OPTIMAL SIMPLE NONLINEAR RULES FOR MONETARY POLICY IN A NEW-KEYNESIAN MODEL

Massimiliano Marzo
Università di Bologna and Johns Hopkins University

Paolo Zagaglia
Stockholm University and Università Bocconi

This version: February 28, 2006

Abstract. We study the role of nonlinear simple rules for monetary policy. We depart from the standard rules proposed by Taylor (1993), and consider a nonlinear rule for the so-called opportunistic approach to disinflation originally proposed by Orphanides and Wilcox (2002) and Aksoy, Orphanides, Small, Wieland, and Wilcox (2002). We set out a model economy with capital accumulation and nominal and real rigidities. Households have weakly-separable preferences along the lines of Chari, Kehoe, and McGrattan (2000). The public sector is modeled as a simple rule for lump-sum taxes like in Leeper (1991). We include three sources of exogenous fluctuations in the form of stochastic shocks to productivity, firms' markup and government spending. We solve the model through the second-order Taylor approach developed by Schmitt-Grohé and Uribe (2004), and maximize a measure of conditional consumer welfare. Our microfounded model represents an improvement over the framework used by Aksoy, Orphanides, Small, Wieland, and Wilcox (2002). Our results support the view that optimal opportunistic monetary policy involves a strong anti-inflationary stance outside the zone of policy inaction, indicating that a large degree of nonlinearity can be desirable from a welfare perspective. We also compare the quantitative and qualitative properties of the model economy under the optimal nonlinear rule with those arising from optimized linear rules.

Keywords: disinflation, monetary policy and policy rules

Jel Classification Code: E31, E61

contents

Acknowledgements: Marzo is grateful to MIUR (research grant) and to Università di Bologna for the support coming from 'Progetto Pluriennale d'Ateneo'. Zagaglia acknowledges partial financial support from Stockholm University and MIUR. The usual disclaimer applies.

1. INTRODUCTION

[to be written]

2. THE MODEL

The structure of the model economy is standard in the New-Keynesian tradition of Woodford (2002). We include money demand through the money-in-the-utility function approach studied by Feenstra (1986), and quadratic capital-adjustment costs like Kim (2000). Nominal price rigidity arises from quadratic-adjustment costs from changing prices.

2.1. *Households*

The model economy is populated by a large number of infinitely-lived agents indexed on the real line, $i \in [0, 1]$, each maximizing the following stream of utility:

$$U_{it} = \sum_{t=0}^{\infty} \beta^t u(c_{it}, m_{it}, \ell_{it})$$

subjected to the following specification for the instantaneous utility function:

$$u\left(c_{it}, m_{it}, \ell_{it}\right) = \frac{1}{1-\frac{1}{\sigma}} \left\{ \left[a c_{it}^{\frac{\mu-1}{\mu}} + (1-a) \left(\frac{M_{it}}{P_t} \right)^{\frac{\mu-1}{\mu}} \right]^{\frac{\mu}{\mu-1}} (1-\ell_{it})^{\xi} \right\}^{\left(1-\frac{1}{\sigma}\right)}$$

The utility function considers money in a weakly separable form with respect to consumption C_{it}. Basically, consumption and real money balances M_{it}/P_t are taken together via a CES aggregator type, as described by Chari, Kehoe, and McGrattan (2000). The advantage of such approach relies on the cross substitution effects between consumption and money derived from the weak separablility betweem money and consumption. It is worth to note that the equivalence between money-in-the utility, transaction costs and cash-in-advance models has been proved by Feenstra (1986).

The i-th household budget constraint (in real terms) is given by:

$$c_{it} + \frac{M_{it} - M_{it-1}}{P_t} + \frac{B_{it}}{P_t} + inv_{it} \left[1 + \frac{\phi_K}{2} \left(\frac{inv_{it}}{k_{it}} \right)^2 \right] \leq q_{it} k_{it} + w_{it} \ell_{it} + R_{t-1} \frac{B_{it-1}}{P_t} - \tau_t^{ls} + \int_0^1 \eta_i(j)\, \Omega_t(j)\, dj$$

Households' income derives from: i) labor income in the form of $w_{it}\ell_{it}$, with w_{it} real wage per unit of labor effort ℓ_{it}; ii) capital income $q_{it}k_{it}$, with q_{it} rental rate on capital stock k_{it}; (iii) proceedings from investment in government bonds $R_{t-1}B_{it-1}/P_t$, where R_t is the gross nominal rate, and B_{it} is the stock of government's bonds held by i-th household. Each agent participates in the profit of the firm producing good j via a constant share $\eta_i(j)$. We assume that this share is constant and out of the control of the single agent.

Households allocate their wealth among money M_{it}, (nominal) bonds B_{it} and (real) investment inv_{it}. In order to reduce the high investment volatility typical of the RBC, we follow the suggestion of Kim (2000) and introduce an investment adjustment cost in the quadratic form. The assumption of quadratic cost of price adjustment simplifies algebra and delivers coherent results. The evolution of capital accumulation is governed by the following equation:

$$k_{it+1} = (1-\delta) k_{it} + inv_{it} \tag{1}$$

Finally, the government levies lump-sum taxes.

Given the existence of differentiated goods and different labor inputs, there exists an intra-temporal optimization program on the final goods-sector.

The are j varieties of final goods produced that are aggregated according to the constant-elasticity of substitution technology proposed by Dixit and Stiglitz (1977):

$$c_{it} = \left[\int_{\omega_2} c_t^i (j)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}}$$

where $\theta > 1$ is the elasticity of substitution between different varieties of goods produced by each j-th firm and $c_t^i(j)$ is the consumption of varieties j by i-th household. The constant elasticity of substitution inverse demand function for j-th variety expressed by i-th household is:

$$\frac{c_t^i (j)}{c_{it}} = \left[\frac{P_t (j)}{P_t} \right]^{-\theta}$$

where $\mathcal{P}_t(j)$ is the price of variety j and P_t is the general price index defined as:

$$P_t = \left[\int_0^1 P_t (j)^{1-\theta} dj \right]^{\frac{1}{1-\theta}}$$

Aggregate consumption is defined as $c_t = \int_{\omega_1} c_{it} di$, after aggregating over the $i \in \omega_1$ households.

The aggregate demand for variety j can be written as:

$$c_t(j) + g_t(j) = y_t(j)$$

such that the individual demand curve takes the form:

$$P_t(j) = \left[\frac{y_t(j)}{y_t} \right]^{-1/\theta} P_t \tag{2}$$

2.2. *Firms*

We assume the existence of a large number of firms indexed by $j \in \omega_2$, each producing a single variety. Each firm acts as a price taker with respect to the varieties supplied by other competitors. The production function is given by:

$$y_{jt} = z_t (k_{jt})^{\alpha} (\ell_{jt})^{1-\alpha} - \Phi_t \tag{3}$$

where k_{jt} and ℓ_{jt} are capital and labor inputs, respectively. We also introduce the exogenous shocks:

$$\log z_t = (1 - \rho_z) \log z + \rho_z \log z_{t-1} + \varepsilon_t^z$$
$$\log \Phi_t = (1 - \rho_\Phi) \log \Phi + \rho_\Phi \log \Phi_{t-1} + \varepsilon_t^\Phi$$

There are quadratic cost of price adjustment *à la* Rotemberg (1982), specified as follows:

$$AC_t^P (j) = \frac{\phi_P}{2} \left(\frac{P_t (j)}{P_{t-1} (j)} - \pi \right)^2 y_t$$

The optimal choice of labor and capital to be hired is described as the maximization of the future stream of profit evaluated with the stochastic discount factor ρ_t:

$$\max_{\{P_t(j), k_t(j), \ell_t(j)\}} E_0 \left[\sum_{t=0}^{\infty} \lambda_t \Omega_t (j) \right]$$

s. t. $\Omega_t(j) = P_t(j)\, y_{jt} - W_t \ell_{jt} - P_t q_t k_{jt} - P_t AC_t^P(j)$

given the demand for differentiated products in (2), and the production function (3).

2.3. *Government*

The government faces a standard flow budget constraint:

$$B_{jt} dj + P_t \tau_t^{ls} + M_{jt} = R_{t-1} B_{jt-1} + P_t g_t + M_{jt-1}$$

Real total taxation is denoted as τ_t, and g_t indicates total government spending. The goverment issues one-period riskless (non-contingent) nominal bonds denoted by D_t. We also specify the intertemporal budget constraint of the government:

$$R_t B_{jt} \leq \sum_{p=0}^{\infty} \mathsf{E}_{t+p} \left(\frac{1}{R_{t+p}} \right)^p \left[M_{jt+p} - M_{jt-1+p} + P_{t+p} \tau_{t+p}^{ls} - P_{t+p} g_{t+p} \right]$$

As it is customary in the literature, we posit an exogenous path to public expenditure, by assuming an AR(1) described by the following equation:

$$\log(g_t) = (1 - \rho_g) \log(g) + \rho_g \log(g_{t-1}) + \varepsilon_t^g$$

with ε_{t+1}^g is i.i.d.$\sim N\left(0, \sigma_g^2\right)$.

The government flow budget constraint in equilibrium can be also re-written by defining the total amount of government's liabilities l_t as follows:

$$l_t := \frac{R_t B_t + M_t}{P_t}$$

In this case, the evolution of total liabilities is represented by the following equation:

$$l_t = \frac{R_t l_{t-1}}{\pi_t} + R_t \left(g_t - \tau_t^{ls} \right) - m_t (R_t - 1)$$

An important feature of the present model, shared by other contributions, consists in a feedback rule for tax revenues of the type suggested by Leeper (1991). In what follows, we introduce the fiscal rule:

$$\tau_t^{ls} = \psi_0 + \psi_1 (l_{t-1} - l) + \psi_2 \left[g_t + \left(\frac{R_{t-1} - 1}{R_{t-1}} \right) \left(\frac{l_{t-1} - m_{t-1}}{\pi_t} \right) \right]$$

The economic interpretation is that the government set taxes in order to stabilize the level of total liabilities l_t in real terms. The particular functional form assumed allows to distinguish between two distinct forms of stabilization: a simple fiscal feedback rule *à la* Leeper (1991), obtained by setting $\psi_2 = 0$, and a balanced budget rule when $\psi_1 = 0$ and $\psi_2 = 1$. In other words, taxes can be adjusted to follow either a 'minimal' adjustment path, enough to avoid that the total amount of government's liabilities to explode, or a 'strong' stabilization path, when taxes are immediately adjusted according to a balanced budget rule.

3. MONETARY POLICY RULES

This section summarizes the simple policy rules we consider in this work. We start out by considering two linear benchmarks, namely the standard backward-looking specification proposed by Taylor (1993) and the modification of Williams et al. The nonlinear rules we consider generalize the linear cases in a Cobb-Douglas form.

3.1. *Linear benchmarks*

The standard formulation of Taylor (1993)'s rule:

$$\hat{i}_t = \alpha_\pi \hat{\pi}_t + \alpha_y \hat{y}_t + \alpha_R \hat{i}_{t-1} \tag{4}$$

where hats and bars denote, respectively, log-deviations and deterministic steady states.

A critique of the previous rules is that their implementation requires an in-depth knowledge of the long-run state of the model economy. Rules in first-differences avoid the problem of unobservability of the deterministic steady states:

$$\ln\left[\frac{i_t}{i_{t-1}}\right] = \alpha_\pi \ln\left[\frac{\pi_t}{\pi_{t-1}}\right] + +\alpha_y \ln\left[\frac{y_t}{y_{t-1}}\right] \tag{5}$$

Since our model includes provides for a role for money demand, it is natural to consider also a simple money-growth target:

$$\hat{m}_t = \rho_m \hat{m}_{t-1} \tag{6}$$

3.2. *Nonlinear generalizations*

Nonlinearity of nominal interest rates can be obtained as a rule of thumb by using a Cobb-Douglas-style specification for the policy rule:

$$\frac{i_t}{\bar{i}} = \left[\frac{\pi_t}{\bar{\pi}}\right]^{\alpha_\pi} \cdot \left[\frac{y_t}{\bar{y}}\right]^{\alpha_y} \cdot \left[\frac{i_{t-1}}{\bar{i}}\right]^{\alpha_R} \tag{7}$$

$$\frac{i_t}{i_{t-1}} = \left[\frac{\pi_t}{\pi_{t-1}}\right]^{\alpha_\pi} \cdot \left[\frac{y_t}{y_{t-1}}\right]^{\alpha_y} \tag{8}$$

3.3. *A rule for opportunistic monetary policy*

Orphanides and Wilcox (2002) formalize the idea underlying the 'opportunistic approach to disinflation' with a policy rule that is both time-dependent and nonlinear. The central bank pursues an intermediate target of inflation $\tilde{\pi}_t$ such that the closer current inflation to $\tilde{\pi}_t$, the stronger the defense of the lower inflation level against past inflation targets. The inflation target $\tilde{\pi}_t$ is a weighted average of long-run inflation $\bar{\pi}$ and inherited past inflation π_t^h:

$$\tilde{\pi}_t := (1-\lambda)\bar{\pi} + \lambda \pi_t^h$$

The term π_t^h is the source of history dependence for monetary policy. Nonlinearity arises from the existence of a range of inflation deviations from the intermediate target within which output stabilization is the primary objective of monetary policy. The larger the deviation of inflation from $\tilde{\pi}_t$, the stronger the focus on price stability.

The oppurtunistic policy rule proposed by Orphanides and Wilcox (2002) takes the form:

$$\left[i_t - \bar{i}\right] = +\kappa_0 \left[y_t - \bar{y}\right] + \mathcal{G}(\pi_t - \tilde{\pi}_t)$$

where hats denote deviations from the deterministic steady states, and $\mathcal{G}(\cdot)$ is represented by the discontinuos function:

$$\mathcal{G}(\pi_t - \tilde{\pi}_t) := \begin{cases} \kappa_1(\pi_t - \tilde{\pi}_t - \kappa_2) & \text{if} \quad (\pi_t - \tilde{\pi}) > \kappa_2 \\ 0 & \text{if} \quad \kappa_2 \geq (\pi_t - \tilde{\pi}) \geq -\kappa_2 \\ \kappa_1(\pi_t - \tilde{\pi}_t + \kappa_2) & \text{if} \quad (\pi_t - \tilde{\pi}) < -\kappa_2 \end{cases}$$

The set of deviations from the inflation target such that $\mathcal{G} = 0$ defines a 'zone of inaction'. If there is a drop in inflation below this zone, the central bank acts to prevent inflation from returning at the higher level of the past. In the intentions of the proponents of the opportunistic approach to disinflation, the zone of inaction defines the scope for opportunism. In the numerical solution of the model, we follow Aksoy, Orphanides, Small, Wieland, and Wilcox (2002), and use the following twice continuously-differentiable approximation of $\mathcal{G}$:

$$\mathcal{G}(\cdot) \approx \kappa_1 \Big[0.05(\pi_t - \tilde{\pi}_t) + 0.475 \left(-\kappa_2 + \pi_t - \tilde{\pi}_t + \left((-\kappa_2 + \pi_t - \tilde{\pi}_t)^2 \right)^{0.51} \right) + 0.475 \left(\kappa_2 + \pi_t - \tilde{\pi}_t - \left((\kappa_2 + \pi_t - \tilde{\pi}_t)^2 \right)^{0.51} \right) \Big]$$

Since we are concerned with the U.S. economy, we assign the same parameter values to the approximated $\mathcal{G}$ that Aksoy, Orphanides, Small, Wieland, and Wilcox (2002) use. There is a slightly-positive slope even when inflation is within the zone of inaction. The numerical algorithm maximizes over a grid for κ_0 and κ_1.

4. EQUILIBRIUM AND AGGREGATION

DEFINITION 1: A symmetric monopolistically-competitive equilibrium consists of stationary sequences of prices $\{\mathbf{P_t}\}_{t=0}^{\infty} := \{\pi_t^*, R_t^*, w_t^*, r_t^*\}_{t=0}^{\infty}$, real quantities $\{\mathbf{Q_t}\}_{t=0}^{\infty} := \{\{\mathcal{Q}_t^h\}_{t=0}^{\infty}, \{\mathcal{Q}_t^f\}_{t=0}^{\infty}, \{\mathcal{Q}_t^g\}_{t=0}^{\infty}\}$, with $\{\mathcal{Q}_t^h\}_{t=0}^{\infty} := \{c_t^*, \ell_t^*, k_{t+1}^*, i_t^*, m_t^*, d_t^*\}_{t=0}^{\infty}$, $\{\mathcal{Q}_t^f\}_{t=0}^{\infty} := \{y_t^*, k_t^*, \ell_t^*\}_{t=0}^{\infty}$, $\{\mathcal{Q}_t^g\}_{t=0}^{\infty} := \{g_t^*, \tau_t^{ls*}, m_t^*, d_t^*\}_{t=0}^{\infty}$ and exogenous shocks $\{\mathcal{E}_t\}_{t=0}^{\infty} := \{\epsilon_t^z, \epsilon_t^g\}_{t=0}^{\infty}$ that aggregate over $\omega_1 = [0,1]$ and $\omega_2 = [0,1]$, that are bounded in a neighborhood of the steady state, and such that:

(i) given prices $\{\mathbf{P_t}\}_{t=0}^{\infty}$ and shocks $\{\mathcal{E}_t\}_{t=0}^{\infty}$, $\{\mathcal{Q}_t^h\}_{t=0}^{\infty}$ is a solution to the representative household's problem;

(ii) given prices $\{\mathbf{P_t}\}_{t=0}^{\infty}$ and shocks $\{\mathcal{E}_t\}_{t=0}^{\infty}$, $\{\mathcal{Q}_t^f\}_{t=0}^{\infty}$ is a solution to the representative firms' problem;

(iii) given quantities $\{\mathbf{Q_t}\}_{t=0}^{\infty}$ and shocks $\{\mathcal{E}_t\}_{t=0}^{\infty}$, $\{\mathbf{P_t}\}_{t=0}^{\infty}$ clears the market for goods, factors of production, money and bonds:

$$y_t^* = \int_{j\in\omega_1} c_t^* + \int_{j\in\omega_1} inv_t^* + \int_{j\in\omega_1} g_t^* + \int_{j\in\omega_1} \mathrm{AC}_t^{P*}$$

$$k_t^* = \int_{\iota\in\omega_2} k_t^* d\iota = \int_{j\in\omega_1} k_t^* dj$$

$$\ell_t^* = \int_{\iota\in\omega_2} \ell_t^* d\iota = \int_{j\in\omega_1} \ell_t^* dj$$

$$m_t^* = \int_{j\in\omega_1} m_t^* dj$$

$$d_t^* = \int_{j\in\omega_1} d_t^* dj$$

(iv) given quantities $\{\mathbf{Q_t}\}_{t=0}^{\infty}$, prices $\{\mathbf{P_t}\}_{t=0}^{\infty}$ and shocks $\{\mathcal{E}_t\}_{t=0}^{\infty}$, $\{\mathcal{Q}_t^g\}_{t=0}^{\infty}$ and satisfy the flow budget constraint of the government;

(v) fiscal policy is set according to a simple rule for lump-sum taxes;

(vi) the central bank sets the nominal interest rate according to a simple policy rule.

5. CALIBRATION

The parameters are calibrated on quarterly data for the US economy. We assume that households have an intertemporal discount rate of 0.996. They devote 1/4 of their time to labour activities at the steady state. The weight on the consumption objective in the consumption objective is 0.993. The calibration of the other parameters in the utility function is consistent with a consumption-output ratio of 0.57, and a money-output ratio of 0.44 in the long run (see table I). The nominal rate of interest is 5% a year, and the inflation rate is 4.2%. Both figures are consistent with the U.S. postwar experience.

We set the investment-output and capital-output ratios as 0.25 and 10.4, respectively. Capital depreciates for 10% a year. Both the parameter ϕ_K in the adjustment cost for capital, and the persistence of the markup shock are from Kim (2000). Capital income has a share of 1/3 in total output. The elasticity of substitution among intermediate goods generates a steady-state markup of approximately 10%. Stochastic productivity shocks are calibrated according to Chari, Kehoe, and McGrattan (2000).

We assume that government spending is 14.8% of GDP. The calibration for the public-spending shock is from Schmitt-Grohé and Uribe (2003). The steady-state ratio between public debt and output is 0.45.

Finally, the monetary-policy rule includes neither an output-gap objective, nor interest-rate smoothing.

6. COMPUTATIONAL ASPECTS

6.1. *Local validity of approximate solutions*

Second-order perturbation methods are defined only around small neighbourhoods of the approximation points, unless the approximated function is globally analytic (see Anderson, Levin, and Swanson, 2004). Since the conditions for an analytic form of the policy function are hardly establishable, the problem of validity of the Taylor expansion remains. we approach this issue at different levels. First, we calibrate the processes for exogenous shocks in such a way that their fluctuations are constrained within small intervals (see also Schmitt-Grohé and Uribe, 2003). Second, we impose an *ad hoc* bound that restricts the stochastic steady state of the nominal interest rate to be arbitrarily close to its deterministic counterpart:

$$\ln\left(\bar{R}\right) > \kappa\sigma_{\hat{R}_t} \tag{9}$$

with a constant κ, and $\sigma_{\hat{R}_t}$ as the unconditional variance of $\hat{R}$. This constraint rules out policies that are excessively aggressive. The reason is that large deviations of the nominal rate of interest from the deterministic steady state are likely to prescribe violations of the zero bound at some point in time. In what follows, we set $\kappa = 2$.

6.2. *Welfare evaluation*

According to the second-order approximation of the policy function, aggregate welfare is defined as the expected lifetime utility conditional on the initial distribution of state varibles s_0:

$$\mathcal{W}_0 := \mathsf{E}\left[\sum_{t=0}^{\infty}\beta^t u(s_{jt})\Big| s_0 \sim (\underline{s}, \underline{\Omega})\right]$$

where $\underline{s}$ and $\underline{\Omega}$ are, respectively, the mean and the covariance matrix of the distribution of the initial state of the economy, and Θ_1 and Θ_2 are suitable matrices.

In order to compare the outcomes of different policies, we compute the permanent change in consumption, relative to the steady state, that yields the expected utility level of the distorted economy. Given steady states of consumption $\bar{c}^{\iota}$ and hours worked $\bar{\ell}$ of the model ι, this translates into the number

Δ_c^ι such that:

$$\sum_{t=0}^{\infty} \beta^t u \left(\left[1 + \Delta_c^\iota \right] \bar{c}_j^\iota, \ \bar{\ell}_j \right) = \mathcal{W}_0^\iota$$

The interpretation of this equation goes as follows. Four elements determine the size of the welfare metric. On the right-hand side of the equality, the deterministic steady state, its stochastic counterpart, and the transition from the deterministic to the stochastic long-run equilibrium of ι. On the left-hand side, instead, the deterministic steady states of the model with respect to which the current distorted economy is compared, i.e. the 'benchmark'.

7. FINAL REMARKS

[to be written]

Massimiliano Marzo: Affiliation and Address here; Phone: — Web: `http://` *— E-mail:*

Paolo Zagaglia: Department of Economics, Stockholm University, Universitetsvägen 10A; SE-106 91 Stockholm, Sweden; Università Bocconi, via Sarfatti 25, 20136 Milano, Italy; Phone: +46-8-16 42 42 — Web: `http://www.ne.su.se/~pzaga` *— E-mail:* `pzaga@ne.su.se`

REFERENCES

Aksoy, Y., A. Orphanides, D. Small, V. Wieland, and D. Wilcox (2002): "A Quantitative Exploration of The Opportunistic Approach to Disinflation," *Journal of Monetary Economics*, (forthcoming).

Anderson, G., A. T. Levin, and E. Swanson (2004): "Higher-Order Perturbation Solutions to Dynamic, Discrete-Time Rational Expectations Models," unpublished manuscript, Federal Reserve Board.

Chari, V. V., P. J. Kehoe, and E. R. McGrattan (2000): "Sticky Price Models of the Business Cycle: Can the Contract Multiplier Solve the Persistence Problem?," *Econometrica*, 68.

Dixit, A., and J. Stiglitz (1977): "Monopolistic Competition and Optimal Product Diversit," *American Economic Review*, 67, 297–308.

Feenstra, R. (1986): "Functional Equivalence Between Liquidity Costs and the Utility of Money," *Journal of Monetary Economics*, 46(2), 271–291.

Kim, J. (2000): "Constructing and Estimating a Realistic Optimizing Model of Optimal Monetary Policy," *Journal of Monetary Economics*, 45, 329–359.

Leeper, E. (1991): "Equilibria under 'Active' and 'Passive' Monetary and Fiscal Policies," *Journal of Monetary Economics*, 27, 129–147.

Orphanides, A., and D. Wilcox (2002): "The Opportunistic Approach to Disinflation," *International Finance*, 5(1).

Schmitt-Grohé, S., and M. Uribe (2003): "Optimal Simple and Implementable Monetary and Fiscal Rules," unpublished manuscript, Duke University.

——— (2004): "Solving Dynamic General Equilibrium Models Using a Second-Order Approximation to the Policy Function," *Journal of Economic Dynamics and Control*, 28(3), 755–775.

Taylor, J. B. (1993): "Discretion versus Policy Rules in Practice," *Carnegie-Rochester Conference Series*, 39.

Woodford, M. (2002): *Foundations of Monetary Theory.*

APPENDIX 1: FIRST-ORDER CONDITIONS

$$\lambda_t = \overline{u}_t^{-1}\left(1-\ell_t\right)^{\xi\left(1-\frac{1}{\sigma}\right)} ac_{it}^{-\frac{1}{\mu}}$$

$$\lambda_t w_t = \overline{u}_{it}\left(1-\ell_t\right)^{\xi\left(1-\frac{1}{\sigma}\right)-1}$$

$$\frac{\lambda_t}{P_t} = \overline{u}_t^{-1}\left(1-\ell_t\right)^{\xi\left(1-\frac{1}{\sigma}\right)}\left(1-a\right) m_{it}^{-\frac{1}{\mu}}\frac{1}{P_t} + \beta\mathsf{E}_t\frac{\lambda_{t+1}}{P_{t+1}}$$

$$\left[q_t + \phi_K\left(\frac{inv_t}{k_t}\right)^3\right]\lambda_t = \mu_t - \beta\left(1-\delta\right)E_t\mu_{t+1}$$

$$\beta E_t\mu_{t+1} = \lambda_t\left[1 + \frac{3\phi_K}{2}\left(\frac{inv_{it}}{k_{it}}\right)^2\right]$$

$$\lambda_t = \beta R_t\mathsf{E}_t\frac{\lambda_{t+1}}{\pi_{t+1}}$$

$$\overline{u}_t := \left[ac_t^{\frac{\mu-1}{\mu}} + \left(1-a\right)\left(m_t\right)^{\frac{\mu-1}{\mu}}\right]^{\left(\frac{\mu}{\mu-1}\right)\left(1-\frac{1}{\sigma}\right)}$$

$$(1-\alpha)\mathrm{mc}_t\frac{y_t+\Phi_t}{\ell_t} = w_t$$

$$\alpha\mathrm{mc}_t\frac{y_t+\Phi_t}{k_t} = q_t$$

$$\frac{(1-\theta)y_t}{P_t} - \phi_P\left(\frac{P_t}{P_{t-1}} - \pi\right)\frac{y_t}{P_{t-1}} + \mathrm{mc}_t\theta\frac{y_t}{P_t} + \beta\mathsf{E}_t\frac{\lambda_{t+1}}{\lambda_t}\left[\phi_P\left(\frac{P_{t+1}}{P_t} - \pi\right)\frac{P_{t+1}}{P_t^2}y_{t+1}\right] = 0$$

APPENDIX 2: STATE-SPACE FORM

This section provides a selected review of second-order perturbation method due to Schmitt-Grohé and Uribe (2004). Suppose that the first-order conditions of a model economy can be arranged in the following way:

$$\mathsf{E}_t\mathcal{H}\left(y_{t+1}, y_t, x_{t+1}, x_t|\sigma\right) = 0$$

where y is a vector of co-state variables. The state variables are collected in x:

$$x_t := \begin{bmatrix} x_{1,t} \\ x_{2,t} \end{bmatrix}$$

with vectors of endogenous state variables $x_{1,t}$, and exogenous state variables $x_{2,t}$:

$$x_{2,t+1} = \Lambda_1 x_{2,t} + \Lambda_2\sigma\epsilon_{t+1}$$

with matrices Λ_1 and Λ_2. The scalar $\sigma \geq 0$ is known.

We define the following:

$$x_{1,t} = [k_t \quad R_{t-1} \quad d_{t-1} \quad m_{t-1}]'$$

$$x_{2,t} = [z_t \quad g_t \quad \Phi_t]'$$

$$y_t = [y_t \quad R_t \quad d_t \quad \mathrm{mc}_t \quad c_t \quad \pi_t \quad \ell_t \quad r_t \quad w_t \quad m_t \quad \varsigma_t \quad \tau_t^c]'$$

$$\Lambda_1 = \begin{bmatrix} 1\,0 \\ 0\,1 \end{bmatrix}$$

$$\Lambda_3 = \begin{bmatrix} 0\,0\,0\,0\,\sigma_\psi & 0 \\ 0\,0\,0\,0 & 0 & \sigma_z \end{bmatrix}'$$

TABLE I:
Calibration of the model

Description	Parameter	Value
Subjective discount factor	β	0.9966
Weight on leisure objective	ξ	0.001
Share of consumption objective	a	0.99
Interest elasticity	μ	0.39
Intertemporal substitution of consumption	σ	0.145
Share of labour effort	ℓ	1/4
Investment-output ratio	$\bar{i}/\bar{y}$	0.25
Capital-output ratio	$\bar{k}/\bar{y}$	10.4
Money-output ratio	$\bar{m}/\bar{y}$	0.44
Steady-state inflation	$\bar{\pi}$	$1.042^{(1/4)}$
Adjustment cost of prices	ϕ_P	60
Adjustment cost of capital	ϕ_K	433
Capital depreciation rate	δ	0.024
Capital elasticity of intermediate output	α	0.33
Elasticity of substitution of interm. goods	θ	10
Persistence of productivity shock	ρ_z	0.98
Steady state of productivity shock	z	1
Standard dev. of productivity shock	σ_z^2	0.055
Persistence of markup shock	ρ_Φ	0.911
Standard dev. of markup shock	σ_Φ^2	0.141
Persistence of government-spending shock	ρ_G	0.97
Standard dev. of government-spending shock	σ_G	0.1
Public spending-output ratio	$\bar{g}/\bar{y}$	0.148
Weight on inflation objective	α_π	1.5
Weight on output objective	α_y	0
Weight on interest-rate smoothing	α_R	0