# Endogenous Tax Evasion and Reserve Requirements: A Comparative Study in the Context of European Economies

Rangan Gupta *

November 12, 2004


## Abstract

Given that data indicates several countries with same, or nearly same, degree of tax evasion but widely different levels of reserve requirements, this paper analyzes the relationship between the "optimal" degree of tax evasion and mandatory cash reserve requirements required to be held by banks using a simple overlapping generations framework. Proceeding on the initial premises that the above observation may be a fallout of the possibilities of multiple levels of tax evasion given the reserve requirements and other policy variables, or that the optimal degree of tax evasion may be completely unaffected by the movements in reserve requirements, we find the latter to be true. The model also suggests the following: (i) An economy with a less corrupt structure will have a higher steady-state of value of reported income; (ii) Increases in the penalty rates of


*This is a revised version of the sixth chapter of my dissertation at the University of Connecticut. I am particularly grateful to my advisors Christian Zimmermann and Dhammika Dharmapala for many helpful comments and discussions. All remaining errors are mine. Email: Rangan.2.Gupta@huskymail.uconn.edu

evading taxes would induce consumers to report greater fraction of their income, while increases in the income-tax rates would cause them to evade greater fraction of their income, and ; (iii) The model does not vindicate the popular belief in the literature that, countries with lower percentage of reported income tend to have higher reserve requirements.

*Journal of Economic Literature* Classification: E26, E52.

Keywords: Reserve requirements; Tax evasion.

# 1 Introduction

This paper analyzes the relationship between the "optimal" degree of tax evasion and mandatory cash reserve requirements required to be held by banks, for eight European economies, using a simple overlapping generations framework.[1] More precisely, the analysis tries to provide a microeconomic foundation to the process of tax evasion given the policy decisions of the social planner. The motivation for such an analysis is simply derived from the observed fact in the data that, there are several countries with same, or nearly same, degree of tax evasion but widely different levels of reserve requirements. Strictly speaking, the paper tries to provide an explanation to this observation, and to the best of our knowledge is first in such an attempt. In addition to this the paper also derives the optimal values of the policy variables given the degree of tax evasion.

The above observation may be a fallout of any two of the following possibilities

(i) There may be multiple levels of tax evasion given the reserve requirements and other policy variables;

(ii) The optimal degree of tax evasion may be completely unaffected by the movements in reserve requirements.

We look into eight European economies and Table 1 compares the level of tax evasion, based on the average size of the underground economy in 1997-98, derived from Schneider and Klinglmair (2004), and the average reserve requirements over the period of 1980-1998.[2] The second column of the table reports the size of the evasion parameter. The value of the evasion parameter lies between 0.115 (U.K) and 0.225 (Greece), which implies that for Greece 22.5 percent of the taxes are evaded and for that of U.K. the value is 11.5 percent. As one can see that there are countries

[1]The analysis is general and can be applied to any country where reserve requirements are used as a monetary policy tool.

[2]For details of the calculations of the degree of tax evasion, see the calibration section of Chapter 2.

with identical (Spain and Portugal, and France and Germany), or near identical (Greece and Italy, and Spain, Portugal and Belgium) degrees of tax evasion, but different reserve requirements. The table clearly vindicates our claim made above and provides us with the motivation of the paper.

**Table 1: Tax Evasion and Reserve Requirements (1980-98)**

| | Evasion Parameter | Reserve-Deposit Ratio |
|---|---|---|
| Spain | 0.190 | 14.10 |
| Italy | 0.214 | 13.70 |
| Greece | 0.225 | 23.50 |
| Portugal | 0.190 | 19.80 |
| Belgium | 0.184 | 1.0 |
| France | 0.130 | 2.0 |
| Germany | 0.130 | 6.0 |
| UK | 0.115 | 2.0 |

Sources: (i) IFS – IMF International Financial Statistics.

(ii) Schneider and Klinglmair (2004).

Notes: (i) Section 6 of Chapter 2.

(ii)Values of reserve-deposit ratio are in percentages.

The paper incorporates endogenous tax evasion in a standard general equilibrium model of overlapping generations. There are two primary assets in the model storage (capital) and fiat money. Storage dominates money in rate of return. An intermediary exists to provide a rudimentary pooling function, accepting deposits to finance the investment needs of the firms, but are subjected to mandatory cash-reserve requirements. There is also an infinitely lived government with two wings: a treasury which finances expenditure by taxing income and setting penalty for tax evasion when caught; and the central bank, which controls the growth rate of the nominal stock of money and the reserve requirements. In such an environment we deduce the optimal degree of tax evasion, derived from the consumer optimization problem, as function of the parameters and policy variables of the model. The paper is organized as follows: Section 2 lays out the economic environment; Section 3, 4, 5 and 6 respectively, are devoted in defining the monetary competitive equilibrium, discussing the process of calibration, analyzing the behavior of the optimal degree of tax evasion corresponding to movements in the reserve-deposit ratio and deriving the optimal values of the policy variables, given the degree of tax evasion. Section 7 concludes and lays out the areas of further research.

## 2 Economic Environment

Time is divided into discrete segments, and is indexed by $t = 1, 2,......$ There are four theaters of economic activities: (i) each two-period lived overlapping generations household (consumer/worker) is endowed with one unit of labor when young, but the agent retires when old. The labor endowment is supplied inelastically to earn wage income, a part of the tax-liability is evaded, with evasion being determined endogenously to maximize utility, and the rest is deposited into banks for future consumption; (ii) each infinitely lived producer is endowed with a production technology to manufacture the single final good, using the inelastically supplied labor, physical capital and credit facilitated by the financial intermediaries; (iii) the banks simply converts one period deposit contracts into loans, after meeting the cash reserve requirements. No resources are assumed to be spent in running the banks, and; (iv) there is an infinitely lived government which meets its expenditure by taxing income, setting penalty for tax evasion when caught, and controlling the inflation tax instruments – the money growth rate and the reserve requirements. There is a continuum of each type of economic agents with unit mass.

The sequence of events can be outlined as follows: When young a household works receives pre-paid wages, evades a part of the tax burden and deposits the rest into banks. A bank, after meeting the reserve requirement, provides a loan to a goods producer, which subsequently manufactures the final good and returns the loan with interests. Finally, the banks pay back the deposits with interests to households at the end of the first period and the latter consumes in the second period.

### 2.1 Consumers

Given that the consumers possess an unit of time endowment which is supplied inelastically, and consumes only when old, formally the problem of the consumer can be described as follows: The utility of a consumer born at $t$ depends on real consumption, $c_{t+1}$, implying that the consumer consumes only when old. The assumptions make computations tractable and is not a bad approximation of the real world.[3] All consumers have the same preferences so that there exists a representative consumer in each generation. The utility function of a consumer born at time $t$ can be written as follows:

$$U_t = u(c_{t+1}) \tag{1}$$

where $U$ is twice differentiable; moreover $u' > 0$ and $u'' < 0$ and $u'(0) = \infty$. The above utility function is maximized subject to the following constraints:

$$\begin{aligned} D_t &\leq q[(1-\beta\tau_t)p_t w_t - \eta(1-\beta)^2 p_t w_t] + (1-q)[(1-\beta\tau_t)p_t w_t - \theta_t\tau_t(1-\beta)p_t w_t - \eta(1-\beta)^2 p_t w_t] && (2) \\ p_{t+1}c_{t+1} &\leq (1+i_{Dt+1})D_t && (3) \end{aligned}$$

where equation (2)[4] is the feasibility (first-period) budget constraint and equation (3) denotes the second period budget constraint for the consumer. $p_t$ ($p_{t+1}$) denotes the money price of the final

[3]See Hall (1988).

[4]This equation implicitly assumes the existence of some sort of an insurance mechanism that always ensures the consumer a certain amount of deposits $d_t$, in our case. Alternatively, we could have assumed the consumer to be

good at $t$ $(t+1)$; $D_t$ is the per-capita nominal deposits; $1-q$ is the probability of getting caught when evading tax; $\beta$ is the fraction of tax paid; $\tau_t$ is the income tax rate at $t$; $\theta_t$ is the penalty imposed, when audited and caught, at $t$; $w_t$ is the real wage at $t$, $\eta > 0$, is a cost parameter, and; $i_{Dt+1}$ is the nominal interest rate received on the deposits at $t+1$.

The constraints can be explained as follows: For the potential evader, there are (ex-ante) two possible situations: "success" (i.e., getting away with evasion) and "failure" (i.e., getting discovered and being convicted). If the consumer is found guilty of concealing an amount of income $(1-\beta)p_t w_t$, then he has to pay the amount of the evaded tax liability, $(1-\beta)\tau_t p_t w_t$ and a proportional fine at a rate of $\theta_t > 1$. Notice we have assumed that the household has to incur transaction costs to evade taxes. These basically involve costs of hiring lawyers to avoid/reduce tax burdens, and bribes paid to tax officials and administrators. The transaction costs are incurred in evading taxes are assumed to be increasing in both degree of tax evasion and the wage income of the household. The form $\eta$ $(1-\beta)^2$ $p_t w_t$ is consistent with our assumptions about the behavior of transaction costs. Note a higher value of $\eta$, would imply a less corrupted economy, implying that it is more difficult to evade taxes. We also endogenize the probability of getting caught, $q$, by assuming it to be an increasing function of the degree of tax evasion. $q$ takes the following quadratic form:

$$1 - q = (1-\beta)^2 \tag{4}$$

The second-period budget constraint is self-explanatory suggesting that the consumer when old consumes out of the interest income from deposits – the only source of income, given that he is retired. The household chooses $\beta$ to maximize his utility from second-period consumption subject

risk-neutral. In that case equation (2) would be the expected value of the deposits obtained. So in some sense we have an observationally equivalent formulation. Our results are, however, independent of whether the consumer is risk-averse or risk neutral, once we assume the existence of an implicit insurance scheme. Our formulation follows Chen (2003).

to the intertemporal budget constraint given as follows:

$$c_{t+1} \quad \leq \quad (1+r_{dt+1})[(1-\beta\tau_t)-\theta_t\tau_t(1-\beta)^3-\eta(1-\beta)^2]w_t \tag{5}$$

where $r_{dt+1}$ is the real interest rate on deposits at period $t+1$. Note $(1+r_{dt+1}) = \frac{1+i_{Dt+1}}{1+\pi_{t+1}}$, where $1+\pi_{t+1} = \frac{p_{t+1}}{p_t}$.

Realizing that both the real interest rate on deposits and the wages would depend on the degree of tax evasion, the first order condition for the consumer is given as follows:

$$\frac{dU}{d\beta} = 0 = u'(c_{t+1})\left[\{\frac{d}{d\beta}(1+r_{dt+1})\}d_t + (1+r_{dt+1})\frac{dd_t}{d\beta}\right] \tag{6}$$

The optimal value of $\beta$ at steady-state is determined below after the equilibrium conditions are imposed and the steady state value of the capital stock is determined.

## 2.2 Financial Intermediaries

At the start of each period the financial intermediaries accept deposits and make their portfolio decision (that is, loans and cash reserves choices) with a goal of maximizing profits. At the end of the period they receive their interest income from the loans made and meets the interest obligations on the deposits. Note the intermediaries are constrained by legal requirements on the choice of their portfolio (that is, reserve requirements), as well as by feasibility. Given such a structure, the intermediaries obtains the optimal choice for $L_t$ by solving the following problem:

$$\max_{L,D} \pi_b \quad = \quad i_{Lt}L_t - i_{Dt}D_t \tag{7}$$

$$s.t. \quad : \quad \gamma_t D_t + L_t \leqslant D_t \tag{8}$$

where $\pi_b$ is the profit function for the financial intermediary, and $M_t \geqslant \gamma_t D_t$ defines the legal reserve requirement. $M_t$ is the cash reserves held by the bank; $L_t$ is the loans; $i_{Lt}$ is the interest rate on loans, and; $\gamma_t$ is the reserve requirement ratio. The reserve requirement ratio is the ratio of required reserves (which must be held in form of currency) to deposits.

To gain some economic intuition of the role of reserve requirements, let us consider the solution of the problem for a typical intermediary. Free entry, drives profits to zero and we have

$$i_{Lt}(1-\gamma_t) - i_{Dt} = 0 \tag{9}$$

Simplifying, in equilibrium, the following condition must hold

$$i_{Lt} = \frac{i_{Dt}}{1-\gamma_t} \tag{10}$$

Reserve requirements thus tend to induce a wedge between the interest rate on savings and lending rates for the financial intermediary. Note, many countries impose a variety of obstacles to proper functioning of the intermediation system. Examples of such impediments include portfolio restrictions, taxes, and requirements that loans to favored sectors be made at interest rates below the market level (popularly called, Priority Sector Lending). To some extent these restrictions can be viewed a wedge between the interest rates goods producing firms pay banks and the rate banks ultimately receive on their loans. This can be easily incorporated in our model, by slightly reformulating the problem of the financial intermediaries.[5] Given that in most of our model economies, interest rates were deregulated in the mid-1980, the reasons for including this wedge factor in our analysis was not very compelling.[6] Moreover, a tax on the interst earnings of the banks introduces a similar type of wedge between the lending and borrowing rates as is obtained through the imposition of reserve requirements.

[5]See Espinosa(1995), Espinosa and Yip (1996) and Chari, Jones and Manuelli (1996) for an explicit theoretical treatment of the above argument.

[6]See Bacchetta and Caminal (1992), for a detailed survey.

## 2.3 Firms

Each firm produces a single final good using a standard neoclassical production function $F(k_t, n_t)$, with $k_t$ and $n_t$, respectively denoting the capital and labor input at time $t$. The production technology is assumed to take the Cobb-Douglas form:

$$Y = F(k, n) = k^{\alpha} n^{(1-\alpha)} \tag{11}$$

where $0 < \alpha \ ((1-\alpha)) < 1$, is the elasticity of output with respect to capital (labor). At date $t$ the final good can either be consumed or stored. Next we assume that producers are capable of converting bank loans $L_t$ into fixed capital formation such that $p_t i_{kt} = L_t$, where $i_t$ denotes the investment in physical capital. Notice that the production transformation schedule is linear so that the same technology applies to both capital formation and the production of consumption good and hence both investment and consumption good sell for the same price $p$. Moreover, we follow Diamond and Yellin (1990) and Chen, Chiang and Wang (2000) in assuming that the goods producer is a residual claimer, i.e., the producer uses up the unsold consumption good in a way which is consistent with lifetime value maximization of the firms. such an assumption regarding ownership avoids the "unnecessary" Arrow-Debreu redistribution from firms to consumers and simultaneously retains the general equilibrium structure.

The representative firm at any point of time $t$ maximizes the discounted stream of profit flows subject to the capital evolution and loan constraint. Formally, the problem of the firm can be outlined as follows

$$\max_{k_{t+1}, n_t} \sum_{i=0}^{\infty} \rho^i [p_t k_t^{\alpha} n_t^{(1-\alpha)} - p_t w_t n_t - (1 + i_{Lt}) L_t] \tag{12}$$

$$k_{t+1} \leqslant (1-\delta_k) k_t + i_{kt} \tag{13}$$

$$p_t i_{kt} = L_t \tag{14}$$

where $\rho$ is the firm owners (constant) discount factor, and $\delta_k$ is the (constant) rate of capital depreciation. The firm solves the above problem to determine the demand for labor and investment.

The firm's problem can be written in the following recursive formulation:

$$V(k_t) = \max_{n,k'}[p_t k_t^{\alpha} n_t^{(1-\alpha)} - p_t w_t n_t - p_t(1+i_{Lt})(k_{t+1} - (1-\delta_k)k_t)] + \rho V(k_{t+1}) \tag{15}$$

The upshot of the above dynamic programming problem are the following first order conditions.

$$k_{t+1} \quad : \quad (1+i_{Lt})p_t = \rho V'(k_{t+1}) \tag{16}$$

$$(n_t) \quad : \quad (1-\alpha)\left(\frac{k_t}{n_t}\right)^{\alpha} = w_t \tag{17}$$

And the following envelope condition.

$$V'(k_t) = p_t[\alpha\left(\frac{n_t}{k_t}\right)^{(1-\alpha)} + (1+i_{Lt})(1-\delta_k)] \tag{18}$$

Optimization, leads to the following efficiency condition, besides (16), for the production firm.

$$(1+i_{Lt}) = \rho(1+\pi_{t+1})[\alpha\left(\frac{n_{t+1}}{k_{t+1}}\right)^{(1-\alpha)} + (1+i_{Lt+1})(1-\delta_k)] \tag{19}$$

Equation (19) provides the condition for the optimal investment decision of the firm. The firm compares the cost of increasing investment in the current period with the future stream of benefit generated from the extra capital invested in the current period. And equation (17) simply states that the firm hires labor up to the point where the marginal product of labor equates the real wage.

### 2.4 Government

As discussed above we have an infinitely lived government with two wings: the treasury and the central bank. The government finances its expenditure $p_t g_t$ through taxation, penalty on consumers when caught evading and the inflation tax (seigniorage). Formally the government budget constraint can be written as follows:

$$p_t g_t = \beta \tau_t p_t w_t + (1-q)\theta_t(1-\beta)\tau_t p_t w_t + (M_t - M_{t-1}) \tag{20}$$

Note throughout the analysis we will assume that money growth is dictated by a rule, $M_t = (1+\mu_t)M_{t-1}$, where $\mu$ is the rate of growth of money. Using, $M_t = \gamma_t D_t$, the government budget constraint in real terms can be rewritten as

$$g_t = [\beta + (1-q)\theta_t(1-\beta)]\tau_t w_t + \gamma_t d_t(1 - \frac{1}{1+\mu_t}) \tag{21}$$

where $d_t = (\frac{D_t}{p_t})$ is the size of deposits in real terms. Skinner and Slemrod (1985) points out that the administrative costs of penalties is usually quite minor, and, hence, for simplicity we ignore them from the government budget constraint.

# 3 Equilibrium

A valid perfect-foresight, competitive equilibrium for this economy is a sequence of prices $\{p_t, i_{Dt}, i_{Lt}\}_{t=0}^{\infty}$, allocations $\{c_t, n_t, i_{kt}\}_{t=0}^{\infty}$, stocks of financial assets $\{m_t, d_t\}_{t=0}^{\infty}$, and policy variables $\{\gamma_t, \mu_t, \tau_t, \theta_t, g_t\}_{t=0}^{\infty}$ such that:

- Taking, $\tau_t$, $g_t$, $\theta_t$, $\gamma_t$, $\mu_t$, $p_t$, the consumer optimally chooses $\beta$ such that (6) holds;
- Banks maximize profits, taking, $i_{Lt}$, $i_{Dt}$, and $\gamma_t$ as given and such that (9) holds;
- The real allocations solve the firm's date–$t$ profit maximization problem, (12), given prices and policy variables.
- The money market equilibrium conditions: $m_t = \gamma_t d_t$ is satisfied for all $t \geqslant 0$.

- The loanable funds market equilibrium condition: $p_t i_{kt} = (1-\gamma_t)D_t$ where the total supply of loans $L_t = (1-\gamma_t)D_t$ is satisfied for all $t \geqslant 0$.

- The goods market equilibrium condition require: $c_t + i_{kt} + g_t = k_t^{\alpha} n_t^{(1-\alpha)}$ is satisfied for all $t \geqslant 0$.

- The labor market equilibrium condition: $(n_t)^d$ =1 for all $t \geqslant 0$.

- The government budget is balanced on a period-by-period basis.

- $d_t$, $(1+r_{dt})$ and $p_t$ must be positive at all dates and $1 + i_{Lt} > 1$.

# 4 Optimal Degree of Tax Evasion

Using the equilibrium conditions, realizing that there is no growth in the model and allowing the government to follow time invariant policy rules, which means the reserve–ratio, $\gamma_t$, the money supply growth–rate, $\mu_t$, the tax–rate, $\tau_t$, and the penalty, $\theta_t$, are constant over time, we have the following set of equations:

$$1 + r_d = (1+r_l)(1-\gamma) + \frac{\gamma}{1+\pi} \tag{22}$$

$$w = (1-\alpha)k^{\alpha} \tag{23}$$

$$(1+r_l) = \frac{\rho\alpha k^{(\alpha-1)}}{1-\rho(1+\pi)(1-\delta_k)} \tag{24}$$

$$(1-\gamma)[(1-\beta\tau) - \theta\tau(1-\beta)^3 - \eta(1-\beta)^2]w = \delta_k k \tag{25}$$

where $r_l$ is the real interest rate on loans. Using (22) to (25) we can solve $k$ in terms of the policy variables, production parameters of the model and $\beta$ and is given by the following equation:

$$k = \left(\frac{(1-\gamma)(1-\alpha)[(1-\beta\tau) - \theta\tau(1-\beta)^3 - \eta(1-\beta)^2]}{\delta}\right)^{\frac{1}{1-\alpha}} \tag{26}$$

Realizing that $\mu = \pi$, from the money market equilibrium condition, the gross real interest rate on deposits $(1 + r_d)$, and real wage $w$ are given by the following expressions:

$$(1 + r_d) = \left(\frac{\rho\alpha\delta_k}{(1-\alpha)(1-\rho(1+\mu)(1-\delta_k))}\right)\left(\frac{1}{[(1-\beta\tau)-\theta\tau(1-\beta)^3-\eta(1-\beta)^2]}\right) + \frac{\gamma}{1+\mu} \quad (27)$$

$$w = (1-\alpha)\left\{(1-\alpha)(1-\gamma)\frac{[(1-\beta\tau)-\theta\tau(1-\beta)^3-\eta(1-\beta)^2]}{\delta_k}\right\}^{\frac{\alpha}{(1-\alpha)}} \quad (28)$$

where $1 + r_l = \left(\frac{\rho\alpha\delta_k}{(1-\gamma)(1-\alpha)(1-\rho(1+\mu)(1-\delta_k))}\right)\left(\frac{1}{[(1-\beta\tau)-\theta\tau(1-\beta)^3-\eta(1-\beta)^2]}\right)$

Using (2) evaluated at steady-state, (27) and (28) we can rewrite equation (6) as follows:

$$\frac{d}{d\beta}\begin{pmatrix} \left\{\frac{\rho\alpha\delta_k}{(1-\alpha)(1-\rho(1+\mu)(1-\delta_k))} + \frac{\gamma}{1+\mu}[(1-\beta\tau)-\theta\tau(1-\beta)^3-\eta(1-\beta)^2]\right\} \\ (1-\alpha)\left\{(1-\alpha)(1-\gamma)\frac{[(1-\beta\tau)-\theta\tau(1-\beta)^3-\eta(1-\beta)^2]}{\delta_k}\right\}^{\frac{\alpha}{(1-\alpha)}} \end{pmatrix} = 0 \quad (29)$$

given that $u'(c_{t+1}) > 0$. Once we derive the optimal value of $\beta$ by solving (29) in terms of the parameters and policy variables we can obtain the reduced form solution to the other endogenous variables in the model.

The derivative of (29) yields a non-linear equation for $\beta$ and needs to be solved in essentially an non-algebraic fashion[7] and hence the choice of parameter values become essential. The following section discuses the process of calibration.

# 5 Calibration

In this section we attribute values to the parameters of our benchmark model using a combination of figures from previous studies and facts about the economic experience for our sample economies between 1980 and 1998.

[7] We can linearize the solution to (29) around the steady-state values of $\beta$, derived from Table 1, to obtain a reduced form solution or plot the implicit function derived for values of $\gamma$ between 0 to 0.99, given the other parameter values. We take the latter approach, since we are not interested in explicitly solving for the other endogenous variables of the model, but study mainly the behavior of the degree of tax evasion in response to change in the reserve-deposit ratio. Besides, the loss of information associated with the linearization is unwarranted since it has bearing on our results.

We follow the standard real business cycle literature in using steady–state conditions to establish parameter values observed in the data. Some parameters are calibrated using country–specific data, while others, without sufficient country–specific evidence over a long period, correspond to prevailing values from the literature. This section reveals the general procedures used. The calibrated parameters are reported in Tables 2. Note unless otherwise stated, the source for all data is the IMF – International Financial Statistics (IFS).

A first set of parameter values is given by numbers usually found in the literature. These are:

- $(1-\alpha)$: since the production function is Cobb-Douglas, this corresponds to the share of labor in income. $(1-\alpha)$ for Spain, Italy, Greece, Belgium, France, Germany and U.K. is derived from Zimmermann (1997) and the value for Portugal is obtained from Correia, Neves and Rebelo (1995). Note for Belgium we use the world average of labor share reported in Zimmermann (1997). The values lie between 53.0 percent (Portugal) and 63.5 percent (Germany);

- $\delta_k$: the depreciation rate of physical capital for Spain, Italy, Greece, Belgium, France, Germany and U.K. is derived from Zimmermann (1997) and the value for Portugal is obtained from Correia, Neves and Rebelo (1995). Note for Belgium we use the world average of capital-output ratio reported in Zimmermann (1997) to arrive at the depreciation rate. The values lie between 3.2 percent (Greece) and 7.6 percent (Germany);

- $UGE$: The parameter measures the size of the underground economy as a percentage of GDP. The values are obtained from Schneider and Klinglmair (2004) and lies between 13.0 percent (U.K) to 29.0 percent (Greece).

- $\theta$: the penalty imposed by the government when the consumer is caught evading is obtained from Chen (2003) and is set to 1.5 for all countries.

- $\eta$: the transaction cost parameter is obtained from Chen (2003) and is set to 0.15.

A second set of parameters is determined individually for each country. Here, we use averages over the whole sample period to find values that do not depend on the current business cycle. These parameters are:

- $\pi$: the annual rate of inflation lies between 2.29 percent (Germany) and 15.16 percent (Greece);
- $\gamma$: the annual reserve–deposit ratio lies between 1 percent (Belgium) and 23.5 percent (Greece);
- $\tau$: the tax rate, calculated as the ratio of tax–receipts to GDP, lies between 22.74 percent (Greece) and 42.10 percent (Belgium);
- $i_{Lt}$: the nominal interest rate on loans lies between 10.01 percent (France) and 22.96 percent (Greece);

The following set of parameters are calibrated from the steady state equations of the model:

- $\mu$: the money growth rate is set equal to the rate of inflation, given the money market equilibrium condition. The annual rate of money growth rate hence, lies between 2.29 percent (Germany) and 15.16 percent (Greece);
- $\rho$: the discount factor of the firms is solved to ensure that equation (24) holds. The value ranges between 0.87 (Greece) to 0.99 (Germany).
- $\beta$: the fraction of reported income is determined by the method outlined in the calibration section of chapter 3 and is not repeated here. We consider this exogenously evaluated value of the reported income parameter as the steady-state value. Note the value hinges critically on the size of the underground economy as a percentage of the GDP. The value of $\beta$ lies between

0.775 (Greece) and 0.885 (U.K), implying that 22.5 percent of the taxes are evaded in Greece and the figure in U.K corresponds to a tx evasion of 11.5 percent.

- $\phi$: the ratio of government expenditure to GDP is obtained using equations (2), (4) and (21). The country specific value lies between 12.27 percent (Greece) and 21.49 percent (Belgium). The resulting values for each of the economies are in fact pretty close to what is observed the data. Except for Germany, the size of the government for all other economies are underestimates. For country-specific sizes of government observed in the data, see Table 4 in Chapter 2.

**Table 2: Calibration of Parameters**

| | $(1-\alpha)$ | $\delta_k$ | $UGE$ | $\pi=\mu$ | $\gamma$ | $\tau$ | $i_L$ | $\rho$ | $\beta$ | $\phi$ |
|---|---|---|---|---|---|---|---|---|---|---|
| Spain | 0.627 | 0.05 | 23.1 | 7.52 | 14.1 | 25.53 | 12.89 | 0.94 | 0.810 | 13.62 |
| Italy | 0.617 | 0.052 | 27.3 | 8.58 | 13.7 | 36.25 | 15.02 | 0.93 | 0.786 | 18.38 |
| Greece | 0.598 | 0.032 | 29.0 | 15.16 | 23.5 | 22.74 | 22.96 | 0.87 | 0.775 | 12.27 |
| Portugal | 0.530 | 0.05 | 23.1 | 13.04 | 19.8 | 27.73 | 19.09 | 0.88 | 0.810 | 12.98 |
| Belgium | 0.618 | 0.046 | 22.5 | 3.59 | 1.0 | 42.10 | 10.71 | 0.97 | 0.816 | 21.49 |
| France | 0.599 | 0.033 | 14.9 | 4.54 | 2.0 | 37.30 | 10.01 | 0.96 | 0.870 | 19.55 |
| Germany | 0.635 | 0.076 | 14.9 | 2.29 | 6.0 | 31.80 | 10.85 | 0.99 | 0.870 | 17.70 |
| UK | 0.631 | 0.043 | 13.0 | 5.77 | 2.0 | 32.90 | 10.06 | 0.96 | 0.885 | 18.45 |

Note: Parameters defined as above.

# 6 Tax Evasion, Reserve Requirements and Tax Penalty

As suggested earlier, given that the non-linearity of the model does not allow us to solve it in an algebraic fashion, we plot the implicit function in the ($\gamma$, $\beta$) plane, obtained from equation (29), to study the behavior of the endogenously determined tax evasion and reserve requirements. Figures 1 to 8 plots the relationship between the reported income and reserve requirements for each of the eight countries, in the order of Spain, Italy, Greece, Portugal, Belgium, France, Germany and U.K. As can be seen, the reported income, and hence the degree of tax evasion, is independent of the reserve requirements, for all eight countries. Below, Table 3 reports the approximate optimal value of the reported income, $\beta$, predicted by the model. Setting the value of $\alpha$ at $\frac{1}{2}$ for Spain, allows us to solve the model algebraically. The solution yields five roots for $\beta$, but the only legitimate solution is found to be a value, 0.64, clearly independent of the reserve-requirements.[8]

Next we study the behavior of the steady-state reported income in the ($\gamma$, $\beta$) plane following a doubling of the transaction cost parameter, namely $\eta$. The comparative static exercise shows an upward shift of the reported income line, for all the eight countries. This suggests that an economy with a less corrupt structure will have a higher steady-state reported income.[9] To save space we present here the figures corresponding to Spain only. When compared to Figure 1, Figure 9 clearly vindicates the point made above. The approximate optimal value of the reported income increases by nearly 9 percent, moving from 0.64 to 0.73. Below, Table 3 reports the approximate optimal value of the reported income, $\beta$, predicted by the model, for all economies when $\eta$ is doubled. Finally, we also study the relationship between the steady-state reported income with the penalty rate when caught evading taxes, $\theta$ and then with the income tax-rate, $\tau$. Each of the eight countries were found to bear a positive relationship between steady-state reported income and penalty rates, while the relationship between steady-state reported income and tax rate is found to be negative.

[8]For Spain, the calibrated value of $\rho$, corresponding to $\alpha = \frac{1}{2}$, was found to be 0.92. Repeating the experiment for all other economies yielded a constant for the optimal value of reported income $\beta$.

[9]Note $\rho$ is the only parameter that needs to be re-calibrated when we change the value of $\eta$. However, since the value of $\rho$ was found to be extremely robust for all countries, with changes observed only in the fourth decimal place, it has not been reported in Table 2.

The model thus suggests that increases in the penalty rates of evading taxes would induce consumers to report greater fraction of their income, while increases in the income-tax rates would cause them to evade greater fraction of their income. As before, to save space we present here the figures corresponding to Spain only. Figures 10 and 11 indicates the nature of the relationship of steady-state reported income with $\theta$ and $\tau$, respectively. Though the relationships between tax-evasion with tax-rate and penalty rate seems standard, however, what is more important, is that we in fact can provide estimates to the strength of such relationships. Table 4 reports the derivative of the reported income with respect to tax rate and the penalty rate, obtained using the implicit function theorem evaluated at the steady-state value of the reported income obtained from the model and the average tax rates derived from the data. To the best of our knowledge, this is the first study that tries to quantify such effects, in a general equilibrium framework. Tax rates are found to have much stronger effects on the reported income than the penalty rates.

The results of the model thus, clearly explains the fact outlined in Table 1, about countries with similar degree of tax evasion having different reserve requirements — the reason being the fact, as suggested by the model, the reported income is found to be independent of the reserve requirements. The model suggests, that given the production parameter, the discount rate, and policy variables, economies can have similar levels of tax evasion but different levels of reserve requirements as a result of the transaction cost parameter of the model. However, to validate such a claim we require more accurate information on country-specific values of this parameter. Besides this, a couple of other observations are noteworthy: (i) When compared to Table 2, as can be seen from Table 3, the model tends to be biased downwards as far as predicting the equilibrium values of reported income are concerned, and; (ii) The model does not match the ranking of countries according to their reported income.

**Table 3: Model Prediction of $\beta$**

| | $\beta^*$ | |
|---|---|---|
| | $\eta$ =0.15 | $\eta$ =0.30 |
| Spain | 0.64 | 0.73 |
| Italy | 0.62 | 0.68 |
| Greece | 0.65 | 0.79 |
| Portugal | 0.63 | 0.72 |
| Belgium | 0.60 | 0.66 |
| France | 0.61 | 0.68 |
| Germany | 0.62 | 0.69 |
| UK | 0.62 | 0.69 |

Note: Derived from equation (29).

**Table 4: Strength of Relationship between $\beta$, $\tau$, and $\theta$; $\eta$ =0.15**

| | $\frac{\delta\beta}{\delta\tau}$ | $\frac{\delta\beta}{\delta\theta}$ |
|---|---|---|
| Spain | -0.37 | 0.09 |
| Italy | -0.23 | 0.10 |
| Greece | -0.44 | 0.08 |
| Portugal | -0.31 | 0.09 |
| Belgium | -0.15 | 0.11 |
| France | -0.20 | 0.11 |
| Germany | -0.25 | 0.10 |
| UK | -0.25 | 0.10 |

Note: Derived from equation (29).

# 7 Optimal Policy Decisions

Given the optimal value of the reported income, the model is well-equipped in determining the optimal values of the policy parameters in such an environment. The social planner maximizes the rate of return on deposits choosing $\tau$, $\gamma$, $\mu = \pi$ and $\theta$, to determine the optimal choices of the policy variables, subject to the set of inequality constraints: $0 \leq \tau \leq 1$, $0 \leq \gamma \leq 1$, $\mu \geq 0$ and, $\theta > 1$, $(1-q)\theta < 1$ and the government budget constraint evaluated at the steady state. Note the constraint $(1-q)\theta < 1$ or alternatively, $(1-\beta)^2\theta < 1$, ensures that the expected penalty rate is less than one.

**Table 4: Optimal Decision Variables ($\eta$=0.15 and 0.30)**

| Countries | $\tau^*$ | | $\mu^*$ | | $\gamma^*$ | | $\theta^*$ | |
|---|---|---|---|---|---|---|---|---|
| | $\eta = 0.15$ | $\eta = 0.30$ | $\eta = 0.15$ | $\eta = 0.30$ | $\eta = 0.15$ | $\eta = 0.30$ | $\eta = 0.15$ | $\eta = 0.30$ |
| Spain | 31.43 | 28.18 | 11.98 | 11.98 | 0.40 | 0.10 | 1.01 | 3.19 |
| Italy | 0.001 | 0.04 | 87.17 | 89.57 | 65.15 | 64.75 | 4.93 | 9.40 |
| Greece | 24.28 | 22.07 | 18.74 | 18.74 | 0.50 | 0.02 | 4.04 | 22.68 |
| Portugal | 0 | 0 | 71.82 | 72.07 | 59.82 | 59.88 | 7.30 | 13.72 |
| Belgium | 1.00 | 0.75 | 112.60 | 110.44 | 65.59 | 67.45 | 6.08 | 8.52 |
| France | 0.001 | 38.86 | 48.76 | 7.72 | 99.98 | 0.4 | 1.00 | 4.89 |
| Germany | 0.01 | 0.02 | 79.83 | 80.75 | 63.69 | 63.55 | 5.26 | 7.74 |
| UK | 45.78 | 40.34 | 8.80 | 0 | 1.0 | 0 | 1.00 | 1.00 |

Notes: All values are in percentages.

The optimal values of the policy variables, corresponding to $\eta$ =0.15 and 0.30 are reported in Table 4. Though we do not observe any specific pattern to the movements in $\tau$ and $\mu$, the penalty rate, is equal to or higher corresponding to $\eta$ =0.30, since the reported income is higher in this case than when $\eta$ = 0.15 and hence reduces the probability of being caught. The optimal value of the reserve requirement decreases for all economies except for Portugal and Belgium, when the optimal value of the reported income increases for $\eta = 0.30$, indicating a lower level of corruption, compared to $\eta = 0.15$. Moreover, the model does not vindicate the popular belief in the literature that, countries with higher tax evasion tend to have higher reserve requirements. Notice, contrary to the general results that emerge out of overlapping generations model deriving optimal values of policy parameters, the model promises high values of the optimal money growth rate but is finite.[10] Note the optimal values not only vary within countries for alternative values of the corruption parameter, but also across countries.

## 8 Conclusion and Areas of Further Research

This paper analyzes the relationship between the "optimal" degree of tax evasion and mandatory cash reserve requirements required to be held by banks, for eight European economies, using a simple overlapping generations framework. More precisely, the analysis tries to provide a microeconomic foundation to the process of tax evasion given the policy decisions of the social planner. The motivation for such an analysis is simply derived from the observed fact in the data that, there are several countries with same, or nearly same, degree of tax evasion but widely different levels of reserve requirements. Strictly speaking, the paper tries to provide an explanation to this observation. As an aside we also derive the optimal values of the policy variables given the degree of tax evasion.

We proceeded based on the initial premises that the above observation may be a fallout of any two of the following possibilities

(i) There may be multiple levels of tax evasion given the reserve requirements and other policy

[10] For a detailed discussion, see Freeman (1987) and Bhattacharya and Haslag (2001).

variables;

(ii) The optimal degree of tax evasion may be completely unaffected by the movements in reserve requirements. As our results indicate it is the latter. The steady-state reported income, and hence the degree of tax evasion, is found to be independent of the reserve requirements, for all of the eight countries. The model also suggests, the following somewhat obvious facts we tend to believe about tax evasion: (i) An economy with a lesser corrupted structure will have a higher steady-state of value of reported income; (ii) Increases in the penalty rates of evading taxes would induce consumers to report greater fraction of their income, while increases in the income-tax rates would cause them to evade greater fraction of their income, and ; (iii) The model does not vindicate the popular belief in the literature that, countries with lower percentage of reported income tend to have higher reserve requirements.

In summary, the model suggests, that given the production parameter, the discount rate, and policy variables, economies can have similar levels of tax evasion but different levels of reserve requirements as a result of the transaction cost and probability parameters of the model. To validate such a claim we, however, require more accurate information on country-specific values of this parameter. Hence future research needs to be oriented along these lines. **Selected References**

[1] Bencivenga, Valerie R., and Smith, Bruce D. (1992), *"Deficits, Inflation, and the Banking System in Developing Countries: The Optimal Degree of Financial Repression"*, Oxford Economic Papers, (44), 767–790.

[2] Bhattacharya, Joydeep., and Haslag, Joseph, H. (2001), *"On the Use of Inflation Tax When Nondistortionary Taxes Are Available"*, Review of Economic Dynamics, (4), 823–841.

[3] ———————————. (2003), *"Is Reserve-Ratio Arithmetic More Pleasant"*,

Economics, (70), 471–491.

[4] Chen, Been-Lon. (2003), *"Tax Evasion in a Model of Endogenous Growth"*, Review of Economic Dynamics, (6), 381–403.

[5] ————., Chiang, Yeong-Yuh, and Wang, Ping. (2000), *"Credit Market Imperfections, Financial Activity and Economic Growth"*, Working Paper, 00-W20, Vanderbilt University.

[6] Diamond, Peter., and Yellin, Joel. (1990), *"Inventories and Money Holdings in a Search Economy"*, Econometrica, 58 (4), 929–950.

[7] Freeman, Scott. (1987), *"Reserve Requirements and Optimal Seigniorage"*, Journal of Monetary Economics, (19), 307–314.

[8] Schneider, Friedrich., and Klinglmair, Robert. (2004), *"Shadow Economies around the World: What Do We Know"*, IZA Discussion Papers, 1043.

[9] Hall, Robert, E. (1988), *"Intertemporal Substitution in Consumption"*, Journal of Political Economy, (96), 339–357.

[10] Skinner, J., and Slemrod, J. (1985), *"An Economic Perspective of Tax Evasion"*, National Tax Journal, (38), 345–353.

[11] Zimmermann, Christian. (1997), *"Technology Innovations and the Volatility of Output: An International Perspective"*, CREFE Working Paper, 34.

Figure 1: Reported Income and Reserve Requirements for Spain, $\eta = 0.15$

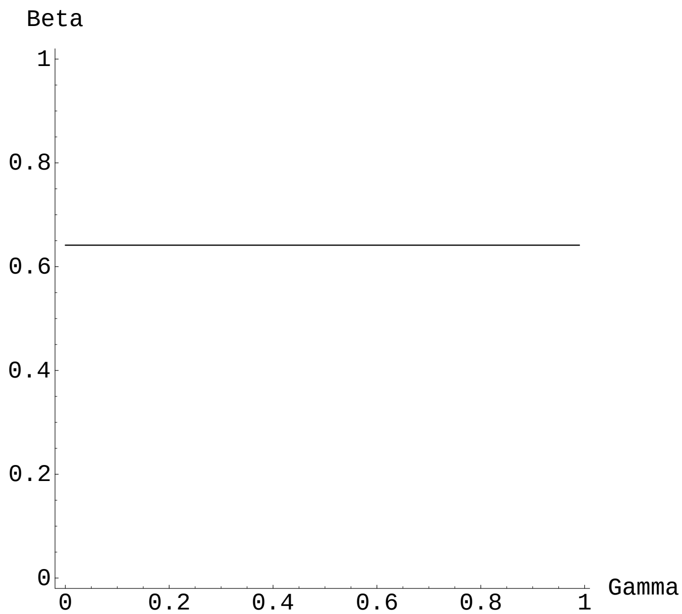


Figure 2: Reported Income and Reserve Requirements for Italy, $\eta = 0.15$

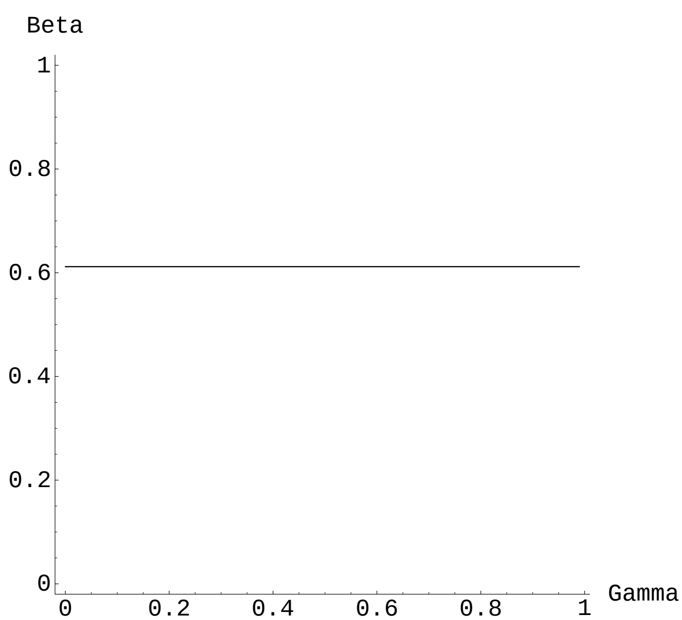

Figure 3: Reported Income and Reserve requirements for Greece, $\eta = 0.15$

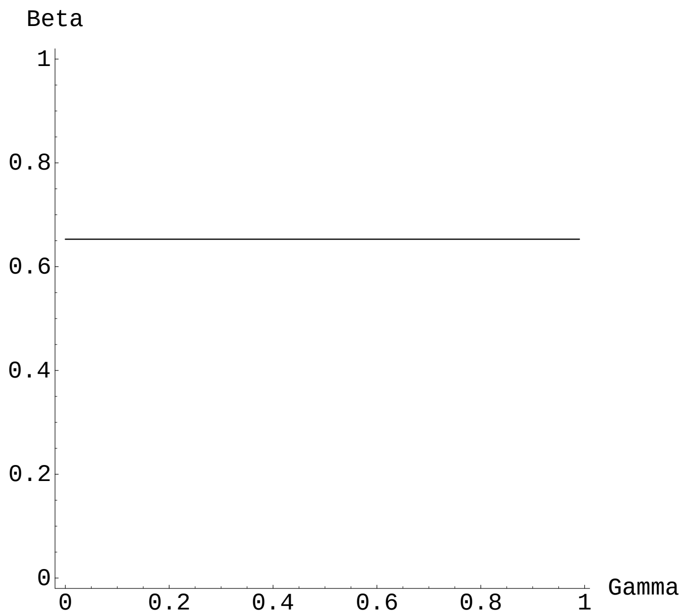


Figure 4: Reported Income and Reserve requirements for Portugal, $\eta = 0.15$

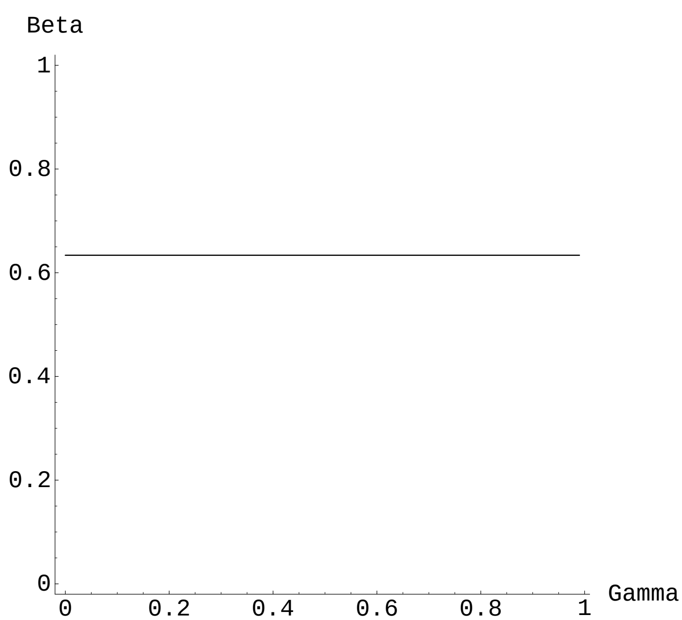

Figure 5: Reported Income and Reserve requirements for Belgium , $\eta = 0.15$

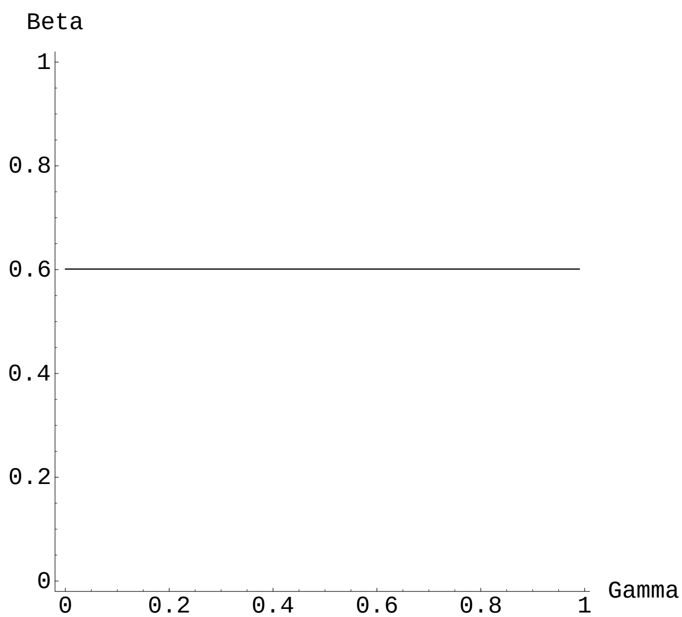


Figure 6: Reported Income and Reserve Requirements for France, $\eta = 0.15$

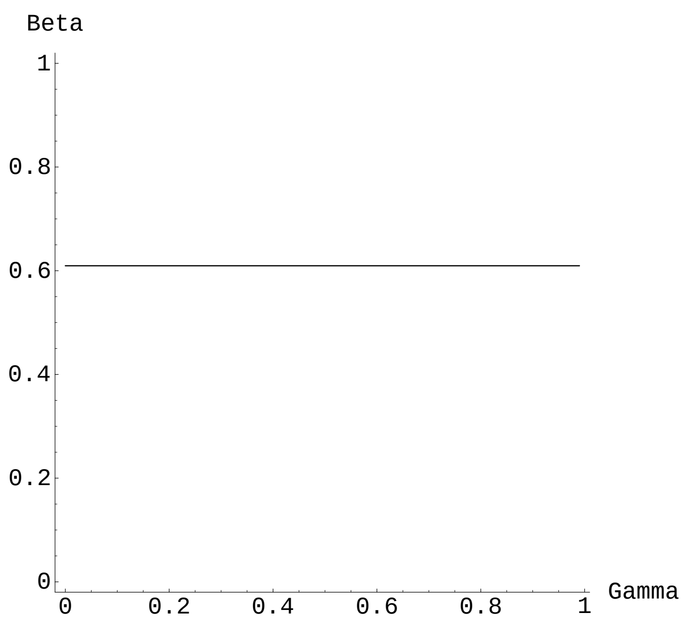

Figure 7: Reported Income and Reserve Requirements for Germany, $\eta = 0.15$

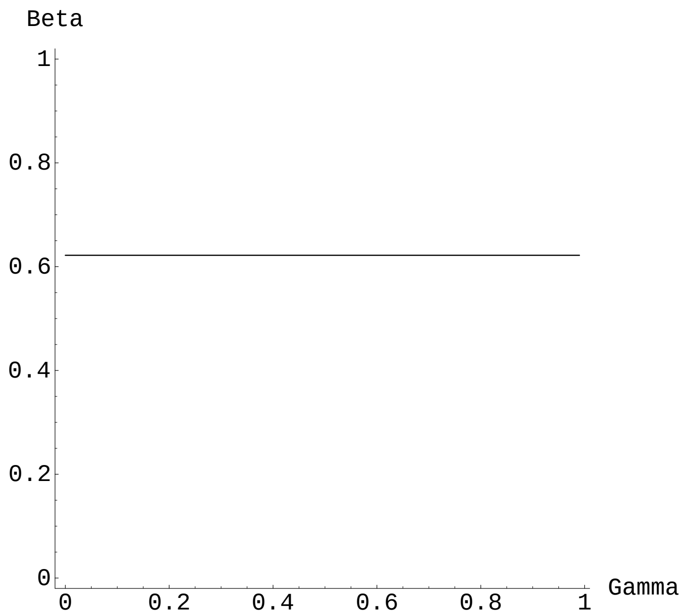


Figure 8: Reported Income and Reserve Requirements for U.K, $\eta = 0.15$

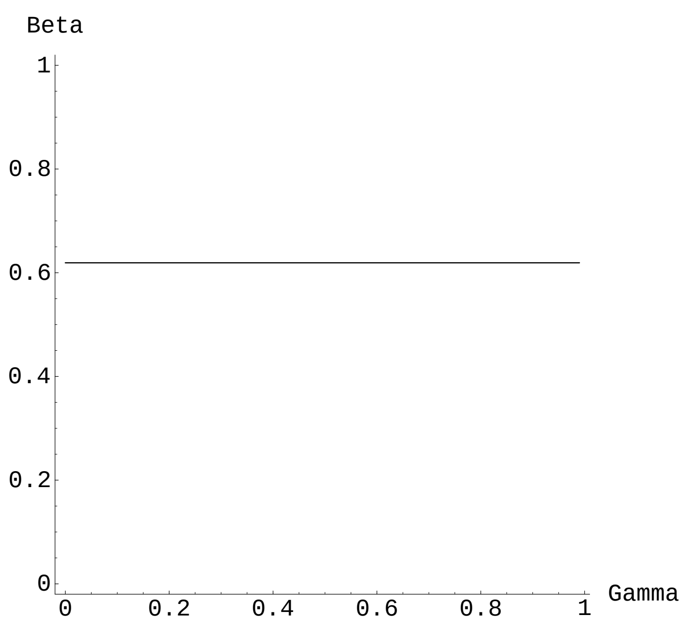

Figure 9: Reported Income and Reserve Requirements for Spain, $\eta = 0.30$

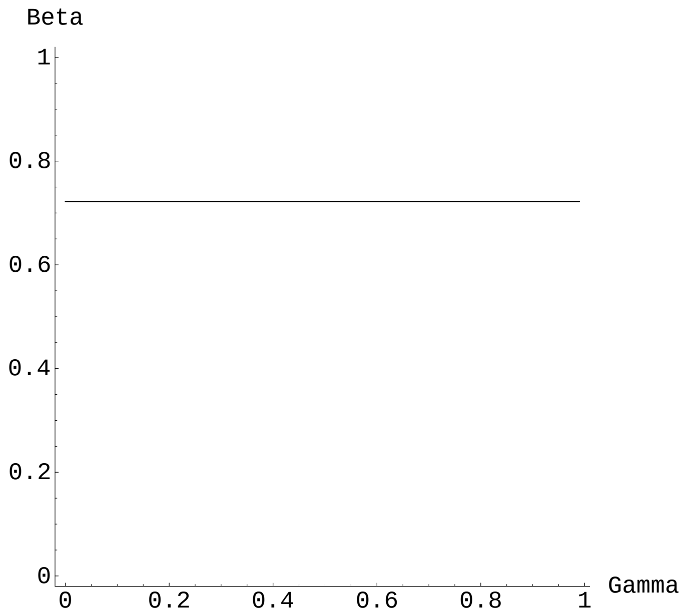


Figure 10: Reported Income and Tax Evasion Penalty for Spain $\eta = 0.15$

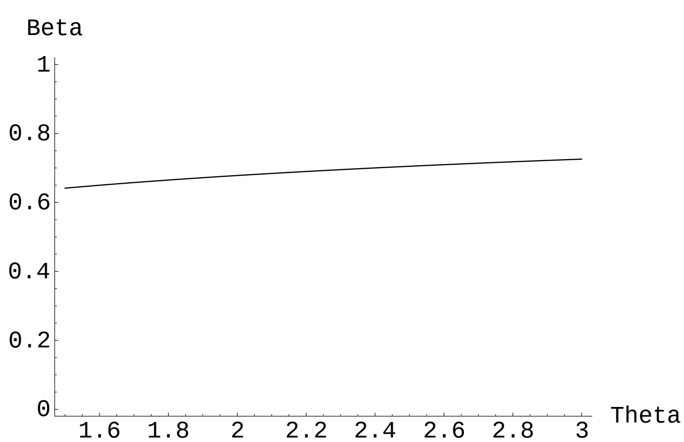

Figure 11: Reported Income and Tax Rate for Spain, $\eta = 0.15$