taken together, their columns constitute all the columns of  $I_m$  and  $V'R_0 = 0$ . The matrix  $Q_0$  is assumed to be positive definite and known. The  $r \times 1$  vector  $\Gamma$  is a vector of unknown random quantities, referred to as the diffuse vector as we let  $\kappa \rightarrow \infty$ . This leads to

$$\alpha_1 \sim N(0, P_1), \quad P_1 = \kappa P_\infty + P_*, \quad (16)$$

where  $P_\infty = VV'$  and  $P_* = R_0Q_0R_0'$ . The Kalman filter is modified to account for this diffuse initialisation implied by letting  $\kappa \rightarrow \infty$  by using the exact initial Kalman filter

<sup>10</sup>The exact elements of the vectors  $y_t, x_t$  and  $\alpha_t$  and the matrices  $Z, A, T, R, H$ , and  $Q$  for both the univariate and the multivariate model are specified in appendix 2.

introduced by Ansley and Kohn (1985) and further developed by Koopman (1997) and Koopman and Durbin (2003).

Subsequently, the Kalman smoother algorithm is used to estimate the distribution of  $\alpha_t$ , for  $t = 1, \dots, n$ , conditional on  $Y_n$  and  $X_n$ , where  $Y_n = \{y_1, \dots, y_n\}$  and  $X_n = \{x_1, \dots, x_n\}$ . Thus, the smoothed state vector  $\hat{\alpha}_t = E(\alpha_t | Y_n, X_n)$  and the smoothed state variance matrix  $\hat{P}_t = Var(\alpha_t | Y_n, X_n)$  are estimated using all the observations for  $t = 1, \dots, n$ . In order to account for the diffuse initialisation of  $\alpha_1$ , we use the exact initial state smoothing algorithm suggested by Koopman and Durbin (2003).

Given the complexity of the multivariate model, we do not use the entire observational vector  $y_t$  in the filtering and smoothing algorithm. Following Koopman and Durbin (2000), the elements of  $y_t$  are introduced into the filtering and smoothing algorithms one at a time, i.e. the multivariate analysis is converted into a univariate analysis. As the data can then be analysed in univariate form, this approach offers significant computational gains, particularly for the treatment of initialisation by diffuse priors.

### 3.3 Bayesian analysis

The filtering and smoothing algorithms both require that  $Z$ ,  $A$ ,  $T$ ,  $R$ ,  $H$ , and  $Q$  are known. In practice, these matrices generally depend on elements of an unknown parameter vector  $\psi$ . One possible approach is to derive, from the exact Kalman filter, the diffuse loglikelihood function for the model under study (see de Jong, 1991; Koopman and Durbin, 2000; Durbin and Koopman, 2001) and replace the unknown parameter vector  $\psi$  by its maximum likelihood estimate. This is not the approach pursued in this paper. First, given the fairly large number of unknown parameters, especially in the multivariate model, the numerical optimisation of the sample loglikelihood function becomes quite cumbersome. Second, most of the unknown parameters in  $\psi$  have been estimated in the past for different countries and samples. Therefore, we analyse the state space models from a Bayesian point of view, i.e. we treat  $\psi$  as a random parameter vector with a known prior density  $p(\psi)$  and estimate the posterior densities  $p(\psi | y, x)$  for the parameter vector  $\psi$  and  $p(\hat{\alpha}_t | y, x)$  for the smoothed state vector  $\hat{\alpha}_t$ , where  $y$  and  $x$  denote the stacked vectors  $(y'_1, \dots, y'_n)'$  and  $(x'_1, \dots, x'_n)'$  respectively, by combining information contained in  $p(\psi)$  and the sample data. Essentially, this boils down to calculating the posterior mean  $\bar{g}$ :

$$\bar{g} = E[g(\psi) | y, x] = \int g(\psi) p(\psi | y, x) d\psi \quad (17)$$

where  $g$  is a function which expresses the moments of the posterior densities  $p(\psi | y, x)$  and  $p(\hat{\alpha}_t | y, x)$  in terms of the parameter vector  $\psi$ .



As  $p(\psi | y, x)$  is not a density with known analytical properties, equation (17) is evaluated using importance sampling. The idea behind this simulation approach is to obtain a sequence  $\psi^{(1)}, \dots, \psi^{(n)}$  of  $n$  random vectors from a density  $g(\psi | y, x)$  which is as close to  $p(\psi | y, x)$  as possible. Such a density is known as an importance density for  $p(\psi | y, x)$ . As an importance density  $g(\psi | y, x)$ , we take a large sample normal approximation to  $p(\psi | y, x)$ , i.e.

$$g(\psi | y, x) = N(\widehat{\psi}, \widehat{\Omega}) \quad (18)$$

where  $\widehat{\psi}$  is the mode of  $p(\psi | y, x)$  obtained from maximising

$$\log p(\psi | y, x) = \log p(y | \psi) + \log p(\psi) - \log p(y) \quad (19)$$

with respect to  $\widehat{\psi}$  and where  $\widehat{\Omega}$  denotes the variance-covariance matrix of  $\widehat{\psi}$  and  $p(y | \psi)$  is given by the likelihood function derived from the exact Kalman filter. Note that we do not need to calculate  $p(y)$  as it does not depend on  $\psi$ .

By Bayes' theorem and after some manipulations, equation (17) can be rewritten as

$$\bar{g} = \frac{\int g(\psi) z^g(\psi, y, x) g(\psi | y, x) d\psi}{\int z^g(\psi, y, x) g(\psi | y, x) d\psi} \quad (20)$$

with

$$z^g(\psi, y, x) = \frac{p(\psi) p(y | \psi)}{g(\psi | y, x)} \quad (21)$$

Using a sample of  $n$  independent draws of  $\psi$ , denoted by  $\psi^{(i)}$ , from  $g(\psi | y, x)$ , an estimate  $\bar{g}_n$  of  $\bar{g}$  can be obtained as

$$\bar{g}_n = \left( \sum_{i=1}^n g(\psi^{(i)}) z^g(\psi^{(i)}, y, x) \right) / \left( \sum_{i=1}^n z^g(\psi^{(i)}, y, x) \right) \quad (22)$$

Geweke (1989) shows that if  $g(\psi | y, x)$  is proportional to  $p(\psi | y, x)$ , and under a number of weak regularity conditions,  $\bar{g}_n$  will be a consistent estimate of  $\bar{g}$  for  $n \rightarrow \infty$ . In drawing from  $g(\psi | y, x)$ , efficiency was improved by the use of antithetic variables, i.e. for each  $\psi^{(i)}$  we take another value  $\tilde{\psi}^{(i)} = 2\widehat{\psi} - \psi^{(i)}$ , which is equiprobable with  $\psi^{(i)}$ . This results in a simulation sample that is balanced for location (Durbin and Koopman, 2001).

## 4 Estimation results

We use quarterly data for the euro area and the United States from 1970Q1 to 2003Q4. The inflation series  $\pi_t$  is the annualised first difference of the log of the seasonally adjusted GDP deflator. For the interest rate,  $i_t$ , we use the annualised central bank key interest rate. This interest rate should be most appropriate to infer changes in the central bank's behaviour.

Real output,  $y_t^r$ , is measured as the log of seasonally adjusted GDP at constant prices. See appendix 3 for a more detailed data description. Given that we work with quarterly data, the number of AR terms in equation (3) is set equal to 4, i.e.  $q = 4$ .

## 4.1 Prior information

Prior information about the unknown parameter vector  $\psi$  is included in the analysis through the prior density  $p(\psi)$ . Where possible, prior information is taken from the literature<sup>11</sup>. If no adequate information is available, we leave considerable uncertainty around the chosen priors. The prior distribution is assumed to be Gaussian for all elements in  $\psi$ , except for the variance parameters which are assumed to be gamma distributed.

**Univariate model** The priors for the AR coefficients  $\varphi_i$  in the univariate model are chosen from studies allowing for a break in the mean of the inflation rate. Levin and Piger (2004) for instance find a value of 0.36 for the sum of the AR coefficients of the United States GDP deflator. Gadzinski and Orlandi (2004) find a somewhat higher figure of 0.6 for the euro area. Finally we choose a prior for the sum of the AR coefficients of 0.4 for both the United States and the euro area. Our prior for  $\delta$  is 0.15, which is the average of the parameter values determining signal extraction in Erceg and Levin (2003) and Kozicki and Tinsley (2003), or sticky information in Mankiw and Reis (2002). The prior for the variance of the inflation target shocks  $\sigma_{\eta_1}^2$  corresponds, on average, to what Kozicki and Tinsley (2003) and Smets and Wouters (2005) find. As at this stage we want to stay quite agnostic about the time series characteristics of inflation, we leave the uncertainty around the priors high and take the same priors for both the euro area and the United States.

**Multivariate model** The priors for the multivariate model come from previous studies estimating variants of the model of Rudebusch and Svensson (1999) as well as the posterior distribution of the univariate model. As a prior for the AR coefficients we chose the posterior means of the univariate model for the euro area and the United States, allowing for more uncertainty than the univariate posterior distributions suggest. The priors for  $\delta$ ,  $\sigma_{\varepsilon_1}^2$  and  $\sigma_{\eta_1}^2$  also correspond to the posterior means of the univariate model. For the impact of the lagged output gap on inflation we choose a value of 0.2. The AR coefficients of the output gap equation are chosen in order to generate a hump-shaped response of output in reaction to a shock. This feature is often found in previous empirical studies (Gerlach and Smets, 1999; Rudebusch and Svensson, 1999; Rudebusch, 2005; Laubach and Williams, 2003). The parameter value for  $\rho_2$  assumes considerable interest rate smoothing (Smets and

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<sup>11</sup>References to the source of prior information for the individual elements of  $\psi$  can be found in Tables 1-4.

Table 1: Parameter estimates univariate model (euro area; 1971Q2:2003Q4)<sup>a</sup>

	Prior reference(s) <sup>b</sup>	Prior distribution <sup>c</sup>			Posterior distribution		
		5 p.c.	Mean	95 p.c.	5 p.c.	Mean	95 p.c.
$\varphi_1$	-	0.04	0.20	0.36	0.14	0.25	0.36
$\varphi_2$	-	-0.06	0.10	0.26	-0.00	0.11	0.22
$\varphi_3$	-	0.01	0.05	0.09	-0.01	0.03	0.07
$\varphi_4$	-	0.01	0.05	0.09	0.02	0.06	0.10
$\sum_{i=1}^4 \varphi_i$	[19],[32]	0.16	0.40	0.64	0.29	0.45	0.61
$\delta$	[16],[29],[33]	0.07	0.15	0.23	0.14	0.20	0.26
$\sigma_{\varepsilon_1}^2$	-	0.35	1.30	2.70	1.40	1.72	2.12
$\sigma_{\eta_1}^2$	[29],[43]	0.01	0.12	0.36	0.10	0.20	0.41

Notes: <sup>a</sup> All variances are expressed at annual rates.

<sup>b</sup> The numbers refer to the numbers in the list of references.

<sup>c</sup> The prior distribution is assumed to be Gaussian for all elements in  $\psi$ , except for the variance parameters which are assumed to be gamma distributed.

Wouters, 2005). The parameter values for  $\rho_1$  and  $\rho_2$  are chosen so that the Taylor (1993) principle  $\left(1 + \frac{\rho_1}{1-\rho_2} = 1.5 > 1\right)$  holds for deviations of  $\pi_t^P$  from  $\pi_t^T$ . The central bank reacts less vigorously  $\left(\frac{\rho_1}{1-\rho_2} = 0.5\right)$  in response to deviations of  $\pi_t$  from  $\pi_t^T$ . This is consistent with the view that an inflation-targeting central bank should only stabilise inflation in the medium run and pay less attention to short-term deviations.

## 4.2 Posterior distributions

In this section we present estimates of the posterior mean  $\bar{\psi} = E[\psi | y, x]$  of the parameter vector  $\psi$  and the posterior mean  $\bar{\alpha}_t = E[\hat{\alpha}_t | y, x]$  of the smoothed state vector  $\hat{\alpha}_t$ . An estimate  $\tilde{\psi}$  of  $\bar{\psi}$  is obtained by setting  $g(\psi^{(i)}) = \psi^{(i)}$  in equation (22) and taking  $\tilde{\psi} = \bar{g}_n$ . An estimate  $\tilde{\alpha}_t$  of  $\bar{\alpha}_t$  is obtained by setting  $g(\psi^{(i)}) = \hat{\alpha}_t^{(i)}$  in equation (22) and taking  $\tilde{\alpha}_t = \bar{g}_n$ , where  $\hat{\alpha}_t^{(i)}$  is the smoothed state vector obtained from the Kalman smoother using the parameter vector  $\psi^{(i)}$ .

We also present the 5th and 95th percentiles of the posterior densities  $p(\psi | y, x)$  and  $p(\hat{\alpha}_t | y, x)$ . Let  $F(\psi_j | y, x) = \Pr(\psi_j^{(i)} \leq \psi_j)$  with  $\psi_j$  denoting the  $j$ -th element in  $\psi$ . An estimate  $\tilde{F}(\psi_j | y, x)$  of  $F(\psi_j | y, x)$  is obtained by setting  $g(\psi^{(i)}) = I_j(\psi_j^{(i)})$  in equation (22) and taking  $\tilde{F}(\psi_j | y, x) = \bar{g}_n$ , where  $I_j(\psi_j^{(i)})$  is an indicator function which equals one if  $\psi_j^{(i)} \leq \psi_j$  and zero otherwise. An estimate  $\tilde{\psi}_j^{5\%}$  of the 5th percentile of the posterior density  $p(\psi | y, x)$  is chosen such that  $\tilde{F}(\psi_j^{5\%} | y, x) = 0.05$ . An estimate  $\tilde{\alpha}_{j,t}^{5\%}$  of the 5th percentile of the  $j$ th element of the posterior density  $p(\hat{\alpha}_t | y, x)$  is obtained by setting  $g(\psi^{(i)}) = \hat{\alpha}_{j,t}^{(i)} - 1.645\sqrt{\hat{P}_{j,t}^{(i)}}$  in equation (22) and taking  $\tilde{\alpha}_{j,t}^{5\%} = \bar{g}_n$ , where  $\hat{\alpha}_{j,t}^{(i)}$  denotes the  $j$ -th element in  $\hat{\alpha}_t^{(i)}$  and  $\hat{P}_{j,t}^{(i)}$  is the  $(j, j)$ th element of the smoothed state variance matrix  $\hat{P}_t^{(i)}$  obtained using the parameter vector  $\psi^{(i)}$ . The 95th percentiles are constructed in a similar way.

Table 2: Parameter estimates multivariate model (euro area; 1971Q2:2003Q4)<sup>a</sup>

	Prior reference(s) <sup>b</sup>	Prior distribution <sup>c</sup>			Posterior distribution		
		5 p.c.	Mean	95 p.c.	5 p.c.	Mean	95 p.c.
$\varphi_1$	univariate euro area	0.09	0.25	0.41	0.17	0.27	0.38
$\varphi_2$	univariate euro area	-0.06	0.11	0.27	0.01	0.11	0.21
$\varphi_3$	univariate euro area	-0.13	0.03	0.20	-0.17	-0.07	0.03
$\varphi_4$	univariate euro area	-0.11	0.06	0.22	0.06	0.16	0.26
$\sum_{i=1}^4 \varphi_i$	univariate euro area	0.12	0.45	0.78	0.28	0.47	0.66
$\delta$	univariate euro area	0.03	0.20	0.36	0.14	0.26	0.38
$\beta_1$	[40],[41],[21]	0.18	0.20	0.22	0.18	0.20	0.22
$\beta_2$	[40],[41],[21]	1.32	1.35	1.38	1.33	1.36	1.40
$\beta_3$	[40],[41],[21]	-0.50	-0.47	-0.44	-0.48	-0.45	-0.42
$\beta_4$	[40],[41],[21]	-0.01	0.15	0.31	0.10	0.15	0.21
$\rho_1$	[45]	0.02	0.05	0.08	0.02	0.04	0.06
$\rho_2$	[45],[43]	0.87	0.90	0.93	0.87	0.90	0.92
$\gamma$	[31]	0.92	1.00	1.08	0.91	0.99	1.07
$\theta$	[31]	0.95	0.97	0.99	0.95	0.97	0.99
$\sigma_{\varepsilon_1}^2$	univariate euro area	0.47	1.72	3.58	1.30	1.59	1.96
$\sigma_{\varepsilon_2}^2$	-	0.21	0.30	0.41	0.25	0.30	0.36
$\sigma_{\varepsilon_3}^2$	[31]	0.11	0.16	0.22	0.08	0.11	0.15
$\sigma_{\eta_1}^2$	univariate euro area	0.05	0.20	0.42	0.07	0.15	0.30
$\sigma_{\eta_2}^2$	-	$3.e-5$	$1.e-4$	$2.e-4$	$1.e-5$	$5.e-5$	$2.e-4$
$\sigma_{\eta_3}^2$	[31]	4.07	5.86	7.88	2.40	3.04	4.16
$\sigma_{\eta_4}^2$	[31]	0.01	0.01	0.01	0.01	0.01	0.01
$\sigma_{\eta_5}^2$	[31]	0.07	0.10	0.14	0.07	0.11	0.14

Notes: see table 1.

#### 4.2.1 Posterior distribution of the parameters

Tables 1-4 present the posterior mean and the 5th and 95th percentile of the posterior distribution of  $\psi$  for the euro area and the United States for both the univariate and multivariate model. Two important conclusions stand out. First, in the univariate model the combination<sup>12</sup> of intrinsic and extrinsic inflation persistence, measured as  $\sum_{i=1}^q \varphi_i$ , amounts to 0.45 for the euro area and 0.67 for the United States. This is considerably lower than estimates from standard AR time series models. The multivariate intrinsic inflation persistence estimates amount to 0.48 and 0.80 for the euro area and the United States, and are in line with the results of the univariate specification. In the case of the United States, intrinsic inflation persistence is somewhat higher than in the euro area. Note that this result is consistent with Galí et al. (2001), who for the United States also find a relatively higher degree of backward-lookingness compared to the euro area. Second, expectations-based persistence, measured by  $(1 - \delta)$ , is at least as high or higher than intrinsic inflation persistence, i.e. higher than 0.75 for both economies across the different models. The persistence in the

<sup>12</sup>Note that since we can not disentangle intrinsic from extrinsic persistence in the univariate model, the AR coefficients measure a combination of both.

Table 3: Parameter estimates univariate model (United States; 1971Q2:2003Q4)<sup>a</sup>

	Prior reference(s) <sup>b</sup>	Prior distribution <sup>c</sup>			Posterior distribution		
		5 p.c.	Mean	95 p.c.	5 p.c.	Mean	95 p.c.
$\varphi_1$	-	0.04	0.20	0.36	0.24	0.36	0.50
$\varphi_2$	-	-0.06	0.10	0.26	0.07	0.19	0.31
$\varphi_3$	-	0.01	0.05	0.09	0.02	0.06	0.10
$\varphi_4$	-	0.01	0.05	0.09	0.02	0.06	0.10
$\sum_{i=1}^4 \varphi_i$	[19],[32]	0.16	0.40	0.64	0.47	0.67	0.87
$\delta$	[16],[29],[33]	0.07	0.15	0.23	0.11	0.18	0.26
$\sigma_{\varepsilon_1}^2$	-	0.36	1.30	2.68	1.10	1.36	1.68
$\sigma_{\eta_1}^2$	[29],[43]	0.01	0.12	0.36	0.03	0.13	0.38

Notes: see table 1.

Table 4: Parameter estimates multivariate model (United States; 1971Q2:2003Q4)<sup>a</sup>

	Prior reference(s) <sup>b</sup>	Prior distribution <sup>c</sup>			Posterior distribution		
		5 p.c.	Mean	95 p.c.	5 p.c.	Mean	95 p.c.
$\varphi_1$	univariate US	0.20	0.36	0.53	0.27	0.36	0.46
$\varphi_2$	univariate US	0.02	0.19	0.35	0.08	0.17	0.25
$\varphi_3$	univariate US	-0.10	0.06	0.22	0.04	0.13	0.22
$\varphi_4$	univariate US	-0.11	0.06	0.22	0.06	0.13	0.21
$\sum_{i=1}^4 \varphi_i$	univariate US	0.34	0.67	1.00	0.68	0.79	0.91
$\delta$	univariate US	0.02	0.18	0.35	0.07	0.21	0.35
$\beta_1$	[40],[41],[21]	0.18	0.20	0.22	0.19	0.20	0.22
$\beta_2$	[40],[41],[21]	1.32	1.35	1.38	1.33	1.36	1.39
$\beta_3$	[40],[41],[21]	-0.50	-0.47	-0.44	-0.48	-0.46	-0.43
$\beta_4$	[40],[41],[21]	-0.01	0.15	0.31	0.11	0.15	0.22
$\rho_1$	[45]	0.02	0.05	0.08	0.03	0.05	0.07
$\rho_2$	[45], [43]	0.87	0.90	0.93	0.85	0.88	0.91
$\gamma$	[31]	0.92	1.00	1.08	0.99	1.00	1.01
$\theta$	[31]	0.95	0.97	0.99	0.95	0.97	0.98
$\sigma_{\varepsilon_1}^2$	univariate US	0.37	1.36	2.83	1.07	1.19	1.35
$\sigma_{\varepsilon_2}^2$	-	0.21	0.30	0.41	0.61	0.69	0.79
$\sigma_{\varepsilon_3}^2$	[31]	0.11	0.16	0.22	0.11	0.14	0.17
$\sigma_{\eta_1}^2$	univariate US	0.03	0.13	0.26	0.07	0.09	0.11
$\sigma_{\eta_2}^2$	-	7.e-5	1.e-4	1.e-4	7.e-5	1.e-4	1.e-4
$\sigma_{\eta_3}^2$	[31]	4.07	5.86	7.88	4.40	5.28	6.39
$\sigma_{\eta_4}^2$	[31]	4.e-3	0.01	0.02	6.e-3	8.e-3	9.e-3
$\sigma_{\eta_5}^2$	[31]	0.01	0.10	0.31	0.29	0.36	0.44

Notes: see table 1.

output gap, measured by the sum of  $\beta_2$  and  $\beta_3$ , amounts to at least 0.9. This implies considerable extrinsic inflation persistence.

#### 4.2.2 Posterior distribution of the states

Figures 3, 4, 5 and 6 show the dynamics of the inflation rate together with the central bank's inflation target and the perceived inflation target. These figures reveal considerable variation in the central bank's inflation target in both the euro area and the United States. The dynamics of the perceived inflation target show that inflation expectations adjust smoothly in response to shifts in the central bank's inflation target. The central bank's inflation target and the perceived inflation target identified in the univariate model are very similar to the ones identified in the multivariate model. This confirms that the permanent shifts in the perceived inflation target identified in the univariate model are indeed driven by shifts in the central bank's inflation target.

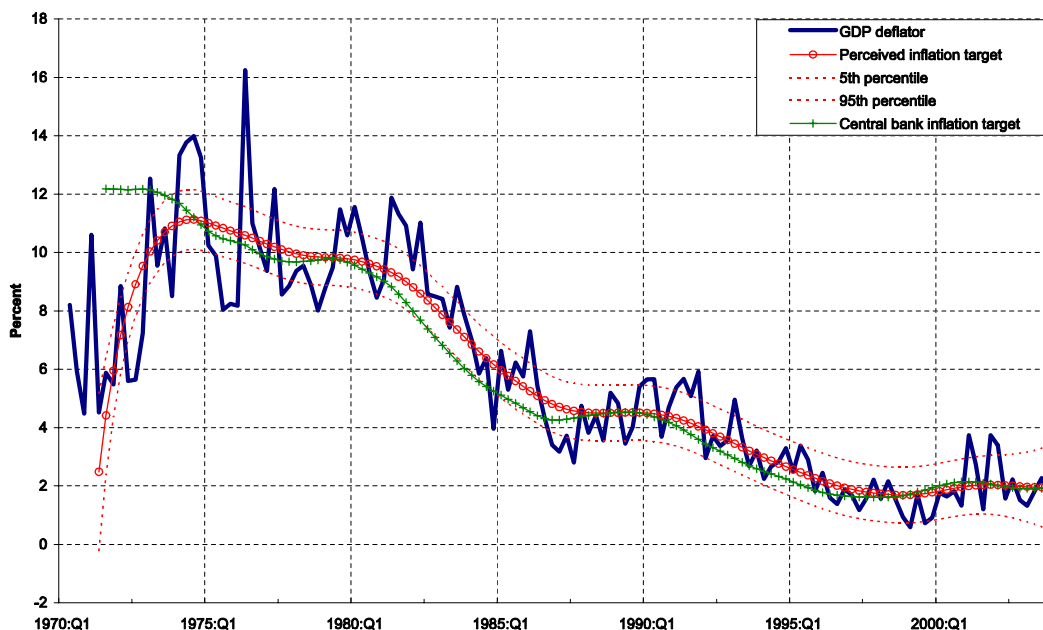


Figure 3: Smoothed univariate states (euro area)

The timing of the shifts in the central bank's inflation target seems to be in line with common knowledge about the historical conduct of monetary policy. A first disinflationary period is present in the early 1980s. In the United States, the univariately estimated inflation target decreased from 7 p.c. in the late 1970s to about 3 p.c. in the mid 1980s. This is matched by the disinflationary policy of Paul Volcker, who was appointed president of the Federal Reserve in 1979. A similar decrease, from about 10 p.c. to about 5 p.c., is

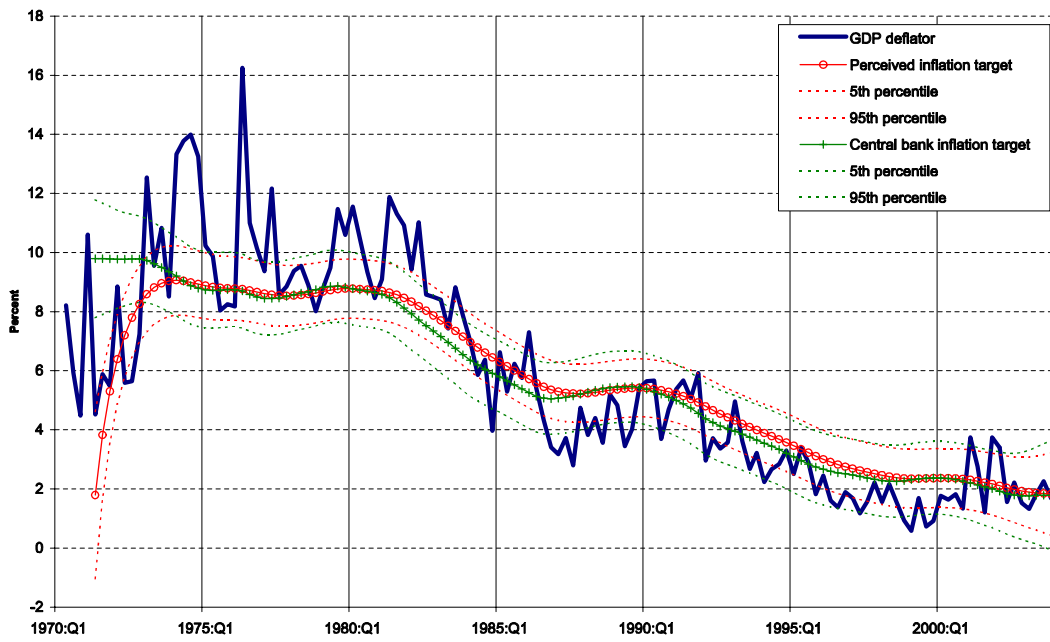


Figure 4: Smoothed multivariate states (euro area)

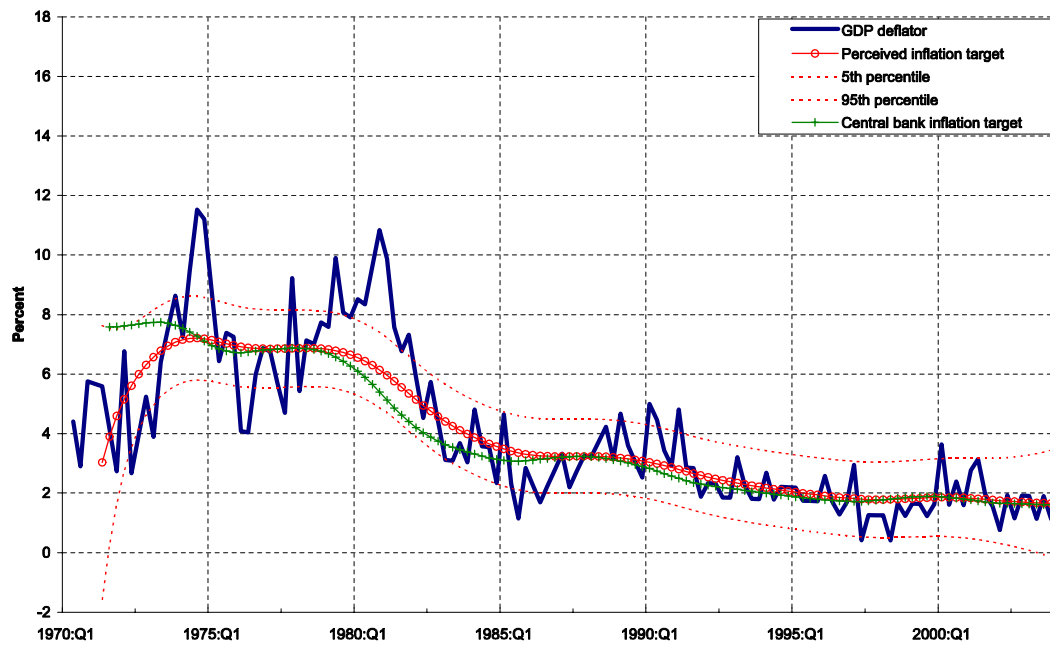


Figure 5: Smoothed univariate states (United States)

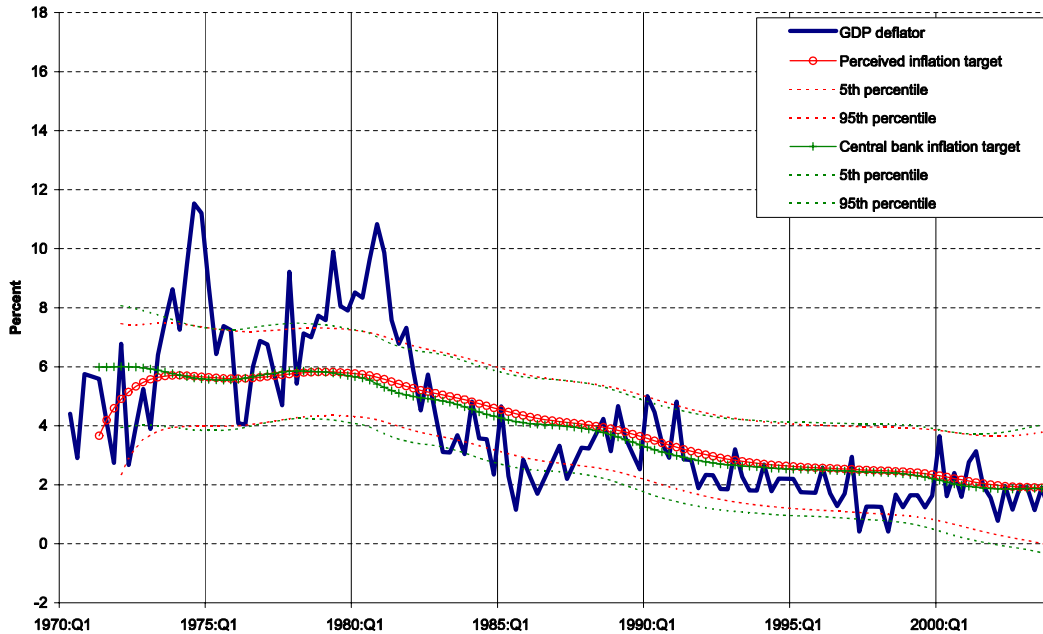


Figure 6: Smoothed multivariate states (United States)

observed for the euro area. This decrease is more difficult to match with narrative evidence, though, as no unified monetary policy existed before 1999. Still, several future euro area member countries (e.g. Austria, Belgium, France, The Netherlands) were disinflating in the beginning of the eighties. For the euro area, a second disinflationary period is also present in the beginning of the nineties. Other future euro area member countries (e.g. Greece, Italy, Portugal, Spain) were then disinflating in order to comply with the Maastricht criteria. In the United States there seems to have been a somewhat less pronounced decrease in the central bank's inflation target over that period.

In an inflation targeting framework, where the short-term interest rate is the primary policy instrument, the natural interest rate provides a metric for the stance of monetary policy. The natural rate of interest varies over time due to shifts in the trend growth of output and other factors such as households' rate of time preference. We took these variations explicitly into account in our model, so that when estimating shifts in the central banks' inflation target the results would not be misleading due the shifts in the benchmark, namely the natural interest rate. From figures 7 and 8 one can see that during the nineties a decrease in the trend growth rate of the euro area has driven the natural real interest rate, whereas this does not seem to be the case for the United States. In addition, especially variations in time preferences have driven the natural real interest rate over the last three decades in both the United States and the euro area.



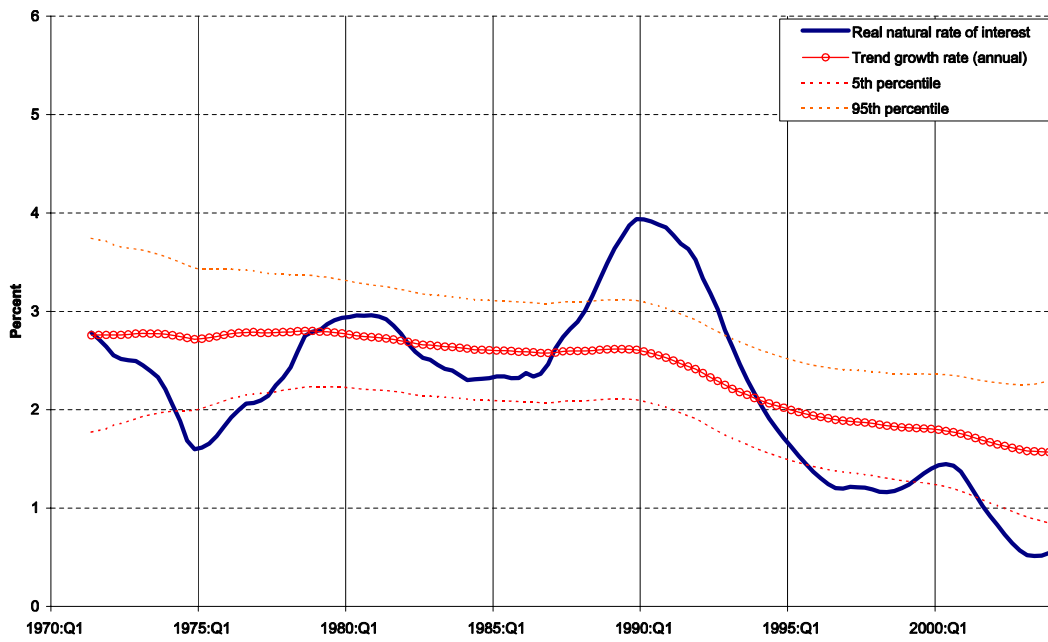


Figure 7: Smoothed multivariate states (euro area)

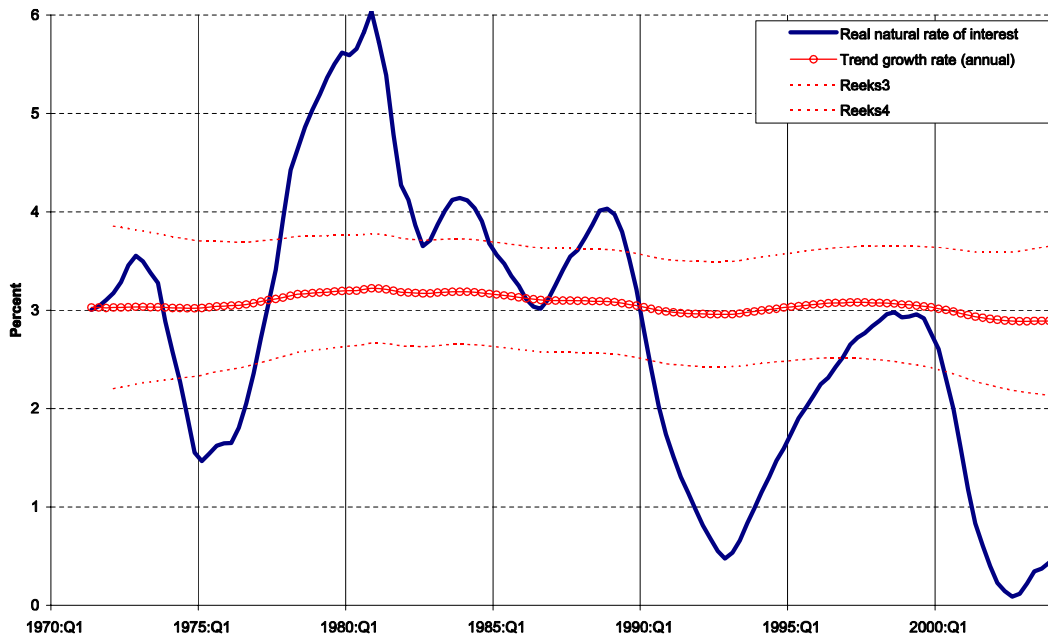


Figure 8: Smoothed multivariate states (United States)

Table 5: Half lives of inflation (quarters)

	Euro area	United States
Temporary inflation shock	1	1
Perceived inflation target shock	8	16
Output gap shock	13	18
Central bank target shock	$\infty$	$\infty$

#### 4.2.3 Half-life and impulse response analysis

An alternative way of analysing inflation persistence in the multivariate model looks at the impulse response functions and the so-called half-life of different shocks to inflation. The latter counts the number of periods for which the effect of a shock to inflation remains above half its initial impact. An important difference with the point estimates of the respective AR coefficients is that with this persistence measure different sources of persistence in response to a shock can reinforce each other. The inflation dynamics in response to a shock will thus not only depend on the persistence in the variable that was shocked, but will also depend on the interaction with other variables. Therefore, also the persistence in the latter will play a role.

Table 5 reports half lives for four shocks to inflation considered in the multivariate model. The half life of a temporary shock ( $\varepsilon_{1t}$ ) is only one quarter. For a shock to the perceived inflation target ( $\eta_{2t}$ ), the half life is 8 and 16 quarters in the euro area and the United States respectively. For a shock to the output gap ( $\varepsilon_{3t}$ ), the half life even amounts to 13 quarters in the euro area and to 18 quarters the United States. Finally, a shock to the inflation target ( $\eta_{1t}$ ) is permanent and therefore its half life is equal to infinity. The latter result is obtained by construction because we assume a random walk process for the shifts in the central bank's inflation target. Still, it shows that ignoring a component with an infinite half life must create a considerable bias in the estimates of the other kinds of persistence.

A similar lesson can be learnt from the impulse response functions in response to a unit shock in Figures 9-10. Both in the euro area and in the United States a shift in the central bank's inflation target ( $\eta_{1t}$ ) has a permanent impact on inflation. Still, it takes various periods before the inflation rate stabilises at the new target. This is to a big extent due to considerable expectations-based persistence that creates persistent deviations of the perceived inflation target from the central bank's inflation target. In case of a shock to the output gap ( $\varepsilon_{3t}$ ) or the perceived inflation target ( $\eta_{2t}$ ), the response of inflation seems to be characterised by a similar degree of persistence. In case of a temporary shock to inflation ( $\varepsilon_{1t}$ ), the convergence to the target goes much faster. Intrinsic and expectations-based persistence measured according to the sum of the AR coefficients are not statistically significantly different. Still, due to the persistence in the reaction of the central bank and

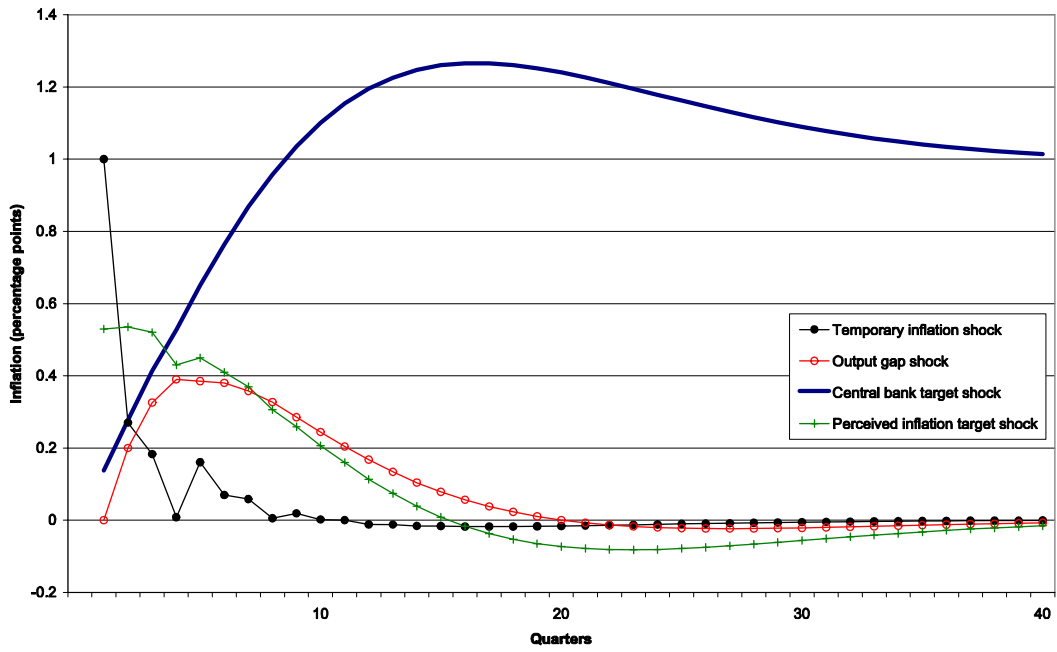


Figure 9: Impulse responses (euro area)

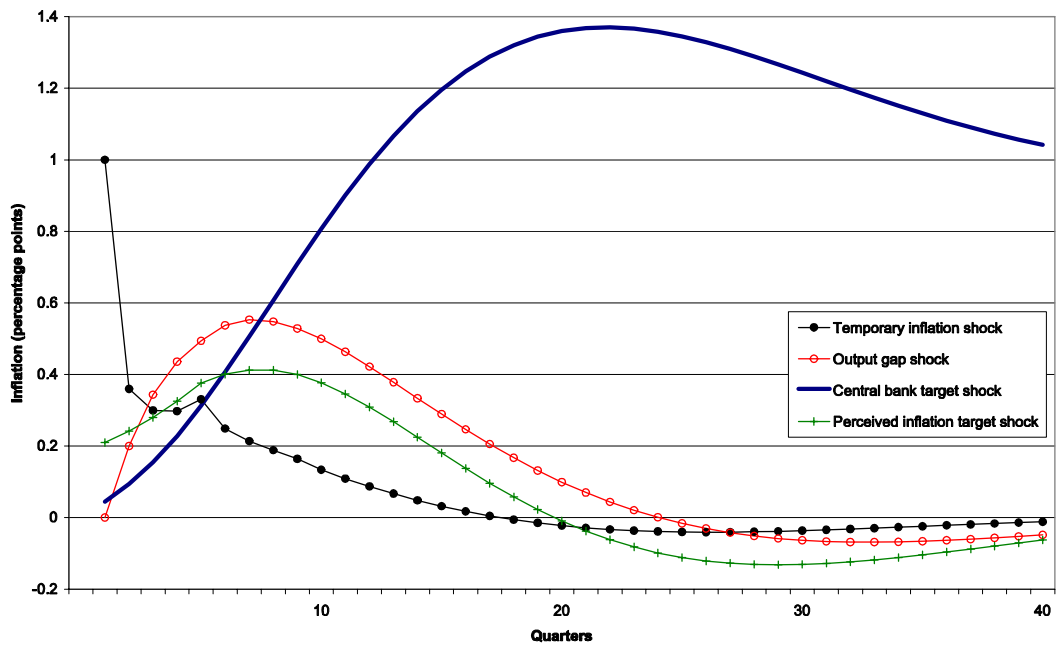


Figure 10: Impulse responses (United States)

the output gap, the number of quarters that inflation is affected by a difference between the perceived and the central bank's inflation target can be considerably higher.

The impulse response functions show that the central bank can play an important role in the adjustment process. The speed and the extent to which the central bank adjusts its policy instrument will determine the speed at which inflation returns to its target level. In case of a difference between the perceived inflation target and the central bank's inflation target we modelled the reaction function of the central bank such that it responds more than in case of deviations from the inflation target caused by other shocks (cf. higher). Still, the interest rate smoothing is the same for all deviations, implying that the speed - and not the extent - at which the central bank adjusts its policy instrument is the same. To accelerate the adjustment to the inflation target a central bank could react more vigorously to a shock to the output gap or the perceived inflation target compared to a temporary shock to inflation.

## 5 Conclusions

This paper aims at measuring different sorts of inflation persistence, i.e. the sluggish response of inflation in response to different macroeconomic shocks. In the literature post war inflation persistence measures are often found to be close to that of a random walk. The main point stressed in this paper is that these unconditional estimates are hard to interpret as the data generating process of inflation can be decomposed in a number of distinct components, each of them exhibiting its own degree of persistence. First, shifts in the central bank's inflation target can induce permanent shifts in the mean inflation rate. Second, imperfect or sticky information implies that private agents have to learn about the true central bank's inflation target. As such, the inflation target perceived by private agents can persistently differ from the true central bank's inflation target. Third, persistence in the various determinants of inflation also introduces persistence in the observed inflation rate. As the first three sources of persistence typically show relatively high inertia, ignoring one of them might create an upward bias in measured intrinsic inflation persistence.

Therefore, we measure inflation persistence in a structural time series model which explicitly models the various components driving inflation. We pursue both a univariate and a multivariate approach. Extracting information from the central bank's key interest rate we find confirmation that shifts in the central bank's inflation target induce a non-stationary component in the inflation rate. In addition, slow adjustment of inflation expectations in response to changes in the central bank's inflation target and persistence of shocks hitting inflation are important factors determining the observed inflation persistence. These com-

ponents explain a large fraction of the high degree of persistence observed in the post-WW II inflation rate. Taking these components into account, intrinsic inflation persistence is found to be lower than the persistence of a random walk, i.e. the sum of the AR coefficients in the data generating process of inflation is estimated to range from 0.45 in the euro area to 0.8 in the United States.

The implications for monetary policy are as follows. Our evidence indicates that in a stable inflation regime, where the central bank's inflation target does not change and where the public perception about this inflation target is well anchored, inflation persistence is relatively lower. The results also imply that in the case monetary policy would again give rise to unstable inflation, it would afterwards be very hard to disinflate due to the slow adjustment of inflation expectations in response to changes in the inflation target. In the case of natural rate misperceptions (Orphanides and Williams, 2004) this might however not be straightforward to avoid.

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## Appendix 1: Deriving an empirical specification for inflation expectations

Equation (4) can be derived using a variant of the sticky information model of Mankiw and Reis (2002) or the signal extraction problem of Erceg and Levin (2003) and Andolfatto et al. (2002). The difference between the two models is the way information about the central bank's inflation target  $\pi_t^T$  arrives to the firms. In the sticky-information model, exact information about  $\pi_t^T$  is available but not all firms update their information about  $\pi_t^T$  every period due to for instance information gathering costs. Therefore, aggregate prices do not respond immediately to changes in  $\pi_t^T$ . In the model of Erceg and Levin (2003) and Andolfatto et al. (2002), exact information about  $\pi_t^T$  is not available. This leads to a signal extraction problem. Aggregate prices will only respond to changes in  $\pi_t^T$  once firms have learned about the new central bank target. If learning is slow, aggregate prices will not respond immediately to changes in  $\pi_t^T$ .

### A sticky-information model

As in Mankiw and Reis (2002) we assume that firms reset their prices every period, but infrequently gather information about the central bank inflation target  $\pi_t^T$ , which is readily available in every period. The log of a firm's optimal price  $p_t^*$  is given by:

$$p_t^* = p_t + \alpha z_t \quad (\text{A.1})$$

$$p_t^* = p_{t-1}^P + \pi_t^T, \quad z_t = 0 \quad \forall t \quad (\text{A.2})$$

where  $p_t$  is the log of the aggregate price level,  $z_t$  is the output gap and  $\alpha$  is a positive coefficient. This equation says that a firm's desired relative price rises in booms and falls in recessions. If we assume that the output gap is always equal to zero, the firms' optimal price  $p_t^*$  will be equal to the aggregate price level  $p_t$  or the sum of the aggregate price level  $p_{t-1}^P$  in the previous period consistent with the perceived inflation target and the central bank's inflation target  $\pi_t^T$  in the current period.

In this model, only a fraction  $\lambda$  of the firms updates its information about  $\pi_t^T$  to calculate a new optimal price. The probability of updating information is the same for each firm, i.e. independent of the timing of the last update. The other firms continue to set their prices based on old information about  $\pi_t^T$ .

A firm that last updated its beliefs about the inflation target  $j$  periods ago sets its price  $x_t^j$ :

$$x_t^j = E_{t-j} p_t^* \quad (\text{A.3})$$

$$= p_{t-1-j}^P + (j+1)\pi_{t-j}^T \quad (\text{A.4})$$

The aggregate price level is the average of the prices of all firms, given by:

$$p_t^P = \lambda \sum_{j=0}^{\infty} (1-\lambda)^j x_t^j \quad (\text{A.5})$$

The perceived inflation target can be calculated from (A.5) as:

$$\pi_t^P = p_t^P - p_{t-1}^P \quad (\text{A.6})$$

$$= \lambda \pi_t^T + (\lambda - 1) p_{t-1}^P + \lambda \sum_{j=0}^{\infty} (1-\lambda)^{j+1} (p_{t-2-j}^P + (j+2)\pi_{t-j-1}^T) \quad (\text{A.7})$$

Substituting out  $p_{t-j}^P$  using (A.5) and rearranging yields:

$$\pi_t^P = \lambda \sum_{j=0}^{\infty} (1-\lambda)^j \pi_{t-j}^T, \quad (\text{A.8})$$

which is equivalent to:

$$\pi_t^P = (1-\lambda)\pi_{t-1}^P + \lambda\pi_t^T \quad (\text{A.9})$$

### A signal extraction problem

Both Erceg and Levin (2003) and Andolfatto et al. (2002) assume that monetary authorities set nominal interest rates in line with their inflation target,  $\pi_t^T$ , using an interest rate rule. Observing the central bank's interest rate, private agents can therefore infer on the central bank's inflation target from their knowledge of the central bank's interest rate rule. An information problem arises from the assumption that the interest rate set by the central bank can shift due to both transitory and permanent monetary policy actions. Transitory policy actions can be interpreted as (i) deviations from the interest rate rule in response to various transitory shocks hitting inflation and/or (ii) imperfect control of monetary authorities over the interest rate. Permanent policy actions are shifts in the central bank's inflation target  $\pi_t^T$ . Consequently, private agents must solve a signal-extraction problem to disentangle transitory and permanent policy actions using shifts in the nominal interest rate. This can be done using the Kalman filter. This optimal filtering solution gives rise to a learning rule that resembles adaptive expectations processes.

In particular, we assume that the central bank's inflation target evolves according to equation (1) while monetary policy is described by the following interest rate rule:

$$i_t = \rho_2 i_{t-1} + (1 - \rho_2) (r_t^* + \pi_{t-1}^P) + \rho_1 (\pi_{t-1} - \pi_t^T) + \varepsilon_{2t} \quad (\text{A.10})$$

More information on this interest rate rule can be found in section 2. Permanent monetary policy actions stem from  $\eta_{1t}$  in equation (1). Transitory policy actions stem from  $\varepsilon_{2t}$  in equation (6). An optimal estimate  $E_t \pi_t^T$  of  $\pi_t^T$  based on the information contained in  $i_t$  can be obtained recursively using the Kalman filter as:

$$E_t \pi_t^T = E_{t-1} \pi_{t-1}^T - k_g \nu_t \quad (\text{A.11})$$

where  $\nu_t$  captures the new information contained in  $i_t$ , i.e.  $\nu_t = i_t - E_{t-1} i_t = \rho_1 (E_{t-1} \pi_t^T - \pi_t^T) + \varepsilon_{2t}$  where for simplicity  $r_t^*$  is assumed to be a constant  $\bar{r}$ .  $k_g$  the Kalman gain parameter that measures the speed at which private agents update their beliefs about the monetary policy target  $\pi_t^T$  in response to the new information contained in  $\nu_t$ . It is given by

$$k_g = \frac{1}{2} \frac{\sigma_{\eta_1}^2}{\sigma_{\varepsilon_2}^2} \left( -\rho_1 + \sqrt{\rho_1^2 + 4 \frac{\sigma_{\varepsilon_2}^2}{\sigma_{\eta_1}^2}} \right) \quad (\text{A.12})$$

Equation (A.12) shows that  $k_g$  is increasing in the signal-to-noise ratio  $\sigma_{\eta_1}^2 / \sigma_{\varepsilon_2}^2$  and decreasing in the reaction  $\rho_1$  of the central bank to deviations of inflation from its target.

As from equation (1) we have that  $E_{t-1} \pi_t^T = E_{t-1} \pi_{t-1}^T$  and setting  $\pi_t^P = E_t \pi_t^T$  using equation (2), equation (A.11) can be rewritten as:

$$\pi_t^P = (1 - \rho_1 k_g) \pi_{t-1}^P + \rho_1 k_g \pi_t^T - k_g \varepsilon_{2t} \quad (\text{A.13})$$

## Appendix 2: State Space representations

### Univariate model

$$y_t = [ \pi_t ]; \alpha_t = [ \pi_t^P \quad \pi_{t-1}^P ]'; x_t = [ \pi_{t-1} \quad \dots \quad \pi_{t-q} ]';$$

$$Z = [ (1 - \sum_{i=1}^q \varphi_i) \quad 0 ]; A = [ \varphi_1 \quad \dots \quad \varphi_q ]; T = \begin{bmatrix} 2 - \delta & \delta - 1 \\ 1 & 0 \end{bmatrix};$$

$$R = [ \delta \quad 0 ]'; \varepsilon_t = [ \varepsilon_{1t} ]; \eta_t = [ \eta_{1t} ]; H = [ \sigma_{\varepsilon_1}^2 ]; Q = [ \sigma_{\eta_1}^2 ]$$

### Multivariate model

$$y_t = [ \pi_t \quad i_t \quad y_t^r ]'; x_t = [ \pi_{t-1} \quad \pi_{t-2} \quad \dots \quad \pi_{t-q} \quad y_{t-1} \quad y_{t-2} \quad i_{t-1} ]';$$

$$\alpha_t = [ \pi_t^T \quad \pi_t^P \quad \pi_{t-1}^P \quad y_t^P \quad y_{t-1}^P \quad y_{t-2}^P \quad \lambda_t \quad \lambda_{t-1} \quad \tau_t \quad \tau_{t-1} ]';$$

$$A = \begin{bmatrix} \varphi_1 & \varphi_2 & \dots & \varphi_q & \beta_1 & 0 & 0 \\ \rho_1 & 0 & \dots & 0 & 0 & 0 & \rho_2 \\ 0 & 0 & \dots & 0 & \beta_2 & \beta_3 & -\beta_4 \end{bmatrix};$$

$$Z = \begin{bmatrix} 0 & (1 - \sum_{i=1}^q \varphi_i) & 0 & 0 & -\beta_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\rho_1 & (1 - \rho_2) & 0 & 0 & 0 & 0 & (1 - \rho_2)\gamma & 0 & (1 - \rho_2) & 0 & 0 \\ 0 & 0 & \beta_4 & 1 & -\beta_2 & -\beta_3 & 0 & \beta_4\gamma & 0 & \beta_4 & \beta_4 \end{bmatrix};$$

$$T = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \delta & (1 - \delta) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \theta & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}; R = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ \delta & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix};$$

$$\varepsilon_t = [ \varepsilon_{1t} \quad \varepsilon_{2t} \quad \varepsilon_{3t} ]'; \eta_t = [ \eta_{1t} \quad \eta_{2t} \quad \eta_{3t} \quad \eta_{4t} \quad \eta_{5t} ]';$$

$$H_t = \begin{bmatrix} \sigma_{\varepsilon_1}^2 & 0 & 0 \\ 0 & \sigma_{\varepsilon_2}^2 & 0 \\ 0 & 0 & \sigma_{\varepsilon_3}^2 \end{bmatrix}; Q_t = \begin{bmatrix} \sigma_{\eta_1}^2 & 0 & 0 & 0 & 0 \\ 0 & \sigma_{\eta_2}^2 & 0 & 0 & 0 \\ 0 & 0 & \sigma_{\eta_3}^2 & 0 & 0 \\ 0 & 0 & 0 & \sigma_{\eta_4}^2 & 0 \\ 0 & 0 & 0 & 0 & \sigma_{\eta_5}^2 \end{bmatrix}$$

### Appendix 3: Data

- **Inflation:** quarterly inflation rate, defined as  $400(\ln P_t - \ln P_{t-1})$ , with  $P_t$  the seasonally adjusted quarterly GDP deflator. Sources: AWM (Fagan et al, 2005) and BIS;
- **Real output:** quarterly  $\ln(\text{GDP}_t)$ , with  $\text{GDP}_t$  the seasonally adjusted quarterly GDP in constant prices. Sources: AWM (Fagan et al, 2005) and BIS. The estimated output gap is expressed in percent deviation of current output from potential output, namely  $100 * (y_t^r - y_t^P)$ ;
- **Key interest rate:** quarterly central bank key interest rate. Sources: NCB and ECB calculations and BIS.