

# Inflation Targeting, Learning and Q Volatility in Small Open Economies

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## Abstract

This paper examines the welfare implications of managing asset-price with consumer-price inflation targeting by monetary authorities who have to learn the laws of motion for both inflation rates. Our results show that the Central Bank can reduce the volatility of consumption and asset price inflation more effectively if it does so with state-contingent preferences than with a Taylor-rule with fixed coefficients. In the state-contingent setup the policy authority reacts to asset price movements only if such movements cross critical thresholds.

*Key words:* Tobin's Q, learning, monetary policy rules, inflation targets

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# 1 Introduction

Many countries now practice inflation targeting, but that has not immunized the economy from experiencing asset price volatility in the form of exchange rate instability in Australia or share-market bubbles in the United States. The practice of controlling changes in goods prices is taken for granted by many Central Banks, but there is no consensus about the management of asset-price inflation, except in the sense that it is not desirable for asset prices to be too high or too volatile.

There is some research which show that central bankers should not target asset prices [see for example Bernanke and Gertler (1999, 2001) for a closed economy study]. At the World Economic Forum in Davos in 2003, Lawrence Summers suggested that policy makers should use other tools, such as margin lending requirements or public jawboning, to combat asset-price inflation. He compared raising interest rates to combat asset-price inflation to a preemptive attack, and stated "it takes enormous hubris to know when the right moment has come to start a war" [Summers (2003), p.1]. However, Cecchetti, Genberg and Wadhvani (2002) have argued that central banks should "react to asset price misalignments". In essence, they show that when disturbances are nominal, reacting to close misalignment gaps significantly improves macroeconomic performance.

In this paper, we consider the rate of growth of Tobin's Q, first introduced by Tobin (1969), as a potential target variable for monetary policy. Our reasoning is that Q-growth would be small when the growth in the market valuation of capital assets corresponds roughly with the growth of replacement costs. Since asset prices (in the market value) are a lot less sticky than good prices (in the replacement cost), the presence of high Q-growth would be indicative of misalignment of market value and replacement cost, in other words an indication of an "excessive" change in the share price. Thus monitoring and targeting Q-growth may be viewed as a proxy policy for monitoring and targeting asset price inflation, but with the advantage that the asset price is evaluated relative to a benchmark (the replacement cost).

The focus on Q is also influenced by Brainard and Tobin (1968, 1977), who argued that Q plays an important role in the transmission of monetary policy both directly via the capital investment decision of enterprises and indirectly via consumption decisions. Thus volatility of Q has implications for inflation and growth. Large swings in Q can lead to systematic overinvestment, and in the open-economy context, over-borrowing and serious capital account deficits.

This paper is concerned with the thought experiment: what happens to

growth and inflation if the central bank monitors  $Q$ ? In particular, we will generate the welfare implications of adopting a stance of monetary policy which includes targeting consumer price inflation as well as changes in  $Q$ . We present the implications for two monetary policy scenarios - inflation targeting with and without reacting to  $Q$ -growth - in two frameworks, first, the well-known Taylor rule framework and secondly, a linear quadratic control framework with state-contingent preferences. In the latter case, monetary policy is more cautious, in the sense that policy makers react to price inflation or  $Q$ -growth only when their forecasts cross critical thresholds, otherwise they refrain from taking action by raising or lowering interest rates, except in a worst-case scenario.

In both the Taylor rule and the linear quadratic "cautious policy" frameworks the central bank has to learn about the nature of the shock as well as the degree of  $Q$ -growth and price inflation. We thus assume that the policy makers are subject to uncertainty about the underlying model and the nature of the shocks.

To anticipate results, we show that incorporating  $Q$ -growth targets with inflation targets reduces the volatility of consumption and  $Q$  growth in both policy frameworks, but the volatility reduction is more effective in the linear quadratic control framework, with state-contingent preferences, when monetary policy is more cautious than in the more familiar Taylor rule framework.

Our approach is similar to a "worst-case" approach to monetary policy design, put forward by Rustem, Wieland, and Žaković (2005). This approach is an example of robust control. They show that under uncertainty this approach leads to more moderate policy responses and represents a form of "cautionary monetary policy" advocated by Brainard (1967), who argued that the degree of policy activism should vary inversely with the extent of uncertainty about policy effectiveness [Rustem, Wieland, and Žaković (2005): p. 15].

The paper is organized as follows. The model is described in Section 2, and the solution algorithm is presented in Section 3. Section 4 contains the simulation results for the alternative policy frameworks. The concluding remarks are in Section 5.

## 2 Model Specification

The framework of analysis contains two modules - a module which describes the behavior of the private sector and a module which describes the behavior of the central bank.

## 2.1 Private Sector Behavior

The private sector is assumed to follow the standard optimizing behavior characterized in dynamic stochastic general equilibrium models.

### 2.1.1 Consumption

The utility function for the private sector “representative agent” is given by the following function:

$$U(C_t) = \frac{C_t^{1-\gamma}}{1-\gamma} \quad (1)$$

where  $C$  is the aggregate consumption index and  $\gamma$  is the coefficient of relative risk aversion. Unless otherwise specified, upper case variables denote the levels of the variables while lower case letters denote logarithms of the same variables. The exception is the nominal interest rate denoted as  $i$ .

The representative agent as “household/firm” optimizes the following intertemporal welfare function, with an endogenous discount factor:

$$W_t = \mathbf{E}_t \left[ \sum_{i=0}^{\infty} \vartheta_{t+i} U(C_{t+i}) \right] \quad (2)$$

$$\vartheta_{t+1+i} = [1 + \bar{C}_t]^{-\beta} \cdot \vartheta_{t+i} \quad (3)$$

$$\vartheta_t = 1 \quad (4)$$

where  $\mathbf{E}_t$  is the expectations operator, conditional on information available at time  $t$ , while  $\beta$  approximates the elasticity of the endogenous discount factor  $\vartheta$  with respect to the average consumption index,  $\bar{C}$ . Endogenous discounting is due to Uzawa (1968) and Mendoza (2000) states that endogenous discounting is needed for the model to produce well-behaved dynamics with deterministic stationary equilibria.<sup>1</sup>

The specification used in this paper is due to Schmitt-Grohé and Uribe (2001). In our model, an individual agent’s discount factor does not depend on their own consumption, but rather their discount factor depends on the average level of consumption. Schmitt-Grohé and Uribe (2001) argue that this simplification reduces the equilibrium conditions by one Euler equation and one state variable, over the standard model with endogenous discounting, it greatly facilitates the computation of the equilibrium dynamics, while delivering “virtually identical” predictions of key macroeconomic variables

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<sup>1</sup>Endogenous discounting also allows the model to support equilibria in which credit frictions may remain binding.

as the standard endogenous-discounting model.<sup>2</sup> In equilibrium, of course, the individual consumption index and the average consumption index are identical. Hence,

$$C_t = \bar{C}_t \quad (5)$$

The consumption index is a composite index of non-tradeable goods  $n$  and tradeable goods  $f$  :

$$C_t = \left(C_t^f\right)^{\alpha_f} \left(C_t^n\right)^{1-\alpha_f} \quad (6)$$

where  $\alpha_f$  is the proportion of traded goods. Given the aggregate consumption expenditure constraint,

$$P_t C_t = P_t^f C_t^f + P_t^n C_t^n \quad (7)$$

and the definition of the real exchange rate,

$$Z_t = \frac{P_t^f}{P_t^n} \quad (8)$$

the following expressions give the demand for traded and non-traded goods as functions of aggregate expenditure and the real exchange rate  $Z$ :

$$C_t^f = \left(\frac{1-\alpha_f}{\alpha_f}\right)^{-1+\alpha_f} Z_t^{-1+\alpha_f} C_t \quad (9)$$

$$C_t^n = \left(\frac{1-\alpha_f}{\alpha_f}\right)^{\alpha_f} Z_t^{\alpha_f} C_t \quad (10)$$

Similarly, we can express the consumption of traded goods as a composite index of the consumption of export goods,  $C^x$ , and import goods  $C^m$ :

$$C_t^f = \left(C_t^x\right)^{\alpha_x} \left(C_t^m\right)^{1-\alpha_x} \quad (11)$$

where  $\alpha_x$  is the proportion of export goods. The aggregate expenditure constraint for tradeable goods is given by the following expression:

$$P_t^f C_t^f = P_t^m C_t^m + P_t^x C_t^x \quad (12)$$

where  $P^x$  and  $P^m$  are the prices of export and import type goods respectively. Defining the terms of trade index  $J$  as:

$$J = \frac{P^x}{P^m} \quad (13)$$

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<sup>2</sup>Schmitt-Grohé and Uribe (2001) argue that if the reason for introducing endogenous discounting is solely for introducing stationarity, “computational convenience” should be the decisive factor for modifying the standard Uzawa-type model. Kim and Kose (2001) reached similar conclusions.

yields the demand for export and import goods as functions of the aggregate consumption of traded goods as well as the terms of trade index:

$$C_t^x = \left( \frac{1 - \alpha_x}{\alpha_x} \right)^{-1 + \alpha_x} J_t^{-1 + \alpha_x} C_t^f \quad (14)$$

$$C_t^m = \left( \frac{1 - \alpha_x}{\alpha_x} \right)^{\alpha_x} J_t^{\alpha_x} C_t^f \quad (15)$$

### 2.1.2 Production

Production of exports and imports is by the Cobb-Douglas technology:

$$Y_t^x = A_t^x (K_{t-1}^x)^{1 - \theta_x} \quad (16)$$

$$Y_t^m = A_t^m (K_{t-1}^m)^{1 - \theta_m} \quad (17)$$

where  $A^x, A^m$  represents the labour factor productivity terms<sup>3</sup> in the production of export and import goods, and  $(1 - \theta_x), (1 - \theta_m)$  are the coefficients of the capital  $K^x$  and  $K^m$  respectively. The time subscripts  $(t - 1)$  indicates that they are the beginning-of-period values. The production of non-traded goods, which is usually in services, is given by the labour productivity term,  $A_t^n$ :

$$Y_t^n = A_t^n \quad (18)$$

Capital in each sector has the respective depreciation rates,  $\delta_x$  and  $\delta_m$ , and evolves according to the following identities:

$$K_t^x = (1 - \delta_x) K_{t-1}^x + I_t^x \quad (19)$$

$$K_t^m = (1 - \delta_m) K_{t-1}^m + I_t^m \quad (20)$$

where  $I_t^x$  and  $I_t^m$  represents investment in each sector.

### 2.1.3 Budget Constraint

The budget constraint faced by the household/firm representative agent is:

$$P_t C_t = \Pi_t + S_t [L_t^* - L_{t-1}^* (1 + i_{t-1}^*)] - [B_t - B_{t-1} (1 + i_{t-1})] \quad (21)$$

where  $S$  is the exchange rate (defined as domestic currency per foreign),  $L_t^*$  is foreign debt in foreign currency, and  $B_t$  is domestic debt in domestic

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<sup>3</sup>Since the representative agent determines both consumption and production decisions, we have simplified the analysis by abstracting from issues about labour-leisure choice and wage determination.

currency. Profits  $\Pi$  is defined by the following expression:

$$\begin{aligned} \Pi_t = & P_t^x \left[ A_t^x (K_{t-1}^x)^{1-\theta_x} - \frac{\phi_x}{2K_{t-1}^x} (I_t^x)^2 - I_t^x \right] \\ & + P_t^m \left[ A_t^m (K_{t-1}^m)^{1-\theta_m} - \frac{\phi_m}{2K_{t-1}^m} (I_t^m)^2 - I_t^m \right] + P_t^n A_t^n \end{aligned} \quad (22)$$

The aggregate resource constraint shows that the firm faces quadratic adjustment costs when they accumulate capital, with these costs given by the terms  $\frac{\phi_x}{2K_{t-1}^x} (I_t^x)^2$  and  $\frac{\phi_m}{2K_{t-1}^m} (I_t^m)^2$ .

The household/firm may lend to the domestic government and accumulate bonds  $B$  which pay the nominal interest rate  $i$ . They can also borrow internationally and accumulate international debt  $L^*$  at the fixed rate  $i^*$ , but this would also include a cost of currency exchange.<sup>4</sup>

The change in bond holdings and foreign debt holdings evolves as follows:

$$P_t^n G_t = B_{t+1} - B_t(1 + i_t) \quad (23)$$

$$(P_t^x X_t - P_t^m M_t) = -S_t (L_{t+1}^* - L_t^*[1 + i_t^*]) \quad (24)$$

where  $G$  is government expenditure (exogenously determined).

### 2.1.4 Euler Equations

The household/firm optimizes the expected value of the utility of consumption (2) subject to the budget constraint defined in (21) and (22) and the constraints in (19) and (20).

$$\begin{aligned} Max : \quad \mathbf{L} = & \mathbf{E}_t \sum_{i=0}^{\infty} \vartheta_{t+i} \{ U(C_{t+i}) \\ & - \Lambda_{t+i} [ C_{t+i} - \frac{P_{t+i}^x}{P_{t+i}} \left( A_{t+i}^x (K_{t-1+i}^x)^{1-\theta_x} - \frac{\phi_x}{2K_{t-1+i}^x} (I_{t+i}^x)^2 - I_{t+i}^x \right) \\ & - \frac{P_{t+i}^m}{P_{t+i}} \left( A_{t+i}^m (K_{t-1+i}^m)^{1-\theta_m} - \frac{\phi_m}{2K_{t-1+i}^m} (I_{t+i}^m)^2 - I_{t+i}^m \right) - \frac{P_{t+i}^n}{P_{t+i}} A_{t+i}^n \\ & - \frac{S_{t+i}}{P_{t+i}} (L_{t+i}^* - L_{t-1+i}^*(1 + i_{t-1+i}^*)) + \frac{1}{P_{t+i}} (B_{t+i} - B_{t-1+i}(1 + i_{t-1+i})) ] \\ & - Q_{t+i}^x [ K_{t+i}^x - I_{t+i}^x - (1 - \delta_x) K_{t-1+i}^x ] \\ & - Q_{t+i}^m [ K_{t+i}^m - I_{t+i}^m - (1 - \delta_m) K_{t-1+i}^m ] \} \end{aligned}$$

<sup>4</sup>The time-varying risk premium is assumed to be zero.

The variable  $\Lambda$  is the familiar Lagrangian multiplier representing the marginal utility of wealth. The terms  $Q^x$  and  $Q^m$ , known as Tobin's  $Q$ , represent the Lagrange multipliers for the evolution of capital in each sector - they are the "shadow prices" for new capital. Maximizing the Lagrangian with respect to  $C_t, L_t^*, B_t, K_t^x, K_t^m, I_t^x, I_t^m$  yields the following first order conditions:

$$\Lambda_t = U'(C_t) \quad (25)$$

$$\vartheta_t U'(C_t)/P_t = \mathbf{E}_t \vartheta_{t+1} U'(C_{t+1})(1 + i_t)/P_{t+1} \quad (26)$$

$$\vartheta_t U'(C_t) S_t/P_t = \mathbf{E}_t \vartheta_{t+1} U'(C_{t+1})(1 + i_t^*) S_{t+1}/P_{t+1} \quad (27)$$

$$[\vartheta_t Q_t^x - \mathbf{E}_t \vartheta_{t+1} Q_{t+1}^x (1 - \delta_x)] = \mathbf{E}_t \vartheta_{t+1} \Lambda_{t+1} \frac{P_{t+1}^x}{P_{t+1}} \left[ A_{t+1}^x (1 - \theta_x) (K_t^x)^{-\theta_x} + \frac{\phi_x (I_{t+1}^x)^2}{2 (K_t^x)^2} \right] \quad (28)$$

$$[\vartheta_t Q_t^m - \mathbf{E}_t \vartheta_{t+1} Q_{t+1}^m (1 - \delta_m)] = \mathbf{E}_t \vartheta_{t+1} \Lambda_{t+1} \frac{P_{t+1}^m}{P_{t+1}} \left[ A_{t+1}^m (1 - \theta_m) (K_t^m)^{-\theta_m} + \frac{\phi_m (I_{t+1}^m)^2}{2 (K_t^m)^2} \right] \quad (29)$$

$$I_t^x = \frac{1}{\phi_x} \left( \frac{Q_t^x}{\Lambda_t} - 1 \right) K_{t-1}^x \quad (30)$$

$$I_t^m = \frac{1}{\phi_m} \left( \frac{Q_t^m}{\Lambda_t} - 1 \right) K_{t-1}^m \quad (31)$$

The above equations (28) and (29) show that the solutions for  $Q_t^x$  and  $Q_t^m$ , which determine investment and the evolution of capital in each sector, come from forward-looking stochastic Euler equations. The shadow price or replacement value of capital in each sector is equal to the discounted value of next period's marginal productivity, the adjustment costs due to the new capital stock, and the expected replacement value net of depreciation.

We also note that the solution for each sector's  $Q$  also gives each sector's investment,  $I$ . Alternatively, if we know the optimal decision rule for investment for each sector, we can obtain the value  $Q$  for each sector:



$$\begin{aligned}
Q_t^x &= \Lambda_t \left( \frac{\phi_x I_t^x}{K_{t-1}^x} + 1 \right) \\
Q_t^m &= \Lambda_t \left( \frac{\phi_m I_t^m}{K_{t-1}^m} + 1 \right)
\end{aligned}$$

In the steady state, of course, the investment/capital ratio is equal to the rate of depreciation for each sector. Thus, the steady state value of  $Q$  for each sector is given by the following expressions:

$$\begin{aligned}
\bar{Q}_t^x &= \bar{\Lambda} (\phi_x \delta^x + 1) \\
\bar{Q}_t^m &= \bar{\Lambda} (\phi_m \delta^m + 1)
\end{aligned}$$

where  $\bar{\Lambda} = U'(\bar{C})$ .

The solution of the model, discussed below, involves finding decision rules for  $C_t, S_t, Q_t^x$ , and  $Q_t^m$  so that the Euler equation errors given in equations 26 through ?? are minimized. Given that we wish to impose non-negativity constraints on  $C_t, S_t, I_t^x, I_t^m$ , we specify decision rules for these variables and solve for the implied values of  $Q_t^x, Q_t^m$ .

### 2.1.5 Exchange rate pass-through and stickiness

The price of export goods is determined exogenously for a small open economy ( $P^{x*}$ ) and its price in domestic currency is  $P^x = SP^{x*}$ . The price of import goods is also determined exogenously for a small open economy  $P^{m*}$ , but, we assume that price changes are incompletely passed-through (see Campa and Goldberg (2002) for a study on exchange rate pass-through and import prices). Using the definition:  $P^m = SP^{m*}$  and assuming partial adjustment, we obtain:

$$p_t^m = \omega(s_t + p_t^{m*}) + (1 - \omega)p_{t-1}^m \quad (32)$$

where  $\omega = 1$  indicates complete pass-through of foreign price changes.

### 2.1.6 Macroeconomic Identities

The market clearing conditions are:

$$\begin{aligned}
\left( Y_t^x - \frac{\phi_x}{2K_t^x} (I_t^x)^2 \right) &= (C_t^x + X_t + I_t^x) \\
\left( Y_t^m - \frac{\phi_m}{2K_t^m} (I_t^m)^2 \right) &= (C_t^m - M_t + I_t^m) \\
Y_t^n &= (C_t^n + G_t)
\end{aligned} \quad (33)$$

Real gross domestic product is given as:

$$y = \frac{1}{P_t} \left[ P_t^x \left( Y_t^x - \frac{\phi_x}{2K_t^x} (I_t^x)^2 \right) + P_t^m \left( Y_t^m - \frac{\phi_m}{2K_t^m} (I_t^m)^2 \right) + P_t^n Y_t^n \right] \quad (34)$$

## 2.2 Terms of Trade

The only shocks explored in this paper comes from the terms of trade. Specifically:

$$p_t^{x*} = .9 p_{t-1}^{x*} + \varepsilon_t^{x*}; \quad \varepsilon_t^{x*} \sim N(0, 0.01)$$

where lower case denotes the log of the world export price,  $p_t^{x*}$ . The evolution of the prices mimic actual data generating processes, with a normally distributed innovation with standard deviation set at 0.01. We assume that  $p_t^{m*}$  is constant, with normalization  $p_t^{m*} = 0$ , so that the stochastic process describes a mean-reverting terms of trade process.

The simulations are also conducted assuming that the domestic price of export goods fully reflect the exogenously determined prices:

$$p_t^x = s_t + p_t^{x*} \quad (35)$$

however, the domestic price of import goods are partially passed on:

$$p_t^m = \omega(s_t + p_t^{m*}) + (1 - \omega)p_{t-1}^m \quad (36)$$

where  $\omega$  is the coefficient of exchange rate pass-through. We consider the case of low ( $\omega = 0.3$ ) and high ( $\omega = 0.9$ ) pass-throughs (see estimates cited in Campo and Goldberg (2002)).

Thus, this is a simulation study about the design of monetary policy for an economy subjected to relative price shocks. The log of the terms of trade ( $j$ ), the real exchange rate ( $z$ ) and the aggregate consumption price deflator ( $p$ ) becomes respectively:

$$j = s_t + p_t^{x*} - \omega(s_t + p_t^{m*}) - (1 - \omega)p_{t-1}^m \quad (37)$$

$$z = \alpha_x(s_t + p_t^{x*}) + (1 - \alpha_x) [\omega(s_t + p_t^{m*}) + (1 - \omega)p_{t-1}^m] \quad (38)$$

$$p_t = \alpha_f.z \quad (39)$$

since we are normalizing on the price of non-traded-goods -  $p_t^n = 0$ . These equations show that when  $\omega = 1.0$ , the terms of trade reflect shocks in both export and import good sectors ( $j = p_t^{x*} - p_t^{m*}$ ) but when  $\omega = 0.0$ , the terms of trade reflect shocks in the export sector only through the exchange rate ( $j = s_t + p_t^{x*} - p_{t-1}^m$ ;  $p_{t-1}^m$  is the starting value for import goods.)

## 2.3 Monetary Authority

We formulate the behavior of the monetary authority in a Taylor rule setup and then in a linear quadratic control framework. In the second approach we assume that the Central Bank adopts practices consistent with optimal control models, but with time-varying preferences.

### 2.3.1 Taylor Rule Framework

We are concerned with two alternative Taylor rules, one with only annualized price inflation targeting, for the desired interest rate,  $\bar{i}_t$ , and one with price and Q-growth. For the pure inflation targeting regime, the desired interest rate has the following form:

$$\bar{i}_t = i^* + \phi_\pi(\hat{\pi}_t - \tilde{\pi}), \quad \phi_\pi > 1 \quad (40)$$

with

$$\pi_t = \left( \frac{P_t}{P_{t-4}} \right) - 1 \quad (41)$$

representing actual inflation, and  $\hat{\pi}_t$  the forecast of inflation based on central bank learning. The desired long run inflation rate is given by  $\tilde{\pi}$ .

The actual interest rate follows the following partial adjustment mechanism:

$$i_t = \theta i_{t-1} + (1 - \theta)\bar{R} \quad (42)$$

This formulation of the Taylor rule is similar to the rule estimated by Judd and Rudebusch (1998).

In the goods price and asset price inflation regime, we change the formulation for the desired interest rate, to include the forecast of Q-growth,  $\hat{\eta}_t$  and a desired target rate,  $\tilde{\eta}$ :

$$\bar{i}_t = i^* + \phi_\pi(\pi_t - \tilde{\pi}) + \phi_\eta(\hat{\eta}_t - \tilde{\eta}), \quad \phi_\pi > 1, \phi_\eta > 0 \quad (43)$$

$$\eta_t = \left( \frac{Q_t}{Q_{t-4}} \right) - 1 \quad (44)$$

For simplicity, with no long run inflation nor trends in terms of trade, we set the targets for inflation and growth to be zero. Hence,  $\tilde{\pi} = \tilde{\eta} = 0$ .

Note that monetary policy in both instances operates symmetrically. The same weight applies to inflation or growth, with different signs, when they are above or below their targets.

### 2.3.2 Linear Quadratic Control with State-Contingent Preferences

In this setup, the Central Bank chooses an optimal interest rate reaction function (47), given its loss function equation (45) and its perception of the evolution of the state variables given by equation (46).

$$\Lambda = \lambda(x_t - x^*)^2 \quad (45)$$

$$x_t = \Gamma_{1t}\mathbf{L}(x_{t-1}) + \Gamma_{2t}i_t + e_t \quad (46)$$

$$\bar{i}_{t+1} = h(\widehat{\Gamma}_t, \lambda)\mathbf{L}(x_t) \quad (47)$$

where  $h(\widehat{\Gamma}_t, \lambda)$  is the solution of the optimal linear quadratic “regulator” problem,  $\mathbf{L}$  is the lag operator,  $e_t$  a random shock to the process determining  $x_t$ , with the control variable  $\bar{i}_t$  solved as a feedback response to the state variables. We assume that once the central bank solves for the control or desired interest rate, it engages in smoothing behavior:

$$i_t = \theta i_{t-1} + (1 - \theta)\bar{i}_{t+1} \quad (48)$$

We assume, perhaps more realistically, that the monetary authority does not know the exact nature of the private sector model. Instead it learns and updates the state-space model equation (46) which underpins its calculation of the optimal interest rate policy period by period.<sup>5</sup> In other words, at each period time  $t$ , the Central Bank updates its information about  $x_t$ , re-estimates the state-space system to obtain new estimates for  $\widehat{\Gamma}_t = \{\Gamma_{1t}, \Gamma_{2t}\}$  which it then uses to determine the optimal interest rate  $i_{t+1}$ , based on its feedback function of the state variables  $x_t$ .

For this paper, two different policy scenarios are considered - a pure inflation targeting policy stance and an inflation-growth policy stance. The weights for inflation and output growth in the loss function depend on the conditions at time  $t$ .

- Pure inflation targeting

In the pure inflation target case, the monetary authority estimates or learns the evolution of inflation as a function of its own lag as well as the interest rate:

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<sup>5</sup>See, Bullard and Mitra (2002) and Evans and Honkapohja (2003) for a discussion of private sector learning.

$$\Lambda_1 = \lambda_{1t}(\pi_t - \pi^*)^2 \quad (49)$$

$$x_t = \sum_{j=0}^k \Gamma_{1t,j} x_{t-j-1} + \Gamma_{2t} i_t + e_t \quad (50)$$

$$\bar{i}_{t+1} = \sum_{j=0}^k h(\hat{\Gamma}_{1t,j}, \Gamma_{2t}, \lambda_{1t}) x_{t-j-1} \quad (51)$$

$$x_t = \pi_t \quad (52)$$

where  $\pi_t = (P_t/P_{t-4}) - 1$  represents an annualized rate of inflation,  $\pi^*$  is the target for inflation, and  $k$  is the number of lags for forecasting the evolution of the state variable,  $x$ .

The weight on the loss function,  $\lambda_t = \{\lambda_{1t}\}$  is chosen to reflect the Central Bank's concerns about inflation and is illustrated in Table I.

Table I: Policy Weights	
Inflation Target	
$ \pi_t  < 0$	$\lambda_1 = 0.0$
$ \pi_t  \geq 0$	$\lambda_1 = 1.0$

In this pure anti-inflation scenario, if the absolute value of inflation is below the target level  $\pi^*$  then the government does not optimize - the desired interest rate remains at its level:  $\bar{i}_{t+1} = \bar{i}_t$ . This is the "no intervention", "do no harm" or cautionary monetary policy case. However, if inflation is above or below the target rate, the monetary authority implements its optimal interest policy  $\bar{i}_{t+1} = h(\hat{\Gamma}_t, \lambda_{1t})x_t$ .

- CPI and asset-price inflation targeting

In the inflation/growth scenario, the central bank learns the evolution of CPI and asset-price inflation as functions of their own lags and the interest rate.

$$\Lambda_2 = \lambda_{1t}(\pi_t - \pi^*)^2 + \lambda_{2t}(\eta_t - \eta^*)^2 \quad (53)$$

$$x_t = \sum_{j=0}^k \Gamma_{1t,j} x_{t-j-1} + \Gamma_{2t} i_t + e_t \quad (54)$$

$$\bar{i}_{t+1} = \sum_{j=0}^k h(\hat{\Gamma}_{1t,j}, \Gamma_{2t}, \lambda_{1t}) x_{t-j-1} \quad (55)$$

$$x_t = [\pi_t, \eta_t] \quad (56)$$

where  $\eta_t = (Q_t/Q_{t-4}) - 1$ , the annualized rate of growth of asset prices,<sup>6</sup> and  $\eta^*$  represents the target for this rate of growth. In this case, we have a bivariate forecasting model for the evolution of the state variables,  $\pi_t$  and  $\eta_t$ , with an equal number of lags. In this case, the coefficient matrix  $\Gamma_{1t,j}$ , for six lags, is a 12 by 2 recursively updated matrix coefficients, representing the effects of lagged inflation and growth on current inflation and growth.

The weights reflecting the Central Bank's preference for inflation and growth in this policy scenario are summarized in Table II.

Table II: Policy Weights		
Inflation and Q-Growth Targets		
Inflation	Q-Growth	
	$ \eta_t  < 0$	$ \eta_t  \geq 0$
$ \pi_t  < 0$	$\lambda_1 = 0.0$	$\lambda_1 = 0.0$
	$\lambda_2 = 0.0$	$\lambda_2 = 1.0$
$ \pi_t  \geq 0$	$\lambda_1 = 1.0$	$\lambda_1 = 0.5$
	$\lambda_2 = 0.0$	$\lambda_2 = 0.5$

In this setup the central banks shows a strong anti-inflation or anti-deflation bias, in both the CPI and in the share price index. In other words, the central bank worries a lot about absolute inflation being above targets, either in the CPI, or in Q, or both. But it worries little about inflation or deflation in either of these variables if the absolute value of the rate is below targets. There are thus four sets of outcomes: (1) if both inflation and asset-price growth are below the target levels, then the government does not optimize, it follows a "do no harm" cautionary approach; (2) if inflation is above the target rate, and asset-price inflation is below the target, the monetary authority puts strong weight on CPI inflation; (3) if both asset-price growth and inflation are above targets, strong weight is put on both variables; and (4) if only asset-price growth is above its target, the central bank puts strong weight on the asset-price growth target.

Uncertainly about the underlying longer-term inflation in the CPI or share price is a rationale for our approach. Swanson (2004), for example, poses the issue as a "signal extraction" problem for a policy-maker, with diffuse-middle priors. In our framework, policymakers are uncertain about the underlying rate of inflation or deflation in the range  $[-2\ 2]$  percent, so they are unwilling to revise estimates within this interval. As observed inflation

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<sup>6</sup>We have obviously assumed that the central bank computes and monitors past values of Tobin's q based on data about market values and replacement costs.

or deflation moves further away from their prior, they respond not at all for small surprises in the realized inflation rate but respond "very aggressively at the margin" [Swanson (2004): p.7]. The main feature of this type of learning is "policy attenuation for small surprises" followed by "increasingly aggressive responses" at the margin [Swanson (2004): p.7].

Corresponding to each scenario, the Central Bank optimizes a loss function  $\Lambda$ , and actively formulates its optimal interest-rate feedback rule. It also acts at time  $t$  as if its estimated model for the evolution of inflation and asset-price growth is true "forever". However, as Sargent (1999) points out in a similar model, the monetary authority's own procedure for re-estimation "falsifies" this pretense as it updates the coefficients  $\{\Gamma_{1t}, \Gamma_{2t}\}$ , and solves the linear quadratic regulator problem for a new optimal response rule of the interest rate to the evolution of the state variables at every point of time  $t$ .

### 3 Calibration and Solution Algorithm

In this section we discuss the calibration of parameters, initial conditions, and stochastic processes for the exogenous variables of the model. We then summarize the parameterized expectations algorithm (PEA) used for solving the model.

#### 3.1 Parameters and Initial Conditions

The parameter settings for the model appear in Table III.

Table III: Calibrated Parameters	
Consumption	$\gamma = 1.5$ , $\beta = 0.009$ $\alpha_x = 0.5$ , $\alpha_f = 0.5$
Production	$\theta_m = 0.7$ , $\theta_x = 0.3$ $\delta_x = \delta_m = 0.025$ $\phi_x = \phi_m = 0.03$

Many of the parameter selections follow Mendoza (1995, 2001). The constant relative risk aversion  $\gamma$  is set at 1.5 (to allow for high interest sensitivity). The shares of non-traded goods in overall consumption is set at 0.5, while the shares of exports and imports in traded goods consumption is 50 percent each. Production in the export goods sector is more capital intensive than in the import goods sector.

The initial values of the nominal exchange rate, the price of non-tradeables and the price of importable and exportable goods are normalized at unity while the initial values for the stock of capital and financial assets (domestic and foreign debt) are selected so that they are compatible with the implied steady state value of consumption,  $\bar{C} = 2.02$ , which is given by the interest rate and the endogenous discount factor. The values of  $\bar{C}^x$ ,  $\bar{C}^m$ , and  $\bar{C}^n$  were calculated on the basis of the preference parameters in the sub-utility functions and the initial values of  $B$  and  $L^*$  deduced. The steady-state level of investment for each sector is equal to the depreciation rate multiplied by the respective steady-state capital stock.

Similarly, the initial shadow price of capital for each sector is set at its steady state value. The production function coefficients  $A^m$  and  $A^x$ , along with the initial values of capital for each sector, are chosen to ensure that the marginal product of capital in each sector is equal to the real interest plus depreciation, while the level of production meets demand in each sector. Since the focus of the study is on the effects of terms of trade shocks, the domestic productivity coefficients were fixed for all the simulations.

Finally, the foreign interest rate  $i^*$  is also fixed at the annual rate of 0.04. In the simulations, the effect of initialization is mitigated by discarding the first 100 simulated values.

### 3.2 Solution Algorithm and Constraints

Following Marcet (1988, 1993), Den Haan and Marcet (1990, 1994), and Duffy and McNelis (2001), the approach of this study is to parameterize decision rules for  $C_{t,t}$ ,  $S_t$ ,  $I_t^x$ ,  $I_t^m$  with nonlinear approximations or functional forms  $\psi^S, \psi^C, \psi^{I^x}$ , and  $\psi^{I^m}$  which minimize the Euler equation errors given in 26 through ??:

$$C_t = \psi^C(\mathbf{x}_{t-1}; \Omega_C) \quad (57)$$

$$S_t = \psi^S(\mathbf{x}_{t-1}; \Omega_S) \quad (58)$$

$$I_t^x = \psi^{I^x}(\mathbf{x}_{t-1}; \Omega_{Q^x}) \quad (59)$$

$$I_t^m = \psi^{I^m}(\mathbf{x}_{t-1}; \Omega_{Q^m}) \quad (60)$$

The symbol  $\mathbf{x}_{t-1}$  represents a vector of observable state variables known at time  $t$ : the terms of trade, the capital stock for exports and manufacturing goods, the level of foreign debt and the interest rate, relative to their steady state values:

$$\mathbf{x}_t = \ln \left[ P_t^{x*} / P_t^{m*}, \frac{K_{t-1}^x}{\bar{K}^x}, \frac{K_{t-1}^m}{\bar{K}^m}, \frac{L_{t-1}^*}{\bar{L}^*}, \frac{1 + i_{t-1}}{1 + \bar{i}} \right] \quad (61)$$



The symbols  $\Omega_\lambda, \Omega_S, \Omega_{Q^x}$ , and  $\Omega_{Q^m}$  represent the parameters for the expectation function, while  $\psi^C, \psi^E, \psi^{Q^x}$  and  $\psi^{Q^f}$  are the expectation approximation functions.

Judd (1996) classifies this approach as a “projection” or a “weighted residual” method for solving functional equations, and notes that the approach was originally developed by Williams and Wright (1982, 1984, 1991). The functional forms for  $\psi^E, \psi^C, \psi^{Q^x}$ , and  $\psi^{Q^f}$  are usually second-order polynomial expansions [see, for example, Den Haan and Marcet (1994)]. However, Duffy and McNelis (2001) have shown that neural networks can produce results with greater accuracy for the same number of parameters, or equal accuracy with fewer parameters, than the second-order polynomial approximation.

We use a neural network specification with two neurons for each of the decision variables. The neurons take on values between  $[0, 1]$  for a logsigmoid function and between  $[-1, 1]$  for a tansigmoid function. The functions were then weighted by coefficients, and an exponent or anti-log function applied to the final value. The functions were multiplied by the steady state values to ensure steady state convergence.

The model was simulated for repeated parameter values for  $\{\Omega_C, \Omega_S, \Omega_{Q^x}, \Omega_{Q^m}\}$  and convergence obtained when the expectation errors were minimized. In the algorithm, the following non-negativity constraints for consumption and the stocks of capital were imposed by the functional forms of the approximating functions:

$$C_t^x > 0, \quad K_t^x > 0, \quad K_t^m > 0 \quad (62)$$

$$I_t^x > 0, \quad I_t^m > 0 \quad (63)$$

$$i_t > 0 \quad (64)$$

The usual no-Ponzi game applies to the evolution of real government debt and foreign assets, namely:

$$\lim_{t \rightarrow \infty} B_t \exp^{-it} = 0, \quad \lim_{t \rightarrow \infty} L_t^* \exp^{-(i^* + \Delta_{S_{t+1}})t} = 0 \quad (65)$$

We keep the foreign asset or foreign debt to GDP ratio bounded, and thus fulfill the transversality condition, by imposing the following constraint on the parameterized expectations algorithm:<sup>7</sup>

$$\sum \left( \frac{|S_t L_t^*| / P_t}{y_t} \right) < \tilde{L}, \quad \sum \left( \frac{|B_t| / P_t}{y_t} \right) < \tilde{B} \quad (66)$$

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<sup>7</sup>In the PEA algorithm, the error function will be penalized if the foreign debt/gdp ratio is violated. Thus, the coefficients for the optimal decision rules will yield debt/gdp ratios which are well below levels at which the constraint becomes binding.

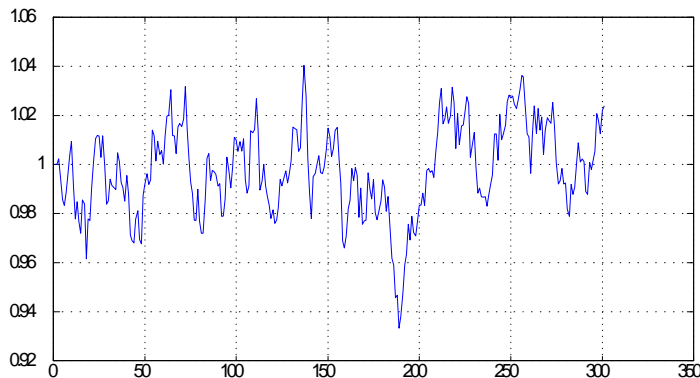


Figure 1: One realization of the terms of trade shocks

where  $\tilde{L}$ , and  $\tilde{B}$  are the critical foreign and domestic debt ratios. In the simulation, the fiscal authority will exact lump sum taxes from non-traded goods sector in order to run a surplus and “buy back” domestic debt if it grows above a critical foreign or domestic debt/GDP ratio. We set the critical level of government debt to be zero, so that government bonds were always repurchased with lump-sum taxes whenever a deficit appeared.

## 4 Simulation Analysis

### 4.1 Base-Line Results

The aim of the simulations is to compare the outcome for inflation, growth and welfare for the two policy scenarios - inflation targeting ( $\pi$ ) and inflation and  $Q$ -growth targeting ( $\pi$  and  $\eta$ ). To ensure that the results are robust, we conducted 1000 simulations (each containing a time-series of 150 realizations of terms of trade shocks) for the case of relatively low pass-through ( $\omega = .3$ ).

Figure 1 shows the simulated paths for one time series realization of the exogenous terms of trade index. Figure 2 pictures time series for consumption, inflation, investment, the current account, the interest rate, and  $Q$  under the Taylor rule setup, while Figure 3 pictures the paths of the same variables under the linear quadratic control setup.

The simulated values for the key variables (consumption, inflation, investment, current account, interest rate, and  $Q$  for the export sector) are well-behaved. Figures 2 and 3 presents the evolution of these variables for the two scenarios, the smooth curves for pure cpi inflation targetings, and the dotted curves for

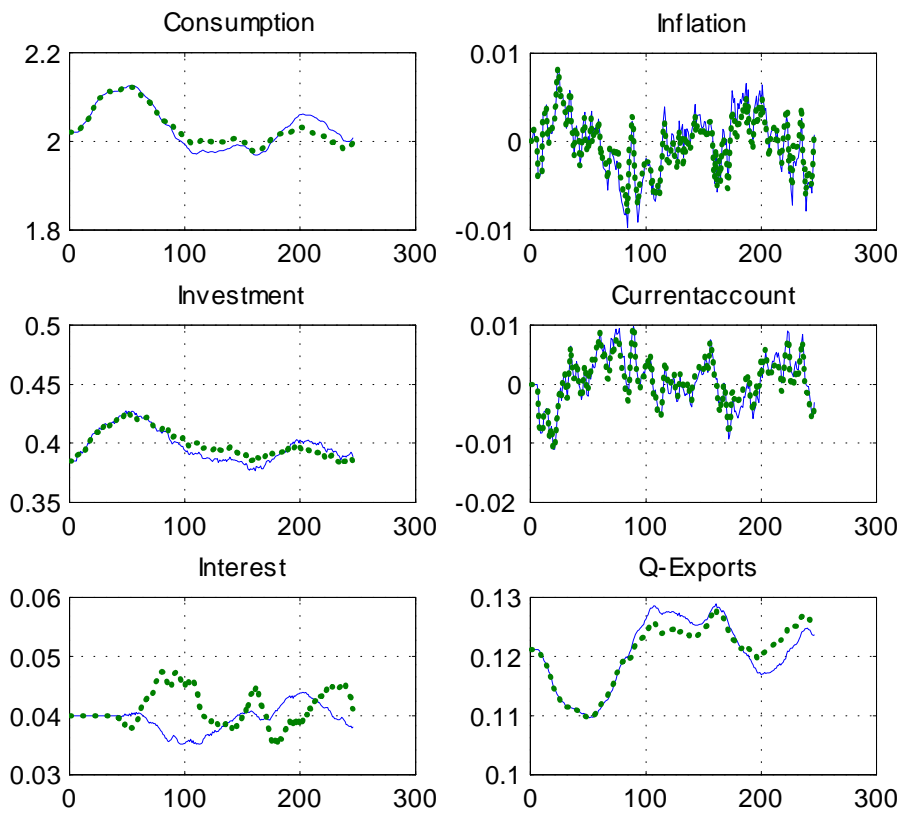


Figure 2: Times Series Under Taylor Rule Framework

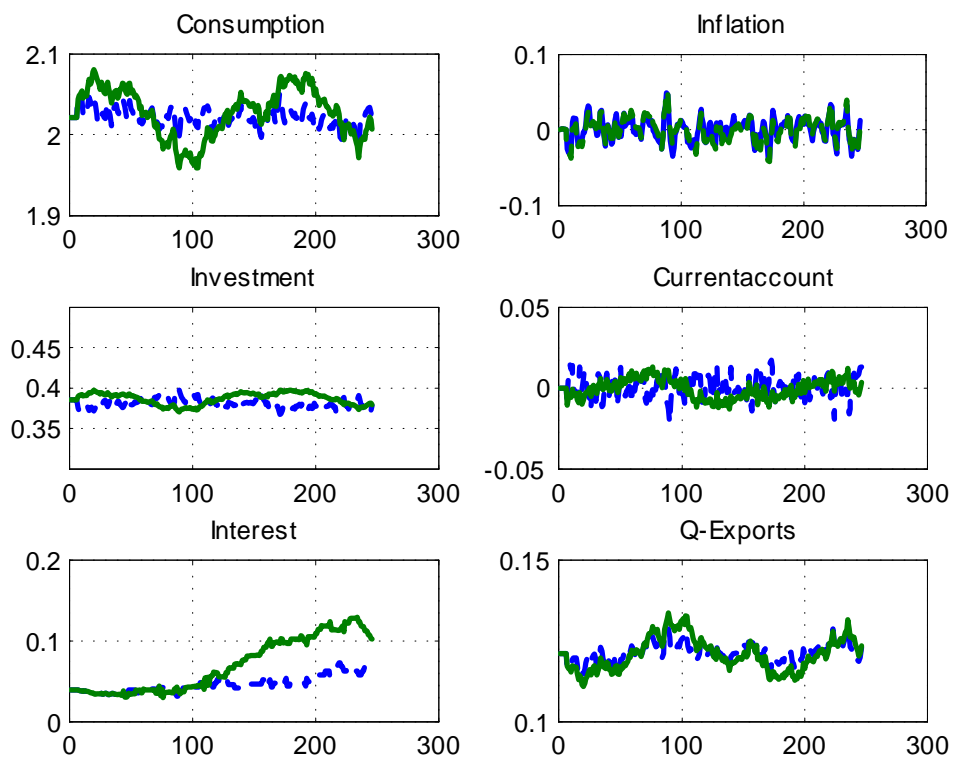


Figure 3: Time Series under Linear Quadratic Control

cpi and Q-inflation targeting, for a relatively low pass-through value, with  $\omega = .3$ . In general, despite the large swings in the terms of trade index, consumption is more stable with the inclusion of Q-growth targeting under the pure Taylor framework and the quadratic control setup. Also, while the introduction of Q-growth targeting for the monetary authority results, not surprisingly, in lower Q, investment and current-account volatility, in the linear quadratic control setup.

To ascertain which policy regime yields the higher welfare value, we examined the distribution of the welfare outcomes of the different policy regimes for 1000 different realizations of the terms of trade shocks. Before presenting these results, we evaluated the accuracy of the simulation results as well as the "rationality" of the learning mechanism.

## 4.2 Accuracy Test

The accuracy of the simulations may be checked by the Judd-Gaspar statistic. This test makes use of the Euler equation error for consumption:

$$\vartheta_{t+1}\Lambda_{t+1}\frac{1}{P_{t+1}}(1+i_t) - \vartheta_t\Lambda_t\frac{1}{P_t} = \nu_t \quad (67)$$

To assess the accuracy of the simulation, Judd and Gaspar propose taking the mean of the absolute value of  $\nu_t$  relative to  $c_t$ . For realization  $j$ , with size  $T$ , we have the following accuracy measure:

$$JG_{mean}^{(j)} = \sum_{t=1}^T \frac{|\nu_t|}{c_t} / T \quad (68)$$

A more stringent measure is to take the maximum value of  $\nu_t$  relative to  $c_t$  for each realization:

$$JG_{max}^{(j)} = \max \left[ \frac{|\nu_t|}{c_t} \right] \quad (69)$$

This statistic is a measure of the mean or maximum error relative to a dollar spent on consumption for each realization.

Table IV and V present the means and standard deviations of the Judd-Gaspar accuracy measures based on the maximum absolute error measures. We see that the average size of the accuracy error is less than a third of a cent per dollar spent on consumption. We see that the mean maximum error measures are in the order of .01 to .03 of one cent spent on consumption.

Table IV(a): Judd-Gaspar Accuracy Statistic: Maximum Absolute Error Taylor Rule Framework Mean and Standard Deviation (in parenthesis)		
Policy Regime	Pass Through Coefficient	
	$\omega = 0.9$	$\omega = 0.3$
Inflation Targeting	1.649e-4 (4.134e-5)	3.111e-4 (8.045e-5)
Inflation/Q-Growth Targeting	1.651e-4 (4.113e-5)	3.165e-4 (7.904e-5)

Table IV(b): Judd-Gaspar Accuracy Statistic: Maximum Absolute Error Linear Quadratic Framework Mean and Standard Deviation (in parenthesis)		
Policy Regime	Pass Through Coefficient	
	$\omega = 0.9$	$\omega = 0.3$
Inflation Targeting	0.0020 (3.5510e-4)	0.0020 (3.6704e-4)
Inflation/Q-Growth Targeting	0.0014 (2.1853e-4)	0.0022 (3.4297e-4)

### 4.3 Learning and Quasi-Rationality

In our model, the central bank learns the underlying process for inflation in the pure inflation-target regime and the underlying processes for inflation and growth in the inflation-growth target regime. The learning takes place by updating recursively the least-squares estimates of a vector autoregressive model.

Marcet and Nicolini (1997) raise the issue of reasonable rationality requirements in their discussion of recurrent hyperinflation and learning behavior. In our model, a similar issue arises. Given that the only shocks in the model are recurring terms of trade shocks, with no abrupt, unexpected structural changes taking place, the learning behavior of the central bank should not depart, for too long, from the rational expectations paths. The central bank, after a certain period of time, should develop forecasts which converge to the true inflation and growth paths of the economy, unless we wish to make some special assumption about monetary authority behavior.

Marcet and Nicolini discuss the concepts of “asymptotic rationality”, “epsilon-delta rationality” and “internal consistency”, as criteria for “boundedly rational” solutions. They draw attention to the work of Bray and Savin (1986). These authors examine whether the learning model rejects serially uncorrelated prediction errors between the learning model and the rational expectations solution, with the use of the Durbin-Watson statistic. Marcet and Nicolini point out that the Bray-Savin method carries the flavor of “epsilon-delta” rationality in the sense that it requires that the learn-

ing schemes be consistent “even along the transition” [Marcet and Nicolini (1997): p.16, footnote 22].

Following Bray and Savin, we use the Durbin-Watson statistic to examine whether the learning behavior is “boundedly rational”. Table VI and VII give the Durbin-Watson statistics for the inflation and Q-growth forecast errors of the central bank, under both policy regimes, for the Taylor rule and Linear Quadratic setup. In all of the cases, we see that the learning behavior for more than 90% of the realizations does not violate near rationality.

Table V(a): Durbin-Watson Statistics for Forecast Errors Taylor Rule Framework Percentage in Lower and Upper Critical Region				
Policy Regime	$\omega = 0.9$		$\omega = 0.3$	
	Inflation	Q-Growth	Inflation	Q-Growth
Inflation Targeting	.001/0	—	.005/0	—
Inflation/ Q-Growth Targeting	0/0	0/0	.018/0	.019/0

Table V(b): Durbin-Watson Statistics for Forecast Errors, Linear Quadratic Framework Percentage in Lower and Upper Critical Region				
Policy Regime	$\omega = 0.9$		$\omega = 0.3$	
	Inflation	Q-Growth	Inflation	Q-Growth
Inflation Targeting	0.038/0	—	0.066/0	—
Inflation/ Q-Growth Targeting	0.071/0	0.040/0	0.040/0	0.063/0

#### 4.4 Comparative Results

This section summarizes the results for 1000 alternate realizations of the terms-of-trade shocks (each realization contains 150 observations), for the Taylor rule and the Linear Quadratic frameworks for conducting monetary policy. Table VIII and IX presents the first two moments of the 1000 sample means for consumption, inflation, investment, the current account, the policy instrument - the interest rate - Q for exports, and the welfare measure, for the high and low pass-through coefficients.<sup>8</sup>

<sup>8</sup>We do not benchmark the welfare effects with respect to the steady state welfare, since the terms of trade realizations may lead to welfare outcomes either greater or less than the steady state welfare.

#### 4.4.1 Taylor Rule Framework

Table VIII and IX presents the first two moments of the 1000 sample means for consumption, inflation, investment, the current account, the policy instrument - the interest rate - Q for exports, and the welfare measure, for the high and low pass-through coefficients.<sup>9</sup>

Table VI(a): Summary Statistics (1000 Simulations): Taylor Rule Framework First and Second Moments (in Parenthesis) of the Sample Means $\omega = .9$		
Policy Regimes		
	$\pi$	$\pi, \eta$
Consumption	2.021 (2.756e-4)	2.021(2.770e-4)
Inflation	1.877e-5 (8.742e-5)	1.880e-5 (8.754e-5)
Investment	0.3842 (0.0016)	0.3842 (0.0016)
Current Account	1.830e-4(3.726e-5)	1.812e-4 (4.013e-5)
Q-Exports	0.121(2.220e-4)	0.121(2.220e4)
Welfare Index	-11.295(0.001)	-11.295(0.001)

Table IX: Summary Statistics (1000 Simulations): Taylor Rule Framework First and Second Moments (in Parenthesis) of the Sample Means $\omega = .3$		
Policy Regimes		
	$\pi$	$\pi, \eta$
Consumption	2.207 (0.014)	2.026 (0.009)
Inflation	-7.165e-5 (3.630e-4)	-4.937e-5 (2.122e-4)
Investment	0.390(0.009)	0.388 (0.006)
Current Account	4.423e-4 (9.758e-4)	4.677e-4 (1.939e-4)
Q-Exports	0.121(.002)	0.121(.001)
Welfare Index	-11.202(0.147)	-11.218(0.154)

We see the the welfare differences between the two regimes are more apparent in the case of  $\omega = .3$  than for  $\omega = .9$ .

<sup>9</sup>We do not benchmark the welfare effects with respect to the steady state welfare, since the terms of trade realizations may lead to welfare outcomes either greater or less than the steady state welfare.



#### 4.4.2 Linear Quadratic Framework

Tables IX and XI present the corresponding results under the Linear Quadratic Framework. We notice one major difference in the welfare results from the Linear Quadratic framework: the standard deviations of consumption, investment, and welfare are lower than the corresponding measures in the pure Taylor rule framework. The reason for this difference is due to the "policy attenuation" effect in the Linear Quadratic framework, in which policy makers do not react to changes in inflation or Q-growth in the interval [-0.02, 0.02].

Table X: Summary Statistics (1000 Simulations): Linear Quadratic Framework First and Second Moments (in Parenthesis) of the Sample Means $\omega = .9$		
Policy Regimes		
	$\pi$	$\pi, \eta$
Consumption	2.0181 (.0093)	2.0214(0.0028)
Inflation	2.6086e-4 (3.2928e-4)	-3.1180e-4 (6.8123e-4)
Investment	0.3828 (0.004)	0.3843 (0.0025)
Current Account	3.0618e-4(2.0698e-4)	4.5049e-4 (2.8020e-4)
Q-Exports	0.1221(26.8056e-4)	0.1221(6.8056e-4)
Welfare Index	-11.3028(0.1088)	-11.2803(0.0347)

Table XI: Summary Statistics (1000 Simulations): Linear Quadratic Setup First and Second Moments (in Parenthesis) of the Sample Means $\omega = .3$		
Policy Regimes		
	$\pi$	$\pi, \eta$
Consumption	2.0220 (0.005)	2.0196 (0.0034)
Inflation	-1.4319e-004(5.4829e-4)	-2.1147e-4(5.7942e-4)
Investment	0.3856(0.0012)	0.3830 (0.0029)
Current Account	-7.3101e-4 (0.0013)	5.5461e-4 (1.3833e-4)
Q-Exports	0.1219(9.6224e-4)	0.1221(6.8056e-4)
Welfare Index	-11.2816(0.0771)	-11.2911(0.0347)

#### 4.4.3 Comparative Volatility Measures

Table XII and XIII give the mean volatility measures (given by the coefficients of variation) for 1000 realizations, for both the Taylor rule and Linear Quadratic policy frameworks. We see the volatility reduction in consumption, inflation, and Q in both policy frameworks if the monetary authority targets Q-growth as well as inflation.

Comparing Tables XII and XIII, we see that switching from a pure Taylor rule framework with pure inflation targeting to a Linear Quadratic framework with inflation and Q-growth targets (with state-contingent preferences) leads to a reduction in volatility in consumption from .018 to .005, and in Q-growth from .0373 to .0203. These results show a volatility correlation between consumption and Q-growth across policy regimes. We also note that the interest rate volatility measure is higher in the Linear Quadratic control case. The reason for this increase is that interest rate movements are much less continuous, due to state-contingent preferences, than in the pure Taylor rule framework.

Table XII: Summary Statistics (1000 Simulations)		
Mean Volatility Measures for Macro Indicators		
Taylor Rule Framework		
$\omega = .3$		
Policy Regimes		
	$\pi$	$\pi, \eta$
Consumption	0.0180	0.0129
Inflation	0.0032	0.0026
Investment	0.0322	0.0222
Current Account	0.0019	0.0018
Interest	0.0386	0.0597
Q-Exports	0.0373	0.0281

Table XIII: Summary Statistics (1000 Simulations)		
Mean Volatility Measures for Macro Indicators		
Linear Quadratic Framework		
$\omega = .3$		
Policy Regimes		
	$\pi$	$\pi, \eta$
Consumption	0.0099	0.0059
Inflation	0.0152	0.0151
Investment	0.0125	0.0152
Current Account	0.0023	0.0033
Interest	0.2654	0.2556
Q-Exports	0.0299	0.0203

## 5 Concluding Remarks

This paper has shown that there are clear trade-offs when the rate of growth of Tobin's Q is incorporated as an additional target to inflation targeting in the conduct of monetary policy. Our results show that the Central Bank can reduce the volatility of consumption if it targets asset-prices as well as consumer price inflation. In other words, compared to a strict inflation targeting regime, monetary policy with inflation and Q-growth targets insulates consumption from adverse terms-of-trade shocks since consumption does not fall as much when negative shocks are realized.

However, across a range of realizations of terms of trade shocks, the welfare effects, measured in terms of the present value of consumption utility, are not much different when Q-targets are incorporated with the inflation targets.

To be sure, we did not introduce shocks in this model in the form of asset price bubbles. We assumed that the driving force for Q growth comes from fundamentals. Given that the Central bank has to learn the laws of motion of Q-growth as well as inflation, and set policy on the basis of longer-term laws of motion of these variables, it seems reasonable to start with Q driven solely by fundamentals. We leave to further research an examination of the robustness of our results to the incorporation of bubbles and other non-fundamental asset-price shocks.

Finally, we admit that our time-varying state-contingent interest-rate rules, coming from uncertainty about the true laws of motion of consumer and asset-price inflation dynamics, generated by a nonlinear stochastic model, is a step away from the design of a nonlinear interest-rate rule, in which the

laws of motion are approximated by nonlinear approximation methods. Non-linear policy rules may show even more beneficial effects from a cautionary monetary policy aimed at asset price as well as consumer price inflation.

E. Gerald Corrigan, former head of the Federal Reserve Bank of New York, stated at the 2002 Per Jacobsson Lecture that "the single-minded pursuit of a policy to burst an asset price bubble at any cost seems to me to be fundamentally incompatible with sound and sensible monetary policy" [Corrigan (2002) p. 10]. In the spirit of Summers' warning, the better way to target asset price inflation, and reduce consumption volatility, is through a less preemptive and more cautious approach. Our results are consistent with the position of Corrigan (2002). While objecting to specific asset-price targeting, Corrigan does admit that a case can be made for a policy "tilt" when there are sharp and persistent increases in asset prices [Corrigan (2002): p. 10]. Such a tilt takes place in our framework when asset price changes cross a critical threshold.

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