On the Benefits of Exchange Rate Flexibility under Endogenous Tradedness of Goods

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Abstract

Previous efforts to compare the costs and benefits of fixed versus flexible exchange rate regimes have ignored the fact that it takes significant resources and time to develop export markets, and they have not included an analysis of the firm-level decision to enter or exit export markets. This paper develops a dynamic stochastic general equilibrium model to analyze the effects of endogenous tradedness of goods on the welfare gains from exchange rate flexibility. The actual range of traded goods in our model depends on the producers who choose to enter and exit export markets taking into account trade costs and relative productivities. A novel feature of the model is that it takes both time and resources to develop export markets and as a consequence expenditure-switching effects can be slow and will depend on a host of factors such as country size, trading costs and the competitive environment that producers face. Interestingly, because the model integrates a model of trade into a monetary business cycle model with sticky prices and wages, it is possible to study the interaction of macro and structural policies. However, in this study we focus initially on how different levels of trading costs can affect the structure of the economy and result in welfare costs of excessive exchange rate volatility.

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1 Introduction

This paper develops a dynamic stochastic general equilibrium model that analyzes how endogenously determined trade integration affects the welfare gains from exchange rate flexibility. The share of trade in output results from the interaction of comparative advantage patterns and trade frictions. We build our model of comparative advantage on the set of tools developed by Naknoi (2004), which is closely related to the classic work on a Ricardian trade model by Dornbusch et al. (1977). The key novelty is that while in principle all goods are exportable, the actual range of exports and nontraded goods is endogenous, depending on production decisions that take into account relative productivities and trading costs. The main difference between our model and alternative models that fix trade shares exogenously is that our approach is capable of generating a much higher long-run elasticity of trade to shocks, because in order to model microeconomic export decisions it becomes necessary to assume high elasticities of substitution between exported varieties, so that both the volume and product range of goods exported respond strongly to shocks. Our goal is to use this model to answer two questions. First, what is the welfare gain of flexible exchange rates over fixed rates, if any? Second, how do the benefits of flexibility vary as we alter the degree of trade integration? Bringing realistic aspects of trade integration into a monetary model to analyze welfare gains from flexible exchange rates is our main contribution.

The literature has avoided specifications such as ours for two reasons: First, its focus has been on business cycle fluctuations; and second, in the short run trade volumes have not been found to be very responsive to shocks. The model of choice has therefore been one with low elasticities of substitution between home and foreign goods. But this strategy gives rise to two problems. First of all, such models are unable to adequately describe microeconomic export decisions including entry and exit from export markets. In addition, given their assumptions of fixed nominal trade shares, they are unable to explain the very large trade responses often observed over medium and long term horizons in response to productivity shocks and reductions in trade costs. To square the short and long run behavior of trade, we assume instead that trade is subject to costly and time consuming frictions. Replacing low elasticities with frictions as the main reason for slow short-run trade responses in turn has crucial implications for monetary policy. A key benefit of exchange rate flexibility (at least under producer currency pricing) is its ability to induce expenditure switching. When such switching is itself subject to costly frictions, the benefits of exchange rate flexibility are likely to be substantially lower. The main objective of this paper is to quantitatively evaluate this hypothesis through welfare analysis.

In general, open economy models formulate exportables, importables and nontradables as a fixed range of goods, through the specification of sub-baskets in utility and production functions. Trade integration in these models is specified as the import share or trade to GDP ratio (see e.g. Laxton and Pesenti (2003)). However, trade integration involves another aspect, namely entry and exit from export markets by individual producers. This aspect of trade has so far been neglected in the literature. In our view, we should be concerned about this aspect of integration for two reasons.

First, several trade economists have identified year-to-year transitions into and out of exporting from plant level data. Bernard and Jensen (2004) studies the export status of a sample of U.S. manufacturers from 1984 to 1992. On average, 13.9 percent of nonexporters begin to export in any given year, and 12.6 percent of exporters stop. Although the export status is found to be persistent, 18 percent of nonexporters begin to export and 20 percent of exporters stop within 3 years. The year-to-year entry and exit rates for Colombian plants reported by Roberts and Tybout (2001) are 2.7 and 11 percent, respectively. Aitken et al. (1997) find that 11.6 percent of their sample of Mexican manufacturers change their export status in the 3-years period from 1986 to 1989. These year-to-year transitions indicate that the exporting decision is not a long run issue. Instead, it is better viewed as a medium run phenomenon with some degree of persistence. Persistent business cycle shocks are therefore very likely to affect these transitions.

Among these studies, Bernard and Jensen (2004) are the first to establish a link between the transitions in and out of exports and industry characteristics. In their study, the probability of entry in exporting is found to be significantly and positively correlated with changes in the industry characteristics of products in the previous year. Moreover, they find that favorable exchange rate shocks increase participation in exporting. This suggests a new channel through which exchange rate dynamics interacts with changes in the types of goods being exported.

Second, in a world with heterogeneous firms, their transitions into and out of export markets create fluctuations in productivity at the aggregate level, as shown in Ghironi and Melitz (2004), Melitz (2003) and Naknoi (2004). This effect generates an additional source of output volatility that monetary policy should aim to stabilize. Without a model where both volume and types of exports respond endogenously to disturbances, we cannot precisely evaluate the welfare gain from exchange rate flexibility.

In this paper we provide a first step in bringing the microeconomic determination of trade together with exchange rate policy analysis in a single framework. Our focus is on the supply side determination of trade, and consequently the three key innovations of the paper are related to firms.

First, as shown in part in Figure 1, the model reflects the complex, multi-stage nature of both production and trading in modern industrial economies. In particular, there are two stages in the production process at which value is added - intermediate and final goods. Countries import (and export) intermediate goods, use them to make final goods, and can re-export the resulting products. Such transactions that break up the value chain tend to be particularly high between countries at different levels of development, such as the accession countries and the euro area. Consumption goods are also finalized in a third stage of production, reflecting the fact that while most firms have a direct relationship with their major suppliers, consumers do not. In addition, goods at each level of production are assumed to be sold to their ultimate users via a distribution sector that is subject to mark-ups and nominal rigidities. [1] The main advantage of this assumption is that it separates the sources of nominal and real rigidities, thereby simplifying the analytic issues involved in imposing both types at the same level of production.

Second, trade in intermediate goods is based on a Dornbusch-Fischer-Samuelson comparative advantage theory of trade in which, in principle, all goods can be produced by both countries, but where actual tradedness is determined endogenously by the interaction between the costs of trading and relative productivity levels between the potential producers of any given good in the two countries (Figure 3).[2] If the price advantage for the more efficient producer at prevailing marginal factor costs and productivity differentials exceeds the costs of trading, the good is made in only one country and traded with the other. If not, then the good is produced in both countries and no trade occurs. As a result, lower trading costs and better technologies can then lead to much more rapid increases in trade than in standard models where the status of a good as traded or nontraded is exogenous. [3]

Third, the model is able to reproduce the gradual response of trade to lower costs or to movements in the real exchange rate by introducing a range of plausible real rigidities. In addition to habit persistence in consumption and time-to-build capital lags for investment, we add a "time-to-build markets" technology for trade. The intuition is similar to that for time-to-build capital - it takes time to build or abandon foreign supplier relationships, so that there is both a time lag between an order decision and actual delivery and a cost of changing the size of deliveries. Time-to-build markets technologies significantly slow down the response of trade to real exchange rate movements, in line with existing empirical

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[1] The real transactions costs of changing prices described in Zbaracki and others (2004) for multi-product firms, due to management time and customer costs, appear to describe such a sector well.

[2] In principle, there is no difficulty in extending endogenous tradability to finished goods trade, although it would add further complexity to an already large model. We decided to dispense with this feature because the main trade expansion in the accession countries did indeed take place in the intermediate goods sector.

[3] Betts and Kehoe (2001) model endogenous tradability in a flexible price two-country framework. Bergin and Glick (2003) use a two-period small open economy model where firms take world prices as given. In both of these studies the source of heterogeneity is product-specific transport costs, whereas this paper emphasizes product-specific levels of producivity. To the best of our knowlege, this is the first model of endogenous tradability with both nominal inertia and significant real rigidities.

evidence. Also, these time-to-build markets technologies are isomorphic to assuming a fixed cost to enter to or exit from export markets as in Ghironi and Melitz (2004) in that they generate realistic short run dynamics in trade volumes. These real rigidities, combined with the nominal rigidities located in the distribution sectors, cascade and cumulate down the production process, so that final goods are more affected by them than intermediate goods.

We perform a preliminary calibration exercise by subjecting the model economy to an interest rate shock in each country, in order to compare properties of our model with the literature. An interest rate hike creates a fall in domestic output, a real appreciation, and trade deficits. The most important aspect of the results is however the very sluggish response of trade volumes to these shocks, due to the time-to-build-markets technology. The main benefit of exchange rate flexibility, the ability to induce immediate and sizeable expenditure switching, is therefore not present in this model. We are therefore presently evaluating the hypothesis that the welfare benefits of exchange rate flexibility are much reduced. We are doing so by exploring the implications of different monetary rules that are consistent with flexible exchange rates, such targeting the CPI, the price of domestic output, etc. Finally, we will compare the results with exchange rate targeting.

Our model is described in detail in the next section. Section 3 reports the short run responses of the volume and pattern of trade to interest rate shocks. Section 4 summarizes our preliminary findings and future work.

2 The Model

The model economy consists of two countries, referred to as Home and Foreign. The countries have identical preferences and technologies and differ only in size, with the population of the home country being α and that of the foreign country $(1 - \alpha)$. We concentrate on the economic decisions of Home agents, as the corresponding decisions of Foreign agents are mirror images.

2.1 Households

Each individual household i maximizes lifetime utility which has three arguments, consumption C^i (which exhibits habit persistence), leisure $(1 - L^i)$ (where L^i is labor effort and 1 is the time endowment), and real money balances $n^i = N^i/P^c$ (where P^c is the consumption based price index). Denoting the intertemporal elasticity of substitution by σ, we have:

$$Max \quad E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{o_t \left(C_t^i - \nu C_{t-1}^i \right)^{1-\frac{1}{\sigma}} - 1}{1 - \frac{1}{\sigma}} + \psi \frac{(1 - L_t^i)^{1-\frac{1}{\sigma}}}{1 - \frac{1}{\sigma}} + \psi_n \frac{(n_t^i)^{1-\epsilon}}{1 - \epsilon} \right\} , \qquad (1)$$

where E_t is the expectation conditional on information available at time t, and o_t is a preference or demand shock. Households' capital accumulation decision involves separate decisions for domestically and foreign produced capital stocks K^{H^i} and K^{F^i}. This is because these are imperfect substitutes in firms' production functions,[4] the aggregate capital stock being given by a CES aggregator with share parameters ξ_k and elasticity of substitution θ_k:

$$K_t^i = \left[\xi_k \left(K_t^{H^i} \right)^{\frac{\theta_k - 1}{\theta_k}} + (1 - \xi_k) \left(K_t^{F^i} \right)^{\frac{\theta_k - 1}{\theta_k}} \right]^{\frac{\theta_k}{\theta_k - 1}} . \qquad (2)$$

Capital accumulation follows time-to-build technologies, with a six-period lag between the investment decision I_t^i and the point at which the investment decision leads to an addition to the productive capital stock:

$$\begin{aligned} K_{t+1}^{H^i} &= (1 - \Delta) K_t^{H^i} + I_{t-5}^{H^i} , \\ K_{t+1}^{F^i} &= (1 - \Delta) K_t^{F^i} + I_{t-5}^{F^i} . \end{aligned} \qquad (3)$$

Furthermore, changes in the level of investment spending are subject to a quadratic adjustment cost paid out of household income (see the budget constraint below). Each investment decision represents a commitment to a spending plan over six periods, starting in the period of the decision and ending one period before capital becomes productive.

4 We are thinking for example of domestic buildings combined with imported machinery.

The shares of the investment project that have to be disbursed in each period are given by $\omega_j,\ j = 0, ..., 5$, with $\Sigma_{j=0}^{5}\omega_j = 1$. Actual investment spending J_t^i is therefore given by

$$\begin{aligned} J_t^{H^i} &= \omega_0 I_t^{H^i} + \omega_1 I_{t-1}^{H^i} + \omega_2 I_{t-2}^{H^i} + \omega_3 I_{t-3}^{H^i} + \omega_4 I_{t-4}^{H^i} + \omega_5 I_{t-5}^{H^i} \ , \\ J_t^{F^i} &= \omega_0 I_t^{F^i} + \omega_1 I_{t-1}^{F^i} + \omega_2 I_{t-2}^{F^i} + \omega_3 I_{t-3}^{F^i} + \omega_4 I_{t-4}^{F^i} + \omega_5 I_{t-5}^{F^i} \ . \end{aligned} \tag{4}$$

In what follows we choose as our numeraire the price P_t of intermediate goods D_t, and lower case price and return variables p and r are in terms of this numeraire. The nominal exchange rate is S_t, and the real exchange rate for intermediates is $s_t = (S_t P_t^*)/P_t$. Households can hold two types of financial assets (apart from money), risk-free bonds issued in Home and denominated in Home currency $\tilde{F}_t^i$ yielding a nominal gross return of i_t, and risk-free bonds issued in Foreign and denominated in Foreign currency F_t^i yielding a nominal gross return of i_t^*, with the corresponding real variables given by $\tilde{f}_t^i = \tilde{F}_t^i/P_t$ and $f_t^i = (S_t F_t^i)/P_t$. The gross rate of currency depreciation is denoted by ε_t, and π_t^* and π_t are the gross foreign and domestic inflation rates for the numeraire good. Furthermore, households own three types of real assets, Home and Foreign produced capital K^{H^i} and K^{F^i}, and fixed factors (such as land) G_t^i. Households' income therefore consists of real wages $w_t L_t^i$, real returns on capital $r_t^{K^H} K_t^{H^i} + r_t^{K^F} K_t^{F^i}$, on fixed factors $r_t^g G_t^i$, on risk-free international bonds $f_{t-1}^i \frac{i_{t-1}^* \varepsilon_t}{\pi_t}$ and on risk-free domestic bonds $\tilde{f}_{t-1}^i \frac{i_{t-1}}{\pi_t}$, in addition to lump-sum government redistributions T_t^i/P_t and profit redistributions Π_t^i/P_t. We assume that agents do not have access to a complete set of internationally tradable state-contingent money claims. It is well-known that this makes net foreign assets nonstationary in linearized versions of the model economy. We therefore impose a small quadratic adjustment cost on deviations from a steady state level of private sector bond holdings (with the latter for simplicity set equal to zero), specifically $\frac{\phi^f}{2}\left(\alpha(f_t + \tilde{f}_t)\right)^2$. Households' expenditure consists of consumption spending $p_t^c C_t^i$ and investment spending $p_t^H J_t^{H^i}$ and $p_t^F J_t^{F^i}$, where p_t^H and p_t^F are the user prices of Home and Foreign produced final goods. Households also face a quadratic cost of adjusting their nominal wage as suggested by Rotemberg (1982), and as extended to costs of adjusting the rate of change of the wage by, among many others, Laxton and Pesenti (2003). Furthermore, following Ireland (2001), the adjustment cost is related to changes in household i's wage inflation relative to the past observed aggregate wage inflation rate. Specifically, the real wage adjustment cost is given by $\frac{\phi^w}{2}\left(\pi_t^{w^i}-\pi_{t-1}^w\right)^2$, where $\pi_t^{w^i}=W_t^i/W_{t-1}^i$ is the (gross) household specific rate of wage inflation and π_t^w is the aggregate rate of wage inflation. This and all other adjustment costs are assumed to be redistributed back to households as lump-sum payments. The period t budget constraint, whose multiplier is denoted by λ_t, is therefore:

$$\begin{aligned} f_t^i+\tilde{f}_t^i+(N_t^i/P_t) &= f_{t-1}^i\frac{i_{t-1}^*\varepsilon_t}{\pi_t}+\tilde{f}_t^i\frac{i_{t-1}}{\pi_t}+(N_{t-1}^i/P_t) \\ &+p_t^g(G_t^i-G_{t+1}^i)+(\Pi_t^i/P_t)+(T_t^i/P_t) \\ &+w_tL_t^i+r_t^{K^H}K_t^{H^i}+r_t^{K^F}K_t^{F^i}+r_t^gG_t^i \\ &-p_t^cC_t^i-p_t^HJ_t^{H^i}-p_t^FJ_t^{F^i} \\ &-\frac{\phi^f}{2}\left(\alpha\left(f_t+\tilde{f}_t\right)\right)^2-\frac{\phi^w}{2}\left(\pi_t^{w^i}-\pi_{t-1}^w\right)^2 \\ &-\frac{\phi^I}{2}p_t^H\frac{\left(I_t^{H^i}-I_{t-1}^{H^i}\right)^2}{I_{t-1}^{H^i}}-\frac{\phi^I}{2}p_t^F\frac{\left(I_t^{F^i}-I_{t-1}^{F^i}\right)^2}{I_{t-1}^{F^i}}\quad. \end{aligned} \tag{5}$$

We assume symmetry by fixing initial holdings of bonds, money, capital and fixed factors to be identical for all households. This implies that each household has the same present discounted value of income, and that all households' marginal conditions are identical, including a synchronization of wage setting behavior. We can therefore drop the index i in the following derivations.[5]

Households maximize (1) subject to (2), (3), (4), (5), and the demand for their labor. The first-order conditions for consumption, bonds and fixed factors are given by[6]

$$o_t\left(C_t-\nu C_{t-1}\right)^{-\frac{1}{\sigma}}-\beta\nu\mathcal{E}_to_{t+1}\left(C_{t+1}-\nu C_t\right)^{-\frac{1}{\sigma}}=\lambda_tp_t^c\ , \tag{6}$$

[5] A market for domestic state-contingent money claims is therefore redundant.

[6] Note that the first-order conditon for money is redundant unless money supply is assumed to follow an exogenous rule, a case that is not considered in this paper.

$$(1+\alpha^2\phi^f f_t) = \beta\mathcal{E}_t\left[\frac{\lambda_{t+1}}{\lambda_t}\frac{i_t^*\varepsilon_{t+1}}{\pi_{t+1}}\right] \quad , \tag{7}$$

$$(1+\alpha^2\phi^f f_t) = \beta\mathcal{E}_t\left[\frac{\lambda_{t+1}}{\lambda_t}\frac{i_t}{\pi_{t+1}}\right] \quad , \tag{8}$$

$$1 = \beta\mathcal{E}_t\left[\frac{\lambda_{t+1}}{\lambda_t}\frac{p_{t+1}^g + r_{t+1}^g}{p_t^g}\right] \quad . \tag{9}$$

Equivalent conditions can be derived for Foreign. Note also that (7) and (8) imply the interest parity condition

$$i_t = i_t^*\left(E_t\varepsilon_{t+1} + \frac{Cov_t\left[\left(\frac{\lambda_t}{\lambda_{t+1}}\frac{1}{\pi_{t+1}}\right), \varepsilon_{t+1}\right]}{E_t\left(\frac{\lambda_t}{\lambda_{t+1}}\frac{1}{\pi_{t+1}}\right)}\right) \quad . \tag{10}$$

The optimality conditions for investment and capital ($j = H, F$) are:

$$\begin{aligned}\omega_0\lambda_t p_t^j + \beta\omega_1\lambda_{t+1}p_{t+1}^j + \beta^2\omega_2\lambda_{t+2}p_{t+2}^j + \beta^3\omega_3\lambda_{t+3}p_{t+3}^j + \beta^4\omega_4\lambda_{t+4}p_{t+4}^j + \beta^5\omega_5\lambda_{t+5}p_{t+5}^j \\ +\lambda_t p_t^j\phi^I\left(\frac{I_t^j - I_{t-1}^j}{I_{t-1}^j}\right) = \beta^5\lambda_{t+5}p_{t+5}^j q_{t+5}^j \\ +\beta\lambda_{t+1}p_{t+1}^j\phi^I\left[\left(\frac{I_{t+1}^j - I_t^j}{I_t^j}\right) + \frac{1}{2}\left(\frac{I_{t+1}^j - I_t^j}{I_t^j}\right)^2\right] \quad ,\end{aligned} \tag{11}$$

$$q_t^j\lambda_t p_t^j = \mathcal{E}_t\left\{\beta\lambda_{t+1}\left[r_{t+1}^{K^j} + p_{t+1}^j q_{t+1}^j(1-\Delta)\right]\right\} \quad . \tag{12}$$

Cost minimization for the aggregate capital stock K_t requires the following conditions:

$$K_t^H = \xi_K K_t\left(\frac{r_t^k}{r_t^{K^H}}\right)^{\theta_k} \quad , \tag{13}$$

$$K_t^F = (1-\xi_K)K_t\left(\frac{r_t^k}{r_t^{K^F}}\right)^{\theta_k} \quad , \tag{14}$$

where

$$r_t^k = \left[\xi_K\left(r_t^{K^H}\right)^{1-\theta_k} + (1-\xi_K)\left(r_t^{K^F}\right)^{1-\theta_k}\right]^{\frac{1}{1-\theta_k}} \quad . \tag{15}$$

We finally consider an individual household's labor supply decision. Firms are assumed to demand labor in terms of an aggregate L_t which is a CES aggregate of all labor varieties supplied by households, with elasticity of substitution θ_w. In choosing its demands for all households' labor varieties, each firm therefore has to solve the following cost minimization problem:

$$\underset{L_t^i, i\in[0,1]}{Min} \int_0^1 W_t^i L_t^i di \;\; s.t. \; L_t = \left(\int_0^1 L_t^{i\,\frac{\theta_w - 1}{\theta_w}} di \right)^{\frac{\theta_w}{\theta_w - 1}} . \tag{16}$$

This gives rise to the set of labor demands

$$L_t^i = \left(\frac{W_t^i}{W_t} \right)^{-\theta_w} L_t \; , \tag{17}$$

where the aggregate wage is given by

$$W_t = \left(\int_0^1 \left(W_t^i \right)^{1-\theta_w} di \right)^{\frac{1}{1-\theta_w}} . \tag{18}$$

Households maximize their utility from leisure subject to this demand (17) and subject to the wage inflation adjustment cost in the budget constraint (5). The optimal wage decision is given by

$$\lambda_t w_t L_t (\theta_w - 1) + \lambda_t \phi^w (\pi_t^w - \pi_{t-1}^w) \pi_t^w - \beta E_t \left[\lambda_{t+1} \phi^w (\pi_{t+1}^w - \pi_t^w) \pi_{t+1}^w \right] = \theta_w \psi L_t (1 - L_t)^{-\frac{1}{\sigma}} . \tag{19}$$

2.2 Intermediates

2.2.1 Varieties (z) of Intermediates

For each variety z there is a continuum of producers who are perfectly competitive price takers in both their input and output markets. They have the following production functions in labor, capital and fixed factors:

$$y_t(z) = a(z) x_t \left[(\xi_v)^{\frac{1}{\theta_v}} \left(l_t(z)^\gamma k_t(z)^{1-\gamma} \right)^{\frac{\theta_v - 1}{\theta_v}} + (1 - \xi_v)^{\frac{1}{\theta_v}} \left(g_t(z) \right)^{\frac{\theta_v - 1}{\theta_v}} \right]^{\frac{\theta_v}{\theta_v - 1}} = x_t a(z) v_t(z) \; . \tag{20}$$

The first two elements of the production function are sector specific productivity levels $a(z)$ and aggregate productivity levels x_t. Aggregate productivity or supply shocks are given by

$$\log(x_t) = \rho^x \log(x_{t-1}) + (1-\rho^x)\log(\bar{x}) + u_t^x \ , \tag{21}$$

$$\log(x_t^*) = \rho^{x^*} \log(x_{t-1}^*) + (1-\rho^{x^*})\log(\bar{x}^*) + u_t^{x^*} \ . \tag{22}$$

The sector specific productivity terms determine the pattern of comparative advantage between countries, a crucial ingredient in making tradedness of intermediate goods endogenous. For each producer, optimality requires that the price of its variety equal marginal cost, where marginal cost is equal to the ratio of marginal factor cost m_t^v (the cost of $v_t(z)$), derived below from the producer's cost minimization problem, and productivity:

$$p_t(z) = \frac{m_t^v}{x_t a(z)} \ . \tag{23}$$

When a good is produced in the Foreign country and shipped to the Home country or vice versa there are iceberg-type proportional trading costs τ_t that are identical across goods.[7] Therefore, in the absence of relative productivity differences, there would be no trade as each country would produce the entire range of consumption goods at home. But as soon as there are sufficiently strong comparative advantage patterns in productivity the effect of trading costs can be overcome, leading to trade. For a given pattern of comparative advantage, lower trade costs lead to more trade, or to a smaller range of nontraded goods. We denote the relative aggregate productivity by $\chi_t = x_t/x_t^*$, and the relative variety-specific productivity between Home and Foreign by $A(z) = a(z)/a^*(z)$. The shape of the function $\chi_t A(z)$, which we will refer to as the comparative advantage schedule, is of crucial importance for our results. This is of course an empirical question, but at this point we are not aware of any evidence to give us guidance. On the grounds of plausibility, we would prefer a negative exponential schedule, with a narrow range of goods over which

[7] Unlike adjustment costs, transport costs are not redistributed back to agents in a lump-sum fashion. They represent an actual loss in transit.

Home has a strong comparative (and possibly an absolute) advantage. For this paper we chose instead a negatively sloped kinked linear schedule, which approximates a negative exponential schedule. More importantly, it has an analytical advantage because the solution of the model requires analytical integration of the comparative advantage schedule over sub-intervals. In the absence of empirical evidence, the best we can (and will) do is to explore the sensitivity of our results to different parameterizations of this schedule, including eventually the adoption of a negative exponential schedule.

We assume that $a^*(z) = 1$ for all z and that the z are ranked from the highest to the lowest relative productivity for Home ($A'(z) < 0$), so that the Home country has a comparative advantage for low end z's and the foreign country for high end z's. We also assume that the comparative advantage schedule is linear and continuous, with a kink at $z = kink$, with $A(z) = \tilde{T} - \tilde{U}z$ for $z \in [0, kink]$, $A(z) = \tilde{V} - \tilde{W}z$ for $z \in [kink, 1]$, and $A(z = kink) = x$, the average home productivity level (see Figure 2). In our base case $kink = \alpha$ (the country's size), $x^* = 1$ and $x = 0.5$, meaning for a good where $a(z) = 1$, the producer in the Home country is half as productive as his counterpart in the Foreign country. The intercept at $z = 0$ is set to $x * \tilde{T} = 1.25$, i.e. the maximum relative productivity for the Home country is 125 percent of Foreign productivity. The intercept at $z = 1$ is set to $x * (1/\tilde{T}) = 0.2$.

Figure 2, which shows $A(z)$ as the solid line at the center of the shaded area, illustrates the determination of the world trade pattern. This pattern depends on the relative prices of Foreign and Home produced goods. A Home firm will produce a given variety only if its price $P_t(z)$ does not exceed the price $(S_t P_t^*(z))/(1 - \tau)$ that an importer of the same variety is able to charge given his marginal cost and trade costs. Given the declining relative productivity pattern in Home there will therefore be a maximum level of z above which Home will rely entirely on imports instead of producing at home. We denote this time-varying level by z_t^h. Equally, there is a minimum z, denoted z_t^l, below which Foreign will rely on imports from Home. We can combine these two conditions on prices with the marginal cost conditions for Home and Foreign producers (23). These include m_t^v and

$m_t^{v^*}$, the marginal factor costs (labor, capital and fixed factors), which are equalized across all varieties in each country. We have

$$\frac{m_t^v}{m_t^{v^*} s_t} \leqq \frac{x_t a(z_t)}{x_t^*} \frac{1}{(1-\tau)} \text{ for } z_t \in [0, z_t^h], \tag{24}$$

with equality at $z_t = z_t^h$, and

$$\frac{m_t^v}{m_t^{v^*} s_t} \geqq \frac{x_t a(z_t)}{x_t^*} (1-\tau_t) \text{ for } z_t \in [z_t^l, 1], \tag{25}$$

with equality at $z_t = z_t^l$. The first expression says that a Home producer's marginal cost $(m_t^v/(x_t a(z)))$ has to be below its Foreign competitor's marginal cost $((m_t^v s_t)/x_t^*)$ to be competitive, but allowing for the fact that a potential Foreign competitor's cost also includes the trading cost hurdle. The second condition expresses the same requirement for the Foreign producer, whose comparative advantage schedule is given by $x_t^*/(x_t a(z))$. In Figure 2, the condition (24) for domestic production to be viable is represented by the upper boundary of the shaded region and the condition (25) for foreign production to be viable by the lower boundary. The solid horizontal line is the relative factor cost, whose intersection with the boundaries of the shaded region determine z_t^l and z_t^h. We define $\delta_t = z_t^h - z_t^l$. The resulting trade pattern is illustrated in Figure 3.

The parametric form of the $A(z)$ schedule is rich enough to allow for the analysis of a variety of different technology shocks. For example, an increase in $\tilde{T}$ or an increase in $kink$ represents a positive productivity shock biased towards a country's export goods, while an increase in x represents a positive productivity shock to all goods. As we will show, the welfare and trade effects of a reduction in trading costs depend crucially on the shape of $A(z)$. For a flat schedule, parameterized as a low $\tilde{T}$, the expansion in trade is very large, but the gains from the extra trade are quite limited because the foreign country does not enjoy a strong productivity advantage. For a steep schedule, while trade may expand by much less, the welfare effects in terms of increased consumption and leisure will generally be higher.

It remains to determine marginal factor cost from the producer's cost minimization problem. Let $u_t(z) = l_t(z)^{\gamma} k_t(z)^{1-\gamma}$, and define $L_{t,H} = \int_0^{z_t^l} l_t(z)dz$, $L_{t,N} = \int_{z_t^h}^{z_t^h} l_t(z)dz$, and similarly for $K_{t,H}$, $K_{t,N}$, $U_{t,H}$, $U_{t,N}$, $G_{t,H}$, and $G_{t,N}$. Also:[8]

$$Y_{t,H} = \int_0^{z_t^l} \frac{P_t(z)}{P_t} y_{t,H}(z)dz \ \ , \ \ Y_{t,N} = \int_{z_t^l}^{z_t^h} \frac{P_t(z)}{P_t} y_{t,N}(z)dz \ \ .$$

Then cost minimization implies the following set of conditions for Home (and an equivalent set of conditions for Foreign):

$$m_t^u = \frac{(w_t)^{\gamma} \left(r_t^k\right)^{1-\gamma}}{\gamma^{\gamma}(1-\gamma)^{(1-\gamma)}} \ \ , \tag{26}$$

$$m_t^v = \left[\xi_v \left(m_t^u\right)^{1-\theta_v} + (1-\xi_v)\left(r_t^g\right)^{1-\theta_v}\right]^{\frac{1}{1-\theta_v}} \ \ , \tag{27}$$

$$w_t L_{t,j} = \gamma m_t^u U_{t,j} \ \ , \ \ j = H, N. \tag{28}$$

$$r_t^k K_{t,j} = (1-\gamma) m_t^u U_{t,j} \ \ , \ \ j = H, N. \tag{29}$$

$$G_{t,j} = U_{t,j} \frac{1-\xi_v}{\xi_v} \left(\frac{m_t^u}{r_t^g}\right)^{\theta_v} \ \ , \ \ j = H, N. \tag{30}$$

$$\left[1 + \frac{1-\xi_v}{\xi_v}\left(\frac{m_t^u}{r_t^g}\right)^{\theta_v - 1}\right] m_t^u U_{t,j} = Y_{t,j} \ \ , \ \ j = H, N. \tag{31}$$

2.2.2 Finished Intermediates

The producer of finished intermediates $D_t^{\Pr od}$ is a price-taker in both his input and his output markets, with his (flexible) output price given by M_t^D. He uses inputs of export goods $D_{t,H}$,[9] nontraded goods $D_{t,N}$ and import goods $D_{t,F}$, with the following CES production

[8] This definition of sectoral real outputs in terms of the overall intermediates price level is a useful analytical "trick" to facilitate aggregation, see below.
[9] These are goods in the varieties range $z \in [0, z_t^l)$ that are both exported and used at home.

function:

$$D_t^{\Pr od} = \left[\int_0^1 y_t(z)^{\frac{\theta_d-1}{\theta_d}} dz\right]^{\frac{\theta_d}{\theta_d-1}} \tag{32}$$

$$= \left[\left(z_t^l\right)^{\frac{1}{\theta_d}} \left(D_{t,H}\right)^{\frac{\theta_d-1}{\theta_d}} + \left(\delta_t\right)^{\frac{1}{\theta_d}} \left(D_{t,N}\right)^{\frac{\theta_d-1}{\theta_d}} + \left(1 - z_t^h\right)^{\frac{1}{\theta_d}} \left(D_{t,F}\right)^{\frac{\theta_d-1}{\theta_d}}\right]^{\frac{\theta_d}{\theta_d-1}} .$$

The sub-baskets of intermediate goods are given by

$$D_{t,H} = \left[\left(\frac{1}{z_t^l}\right)^{\frac{1}{\theta_d}} \int_0^{z_t^l} \left(y_t^h(z)\right)^{\frac{\theta_d-1}{\theta_d}} dz\right]^{\frac{\theta_d}{\theta_d-1}} , \tag{33}$$

$$D_{t,N} = \left[\left(\frac{1}{\delta_t}\right)^{\frac{1}{\theta_d}} \int_{z_t^l}^{z_t^h} \left(y_t^n(z)\right)^{\frac{\theta_d-1}{\theta_d}} dz\right]^{\frac{\theta_d}{\theta_d-1}} ,$$

$$D_{t,F} = \left[\left(\frac{1}{1-z_t^h}\right)^{\frac{1}{\theta_d}} \int_{z_t^h}^{1} \left(y_t^f(z)\right)^{\frac{\theta_d-1}{\theta_d}} dz\right]^{\frac{\theta_d}{\theta_d-1}} ,$$

where the price sub-indices for each of these baskets can be shown to be

$$P_{t,H} = \left[\frac{1}{z_t^l} \int_0^{z_t^l} P_t(z)^{1-\theta_d} dz\right]^{\frac{1}{1-\theta_d}} , \tag{34}$$

$$P_{t,N} = \left[\frac{1}{\delta_t} \int_{z_t^l}^{z_t^h} P_t(z)^{1-\theta_d} dz\right]^{\frac{1}{1-\theta_d}} ,$$

$$P_{t,F} = \left[\frac{1}{1-z_t^h} \int_{z_t^h}^{1} P_t(z)^{1-\theta_d} dz\right]^{\frac{1}{1-\theta_d}} .$$

Using our results on the pricing of individual varieties, and dividing through by the numeraire price level, we can rewrite these price indices in terms of aggregate variables as

$$p_{t,H} = \frac{m_t^v}{x_t a_{t,H}} , \tag{35}$$

$$p_{t,N} = \frac{m_t^v}{x_t a_{t,N}} ,$$

$$p_{t,F} = p_{t,F}^* \frac{s_t}{1-\tau_t} ,$$

and similarly for Foreign. Here we have used definitions of sectorial productivities derived through analytical integration over the appropriate sub-intervals of goods varieties. Assuming[10] that $kink = \alpha$, and that $z_t^l < \alpha < z_t^h$, these are given by

$$a_{t,H} = \left[\frac{\tilde{T}^{\theta_d} - \left(\tilde{T} - \tilde{U} z_t^l \right)^{\theta_d}}{\theta_d \tilde{U} z_t^l} \right]^{\frac{1}{\theta_d - 1}} , \tag{36}$$

$$a_{t,N} = \left[\frac{\left(\tilde{T} - \tilde{U} z_t^l \right)^{\theta_d} - \left(\tilde{T} - \tilde{U} \alpha \right)^{\theta_d}}{\theta_d \tilde{U} \delta_t} + \frac{1 - \left(\tilde{V} - \tilde{W} z_t^h \right)^{\theta_d}}{\theta_d \tilde{W} \delta_t} \right]^{\frac{1}{\theta_d - 1}} .$$

Producers of the finished intermediate good $D_t^{\Pr od}$ face a cost of adjusting the individual sub-components $D_{t,H}$, $D_{t,N}$ and $D_{t,F}$. In the present version of the model this cost is simply a quadratic adjustment cost, but future versions will also include delivery time-lags. Let the nominal discount factor be $I_t^{DF} = 1$ for $t = 0$ and $I_t^{DF} = \Pi_{j=0}^{t-1}(1/i_j)$ for $t \geqq 1$. Then producers solve the following problem:

$$Max\ E_0 \sum_{t=0}^{\infty} I_t^{DF} \left\{ M_t^D \left[\left(z_t^l \right)^{\frac{1}{\theta_d}} \left(D_{t,H} \right)^{\frac{\theta_d - 1}{\theta_d}} + \left(\delta_t \right)^{\frac{1}{\theta_d}} \left(D_{t,N} \right)^{\frac{\theta_d - 1}{\theta_d}} \right. \right. \tag{37}$$

$$\left. \left. + (1 - z_t^h)^{\frac{1}{\theta_d}} \left(D_{t,F} \right)^{\frac{\theta_d - 1}{\theta_d}} \right]^{\frac{\theta_d}{\theta_d - 1}} - \sum_{j=H,N,F} P_{t,j} \left[D_{t,j} + \frac{\phi^d}{2} \frac{\left(D_{t,j} - D_{t-1,j} \right)^2}{D_{t-1,j}} \right] \right\} .$$

The solution to this problem for $D_{t,H}$ is

$$m_t^D \left(z_t^l \right)^{\frac{1}{\theta_d}} \left(\frac{D_t^{\Pr od}}{D_{t,H}} \right)^{\frac{1}{\theta_d}} = p_{t,H} \left(1 + \phi_H^d \left(\frac{D_{t,H} - D_{t-1,H}}{D_{t-1,H}} \right) \right) \tag{38}$$

$$- \frac{\pi_{t+1}}{i_t} p_{t+1,H} \left(\frac{\phi_H^d}{2} \left(\frac{D_{t+1,H} - D_{t,H}}{D_{t,H}} \right)^2 + \phi_H^d \left(\frac{D_{t+1,H} - D_{t,H}}{D_{t,H}} \right) \right) ,$$

and similarly for $D_{t,N}$ and $D_{t,F}$. The homogenous final output is sold by the finished intermediates producer to a continuum of distributors.

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[10] This is of course verified in the course of numerical simulations.

2.2.3 Distributed Intermediates

Intermediates distributors are price takers in their input market, taking M_t^D as given, and monopolistic competitors in their output market, selling at the numeraire price level P_t to producers of output Ω_t. The latter demand a composite of distributed varieties with elasticity of substitution θ_d^D:

$$D_t = \left[\int_0^1 D_t^{\mathrm{Pr}\,od}(k)^{\frac{\theta_d^D - 1}{\theta_d^D}} dz \right]^{\frac{\theta_d^D}{\theta_d^D - 1}} .$$

This implies goods demands

$$D_t^{\mathrm{Pr}\,od}(k) = \left(\frac{P_t(k)}{P_t} \right)^{-\theta_d^D} D_t .$$

Each distributor faces a quadratic adjustment cost of changing the rate of change of his prices. In particular, it is costly to set a firm-specific inflation rate that differs from the observed lagged inflation rate for the entire sector, similar to the specification of wage rigidities above. The optimization problem therefore takes the following form:

$$\underset{P_t(k)}{Max} \left(\frac{P_t(k) - M_t^D}{P_t} \right) \left(\frac{P_t(k)}{P_t} \right)^{-\theta_d^D} D_t - \frac{\Phi^d}{2} \left(\frac{P_t(k)}{P_{t-1}(k)} - \frac{P_{t-1}}{P_{t-2}} \right)^2 \tag{39}$$
$$-E_t \left[\frac{\pi_{t+1}}{i_t} \frac{\Phi^d}{2} \left(\frac{P_{t+1}(k)}{P_t(k)} - \frac{P_t}{P_{t-1}} \right)^2 \right] .$$

All firms face an identical problem and therefore behave identically. In equilibrium we therefore have $P_t(k) = P_t$. The first-order condition for this problem is therefore as follows:

$$D_t \left((1 - \theta_d^D) + \theta_d^D m_t^d \right) = \Phi^d \pi_t (\pi_t - \pi_{t-1}) - \frac{\pi_{t+1}}{i_t} \Phi^d \pi_{t+1} (\pi_{t+1} - \pi_t) \; . \tag{40}$$

2.3 Output

2.3.1 Finished Output

Producers of finished output are perfectly competitive price takers in both their input markets and their output market. They sell output $\Omega_t^{\mathrm{Pr}\,od}$ at the price M_t^o to a distribution sector. Producers use inputs of intermediates D_t and of second stage value added $Y_{t,O}$, with the CES production function given by

$$\Omega_t^{\Pr od} = \left(\xi_o^{\frac{1}{\theta_o}} \left(D_t\right)^{\frac{\theta_o - 1}{\theta_o}} + (1-\xi_o)^{\frac{1}{\theta_o}} \left(Y_{t,O}\right)^{\frac{\theta_o - 1}{\theta_o}} \right)^{\frac{\theta_o}{\theta_o - 1}} \quad . \tag{41}$$

The production function for second-stage value added $Y_{t,O}$ has the same form as (20), except for the absence of the varieties-index z and of the variety specific productivity term $a(z)$. The conditions (28)-(30) for optimal value added input choices are therefore identical to those for intermediates varieties production, while (31) is replaced by

$$\left[1 + \frac{1-\xi_v}{\xi_v} \left(\frac{m_t^u}{r_t^g} \right)^{\theta_v - 1} \right] m_t^u U_{t,O} = \frac{m_t^v}{x_t} Y_{t,O} \quad , \quad j = H, N. \tag{42}$$

Cost minimization furthermore implies the following producer price of finished goods

$$m_t^o = \left[\xi_o + (1-\xi_o) \left(\frac{m_t^v}{x_t} \right)^{1-\theta_o} \right]^{\frac{1}{1-\theta_o}} \quad , \tag{43}$$

and the cost-minimizing demands

$$D_t = \xi_o \Omega_t^{\Pr od} \left(m_t^o\right)^{\theta_o} \quad , \tag{44}$$

$$Y_{t,O} = (1-\xi_o) \Omega_t^{\Pr od} \left(\frac{m_t^o}{(m_t^v / x_t)} \right)^{\theta_o} \quad . \tag{45}$$

2.3.2 Distributed Output

The optimization problem of this sector is identical in nature to (39), with the appropriate change of notation. Specifically,

$$\Omega_t = \left[\int_0^1 \Omega_t^{\Pr od}(k)^{\frac{\theta_o^D - 1}{\theta_o^D}} dz \right]^{\frac{\theta_o^D}{\theta_o^D - 1}} \quad .$$

We therefore obtain the optimality condition

$$\Omega_t \left((1-\theta_o^D) + \theta_o^D \left(\frac{m_t^o}{p_t^o} \right) \right) = \Phi^o \pi_t^o (\pi_t^o - \pi_{t-1}^o) - \frac{\pi_{t+1}^o}{i_t} \Phi^o \pi_{t+1}^o (\pi_{t+1}^o - \pi_t^o) \quad , \tag{46}$$

where Ω_t is final output sold by the distribution sector and p_t^o/π_t^o are the relative price/inflation rate for that output. Final output is sold either as an investment good, to domestic or foreign households, or as a consumption good, to domestic or foreign producers of consumption goods.

2.4 Consumption Goods

2.4.1 Finished Consumption Goods

Producers of finished consumption goods are perfectly competitive price takers in their input markets and output market. They sell output $C_t^{\Pr od}$ at the price M_t^C to a distribution sector. Producers use inputs of Home final output C_t^H and of Foreign final output C_t^F, at prices $p_t^H = p_t^o$ and $p_t^F = (p_t^{o^*} s_t)/(1-\tau_t)$. Note that imports at this level are assumed to be subject to the same transport costs as imports of intermediates further up the production chain. The overall production function for finished consumption goods is given by

$$C_t^{\Pr od} = \left[(\xi_c)^{\frac{1}{\theta_c}} \left(C_t^H \right)^{\frac{\theta_c - 1}{\theta_c}} + (1-\xi_c)^{\frac{1}{\theta_c}} \left(C_t^F \right)^{\frac{\theta_c - 1}{\theta_c}} \right]^{\frac{\theta_c}{\theta_c - 1}} . \tag{47}$$

Producers of $C_t^{\Pr od}$ face a cost of adjusting the individual sub-components C_t^H and C_t^F. As above, in the present version of the model this cost is simply a quadratic adjustment cost, but future versions will also include delivery time-lags. Then producers solve the following problem:

$$Max\ E_0 \sum_{t=0}^{\infty} I_t^{DF} \left\{ M_t^C \left[(\xi_c)^{\frac{1}{\theta_c}} \left(C_t^H \right)^{\frac{\theta_c - 1}{\theta_c}} + (1-\xi_c)^{\frac{1}{\theta_c}} \left(C_t^F \right)^{\frac{\theta_c - 1}{\theta_c}} \right]^{\frac{\theta_c}{\theta_c - 1}} \right. \tag{48}$$
$$\left. - \sum_{j=H,F} P_t^j \left[C_t^j + \frac{\phi_j^c}{2} \frac{\left(C_t^j - C_{t-1}^j \right)^2}{C_{t-1}^j} \right] \right\} .$$

The solution to this problem for C_t^H is

$$m_t^C (\xi_c)^{\frac{1}{\theta_c}} \left(\frac{C_t^{\Pr od}}{C_t^H} \right)^{\frac{1}{\theta_c}} = p_t^H \left(1 + \phi_H^c \left(\frac{C_t^H - C_{t-1}^H}{C_{t-1}^H} \right) \right) \tag{49}$$
$$- \frac{\pi_{t+1}}{i_t} p_{t+1}^H \left(\frac{\phi_H^c}{2} \left(\frac{C_{t+1}^H - C_t^H}{C_t^H} \right)^2 + \phi_H^c \left(\frac{C_{t+1}^H - C_t^H}{C_t^H} \right) \right) ,$$

and similarly for C_t^F. The homogenous final output is sold by the finished intermediates producer to a continuum of distributors.

2.4.2 Distributed Consumption Goods

The optimization problem of this sector is identical in nature to (39), again with the appropriate change of notation. Specifically,

$$C_t = \left[\int_0^1 C_t^{\Pr od}(k)^{\frac{\theta_c^D - 1}{\theta_c^D}} dz \right]^{\frac{\theta_c^D}{\theta_c^D - 1}} .$$

We therefore obtain the optimality condition

$$C_t \left((1 - \theta_c^D) + \theta_c^D \left(\frac{m_t^c}{p_t^c} \right) \right) = \Phi^c \pi_t^c (\pi_t^c - \pi_{t-1}^c) - \frac{\pi_{t+1}^c}{i_t} \Phi^c \pi_{t+1}^c (\pi_{t+1}^c - \pi_t^c) \ , \qquad (50)$$

where C_t is final output sold by the distribution sector and p_t^c/π_t^c are the relative price/inflation rate for that output.

2.5 Government

Fiscal policy in both countries is monetary dominant in that fiscal lump-sum transfers are endogenous to the implications of monetary policy choices. Monetary policy is characterized by interest rate feedback rules. For the analysis we employ a simple inflation-forecast-based (IFB) rule where the short-term interest rate (i_t) depends on its own lag, as well as a 3-quarter-ahead model-consistent forecast of year-on-year inflation,

$$\log(i_t) = \lambda_i \log(i_{t-1}) + (1 - \lambda_i) \log(\pi 4_{w,t+3}/\beta) + \lambda_\pi \log(\pi 4_{w,t+3}/\overline{\pi}) + u_t^i, \qquad (51)$$

where $\pi 4_{w,t}$ is a weighted sum of the rate of change in consumer prices and the price of domestic output, $\overline{\pi}$ is a fixed long-term inflation objective, and u_t^i is stochastic disturbance term. Relative to other IFB rules used in the literature, the only novel feature of this form of the rule is that it allows for the possibility that interest rates respond to expected movements in headline CPI inflation (π_t^c) in addition to a measure of domestic inflation (π_t^o):

$$\log(\pi 4_{w,t}) = w_c \log(\pi_t^c \pi_{t-1}^c \pi_{t-2}^c \pi_{t-3}^c)/4 + (1 - w_c) \log(\pi_t^o \pi_{t-1}^o \pi_{t-2}^o \pi_{t-3}^o)/4 \qquad (52)$$

These types of rules have been employed extensively in central bank models to characterize monetary policy because interest rates settings are typically based on forecasts of measures of underlying inflationary pressures.[11] They can be augmented with a measure of the output gap, but for simplicity, we ignore that in this paper.

2.6 Market Clearing Conditions and the Current Account

The following relationships hold between intermediate varieties sectorial output levels Y and finished intermediates sectorial input levels D (always with corresponding relationships for Foreign):

$$Y_{t,H} = p_{t,H} D_{t,H} + p^*_{t,H} D^*_{t,H} s_t \quad , \tag{53}$$

$$Y_{t,N} = p_{t,N} D_{t,N} \quad . \tag{54}$$

Factor market clearing conditions are:

$$\alpha L_t = L_{t,H} + L_{t,N} + L_{t,O} \quad , \tag{55}$$

$$\alpha K_t = K_{t,H} + K_{t,N} + K_{t,O} \quad , \tag{56}$$

$$\alpha G = G_{t,H} + G_{t,N} + G_{t,O} \quad . \tag{57}$$

And the output market clearing condition is:

$$\Omega_t = \alpha (C_t^H + J_t^H) + (1-\alpha) \frac{(C_t^{H^*} + J_t^{H^*})}{(1-\tau_t)} \quad . \tag{58}$$

[11] Because IFB rules provide a reasonable summary of the entire dynamics in a forecast, they are usually found to be more robust than Taylor rules, which respond to "observed" measures of year-on-year inflation and the output gap—see Levin, Wieland and Williams (2001). This will be the case in models with richer sources of dynamics that are difficult to summarize adequately in the current "observed" values of some measure of inflation and the output gap. IFB rules have been used extensively by many central banks with either explicit and implicit inflation-targeting frameworks and have been relied upon in some cases for well over a decade—see Laxton, Rose and Tetlow (1993).

After consolidating the budget constraints of domestic households and their government, and taking account of transfers, we obtain the following aggregate flow resource constraint of Home vis-a-vis Foreign:

$$\alpha f_{t-1}\frac{i^*_{t-1}\varepsilon_t}{\pi_t}+p^*_{t,H}D^*_{t,H}s_t+(1-\alpha)s_t p_t^{H^*}(C_t^{H^*}+J_t^{H^*}) = \alpha f_t+p_{t,F}D_{t,F}+\alpha p_t^F(C_t^F+J_t^F) \quad (59)$$

Then the bond market clearing condition is given by:

$$\alpha f_t + (1-\alpha)f_t^* s_t = 0 \ . \quad (60)$$

3 Calibration

The model's parameters have been calibrated to be consistent with those employed in the literature. We assume that the size of the Home country represents only 5 percent of that of the Foreign country. In other words, the Home country represents a small open emerging market economy, and the Foreign country represents a large industrialized nation such as the United States of the euro area.

3.1 Base-Case Parameter Values

Table 1 reports on a number of fundamental parameters which are assumed to be the same across the two countries. Consumers discount the future at the rate of 1 percent per quarter (4 percent per year) ($\beta = 0.99$), while firm's capital depreciates by 2.5 percent (10 percent) over the same time frame ($\Delta = 0.025$).

The intertemporal elasticity of substitution (σ) and the degree of habit persistence (ν) are 0.83 and 0.72, respectively. These estimates are taken from a study by Juillard and others (2004), although they are somewhat higher than those estimated by Smets and Wouters (2002b). These coefficients, together with adjustment costs on the components of consumption expenditures, generate the lagged and hump-shaped responses to interest hikes typically found in empirical models.[12]

[12] Without the adjustment costs, even higher parameter estimates may be needed. For example, Bayoumi,

Given the paucity of evidence on mark-ups in emerging market economies, elasticities of substitution (EOS) across firms and workers are set at 5, a typical value used for industrial countries, which implies markups of 25 percent for labor and for distribution sectors.[13] In the analysis we also consider cases where these elasticities are higher and lower, and are asymmetric across to the two economies. The EOS between imported and domestically produced capital (θ_k) is set at one in the baseline, implying fixed nominal shares are spent on these goods, and we consider alternative cases where it is both higher ($\theta_k = 1.50$) and lower ($\theta_k = .50$). The EOS between capital and labor is one, the EOS between capital/labor and land (θ_v) is 0.50 in the baseline, and we again examined alternative cases where it is both higher ($\theta_v = 1$) and lower ($\theta_v = .25$).

There is little reliable evidence about the magnitude of wage and price rigidities in developing countries, but they are generally assumed to be smaller than in the industrialized nations. For our base-case, coefficients defining wage and price stickiness parameters have all been set to 400 in the Home country, half of the value in the Foreign country. These values were chosen to produce plausible impulse responses for interest rate shocks.

Turning to time-to-build lags, following Murchison, Rennison and Zhu (2004), we assume that it takes one quarter to plan an investment project and 5 quarters to complete it.[14] In addition, we set the adjustment cost parameters that govern investment dynamics to be consistent with the hump-shaped pattern seen in response to interest rate cuts that peak at

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Laxton and Pesenti (2004) show that estimates as high as 5.0 and 0.97 are required for σ and ν to generate the hump-shaped responses to interest rate shocks that can be found in the ECB's Area-Wide Model (AWM) of the monetary transmission mechanism—see Fagan, Henry and Mestre (2001).

13 In reviewing existing empirical work on markups for the euro area, Bayoumi, Laxton and Pesenti (2004) employ a price markup of 35 percent and a wage markup of 30 percent. They argue that these are significantly higher than price and wage markups in the United States, which they argue are closer to 23 percent and 16 percent, respectively.

14 Time-to-build dynamics are becoming an important feature of the new generation of macro models that are being designed inside central banks. For example, the work by Murchison, Rennison and Zhu (2004) at the Bank of Canada builds on earlier work at the Fed by Edge (2000a, 2000b). For more information on the importance of time-to-build dynamics for the internal propagation mechanism of DSGE models, see Casares (2004). In particular, Casares (2004) provides a very useful study showing the effects on macroeconomic dynamics of adding time-to-build lags that range between 1 and 8 quarters.

around 4-6 quarters and, in the case of accession countries, the relatively long-lived nature of the recent boost to the investment to GDP ratio. To reflect the greater difficulties of building and maintaining international supplier relationships, we set the adjustment parameter on imported capital goods to be twice as high as on domestically produced capital goods. We have imposed adjustment costs on imports of intermediate inputs and consumption goods in a similar manner. The model therefore generates moderate changes in trade volumes in response to short run real exchange rate fluctuations but large changes in response to permanent shocks, as has been observed in the transition economies—see Erceg, Guerrieri, and Gust (2003) and Laxton and Pesenti (2003).

Finally, we set the parameters that determine the endogenous risk premium on bonds to ensure that changes in the risk premium are sufficient to prevent implausibly large current account deficits.

We wish to calibrate the initial steady state so that Home corresponds to a typical emerging market economy in the mid-1990s. In the initial equilibrium, per capita consumption (measured at purchasing power parity) in the Home country is assumed to be just over half of the value in Foreign. We assume that the same proportion of time is allocated to work in both countries, but that total factor productivity in the Home country is only half that in the Foreign country.

Turning to trade, we assume the baseline parameterization of relative productivity $\chi A(z)$ reported in Table 2. The interaction of the aggregate term $\chi = x/x^*$ (where the Home country is assumed to be only half as productive as the Foreign country) with the industry-specific term $A(z)$ implies that Home enjoys a 25 percent productivity advantage in its most productive industry (at $z = 0$) while Foreign is five times more productive than Home in its most productive sector (at $z = 1$).

As reported in Table 3, for the Home country both the import-to-GDP and export-to-GDP ratios are assumed to be 30 percent, with trade in intermediate inputs comprising half of the total and the remainder being allocated equally between final consumption and investment goods.[15] The values of trade flows in the Foreign country reflect the mirror image of these values, and hence they are considerably smaller as a percentage of overall activity as Foreign is assumed to be large relative to Home.

Finally, the steady-state value of labor income has been set at 64 percent of nominal GDP in both economies, roughly the share of labor income in the Foreign country. With no government sector, the savings rate was set at 30 percent, approximately the average value in the Foreign country after excluding government output from nominal income.

3.2 Responses to Monetary-Induced Interest Rate Increases

To illustrate the dynamic properties of the model, Figure 4 reports results for a 1/2 percentage point increase in the foreign interest rates on its economy. This allows us to compare the model's impulse responses with existing one- or two-country monetary models. This calibration exercise can be done with standard tools such as DYNARE.

In our model, real GDP and CPI inflation decline and reach troughs of about one quarter percent below baseline after 3-4 quarters and one third percentage points below baseline after 4 quarters, respectively, while the real exchange rate appreciates by slightly over 1 percent on impact. Consumption and investment responses are hump-shaped, reflecting habit persistence, time-to-build, and costs of adjustment. Reassuringly, these results are relatively similar to those from the ECB's Area Wide Model (AWM), although the monetary transmission mechanism is somewhat faster and inflation responds more in this model than AWM—see Bayoumi, Laxton and Pesenti (2004) for a discussion of AWM dynamics in response to interest rate hikes. Results from the same experiment for the Home country are reported in Figure 5. Output responds more in the open economy because the appreciation in the real exchange rate has a larger impact on net exports in the more open economy. This is consistent with previous work that indicates the monetary transmission mechanism may

[15] These values were obtained by appropriate coefficient restrictions on final consumption and investment demands as well as trade costs. The implied restriction for trade costs is consistent with some empirical estimates that suggest they represent about one third of the value of goods.

be faster and stronger in small open economies than in relatively larger and more closed economies like the Foreign country.

The most important aspect of the results is however the very sluggish response of trade volumes to these shocks, due to the time-to-build-markets technology. We note that this is even more pronounced in versions of the model using the full time-to-build-markets technology with delivery time lags (not yet reported here). The main benefit of exchange rate flexibility, the ability to induce immediate and sizeable expenditure switching, is therefore not present in this model. We are therefore presently evaluating the hypothesis that the welfare benefits of exchange rate flexibility are much reduced. We are doing so by exploring the implications of different monetary rules that are consistent with flexible exchange rates, such targeting the CPI or the price of domestic output, and we will compare the results with exchange rate targeting. Finally, we will explore how the benefits of exchange rate flexibility vary as the degree of trade integration increases. This is an old question in open economy macroeconomics: do more open economies benefit relatively more or less from exchange rate flexibility. The advantage of our model is that it allows us to answer it in a fully structural framework.

4 Summary

The paper we have presented combines a microeconomic model of international trade decisions at the level of heterogeneous firms with a macroeconomic model of nominal rigidities and monetary policy. This allows us to analyze the effect of trade integration on monetary policy decisions in open economies, and we are completing that work using welfare analysis based on second-order approximations of the model.

Thinking through the modeling of microeconomic trade decisions has forced us to confront a well-known puzzle in open economy macroeconomics. This is that in the short run trade volumes respond little to real exchange rate movements, but in the long run very large changes in trade volumes can be observed, especially in response to reductions in trading costs. Models that are consistent with one of these facts typically have difficulty with the other. Models that focus on the business cycle frequency are made consistent with the sluggish short-run trade response through low elasticities of substitution between domestic and foreign goods, but they are unable to generate very large expansions in trade volumes. We find that our attempts to model microeconomic trade decisions at the firm level necessitate a model with very high elasticities of substitution between domestic and foreign goods, the opposite of the conventional approach. But our time-to-build markets technology in international trade means that we nevertheless generate sluggish short-run trade responses, while at the same time retaining the advantage that very large long-run trade expansions are possible. But in this model the effects of exchange rate flexibility are quite different from the conventional result. Even in a model with producer currency pricing, exchange rate flexibility is unable to induce large expenditure switching in the short-run, and any rapid switching that does take place is costly. In the welfare analysis that we are completing at this time, the welfare benefits of exchange rate flexibility therefore stand to be significantly reduced.

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Figure 1: Detailed Structure of Production

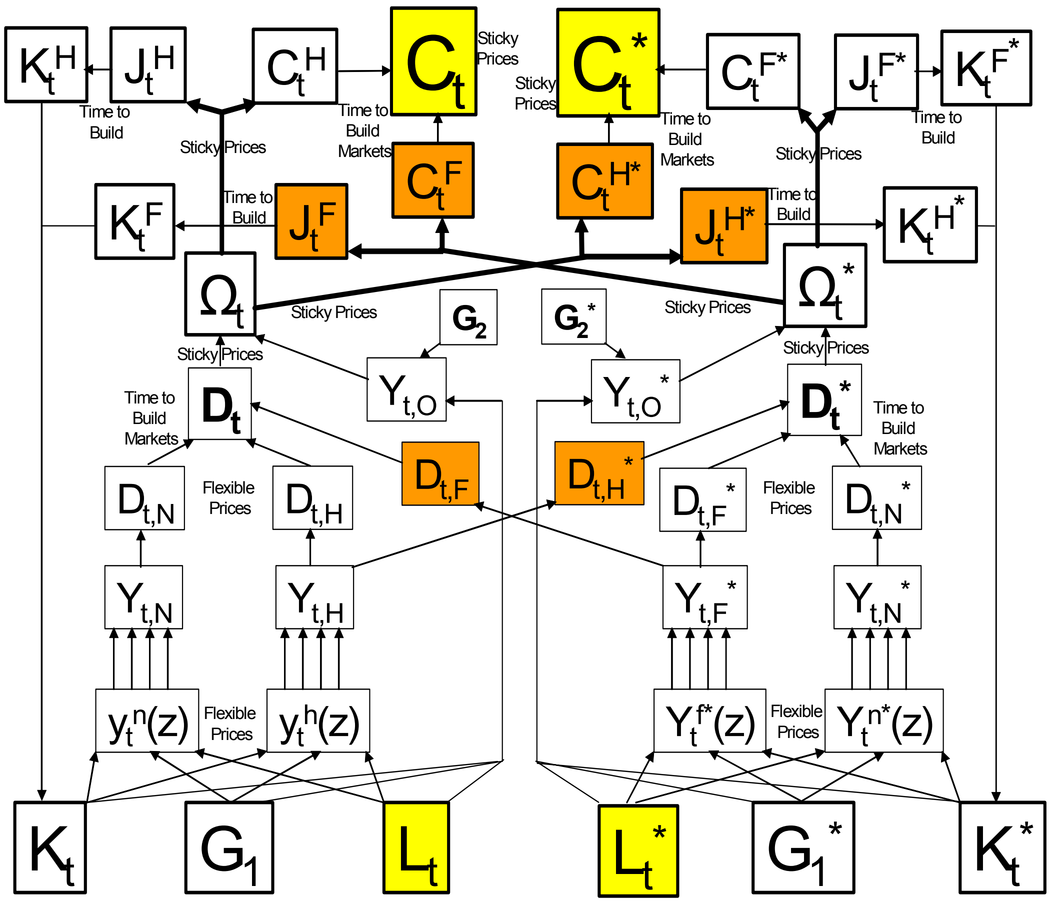

Figure 2: Base-Case Comparative Advantage Schedule

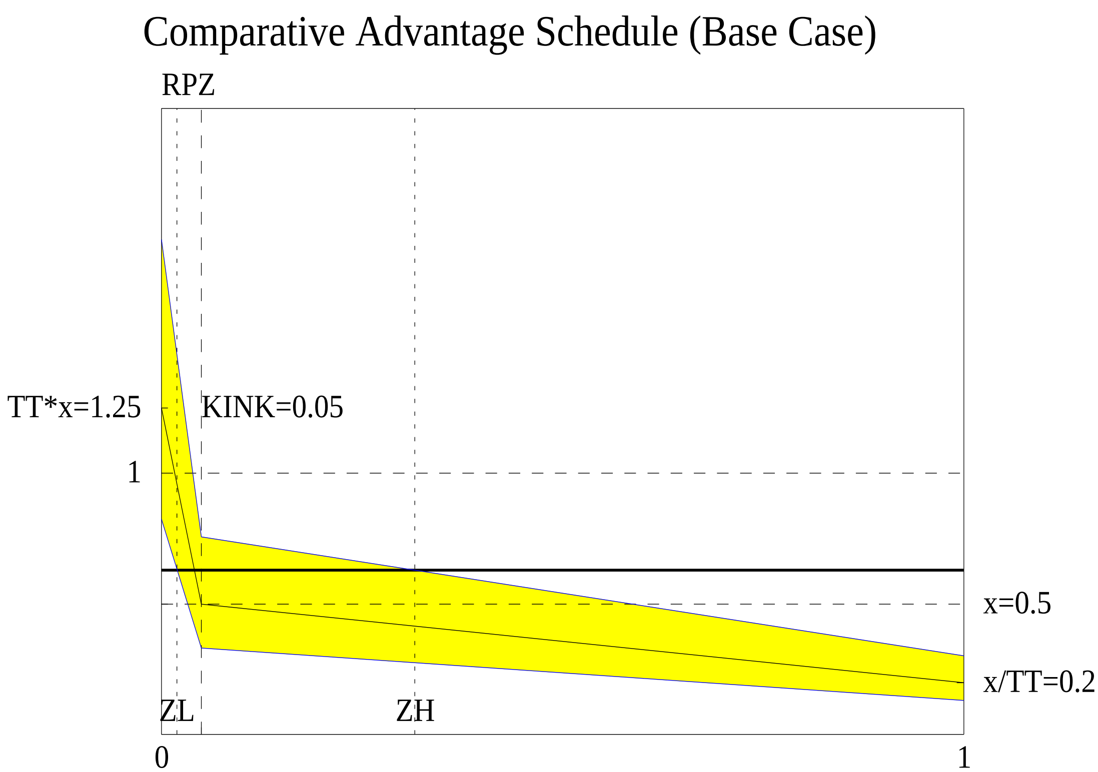

Figure 3: Trade Pattern

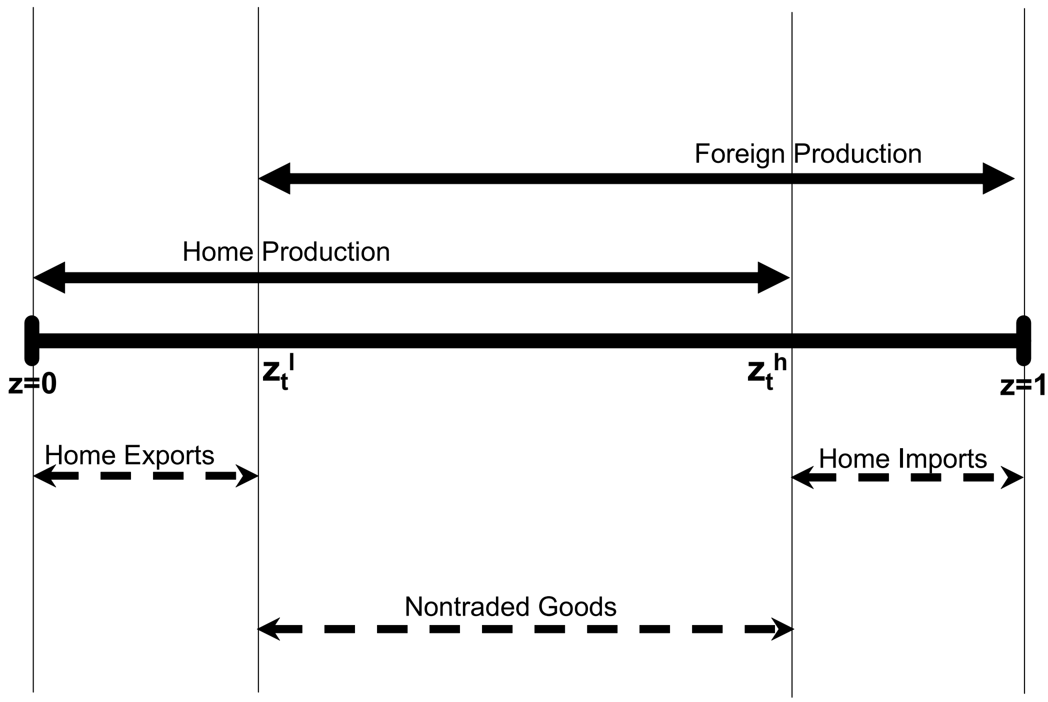

Figure 4: Foreign Responses to a Monetary Induced-Interest Rate Hike in the Foreign Economy

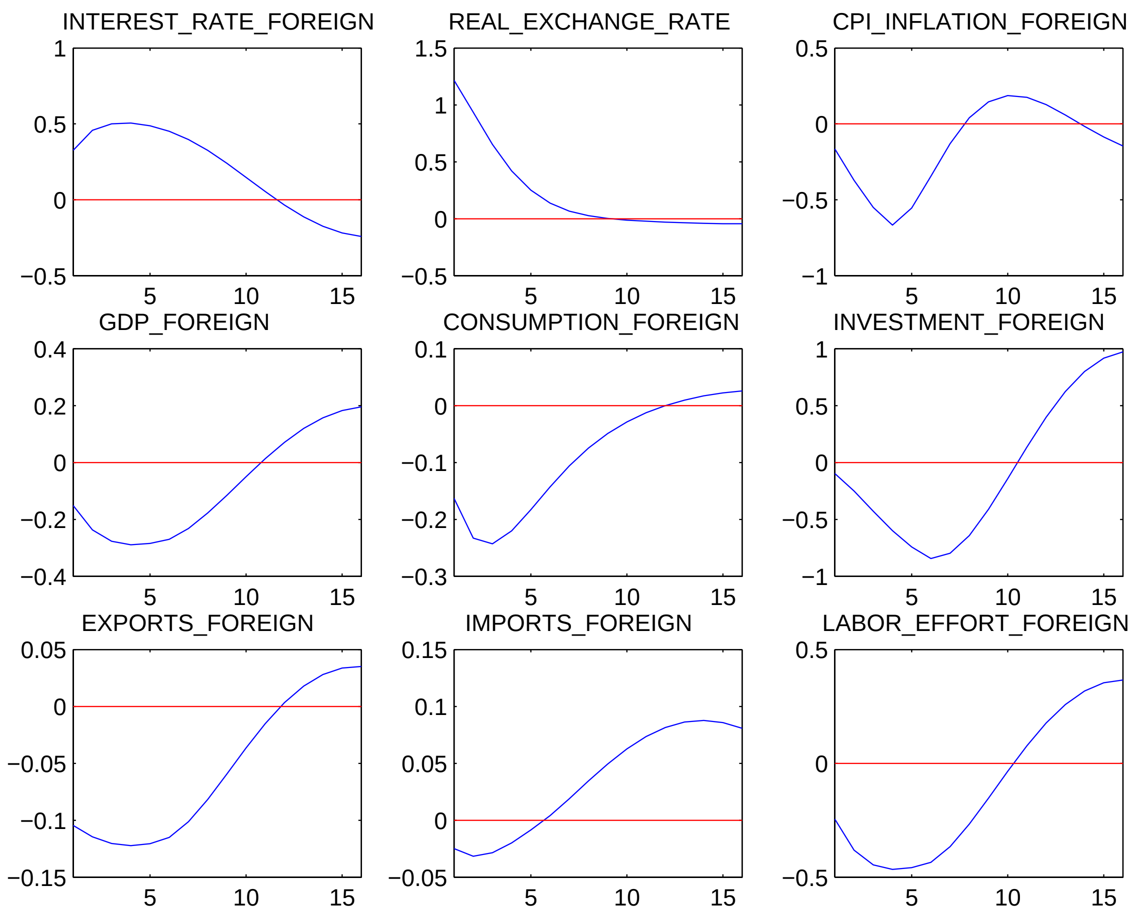

Figure 5: Home Responses to an Interest Rate Hike in the Home Economy

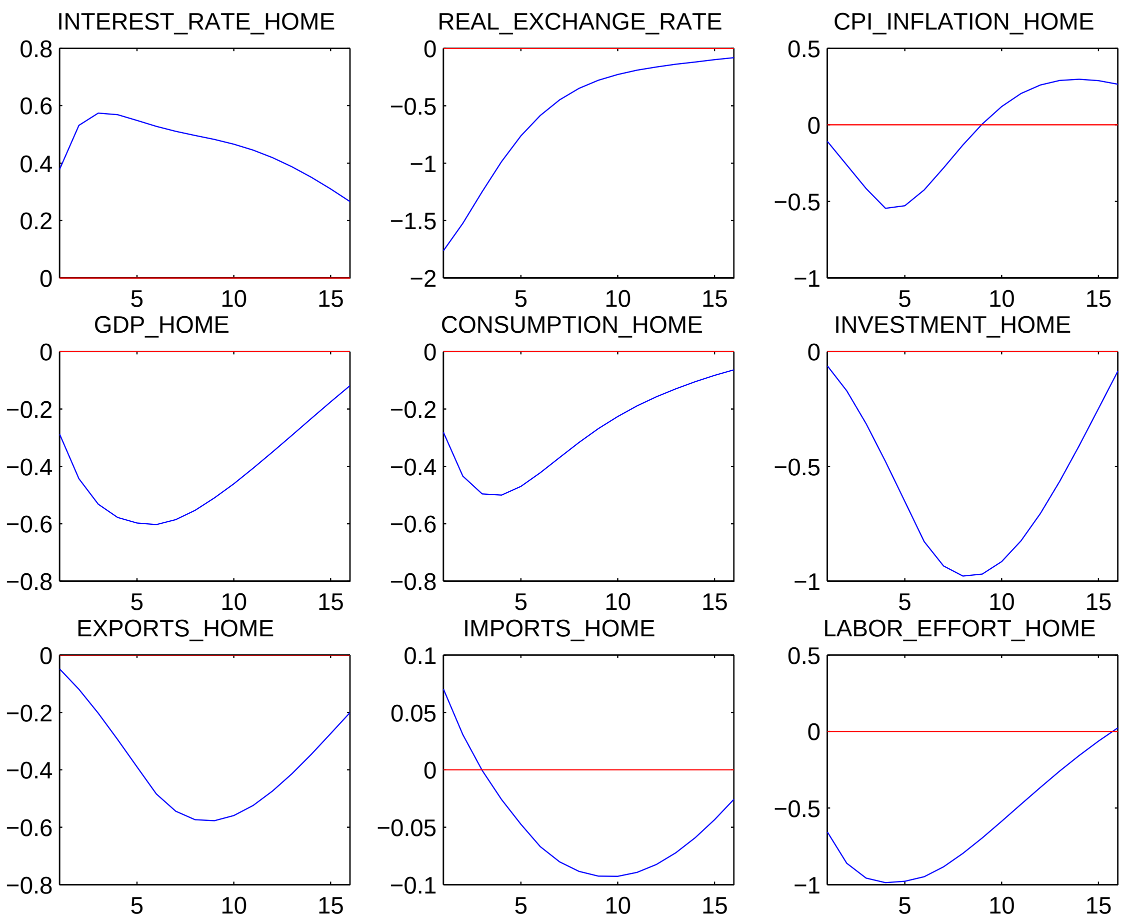

Table 1: Key Behaviorial Parameters

	Home	Foreign
Size α	0.05	0.95
Discount Rate β	0.99	0.99
Depreciation Rate on Capital Δ	0.025	0.025
Habit Persistence Parameters ν	0.55	0.55
Intertemporal EOS: σ	0.80	0.80
EOS: Final Goods Bundle θ_c	5.00	5.00
EOS: Final Goods Bundle θ_{cd}	5.00	5.00
EOS: Final Goods Bundle θ_{od}	5.00	5.00
EOS: Intermediates θ_d	5.00	5.00
EOS: Domestic Final Output θ_{dd}	5.00	5.00
EOS: Labor η	5.00	5.00

Table 2: Determinants of Per Capita Income

	Home	Foreign
Labor Effort	0.33	0.33
Aggregate Productivity (x)	0.50	1.00
TT	2.50	2.50
KINK	0.05	0.05
Trading Costs	0.34	0.34
Per Capita Consumption	1.47	2.68

Table 3: Steady-State Flows

Percent of Nominal GDP		
	Home	Foreign
Exports:	*30.0*	*1.5*
...Intermediate Inputs	*15.0*	*0.8*
...Final Consumption Goods	*7.5*	*0.4*
...Final Investment Goods	*7.5*	*0.4*
Imports:	*30.0*	*1.5*
...Intermediate Inputs	*15.0*	*0.8*
...Final Consumption Goods	*7.5*	*0.4*
...Final Investment Goods	*7.5*	*0.4*