An Integrated Approach For Stock Price Forecasting

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Abstract

This article faces the problem of stock price forecasting through an integrated approach in which the modeling of high frequency financial data (duration, volume and bid-ask spread) uses a contemporaneous ordered probit model. Here, the formulation introduced by Raposo and Veiga (2004) - EMACM - is used in order to capture the dynamic that high frequency variables present, and its forecasting function is taken as proxy to the contemporaneous information necessary to the price model.

In that context, the main purpose of the article is to test the performance of the proposed model and compare it with the results obtained based on a NAIVE rule.

Keywords: High frequency data, ordered probit model, EMACM, nonlinear time series.

1 Introduction

The challenge of understanding market price dynamics has been enforced by recent technological developments. Nowadays, with the existence of "high frequency" database, it's possible to understand the microstructures dynamics and asset price behaviors. Here, the question is how the environment (negotiation systems and investors) could influence asset market price, bearing in mind that, in the real world, financial market imperfections exists and, in reality, are investigated and considered by the participants.

Among the objectives of that paper, could be pointed:

- Understand and model the dynamic of high frequency data, in particular: duration, volume and spread;
- Model the price-making process measuring the influence of the microstructures asset price changes distribution and understand the rationality behind buy and sell orders;
- Analyze the evidences of forecasting capability regarding financial asset prices.

That article is divided as follows. Section 2 presents the EMACM and Ordered Probit Model formulation. Section 3 brings an empirical example and, finally, Section 4 concludes.

2 The Model

In order to model the dynamic behind the microstructures, that article uses the methodology introduced by Raposo and Veiga (2004) – EMACM. That formulation is appropriated due to the capability to deal with causality among variables and the forecasting function that can be directly used in the ordered probit formulation, providing price change estimation.

2.1 EMACM:

A set of high-frequency variables (duration $-x_i$, volume $-v_i$ and bid-ask spread $-s_i$) follows an Exponential Multivariate Autoregressive Conditional High Frequency Data Model (EMACM), if:

 $x_i = \psi_i \cdot \varepsilon_i \quad \to \quad \varepsilon_i \sim i.i.d. \exp(1) \tag{2.1.1}$

$$v_i = \phi_i \cdot \eta_i \quad \to \quad \eta_i \sim i.i.d.\exp(1) \tag{2.1.2}$$

$$s_i = \varphi_i \cdot \overline{\omega}_i \rightarrow \overline{\omega}_i \sim i.i.d.\exp(1)$$
 (2.1.3)

Where, the conditional mean is as follows:

$$\ln(\mu_{i}) = \gamma + A_{1} \ln(\mu_{i-1}) + \dots + A_{q} \ln(\mu_{i-q}) + B_{0} \ln(\tau_{i}) + B_{1} \ln(\tau_{i-1}) + \dots + B_{p} \ln(\tau_{i-p})$$
(2.1.4)

 $\mu_i' = (\psi_i \phi_i \phi_i), \tau_i' = (d_i \upsilon_i s_i), \gamma$ is the vector of coefficients and $A_1, ..., A_q$ and $B_1, ..., B_p$ are coefficients matrices of each stochastic processes.

So, the general formulation of the complete model can be written as:

Observation equation:

$$\begin{bmatrix} x_i \\ v_i \\ s_i \end{bmatrix} = \begin{bmatrix} \psi_i & 0 & 0 \\ 0 & \phi_i & 0 \\ 0 & 0 & \varphi_i \end{bmatrix} \cdot \begin{bmatrix} \varepsilon_i \\ \eta_i \\ \overline{\sigma}_i \end{bmatrix}$$
(2.1.5)

Where, ε_i , $\upsilon_i e \omega_i \sim exp(1)$.

□ State equation:

$$\ln \begin{bmatrix} \boldsymbol{\psi}_{i} \\ \boldsymbol{\phi}_{i} \\ \boldsymbol{\varphi}_{i} \end{bmatrix} = \begin{bmatrix} a_{0} \\ b_{0} \\ c_{0} \end{bmatrix} + \sum_{l=1}^{q} \begin{bmatrix} a_{1}^{(l)} & a_{2}^{(l)} & a_{3}^{(l)} \\ b_{1}^{(l)} & b_{2}^{(l)} & b_{3}^{(l)} \\ c_{1}^{(l)} & c_{2}^{(l)} & c_{3}^{(l)} \end{bmatrix} \ln \begin{bmatrix} \boldsymbol{\psi}_{i-l} \\ \boldsymbol{\phi}_{i-l} \\ \boldsymbol{\varphi}_{i-l} \end{bmatrix} + \left[\begin{bmatrix} 0 & 0 & 0 \\ b_{4} & 0 & 0 \\ c_{4} & c_{5} & 0 \end{bmatrix} \ln \begin{bmatrix} x_{i} \\ \boldsymbol{v}_{i} \\ s_{i} \end{bmatrix} + \sum_{m=1}^{p} \begin{bmatrix} a_{4}^{(m)} & a_{5}^{(m)} & a_{6}^{(m)} \\ b_{5}^{(m)} & b_{6}^{(m)} & b_{7}^{(m)} \\ c_{6}^{(m)} & c_{7}^{(m)} & c_{8}^{(m)} \end{bmatrix} \ln \begin{bmatrix} x_{i-m} \\ v_{i-m} \\ s_{i-m} \end{bmatrix} \right]$$
(2.1.6)

2.2 Ordered Probit Model:

In 1992 Hausman, Lo and MacKinlay proposed an alternative statistical model to capture market price changes, based on the techniques usually employed in empirical studies of naturally ordered enumerable dependent variables.

Heuristically, the ordered probit model corresponds to a generalization of the linear regression model, in which the dependent variable is discrete. It is the only specification that captures, in a simple way, both the impact of the variables considered in the formulation and the irregular time interval between trades.

Basic specification:

Let a certain transaction price sequence as being: $P(t_0)$, $P(t_1)$, ..., $P(t_n)$ regarding time t_0 , t_1 , ..., t_n , and consider Y_1 , Y_2 , ..., Y_n the observed price changes $Y_k = P(t_k) - P(t_{k-1})$. Due to the discreteness of price, Y_k could be represented as a multiple of the tick¹. Taking Y_k^* as being a certain non-observed continuous stochastic variable:

$$Y_k^* = X_k' \cdot \beta + \varepsilon_k \tag{2.2.1}$$

$$E[\boldsymbol{\varepsilon}_k | \boldsymbol{X}_k] = 0 \tag{2.2.2}$$

$$\varepsilon_k \sim INID N(0, \sigma_k^2)$$
 (2.2.3)

¹ Tick: minimum amount in which the assets are quoted.

Where, the qx1 vector $X_k = [X_{1k} ... X_{qk}]$ ' corresponds to the variable set being considered (Y_k^* 's conditional mean formulation) and "INID" indicates that the residual sequence (ε_k 's) is independently but non-identically distributed. This is one of the most important differences between ordered probit and the standard econometric formulations.

The main characteristic of the ordered probit models is the assumption that the observed price changes (Y_k) are related to the variable Y_k^* , through the following probabilistic model:

$$Y_{k} = \begin{cases} s_{1} \rightarrow Y_{k}^{*} \in A_{1} \\ s_{2} \rightarrow Y_{k}^{*} \in A_{2} \\ \vdots \\ s_{m} \rightarrow Y_{k}^{*} \in A_{m} \end{cases}$$
(2.2.4)

Where, A_j 's are state space partitions S^* of Y_k^* , or, $S^* = \bigcup_{j=1}^m A_j$ and $A_i \bigcap A_j = \emptyset$ to $i \neq j$ and s_j 's are discrete values that comprehend the state space S of Y_k . In order to simplify the subsequent procedures, the partitions of the space state S^* could be taken as fixed intervals.

$$A_{1} \equiv \left(-\infty, \alpha_{1}\right) \tag{2.2.5}$$

$$A_2 \equiv (\alpha_1, \alpha_2] \tag{2.2.6}$$

$$A_i \equiv (\alpha_{i-1}, \alpha_i]$$
(2.2.7)

$$A_m \equiv (\alpha_{m-1}, \infty) \tag{2.2.8}$$

Despite the fact that asset price changes could be any integer number, it's assumed that "m" (2.2.4) is finite, limiting the parameters set that will be estimated. Such procedure does not introduce problems to the modeling, since that states could represent multiple values of the observed price changes. For example, let s_k as being the price change of -5 ticks or less, and s_i as the price change of 6 ticks or more; in that way, the model does not distinguish between price changes of 6 or more and -5 or less.

Regarding the number of states, the choice will depend on the type of analysis and it won't be connected to model's accuracy, when considering large sample. In the case of small samples, the addition of extra states could represent a problem, especially for the estimation process.

In reality, the data will impose the limits to the number of states to be considered, once there won't be any observation that lies into the "extreme states", making its estimation impossible.

Conditional distribution of price changes:

As mentioned before, the residuals ε_k 's (2.2.3) are not identically distributed, when conditioned to a certain state (X_k's). The main reason for that assumption is the irregular and random form of time distance between successive trades. If, for example, the price changes (Y_k^{*}'s) could be described by the Arithmetic Brownian Motion (as proposed by Marsh e Rosenfeld - 1986) with variance proportional to $\Delta t_k = t_k - t_{k-1}$, so σ_k^2 would be a linear function of Δt_k , which varies from transaction to transaction.

In order to deal with heteroskedasticity, σ_k^2 will be taken as a linear function of a predetermined variables vector $W_k = [W_{1k} \dots W_{Lk}]'$, such that:

$$E[\varepsilon_k | X_k, W_k] = 0, \qquad \varepsilon_k INID \ N(0, \sigma_k^2)$$
(2.2.9)

$$\sigma_k^2 = \gamma_0^2 + \gamma_1^2 \cdot W_{1k} + \dots + \gamma_L^2 \cdot W_{Lk}$$
(2.2.10)

Where, (2.2.9) and (2.2.10) substitute the hypothesis embedded in equations (2.2.1), (2.2.2) and (2.2.3), and the conditional volatility coefficients (γ_i) are squared, what guarantees the non-negativity. In that generic formulation, the propose of Marsh e Rosenfeld (1986) could be easily considered, being necessary the following substitutions:

$$X_k \cdot \beta = \mu \cdot \Delta t_k \tag{2.2.11}$$

$$\sigma_k^2 = \gamma^2 \cdot \Delta t_k \tag{2.2.12}$$

In that case, W_k has just one variable (Δt_k) . The fact of the same variable has been considered in X_k e W_k does not causes perfect multicolinearity, because the first affects the conditional mean of Y_k^* , and the other influences the conditional variance.

The structure of dependency imposed in observed price changes process (Y_k) is clearly connected to Y_k^* and the definition of A_j 's.

$$P(Y_{k} = s_{j} | Y_{k-1} = s_{i}) = P(Y_{k}^{*} \in A_{j} | Y_{k-1}^{*} \in A_{i})$$
(2.2.13)

As a consequence, if X_k and W_k are independent through time, the process Y_k (observed) will be too. That assumption is less restrictive, and does not make any of the subsequent statistical inferences invalid. The only assumption that must be preserved is related to the conditional independency of residuals (ε_k 's). In that way, the dynamic (serial dependency) observed in the variable is captured by X_k and W_k . Consequently, the independence of ε_k 's does not necessarily imply that Y_k^* 's are independently distributed, once none restriction about serial dependency of X_k 's e W_k 's is made.

The observed price change (Y_k) distribution, conditioned to X_k e W_k , could be determined considering the partitions boundaries and the probability distribution function of ε_k . In case of normally distributed residuals (Gaussian distribution), the conditional distribution will be:

$$P(Y_k = s_i | X_k, W_k) = P(X_k : \beta + \varepsilon_k \in A_i | X_k, W_k)$$
(2.2.14)

$$P(Y_{k} = s_{i} | X_{k}, W_{k}) = \begin{cases} P(X_{k}^{'} \cdot \beta + \varepsilon_{k} \leq \alpha_{1} | X_{k}, W_{k}) \rightarrow i = 1\\ P(\alpha_{i-1} < X_{k}^{'} \cdot \beta + \varepsilon_{k} \leq \alpha_{i} | X_{k}, W_{k}) \rightarrow 1 < i < m \quad (2.2.15)\\ P(\alpha_{m-1} < X_{k}^{'} \cdot \beta + \varepsilon_{k} | X_{k}, W_{k}) \rightarrow i = m \end{cases}$$

$$P(Y_{k} = s_{i} | X_{k}, W_{k}) = \begin{cases} \Phi\left(\frac{\alpha_{i} - X_{k}^{'} \cdot \beta}{\sigma_{k}(W_{k})}\right) \rightarrow i = 1\\ \Phi\left(\frac{\alpha_{i} - X_{k}^{'} \cdot \beta}{\sigma_{k}(W_{k})}\right) - \Phi\left(\frac{\alpha_{i-1} - X_{k}^{'} \cdot \beta}{\sigma_{k}(W_{k})}\right) \rightarrow 1 < i < m \quad (2.2.16)\\ 1 - \Phi\left(\frac{\alpha_{m-1} - X_{k}^{'} \cdot \beta}{\sigma_{k}(W_{k})}\right) \rightarrow i = m \end{cases}$$

Where,

 $\sigma_k(W_k)$ – conditional standard-deviation as function of W_k 's;

 $\Phi(.)$ – accumulated probability distribution function (standard Normal).

It could be noticed that the probability associated with a specific price change is determined from the position of the conditional mean regarding the state space partitions boundaries. Besides that, given a certain value for the conditional mean X_k ' β , any change in the position of the partitions, will affect the probability of each state.

As mentioned before, the ordered probit model can be used with different probability distribution functions – residual term.

In that way, given the partition's boundaries, a big value for the conditional mean, would indicate a high probability of an "extreme state" being observed. But, the denomination of the state can be "occult" (choice of the number of partitions).

Another advantage of the model presented here is related to the use of economic variables in vectors X_k and W_k , making possible the determination of the type and magnitude of their influence.

As the estimation process of partition's boundaries α , β coefficients and conditional variance σ_k^2 is based on the sample's information (data-driven), then ordered probit model captures empirical relation between the non-observed continuous state space S^{*} and the observed discrete state space S, as function of economic variables X_k e W_k.

Estimation process (maximum likelihood method):

Let $I_k(i)$ as being an indicative variable, which assumes an unitary value, when the realization of k-th observation of Y_k corresponds to the i-th state s_i , and zero otherwise. Thus, the conditional log-likelihood function L of the vector of price changes $Y = [Y_1 \ Y_2 \ ... \ Y_n]'$, conditioned on the explicative variables vector $X = [X_1 \ X_2 \ ... \ X_n]'$ e $W = [W_1 \ W_2 \ ... \ W_n]'$ will be:

$$L(Y|X,W) = \sum_{k=1}^{n} \{A_k + B_k + C_k\}$$
(2.2.17)

Where,

$$A_{k} = I_{k}(1)\log\Phi\left(\frac{\alpha_{1} - X_{k}^{'}\beta}{\sigma_{k}(W_{k})}\right)$$
(2.2.18)

$$B_{k} = \sum_{i=2}^{m-1} I_{k}(i) \log \left[\Phi\left(\frac{\alpha_{i} - X_{k}^{\dagger} \beta}{\sigma_{k}(W_{k})}\right) - \Phi\left(\frac{\alpha_{i-1} - X_{k}^{\dagger} \beta}{\sigma_{k}(W_{k})}\right) \right]$$
(2.2.19)

$$C_{k} = I_{k}(m) \log \left[1 - \Phi \left(\frac{\alpha_{m-1} - X_{k}^{\dagger} \beta}{\sigma_{k}(W_{k})} \right) \right]$$
(2.2.20)

Despite of the fact that σ_k^2 has been defined as a linear function of W_k , there are some restrictions that must be imposed to the parameters, making possible the identification of them. For example, the log-likelihood function value would remain the same, if a certain constant K multiplied the values of α 's, β 's and σ_k^2 . A typical procedure is to define $\gamma_0 = 1$.

There are three basic steps to be followed before initialize the estimation process:

- The determination of the number of states m;
- The definition of the regressors being considered;
- The specification of the conditional variance σ_k^2 .

As commented already, the definition of the number of states should be based on the observed price changes empirical distribution. Such procedure will avoid the adoption of a state with no observations.

Regarding the regressors definition, it will depend on the type of analysis and objectives. If they are related to forecasting, in general, the use of lagged price changes and market indices contribute to good results.

Price model (Ordered Probit Model):

Let z_i as the price changes of a specific asset expressed in tick units. Thus, based on equations (2.2.1) and (2.2.10), the proposed formulation is defined as follows:

$$z_i \sim N(\mu_i, \sigma_i^2) \tag{2.2.21}$$

$$\mu_{i} = \sum_{k=0}^{3} \left(a_{k} \cdot x_{i-k} + b_{k} \cdot v_{i-k} + c_{k} \cdot s_{i-k} \right) + \sum_{n=1}^{3} \left(d_{n} \cdot z_{i-n} \right)$$
(2.2.22)

$$\boldsymbol{\sigma}_{i}^{2} = \boldsymbol{\gamma}_{0} \cdot \boldsymbol{x}_{i} + \boldsymbol{\gamma}_{1} \cdot \boldsymbol{s}_{i-1}$$
(2.2.23)

Where, $(a_0, ..., a_3, b_0, ..., b_3, c_0, ..., c_3, d_1, ..., d_3, \gamma_0, \gamma_1)$ are the parameters to be estimated, x_i is the duration, v_i is the volume and s_i the bid-ask spread of the i-th event.

3 Empirical Analyses

3.1 Data Base:

The database used in the empirical analysis presented in this article was built by Joel Hasbrouck e NYSE – Trades, Orders Reports and Quotes (TORQ). The data reflect the trades of IBM stocks, occurred between November 1st, 1990 and November 16th, 1990.

The database comprehends all the relevant information embedded in financial transactions – buys or sells (i.e., bid price, ask price, transaction price, time and volume) registered during regular financial market time – 9:30 AM - 4:00 PM (after-market is not considered).

In order to get the basis for the development of the study, some changes were implemented in the original data. Such procedure was necessary, due to the nature of the existent information.

Each one of the registers in the original database refers to a transaction that has effectively occurred. Since that study will focus on the modeling of tick-by-tick (price change) data and not in an event basis analysis, it was necessary to group the relevant information.

- Duration:

- If the price of transaction "i" is equal to the price of transaction "i-1", so duration "i" is equal to the sum of durations "i" and "i-1";
- If a certain transaction (with price change) presents duration equal to zero, so that register is discarded.

- Volume:

• If the price of transaction "i" is equal to the price of transaction "i-1", so volume "i" is equal to the mean of the volumes of transactions "i" and "i-1".

- Spread:

- Spread "i" is equal to the difference between bid price "i" and ask price "i";
- If the price of transaction "i" is equal to the price of transaction "i-1", so spread "i" is equal to the mean of the spreads of transactions "i" and "i-1" weighted by the volumes of such transactions (bid-ask spread may change according to the volume being traded through an specific market-maker).

Relevant changes and considerations:

 \circ The tick-by-tick series has the unity value of the tick as reference (US\$ 0.125);

- The transaction occurred during the first twenty minutes of trading day were not considered for estimation purposes (9:30 AM - 9:50 AM) – opening postponing problems and "first trades" effects;
- For each day, the conditional mean of each one of the variables of the system (deseasonalized series) will be taken as the mean of the respective values observed between 9:50 AM e 10:00 AM (if there are no observations, the conditional mean is taken as one).

3.2 Empirical tests:

The empirical tests could be divided into different stages with distinct objectives.

 Five consecutives trading days are selected from the database (01/11/1990 – 08/11/1990) and EMACM (2,2) is estimated just considering the variables existent in the price model (duration, bid-ask spread and volume). Figures 1 – 5 present the results of the in-sample analysis.

- ACF

0

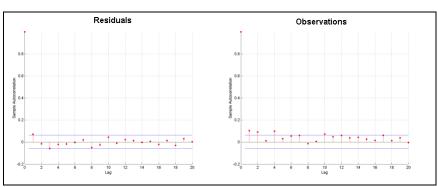


Figure 1 - ACF of duration (residuals x observations)

• Volume

Duration

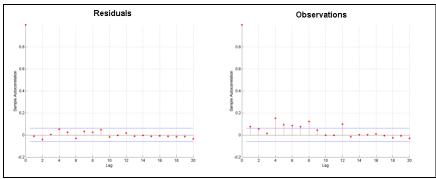


Figure 2 - ACF of volume (residuals x observations)

• Spread

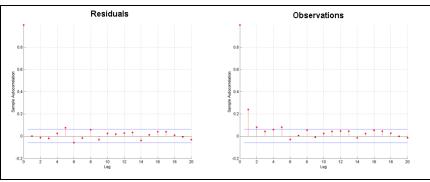


Figure 3 - ACF of spread (residuals x observations)

- Ljung-Box:

		Accept H0 (95%)	P-Value	Ljung-Box	Critical Value
Observations	Duration	Reject	0,00%	56,52	25,00
	Volume	Reject	0,00%	88,94	25,00
	Spread	Reject	0,00%	94,68	25,00
Residual (08/11/1990)	Duration	Accept	29,89%	17,34	25,00
	Volume	Accept	67,09%	12,10	25,00
	Spread	Accept	11,99%	21,55	25,00

Table 1 – Linear dependency analysis (Ljung-Box test)

- 2) Ordered Probit Model as described in (2.2.21), (2.2.22) and (2.2.23) is estimated. Through the use of EMACM forecasting function, the one-step-ahead forecast is generated and its results are used as substitutes to the contemporaneous variables value in the Ordered Probit Model formula (price model's forecasting function). The partitions selected by the price model's forecasting function are then checked against the real data and the one selected by NAIVE rule². In this part, the time period from 08/11/1990 until 16/11/1990 is tested based on a five days period. The main objective is to present the out-of-sample price model forecasting results (Appendix I brings the main results observed).
- Unconstraint model: the econometric model estimates the partitions boundaries and coefficient matrices. Figure 4 present the results found. Here, the number of right choices (one-step-ahead price movements) of the proposed model is compared against NAIVE, as described before.

 $^{^{2}}$ NAIVE rule: if a certain partition X is selected in a given event "i", then, for the subsequent event, the partition will remain the same.

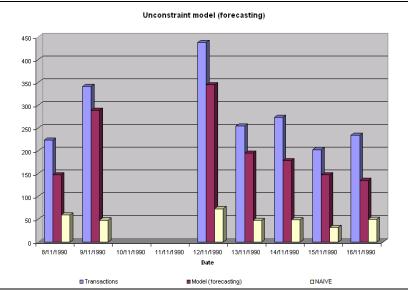


Figure 4 – Number of forecasted price movements (direction and magnitude)

• **Direction:** the main objective of that procedure is to test the capability to forecast just the direction of the prices movements and not the magnitude of them. In that experiment the partition is fixed in zero and the only the equation's parameters have to be estimated.

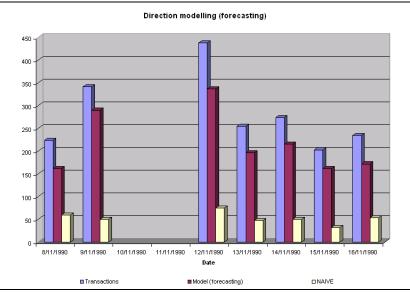


Figure 5 – Number of forecasted price movements (direction)

As can be notice the proposed method captures quite satisfactory the intra-day pattern and the dynamic embedded in price changes. Figure 6 summarizes the results.

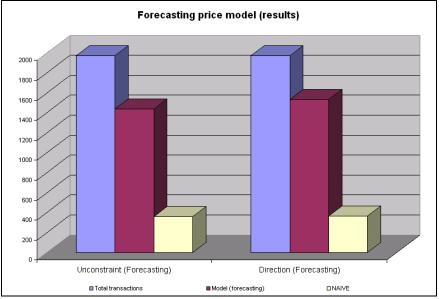


Figure 6 – Total number of forecasted price movements

The results point to the existence of a intra-day pattern that is captured by the proposed model. Despite of the fact that trades, when analyzed together, do not present significant asymmetry (see results in Appendix I). When the chronological sequence of events is taken and modeled properly, intra-day behavior could conduct to some predictability when considering price movements (conditional distribution).

4 Conclusion and Final Comments

In this article, the challenge of stock price forecasting is faced through the use of an integrated approach in which the modeling of high frequency financial data (duration, volume and bid-ask spread) uses a contemporaneous ordered probit model in which price changes (measured in numbers of ticks) are the interest variable. Here, the formulation introduced by Raposo and Veiga (2004) – EMACM – was used in order to capture the dynamic that high frequency variables present, and its forecasting function is taken as proxy to the contemporaneous information necessary to the proposed price model.

Regarding high frequency data model, excellent results were obtained when considering the linear dependence observed in the original series. If compared with the results presented by the authors in the original article (one month sample), these could be considered better in terms of fitting. Another interesting point is that non-linear dependencies (excess of dispersion) weren't so significant.

When the method (high frequency data and ordered probit models) is tested against NAIVE rule, the results show that the use of high frequency variables in order to forecast intraday price changes is really effective. Both, the intra-day pattern and variables dynamics are satisfactory captured.

In general lines, the main objective of the paper was achieved and the integrated approach proposed here shows itself robust when dealing with one of most exciting questions.

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Appendix I

Date	Model type	Model specification	Partition	Observations	Model (real data)	NAIVE	Observations	Model (forecasting)	NAIVE
08/11/1990	Unconstraint	EMACM(2,2)]-Inf4,00[[-4,00 -0,54[[-0,54 3,00[[3,00 +Inf.[0 109 116 1	147	60	0 108 115 1	148	59
	Direction	EMACM(2,2)]-Inf. 0,00[[0,00 +Inf.]	109 117	163	60	108 116	162	59
09/11/1990	Unconstraint	EMACM(2,2)]-Inf4,08[[-4,08 -0,34[[-0,34 3,09[[3,09 +Inf.[1 162 180 1	300	48	1 161 179 1	289	48
	Direction	EMACM(2,2)]-Inf. 0,00[[0,00 +Inf.]	163 181	297	50	162 180	290	50
12/11/1990	Unconstraint	EMACM(2,2)]-Inf4,00[[-4,00 -0,10[[-0,10 3,47[[3,47 +Inf.[3 214 219 4	364	73	3 213 218 4	345	73
	Direction	EMACM(2,2)]-Inf. 0,00[[0,00 +Inf.]	217 223	361	75	216 222	337	75
13/11/1990	Unconstraint	EMACM(2,2)]-Inf3,63[[-3,63 0,40[[0,40 4,84[[4,84 +Inf.]	1 130 125 1	208	47	1 129 124 1	195	47
	Direction	EMACM(2,2)]-Inf. 0,00[[0,00 +Inf.]	131 126	206	47	130 125	196	47
14/11/1990	Unconstraint	EMACM(2,2)]-Inf1,00[[-1,00 1,00[[1,00 7,01[[7,01 +Inf.[8 128 140 0	197	49	8 127 139 0	179	49
	Direction	EMACM(2,2)]-Inf. 0,00[[0,00 +Inf.]	136 140	220	50	135 139	215	50
15/11/1990	Unconstraint	EMACM(2,2)]-Inf3,89[[-3,89 -0,54] [-0,54 3,15] [3,15 +Inf.[0 105 100 0	150	32	0 104 99 0	148	32
	Direction	EMACM(2,2)]-Inf. 0 ,00[[0 ,00 +Inf.]	105 100	163	32	104 99	162	32
16/11/1990	Unconstraint	EMACM(2,2)]-Inf1,01[[-1,01 1,00[[1,00 4,13] [4,13 +Inf.[10 110 116 1	153	50	10 109 115 1	135	50
	Direction	EMACM(2,2)]-Inf. 0,00[[0,00 +Inf.]	120 117	182	53	119 116	172	53