Aiming for the Bull's Eye: Uncertainty and Inertia in Monetary Policy*

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Abstract

We study the implications of uncertainty for inflation targeting. We apply multiplicative uncertainty to a standard forward looking model and demonstrate Brainard's attenuation effect. But the result as monetary authorities become naturally more cautious at the same time monetary objectives are seldom achieved. We therefore attempt to find a monetary rule that reaches the objectives set more often and improves the welfare of the Central Bank. To do that, we assume that private sector expectations are subject to differentiated information, thereby introducing inertia in the system. Such a rule is the result of a new algorithm that we put forward, in which the inflation target is state contingent. The Central Bank sets therefore (as an auxiliary step), a variable inflation target that depends on both the degree of uncertainty as well as the shocks that occur each time. We show that such an optimisation procedure helps the CB attain its objectives more often, thereby reducing the losses incurred. Moreover, and as a corollary to such an approach, the rule derived is *ex ante* neutral to the degree of uncertainty.

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1 Introduction

The benefits of inflation targeting in the Svensson (1999) sense amount to providing a nominal anchor for the private sector to infer policies with, in order to formulate expectations with greater accuracy. For the Central Bank (CB) on the other hand, inflation targeting provides an implicit commitment mechanism which increases its cost of deviating from announced targets and hence discourages it from doing so. The economy on the whole benefits from greater transparency because it leads to greater credibility and by consequence to effective monetary policies. From a political economy standpoint therefore, the literature associates the concept of inflation targeting with greater transparency and hence with more credible and effective policies. By the same token, a central bank that fails to achieve the target that it sets (and announces) will be penalised with a loss in credibility and hence a subsequent reduction in the ability to pursue its objectives. "It appears that for monetary policy makers, announcements alone are not enough; the only way to gain credibility is to earn it", (Bernanke and Mishkin, 1997).
We analyse the effects of inflation targeting in an economy characterised by parameter uncertainty, as modelled by Brainard, (1967). In describing the attenuation effect put forward by Brainard, we observe that as the contribution of policy reduces in the presence of multiplicative uncertainty, that of expectations increases proportionally to the prevailing degree of that uncertainty. This in turn implies that the role expectations are formed becomes important in the presence of uncertainty. Naturally, if the Central Bank operates under commitment, then expectations are anchored by the level of inflation the Central Bank aims at. However, when analysing monetary policy in a discretionary environment, in which the Central Bank operates period by period, then expectations are parametric to its actions. The assumption then of rational expectations is going to"force" the private sector to adjust their expectations such that the outcome is consistent with the intentions of the Central Bank. However, we will introduce the argument put forward by Morris and Shin (2006) in which they argue that in the presence of differential information, even if a very small proportion of people is backward looking in the way they form expectations, then their beliefs prevail. This implies that expectations are then backward looking altogether and policy is unable to close the gap between current inflation and the objective. We will add to that, that this inability is made worse in the presence of uncertainty and therefore, attaining the target becomes increasingly more difficult. As a result, we analyse two issues: *first*, if there is some value in attaining the target, then we aim to find an algorithm that will both achieve it on average, as well as still operate in an optimisation framework, such that the procedure remains transparent to the public. We will thus identify a two-step algorithm. In the first step, the central bank deviates from the target in order to reactivate the instrument and only in the second does it aim for the actual target itself. The two-step procedure amounts therefore, to the Central bank aiming *for* the bull's eye, and not directly *at* it. *Second*, we identify the conditions of uncertainty under which such an algorithm can prove superior to

the Brainard result. This is important in an inflation targeting framework as announcing a target that is unlikely to be achieved is not necessarily increasing one's credibility (Posen, 2002).
The paper is organised as follows. Sections 2 and 3 discuss the model under certainty and multiplicative uncertainty respectively. Section 4 introduces agents with differential information and how that affects their expectations. Section 5 then derives a two-step algorithm to inflation targeting and with the aid of numerical simulations, section 6 discusses when such an algorithm is beneficial. Section 7 concludes.

2 The Model

Most of the attempts to examine the effects of uncertainty in a dynamic framework rely on a backward looking set-up (Söderstöm, 2002, Srour, 1999). The somehow surprising result, from the point of view of inflation targeting proponents, is that uncertainty in the structure of the economy implies that achieving the target is not optimal as it may lead to instability in the system. This seems at odds with the general perception that the main advantage of inflation targeting is that it stabilises expectations (and the evidence which verifies that, see Johnson, 2002 and Levin et al, 2003). But this is relevant, as already mentioned, only if expectations are an important determinant in the economic system. Applying a standard forward looking New Keynesian model on the other hand, as developed in Clarida Gali and Getler (1999) and Woodford (2004) and used in similar context by Giannoni (2002), expectations play again an active role (Woodford, 2003). We this approach therefore, where our economy is thus described by the following pair of log-linear relations in deviation from the steady state:

$$\pi_t = \beta E_t \pi_{t+1} + \alpha y_t + \varepsilon_t \tag{1}$$
$$y_t = E_t y_{t+1} - \gamma \left(i_t - E_t \pi_{t+1} \right) + \xi_t \tag{2}$$

where (1) is an expectations-augmented "AS" relation in which present inflation is a function of the private sector expectations of inflation one period ahead, and (2) is an intertemporal "IS" relation. The coefficients satisfy, $\alpha, \gamma > 0$, and $0 < \beta \leq 1$ (we assume here$\beta = 1$ for simplicity but will reintroduce the parameter in the section on simulations). The shocks are uncorrelatred autoregressive processes, i.e.:

$$\varepsilon_{t+1} = \rho \varepsilon_t + v_t, \qquad 0 < \rho < 1$$

We define the Central Bank is aiming to minimise the following objective function[1]:

$$L = \frac{1}{2} E \left\{ \left(\pi_t - \pi^* \right)^2 + y_t^2 \right\} \tag{3}$$

[1] We imply losses conditional on shocks, i.e. $L \equiv L|_{\varepsilon}$.

In evaluating policy, expectations are treated as parametric (Currie and Levine 1999) and the discretionary solution reduces the problem to a period-by-period optimization of the loss function under and (1) and (2). The discretionary solution of the problem is therefore the following:

$$L = \frac{1}{2} E \left\{ \left(E_t \pi_{t+1} + \alpha y_t + \varepsilon_t - \pi^* \right)^2 + y_t^2 \right\}$$

We solve under the AS constraint only and then identify the i that is implied by the aggregate demand curve. The FOC is then:

$$\begin{aligned} \frac{\partial L}{\partial y} &= \alpha \left(E_t \pi_{t+1} + \alpha y_t + \varepsilon_t - \pi^* \right) + y_t = 0 \\ y_t \left(1 + \alpha^2 \right) &= -\alpha \left(E_t \pi_{t+1} + \varepsilon_t - \pi^* \right) \\ y_t &= \frac{\alpha}{1+\alpha^2} \pi^* - \frac{\alpha}{1+\alpha^2} \left(E_t \pi_{t+1} + \varepsilon_t \right) \end{aligned} \tag{4}$$

Substituting (4) in (1), we obtain the discretionary level of inflation:

$$\begin{aligned} \pi_t &= E_t \pi_{t+1} + \alpha \left[\frac{\alpha}{1+\alpha^2} \pi^* - \frac{\alpha}{1+\alpha^2} \left(E_t \pi_{t+1} + \varepsilon_t \right) \right] + \varepsilon_t \\ &= \frac{\alpha^2}{1+\alpha^2} \pi^* + \frac{1+\alpha^2-\alpha^2}{1+\alpha^2} E_t \pi_{t+1} + \frac{1}{1+\alpha^2} \varepsilon_t \end{aligned}$$

or

$$\pi_t = \frac{\alpha^2}{1+\alpha^2} \pi^* + \frac{1}{1+\alpha^2} E_t \pi_{t+1} + \frac{1}{1+\alpha^2} \varepsilon_t \tag{5}$$

This equation can be solved forward to obtain (under $\varepsilon_{t+1} = \rho \varepsilon_t$ and calling $\frac{1}{1+\alpha^2} = A$) a solution for inflation

$$\begin{aligned} \pi_t &= A\alpha^2 \pi^* + A E_t \pi_{t+1} + A \varepsilon_t \\ \pi_{t+1} &= A\alpha^2 \pi^* + A E_t \pi_{t+2} + A \rho \varepsilon_t \end{aligned}$$

then it follows that

$$\begin{aligned} \pi_t &= A\alpha^2 \pi^* + A \left(A\alpha^2 \pi^* + A E_t \pi_{t+2} + A \rho \varepsilon_t \right) + A \varepsilon_t \\ &= A\alpha^2 \left[1 + A + A^2 + ... \right] \pi^* + A \varepsilon_t \left[1 + A\rho + A^2 \rho^2 + ... \right] \end{aligned}$$

The two geometric series inside the square brackets are respectively equal to:

$$\begin{aligned} 1 + A + A^2 + ... &= \frac{1}{1-A} \\ 1 + A\rho + A^2\rho^2 + ... &= \frac{1}{1-A\rho} \end{aligned}$$

Therefore, equilibrium inflation is equal to

$$\pi_t = \frac{A\alpha^2}{1-A}\pi^* + \frac{A}{1-A\rho}\varepsilon_t$$

Substituting for $A = \frac{1}{1+\alpha^2}$, we obtain

$$\begin{aligned} \pi_t &= \frac{\frac{1}{1+\alpha^2}\alpha^2}{1-\frac{1}{1+\alpha^2}}\pi^* + \frac{\frac{1}{1+\alpha^2}}{1-\frac{1}{1+\alpha^2}\rho}\varepsilon_t \\ &= \frac{\frac{\alpha^2}{1+\alpha^2}}{\frac{1+\alpha^2-1}{1+\alpha^2}}\pi^* + \frac{\frac{1}{1+\alpha^2}}{\frac{1+\alpha^2-\rho}{1+\alpha^2}}\varepsilon_t \end{aligned}$$

and therefore,

$$\pi_t = \pi^* + \frac{1}{1+\alpha^2-\rho}\varepsilon_t \tag{6}$$

Under the assumption of Rational Expectations, then the level of inflation will be equal to the objective that the Central Bank sets and the persistence ρ of the supply shock ε at time t. We see next that the role of expectations in affecting the final outcome, increases with uncertainty while the role of policy diminishes, in line with Brainard's attenuation effect.

(**still to be added: derivation of interest rate rule and Taylor principle**)

3 Multiplicative uncertainty

If there is now limited knowledge about the monetary transmission mechanism then parameter α has stochastic properties which we assume to be $\alpha_t \rightarrow N\left(\bar{a}, \sigma_\alpha^2\right)$ .As previously, the CB's instrument is the nominal interest rate. The objective of the CB is expressed in term of the standard quadratic objective function in deviation of inflation from a target and output gap. The existence of uncertainty implies that (3) can now be expressed as follows:

$$L = \frac{1}{2}E\left\{\left(\bar{\pi}_t - \pi^*\right)^2 + y_t^2\left(1+\sigma_\alpha^2\right)\right\} \tag{7}$$

where $\bar{\pi}_t = \beta E_t\pi_{t+1} + \bar{\alpha}y_t + \varepsilon_t$. The last term in parenthesis is the extra cost that the CB pays for the uncertainty in the parameter structure of the model (see appendix A for derivation). Again expectations are treated as parametric and the discretionary solution reduces the problem to a period-by-period optimization of the loss function (7) under (1) and (2). The discretionary solution of the problem is based on:

$$L = \frac{1}{2}E\left\{\left(E_t\pi_{t+1} + \bar{\alpha}y_t + \varepsilon_t - \pi^*\right)^2 + y_t^2\left(1+\sigma_\alpha^2\right)\right\}$$

Like above, we solve under the AS constraint only and then identify the i that is implied by the aggregate demand curve. The FOC is:

$$
\begin{aligned}
\frac{\partial L}{\partial y} &= \bar{\alpha}\left(E_t\pi_{t+1} + \bar{\alpha}y_t + \varepsilon_t - \pi^*\right) + y_t\left(1+\sigma_\alpha^2\right) = 0 \\
y_t\left(1+\sigma_\alpha^2 + \bar{\alpha}^2\right) &= -\bar{\alpha}\left(E_t\pi_{t+1} + \varepsilon_t - \pi^*\right) \\
y_t &= \frac{\bar{\alpha}}{1+\bar{\alpha}^2+\sigma_\alpha^2}\pi^* - \frac{\bar{\alpha}}{1+\bar{\alpha}^2+\sigma_\alpha^2}\left(E_t\pi_{t+1} + \varepsilon_t\right) \qquad (8)
\end{aligned}
$$

Substituting (8) in (1), we obtain the discretionary level of inflation:

$$
\begin{aligned}
\pi_t &= E_t\pi_{t+1} + \bar{\alpha}\left[\frac{\bar{\alpha}}{1+\bar{\alpha}+\sigma_\alpha^2}\pi^* - \frac{\bar{\alpha}}{1+\bar{\alpha}^2+\sigma_\alpha^2}\left(E_t\pi_{t+1}+\varepsilon_t\right)\right] + \varepsilon_t \\
&= \frac{\bar{\alpha}^2}{1+\bar{\alpha}+\sigma_\alpha^2}\pi^* + \frac{1+\bar{\alpha}^2+\sigma_\alpha^2-\bar{\alpha}^2}{1+\bar{\alpha}^2+\sigma_\alpha^2}E_t\pi_{t+1} + \frac{1+\bar{\alpha}^2+\sigma_\alpha^2-\bar{\alpha}^2}{1+\bar{\alpha}^2+\sigma_\alpha^2}\varepsilon_t
\end{aligned}
$$

and therefore,

$$
\pi_t = \frac{\bar{\alpha}^2}{1+\bar{\alpha}^2+\sigma_\alpha^2}\pi^* + \frac{1+\sigma_\alpha^2}{1+\bar{\alpha}^2+\sigma_\alpha^2}E_t\pi_{t+1} + \frac{1+\sigma_\alpha^2}{1+\bar{\alpha}^2+\sigma_\alpha^2}\varepsilon_t \qquad (9)
$$

To solve of expectations, under the assumption of rational expectations, we iterate the equation forward (and assume just like above that $\varepsilon_{t+1} = \rho\varepsilon_t$ and calling $\frac{1}{1+\bar{\alpha}^2+\sigma_\alpha^2} = \Psi$)

$$
\begin{aligned}
\pi_t &= \Psi\bar{\alpha}^2\pi^* + \Psi\left(1+\sigma_\alpha^2\right)E_t\pi_{t+1} + \Psi\left(1+\sigma_\alpha^2\right)\varepsilon_t \\
\pi_{t+1} &= \Psi\bar{\alpha}^2\pi^* + \Psi\left(1+\sigma_\alpha^2\right)E_t\pi_{t+2} + \Psi\left(1+\sigma_\alpha^2\right)\rho\varepsilon_t
\end{aligned}
$$

then we can substitute and iterate forward

$$
\begin{aligned}
\pi_t &= \Psi\bar{\alpha}^2\pi^* + \left(1+\sigma_\alpha^2\right)\Psi\left[\Psi\bar{\alpha}^2\pi^* + \Psi\left(1+\sigma_\alpha^2\right)E_t\pi_{t+2} + \Psi\left(1+\sigma_\alpha^2\right)\rho\varepsilon_t\right] + \Psi\left(1+\sigma_\alpha^2\right)\varepsilon_t \\
\pi_t &= \Psi\bar{\alpha}^2\left[1 + \left(1+\sigma_\alpha^2\right)\Psi + \left(1+\sigma_\alpha^2\right)^2\Psi^2..\right]\pi^* + \Psi\left(1+\sigma_\alpha^2\right)\left[1 + \Psi\left(1+\sigma_\alpha^2\right)\rho + \Psi^2\left(1+\sigma_\alpha^2\right)^2\rho^2\right]\varepsilon
\end{aligned}
$$

The two geometric series inside quadratic brackets are equal to

$$
\begin{aligned}
\left[1 + \left(1+\sigma_\alpha^2\right)\Psi + \left(1+\sigma_\alpha^2\right)^2\Psi^2...\right] &= \frac{1}{1-\left(1+\sigma_\alpha^2\right)\Psi} \\
\left[1 + \Psi\left(1+\sigma_\alpha^2\right)\rho + \Psi^2\left(1+\sigma_\alpha^2\right)^2\rho^2\right] &= \frac{1}{1-\Psi\left(1+\sigma_\alpha^2\right)\rho}
\end{aligned}
$$

Therefore, equilibrium inflation is equal to

$$
\pi_t = \frac{\Psi\bar{\alpha}^2}{1-\left(1+\sigma_\alpha^2\right)\Psi}\pi^* + \frac{\Psi\left(1+\sigma_\alpha^2\right)}{1-\left(1+\sigma_\alpha^2\right)\Psi\rho}\varepsilon_t
$$

Substituting back the value for $\Psi = \frac{1}{1+\bar{\alpha}^2+\sigma_\alpha^2}$ we obtain

$$\begin{aligned}
\pi_t &= \frac{\frac{1}{1+\bar{\alpha}^2+\sigma_\alpha^2}\bar{\alpha}^2}{1-\left(1+\sigma_\alpha^2\right)\frac{1}{1+\bar{\alpha}^2+\sigma_\alpha^2}}\pi^* + \frac{\frac{1}{1+\bar{\alpha}^2+\sigma_\alpha^2}\left(1+\sigma_\alpha^2\right)}{1-\frac{1}{1+\bar{\alpha}^2+\sigma_\alpha^2}\left(1+\sigma_\alpha^2\right)\rho}\varepsilon_t \\
&= \frac{\frac{\bar{\alpha}^2}{1+\bar{\alpha}^2+\sigma_\alpha^2}}{\frac{1+\bar{\alpha}^2+\sigma_\alpha^2-1-\sigma_\alpha^2}{1+\bar{\alpha}^2+\sigma_\alpha^2}}\pi^* + \frac{\frac{\left(1+\sigma_\alpha^2\right)}{1+\bar{\alpha}^2+\sigma_\alpha^2}}{\frac{1+\bar{\alpha}^2+\sigma_\alpha^2-\left(1+\sigma_\alpha^2\right)\rho}{1+\bar{\alpha}^2+\sigma_\alpha^2}}\varepsilon_t
\end{aligned}$$

and therefore,

$$\pi_t = \pi^* + \frac{1+\sigma_\alpha^2}{\bar{\alpha}^2+\left(1+\sigma_\alpha^2\right)(1-\rho)}\varepsilon_t \tag{10}$$

To derive the interest rate rule that corresponds to the above solution, we rearrange the IS curve (2) in terms of the interest rate and substitute the solution from above, (**still to be added: derivation of interest rate rule and Taylor principle**)

3.1 The role of parameter uncertainty

Proposition 1 *The existence of parameter uncertainty reduces the role for policy and emphasises the role of expectations in determining the final outcome.*

To see this we need to compare (5) and (9)

Table 1: The Role of Policy and Expectations

	π^*	$E_t\pi_{t+1}$	ε_t
Certainty	$\frac{\alpha^2}{1+\alpha^2}$	$\frac{1}{1+\alpha^2}$	$\frac{1}{1+\alpha^2}$
Brainard Uncertainty	$\frac{\bar{\alpha}^2}{1+\bar{\alpha}^2+\sigma_\alpha^2}$	$\frac{1+\sigma_\alpha^2}{1+\bar{\alpha}^2+\sigma_\alpha^2}$	$\frac{1+\sigma_\alpha^2}{1+\bar{\alpha}^2+\sigma_\alpha^2}$

Table 1 shows that the coefficient on π^* reduces in uncertainty (i.e., the Central Bank is more cautious) whereas the contribution of expectations is enhanced. At the limit when uncertainty is infinite ($\sigma_\alpha^2 \to \infty$), it is straightforward to show that the action of the Central Bank becomes irrelevant and all that matters is private sector expectations (naturally shocks always play a role).
However, this shift in allocated importance of each of the terms0 contribution does not become visible under the assumption of Rational Expectations as seen on both (6) and (10). This occurs because expectations $E_t\pi_{t+1}$ act as a jump variable that will always move to compensate for any shortcomings in the policy action, in order to bring the inflation outcome in line with the objective. For the relevance of expectations to come to the fore however, one needs to address an expectation formation process that departs from this immediate adjustment to the desired level.

4 Introducing Differential Information

We introduce next the concept of Differential Information as presented by Morris and Shin (2006) which builds on some of their previous work (Morris and Shin, 2002a and 2002b). When applied to monetary policy, the idea behind this is as follows: the current state of inflation is π_0 but the Central Bank wants to move it to a new state π^* within a given time horizon. It announces therefore, the desired level of inflation π^*, which under the assumption of differential information, is not automatically believed. A proportion of the private sector μ will automatically adjust expectations to that level at time t, whereas, $1-\mu$ will not. In the eyes of those who form expectations, inflation state π_t therefore evolves in the following way:

$$\pi_t = \begin{cases} \pi_0 & \text{for } t \leq 0 \\ \pi^* & \text{for } t \geq 1 \end{cases}$$

If agents are subject to differential information, only a proportion μ_t of the agents know the true value of π_t at time $t \geq 1$. This proportion is increasing over time, such that eventually $\mu_t = 1$, and therefore, all agents learn what the true value of π_t is in the sufficiently distant future (as $t \to \infty$). Note that this latter assumptions implies that as agents learn the value of the state variable, this set-up is consistent with Rational Expectations. Moreover, MS assume that μ_1 is very close to one to start with, such that the informational friction is sufficiently small by comparison to the occasion of no differential information. Looking at the agents individually, it is the case that informed agents form expectations as:

$$E_{i,t}\begin{bmatrix} \pi_0 \\ \pi_{t+h} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \pi_0 \\ \pi^* \end{bmatrix} \tag{11}$$

whereas uniformed agents form expectations as:

$$E_{i,t}\begin{bmatrix} \pi_0 \\ \pi_{t+h} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \pi_0 \\ \pi^* \end{bmatrix} \tag{12}$$

The assumption therefore, is that everyone knows the current or past state of inflation. Then, since a proportion μ_t are informed at time t, average expectation for inflation is

$$\bar{E}_t\begin{bmatrix} \pi_0 \\ \pi_{t+h} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1-\mu_t & \mu_t \end{bmatrix} \begin{bmatrix} \pi_0 \\ \pi^* \end{bmatrix} \tag{13}$$

Similarly,

$$
\begin{aligned}
\bar{E}_{t-1}\bar{E}_t \begin{bmatrix} \pi_0 \\ \pi_{t+h} \end{bmatrix} &= \bar{E}_{t-1} \begin{bmatrix} 1 & 0 \\ 1-\mu_t & \mu_t \end{bmatrix} \begin{bmatrix} \pi_0 \\ \pi^* \end{bmatrix} \\
&= \begin{bmatrix} 1 & 0 \\ 1-\mu_t & \mu_t \end{bmatrix} \bar{E}_{t-1} \begin{bmatrix} \pi_0 \\ \pi^* \end{bmatrix} \\
&= \begin{bmatrix} 1 & 0 \\ 1-\mu_t & \mu_t \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1-\mu_{t-1} & \mu_{t-1} \end{bmatrix} \begin{bmatrix} \pi_0 \\ \pi^* \end{bmatrix} \\
&= \begin{bmatrix} 1 & 0 \\ 1-\mu_{t-1}\mu_t & \mu_{t-1}\mu_t \end{bmatrix} \begin{bmatrix} \pi_0 \\ \pi^* \end{bmatrix}
\end{aligned}
$$

In a forward looking world, the present is a function of the sequence of all future expectations. In other words, to derive the appropriate expectation one needs to iterate this forward such that

$$
\bar{E}_1\bar{E}_2...\bar{E}_t \begin{bmatrix} \pi_0 \\ \pi_{t+h} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1-\prod_{s=1}^{t}\mu_s & \prod_{s=1}^{t}\mu_s \end{bmatrix} \begin{bmatrix} \pi_0 \\ \pi^* \end{bmatrix}
$$

and therefore,

$$
\begin{aligned}
\bar{E}_1\bar{E}_2...\bar{E}_t\,(\pi_0) &= \pi_0 && (14)\\
\bar{E}_1\bar{E}_2...\bar{E}_t\,(\pi_{t+h}) &= \left(1-\prod_{s=1}^{t}\mu_s\right)\pi_0 + \left(\prod_{s=1}^{t}\mu_s\right)\pi^* && (15)
\end{aligned}
$$

This implies that the higher order expectation for a given timing $t+h$ (where $h>0$) depends on the limiting property of $\prod_{s=1}^{t}\mu_s$.

4.1 Uncertainty and Differential Information

We turn next, back to the monetary policy problem. We know that the inflation outcome is a function of the action of the CB and the expectations formed as can be seen in (5) and (9), and their relative contributions are affected by the level of uncertainty. Under pure Rational Expectations, expectations move to compensate for any policy shortcomings. However, the role of expectations changes when we introduce the concept of differential information. To allow for differential information, we now use (15) to substitute for expectations in (9). This give us

$$
\begin{aligned}
\pi_t = {} & \frac{\bar{\alpha}^2}{1+\bar{\alpha}^2+\sigma_\alpha^2}\pi^* + \frac{1+\sigma_\alpha^2}{1+\bar{\alpha}^2+\sigma_\alpha^2}\left[\left(1-\prod_{s=1}^{t}\mu_s\right)\pi_0 + \left(\prod_{s=1}^{t}\mu_s\right)\pi^*\right] \qquad (16)\\
& + \frac{1+\sigma_\alpha^2}{1+\bar{\alpha}^2+\sigma_\alpha^2}\varepsilon_t
\end{aligned}
$$

Three cases summarise the points of interest:

A. No differentiated information ($\mu_s = 1$).
We apply first the assumption of $\mu_s = 1$ to (16):

$$\begin{aligned} \pi_t &= \frac{\bar{\alpha}^2}{1+\bar{\alpha}^2+\sigma_\alpha^2}\pi^* + \frac{1+\sigma_\alpha^2}{1+\bar{\alpha}^2+\sigma_\alpha^2}\pi^* + \frac{1+\sigma_\alpha^2}{1+\bar{\alpha}^2+\sigma_\alpha^2}\varepsilon_t \\ &= \pi^* + \frac{1+\sigma_\alpha^2}{1+\bar{\alpha}^2+\sigma_\alpha^2}\varepsilon_t \qquad (17) \end{aligned}$$

This is equivalent to the Rational expectations outcome.

B. Differentiated information ($\mu_s < 1$).
The assumption now is that $\mu_s < 1$, and therefore, $\prod_{s=1}^{t} \mu_s \to 0$ as $t \to \infty$. In turn, projecting expectations to the future and substituting into (16) produces now:

$$\pi_t = \frac{\bar{\alpha}^2}{1+\bar{\alpha}^2+\sigma_\alpha^2}\pi^* + \frac{1+\sigma_\alpha^2}{1+\bar{\alpha}^2+\sigma_\alpha^2}\pi_0 + \frac{1+\sigma_\alpha^2}{1+\bar{\alpha}^2+\sigma_\alpha^2}\varepsilon_t \qquad (18)$$

This shows that the system exhibits inertia and therefore, policy will not be able to bring inflation back in line with the CB's objective, but will fall at level between current and desired inflation, in proportion to the level of prevailing uncertainty.

C. The role of uncertainty, i.e. ($\sigma_\alpha^2 \to \infty$)
This naturally leads then to consider the role of uncertainty. It is then straight forward to show from (18) even if a little differentiated information is introduced, the existence of uncertainty emphasises the role of expectations in terms of determining the outcome and de-emphasises that of policy. At the limit when uncertainty is infinite, the central bank is unable to move the economy to its objective. In other words,

$$\lim_{\sigma_\alpha^2 \to \infty} \pi_t = \pi_0 + \varepsilon_t \qquad (19)$$

This implies that in the presence of uncertainty, it becomes increasingly difficulty for policy to achieve its objective and the system is characterised by full inertia.

5 Two-Step Inflation Targeting

In the present of differentiated information, we observe that the role of policy is reduced. Our objective now is to re-introduce a role for policy but still remain within an optimisation framework. We argue that the CB can improve on the previous result by using its information advantage, namely the knowledge of the shock that has hit the economy[2]. We put forward a two-step optimisation procedure according to which the Central Bank, operates as thought its objective was $\pi^* + \theta$, instead of π^* in the first step, in order to moderate Brainard's attenuation effect. However, it does this optimally, in the sense that the level of deviation θ applied, is a function of the shocks hitting the economy as well as the degree of prevailing uncertainty. In other words, in the second step, the Central Bank optimises with respect to this deviation, aiming to close the gap from its objectives which arise due to the existence of uncertainty. The following two sections describe the two-step procedure in greater detail.

5.1 Step 1

In the first step, and after the shock has occurred, the monetary policy authority identifies the optimal policy rule as a function of an auxiliary target $(\pi^* + \theta)$. Formally this means optimising the following objective function

$$\min_y E(L) = \frac{1}{2}\left\{[\bar{\pi}_t - (\pi^* + \theta)]^2\right\} + y_t^2\left(1 + \sigma_\alpha^2\right) \tag{20}$$

subject to the system of equations (1) and (2) where we assume the latter exogenous for the moment[3]. Optimising (20) produces an optimal rule as a function of θ:

$$y_t = \frac{\bar{\alpha}}{1 + \bar{\alpha}^2 + \sigma_\alpha^2}(\pi^* + \theta) - \frac{\bar{\alpha}}{1 + \bar{\alpha}^2 + \sigma_\alpha^2}(E_t \pi_{t+1} + \varepsilon_t) \tag{21}$$

and

$$\pi_t = \frac{\bar{\alpha}^2}{1 + \bar{\alpha}^2 + \sigma_\alpha^2}(\pi^* + \theta) + \frac{1 + \sigma_\alpha^2}{1 + \bar{\alpha}^2 + \sigma_\alpha^2}(E_t \pi_{t+1} + \varepsilon_t) \tag{22}$$

The above two equations imply that for a given level of uncertainty, the CB will choose to deviate, at first instance, from its ultimate target π^* by a parameter θ.

5.2 Step 2

The degree of deviation θ is chosen optimally. In other words, the CB chooses θ in full knowledge of the extent of uncertainty and the size of the shock, and aims to maximise the probability of achieving its true objectives. In other words, since inflation expectations move away from the target as uncertainty increases, the deviation term θ will move to close that gap. Similarly, the instrument will

[2] As already mentioned, this presumes that the private sector forms expectations first, a shock occurs next and the CB reacts by choosing that interest rate which optimises the conditional expectation of its loss function.

[3] The expected value of the objective function is conditional on the shocks, omitted here for simplicity.

In that respect θ is therefore, an auxiliary step, necessary in order to make full use of the information available to the bank. The derived rules from Step 1 (21) and (22) are now substituted into the objective function of the Central Bank:

$$\min_{\theta} E(L) = E_t \left[\frac{1}{2} (\pi_t - \pi^*)^2 + \frac{1}{2} y^2 \right] \tag{23}$$

to produce

$$\min_{\theta} E(L) = f(\theta, \sigma_a^2, y_t, \pi_t) \tag{24}$$

Given the rules, the aim of the CB is to find the optimal value for θ, contingent on the economy's past history and the perceived uncertainty of the transmission of policies, i.e.:

$$\theta(\sigma_c^2, y_t, \pi_t) = \arg \min_{\theta} E(L)$$

which in its analytical form is

$$\theta = \frac{\sigma_a^2}{1 + \bar{\alpha}} \left[\pi^* - E_t \pi_{t+1} - \varepsilon_t \right] \tag{25}$$

As uncertainty decreases, the deviations from π^* decrease as well, such that at the limit they become zero, i.e.

$$\lim_{\sigma_a^2 \to 0} (\theta) = 0$$

Proposition 2 *Applying a two-step procedure in which θ is contingent on the shocks that hit the economy, the existing uncertainty and the inflation target, neutralises the ex ante effects of uncertainty on the policy rules.*

Proof 1: Substituting the analytical solutions for θ into (21) and (22) produces the *two-step* target rules that a Central Bank needs to apply under uncertainty.

$$y_t = \frac{\bar{\alpha}}{1 + \bar{\alpha}^2} \pi^* - \frac{\bar{\alpha}}{1 + \bar{\alpha}^2} (E_t \pi_{t+1} + \varepsilon_t) \tag{26}$$

$$\pi_t = \frac{\bar{\alpha}^2}{1 + \bar{\alpha}^2} \pi^* + \frac{1}{1 + \bar{\alpha}^2} (E_t \pi_{t+1} + \varepsilon_t) \tag{27}$$

The rules achieved are similar to those attained with no uncertainty (with α replaced by $\bar{\alpha}$). This demonstrates that by varying the target optimally, uncertainty in the transmission process is neutralised[4]. This however, is an *ex ante* result. As we will show next, this happens at the expense of using y_t more actively, thereby introducing greater variability in the system and therefore, in a

[4] Our approach is in fact equivalent to introducing an extra instrument while the number of targets remains the same. As Hughes Hallett (1989) mentions "...all the instruments will be needed to combat uncertainty even when there are only a few targets compared to the number of instruments".

period-by-period optimisation Brainard is always optimal. However, this is not necessarily the case when looking at the dynamic properties of the rule, where the benefits of hitting the target can be more than enough to compensate the increased volatility. We will show this in the sections of simulations.

5.3 Two-Steps and Differentiated Information

We can replace the term for expectations with the equivalent way they evolve. We apply (15) in (26) and (27). For output

$$\begin{aligned} y_t &= \frac{\bar{\alpha}^2}{1+\bar{\alpha}^2}\pi^* - \frac{\bar{\alpha}}{1+\bar{\alpha}^2}\left(\left[\left(1-\prod_{s=1}^{t}\mu_s\right)\pi_0 + \left(\prod_{s=1}^{t}\mu_s\right)\pi^*\right] + \varepsilon_t\right) \\ &= \frac{\bar{\alpha}^2}{1+\bar{\alpha}^2}\pi^* - \frac{\bar{\alpha}}{1+\bar{\alpha}^2}\left(\left[\left(1-\prod_{s=1}^{t}\mu_s\right)\pi_0 + \left(\prod_{s=1}^{t}\mu_s\right)\pi^*\right] + \varepsilon_t\right) \end{aligned} \quad (28)$$

and inflation

$$\begin{aligned} \pi_t &= \frac{\bar{\alpha}^2}{1+\bar{\alpha}^2}\pi^* + \frac{1}{1+\bar{\alpha}^2}\left(\left[\left(1-\prod_{s=1}^{t}\mu_s\right)\pi_0 + \left(\prod_{s=1}^{t}\mu_s\right)\pi^*\right] + \varepsilon_t\right) \\ &= \frac{\bar{\alpha}^2}{1+\bar{\alpha}^2}\pi^* + \frac{1}{1+\bar{\alpha}^2}\left[\left(1-\prod_{s=1}^{t}\mu_s\right)\pi_0 + \left(\prod_{s=1}^{t}\mu_s\right)\pi^*\right] + \frac{1}{1+\bar{\alpha}^2}\varepsilon_t \end{aligned} \quad (29)$$

and as $\mu_s < 1$, and therefore, $\prod_{s=1}^{t}\mu_s \to 0$ as $t \to \infty$.

$$\begin{aligned} y_t &= \frac{\bar{\alpha}^2}{1+\bar{\alpha}^2}\pi^* - \frac{\bar{\alpha}}{1+\bar{\alpha}^2}\pi_0 - \frac{\bar{\alpha}}{1+\bar{\alpha}^2}\varepsilon_t \\ \pi_t &= \frac{\bar{\alpha}^2}{1+\bar{\alpha}^2}\pi^* + \frac{1}{1+\bar{\alpha}^2}\pi_0 + \frac{1}{1+\bar{\alpha}^2}\varepsilon_t \end{aligned}$$

equivalent to the solution under no uncertainty. As argued earlier, the advantage of this algorithm is that it neutralises uncertainty such that the CB's policy action can bring inflation back to the intended target. This however, implies that there will be greater variability introduced in the system, as the instrument is applied more than to what Brainard uncertainty recommends. It is important to examine next, whether this benefits of closing the inflation gap, compensate for the variability introduced, and under which levels of uncertainty.

6 Numerical Simulations

We illustrate next the welfare implications of the two alternative procedures through Monte Carlo simulations. We design the simulations as follows: the solutions for output for the two different regimes,

$$y_t^{BR} = \frac{\bar{\alpha}}{1+\bar{\alpha}^2+\sigma_\alpha^2}\pi^* - \frac{\bar{\alpha}}{1+\bar{\alpha}^2+\sigma_\alpha^2}E_t\pi_{t+1} - \frac{\bar{\alpha}}{1+\bar{\alpha}^2+\sigma_\alpha^2}\varepsilon_t \tag{30}$$

$$y_t^{TS} = \frac{\bar{\alpha}}{1+\bar{\alpha}^2}\pi^* - \frac{\bar{\alpha}}{1+\bar{\alpha}^2}E_t\pi_{t+1} - \frac{\bar{\alpha}}{1+\bar{\alpha}^2}\varepsilon_t \tag{31}$$

are substituted in the equation for prices

$$\pi_t = \beta E_t\pi_{t+1} + \alpha_i y_t + \varepsilon_t$$

where parameter α_i is drawn from a distribution $N\left(\bar{\alpha}, \sigma_\alpha^2\right)$. Further expectations are backward looking and errors excibit a certain degree of persistence, i.e.:

$$\begin{aligned} E_t\pi_{t+1} &= \pi_{t-1} \\ \varepsilon_t &= \rho\varepsilon_{t-1} + v_t \qquad v_t \simeq N(0,1) \end{aligned} \tag{32}$$

As we operate in a discretionary framework we calculate losses period by period as measured by

$$\begin{aligned} L_{BR,t} &= \frac{1}{2}\left\{(\pi_t - \pi^*)^2 + y_{BR,t}^2\right\} \\ L_{TS,t} &= \frac{1}{2}\left\{(\pi_t - \pi^*)^2 + y_{TS,t}^2\right\} \end{aligned}$$

We then calculate the cumulative losses for a certain number of years, discounted by the appropriate discount factor, i.e.

$$\sum_{t=1}^{n}\beta^t L_{j,t} \qquad \forall \quad j = BR, TS \quad \text{and} \quad n = 10 \tag{33}$$

We apply the following parameterisation[5]:

$$\begin{aligned} \beta &= 0.99 \\ \alpha &\simeq N\left(0.5, \sigma_\alpha^2\right) \\ \rho &= 0.8 \\ \pi^* &= 1, \quad \pi_0 = 0 \end{aligned}$$

[5] Note that for $\beta < 1$ then the rules become

$$\begin{aligned} y_t^{BR} &= \frac{\overline{\alpha}}{1+\overline{\alpha}^2+\sigma_\alpha^2}\pi^* - \frac{\overline{\alpha}}{1+\overline{\alpha}^2+\sigma_\alpha^2}\beta E_t\pi_{t+1} - \frac{\overline{\alpha}}{1+\overline{\alpha}^2+\sigma_\alpha^2}\beta\varepsilon_t \\ y_t^{TS} &= \frac{\overline{\alpha}}{1+\overline{\alpha}^2}\pi^* - \frac{\overline{\alpha}}{1+\overline{\alpha}^2}\beta E_t\pi_{t+1} - \frac{\overline{\alpha}}{1+\overline{\alpha}^2}\beta\varepsilon_t \end{aligned}$$

The average value of α applied is somewhat higher than what exists in the literature, where it ranges from a minimum of 0.024 in Woodford (1999) to a maximum of 0.3 in McCallum and Nelson (1999). However, the qualitative nature of the results is dependent only on the coefficient of variation of α and not its mean, the choice of value α we apply facilitates presentation. The updating of expectations in equation (32) is consistent with Morris and Shin's (2006) definition of expectations inertia. The model is similar to the model used by Svensson (1999) and Söderstrom (2002), although the timing of policy responses and effectiveness is different. In the model applied, the policy response is contemporaneous to the supply shock and to the realisation of inflation. The lag response of the system to the policy action observed is due to the inertia in expectations formation imposed and there is no built-in lag in the monetary transmission mechanism. In the first period the economy is subjected both to a supply shock ε_t and an inflation target shift to π^*. Note that the numerical value of the target does not influence the qualitative nature of the results. We then apply the optimal targeting rule and calculate the impulse response functions for y and π. Cumulative losses for ten periods are calculated in deviation from the targets. Before presenting a detailed welfare analysis, figures (1) and (2) show a typical path for output and inflation produced by the simulations.

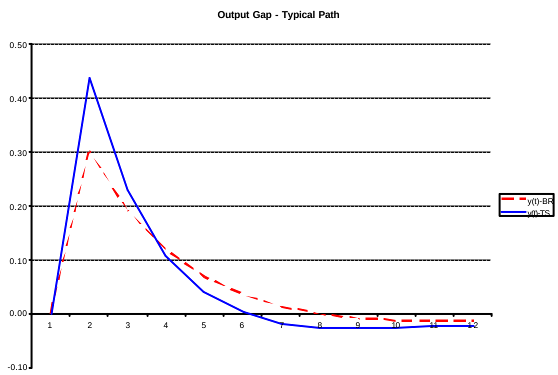

Figure 1:

Figure 1 shows a typical path of y, the instrument in our targeting rule. To achieve the inflation target the economy is subjected to a higher real variability in the early periods of the policy plan, in the *two/step* regime. As can be seen in figure 2, once inflation and inflation expectations converge towards the target, the *two-step* policy rule produces both lower real variability as well as a path of inflation closer to the target, relative to the cautious Brainard policy rule.
Table 2 presents then the average cumulative losses of 10,000 stochastic simulations for the two regimes for different degrees of uncertainty captured by the coefficient of variation (CV$\equiv \frac{\sigma_\alpha}{\bar{\alpha}}$). Ccumulative losses are lower in the TS regime for coefficient of variations equal to 0.5 and 1. For higher than that lev-

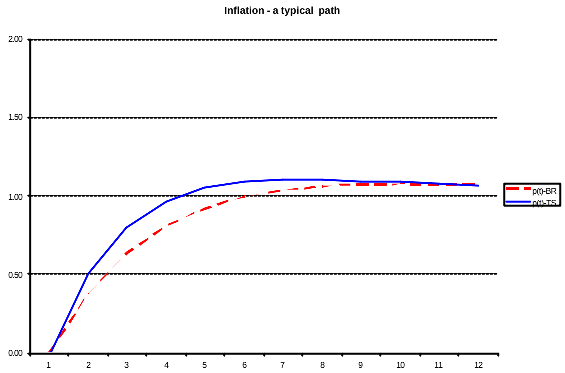

Figure 2:

els of uncertainty, the gains in convergence no longer compensate for the early losses, relative to Brainard's cautious approach.

Table 2. Cumulative Losses

CV	$L_{BR} * 10$	$L_{TS} * 10$
0.5	260.7	252.7
1	316.6	292.1
1.5	573.1	1032.0

Table 3 instead shows the first period losses for the two policy regime. This is also confirmed analytically in appendix B. In order to stabilise the system around the target, the two steps regime introduces greater variability in the early periods, thus increasing early losses[6].

Table 3. First Period Losses

CV	$L_{BR} * 10$	$L_{TS} * 10$
0.5	11.9	12.1
1	11.6	12.2
1.5	11.4	12.7

It is the case therefore, that when evaluating the benefits of two regimes in terms of their dynamic properties, then there exist levels of uncertainty when it is better to ignore the prevailing level of uncertainty and aim to achieve the objectives set. The variability introduced as result is more than compensated by the benefits of achieving them.

[6] This also means that the results are a function of the discount rate applied. A very myopic policy maker will be always cautious

7 Conclusions

Our motivation stems from the observation that as policy becomes less active under uncertainty, this has the consequence that the objectives set out by monetary policy authorities are also seldom achieved. This goes against the advantage of the institutional set up of inflation targeting where the explicit quantification of those objectives help tie down expectations better and thus attain them with greater certainty. We assume that expectations are subject to differentiated information and thus the private sector requires sometime to learn and therefore, gradually converges to the objectives set by the Central Bank. This introduces by itself inertia to the system which is worsened in the presence of multiplicative uncertainty. Following this, we identify a two-step algorithm that aims to reintroduce the relevance of policy. This has the advantage that the Central Bank is able to achieve its objectives quicker, but at the expense of introducing greater variability in the system. Our simulation section then shows (and our appendix shows that also analytically), that in a one-period framework, Brainard does indeed better on average; however, as the TS algorithm attains the targets quicker, there are levels of uncertainty where the benefits of hitting the "bull's eye" outweigh the costs of greater variability. Furthermore, as this regime is done within an optimisation framework that accounts for the level of prevailing uncertainty, the rules derived are easier to communicate to the public and are in line with the degree of transparency associated with inflation targeting.

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APPENDICES

A Objective function with Uncertainty in α

Under uncertainty, where $\alpha_t \rightarrow N\left(\bar{\alpha}, \sigma_\alpha^2\right)$, losses conditional on shocks ε, we can express the objective function of the CB in terms of the moments of α.

$$
\begin{aligned}
L &= \frac{1}{2}E\left\{\left(\pi_t-\pi^*\right)^2+y_t^2\right\} \\
&= \frac{1}{2}E\left(\pi_t-\pi^*\right)^2+\frac{1}{2}E\left(y_t^2\right) \\
&= \frac{1}{2}E\left\{E_t\pi_{t+1}+\alpha y_t+\varepsilon_t-\pi^*\right\}^2+\frac{1}{2}E\left(y_t^2\right) \\
&= \frac{1}{2}E\left\{\left(E_t\pi_{t+1}+\varepsilon_t-\pi^*\right)^2+\left(\alpha y_t\right)^2\right. \\
&\quad \left.+2\left(E_t\pi_{t+1}+\varepsilon_t-\pi^*\right)\left(\alpha y_t\right)\right\}+\frac{1}{2}E\left(y_t^2\right) \\
&= \frac{1}{2}\left\{\left(E_t\pi_{t+1}+\varepsilon_t-\pi^*\right)^2+E\left(\alpha y_t\right)^2\right. \\
&\quad \left.+2\left(E_t\pi_{t+1}+\varepsilon_t-\pi^*\right)E\left(\alpha y_t\right)\right\}+\frac{1}{2}E\left(y_t^2\right)
\end{aligned}
$$

but since $E\left(\alpha y_t\right)^2=y_t^2E\left(\alpha\right)^2$and $E\left(\alpha^2\right)=\sigma_\alpha^2+\bar{\alpha}^2$ then,

$$
\begin{aligned}
L &= \frac{1}{2}\left\{\left(E_t\pi_{t+1}+\varepsilon_t-\pi^*\right)^2+y_t^2E\left(\alpha\right)^2\right. \\
&\quad \left.+2\left(E_t\pi_{t+1}+\varepsilon_t-\pi^*\right)\bar{\alpha}y_t\right\}+\frac{1}{2}E\left(y_t^2\right) \\
&= \frac{1}{2}\left\{\left(E_t\pi_{t+1}+\varepsilon_t-\pi^*\right)^2\right. \\
&\quad \left.+y_t^2\left(\sigma_\alpha^2+\bar{\alpha}^2\right)+2\left(E_t\pi_{t+1}+\varepsilon_t-\pi^*\right)\bar{\alpha}y_t\right\}+\frac{1}{2}E\left(y_t^2\right) \\
&= \frac{1}{2}\left\{\left(E_t\pi_{t+1}+\varepsilon_t-\pi^*\right)^2+y_t^2\bar{\alpha}^2\right. \\
&\quad \left.+2\left(E_t\pi_{t+1}+\varepsilon_t-\pi^*\right)\bar{\alpha}y_t+y_t^2\sigma_\alpha^2\right\}+\frac{1}{2}E\left(y_t^2\right)
\end{aligned}
$$

From this, it follows that

$$\bar{\pi}_t \equiv E(\pi_t) = E_t\pi_{t+1}+\bar{\alpha}y_t+\varepsilon_t$$

and therefore,

$$
\begin{aligned}
L &= \frac{1}{2}\left\{\left(\bar{\pi}_t-\pi^*\right)^2+y_t^2\sigma_\alpha^2+y_t^2\right\} \\
&= \frac{1}{2}\left\{\left(\bar{\pi}_t-\pi^*\right)^2+y_t^2\left(\sigma_\alpha^2+1\right)\right\}
\end{aligned}
$$

B Ex Ante losses comparison

Static losses are evaluated based on

$$L = \frac{1}{2} E \left\{ (\bar{\pi}_t - \pi^*)^2 + y_t^2 \left(1 + \sigma_\alpha^2\right) \right\}$$

The solutions for Brainard are:

$$\begin{aligned} y_{BR,t} &= \frac{\bar{\alpha}}{1 + \bar{\alpha}^2 + \sigma_\alpha^2} \pi^* - \frac{\bar{\alpha}}{1 + \bar{\alpha}^2 + \sigma_\alpha^2} \pi_0 - \frac{\bar{\alpha}}{1 + \bar{\alpha}^2 + \sigma_\alpha^2} \varepsilon_t \\ \pi_{BR,t} &= \frac{\bar{\alpha}^2}{1 + \bar{\alpha}^2 + \sigma_\alpha^2} \pi^* + \frac{1 + \sigma_\alpha^2}{1 + \bar{\alpha}^2 + \sigma_\alpha^2} \pi_0 + \frac{1 + \sigma_\alpha^2}{1 + \bar{\alpha}^2 + \sigma_\alpha^2} \varepsilon_t \end{aligned}$$

and similarly for the TS solution:

$$\begin{aligned} y_{TS,t} &= \frac{\bar{\alpha}^2}{1 + \bar{\alpha}^2} \pi^* - \frac{\bar{\alpha}}{1 + \bar{\alpha}^2} \pi_0 - \frac{\bar{\alpha}}{1 + \bar{\alpha}^2} \varepsilon_t \\ \pi_{TS,t} &= \frac{\bar{\alpha}^2}{1 + \bar{\alpha}^2} \pi^* + \frac{1}{1 + \bar{\alpha}^2} \pi_0 + \frac{1}{1 + \bar{\alpha}^2} \varepsilon_t \end{aligned}$$

Substituting then the solutions to the objecitve functions we calculate losses for any given shock ε_t:

$$\begin{aligned} L_{BR,t} &= \frac{\left(1 + \sigma_\alpha^2\right) \left[\varepsilon_t + \pi_0 - \pi^*\right]^2}{2 \left(1 + \bar{\alpha}^2 + \sigma_\alpha^2\right)} \\ L_{TS,t} &= \frac{\left[1 + \bar{\alpha}^2 \left(1 + \sigma_\alpha^2\right)\right] \left[\varepsilon_t + \pi_0 - \pi^*\right]^2}{2 \left(1 + \bar{\alpha}^2\right)^2} \end{aligned}$$

When are losses for Brainard bigger than for TS?

$$L_{BR,t} = \frac{\left(1 + \sigma_\alpha^2\right) \left[\varepsilon_t + \pi_0 - \pi^*\right]^2}{2 \left(1 + \bar{\alpha}^2 + \sigma_\alpha^2\right)} > L_{TS,t} = \frac{\left[1 + \bar{\alpha}^2 \left(1 + \sigma_\alpha^2\right)\right] \left[\varepsilon_t + \pi_0 - \pi^*\right]^2}{2 \left(1 + \bar{\alpha}^2\right)^2}$$

and therefore,

$$\begin{aligned} \frac{\left(1 + \sigma_\alpha^2\right) \left[\varepsilon_t + \pi_0 - \pi^*\right]^2}{2 \left(1 + \bar{\alpha}^2 + \sigma_\alpha^2\right)} &> \frac{\left[1 + \bar{\alpha}^2 \left(1 + \sigma_\alpha^2\right)\right] \left[\varepsilon_t + \pi_0 - \pi^*\right]^2}{2 \left(1 + \bar{\alpha}^2\right)^2} \\ \bar{\alpha}^2 &> \bar{\alpha}^2 + \sigma_\alpha^2 \end{aligned}$$

This is never true for $\sigma_\alpha^2 > 0$ and therefore $L_{BR,t} < L_{TS,t}$ holds for $t = 1$ always.

B.1 Cumulative losses

However, for $t = 2$, in other words, in the second period after the shock has occurred, as TS is more aggressive it will have managed to close more of the distance between actual inflation and the target i.e. $\pi_2^{TS} - \pi^* < \pi_2^{BR} - \pi^*$ and therefore $\left[\varepsilon_t + \pi_2^{TS} - \pi^*\right]^2 < \left[\varepsilon_t + \pi_2^{BR} - \pi^*\right]^2$. This implies that in the next period losses with Brainard can be worse if the following holds.

$$\frac{\left(1+\sigma_\alpha^2\right)\left[\varepsilon_t + \pi_2^{BR} - \pi^*\right]^2}{2\left(1+\bar{\alpha}^2+\sigma_\alpha^2\right)} > \frac{\left[1+\bar{\alpha}^2\left(1+\sigma_\alpha^2\right)\right]\left[\varepsilon_t + \pi_2^{TS} - \pi^*\right]^2}{2\left(1+\bar{\alpha}^2\right)^2}$$

$$\frac{\left(1+\sigma_\alpha^2\right)\left(1+\bar{\alpha}^2\right)^2}{\left(1+\bar{\alpha}^2+\sigma_\alpha^2\right)\left[1+\bar{\alpha}^2\left(1+\sigma_\alpha^2\right)\right]} > \frac{\breve{\pi}_2^{TS}}{\breve{\pi}_2^{BR}}$$

where $\breve{\pi}_2^{TS} = \left[\varepsilon_t + \pi_2^{TS} - \pi^*\right]^2$ and $\breve{\pi}_2^{BR} = \left[\varepsilon_t + \pi_2^{BR} - \pi^*\right]^2$. In general, for any period n this condition is

$$\frac{\left(1+\sigma_\alpha^2\right)\left(1+\bar{\alpha}^2\right)^2}{\left(1+\bar{\alpha}^2+\sigma_\alpha^2\right)\left[1+\bar{\alpha}^2\left(1+\sigma_\alpha^2\right)\right]} > \frac{\pi_n^{TS}}{\pi_n^{BR}}$$

We demonstrate through simulations for which values for the coefficient of variation this happens and then compare the cumulative losses implied by the two methods.