Numerical investigation on bottom-up fluctuation of investment

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Abstract

This paper investigates the aggregate fluctuations in production and demand components when a firm's investment decision takes the form of (S,s) policies. In the field of large-dimensional non-linear dynamical systems, it is a commonly accepted view that a system of coupled non-linear oscillators can exhibit sizable aggregate fluctuations in principle even when the coupling is relatively weak. It has been recognized that this mechanism can take effects in a dynamic general equilibrium model in which a firm's investment is lumpy and strategically complement with each other (Nirei, Journal of Economic Theory, forthcoming). In this paper we explore the possibility of such a bottom-up fluctuation which is driven by the endogenous synchronization of firms' lumpy investments. We consider an economy which is disaggregated up to the SIC 4-digit industries (about 500 sectors). We calibrate the magnitude and periodicity of the sectoral fluctuations by using data on the U.S. 4-digit manufacturing sectors. Then we compute the equilibrium paths with various parameters for preference and technology. We observe considerable aggregate fluctuation which is compatible with the U.S. aggregate fluctuations in size for a range of parameters. We quantify the dependence of the magnitude of fluctuations and the autocorrelation of the aggregate series on parameters such as the elasticity of intertemporal substitution and the imperfect competition of the product markets.
Keywords: Business cycles, investment, aggregate fluctuations, microscopic non-linearity, (S,s) policy, stochastic clustering, synchronization, globally coupled map, dynamic general equilibrium

1 Introduction

This paper concerns a propagation mechanism in investment across sectors. The large fluctuation in investment is often considered as a driving force of business cycles. Also the investment fluctuation is characterized by the synchronized

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oscillation across sectors. We propose a model of investment propagation which quantitatively explains this phenomenon and identifies the parameters at work.

Quantitatively, we ask the following question: given the magnitude of sectoral oscillations in the U.S. economy, how do the sectoral fluctuations add up to the aggregate fluctuations? It is immediately clear that just summing up the independent series of the sectoral oscillations do not amount to the aggregate fluctuations observed in the U.S. production. There must be some sectoral comovements. Simulations show that the general equilibrium path of our model matches the magnitude and pattern of the aggregate fluctuations observed in the U.S. when the responses of real wage and real intrest rate to aggregate production are modest.

Traditional macroeconomics as well as the benchmark real business cycle theory supposes the aggregate shocks, such as money supply, aggregate productivity, or animal spirits of investors, as the fundamental shock. Without apparent evidence of such aggregate shocks as the consistence cause of business cycles [6, 5], however, the literature is in search of the mechanism which propagates and amplifies the shocks on disaggregated parts of economies [19, 13, 11]. The disaggregated model of the aggregate fluctuations turns out to face the law of large numbers: the tendency that disaggregated shocks cancel out each other. In many models the tendency is so strong that a realistic magnitude of an individual shock does not generate aggregate fluctuations large enough to match empirical fluctuations. For example, Long and Plosser [14] show that a general equilibrium model can in principle generate comovement across sectors when sectors bear idiosyncratic productivity shocks. In a successive research, however, Dupor [10] establishes that their model cannot generate the aggregate fluctuations unless the individual shock is of order the size of the number of individuals in the economy.

Another line of research on investment fluctuations focused on the endogenous fluctuations which result from non-linearity of economic dynamics. The models of multiple equilibria, chaos, or self-fulfilling expectation show the possibility that the aggregate fluctuations occur in a deterministic environment of economic fundamentals if the non-linearity is sufficiently strong. This paper explores a new approach along this line, in which an interaction of many small non-linear behaviors causes a deterministic fluctuation. We suppose that individual sectors follow a deterministic pattern of capital oscillations with occasional large adjustments and periods of inertial depreciation. The sectors monopolistically compete each other, so an increase in production in a sector induces other sectors to increase their production (and cut prices). Thus the timing of occasional capital adjustments may be endogenously synchronized. This interrelation makes the product markets a multi-dimensional non-linear dynamical system which in principle is capable of generating an endogenous complex fluctuation. The model investigated in this paper is closely connected to the self-organized critical fluctuations demonstrated by Bak, Chen, Scheinkman, and Woodford [1] in particular. They show a power-law distribution of production propagation in a network of locally interacting producers. We implement a similar propagation mechanism in an equilibrium model of globally interacting sectors.

This paper also addresses the question of whether a micro discrete choice, in particular an (S,s) behavior, is relevant in aggregate fluctuations. The seminal paper by Doms and Dunne [9] found that an establishment level capital is adjusted only occasionally but by a jump in size. A series of research, among others Cooper, Haltiwanger, and Power [7], has stressed the role of the lumpy adjustments played in business cycles. Theoretical and numerical studies on aggregation of (S,s) behaviors, for example Caplin and Spulber [4] and Caballero and Engel [3], have largely found that such an individual lumpiness does not contribute to aggregate fluctuations. Again, the law of large number is the logic: the individual lumpiness tends to cancel out each other. To the contrary, this paper shows that the (S,s) behavior can generate a considerable magnitude of aggregate fluctuations.

We consider the following mechanism. The sectors are linked each other by derived factor demand when each sector uses other sectors' product as intermediate inputs. Their interaction forms a positive feedback in capital adjustments in the network of input-output relations. Suppose that a capital adjustment takes a form of discrete decision. Then there is a chance of a chain-reaction of investment in which an investment in one sector triggers an investment in another sector, and so on. Our previous study [15] has shown that this chain-reaction turns out to be represented by a branching process. It established that the total size of the chain-reaction can exhibit a very large variance in a partial equilibrium of product markets. This means that the law of large numbers in the disaggregated economy can be overcome. It has analytically shown that the propagation distribution in our model follows a truncated power-law. The propagation also exhibits critical fluctuations in which the variance of aggregate growth rates does not converge to zero when the number of sectors in the economy tends to infinity in the limiting case at which wage and interest rate are determined independently from the product market. This proposition assures that any plausible magnitude of aggregate fluctuation can be obtained in our model when the price response to aggregate product is sufficiently slow.

In this paper, we simulate a general equilibrium calibrated by a finely disaggregated sectoral data to examine under what conditions the model generates the right magnitude of fluctuations. As argued by Thomas [20], the general equilibrium effects via wage and interest rate dampen the fluctuation effects due to the (S,s) behavior. We examine this dampening effect by simulating the general equilibrium paths. Simulations show that a realistic magnitude of fluctuations is obtained when the intertemporal substitution of consumption as well as leisure is large. We also show that the autocorrelation and correlation structure of the production and demand components matches the empirical business cycle patterns.

There have been successful attempts in reproducing the production fluctuations by simulating coupled oscillators (see [18] for example). Here we embed the coupled oscillators in a general equilibrium framework which incorporates a representative household's response to prices, construct the coupling parameters by the fundamentals such as technology and preference, and examine the structure of fluctuations among aggregate variables.

The rest of the paper is organized as follows. The next section presents the model. Section 3 numerically examines the quantitative properties of the propagation and the business cycle fluctuations. Section 4 concludes the paper.

2 Model

This section draws on the model presented in [15]. The product market consists of N monopolists and a representative household. Each monopolist j produces a differentiated good y_j, using capital k_j and labor h_j. The production function is a Cobb-Douglas:

$$y_{j,t} = Ak^{\alpha}_{j,t}h^{\gamma}_{j,t}. \tag{1}$$

The capital is accumulated over time as:

$$k_{j,t+1} = (1-\delta_j)k_{j,t} + i_{j,t} \tag{2}$$

where δ_j is an industry specific depreciation rate. Investment $i_{j,t}$ is a composite good produced by combining all the goods symmetrically as:

$$i_{j,t} = N\left(\sum_{l=1}^{N}(z^I_{l,j,t})^{\frac{1}{\mu}}/N\right)^{\mu} \tag{3}$$

where $\mu - 1 > 0$ denotes the mark-up rate. The mark-up is determined by the elasticity of substitution between inputs in the production of investment good, $\mu/(\mu-1)$. The production technology is allowed to exhibit increasing returns to scale as far as $\alpha + \gamma < \mu$ is satisfied.

We assume that the investment rate is chosen from a discrete set. Specifically, we assume that:

$$\frac{i_{j,t}}{k_{j,t}} \in \left\{(1-\delta_j)(\lambda_j^{\kappa_t}-1)\right\}_{\kappa_t=0,\pm1,\pm2,\dots} \tag{4}$$

where $\lambda_j > 1$. Note that the choice space for $k_{j,t}$ is independent of the path: $k_{j,t} \in \{(1-\delta_j)^t k_{j,0}\lambda_j^{\tilde{\kappa}_t}\}_{\tilde{\kappa}_t=0,\pm1,\pm2,\dots}$. The assumption implies that the next period capital $k_{j,t+1}$ has to be either the depreciated level $k_{j,t}(1-\delta_j)$ or its multiplication or division of λ_j. By this assumption, the producer is forced to invest in a lumpy manner. Thus this constraint is a shortcut for the lumpy behavior which typically occurs when a fixed cost incurs in investment. This is the only modification from the usual model of monopolistic economies. The main objective of this paper is to examine the aggregate consequence of a non-linear behavior of producers induced by the discreteness constraint.

Let $p_{j,t}$ denote the price of good j at t. Define a price index $P_t \equiv (\sum_{j=1}^{N} p_{j,t}^{1/(1-\mu)}/N)^{1-\mu}$ and normalize it to one. Let w_t denote a real wage for an efficiency unit of labor. Then the monopolist's profit at t is written as:

$$\pi_{j,t} \equiv p_{j,t}y_{j,t} - w_t h_{j,t} - \sum_{l=1}^{N} p_{l,t} z^I_{l,j,t} \tag{5}$$

The demand function for good j is derived by usual procedure [8]. Let us suppose that the representative household has a preference over the sequence of consumption and hours worked:

$$\sum_{t=0}^{\infty} \beta^t U(C_t, H_t) \tag{6}$$

where C_t is an average composite consumption good produced identically as the investment good:

$$C_t = \left(\sum_{l=1}^{N} (z_{l,t}^C)^{\frac{1}{\mu}} / N \right)^{\mu}. \tag{7}$$

Note that we normalize the consumption C_t by N so that the disaggregation level does not affect the level of average aggregate variable. The same normalization applies to H_t as well as I_t, Y_t, Π_t as we define shortly. The representative household maximizes the utility function subject to the sequence of budget constraints:

$$\sum_{j=1}^{N} p_{j,t} z_{j,t}^C = w_t H_t + \Pi_t. \tag{8}$$

where Π_t is the average dividend from firms: $\Pi_t \equiv \sum_{j=1}^{N} \pi_{j,t}/N$.

The cost minimization of the consumer given the level of consumption C_t implies: $z_{j,t}^C = p_{j,t}^{-\mu/(\mu-1)} C_t$ and a relation $\sum_{j=1}^{N} p_{j,t} z_{j,t}^C / N = C_t$. Similarly, the derived demand for good j by the monopolist l given the level of investment $i_{l,t}$ is obtained as $z_{j,l,t}^I = p_{j,t}^{-\mu/(\mu-1)} i_{l,t}/N$ and $\sum_{j=1}^{N} p_{j,t} z_{j,l,t}^I = i_{l,t}$. With the equilibrium condition for good j, $y_{j,t} = z_{j,t}^C + \sum_{l=1}^{N} z_{j,l,t}^I$, these yield the demand function for good j as: $y_{j,t} = p_{j,t}^{-\mu/(\mu-1)}(C_t + I_t)$ where $I_t \equiv \sum_{j=1}^{N} i_{j,t}/N$. Define a production index $Y_t \equiv (\sum_{j=1}^{N} y_{j,t}^{1/\mu}/N)^{\mu}$. Then we have an equilibrium relation $\sum_{j=1}^{N} p_{j,t} y_{j,t} = Y_t$. Combining with the consumer's budget constraint (8) and the equilibrium condition for labor, $H_t = \sum_j h_{j,t}/N$, we obtain the demand function:

$$y_{j,t} = p_{j,t}^{\frac{-\mu}{\mu-1}} Y_t \tag{9}$$

The monopolist maximizes its discounted future profits as instructed by the representative household. The discount rate, r_t^{-1}, is the marginal rate of intertemporal substitution of consumption. Then the monopolist's problem is defined as follows.

$$\max_{\{y_{j,t}, k_{j,t+1}, h_{j,t}, i_{j,t}, z_{l,j,t}^I\}} \sum_{t=0}^{\infty} (r_1 \cdots r_t)^{-1} \pi_{j,t} = \sum_{t=0}^{\infty} (r_1 \cdots r_t)^{-1} \left(p_{j,t} y_{j,t} - w_t h_{j,t} - \sum_{l=1}^{N} p_{l,t} z_{l,j,t}^I \right) \tag{10}$$

subject to the production function (1,3), the capital accumulation (2), the discreteness of investment rate (4), and the demand function (9).

Let us define the average capital index K_t as follows.

$$K_t \equiv \left(\sum_{j=1}^{N} k_{j,t}^{\rho} / N \right)^{\frac{1}{\rho}} \tag{11}$$

where $\rho \equiv \alpha/(\mu - \gamma)$. Note that $\rho < 1$ holds by the assumption $\alpha + \gamma < \mu$. By using the optimality condition for $h_{j,t}$, the profit at t is reduced to a function of $(k_{j,t}, k_{j,t+1})$ as:

$$\pi_{j,t} = D_0 w_t^{\frac{-\gamma}{1-\gamma}} K_t^{\frac{\rho(\mu-1)}{1-\gamma}} k_{j,t}^{\rho} - k_{j,t+1} + (1 - \delta_j) k_{j,t} \tag{12}$$

where $D_0 \equiv (1 - \gamma/\mu)(A(\gamma/\mu)^{\gamma})^{1/(1-\gamma)}$. The profit is concave in $k_{j,t}$ by $\rho < 1$. Thus the optimal policy is characterized by an inaction region in $k_{j,t}$ with a lower bound $k_{j,t}^*$ and an upper bound $\lambda_j k_{j,t}^*$. Consider two sequences of $k_{j,s}$ which are identical except at t. Such sequences can be constructed by assigning a positive investment at $t-1$ and zero investment at t in one sequence and zero investment at $t-1$ and a positive investment at t in the other. Then the lower bound of the inaction region is derived by solving for $k_{j,t}^*$ at which the two sequences yield the same discounted profit. Namely, if $k_{j,t}$ is strictly less than $k_{j,t}^*$, the producer is better off by adjusting it upward rather than waiting. Let one sequence with zero investment at $t-1$ and one tick of investment at t have $k_{j,t} = k_{j,t}^*$, and let the other sequence have $k_{j,t} = \lambda_j k_{j,t}^*$. Then the both sequences have the same amount of capital at $t-1$ and $t+1$: $k_{j,t-1} = (1/(1-\delta_j))k_{j,t}^*$ and $(\lambda_j(1-\delta_j))k_{j,t}^*$. Solving for $k_{j,t}^*$ which equates the discounted profits of the two sequences, we obtain:

$$k_{j,t}^* = \left(\frac{D_0(\lambda_j^{\rho} - 1)}{\lambda_j - 1} \right)^{\frac{1}{1-\rho}} (r_t - 1 + \delta_j)^{\frac{-1}{1-\rho}} w_t^{\frac{-\gamma}{(1-\gamma)(1-\rho)}} K_t^{\zeta} \tag{13}$$

where,

$$\zeta \equiv \frac{\rho(\mu - 1)}{(1-\rho)(1-\gamma)} = \left(\frac{\alpha}{1-\gamma} \right) \left(\frac{\mu - 1}{\mu - (\alpha + \gamma)} \right). \tag{14}$$

Equation (13) expresses the strategic complementarity between the average capital level K_t and the threshold for an individual capital level $k_{j,t}$. The degree of the complementarity is represented by ζ. A percentage change in average capital induces ζ percent change in the individual threshold $k_{j,t}^*$. In particular, the movement in K_t and $k_{j,t}^*$ coincides if $\zeta = 1$. A simple manipulation reveals that $\zeta \geq 1$ holds if and only if $\alpha + \gamma \geq 1$. The complementarity effect is solely determined by the returns to scale, and is not dependent on the competitiveness of the market, μ.

The feedback effect on $k_{j,t}$ is non-linear because of the threshold behavior. The mean capital level K_t affects the threshold of the inaction region, but it may or may not induce the adjustment of $k_{j,t}$. Hence the correlation of the firms' capital choice may be summarized as local inertia and global strategic complementarity of the individual behavior. The individual capital is insensitive to a small perturbation in the mean capital level, while it perfectly synchronizes with the average capital if the perturbation is large.

In this setup, a previous study [15] has shown that the average capital can show endogenous fluctuations even when the number of firms N tends to infinity, for a partial equilibrium in which w_t and r_t are held fixed. The fluctuation results hold in a more generic setup [16]. In the present paper, we investigate the fluctuations in a general equilibrium setup numerically.

We use a utility specification:

$$U(c_t, h_t) = \frac{c_t^{1-\sigma}}{1-\sigma} - \frac{h_t^{1+\nu}}{1+\mu} \tag{15}$$

where $\sigma \geq 0$ and $\nu \geq 0$. From the utility specification we obtain the equilibrium price conditions immediately:

$$w_t = c_t^\sigma h_t^\nu \tag{16}$$

$$r_t = (c_{t+1}/c_t)^\sigma / \beta \tag{17}$$

A contemporaneous equilibrium (y_t, c_t, h_t, w_t) given k_t, i_t, r_t is determined by (16) and:

$$Y_t = (A(\gamma/\mu)^\gamma)^{1/(1-\gamma)} w_t^{-\gamma/(1-\gamma)} K_t \tag{18}$$

$$w_t h_t = (\gamma/\mu) Y_t \tag{19}$$

$$y_t = c_t + i_t \tag{20}$$

The first equation is derived by aggregating the optimal production level when the capital is given. The second equation is obtained by aggregating the optimal employment given capital. It shows that the labor share is equal to γ/μ. The third equation is a product market equilibrium condition. Given these equilibrium relations, the equilibrium path (k_t, i_t, r_t) is determined by the capital accumulation (2), the equilibrium interest rate (17), and the selection algorithm for i_t with the optimal threshold rule (13).

3 Business Cycles Simulation

In this section we examine quantitative properties of the equilibrium fluctuation by numerical simulations. We ask whether the sectoral oscillations of magnitude exhibited by the U.S. manufacturing sectors would add up in our model to the observed aggregate fluctuations and generate the business cycle patterns. The answer is affirmative when the intertemporal substitutions of aggregate consumption and leisure are close to perfect. If this is the case, the (S,s) policy at the individual level generates an endogeneous fluctuation of the aggregates. Our aim is to reproduce the second moment structure of aggregate variables in interest. In particular, we attempt to explain the mechanism for the positive autocorrelation of the business cycle variables and the positive correlation between production and demand components. The parameter range we work in is in the vicinity of the fixed price regime.

Let us start from estimating the fluctuation magnitude of U.S. manufacturing sectors. We use the 4-digit SIC annual data compiled by Bartelsman and Gray [2]. We remove the trend by Hodrick-Prescott filter with smoothing parameter $\lambda = 100$. We estimate a second order autoregressive process of the detrended log sectoral capital as:

$$y_{j,t} = \phi_{1,j} y_{j,t-1} + \phi_{2,j} y_{j,t-2} + \epsilon_{j,t} \tag{21}$$

The regression shows that 434 sectors out of total 459 sectors exhibit a damped oscillation phase $\phi_{1,j}^2 + 4\phi_{2,j} < 0$. A second order autoregressive process with a damped oscillation displays a pseudo-periodic behavior. The pseudo-periodicity is calculated as $1/\mu_j \equiv 2\pi/\cos^{-1}(\phi_{1,j}/(2\sqrt{-\phi_{2,j}}))$, following the procedure similar to Yoshikawa and Ohtake [21].

We emulate this oscillation by our lumpy behavior of sectoral investments. The presumption is that a sector has to commit to a sizable investment if it invests at all. If it does not invest, then the gap between the actual and desired level of capital increases as capital depreciation and technological progress takes effect. The lumpy adjustment generates a non-harmonic oscillation which is familiar in the (S,s) literature such as the Baumol-Tobin cash balance dynamics. It is more likely that the committed amount of investment is executed in several periods, if we consider the time to build. By incorporating the time to build, the sectoral oscillation exhibits a more realistic harmonic oscillation, but the basic properties of aggregate behavior does not change by this modification. Individual sectors may fluctuate for various reasons in reality such as technological improvement or strategic complementarity among firms' behaviors within the industry. It is for convenience of analysis that we assume the lumpy behavior of monopolists.

We derive λ_j and δ_j from the observed oscillations μ_j and σ_j in the way that the periodicity and magnitude of oscillation the data shows are maintained. From the periodicity we have a relation $1/\mu_j = \log \lambda_j / |\log(1-\delta_j)|$. Also, we numerically calculate the standard deviation of the model oscillation for $\log \lambda = 1$ and δ_j. Then $\log \lambda_j$ is derived by dividing σ_j by the calculated standard deviation. Thus we obtain λ_j and δ_j.

Figure 1 shows the estimated periodicity in the first panel. The periodicity is distributed with mean 8.2 years and standard deviation 3.3. The second and third panels show the calibrated discreteness λ_j and the annual depreciation rate δ_j that match with the estimated parameters for oscillations. The mean of λ_j is 2.5 and standard deviation is 2, and the mean of δ_j is 0.09 and standard deviation 0.07. Let us notice the considerable heterogeneity shown in the periodicity. It casts a doubt on the view that the sectoral fluctuation is merely a reflection of aggregate fluctuations. It is worth exploring the possibility that a pseudo-random propagation effect across sectors causes the aggregate fluctuations.

We will show that our model of investment propagation is capable of reproducing the basic business cycle structure: the standard deviation of GDP around 1.7%, the positive correlations between production and demand components, and the strong autocorrelations of the production and demand components.

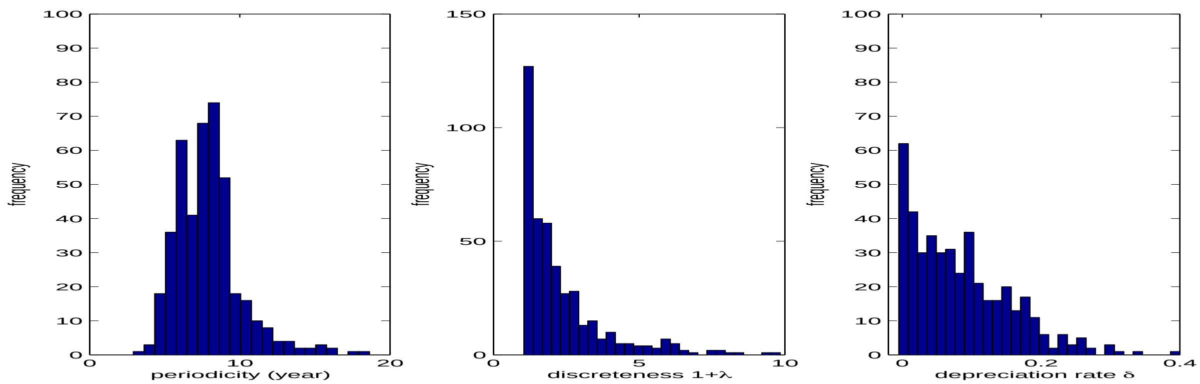

Figure 1: Properties of sectoral oscillations

Our model shares the basic quantitative characteristics of monopolistic models studied previously [12, 17]. In the following we concentrate on the investment fluctuation and its effect on production and consumption.

We resort to numerical simulations to solve the equilibrium path. In the simulation, we assume that the representative household and monopolists have a static expectation on future investment. Namely, the expected future investment is set at the time average level $\sum_j \delta_j \tilde{k}_j$. Computational difficulty is the reason we do not solve for a perfect foresight equilibrium. Since the investment crucially depends on the details of the configuration of producers capital positions, solving the perfect foresight path requires prohibiting computational loads. Also, it is not realistic to suppose that the agents are able to form a perfect foresight. Besides the computational problem, the agents would have to have precise information about the capital configuration of the entire economy. When the economy has attained the stationary level, a noisy information would not contribute to the accuracy of prediction very much in our setting. We also tried another expectation formulation based on an AR(1) estimate of the past investment path. We confirmed that the basic property of the fluctuations does not change, although we noted that the convergence to the rationally expected AR(1) parameters can be fragile depending on the fundamental parameters. Another issue in the simulation is the finiteness of the agents. The existence of equilibrium is shown in the previous section as an asymptotic property when the number of sectors N tends to infinity. When N is finite, with a positive probability the best response dynamics does not reach an equilibrium. We impose a rule that the dynamics stops either when all the sectors adjust upward or all the sectors which adjust at the initial step re-adjust downward. This case happens in the early periods of simulated paths. We did not observe this case once the equilibrium path is converged to a stationary state level.

Table 1 summarizes the simulation result on the second moments. The standard deviations of the estimated second moments in 500 runs are shown in

	GDP	Investment	Consumption	Capital	Hours	Wage
standard deviation (%)	1.83 (0.52)	11.63 (1.07)	1.80 (0.50)	1.88 (0.54)	1.82 (0.52)	0.02 (0.01)
correlation with GDP	1 –	0.49 (0.05)	0.77 (0.09)	1.00 (0.00)	1.00 (0.00)	0.77 (0.09)
autocorrelation	0.89 (0.05)	0.61 (0.07)	0.52 (0.21)	0.88 (0.05)	0.89 (0.05)	0.52 (0.21)

Table 1: Simulated business cycle statistics

parentheses. The parameter values are set as $\sigma = 0.01$, $\nu = 0$, returns to scale $\alpha+\gamma = 1$, labor share $\gamma/\mu = 0.58$, mark-up rate $\mu-1 = 1/3$, and annual discount rate $\beta = 0.96$. Although the correlation between production and investment is not strong enough, the simulation captures the basic feature of business cycles such as the magnitude of fluctuations in GDP, investment, and consumption, strong autocorrelations in GDP, positive correlations between production and demand components and input components, and small wage fluctuations.

Figure 2 shows typical paths of the simulated production and investment for the same parameter set. The variables are normalized by the stationary level GDP after convergence. The top left panel shows the entire paths of the GDP and the aggregate investment. The simulated path converges to a certain level quickly and exhibits persistent fluctuations thereafter. The investment-production rate converges to a realistic 9.6%. The bottom left panel shows the capital paths of individual sectors. We observe an (S,s) behavior of the sectors. The right panels show the magnified plots of the same aggregate paths in a shorter time horizon. We observe a chaotic fluctuation (in the sense that the deterministic path appears random) with a certain degree of periodicity. Also we see a strong correlation between the production and investment.

The correlation structure shown in Table 1 exhibits a limited robustness in parameters. Figure 3 shows the admissible range of parameters. For each parameter alignment, we take an average of estimates from 15 simulation runs. We plot a circle when the standard deviation of GDP is more than 1% and less than 3%, a cross when investment correlates with production, and a plus when consumption correlates with production. The plots show that our results depend on the preference specifications (σ and ν) sensitively but not on the markup rate ($\mu - 1$). In the left panel, there exists an admissible range of σ for the markup rate larger than 30%. The larger the markup rate is, the larger and the broader the admissible range of σ is. We observed that the business cycle patters obtain also for a smaller markup rate ($\mu - 1 < 0.1$). It is not certain, however, if the business cycle pattern in this region is generated by the mechanism we analytically identified.[1] The right panel shows that the

[1]For a small markup, the production goods are easily substitutable and the firms are competitive. Hence the price responds sensitively to the initial shock in the best response dynamics. The subsequent adjustment process occurs not in the direction to amplify the

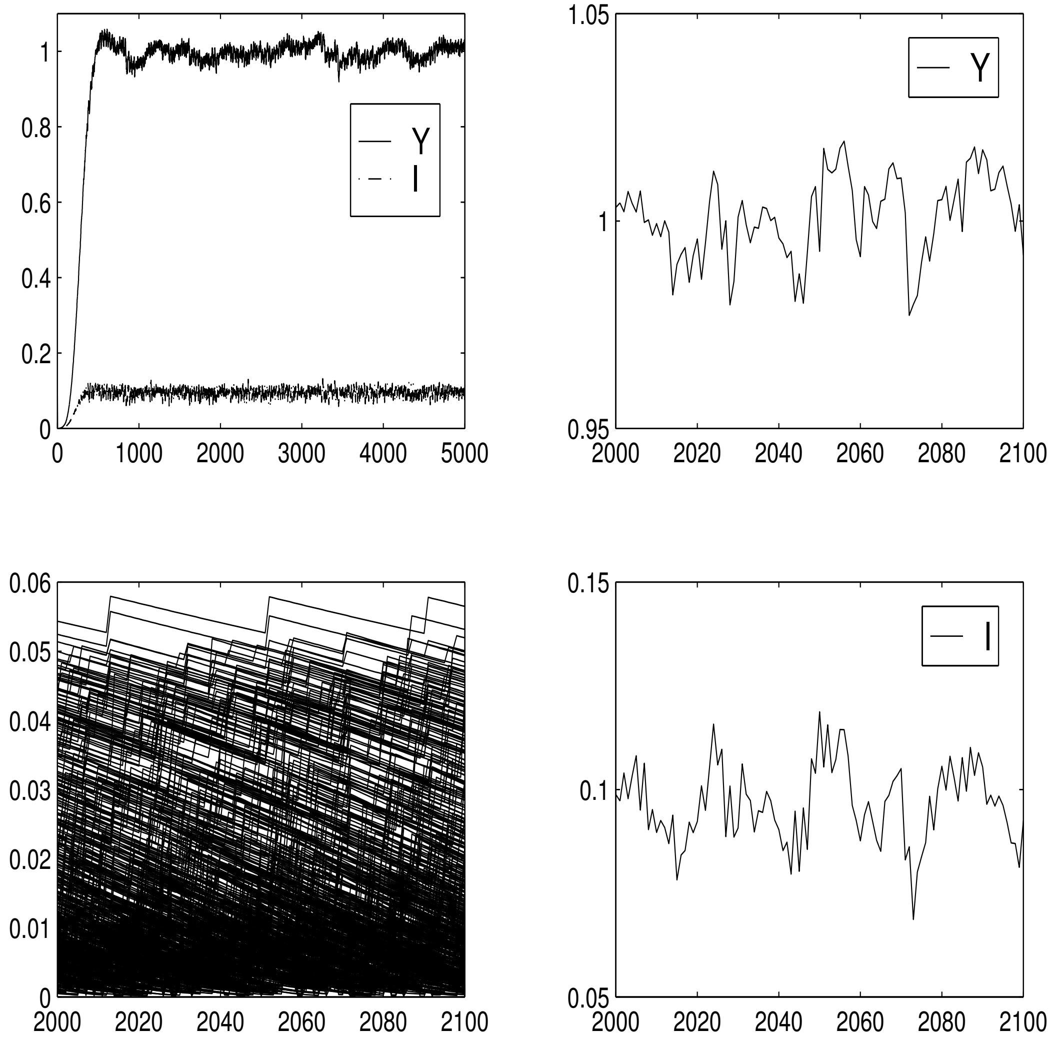

Figure 2: A simulation path of GDP and investment. X axis shows quarters. Y axis is scaled by the stationary level GDP.

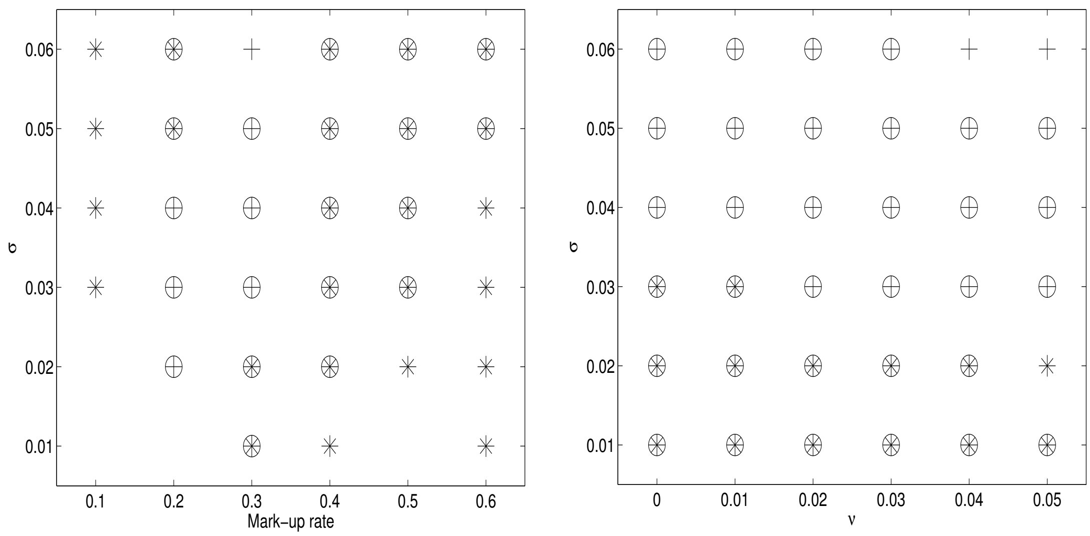

Figure 3: Admissible range of parameters

admissible range for preference specifications (σ and ν). We obtain the aggregate fluctuations large enough when $\sigma + \nu$ is small enough. To obtain a meaningful stochastic propagation effect, the representative household needs to be sensitive enough to interest rate or wage. We also observe in the plot that σ needs to be small for the correlation between production and investment to obtain. When σ is larger, an investment by a sector increases the interest rate more, and dampens the propagation effect.

The simulation replicates well the mean behavior of the pairwise correlation between sectoral production and GDP. The comovement of the sectoral production (and hence sectoral and aggregate production) is a defining characteristic of business cycles. However, the comovement is far from a perfect mode locking. The left panel of Figure 4 shows the histogram of the correlations in data (shown by a bar). The correlation between a sector and aggregate is only modest. This fact agrees with another fact we noted that the periodicity of sectoral oscillations varies much. These suggest contrary to the view that the business cycles are mainly driven by an aggregate factor and the sectoral movements are only a noise-ridden version of the same cycles. The modest correlation between the sectoral and aggregate production is captured by our simulation well. The histogram of the simulated correlations under our benchmark parameter set (as for Table 1) is drawn by a real line. The simulated histogram is more centered than the real histogram, which is a natural consequence of our symmetric modeling of

initial shocks but in the direction to mitigate the initial response. Hence our analysis does not apply to this case. It is nonetheless interesting that a competitive setting also generates an endogeneous fluctuation.

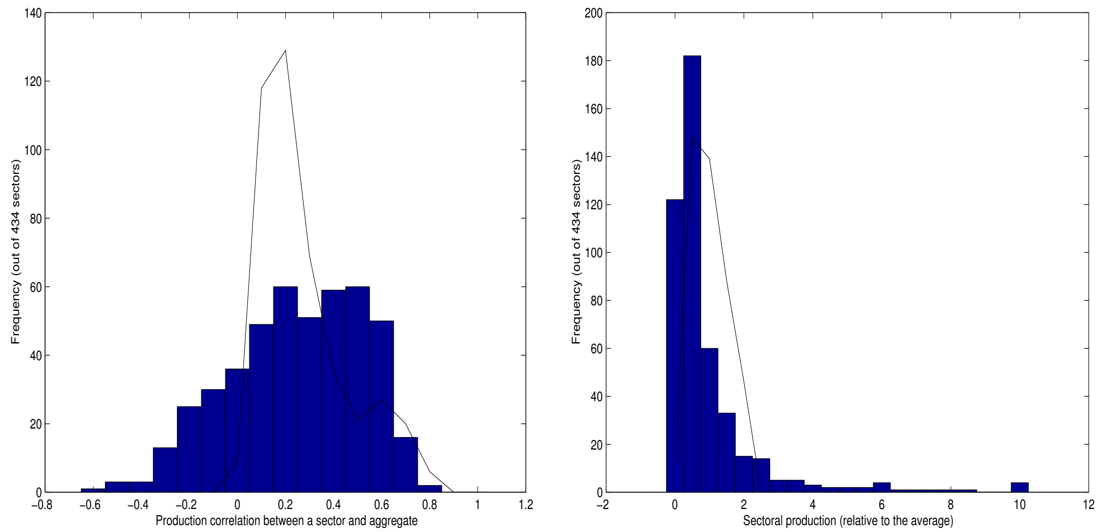

Figure 4: Correlation between sectoral and aggregate production (left) and cross sectional distribution of production relative to average (right). The bar and line respectively show the actual data and simulation.

sectoral interactions. The real input-output matrix is far from symmetric [13], and the asymmetric input-output relation will generate more heterogeneity in the comovement structure across sectors. The mean of the correlation (0.24) is reproduced well by our simulation, however. This suggests that the symmetric modeling may be satisfactory insofar as the aggregate fluctuations is concerned. The right panel of Figure 4 shows the histograms of sector size in data (bar) and in simulation (line). The only source of heterogeneity in the model is depreciation rate (δ_j) and lumpiness (λ_j). The heterogeneity of the sector size is reproduced fairly well. This excludes the case in which the different variety in comovement stems from the different sector size distributions. Also this assures that the model fluctuation we observe does not result from an unrealistic distribution of sector size.

It is not trivial in our model to have correlations between production and demand components. In the standard real business cycle model, the fluctuation in total factor productivity causes the procyclical movement of both consumption and investment. Instead, the investment fluctuates relatively independently from the economic environment in our model. This aspect gives the model a different mechanism for the procyclical movement of the consumption and investment. An increase in investment demand induces the monopolistic producers to produce more on one hand. On the other hand, since the capital level is predetermined, an increase in investment competes with the contemporaneous consumption given the production level. By using the equilibrium relations given

k_t for the case $\alpha+\gamma=1$, we obtain $dy_t/di_t = 1/(1+(\alpha+\nu)/(\sigma(1-\alpha)(c_t/y_t)))$, which is always between 0 and 1. Hence, given the capital level, an investment has a positive effect on production, but the effect is no more than 1. Hence there is no multiplier effect of the investment demand on the production. The correlation between consumption and production rather stems from the fluctuations of accumulated capital. We also obtain $(dy_t/y_t)/(dk_t/k_t) = (1+\nu)\alpha(c_t/y_t)/(\sigma(1-\alpha)+(\alpha+\nu)(c_t/y_t))$ at equilibrium. This takes values between zero and one, and is close to one when σ and ν are close to zero, agreeing with our benchmark simulation. Since the investment is determined partly by an independent process of best response dynamics across producers which the representative household cannot predict deterministically, large production due to large capital can result in large consumption. The Keynesian multiplier effect would increase the correlations between production and demand components. This would be the case when the consumption function is more sensitive to income than our baseline model. If a significant number of consumers face liquidity constraint, for example, it would contribute to more synchronous movements between production and demands.

The autocorrelation of production is partly generated by the demand-smoothing effect of the real interest rate. In the previous paper we saw that an increase in the interest rate sensitivity dampens the instantaneous investment propagation. In a dynamic setting, this dampening effect only postpones the investment propagation to the subsequent periods. Suppose that the interest rate is now above the time average level due to a large concentration of sectors near the adjustment threshold. In the next period, the interest rate would decrease to the time average level if the investment is at the time average level. This decrease in interest rate increases the threshold for capital adjustment. Hence the investment in the next period tends to be larger than the time average level. This is the mechanism for the autocorrelation in investment. In this mechanism, the effect of delaying the investment is strong when the interest rate sensitivity is large, and a large sensitivity follows a small intertemporal substitution in consumption, $1/\sigma$. The autocorrelation in investment generates the autocorrelation in production in two routes: a contemporaneous effect on aggregate demand and subsequent effects on aggregate supply via capital accumulation.

It is helpful to examine our economy's smooth counterpart to understand the fundamental condition when the fluctuations occur. Suppose that there is no discreteness constraint (4); then any capital level can be chosen. The producers' optimal choice of capital yields an optimality condition which is linear in aggregate capital as in (13). By aggregating the optimality condition, we find that the aggregate capital level k_t is indeterminate in the product market. The capital level is thus solely determined by the consumer's choice between leisure and consumption. In our model, the time average capital level (normalized by the total factor productivity) is also determined independently from technology. However, the investment is determined uniquely in the best response dynamics across producers. We saw that the propagation exhibits an extreme variance when the wage and interest rate are fixed. This corresponds to the indeterminacy of capital level in the smooth economy. When the wage and interest rate are not fixed, the aggregate capital does have a unique time average level. However, the attraction power of the time average in the dynamics of aggregate capital is vanishingly small as the wage and interest rate bacomes insensitive to production.

4 Conclusion

This paper explores a mechanism of investment propagation as a fundamental shock to the business cycle fluctuations. We consider industrial sectors which are characterized by constant returns to scale technology and monopolistic pricing. Demand for intermediate inputs forms a positive feedback of capital adjustment in the interindustrial relations. We suppose that the sectoral capital exhibits an intermittent adjustment where a large investment occurs occasionally. Under this environment, our previous studies [15, 16] have shown that the propagation size has a large variance is a partial equilibrium of product markets. When the returns-to-scale is constant, the variance of capital growth rates does not depend on the level of dissaggregation. In the present paper, we study the fluctuation of the economy in a general equilibrium in which the wage and interest are determined within the model.

Simulations show that the investment propagation mechanism can explain the aggregate fluctuations of the U.S. economy quantitatively. We specify the representative household's utility as a separable function in leisure and consumption and solve for the equilibrium paths. The results show that the standard deviation, the correlations between production and investment and consumption, and the autocorrelation of production, investment, and consumption match the U.S. postwar business cycles well. Thus we show that, given the magnitude of oscillations that a manufacturing sector exhibits, the sectoral oscillations can add up to the aggregate fluctuations through the investment propagation mechanism with the correct second moments of the business cycle variables.

The paper leaves three points for further explorations. First, the simulation shows that the correlations between two demand components and production are not strong enough simultaneously. It stems from that the consumption responds weakly to income when capital level is fixed. The behavior of representative household needs to be modified in such a way that the income effect becomes strong, for example by incorporating the liquidity constraint. Secondly, the intertemporal substitution of consumption in the simulation is set larger than the evidence for the U.S. economy suggests. Thirdly, the deterministic oscillation of sectoral capital is assumed. It is no doubt an over-simplification that a sectoral capital jumps in one period and depreciate capital over many years. Incorporating the time-to-build of capital would make the sectoral oscillations more realistic with keeping the results of the paper unaltered. Yet it is not obvious that a sectoral capital accumulation process incurs such degree of inflexibility. This leads to the question as to whether the business cycle patterns still obtain if we disaggregate the economy to the establishment level. Our analytics shows that the large aggregate fluctuations can occur in principle regardless of the number of agents, but a quantitative demonstration of the theoretical possibility is left open.

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