Agent-based simulation of power exchange with heterogeneous production companies

Eric Guerci, Stefano Ivaldi, Marco Raberto, Silvano Cincotti *

DIBE-CINEF, Università di Genova, Via Opera Pia 11a, 16145 Genova, Italy

Abstract

Electricity market are worldwide transitioning from centrally regulated systems to decentralized markets. The specific characteristic of power exchange, i.e., reduced number of producers, requires proper attention in the policy design in order to guarantee market efficiency. In particular, this paper investigates the nature of the clearing mechanism comparing two different methods, i.e., discriminatory and uniform auctions. The theoretical framework used to perform the analysis is the theory of learning in games. We consider an inelastic demand faced by sellers which use learning algorithms to understand proper for increasing their profits. We model the auction mechanism in two different duopolistic scenario. A low demand situation, where one seller can clear all the demand, and a high demand situation, where both sellers are requested. Heterogeneity in the linear cost function is considered. Consistent results are achieved with the two different learning algorithms.

Key words: Agent-based simulation, power-exchange market, market power, reinforcement learning, competitive equilibrium.

1 Introduction

Electricity market have been characterized by a progressive liberalization around the world. The deregulation of the electricity market brings competition to the previously monopolistic market. The sale of electric power now is performed or through bilateral contracts or more and more through organized markets, i.e., Power Exchanges (PEs). PEs are markets which aggregate the effective supply and demand of electricity. Usually spot-price market are Day Ahead Market (DAM) and are requested in order to provide an indication for the hourly unit commitment. This first session of the complex daily energy market collects and orders all the offers, determining the market price by

* to whom correspondence should be addressed.
Email: cincotti@dibe.unige.it Url: www.cinef.it

matching the cumulative demand and supply curves for every hour of the day after according to a merit order rule. Subsequent market sessions (also online) operate in order to guarantee the feasibility and the security of this plan.
The electric market is usually characterized by a reduced number of competitors, thus oligopolistic scenario may arise. Understanding how electricity prices depend on oligopolistic behavior of suppliers and on production costs has become a very important issue. Several restructuring proposal for the electric power industry have been proposed. Main goal is to increase the overall market efficiency, trying to study, to develop and to apply different market mechanisms. Auction design is the standard domain for commodity markets (5; 8). However, properties of different auction mechanism must be studied and determined correctly before their appliance. Generally speaking, different approaches have been proposed in the literature (3; 11; 1; 2; 7). Game theory analysis has provided an extremely useful instrument to study and derive properties of economic "games", such as auctions. Within this context, an interesting computational approach, for studying market inefficiencies, is the theory of learning in games. This methodology is useful in the context of infinitely repeated games (13; 11; 6).

The paper is organized as follows: Section 2 introduces the models of auction adopted in this work: discriminatory and uniform. We consider both mechanism, with the specific aim to compare market inefficiencies arising from repeated interaction among players. Section 3 illustrates how the decision-making process of the seller has been modeled in the framework of learning algorithms. Section 4 describes the characteristic of the computational settings used to study both auction mechanism in two different duopolistic scenarios. Section 5 reports the results of our research.

2 Auction models

In many power exchanges, buyers and sellers exchange repeatedly quantities of electric power through the submission of sale and of purchase bids to a clearinghouse double auction. This market mechanism is worldwide considered as the restructuring proposals for wholesale electricity market, in particular for the so called Day Ahead Market (DAM). The goal of this first market session is to establish the hourly price of the electricity and to define the dispatched power plants for the next day. The DAM simultaneously and separately collects all bid and ask offers for every hour of the following day. 24 clearinghouse double-auctions independently operate in order to set market prices for the 24 hours of the next day, matching buyer and seller offers according to a price-merit criterion. At the beginning of each round, buyers and sellers submit offers for a specific hour, i.e., one limit-price order together with the corresponding quantity of energy. Matching procedures between supply and demand can be either "discriminatory" or "uniform". The former sets individual prices for each matched buyer-seller pair according to a pay-as-bid rule, whereas the latter sets a system marginal price at the intersection of demand and supply curves. In this paper, both double-auctions have been modeled according to realistic principles. Simple offer mechanism are considered, i.e. each sale/purchase bid is a couple of values corresponding to a limit price and a quantity of energy. More complex offer solutions, e.g., multiple offers, are not currently included. Demand is assumed to be inelastic to price, i.e., it is modeled through a representative buyer offering a constant and inelastic demand at each round.

2.1 Uniform Auction

In a uniform-auction, the auctioneer matches the empirical supply and demand curves determining the crossing point of the two curves, thus establishing the marginal price. The marginal price corresponds to the priced sale bid made by the last production unit whose entry into the system was required to satisfy the demand for electric power. Figure 1 shows the matching mechanism. Demand $Q^d(p)$ and supply $Q^s(p)$ curves are determined aggregating buyer (q_i^d) and seller (q_j^s) quantity offers for a specific hour, i.e.,

$$
\begin{aligned}
Q^d(p) &= \sum_{i|p_i^d \geq p} q_i^d \\
Q^s(p) &= \sum_{j|p_j^s \leq p} q_j^s
\end{aligned}
$$

Hourly marginal price $P(t)$ is found at the intersection of the demand and supply curves and 24 marginal prices are determined independently and in parallel for every hour of the subsequent day. Accepted bid and ask offers are price offers greater or equal to the marginal price for sellers and equal or lower to the marginal price for buyers, respectively. Since aggregate electric power production and demand curves are discrete stepped curves, their crossing point may give rise to indeterminacy in the assignment of electric power. This requires the application of a distribution criterion that may correspond to electric power purchase/sale bids. Generally speaking, in the presence of a surplus of electric power supply, the criterion adopted in this work, randomly selects some sellers, among those who offered at the marginal price, so that the offer satisfy entirely or partially the demand at the marginal price.

2.2 Discriminatory Auction

The discriminatory double-auction mechanism considered in this work, corresponds to a simple pay as bid procedure. The auctioneer collects all the purchase and sale bids for a specific hour and creates the demand and supply curve in order to match demand and supply according to a price-merit criterion. Furthermore she establishes which bids are accepted according to the crossing-point. The highest buyers price-bid is matched with the lowest sellers price-bid. This procedure is iterated until demand and supply curves intersect. We consider an inelastic demand, so every sellers accepted is matched with the same ask-price. Finally, we use the bid-price as market price and not the mid-point price as usually considered.

3 Learning

Real electricity markets are usually composed by a reduced number of producers. Repeated interactions through the auction mechanism might cause the occurrence of implicit collusive behavior (11). Thus, sellers can try to behave opportunistically, i.e., increasing their profits, and sell at prices different from their true marginal costs. Generally speaking, analytical tools in

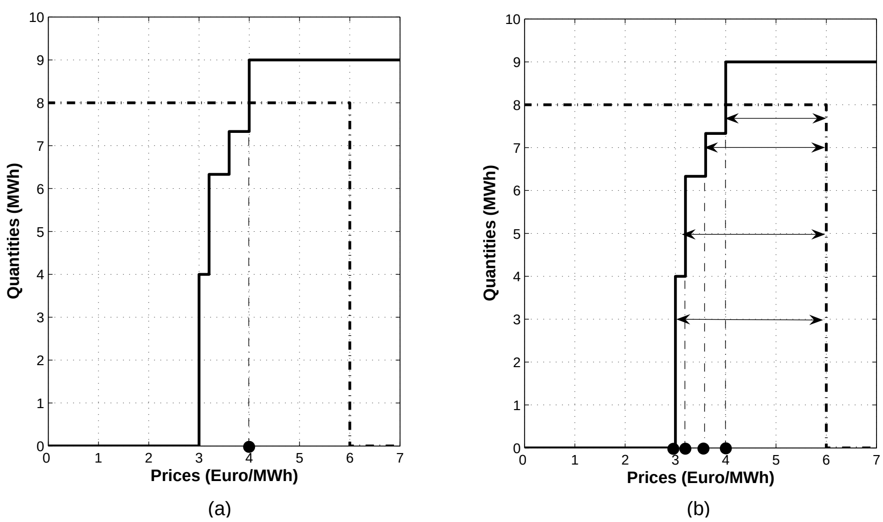

Fig. 1. Example of clearing-house with a single representative buyer. (a): the matching procedures in the uniform auction where the demand curve intersects supply curve in the point p=4 and Q=8. the black spot highlights the market clearing price. (b): the matching procedures in the discriminatory auction coincides with the supply curve in the price interval p $\in [4, 6]$ and Q=8. In both cases the marginal price P is set to 4.

single-round game have been largely used to investigate the nature of market inefficiencies (14), but many interesting properties, such as implicit collusive behavior, require to investigate repeated games (13). Two main approaches can be considered in this framework, i.e., evolutionary game theory (9) or learning in games theory(6). In this paper, we have used the latter. Seller decision-making process is modeled through an homogenous individual learning behavior, whereas a representative buyer determines the constant and inelastic demand curve. Two learning algorithms, which can be implemented under the same behavioral hypothesis, are compared in order to highlight consistent results studying the two auction mechanism. The algorithms are: Roth and Erev reinforcement learning algorithm, henceforth RE, and Marimon and McGrattan adaptive evolutionary learning algorithm, henceforth MM. Both approaches requires minimal information for players' knowledge. Both models perform a stochastic search among the strategy space in order to identify the high profitable strategy for every player. Players gain knowledge only from their own actual and past selected actions and their associated realized profit. Some sellers properties are common to both algorithm and are the following. Each i-th seller is characterized by a fixed productive capacity $\mathcal{Q}_i^s$ and constant marginal costs $c_{m,i}$. Seller's strategy space $\mathcal{A}_i$ is a discrete bi-dimensional space defined by couples (q_i, p_i) for admissible sell orders. $\mathcal{A}_i$ is constrained by maximal price P^* along the price-axis and by productive capacity along the quantity-axis, i.e., $\mathcal{A}_i := \{(p,q) | q \leq \mathcal{Q}_i^s \quad \text{and} \quad p \leq P^*\}$. At auction round t, i-th seller realizes a positive or null profit $\Pi_{i,\hat{a}_i}(t)$ depending only on opponents bids and on her strategy $\widehat{a}_i(t)$ selected at time t, i.e., $\widehat{a}_i(t) = (\widehat{p}_i(t), \widehat{q}_i(t))$. Generally speaking, realized profit $\Pi_{i,\hat{a}_i}(t)$ differs according to the specific auction mechanism. In this paper we consider both heterogenous linear cost functions for sellers and strategy's space $\mathcal{A}_i$ defined by the value of both q_i and p_i. Thus, a pure rational player would not consider q-axis, but only her total capacity as a rational choice for q-strategies available. However, our aim is to test if the proposed framework is able to generalize over a bi-dimensional strategy space in order to allow further experiments for non-linear cost functions. In the following the complete mathematical formulation for each algorithm is presented.

3.1 Marimon and McGrattan adaptive evolutionary learning algorithm

Marimon and McGrattan (10) aimed is to provide a behavioral foundation for equilibrium theory in the theoretical context of bounded rationality. They introduced a useful and general classification for adaptive learning algorithms, where players have not perfect knowledge of the consequences of their actions and they need to learn the economic environment in order to define their "best" strategy. Inside this category of learning algorithms, they propose a distinction among three subclasses according to agents' information of the past history play and of opponent's plays. In this work, we focus on the third class of algorithms proposed by their original classification which is referred to adaptive evolutionary learning subclass. In this framework, players have minimal information about the evolution of the game. They keep track only of their own realized payoffs and on the number of pure strategies played by themselves on the recent past and they do not consider the strategic consequences of their actions. They introduce three properties in the algorithm in order to replicate some human learning characteristics: adaptation, experimentation and inertia. Adaptation stands for the tendency to exploit strategies which performed better in the past, those strategies more likely will be played in the future. Experimentation corresponds to the fact that mixed strategies will always keep positive probabilities over every pure strategy. This implies that all the pure strategies will always have a minimum probability of being played at every time. Inertia is a mechanism which allows players to keep constant their mixed strategies over a certain period without updating them in order to better test the evolving environment. This mechanism is conceived to be has no correlation among the player. The mathematical formulation of the algorithm is the following: each seller assigns a strength to every strategy and keep memory of its value in order to perform an update according to the realized profits. Strength vector can be interpreted as a measure of performance for every strategy subsequent to experimentation.

$$S_{i,t}(a_i) = \begin{cases} S_{i,t-1}(a_i) - \frac{1}{\eta_{i,t-1}(a_i)} \cdot [(S_{i,t-1}(a_i) - \Pi_{i,t-1}(a_i))] & \text{if i plays } a_i \\ S_{i,t-1}(a_i) & \text{otherwise} \end{cases}$$

where $\eta_{i,t}(a_i)$ is the number of times that strategy a_i was played between the period of inertia of i-th player, whose updating value is:

$$\eta_{i,t}(a_i) = \begin{cases} \eta_{i,t-1}(a_i) + 1 & \text{if i plays } a_i \\ \eta_{i,t-1}(a_i) & \text{otherwise} \end{cases}$$

The inertia is determined according to parameter $\rho_{i,t}$. The sequence establishes whether or not i-th player will update her mixed strategies. The updating formula is the following:

$$\bar{\sigma}_{i,t}(a_i) = \begin{cases} \sigma_{i,t-1}(a_i) \cdot \frac{e^{S_{i,t-1}(a_i)}}{\sum \sigma_{i,t}(a_i) e^{S_{i,t-1}(a_i)}} & \text{with probability } \rho_{i,t} \\ \sigma_{i,t-1}(a_i) & \text{with probability 1-}\rho_{i,t} \end{cases}$$

This updating rule can converge to a zero probability for some strategies. An important feature of this algorithm is to allow the player to have positive probability for every strategy everytime. This mechanism is called experimentation and is described by:

$$\sigma_{i,t}(a_i) = \begin{cases} \epsilon_{i,t} & \text{if } \bar{\sigma}_{i,t}(a_i) \leq \epsilon_{i,t} \\ \frac{\bar{\sigma}_{i,t}(a_i)}{\sum \bar{\sigma}_{i,t}(a_i)}(1 - \bar{\epsilon}_{i,t}) & \text{otherwise} \end{cases}$$

where $\bar{\epsilon}_{i,t} = \epsilon_{i,t} \cdot |\{\bar{\sigma}_{i,t}(a_i) \leq \epsilon_{i,t}\}|$, $\epsilon_{i,t} \in (0, 1)$, $\sum_t \epsilon_{i,t} = +\infty$.
$\epsilon_{i,t}$ corresponds to the minimum probability assigned to a strategy. A random draw, according to the mixed strategies, determines which strategy is going to be selected at round $(t + 1)$.

3.2 Roth and Erev reinforcement learning algorithm

The second algorithm was inspired by the original work of Roth and Erev (12; 4) and subsequently by *Nicolaisen et alii* (11). Roth and Erev studied extensively individual human learning in multiagent experimental games with unique equilibria. They came up with a concise and robust model which takes into account different aspects of human learning in the context of decision-making behavior. In their model, three psychological aspects are considered: the power law of practice, i.e., reinforcement learning), the recency effect (i.e., forgetting effect) and an experimentation effect (i.e., not only experimented action but also similar strategies are reinforced). *Nicolaisen et alii* already adopted RE algorithm in order to study possible market power occurrence in the context of the discriminatory auction. The paper proposed some modifications to the original RE algorithm in order to play a game with zero and negative payoffs. Inspired by that work, we propose some slightly more modifications both to the algorithm and to the strategy space (we use a bi-dimensional strategy space). For each strategy $a_i \in \mathcal{A}_i$, a propensity f_i is defined. At every round t, propensities $f_{i,t-1}$ are updated for all i according to the learning algorithm which maps the current state of the world, i.e., marginal price P_t realized in the market, to a new set of propensities $f_{i,t}$. The following formula holds:

$$f_{i,t}(a_i) = (1 - r) \cdot f_{i,t-1}(a_i) + E_{i,t-1}(a_i) \tag{1}$$

where $r \in [0, 1]$ is the recency parameters which contributes to decrease exponentially the effect of past results. The second term is the experimentation function.

$$E_{i,t}(a_i) = \begin{cases} R_{i,t-1}(a_i) \cdot (1 - e) \\ f_{i,t}(a_i) \cdot \frac{e}{n-1} \end{cases}$$

where $e \in [0,1]$ is an experimentation parameter which allows to assign different weight to the played action compared to the other actions.
This term is similar to the strength vector in MM algorithm. Propensities are normalized in order to provide a probabilistic scheme of sellers strategies, reproducing the mixed strategies of the i-th player, i.e.,

$$\sigma_{i,t}(a_i) = \frac{f_{i,t+1}(a_i)}{\sum\limits_{a_i} f_{i,t+1}(a_i)} \tag{2}$$

A random draw at time $t+1$ according to the mixed strategies then determines which strategy is going to be selected at round $(t+1)$.

4 Computational setting

The proposed model is based on a duopoly framework. In this contest, a direct application of classical game theory allows one to find the Nash equilibria in pure-strategy. According to (**?**), two generic scenarios has been considered: "Low-Demand" (LD), in which one producer can meet the whole of demand, i.e., the demand is less than the capacity of the smallest seller, and "High-Demand" (HD), in which both producers will share the demand, i.e., the demand is greater than the capacity of greatest seller.
The price is fixed by means of two rules: uniform auction and discriminatory auction. Moreover, every agent decides her offer trough two learning algorithm: Roth and Erev reinforcement learning algorithm (RE) and Marimon and McGrattan adaptive evolutionary learning algorithm (MM).

4.1 Market participants

There is a buyer agent representing the whole inelastic demand, i.e., at each negotiation round, she asks a fixed energy quantity Q^d.
The seller population consists of two independent agents each having constant, but different marginal costs $c_{m,i} \geq 0$ $(i = 1,2)$ and identical production capacity. At each round, every single agent may choose a price-quantity pair (p_i^s, Q_i^s) in the strategy space, $0 \leq p_i^s \leq P^*$ and $0 \leq Q_i^s \leq Q^*$, with unit quantization. The market price depend on the specific auction mechanism.
For the simulations, each seller have been initialized with a quantity $Q_i^s = 5$ and starting price $p_i^s = 8$, the marginal costs have been set $c_{m,1} = 7, c_{m,2} = 4$ and the fixed cost $c_{f,i} = 0$.
The Table 1 shows the parameter values of the learning algorithms in different simulations.

RE algorithm			MM algorithm	
e	r	r	ρ	η
0.9700	0.04	0.08	0.10	0.0001

Table 1
Parameter values of learning algorithms.

Finally, the results have been judged by the price that the buyer must pay and the pure-strategy equilibrium frequency.

4.2 Market mechanism

Different simulations have been formed for all combinations of auction types and learning algorithms. The number of negotiation has been fixed to 1000 for every 1000 games.
In the LD case, at each round negotiation, the agent offering the least price will be accepted to supply the whole demand. Therefore, there exist pure-strategy equilibria, i.e., system price equals the greater marginal cost. Thus, only the agent having the least marginal cost is called to produce.
The payoff matrix in pure-strategy of classical game theory, is the same in both auction cases, i.e., uniform and discriminatory. However, Table 2, points out the existence of other pure-strategy equilibria not assuming multiple unit. The analysis will be focused on the pure-strategy equilibria: $(7, 8)$ and $(6, 7)$.

		II				
		10	9	8	7	6
I	10	(12,6)	(0,$\bar{8}$)	(0,4)	(0,0)	(0,-4)
	9	($\underline{20}$,0)	(10,$\bar{4}$)	(0,$\bar{4}$)	(0,0)	(0,-4)
	8	(16,0)	($\underline{16}$,0)	(8,$\bar{2}$)	(0,0)	(0,-4)
	7	(12,$\bar{0}$)	(12,$\bar{0}$)	($\underline{12}$,$\bar{0}$)	(6,$\bar{0}$)	(0,-4)
	6	(8,$\bar{0}$)	(8,$\bar{0}$)	(8,$\bar{0}$)	($\underline{8}$,$\bar{0}$)	($\underline{4}$,-2)

Table 2
Payoff Matrix in low demand case ($Q_1^s = Q_2^s = Q^s > Q^d = 4$), for uniform auction ($c_{m,1} = 4, c_{m,2} = 7$).

In the HD case, if the system price is determined by uniform auction, there exist pure-strategy equilibria satisfying either $p_1^s = P^*$ and $p_2^s < P^*$ or $p_1^s < P^*$ and $p_2^s = P^*$. This is confirmed by Table 3 that shows the payoff matrix in pure-strategy and points out such equilibria.

If the system price is determined by discriminatory auction, the Table 4

		II		
		10	9	8
I	10	(33,16.5)	$(\underline{30},\overline{18})$	$(\underline{30},\overline{18})$
	9	$(\underline{36},\overline{15})$	(27.5,11)	(25,12)
	8	$(\underline{36},\overline{15})$	$(\underline{30},10)$	(22,5.5)

Table 3
Payoff Matrix in high demand ($Q^d = 11$), for uniform auction ($c_{m,1} = 4, c_{m,2} = 7$).

shows the payoff matrix. It is worth noting that then exists one pure-strategy equilibrium in $(10, 10)$, i.e., when both agents offer at maximum price.

		II		
		10	9	8
I	10	$(\underline{33},\overline{16.5})$	$(\underline{30},12)$	$(\underline{30},6)$
	9	$(30,\overline{15})$	(27.5,11)	(25,6)
	8	$(24,\overline{15})$	(24,10)	(22,5.5)

Table 4
Payoff Matrix in high demand ($Q^d = 11$), for discriminatory auction ($c_{m,1} = 4, c_{m,2} = 7$).

5 Simulation results

5.1 Low demand analysis

The duopolistic scenario, in the case of low demand, reproduces the characteristic of the Bertrand oligopoly. However, in our framework the two players competes both with prices and quantities. Anyway, we present also the results for the pure Bertrand oligopolistic conjecture. Figure 2 shows the results for an auction "game" played restricting the strategy space to a uni-dimensional space, i.e., price-axis. Similar results are obtained for both algorithms. The results confirm that players, in both auction mechanisms, face the same payoff matrix (Table 4.2). In both contexts, they learn to play in the same way. The plot in the center shows the profits of the more efficient producers compared to the profits obtained playing the nash equilibrium. A value of zero would mean that the player at that time got a payoff equal to the one received playing the nash equilibrium strategy.

In figure 3 and 4 are presented the results of the simulations using the bi-dimensional strategy space. The figures show the same graphs, but obtained with different learning algorithms. Figure 3 refers to the results with the RE algorithm, conversely figure 4 with the MM algorithm. The results show a consistent behavior between the learning algorithms. The difference is in the average number of times in which the nash equilibrium strategy is played. In the long run, the RE algorithm plays more often this strategy. In both simulations there is a clear difference between the two auction mechanisms. The Nash equilibrium strategy is more often played in the Discriminatory auction (DA) rather than in the Uniform auction (UA). It also results easier for the sellers to get an higher profits in the Uniform rather than in the Discriminatory auction. The results seem to indicate that for those "bounded rational" sellers it is easier to learn to collude in the uniform rather than in the discriminatory auction context.

5.2 High demand analysis

Acknowledgments

This work has been partially supported by the University of Genoa and by the Italian Ministry of Education, University and Research (MIUR) under grants FIRB 2001 and COFIN 2004.

References

[1] R. Baldick, “Electricity market equilibrium models: the effect of parametrization,” *IEEE T Power Syst*, vol. 17, no. 4, pp. 1170–1176, November 2002.

[2] D. W. Bunn and F. S. Oliveira, “Agent-based simulation - an application to the new electricity trading arrangements of england and wales,” *IEEE Trans Evolut Comput*, vol. 5, no. 5, pp. 493–503, October 2001.

[3] M. J. Denton, S. J. Rassenti, and V. L. Smith, “Spot market mechanism design and competitivity issues in electric power,” *J Econ Behav Organ*, vol. 44, pp. 435–453, October 2001.

[4] I. Erev and A. K. Roth, “Predicting how people play games: Reinforcement learning in experimental games with unique, mixed strategy equilibria,” *Am Econ Rev*, vol. 88, no. 4, pp. 848–881, September 1998.

[5] D. Friedman, “The double auction institution: A survey,” in *The Double Auction Market: Institutions, Theories and Evidence*, D. Friedman and J. Lust, Eds. Sante Fe Institute Studies in the Sciences of Complexitiy: Addison-Wesley, 1993, pp. 3–25.

[6] D. Fudenberg and L. D.K., *The theory of learning in games.* The MIT Press, 1999.

[7] E. Guerci, S. Ivaldi, S. Pastore, and S. Cincotti, “Modeling and implementation of an artificial electricity market usign agent-based techniques,” *Physica A*, 2005, forthcoming.

[8] K. Klemperer, “Auction theory: A guide to the literature,” *The Economic Theory of Auction*, pp. 3–62, 2000.

[9] S. L., *Evolutionary Games and Equilibrium Selection.* The MIT Press, 1997.

[10] R. Marimon and McGrattan, *Learning and rationality in economics.* Basil Blackwell, 1995, pp. 63–101.

[11] J. Nicolaisen, V. Petrov, and L. Tesfatsion, “Market power and efficiency in a computational electricity market with discriminatory double-auction pricing,” *IEEE T Evolut Comput*, vol. 5, no. 5, pp. 504–523, October 2001.

[12] A. E. Roth and I. Erev, “Learning in extensive form games: Experimental data and simple dynamic models in the inteermediate term,” *Games Econ Behav*, vol. 8, no. 1, pp. 164–212, January 1995.

[13] F. Vega-Redondo, *Economics and the Theory of Games.* Cambridge University Press, 2003.

[14] H. D. von der Fehr N.H.M., “Spot market competition in the uk electricity industry.” *The economic Journal*, vol. 103, pp. 531–546, May 1993.

6 Figures

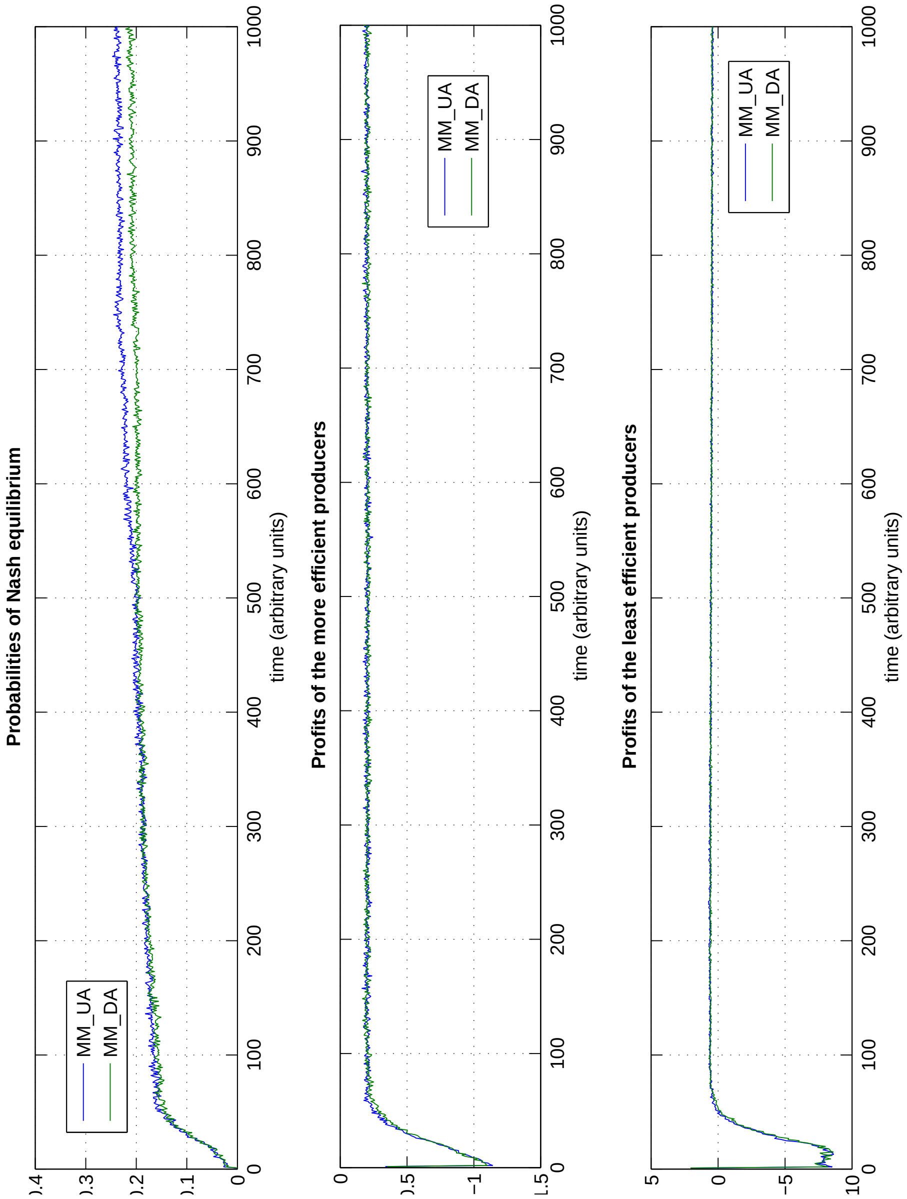

Fig. 2. This figure presents the results for a simulations in which the strategy space of both sellers is reduced to a single axis, the price axis. The learning algorithm used is the RE. In every plot, the green line refers to the results of the Discriminatory auction (DA) and the blue line to the Uniform auction (UA).

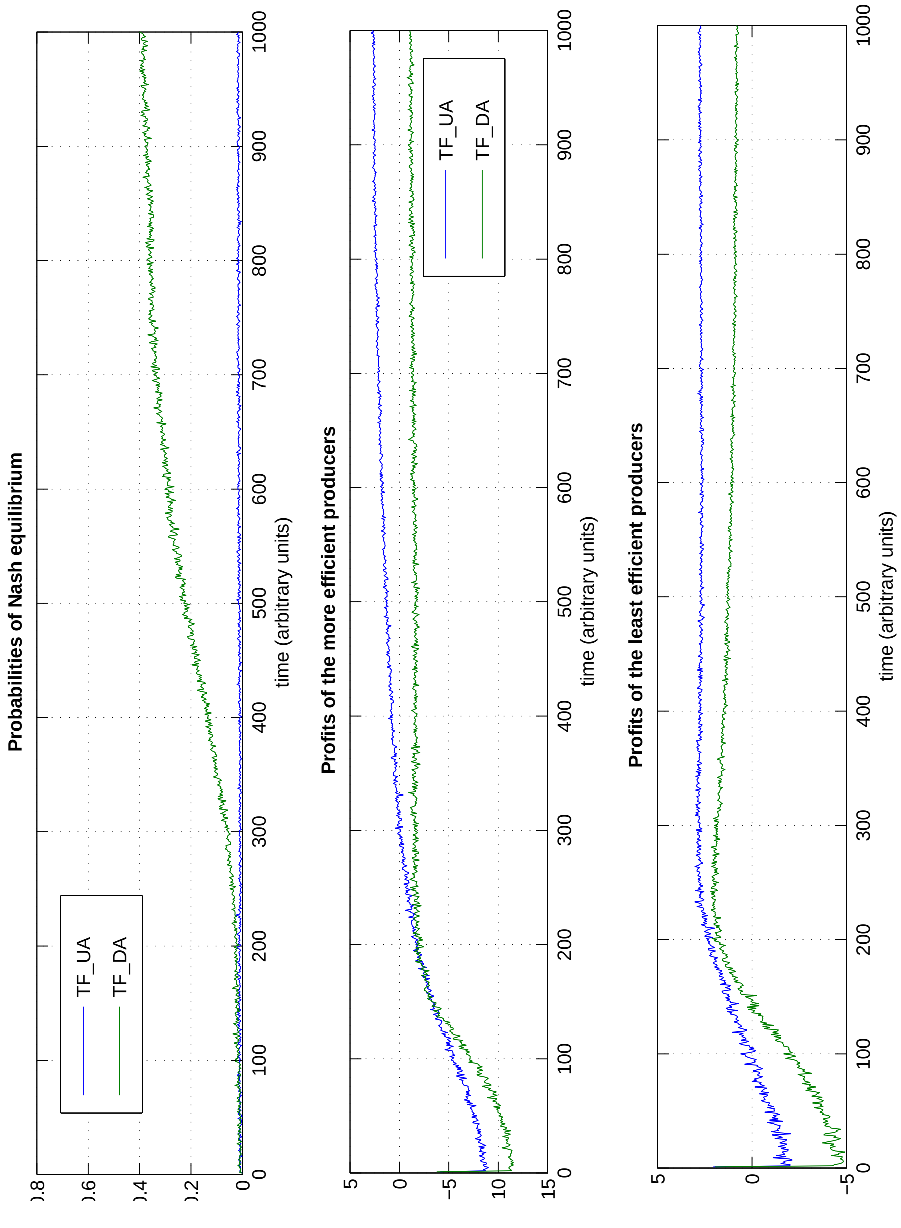

Fig. 3. This figure presents the results for a simulations in which the strategy space of both sellers is bi-dimensional. The learning algorithm used is the RE. In every plot, the green line refers to the results of the Discriminatory auction (DA) and the blue line to the Uniform auction (UA).

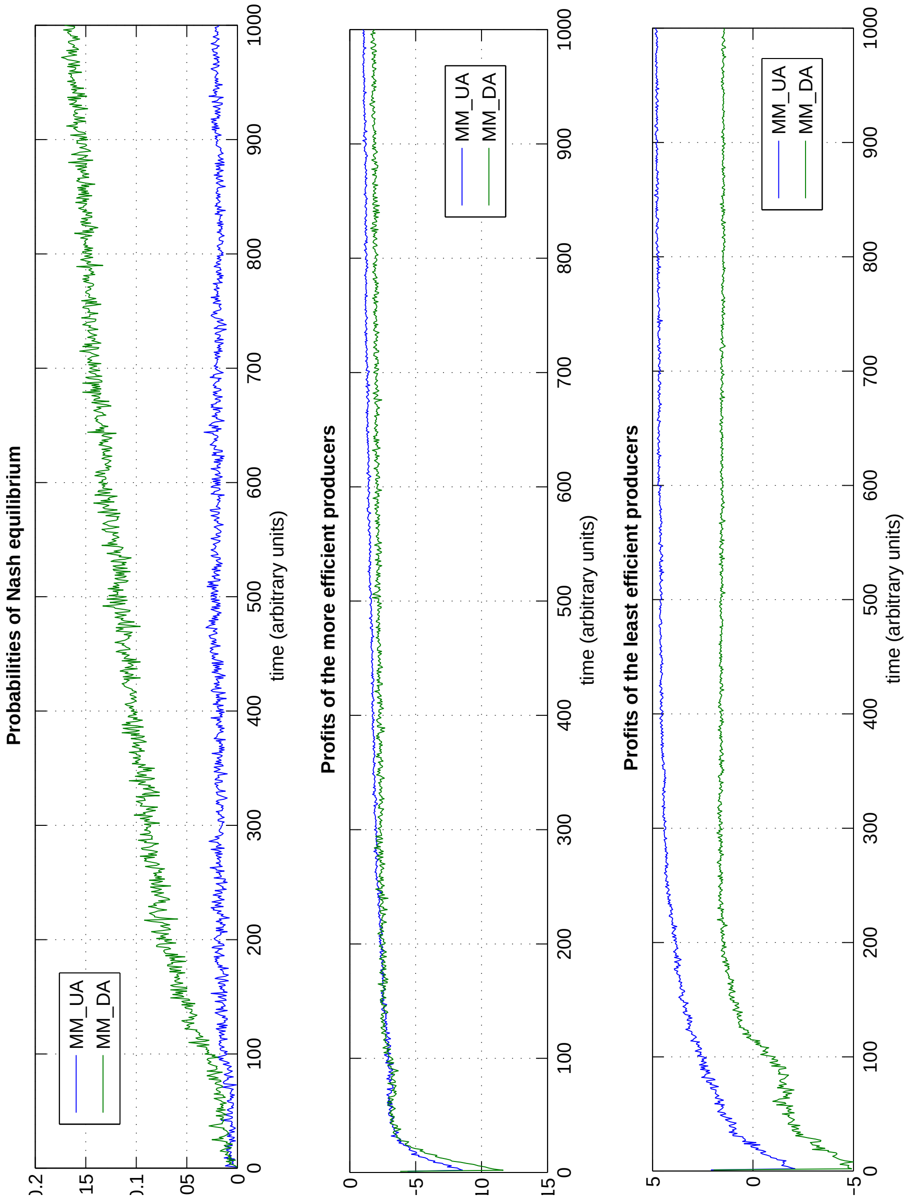

Fig. 4. This figure presents the results for a simulations in which the strategy space of both sellers is bi-dimensional. The learning algorithm used is the MM. In every plot, the green line refers to the results of the Discriminatory auction (DA) and the blue line to the Uniform auction (UA).